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## A DIRECT DERIVATIVE METHOD TO CALCULATE RESISTANCE SENSITIVITY FOR MINE VENTILATION NETWORKS

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### ABSTRACT

A reliable and stable ventilation system is essential to the safe operation of underground mines. The stability of a mine ventilation system becomes extremely critical while responding to a fire incident since an unstable ventilation system will pose a risk of airflow reversal. The reversed airflow could bring the fire contaminants such as toxic gases and smoke unexpectedly to working areas. In the past few years, there has been a growing interest in the study of ventilation network stability using the concept of resistance sensitivity, which is described as an indicator of how the airflow in an airway is reacting to a resistance change of other airways. Several methods of calculating the resistance sensitivity in a mine ventilation network have been carried out by researchers and scholars around the world. However, the proposed methods heavily rely on a vast amount of mine ventilation simulations, which are very time and computer-power consuming, especially for a large-scale mine ventilation network. In this paper, a direct derivative method calculating the resistance sensitivities with a single mine ventilation simulation has been developed and implemented into a mine fire simulation software, MFIRE. The results from the direct derivative method were verified against the results from a traditional method. The direct derivative method has been proved to be reliable and accurate.

**Key words**: Mine ventilation; Ventilation network; Ventilation stability; Resistance sensitivity

### INTRODUCTION

Ventilation is the lifeblood of an underground mine for providing fresh air for miners to breath and carrying away unwanted contaminants such as respirable dusts, explosive gas mixtures, and other harmful airborne particles. A healthy and safe ventilation system is essential to ensuring safe and efficient underground mining operation. However, a mine ventilation system could be disturbed adversely under fire influence. Fire pressure due to the rapid change in heat, natural ventilation, and air volume expansion can adversely affect the stability of a ventilation network by reversing airflows and consequently carrying contaminants unforeseen to working sections or primary escape ways. Therefore, the stability of a ventilation system under fire influence is of utmost important to the safety of the mine.

The stability of a ventilation network has been recognized as a key factor in determining how the ventilation network responds to anomalous changes, such as those arising from a fire emergency. In recent years, there has been a growing interest in the study of the stability assessment of mine ventilation networks through the concept of sensitivity (Dziurzyński et al., 2017; Griffith & Stewart, 2019; Semin & Levin, 2019; Jia et al., 2000). The sensitivity in a ventilation network refers to the degree of dependency of airflows in an airway to the resistance change of another airway, heat change, or other conditions. The sensitivity to resistance change has been considered more important than the other conditions. This paper will focus on the resistance sensitivity which reflects how sensitive the airflow in an airway is in responding to the resistance changes of other airways. Besides the application in the ventilation network stability study, the resistance sensitivity has been also used as an assistant tool to predict the airflow distributions in a ventilation network when underground conditions and ventilation controls are changed for the ventilation-on-demand system (Li et al., 2011) and to identify the causes of abnormal airflows in a mine ventilation system (Zhou et al., 2007). In this paper, the resistance sensitivity is referred to simply as "the sensitivity" for ease of use.

Several methods and algorithms have been proposed for calculating the sensitivity in the past. Dziurzyński et al. (2017) proposed an algorithm to produce the sensitivity matrix by solving the approximation expression of the derivative of the airflow to the resistance. Jia et al. (2000) developed a so-called Cross iterative method which combines the Hardy Cross method for network solution and iterative method for sensitivity calculation to determine the sensitivity. The calculation of sensitivity in Li et al.'s (2011) paper is fairly straightforward with the direct resistance change to a certain airway and then solving the ventilation network for airflow changes. Griffith and Stewart (2019) proposed a method to determine the sensitivity by running multiple samples of airflow simulation with randomly perturbed resistances for each resistance. Despite the various methods for calculating resistance sensitivity, one thing these methods have in common is that they all require a large number of mine ventilation simulations. It is certainly time consuming and computationally expensive for a large and complicated mine ventilation network. In this paper, the authors are proposing a direct derivative method to calculate the resistance sensitivity with only a one-time mine ventilation simulation.

### DIRECT DERIVATIVE METHOD

#### Definition of resistance sensitivity

The resistance sensitivity of an airway is an indicator of how the airflow in the airway is affected by the change of resistance in a certain airway. It is mathematically defined as (Zhou et al., 2007; Jia et al., 2020):

$$s_{ij} = \lim_{|\Delta R_j \to 0|} \frac{\Delta Q_i}{\Delta R_j} = \frac{\partial Q_i}{\partial R_j} \quad (1)$$

Where $S_{ij}$ is the sensitivity of airway i to the change of resistance in airway j; $R_j$ is the resistance in airway j, and $Q_i$ is the volumetric airflow rate in the airway i. Li et al. (2011) and Dziurzynski et al. (2017) also have very similar definitions of the resistance sensitivity but with slightly different expressions than Equation (1).

Given a ventilation network with N airways and J nodes, each airway can have the number of N sensitivities to the rest of the airways and itself. Therefore, an $N \times N$ sensitivity matrix S can be formed for the ventilation network:

$$S = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1N} \\ s_{21} & s_{22} & \cdots & s_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ s_{N1} & s_{N2} & \cdots & s_{NN} \end{bmatrix} \quad (2)$$

In the above N by N sensitivity matrix, each row represents that the airflow change of each airway is responding to the resistance change of all the airways. For instance, the second row is the sensitivity of Airway #2 to all the airways, and the element $S_{23}$ indicates the airflow change of Airway #2 to the resistance change in Airway #3.

#### Useful terms of the ventilation network

Prior to explaining how to obtain the sensitivity matrix through a direct directive method for a ventilation network, it is often necessary to address several commonly used terms in ventilation network


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analysis, such as basic mesh, spanning tree, and chord (Hartman, et al., 1997).

Figure 1 displays an example of a simple network with six airways and four nodes, where an airway, often interchangeably used with branch, is a connecting line between two nodes. A mesh is a closed loop of airways in the ventilation network. A good number of meshes can be formed in the ventilation network as shown in Figure 1, but there are only three basic meshes. The basic mesh refers to the mesh containing one airway which is not included in any other meshes. For a ventilation network with N airways and J nodes, N-J+1 basic meshes will be formed. Hence, the ventilation network shown in Figure 1 has three basic meshes. In a ventilation network, the spanning tree is a connected graph containing airways that connect all the nodes but creates no meshes. For example, airways 1, 2, and 5 (drawn in blue) is one set of spanning trees for the example network since they connect all the four nodes without forming any meshes. A chord is an airway contained in the network but not in a given spanning tree. A ventilation network could have multiple sets of spanning trees and will generate a unique set of chords for a particular spanning tree. In the example network, Airways 3, 4, and 6 are the chords corresponding to the spanning tree of airways 1, 2, and 5. For this particular set of spanning trees, the three basic meshes can be formed as 6-2-5, 1-2-3, and 1-2-5-4. Each basic mesh only contains one chord, which is Airways 6, 3, and 4 respectively. In this paper, the airways other than the chord in a basic mesh are referred as spanning tree airways. In short, a basic mesh is made out of one chord and a few spanning tree airways.

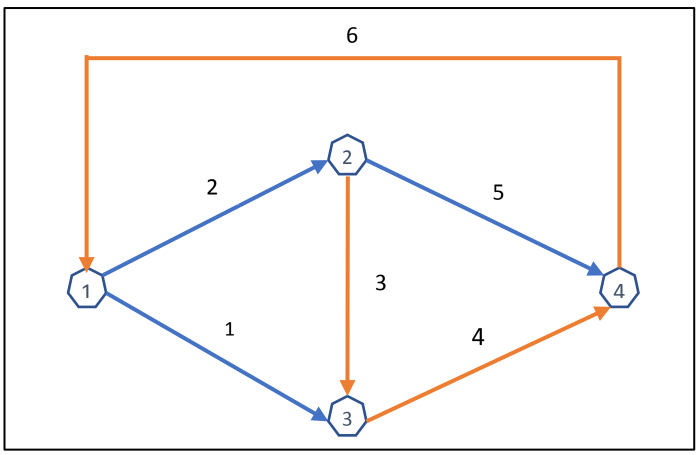


**Figure 1.** An example of a simple network.

**Mathematical representation of resistance sensitivity based on ventilation network analysis theory**

Kirchhoff's First and Second Laws are extensively applied in ventilation network analysis, though they were initially developed as two fundamental laws governing the behavior of electrical circuits. For a ventilation network, Kirchhoff's first law states that the quantity of air leaving a node must equal the quantity of air entering a node. As an example, for a ventilation network with N airways and J nodes, the following equation (Eq.3) can be used for expressing Kirchhoff's first law:

$$\sum_{j=1}^{N} a_{ij}Q_j = 0 \quad (i = 1,2,\dots,J) \tag{3}$$

Where $a_{ij}$ is a constant defined as follows:

$a_{ij} = 1,$ if the end node of airway j is equal to node i (airflow entering node i)
$a_{ij} = -1,$ if the starting node of airway j is equal to node i (airflow leaving node i)
$a_{ij} = 0,$ if neither of the nodes of airway j is equal to node i

Kirchhoff's second law applies to the meshes in the ventilation network with a statement that the algebraic sum of all pressure drops around a mesh in the network must be zero. For a ventilation network with N airways and M basic meshes, Kirchhoff's second law can be expressed as:

$$\sum_{j=1}^{N} b_{ij}R_jQ_j^2 - H_{Fi} - H_{Ni} = 0 \quad (i = 1,2,\dots.,M) \tag{4}$$

Where $H_{Fi}$ is the fan pressure if there is one in mesh i, and $H_{Ni}$ is the natural ventilation pressure in mesh i. $b_{ij}$ is a constant defined as follows:

$b_{ij} = 1,$ if branch j is in mesh i and has the same direction with the chord of mesh i
$b_{ij} = -1,$ if branch j is in mesh i and has the opposite direction with the chord of mesh i
$b_{ij} = 0,$ if branch j is not in mesh i

In a ventilation network, if the airflows of all the chords are known, the airflow of each spanning tree airway can be obtained with known chord airflows by applying the concept of basic meshes and chords to Kirchhoff's first law. Given a ventilation network with N airways and J nodes, it will form M=N+J-1 basic meshes with M chords and N−M (N minus M) spanning tree airways. If the airflows of the M chords are known, the airflows of the N-M tree branches can be expressed using the chord airflows:

$$Q_j = \sum_{i=1}^{M} c_{ij}Q_i \ (j = 1,2,\dots,N-M) \tag{5}$$

Where $Q_j$ is the airflow of spanning tree airway j; $Q_i$ is the airflow of the chord in mesh i; $c_{ij}$ is a constant defined as follows:

$c_{ij} = 1,$ if branch j is contained in mesh i and has the same direction with the chord of mesh i
$c_{ij} = -1,$ if branch j is contained in mesh i and has the opposite direction with chord of mesh i
$c_{ij} = 0,$ if branch j is not contained in mesh i

To substitute Equation (5) into Equation (4), we can obtain the following equation (Equation 6):

$$\sum_{j=1}^{N} b_{ij}R_j\left(\sum_{i=1}^{M} c_{ij}Q_i\right)^2 - H_{Fi} - H_{Ni} = 0 \quad (i = 1,2,\dots,M) \tag{6}$$

As it has been defined in Equation (1), for a given airway k with resistance $R_k$, the sensitivity of any airway i to the resistance change in airway k is denoted as $\partial Q_i/\partial R_k$. Differentiating Equation (6) on both sides with respect to $R_k$ leads to the following set of equations:

$$\sum_{j=1}^{N} 2b_{ij}R_jQ_j\left(\sum_{i=1}^{M} c_{ij}\frac{\partial Q_i}{\partial R_k}\right) - \frac{\partial H_{Fi}}{\partial R_k} - \frac{\partial H_{Ni}}{\partial R_k} = 0$$
$$(i = 1,2,\dots,M) \ (k \neq j)$$

$$\sum_{j=1}^{N} 2b_{ij}R_jQ_j\left(\sum_{i=1}^{M} c_{ij}\frac{\partial Q_i}{\partial R_k}\right) + \sum_{j=1}^{N} b_{ij}{Q_j}^2 - \frac{\partial H_{Fi}}{\partial R_k} - \frac{\partial H_{Ni}}{\partial R_k} = 0 \quad (i = 1,2,\dots,M) \ (k = j) \tag{7}$$

For a given ventilation network, a system of linear equations with M (number of basic meshes as well as chords) unknowns ($\partial Q_i/\partial R_k$) will be obtained since one basic mesh will yield one linear equation. Therefore, the system of M linear equations with M unknowns is solvable. Neither the fan pressure nor the natural pressure in a mesh is affected significantly by resistance changes; therefore, those two related terms in the equation can be ignored. Write the above system of linear equations into the matrix equation (as shown in Equation (8)):

$$DX_k = e_k \tag{8}$$

Where D is a $M \times M$ matrix with the element $d_{ij} = \sum_{n=1}^{N} 2b_{in}R_nQ_n$ ; $X_k$ is a column vector with the partial derivatives of the volume flowrate of the chord in each basic mesh to the resistance $R_k$: $X_k = (\frac{\partial Q_1}{\partial R_k}, \frac{\partial Q_2}{\partial R_k}, \dots, \frac{\partial Q_M}{\partial R_k})$; $e_k$ is a column vector with M entries, where $e_{ki} = -\sum_{j=1}^{N} b_{ij}{Q_j}^2$ if branch k is in the mesh i and $e_{ki} = 0$ if branch k is not in the mesh i. Solving the system of linear equations, M partial derivatives of the chords to the resistance $R_k$ will be obtained. The remaining derivatives of N-M tree airways to the resistance $R_k$ can be determined by differentiating both sides of Equation (5). As of now, the sensitivity $s_{k1}, s_{k2}, \dots, s_{kN}$, which is the row k in the sensitivity matrix S, is obtained. Following the same procedure, the derivatives of all the branches to each resistance in the ventilation network can be



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calculated. Therefore, the sensitivity matrix S of a given ventilation network is completed.

### IMPLEMENTATION IN MFIRE

To realize the mathematical model of the sensitivity, the above mathematical model is implemented into the open-source mine fire simulation software, MFIRE, using programming language Visual C#.

Figure 3 is a screenshot of Sensitivity Tab from MFIRE which displays the sensitivity matrix using an example of a 49-airway ventilation network (Figure 2). The left frame in Figure 3 displays the 49 by 49 resistance sensitivity matrix. A *Save to Excel* feature was created to export the matrix data to an Excel file for data saving and further data analysis utilizing Excel functions. The right frame of the Sensitivity tab was designed for users to check the airways affecting or affected by the airway of interest.

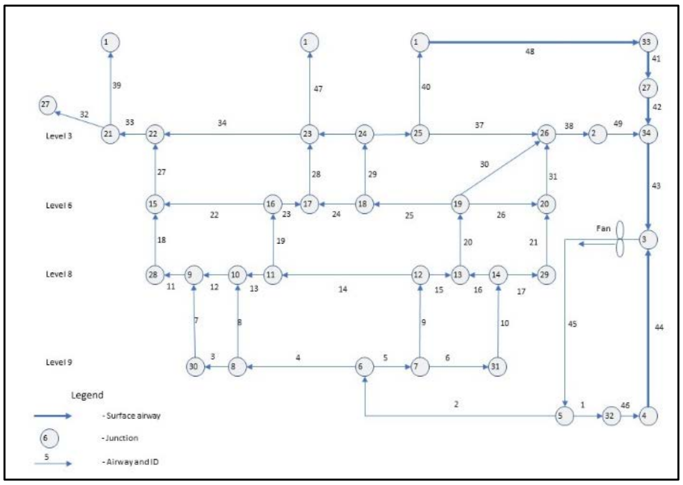


**Figure 2.** The schematic layout of the 49-airway network (source: Chang, et al., 1990).

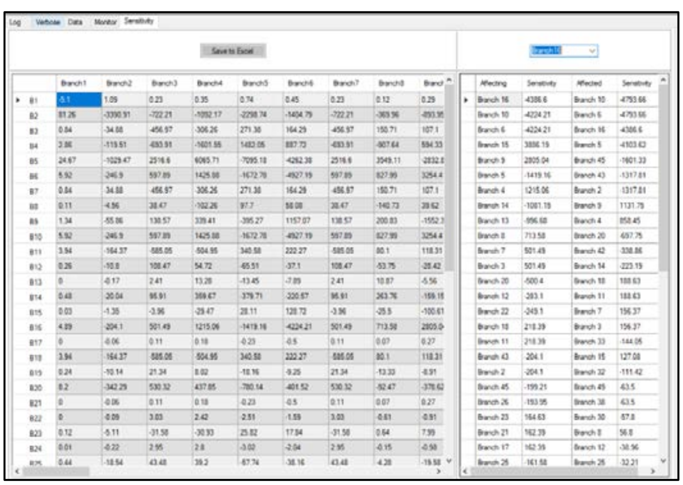

**Figure 3.** Sensitivity matrix and affected and affecting branch list to a selected branch in MFIRE.

As stated before, the sensitivity indicates the airflow change in an airway because of the resistance change in any airway, including itself, in a ventilation network. In the displayed sensitivity matrix, the K Row represents the sensitivity of each airway corresponding to the resistance change in the K airway, while the K Column displays the airflow changes in the K airway in response to the resistance change in each airway. For example, the value of the cell at the cross line of row B4 and column Branch5 is 1482.05. This indicates that the change of airflow in Airway 5 is 1482.05 $ft^3$/min responding to the resistance increase or decrease of Airway 4. The positive sign of the sensitivity means the airflow change is in the same direction of the resistance change. In other words, the airflow increases with the resistance increase, and vice versa. Likewise, the negative sign of the sensitivity indicates that the airflow of an airway changes in the opposite direction of the resistance change in the other branch. As a fact, the sensitivity of an airway is always positive to its paralleled connecting airway and negative to its series connecting airway. It is easy to understand that the resistance increase (decrease) of an airway will lead to the airflow decrease (increase) of the other branches in a series circuit and the other way around in a parallel circuit.

The right frame of the Sensitivity tag page shown in Figure 3 is a sorted sensitivity list of the affecting and affected airways to an airway of interest. When an Airway of Interest is selected, the first two columns will list the airway numbers and their sensitivities which the resistance change of the selected airway affects from the most to the least, while the second two columns will display the affected branches of which the selected branch is affected by the resistance changes in these branches in a descending order. Taking the example shown in Figure 3, the resistance increase (decrease) in the selected Branch 16 will affect itself the most by a 4386.6 ft3/min decrease (increase) in airflow, and then Branch 10, Branch 6, and the rest of airways. Similarly, the airflow of Branch 16 will be affected most by the resistance changes in Branch 10 and 6, and then itself as shown in the second two columns. Using this feature, it will be easy to find out how sensitive a certain branch is to other branches in a ventilation network. This could allow users to have an insight about the mutual dependence of airways. Users can utilize this feature to potentially diagnose the cause of abnormal airflow, develop a ventilation regulating plan, and evaluate the ventilation stability, etc.

The sensitivity displayed in Figure 3 is the value of airflow change in response to a resistance change, which can be called absolute sensitivity. It is useful to display the relative sensitivity which expresses how large the airflow rate change in an airway is compared to its original airflow rate. The relative sensitivity is the sensitivity divided by the original airflow rate and displayed in the form of a percent (Figure 4).

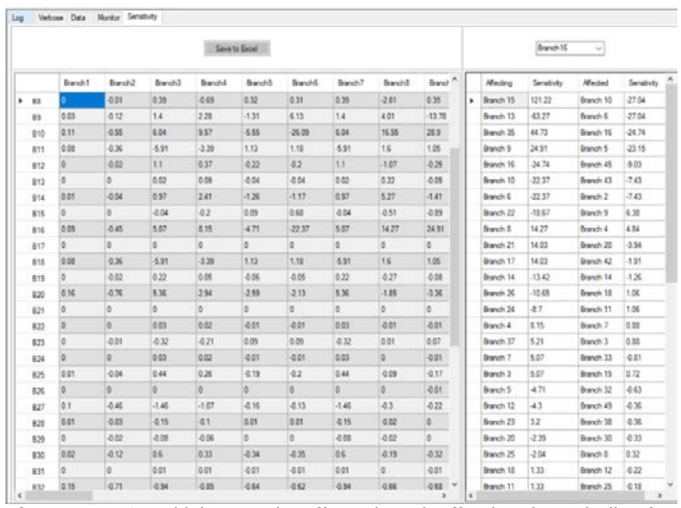

**Figure 4.** Sensitivity matrix affected and affecting branch list for relative sensitivity (%).

### VERIFICATION OF THE DIRECT DERIVATIVE METHOD

As the sensitivity reflects the airflow change in an airway in response to the resistance change in another branch, the sensitivity matrix can be directly obtained by changing the resistance of each branch one at a time and running the ventilation simulation for each, which is the approach Li et al. (2011) employed in their work. This approach requires one ventilation simulation for each resistance change. The sensitivity results from this approach can be used to compare them with the outcome from the direct derivative method proposed in this study. Taking an example of Branch 19, we


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calculated the airflow changes in each branch while the resistance of Branch 19 changes by 0.1, 0.2, and 0.8 ($10^{-10}$ * in.w.g./cfm$^2$) respectively. To better understand the correlation of the results, the normalized scatterplot is used to display the relationship between the sensitivity from the derivative method and the results from the three fixed resistance changes. It can be seen from Figure 5 that the sensitivity calculated from the derivative method is linearly related to the results obtained from the various resistance change. The Pearson Coefficients calculated for the three cases are 0.999, which indicates that they are positively linear related. As of now, we can say the resistance sensitivity calculated from the proposed direct derivative method is verified. It also needs to be noted that the direct derivative method may yield different magnitudes of the sensitivity comparing to other methods (Dziurzyński et al., 2017; Griffith & Stewart, 2019; Semin & Levin, 2019; Jia et al., 2000), but it will not change the fact that the sensitivity is an indicator of how a resistance change in one branch could affect the airflow of each branch in a mine ventilation network.

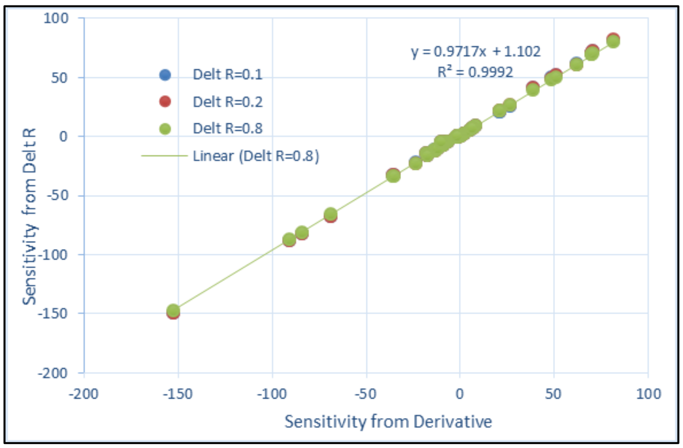


**Figure 5.** The correlation between the sensitivity from the derivative method and normalized resistance changes.

## CONCLUSIONS

Resistance sensitivity of a ventilation network is a valuable tool to understand the degree of inter-dependency of each branch and overall stability of the network. This paper presents a direct derivative method to calculate the sensitivity without the need of numerous simulations of ventilation network as used in other methods. As compared to other methods, the advantage of the direct derivative method is the time saving and computer resource saving since it only requires running the ventilation network solution one time. The proposed method has been implemented into NIOSH's mine fire simulation program, MFIRE, and has proven to provide a useful addition to the software features. The verification study of the direct derivative method has shown that the calculated sensitivity results are in excellent agreement with the airflow rate changes from the example cases with a manual resistance change. The sensitivity matrix of a ventilation network provides a good picture of how the airways correlate with each other. It can help mine operators to perform ventilation control and diagnose ventilation problems such as abnormal airflow.

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