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Predicting glass furnace output using statistical and neural computing methods

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This paper describes the development of predictive models for glass production at a regional manufacturing company. The objectives of the models are to predict the actual batch tonnage produced per week from the glass furnace based on the planned production schedule. Four modelling methods were explored: (i) linear regression; (ii) nonlinear regression; (iii) artificial neural network using back-propagation; and (iv) radial basis function neural network. Using 175 cases of production schedule data and subsequent furnace output, the two neural network-based prediction models resulted in lower average absolute error and lower maximum absolute error than the linear or nonlinear regression models. Accurate neural network-based prediction models of furnace output will subsequently be used in the overall production planning system by utilizing estimates of furnace output to determine the necessary energy, raw material, repair and personnel requirements of the glass manufacturing facility.

1. Background

1.1. *The modern glass-making industry in the United States*

Following the steep rise in energy, environmental, capital and labour costs in the 1970s and 1980s, many of the glassware producers in the US were forced to close. Today, only ~ 20 shops nationwide remain in business. Given the increases in operating expenses, glassware companies continue to search for advances in manufacturing technology that will allow them to increase production while limiting costs due to energy, raw material, repair and personnel requirements. Although various types of glass production operations exist, their manufacturing process can be summarized in several basic steps, as seen in figure 1.

Raw materials, primarily sand, lime, soda ash and recycled glass (called cullet) are automatically fed into the melting furnace as needed. Once charged into the furnace, the raw materials are melted using a combination of natural gas, preheated air and pure oxygen. The molten glass is removed from the furnace and distributed to various production areas (called 'shops') within the facility. Each shop produces one or more types of glass products (e.g. pitchers, bowls). Within a shop, a punty rod is used to pour the molten glass into an open mould. The worker then cuts off the amount of glass necessary to fill the mould and presses it manually. The moulded piece is then subjected to glazing and finishing.

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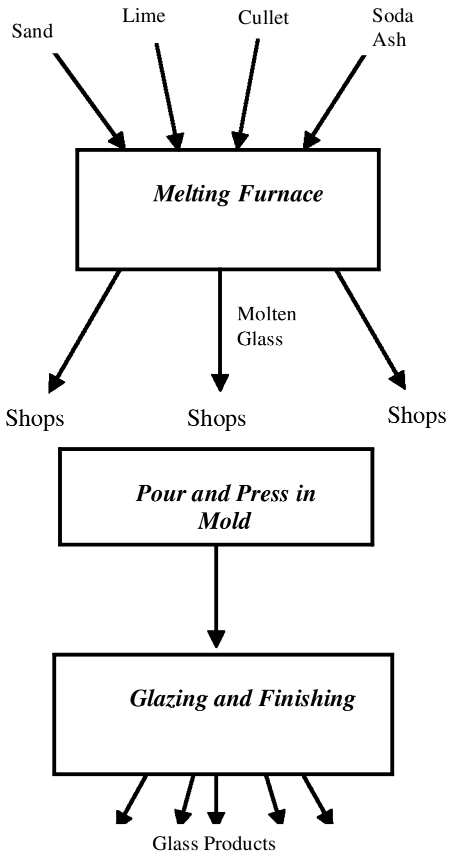

Figure 1. Basic steps in glass-making process.

1.2. *The project environment*

This research project was conducted at the Louie Glass Company, founded in 1926 by Louie Wohinc, an immigrant glass blower from Eastern Europe. Louie Glass survived a downturn in the industry during the 1970s and 1980s in part due to its being acquired by Princess House. Princess House has implemented computer control into the glass melting process. Molten glass is removed from the plant's furnace and distributed to 15 'shops' within the plant. Each shop uses the molten glass to manufacture various products, as listed in table 1.

With the exception of the Press Item Shop, that uses robots, the glass products produced in the shops are manufactured by hand. The production schedule for this process (i.e. man-hours worked in each of these shops per week) is determined by the product orders that the factory must fill.

2. Problem statement

One of the major difficulties that Princess House faces in its day-to-day operations is deciding how much molten glass the furnace should produce to adequately meet the weekly demands of each of its shops. As the production schedule changes from week to week, the rate at which each shop gathers molten glass from the

Shop name	Products produced
German stem	Cake plates, compote bowls, punch bowls, bridal flute, brandy snifter
German jug	Pitchers and jugs of various shapes and sizes
German punch	Vases, serving bowls, jars, hurricane shades, storage cylinders, pot-pourri bowls
Stem	Wine glasses, toasting flutes, martini glasses, candle sticks, shrimp cocktail bowls, dessert glasses
Handle punch	Water pitches, creamers, sugar bowls
Punch	Small vases, bowls, jars, drinking glasses and candle shades
Lid	Covers for candy jars, sugar bowls and storage jars
Hand craft/Hand shop	Paper weights, pieces for fruit
Press items 1–6	Cake plates, luncheon plates, dinner plates, serving pieces

Table 1. Glass items produced in the shops of Princess House Manufacturing.

furnace also changes. These changes cause large temperature variations within the furnace's glass gathering area. Fluctuations in the temperature can result in 'cords' developing in a glass product. These cords are ribbons of non-homogeneous glass that can run through a finished glass piece. According to the company, the number one reason for rejecting a piece of glass is due to the presence of cords. These changes in the furnace temperature and defective products increase operational costs. Currently, Princess House does not use any systematic analytical method for accurately predicting molten glass demands of the furnace. Adjustment in furnace temperature is purely reactive and made only when a temperature change is detected. The quality control problem of cords is managed primarily through the use of inspection and rejection of flawed finished products. Various parameters that focus around the product characteristics being manufactured, including the colour of each product, quantity of each product needed, the cumulative specific weight of each product, and the thickness of each product produced, typical rejection and rework rates at each shop can be used as independent variables to predict the required glass furnace output. However, because the operation produces a large variety of products, the resources required to collect such data were prohibitive.

Alternatively, the man-hours that are worked in each shop could be used as independent variables for furnace output prediction. Although collecting such data would be easier, using these variables to predict glass furnace output might prove more difficult due to nonlinear relationships between the shop hours and the amount of molten glass required, as can be seen in figure 2.

2.1. *Objective of study*

This project explored various methods to translate the production schedule into a melting requirement (batch tonnage tons per week). Knowing the effect of changes in the production schedule on the required furnace output ahead of time will allow Princess House to gradually adjust the furnace temperature rather than altering the furnace temperature drastically to accommodate changes in demand. It is believed that these gradual changes in furnace temperature will reduce utility costs and the number of cords found in finished glass pieces. Accurate estimates of the furnace's weekly batch tonnage will also permit management to determine: (i) what the furnace's energy requirements will be for the week; (ii) the amount of raw materials that will have to be purchased in order to run the furnace; (iii) the human resources and overtime

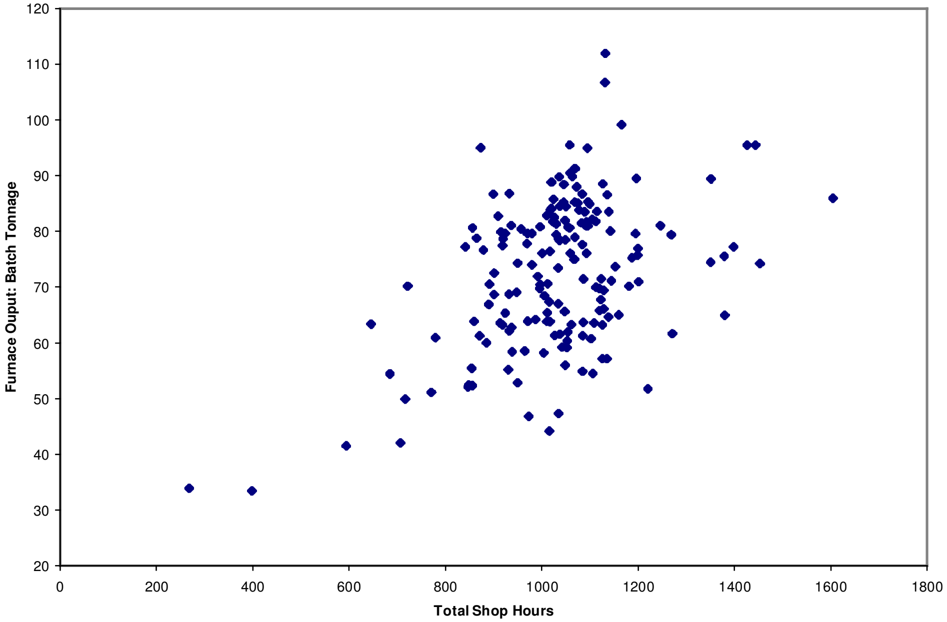

Figure 2. Batch tonnage as a function of total shop hours.

requirements needed to operate the furnace; and (iv) how frequently repairs and preventive maintenance will have to be made on the furnace and machinery that supports furnace operation. The relationship between the production schedule, the furnace and requirements of the glass manufacturing operation are summarized in figure 3.

From an operations research perspective, developing a model to estimate the required batch tonnage that the furnace must produce can be considered to be an optimization problem. The model will use the man-hours worked in each shop as inputs (i.e. independent variables) to predict the required furnace output (i.e. the dependent variable) in batch tons. Optimization of the furnace output prediction model entails minimizing the mean squared error (MSE). This error is represented by the squared difference between the actual batch tonnage that the furnace must produce and the predicted batch tonnage produced by the estimation model. This

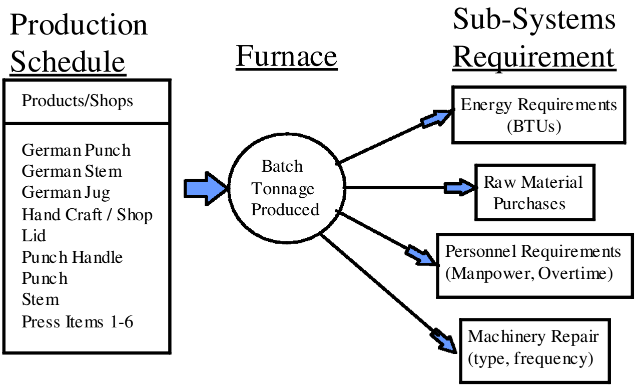

Figure 3. Relationship between production schedule and sub-system requirements.

model can be represented mathematically as shown below in equation (1).

$$\text{Minimize} \quad MSE = (1/N)\sum_{k=1}^{N}(BT_{\mathrm{A}(k)} - BT_{\mathrm{P}(k)})^2 \tag{1}$$

where: MSE is the mean squared error; N is the total number of training samples; $BT_{\mathrm{A}(k)}$ is the actual batch tonnage that the furnace must produce for the kth sample; and $BT_{\mathrm{P}(k)}$ is the predicted batch tonnage that the furnace must produce for the kth sample.

3. Methodology

3.1. *Background—modern glass making using computer-based control methods*

In developing a solution methodology for a predictive model of furnace output the authors could either focus on one of two approaches.

(1) A traditional analytical approach using regression. This approach is typically used when a deeper understanding of the underlying variables is sought and when dependent process variables are known.
(2) An artificial intelligence-based technique, e.g. an expert system, fuzzy logic or an artificial neural network. With processes that are more difficult to characterize, and when a 'black box' approach to modelling will suffice, this approach has had some success. For the type of data available (deterministic, quantitative), neural networks seemed the most appropriate.

The authors chose to evaluate both approaches here: two types of regression and two types of neural networks were applied to the problem. Archival literature abounds with examples of the application of regression methods to process modelling. Thus, our assessment of computer-based control methods in glass making and related areas focus on the application of neural networks to similar problems.

The modern glass-making process combines traditional glassware production techniques with new computer-controlled automation. Although the gathering and shaping of molten glass is performed totally by hand, advanced computer-based control methods have been used to increase the output and limit the operating costs of the glass furnace itself. An example of such a method was described by Liu and Lyons (1993) who attempted to use an artificial neural network to enhance the glass furnace performance and reliability while simultaneously limiting its instrumentation costs. As described by Zahedi (1991) and Fausett (1994), artificial neural networks (ANN) are informational processing mathematical models that can learn from past examples or training data. Based upon the literature review by Sharda (1994), ANNs can be used to perform classification, clustering and control tasks. ANNs have also been successfully applied to function approximation (Billings and Chen 1992, Haykin 1994, Hassoun 1995), suggesting that they would be good candidates for comparison to linear and nonlinear regression techniques. In fact, the use of ANNs provides an alternative modelling methodology to classical regression analysis (Haykin 1994). As with regression, the goal is to develop a network with a high R^2 value and a low root mean square error. However, another goal of neural networks is to develop a model that can generalize and not simply memorize its training and testing data. A primary characteristic of ANNs is that they allow inputoutput mappings without understanding of the functional relationships between variables (Pollard *et al.* 1992).

The purpose of the ANN developed by Liu and Lyons (1993) was to predict the specific gravity of a propane/air gas mixture used to fuel the glass furnace. This prediction was to be made based on the differential gas pressure and valve inlet pressure. Using the gas mixture's specific gravity, the flow and pressure control valves can be adjusted to set the amounts of air and propane to their appropriate levels. The ANN was able to predict the gas mixture's specific gravity with a mean absolute error of 1.7% and a maximum absolute error of 4.4%. Results of this study suggest that an ANN approach may prove valuable for predicting certain operating parameters of the furnace. This information in turn can be used to properly regulate the glass-manufacturing process.

3.2. *Approach*

As mentioned earlier, four modelling methods were used to predict weekly batch tonnage output: (i) linear regression; (ii) nonlinear regression; (iii) artificial neural network using backpropagation; and (iv) radial basis function neural network. The inputs for each of these models were the weekly man-hours recorded in each of the manned production shops. The key melting process variables needed to determine the input and output parameters were: the required product mix; number of production shops; the production rate on the automatic press; and the day-to-day and week-to-week changes in molten glass demand on the furnace.

3.3. *Data collection*

The project data are historical in nature and were collected on a weekly basis from May 1993 to January 1996. The data were preprocessed to remove outliers resulting from holidays, sick days, planned downtimes, and unplanned downtimes. After preprocessing, 175 data points were available for use by all of the models.

3.4. *Descriptive statistics*

Descriptive statistics for the 16 variables examined in the models are provided in table 2. The first variable BATCH represents the batch tonnage produced per week

Variable	*N*	Minimum	Maximum	Mean	Stand.	Variance
BATCH	175	38.43	118.720	83.77	15.00	225.00
German	175	0.000	575.184	318.57	98.27	9656.95
German Stem	175	0.000	328.129	56.104	49.277	2428.27
German Jug	175	0.000	299.759	65.638	56.879	3235.22
Hand Craft	175	0.000	66.70	11.029	18.746	351.418
Hand Shop	175	0.000	101.39	20.329	27.114	735.144
Lid	175	0.000	342.31	101.902	60.851	3702.856
Punch	175	54.326	864.409	319.522	128.475	16506.00
Punch Handle	175	0.000	322.759	29.382	37.239	1386.765
Stem	175	95.22	498.709	230.13	69.011	4762.619
Press Item 1	175	0.000	62.100	17.155	23.505	552.478
Press Item 2	175	0.000	104.075	6.023	18.925	358.152
Press Item 3	175	0.000	127.65	7.742	24.514	600.963
Press Item 4	175	0.000	97.75	2.3591	12.346	132.541
Press Item 5	175	0.000	60.95	1.327	6.832	46.678
Press Item 6	175	0.000	9.775	0.0644	0.746	0.556

Table 2. Descriptive statistics for independent and dependent variables.

by the furnace. The remaining 15 variables represent the man-hours worked per week in each of the shops.

By collapsing across different shops, one can plot the batch tonnage produced per week as a function of total shop hours worked. As seen in figure 2, a high degree of variability exists in the relationship between the amount of shop hours worked and the batch tonnage of the furnace. It is due to this variation that alternatives to linear regression (nonlinear regression, neural networks) were utilized in the current analysis.

3.5. *Cross-validation*

A fivefold group cross-validation strategy (Myers 1990, Haykin 1994) was utilized in order to develop six replications for each predictive model. Each replication entailed constructing the predictive model using one portion of the data set and then testing the model using the remaining portion of the data set. Replications 1–5 use 140 data points (80%) randomly selected for training and 35 data points (20%) set aside for model testing. Selection is carried out in such a way that each data point is used for testing only once. This strategy was selected in order to more accurately assess the performance differences between each of the four models. A sixth replication included all 175 data points for training and testing. Upon testing, the average absolute error and maximum absolute error was recorded for each replication.

3.6. *Methods for furnace output prediction*

The four methods used to model the relationships between the data (linear regression, nonlinear regression, backpropagation neural network, and radial basis function neural network) are now described.

3.6.1. *Linear regression*

Based on partial correlation coefficients, the following independent variables were selected for inclusion in the linear regression model: German Punch (*Gp*), German Stem (*Gs*), German Jug (*Gj*), Lid (*L*), and Stem (*S*).

The six models constructed using linear regression were based on the following format:

$$\text{Batch tonnage} = (\beta_1 \times Gp) + (\beta_2 \times Gs) + (\beta_3 \times Gj) + (\beta_4 \times L) + (\beta_5 \times S) \tag{2}$$

Each independent variable represents the man-hours worked in the shop over 1 week's time and the coefficients β_i represent the corresponding constant.

3.6.2. *Nonlinear regression*

Given that the relationship between the shop hours and glass furnace output may be nonlinear in nature, the Box–Tidwell transformation (Box and Tidwell 1962) was utilized to construct a predictive model. This transformation can be employed for estimating exponents α_j in a model of the type:

$$y = \beta_0 + \beta_1 w_1 + \ldots + \beta_k w_k + \varepsilon \tag{3}$$

where

$$w_j = \begin{cases} x_j^{\alpha_j} & \text{if } \alpha_j \neq 0 \\ \ln(x_j) & \text{if } \alpha_j = 0 \end{cases}$$

This method accommodates exponents on one or more regressor variables and is easy to use with a multiple linear regression software package. The following format was used for the nonlinear regression model.

$$\text{Batch tonnage} = (\beta_1 \times Gp^{\alpha_1}) + (\beta_2 \times Gs^{\alpha_2}) + (\beta_3 \times Gj^{\alpha_3}) \qquad (3)$$

Each variable represents the man-hours worked in the shop over 1 week's time (raised to the power of α_j) and β_i represents its corresponding constant.

3.6.3. *Backpropagation (BP) neural network*

Through the use of a systematic approach, the architecture for a BP network was developed for the glass furnace output problem. The resulting network possessed two hidden layers with 15 nodes in the first layer and 10 nodes in the second layer. The connective weights were trained using a backpropagation algorithm (Wasserman 1989) at a learning rate of 0.35 until the R^2 value under the training set data was equal to 0.995 (considered by the authors to be an acceptable level of minimum error). A binary sigmoid transfer function was used for the output node.

3.6.4. *Radial basis function (RBF) neural network*

Based upon a systematic approach, the architecture for a RBF network was developed for the glass furnace output problem. The resulting network possessed 30 nodes in the cluster layer. The weights connecting the input layer to the cluster layer were trained using a learning rate that decreased from 0.99 to 0.01 across 1000 epochs. For training the weights, which connected the cluster layer to the output node, backpropagation was used at a learning rate of 0.35. Training continued until the R^2 value under the training set data was equal to 0.995. A binary sigmoid transfer function was used for the output node.

4. Results

The present section provides a broad picture of the methods employed in this project and their relative performance with respect to one another. In table 3, it can be observed that both of the ANN models generally outperformed both the regression models (except in the case of Model 4). The BP network-based model resulted in lower average absolute error than the RBF network model. Generally, the RBF network had lower maximum absolute error than those constructed using the backpropagation network.

When using cross-validation as part of a boot strapping procedure, one will choose a model that performed well across various data test sets. The model will

Replication	Linear regression	Nonlinear regression	Backpropagation network	Radial basis network
1	7.49 (28.54)	8.80 (26.08)	6.30 (21.79)	6.30 (15.00)
2	6.10 (25.74)	9.14 (33.36)	7.52 (26.36)	9.90 (23.80)
3	11.25 (34.91)	9.39 (28.39)	6.94 (21.25)	7.50 (19.24)
4	8.41 (19.22)	7.55 (23.54)	7.44 (20.30)	12.04 (29.80)
5	7.92 (28.44)	7.16 (22.20)	5.48 (16.91)	5.50 (14.51)
6†	7.56 (33.25)	7.63 (28.56)	6.49 (22.96)	6.71 (20.44)

† The sixth data set represents all 175 cases.

Table 3. Model accuracy with regard to average absolute error and (maximum absolute error).

Source of variation	Sum of squares	Degrees of freedom	Mean square error	F Statistic	P-value
Model	942 553	5	188 511	1850.324	< 0.001
Error	17 319.6	170	101.880		
Total	959 872	175			

Table 4. ANOVA table for linear regression model.

then be retrained using all the available data (training and test set). This model represents replication #6. Given that the learning and prediction accuracy characteristics were similar across replications, replication #6 will be used to describe the performance of each predictive method. The linear regression equation constructed based on replication #6 provided a model that accounted for 98% of the variability in the batch tonnage output of the furnace, as seen in table 4.

$$\text{Batch tonnage} = (0.121 \times Gp) + (0.0105 \times Gs) + (0.165 \times Gj) + (0.35 \times L) + (0.105 \times S) \tag{4}$$

Table 4 represents the ANOVA for the linear regression model. Given the value of the F test statistic, one would conclude that the coefficients of the model do not equal zero and that batch tonnage is related to the hours worked in the German Punch, German Stem, German Jug, Lid, and Stem shops. The P-value for this test is less than 0.001, suggesting that the probability of obtaining an F test statistic this large (given there was no relationship between batch tonnage and shop hours) is extremely small. The nonlinear regression equation constructed based on replication #6 also provided a model that accounted for 98% of the variability in the batch tonnage output, as seen in table 5.

$$\text{Batch tonnage} = (6.942 \times Gp^{0.4}) + (0.00021 \times Gs^{2.172}) + (0.079 \times Gj^{1.055}) \tag{5}$$

When using the Box–Tidwell transformation, regression analysis revealed that the coefficients of the two independent variables (Lid and Stem) were non-significant (i.e. their confidence intervals for both coefficients included 0). Given this finding, these two variables were not included in the final regression model. Table 5 represents the ANOVA for the nonlinear regression model. Given the value of the F test statistic, one would conclude that the coefficients of the model do not equal zero and that batch tonnage is related to the hours worked in the German Punch, German Stem and German Jug shops. The P-value for this test is less than 0.001, suggesting that the probability of obtaining an F test statistic this large (given there was no relationship between batch tonnage and shop hours) is extremely small.

Source of variation	Sum of squares	Degrees of freedom	Mean square error	F Statistic	P-value
Model	944 153	3	314 718	3443.631	< 0.001
Error	15 719.3	172	91.391		
Total	959 872	175			

Table 5. ANOVA table for nonlinear regression model.

Although the performances for the RBF and BP network were comparable, the amount of training that each network required to reach the target minimum average sum squared error was substantially different. Using all 175 cases (replication 6), the BP network reached the target minimum average sum squared error in ~ 43 epochs, that was over six times faster than the RBF network that reached its target only after 290 epochs. These learning trend differences were similar across all six replications. This behaviour is consistent with previous research that reports that an RBF

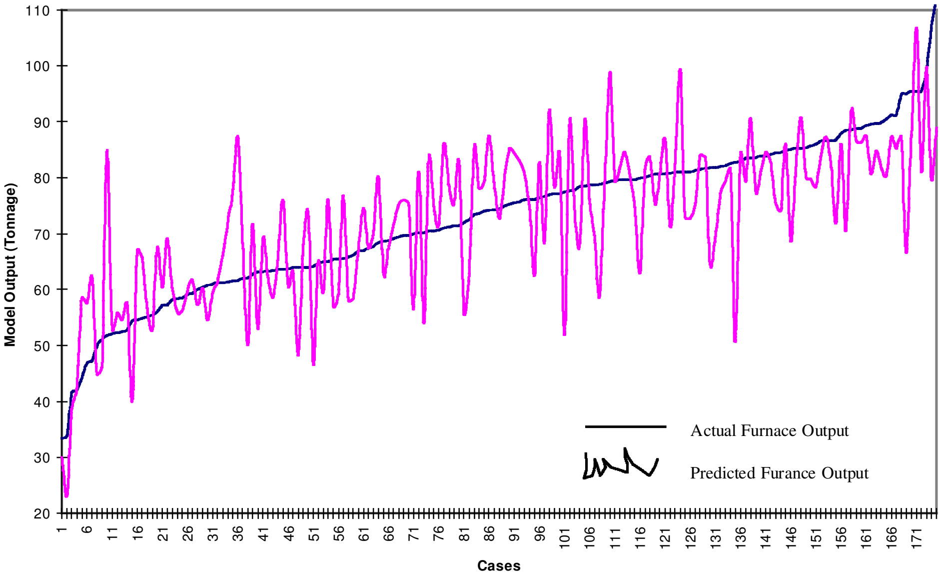

Figure 4. Actual output versus linear regression model predicted output.

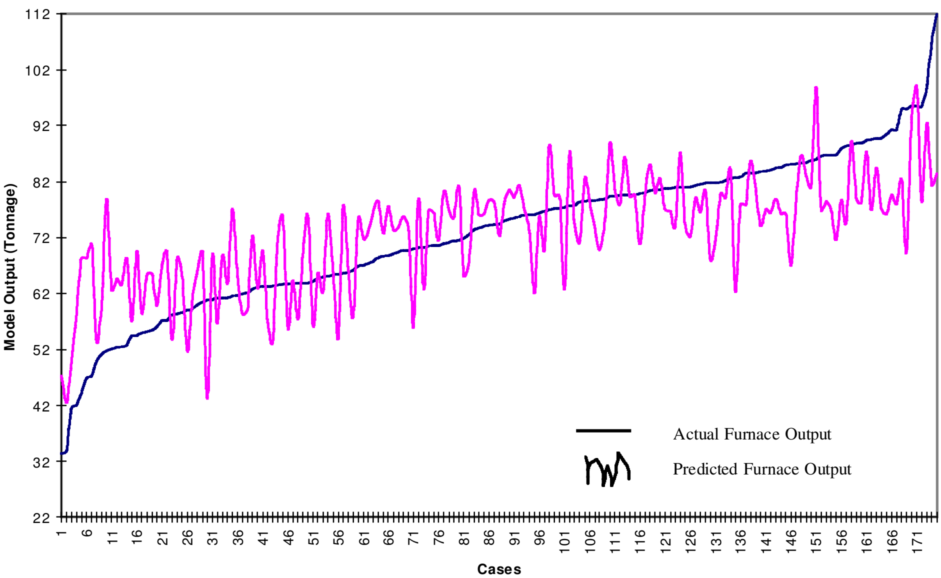

Figure 5. Actual output versus nonlinear regression model predicted output.

network generally requires more training than the backpropagation network to achieve the same level of accuracy (Hassoun 1995).

Using the entire data set (175 cases), an ordered plot was constructed for the linear, nonlinear, RBF network and backpropagation network methods (as shown in figures 4–7, respectively). For each ordered plot, the actual glass furnace output was plotted from the lowest to the highest value. Alongside each actual data point, its corresponding predicted value was also plotted. As shown in the figures,

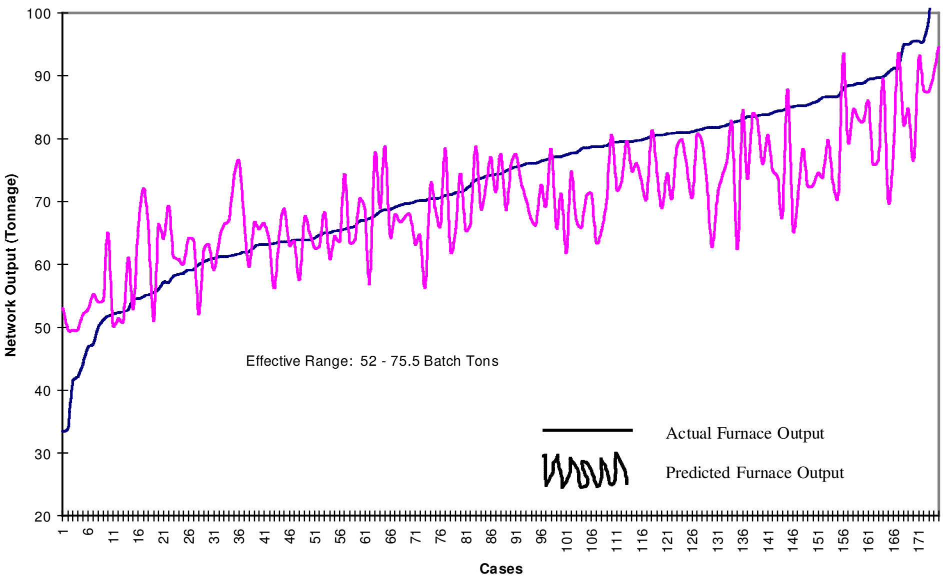

Figure 6. Actual output versus RBF network predicted output.

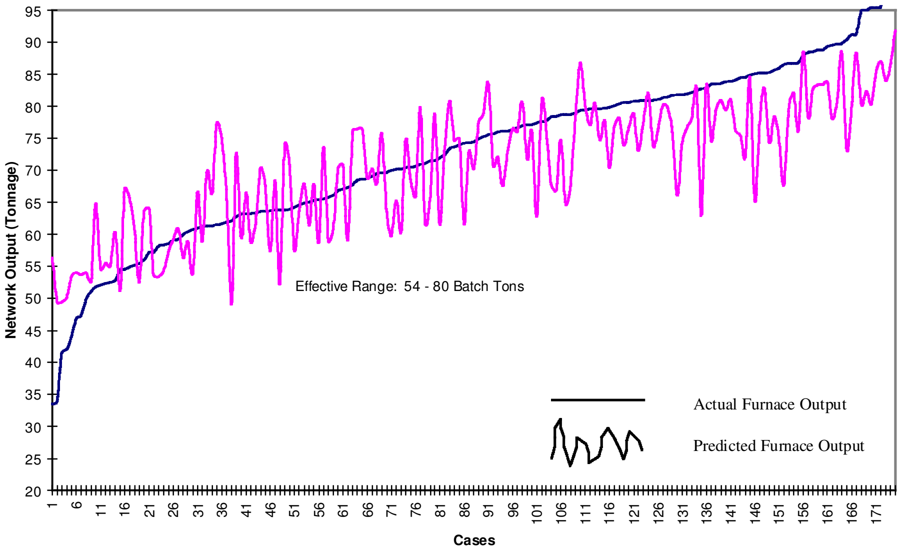

Figure 7. Actual output versus backpropagation network predicted output.

the variation between actual and predicted output in the two regression methods is quite large (when compared to the ANN models).

At roughly 80 tons of output, both the linear and nonlinear regression method tend to underestimate actual batch tonnage. The RBF network is reasonably accurate from 52 to 75 tons of output. A similar trend is seen with the backpropagation network that predicts well when the furnace output is between 54 and 80 tons.

Review of the residual histograms for the linear, nonlinear, RBF network and backpropagation network methods (figures 8–11, respectively) indicates that both the neural network models had residuals that were normally distributed, although somewhat negatively skewed. This skewness was also observed in the regression models that contain residual distributions that were far less symmetric.

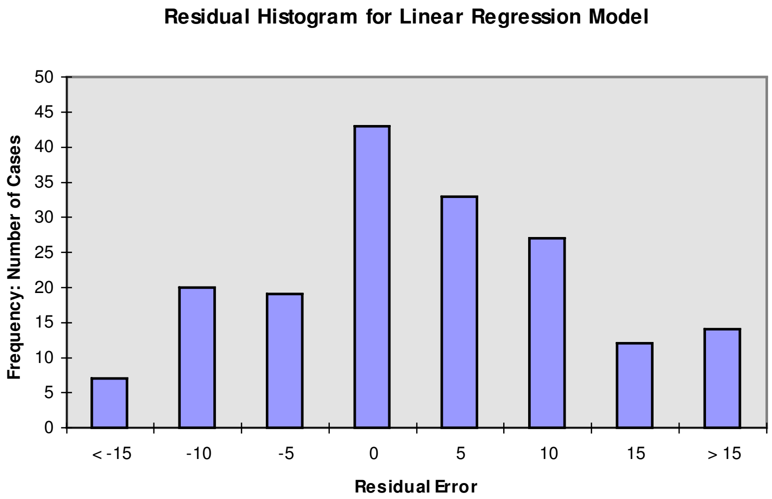

Figure 8. Residual histogram for linear regression method ($N = 175$).

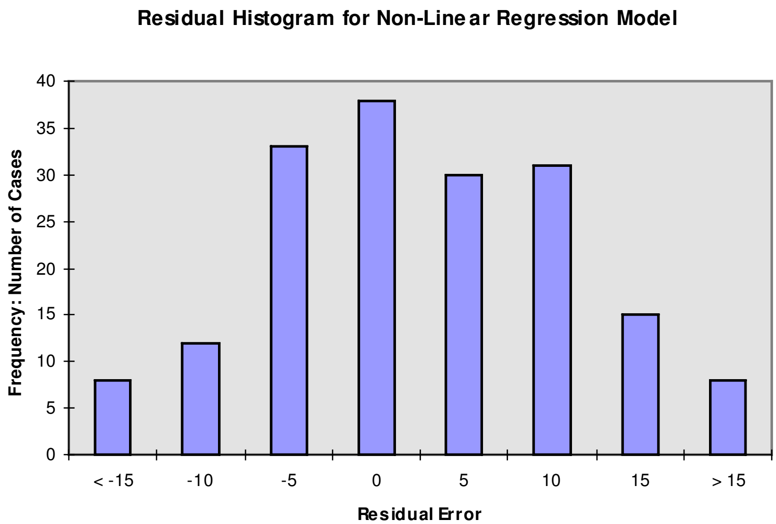

Figure 9. Residual histogram for nonlinear regression method ($N = 175$).

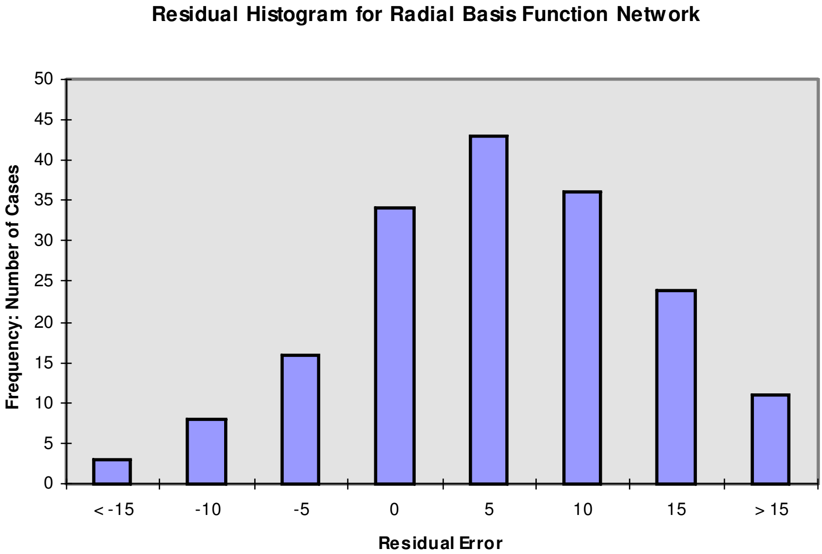

Figure 10. Residual histogram for RBF method ($N = 175$).

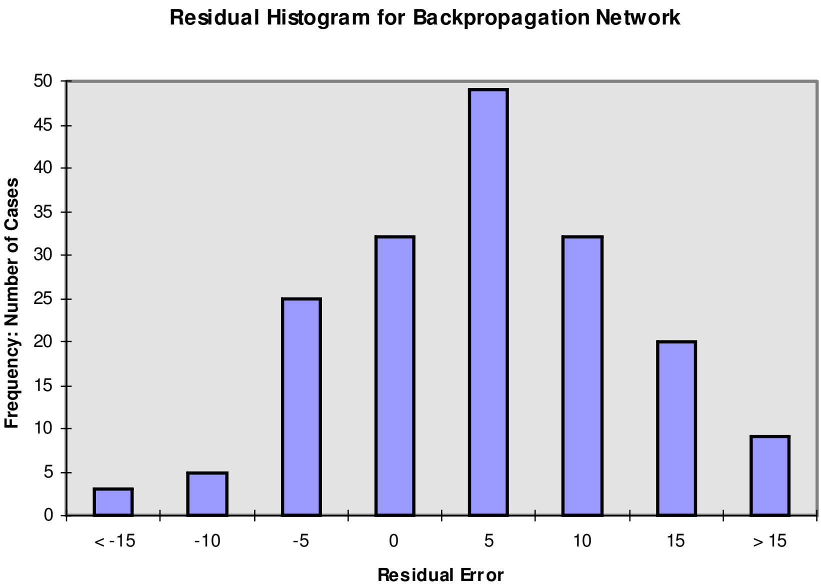

Figure 11. Residual histogram for backpropagation method ($N = 175$).

These differences are summarized in table 6. Results of the backpropagation network performance revealed that in 138 of the 175 cases (78.9%) the network provided a prediction of batch tonnage that was within 10 tons of the actual output value. Only in 12 cases (6.9%) did the backpropagation network provide a prediction that was more than 15 tons from the actual output. Linear regression was much less accurate with only 122 of the 175 cases (69.7%) predicted within 10 tons of the actual output, and 21 cases (12%) for which the predicted value was more

Prediction model	±5 tons to target value	±10 tons to target value	±15 tons to target value	> ±15 tons to target value
Linear	76 (43.4)	122 (69.7)	154 (88.0)	21 (12.0)
Nonlinear	68 (38.9)	134 (76.6)	159 (90.9)	16 (9.14)
RBF	77 (44.0)	129 (73.7)	161 (92.0)	14 (8.00)
BP	81 (46.3)	138 (78.9)	163 (93.1)	12 (6.86)

Using entire data set, $N = 175$.

Table 6. Frequency distribution of residuals (and percentage of the 175 cases) for the four predictive methods.

than 15 tons from the actual output value. Nonlinear regression and the RBF network were comparable with regards to delivering predictions that were within 15 tons of the actual furnace output. Based on minimum batch tonnage output recorded at the plant, a model with a predictive error of greater than 15 tons could underestimate or overestimate actual furnace output by as much as 44%. As seen in table 6, errors of this magnitude were much more frequent using the linear regression model as compared to the BP network.

5. Conclusions and future work

Applying neural computing techniques to the prediction of glass furnace output resulted in better predictions with regard to accuracy and stability as compared to statistical regression. In particular, the number of extreme errors in prediction (> 15 tons above or below actual output) was far less for backpropagation than for linear regression. The backpropagation method, however, requires much more input information (man-hours for 15 shops) than the linear regression method (man-hours for five shops). Which method to use may depend in part on the ease in which the shop hour information is collected. Both ANN approaches tended to underestimate the glass furnace output when actual batch tonnage was above 80 tons. This tendency may be attributed to the back orders incurred during normal plant operations. Specifically, there may be some circumstances in which back orders of glass products are sent out from the various shops along with the current orders. In this case, the batch tonnage output would be artificially inflated and not representative of the actual hours worked. To deal with this potential bias, it may be advantageous in future work to develop three separate neural networks. The first network would be trained to classify shop hour input vectors as being associated with a batch tonnage output of 80 tons or more, or classify the vector being associated with a batch tonnage output of less than 80 tons. The remaining two neural networks could be trained to predict glass furnace output for each group.

The negative skewness observed in the residual histograms of each predictive model may also be attributed to plant activities not taken into account. For example, throughout the year, plant employees will engage in day-long classes on industrial safety or quality control. Under these circumstances, employee shop hours would still be counted and recorded, but the glass tonnage output would be decreased. To account for these instances, future models might include the number of shop hours per week (or per month) spent in training sessions. Improvements in the linear/nonlinear regression models may be achieved by performing ridge regression or principle component regression on the five shop hour inputs. Ridge regression (Neter *et al.* 1990) and principle component regression (Myers 1990) develop models that account for correlation that may exist between the independent variables. This correlation can lead to large predictive errors during testing. The existence of these correlations may also subsequently explain the asymmetry found in the residual histograms of both the linear and nonlinear regression models. Finally, future predictive models could combine both classical statistical and neural computing methods. As proposed by Roadknight *et al.* (1997), principle component analysis was used to identify the inputs for an ANN model.

In summary, both classical statistical and neural computing methods were used to develop models that would predict glass furnace output given the man-hours worked in various production shops. The neural computing methods possessed less average absolute error, less maximum absolute errors, and fewer extreme errors than linear or nonlinear regression. Furnace output estimates from these models could be used to determine the energy and material requirements of the glass manufacturing operation.

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