Note

A SIMPLE CALCULATION FOR THE BUILDUP AND DECAY OF RADON PROGENY

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***Abstract*—A Markov chain in conjunction with an electronic spreadsheet is used to generate the buildup and decay patterns of radon progeny in a procedure involving only multiplication and addition. Markov chains are highly suitable for calculations of this type. Because the Markov matrix may not be readily available, and because some health physicists may not be familiar with this procedure, we give the values of a Markov matrix and demonstrate how it can be used to produce a Markov chain describing the buildup and decay of radon progeny.**
Health Phys. 80(6):616–617; 2001

Key words: radon progeny; decay chain; computer calculations; matrices

INTRODUCTION

Currently we are examining the effect of radon progeny on the response of a radon monitor. For our analysis we need to calculate the buildup and decay of these progeny, and we initially thought that the only procedure for this calculation is to apply the Bateman equations. However, using a Markov chain in conjunction with an electronic spreadsheet, we were able to generate the desired buildup and decay patterns of radon progeny with a procedure involving only multiplication and addition. Markov chains are highly suitable for calculations of this type; but because the Markov matrix may not be readily available, and because some health physicists may not be familiar with this procedure, we give the values of a Markov matrix and demonstrate how it can be used to produce a Markov chain describing the buildup and decay of radon progeny.

PROCEDURE

The Markov matrix used here is the 3 × 3 set of numbers given in Table 1. In keeping with the use of this matrix in a spreadsheet, the columns are designated by letters and the rows by numbers. In this matrix, the first row of numbers gives the activities of ^{218}Po, ^{214}Pb, and ^{214}Bi, respectively, that would be found after a 1-min decay of a sample of ^{218}Po having initially a unit activity. The second row gives the corresponding results assuming an initial unit activity of ^{214}Pb. As ^{214}Pb does not decay to give ^{218}Po, the first element in this row has a value of zero. The final row, giving the activities of the progeny of ^{214}Bi, has two null elements, as ^{214}Bi cannot decay to either ^{218}Po or ^{214}Pb.

The columns in Table 2 represent the time and the activities of ^{218}Po, ^{214}Pb, and ^{214}Bi, respectively. The first row gives the initial concentrations of the radon progeny at $t = 0$. The selection of these initial values is purely arbitrary; we chose a sample having no ^{214}Pb, ^{214}Bi, and an activity of 100 for ^{218}Po. Using the common mathematical terminology, these three progeny activities at any given time can be treated as a horizontal vector. Multiplying this vector by the Markov matrix results in another vector, a vector representing the activity of the progeny 1 min later than the activities given by the previous vector. This process is repeated to give the activities of the progeny for as long a time span as desired.

Carrying through the matrix multiplication gives the activity of ^{218}Po at the n^{th} minute as

$$[^{218}\text{Po}]_n = [\text{A1}][^{218}\text{Po}]_{n-1} = 0.799676[^{218}\text{Po}]_{n-1}\,. \quad (1)$$

The activity of ^{214}Pb at the n^{th} minute is given as

$$[^{214}\text{Pb}]_n = [\text{B1}][^{218}\text{Po}]_{n-1} + [\text{B2}][^{214}\text{Pb}]_{n-1} = 0.022864[^{218}\text{Po}]_{n-1} + 0.974473[^{214}\text{Pb}]_{n-1}\,, \quad (2)$$

and the activity of ^{214}Bi at the n^{th} minute is given as:

$$[^{214}\text{Bi}]_n = [\text{C1}][^{218}\text{Po}]_{n-1} + [\text{C2}][^{214}\text{Pb}]_{n-1} + [\text{C3}][^{214}\text{Bi}]_{n-1} = 0.00041[^{218}\text{Po}]_{n-1} + 0.033783[^{214}\text{Pb}]_{n-1} + 0.965775[^{214}\text{Bi}]_{n-1}\,, \quad (3)$$

where the [A1], [B1], etc., refer to the cells in Table 1.

This repetitive procedure is very stable. We compared the spreadsheet values of the progeny computed by this procedure with those determined through the

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(*Manuscript received* 30 *May* 2000; *revised manuscript received* 25 *October* 2000, *accepted* 15 *February* 2001)

0017-9078/01/0

Table 1. The Markov matrix for radon progeny.[a]

Row	Column A	Column B	Column C
1	0.799638	0.022868	0.000410
2	0.000000	0.974468	0.033790
3	0.000000	0.000000	0.965768

[a] The above table was calculated from the Bateman equations assuming half-lives of 3.1, 26.8, and 19.9 min for ^{218}Po, ^{214}Pb, and ^{214}Bi, respectively. The general Bateman formula for the activity of the n^{th} progeny, for the case in which only atoms of the parent isotope are present initially, is

$$A_n = c_1 e^{-\lambda_1 t} + c_2 e^{-\lambda_2 t} + \cdots c_n e^{-\lambda_n},$$

where

$$c_1 = \frac{\lambda_2\lambda_3\lambda_4 \cdots \lambda_n}{(\lambda_2 - \lambda_1)(\lambda_3 - \lambda_1)\cdots(\lambda_n - \lambda_1)} A_{1,0}$$

$$c_2 = \frac{\lambda_2\lambda_3\lambda_4 \cdots \lambda_n}{(\lambda_1 - \lambda_2)(\lambda_3 - \lambda_2)\cdots(\lambda_n - \lambda_2)} A_{1,0},$$

and $A_{1,0}$ = activity of the parent isotope at $t = 0$; λ_I = decay constant of the I^{th} isotope.

Table 2. Calculated activity of radon progeny at a time, *t*, after their initial measurement.

Time, *t* min	Activity ^{218}Po	Activity ^{214}Pb	Activity ^{214}Bi
0	100	0	0
1	79.963823	2.286802	0.041013
2	63.942130	4.057029	0.149677
3	51.130572	5.415674	0.307867
4	40.885960	6.446655	0.501297
5	32.693977	7.217039	0.718741
6	26.143354	7.780419	0.951413
7	20.905225	8.179616	1.192470
8	16.716617	8.448834	1.436616

Bateman equations and found no difference in the values computed for the first 500 min.

It should be noted that the Markov matrix used in this calculation differs from the Markov matrix commonly given in the literature (as for example, in Bronson 1982) in that the sums of the elements in the individual rows are not equal to unity. This is because activity is not a conserved value; e.g., a unit activity of ^{238}U eventually decays into ^{206}Pb having zero activity. Had the Markov matrix been used to calculate the number of atoms of the progeny, and further had the matrix been enlarged to include all progeny down to and including the stable isotope, ^{206}Pb, then the sums of all the rows would have become unity.

A related problem that can be easily solved by this procedure is "given a mixture of radon progeny, what was the activity of these progeny at some specific earlier time?" We needed this calculation to correct our instruments for the effects of radon progeny for times prior to the point where an assessment of progeny composition had been made. The procedure used is to multiply the vector giving the progeny composition at a time, *t*, by the *inverse* of the Markov matrix given in Table 1. The result of this multiplication is the composition of these progeny 1 min earlier. Tables 3 and 4 show the inverse matrix and sample results from using this repetitive procedure, respectively. Note that after 8 min, the result of this calculation is the pure ^{218}Po that the decay process had at its beginning. Continuing this procedure for an additional minute leads to nonsensical results, as it is impossible for ^{218}Po to decay at some earlier time and leave only ^{218}Po.

Table 3. The inverse Markov matrix for determining the activity of radon progeny.

Row	Column A	Column B	Column C
1	1.250566	−0.029347	0.000496
2	0.000000	1.026201	−0.035905
3	0.000000	0.000000	1.035445

Table 4. Calculated activity of radon progeny at a time, *t*, before their initial measurement.

Time, *t*, min	Activity ^{218}Po	Activity ^{214}Pb	Activity ^{214}Bi
0	16.716617	8.448834	1.436616
1	20.905225	8.179616	1.192470
2	26.143354	7.780419	0.951413
3	32.693977	7.217039	0.718741
4	40.885960	6.446655	0.501297
5	51.130572	5.415674	0.307867
6	63.942130	4.057029	0.149677
7	79.963823	2.286802	0.041013
8	100.000000	0.000000	0.000000
9	125.056552	−2.934725	0.049573

In our tests, the activity of the 21-y half-life ^{210}Pb is negligible, and thus we ended the calculation with ^{214}Bi. However, the initial buildup of ^{210}Pb from the short-lived radon progeny can be estimated through eqn (4):

$$^4A = \frac{(^1t_{1/2}(^1A_{t=0} - {}^1A) + {}^2t_{1/2}(^2A_{t=0} - {}^2A) + {}^3t_{1/2}(^3A_{t=0} - {}^3A)}{^4t_{1/2}} \quad (4)$$

where

A = Activity at a time, t;
$A_{t=0}$ = 0 initial activity;
$t_{1/2}$ = half life; and

the superscripts 1, 2, 3, and 4 refer to ^{218}Po, ^{214}Pb, ^{214}Bi, and ^{210}Pb, respectively.

Eqn (4) is similar to the algorithm given for the activities of the earlier radon progeny in that it too is very simple to implement with the help of a spreadsheet. This is consistent with the objective of this note, which is to give a simple, straightforward procedure for determining the activities of the radon progeny.

REFERENCE

Bronson, R. Schaum's outline of theory and problems of operations research. New York: McGraw-Hill Book Company; 1982.

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