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Combined Effects of Rock Bedding Orientation and Topography on Stresses Around Mine Openings

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ABSTRACT

In this paper we introduce a realistic model for mine openings that explicitly considers the effects of local irregular topography. Also, we consider the effect of entry orientation and rock anisotropy on the induced stress fields around mine openings under the given *in-situ* stress field. A triangular mesh formulation is derived for representing complicated irregular surface topography and underground openings. Each element contains three internal collocation nodes. Parts of the surface which are far from the mine openings are discretized by infinite elements. The rectangular infinite elements with three internal nodes are introduced specifically for this problem so they are compatible with the triangular elements. With the corresponding boundary element method (BEM) formulation, we then investigate the combined effect of the topography and entry orientation on the stress fields induced by openings in anisotropic rocks under *in-situ* stresses. A single rectangular opening in the anisotropic ground with topography and *in-situ* stresses is simulated using data from the Carroll Hollow mine in Ohio. The results of these simulations show that topography, rock anisotropy, orientation of the regional stress field and mine opening geometry all have appreciable effects on the stress fields around underground openings, yet these factors can all be accounted for in an efficient code with no need to discretize the model domain. Future applications using this new BEM formulation will provide useful guidance on selecting the optimal opening orientation in the mine in order to minimize undesirable stress concentrations around underground mine workings.

INTRODUCTION

Design of mine openings requires prior knowledge on the strength of the nearby Earth's ground materials as well as the quantification of stress states at the walls and around the opening. While *in-situ* experimental analyses are vital for all mine opening excavations, analytical methods can be utilized to guide mine design and analysis. The boundary element method (BEM) is a powerful computational tool for analyzing stresses in infinite and semi-infinite regions (Liu et al., 2011; Tonon, Pan and Amadei, 2001). The use of Green's functions for half-space or half-plane problems has found many applications in the field of rock mechanics with several important applications including hydraulic fracturing (Pan et al., 2013), earthquake measurements (Irikura,

1963), and mine exploration (Tonon and Amadei, 2002). For a semi-infinite space (or semi-infinite plane for two-dimensional problems) in particular, half-space or half-plane Green's functions are commonly implemented into the BEM formulation to remove the need for discretization along the surface of the half-space (Pan, Amadei and Kim, 1998). However, there are situations for which it is better to solve a half-space problem using full-space Green's functions. To do so, one needs to define an infinite element to discretize the infinite part of the boundary surface (i.e., the portion of the boundary surface which extends beyond the local region of interest). Liang and Liew (2001) used Kelvin's solution for modeling three-dimensional (3D) half-space problems. The advantage of their method was avoiding the strongly singular line integral, and it also led to a more accurate result with less computation time. Liew and Liang (2003) also used this method to set up the BEM formulation for a transversely isotropic piezoelectric half-space. They used both half-space and full-space Green's functions to overcome the computational difficulties.

For underground mine openings with irregular surface topography, the topography needs to be discretized; therefore, it is necessary to use the full-space Green's function and discretize the irregular surface of the half-space with suitable elements. Furthermore, when representing underground openings and excavations near the surface, the geometry of mine openings with respect to the local topography and entry is of particular importance. A poor choice for orientation of the excavation could result in dangerous perturbations in the *in-situ* stress field, introducing unsafe mining conditions.

The objective of this research was two-fold. First, we introduced a realistic model for underground mine openings considering the perturbing effect of local irregular topography. Second, we also considered the effect of entry orientation and rock anisotropy on the induced stress fields around mine openings under the given *in-situ* stress field. In this study, a triangular mesh formulation was derived for representing complicated irregular surface topography and underground openings.

Each element contains three internal collocation points. Parts of the surface which are far from the mine openings are discretized using infinite elements. Because we used triangular elements with three internal nodes, we developed a new infinite element with

three internal points and suitable shape functions which decay to zero at infinity. Using the corresponding BEM formulation, we then investigated the combined effect of the topography and entry orientation on the stress fields induced by openings under *in-situ* stresses. One rectangular opening in the ground with topography and *in-situ* stresses was simulated using data from the Carroll Hollow Mine in Ohio. Future applications using this new BEM formulation will provide useful guidance on selecting the optimal opening orientation in order to minimize undesirable stress concentrations around underground mine workings.

This paper has four remaining sections. First, we will present the problem and solution methodology and the BEM formulation for solving the problem. Next, we introduce the numerical scheme used to solve the problem. Two different types of elements and their corresponding shape functions are introduced in this section. Then we will provide example solutions for selected problems. In this section, all the stress components on the top side (roof) of the opening are derived and compared with each other. The stress components are calculated for different orientations of the opening with/without considering the effect of topographic surface of half-space. The effect of anisotropy of rock is also investigated in this portion of our work. Finally, we will summarize the works and present our key conclusions.

BEM FORMULATION

In a linearly elastic medium, the total displacements, stresses, and tractions can be expressed as follows:

$$u_i^t = u_i^h + u_i^p; \quad \sigma_{ij}^t = \sigma_{ij}^h + \sigma_{ij}^p; \quad T_i^t = T_i^h + T_i^p$$

where superscripts t , p , and h denote the total solution, a particular solution corresponding to the body forces and the far field stresses, and the homogenous solution, respectively.

Pan and Amadei (1996) have shown that the total internal displacement solution at one point x_p can be expressed as equation 2 below:

$$\begin{aligned} u_i^t(x_p) = & - \int_{S_o \cup S_i \cup S_f} T_{ij}^*(x_p, x_s) u_j^t(x_s) dS(x_s) \\ & + \int_{S_o \cup S_i \cup S_f} U_{ij}^*(x_p, x_s) T_j^t(x_s) dS(x_s) \\ & + \int_{S_o \cup S_i \cup S_f} T_{ij}^*(x_p, x_s) [u_j^p(x_s) - u_j^p(x_o)] dS(x_s) \\ & - \int_{S_o \cup S_i \cup S_f} U_{ij}^*(x_p, x_s) T_j^p(x_s) dS(x_s) \end{aligned}$$

where U_{ij}^* and T_{ij}^* are the displacement and traction Green's functions respectively and are given exactly for a transversely isotropic solid with arbitrarily oriented isotropic plane as can be found in Pan and Yuan (2000). S_o , S_i , and S_f are different parts of the surface which are shown in Figure 1, and the subscripts s and p indicate points on the surface and within the underground, respectively. S_o is the surface of the opening in the ground, S_i is the topographic surface which is finite and above the opening, and S_f is that part of the topographic surface which is far enough from

the opening and is extended from the edges of S_i to infinity and is assumed to be horizontal.

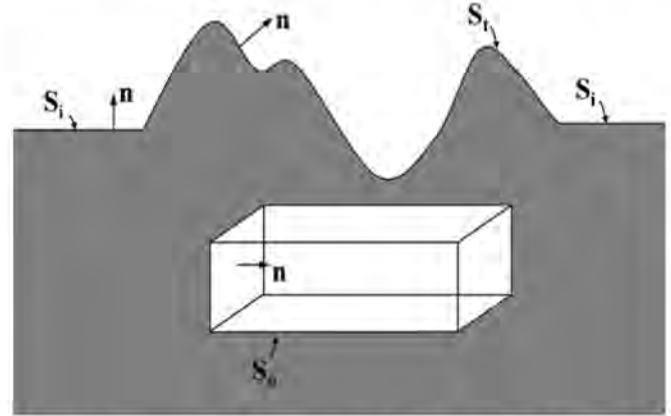


Figure 1. Geometry of the problem. S_o is surface of the opening, S_i is the finite part of topographic surface above the opening and S_f is the part of topographic surface which extends from the edges of S_i to infinity.

The following boundary integral equation for displacements can be derived from Equation 2 when the internal point x_p approaches point y_s on one of the surfaces S_o , S_i , or S_f :

$$\begin{aligned} b_{ij} u_j^t(y_s) + \int_{S_o \cup S_i \cup S_f} T_{ij}^*(y_s, x_s) u_j^t(x_s) dS(x_s) = \\ \int_{S_o \cup S_i \cup S_f} U_{ij}^*(y_s, x_s) T_j^t(x_s) dS(x_s) \\ + \int_{S_o \cup S_i \cup S_f} T_{ij}^*(y_s, x_s) [u_j^p(x_s) - u_j^p(y_s)] dS(x_s) \\ - \int_{S_o \cup S_i \cup S_f} U_{ij}^*(y_s, x_s) T_j^p(x_s) dS(x_s) \end{aligned}$$

In Equation 3 above, b_{ij} are coefficients that depend only on the local geometry of the boundary at point y_s . All terms on the right-hand side of the above equation have weak singularities and are integrable. The second term on the left-hand side of Equation 3 has a strong singularity which can be treated by the rigid-body motion method avoiding further calculation of b_{ij} .

NUMERICAL SCHEME

Two different types of elements have been used to discretize all surfaces. Triangular elements with three internal nodes are used to discretize S_o and S_i and rectangular infinite elements with three internal nodes are used to discretize S_f (Figure 2). For each element, the displacement and traction can be approximated by their nodal values.

$$u_i = \sum_{k=1}^3 \phi_k u_i^k; \quad T_i = \sum_{k=1}^3 \phi_k T_i^k; \quad i = 1, 2, 3$$

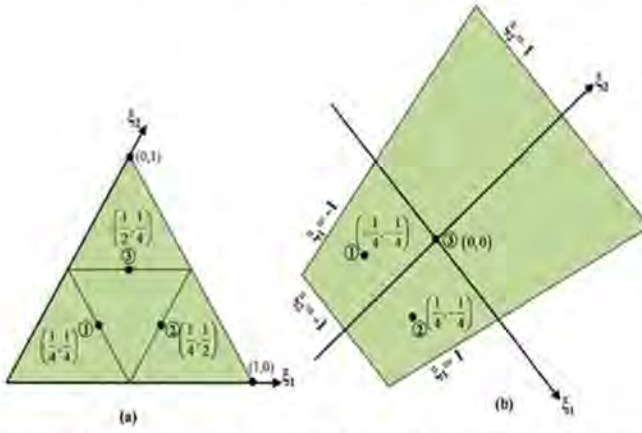


Figure 2. The shape of the elements and internal nodes and local coordinates ξ_1 and ξ_2 . (a) Triangular element with three internal nodes which is used to discretize S_o and S_t . (b) Rectangular infinite element with three internal nodes which is used to discretize S_i .

where ϕ_k are the shape functions as defined in Equations 5 and 6 below. u_i^k and T_i^k are the nodal displacements and tractions at node k , respectively. Shape functions for the triangular element are:

$$\phi_1^t = -4\xi_1 - 4\xi_2 + 3; \quad \phi_2^t = 4\xi_1 - 1; \quad \phi_3^t = 4\xi_2 - 1$$

The intrinsic coordinates ξ_1 and ξ_2 are defined in Figure 2. For the rectangular infinite element, which is used to discretize the boundary S_i , the shape functions can be defined in equation 6 as follows:

$$\begin{aligned} N_1 &= -2\xi_1 - 2\xi_2; & N_2 &= 2\xi_1 - 2\xi_2; & N_3 &= 4\xi_2 + 1 \\ M_1 &= \left(\frac{9(\xi_2 - 1)}{5(2 - \xi_2)} \right)^2; & M_2 &= \left(\frac{9(\xi_2 - 1)}{5(2 - \xi_2)} \right)^2; & M_3 &= \left(\frac{2(\xi_2 - 1)}{(2 - \xi_2)} \right)^2 \\ \phi_1^i &= N_1 \times M_1; & \phi_2^i &= N_2 \times M_2; & \phi_3^i &= N_3 \times M_3 \end{aligned}$$

N_1 , N_2 , and N_3 are regular-shape functions which satisfy the condition that equals to one at their corresponding node and zero at the other two nodes, and M_1 , M_2 and M_3 are decaying functions which are unit at their corresponding node and approach zero when ξ_2 approaches one. The final shape functions are multiplications of the corresponding N and M functions. The shapes of the elements and the internal nodes are shown in Figure 2.

NUMERICAL RESULTS AND DISCUSSION

To investigate the effects of the topographic surface and the orientation of the opening, the following example was designed. One opening with the length of 10m, width of 2m and height of 2m was assumed in a half-space. The half-space was assumed to be transversely isotropic with $E=10\text{GPa}$, $E'=30\text{GPa}$, $\nu=\nu'=0.25$ and $G'=4\text{GPa}$. These material properties can be considered to be approximately representative of a highly anisotropic package of sedimentary rocks, such as alternating layers of shale, coal, sandstone, and limestone present in the Allegheny Group, the rocks found at the site of the Carroll Hollow Mine.

To consider the effect of topography, two different situations were assumed. First, the surface of half-space was flat at $z = 0$ (Figure 3) and, second, we used data from the Carroll Hollow Mine in Ohio to introduce the topographic surface (Figure 4). According to the dimension of opening the surface of half-space was divided into two parts (S_i and S_j). The top surface of the opening is $2\text{m} \times 10\text{m}$, consequently a $6\text{m} \times 30\text{m}$ rectangle on the half-space surface immediately above the opening was discretized with triangular elements and the rest of the half-space surface was discretized with rectangular infinite elements. Since these two cases are subjected to comparison, the depth of opening was set in such a way that the distance of opening to S_i on the surface of half-space was the same for both cases. For the topographic surface, the mean elevation of points on S_i was used to set the depth of the opening.

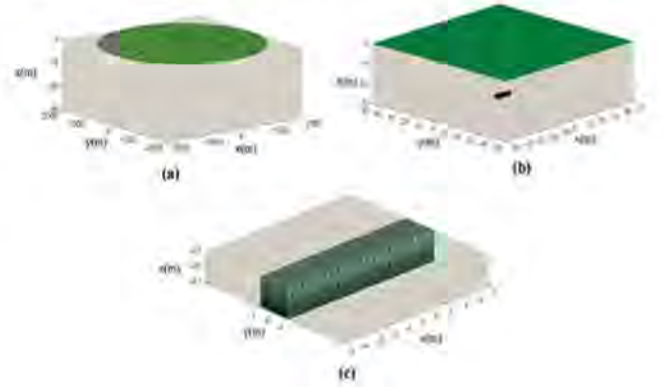


Figure 3. Case 1: Opening in half-space with flat surface. a) The surface of the half-space, b) The surface of half-space above the opening, c) the discretized opening.

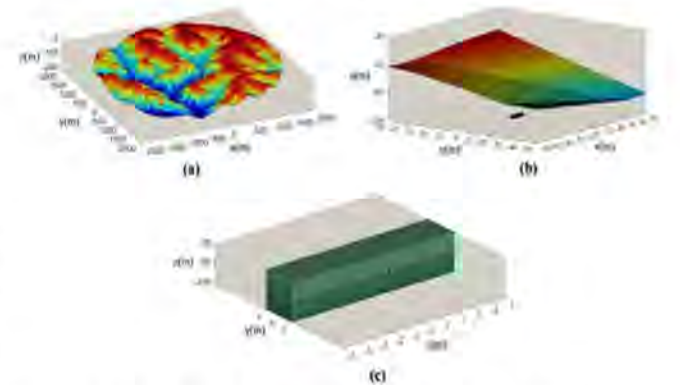


Figure 4. Case 2: Opening in half-space with topographic surface. a) The surface of the half-space, b) The surface of half-space above the opening, c) the discretized opening.

To investigate the effect of orientation of the opening, three cases for the far-field stresses were considered. The three cases avoid the need to rotate the opening within the model. All the combined situations are presented in Table 1. In this table, two cases (1d and 2d), which are used to investigate the effect of rock anisotropy, will be discussed at the final part of this section.

The results for stress components on the roof of the opening for different cases are shown in Figures 5, 6 and 7. Figure 5 shows stress components on the roof of the opening for Case 1a

and 2a, where there is a far-field stress in x -direction only. This figure shows that the topographic surface has a small effect on the magnitude of stress components. Also, it does not change the distribution of σ_{xx} , but it affects the shape of contours for σ_{yy} and σ_{xy} . The extent of effects is expected to vary, depending on the placement of the opening with respect to undulations in the overlying topography.

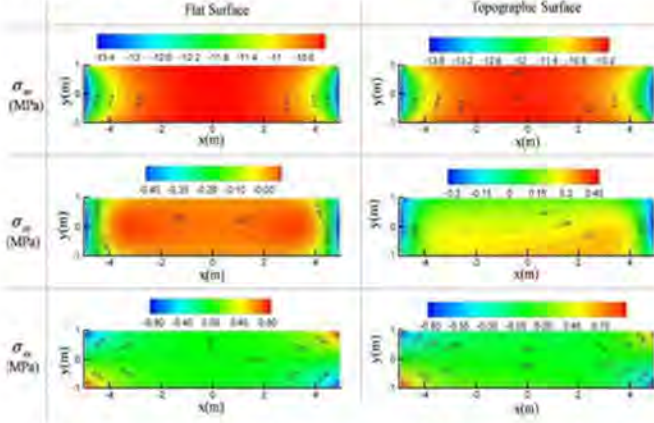


Figure 5. Stress components on the roof of the opening for Case 1a on the left column and Case 2a on the right column.

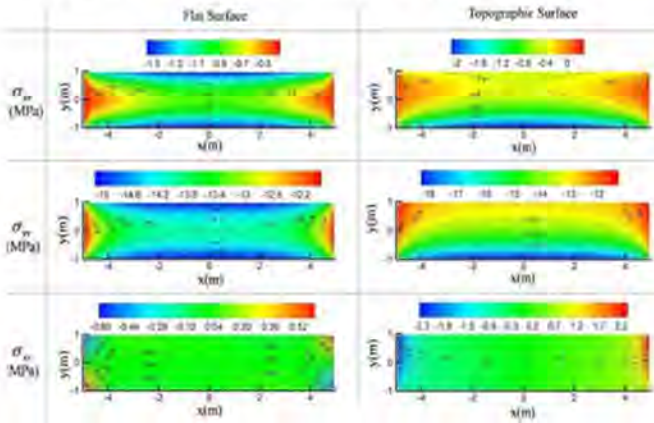


Figure 6. Stress components on the roof of the opening for Case 1b on the left column and Case 2b on the right column.

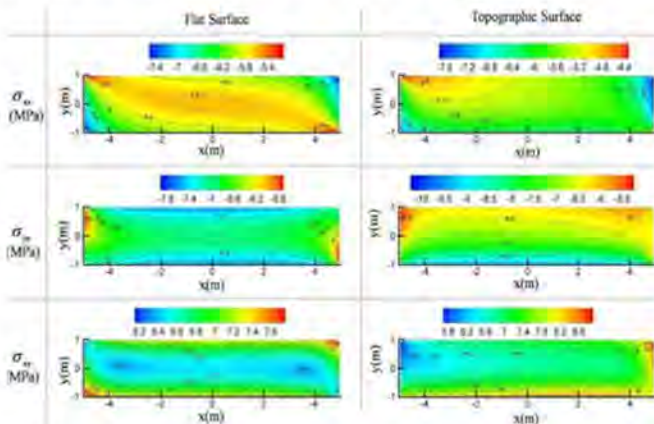


Figure 7. Stress components on the roof of the opening for Case 1c on the left column and Case 2c on the right column.

In Figure 6, the stress components on the roof of the opening are shown for Cases 1b and 2b. In these cases, the non-zero far-field stress is along the y axis. It is observed from this figure that the topographic surface has a noticeable effect on both the shape of the contours and their magnitude. We can also see the same effects for Figure 7, which is for Cases 1c and 2c, where the far-field stress is rotated by 45° .

Comparing the three figures with each other, one can investigate the combined effect of topography and entry orientation. These figures indicate that the topographic surface and orientation of the opening can have a drastic effect on the distribution and magnitude of stress components and should be considered wisely.

The results of the stresses on the roof of the opening that are shown in Figures 5-7 are for a transversely isotropic rock with a horizontal plane of isotropy. Since we show the results on the top roof of the opening, which is a horizontal surface, the contours shapes are expected to be the same as in the corresponding isotropic half-space. To investigate the effect of rock anisotropy, two new cases were defined (Cases 1d and 2d). One opening with the same geometry as above examples is considered: a half-space with flat surface and a half-space with a topographic surface; however, the plane of isotropy was rotated by the angle of 45° about the y axis ($\psi=45^\circ$, $\beta=0$). The non-zero far-field stress is assumed to be compression and along the x axis. Similar to the work by Wang et al. (2012), the transform matrix is introduced in Equation 7 below:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\cos\psi \sin\beta & -\cos\psi \cos\beta & \sin\psi \\ \cos\beta & -\sin\beta & 0 \\ \sin\psi \sin\beta & -\sin\psi \cos\beta & \cos\psi \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$$

where the angles ψ and β and the global axes x , y , and z as well as the local material axes x_1 , y_1 , z_1 (with z_1 being the symmetry axis of the material) are defined in Figure 8. The stress components on the roof of the opening for these two cases are shown in Figure 9.

Comparing Figure 9 and Figure 5 shows that the rock anisotropy could drastically affect the magnitude and distribution of σ_{yy} , however other stress components does not change noticeably. It demonstrates that these results could depend closely on the size and orientation of the opening, and also the shape of the topographic surface. Therefore, this work demonstrates the fact that both topography and elastic anisotropic properties can exert significant influence on the 3D distribution of the stress in the roof of underground openings. This work further shows the importance of selecting the entry orientation because some orientations may lead to safer designs for mine tunnels. The new code based on the proposed BEM formulation allows us to consider the effects of topography and rock anisotropy on the stress field around underground openings efficiently, with no need to discretize the model domain. Future development of this code will allow for simulation of stresses around realistic mine geometries.

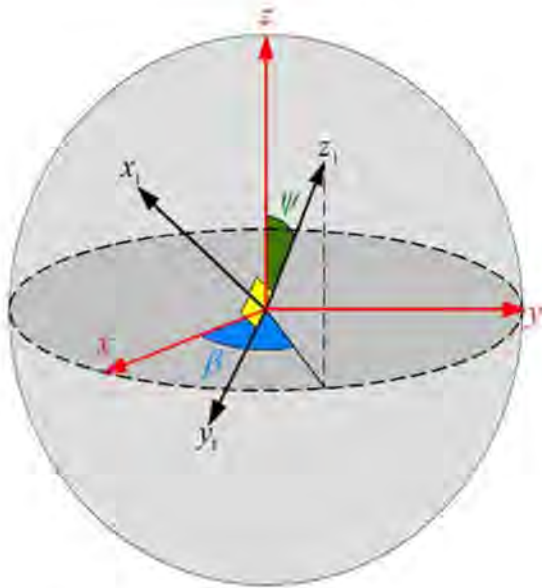


Figure 8. Relation between the global (x, y, z) and local material (x1, y1, z1) coordinates.

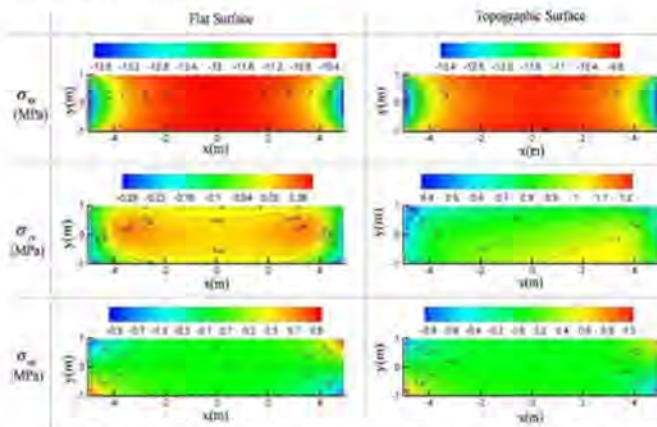


Figure 9. Stress components on the roof of the opening for Case 1d on the left column and Case 2d on the right column.

CONCLUSIONS

In this paper, we investigated the combined effect of entry orientation and topography on stress components on the roof of a mine opening in the anisotropic rock. We used the BEM formulation to find the stresses in the domain. The boundary of the problem was discretized using two types of elements: triangular and rectangular, both with three internal nodes. The triangular element was used to mesh the boundary of the opening and the finite part of the half-space surface above the opening. The rectangular element was used to discretize the infinite part of the half-space surface, which was suitably far from the opening.

The results indicate that if the mine opening is close enough to the earth's surface, the surface topography can have a drastic effect on the magnitude and distribution of stress components. Also, the orientation of mine tunnels could change the shape and magnitude of stress components completely. Furthermore, we investigated the

effect of rock anisotropy by rotating the plane of isotropy, and the results indicate that its effect can be significant. As a result, to have a realistic estimation of the local stress fields around underground mines, the topographic effect of the surface as well as the rock anisotropy should be considered. Safe design for mining tunnels requires the orientation of tunnels with respect to the regional stress field, with respect to the rock anisotropy, and the overlying topography to be selected wisely.

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