SUPPLEMENTARY DATA

We constructed dynamic models consisting of systems of difference equations. The U.S. population ages 0 to 19 years is modeled at one year intervals starting at year t = 0 (2001) and ending at year t = 49 (2050). Specifically, we define numbers of people in various states , rates, and flows. All rates are annual and flows occur during year t, i.e., in the interval (t-1,t]. Moreover, the notation and model suppresses the ten race-sex combinations, i.e., there are ten separate models described by the following. First we characterize the Census population projections.

$N(a,t)$ = the total number of people age = a at year t coming from Census projections.

$b(t)$ = Census projection of the number of births in year t.

$m(a,t)$ = Census projection of the number of people age = a migrating into the U.S. in year t.

$d(a\text{-}1,t)$ = Census projected death rate among all people age = a-1 in year t.

$D(a,t)$ = the number of deaths in year t among people age = a-1 at the beginning of the year. Note that $D(a,t) = d(a\text{-}1,t)N(a\text{-}1,t\text{-}1)$.

Using the above notation

$$N(0,t) = b(t) + m(0,t)$$

$$N(a,t) = N(a-1,t-1) - D(a,t) + m(a,t)$$

with a = 1,2…,19 and t = 1,…,49. For the diabetes projections let

$X(a,t)$ = the number of people age = a without diabetes at year t.

$Y(a,t)$ = the number of people age = a with Type 1 diabetes at year t.

$Z(a,t)$ = the number of people age = a with Type 2 diabetes at year t.

$\delta(a\text{-}1,t)$ = non-diabetes death rate among $X(a\text{-}1,t\text{-}1)$ in year t.

$\delta(a\text{-}1,t)$ = diabetes death rate among $Y(a\text{-}1,t\text{-}1)$ in year t; $r_2\delta(a\text{-}1,t)$ = diabetes death rate among $Z(a\text{-}1,t\text{-}1)$ in year t; r_1 and r_2 are relative risks which are assumed to be age and time invariant.

$\beta(a\text{-}1,t)$ = incidence rate of Type 1 diabetes among $X(a\text{-}1,t\text{-}1)$ in year t, estimated from SEARCH data as described in the previous section.

$\gamma(a\text{-}1,t)$ = incidence rate of Type 2 diabetes among $X(a\text{-}1,t\text{-}1)$ in year t, estimated from SEARCH data as described in the previous section.

$f_x(t)$ = proportion of $b(t)$ without diabetes.

$f_y(t)$ = proportion of $b(t)$ with Type 1 diabetes.

$f_z(t)$ = proportion of $b(t)$ with Type 2 diabetes.

$g_x(a,t)$ = proportion of $m(a,t)$ without diabetes.

$g_y(a,t)$ = proportion of $m(a,t)$ with Type 1 diabetes.

$g_z(a,t)$ = proportion of $m(a,t)$ with Type 2 diabetes.

Define $\theta(a,t) = \dfrac{Y(a,t)+Z(a,t)}{X(a,t)+Y(a,t)+Z(a,t)}$, the prevalence of diabetes among people age = a at year t;

$\theta_1(a,t) = \dfrac{Y(a,t)}{X(a,t)+Y(a,t)+Z(a,t)}$, the prevalence of Type 1 diabetes among people age = a at year t;

$\theta_2(a,t) = \dfrac{Z(a,t)}{X(a,t)+Y(a,t)+Z(a,t)}$, the prevalence of Type 2 diabetes among people age = a at year t, noting that $\theta(a,t) = \theta_1(a,t) + \theta_2(a,t)$. We impose the constraints $X(a,t) + Y(a,t) + Z(a,t) = N(a,t)$ so that the diabetes projections are consistent with the Census population projections.

To this end, we first set the initial conditions

$$X(a,0) = [1 - p_1(a) - p_2(a)]N(a,0)$$

$$Y(a,0) = p_1(a)N(a,0)$$

$$Z(a,0) = p_2(a)N(a,0)$$

$a = 0,1,\ldots,19$, with $p_1(a)$ and $p_2(a)$ the 2001 prevalences of Type 1 and Type 2 diabetes estimated from the SEARCH data as described in the second section and $N(a,0)$ is the 2001 population from the Census data. We also need the relations $f_x(t) + f_z(t) + f_y(t) = 1$ and $g_x(a,t) + g_y(a,t) + g_z(a,t) = 1$ to guarantee that incoming flows match. Lastly, we need the deaths in the diabetes model to equal deaths in the Census projections model, or

$$\begin{aligned} &\delta(a-1,t)X(a-1,t-1)+ \\ &r_1\delta(a-1,t)Y(a-1,t-1)+ \\ &r_2\delta(a-1,t)Z(a-1,t-1) \\ &= d(a-1,t)N(a-1,t-1) \end{aligned}$$

which implies (with a little algebra)

$$\delta(a-1,t) = \frac{d(a-1,t)}{[1-\theta(a-1,t-1)] + [r_1\theta_1(a-1,t-1) + r_2\theta_2(a-1,t-1)]}.$$

Consider the transition matrix

$$\begin{array}{lcccc} & X(a,t): & Y(a,t): & Z(a,t): & D(a,t): \\ X(a-1,t-1): & \alpha(a-1,t) & \beta(a-1,t) & \gamma(a-1,t) & \delta(a-1,t) \\ Y(a-1,t-1): & 0 & [1-r_1\delta(a-1,t)] & 0 & r_1\delta(a-1,t) \\ Z(a-1,t-1): & 0 & 0 & [1-r_2\delta(a-1,t)] & r_2\delta(a-1,t) \\ D(a-1,t-1): & 0 & 0 & 0 & 1 \end{array} \qquad (1)$$

where $a = 1,2,\ldots,19$ and $t = 1,2,\ldots,49$. Note that the rows of this matrix display the distribution of the beginning year stocks (age = a-1) to the ending year stocks (the columns, age = a), and thus the transition rates in each row must be nonnegative and add to unity for each year t. Also, some assumptions about transition rates are apparent. First, people cannot move from diabetes to non-diabetes. Second, people cannot move from one diabetes state to another diabetes state. Third, as stated above, the relative risks of death for the two diabetes states to the non-diabetes state are constant by age over time. The transition matrix (1) and the inflows lead to the system of difference equations

$$\begin{aligned} X(0,t) &= f_x(t)b(t) + g_x(0,t)m(0,t) \\ X(a,t) &= \alpha(a-1,t)X(a-1,t-1) + g_x(a,t)m(a,t) \\ Y(0,t) &= f_y(t)b(t) + g_y(0,t)m(0,t) \\ Y(a,t) &= \beta(a-1,t)X(a-1,t-1) + [1-r_1\delta(a-1,t)]Y(a-1,t-1) \\ &\quad + g_y(a,t)m(a,t) \qquad (2) \\ Z(0,t) &= f_z(t)b(t) + g_z(0,t)m(0,t) \\ Z(a,t) &= \gamma(a-1,t)X(a-1,t-1) + \left[1-r_2\delta(a-1,t)\right]Z(a-1,t-1) \\ &\quad + g_z(a,t)m(a,t) \end{aligned}$$

where, again, $a = 1,2,\ldots,19$ and $t = 1,2,\ldots,49$. Initial conditions for (2) are $X(a,0)$, $Y(a,0)$, $Z(a,0)$ as defined earlier. Finally, in all our projections we set $f_x(t) = 1$, $f_y(t) = 0$, $f_z(t) = 0$, or incoming births are non-diabetic, and

SUPPLEMENTARY DATA

$$
\begin{aligned}
g_x(a,t) &= [1-\theta_1(a,t-1)-\theta_2(a,t-1)],\\
g_y(a,t) &= \theta_1(a,t-1),\\
g_z(a,t) &= \theta_2(a,t-1)
\end{aligned}
.
$$

All diabetes projections were calculated using the programming language GAUSS.[1]

We finish this section with a matrix characterization of the system (2). Let $\mathbf{T}(a,t)$ be the 3x3 matrix of transition rates (excluding death rates) defined in the transition matrix (1) and set

$$
\mathbf{T}(t) = \begin{bmatrix}
\mathbf{O} & \mathbf{T}(0,t) & \mathbf{O} & \cdots & \mathbf{O}\\
\mathbf{O} & \mathbf{O} & \mathbf{T}(1,t) & \cdots & \mathbf{O}\\
. & . & . & . & .\\
\mathbf{O} & \mathbf{O} & \cdots & \mathbf{O} & \mathbf{T}(18,t)\\
\mathbf{O} & \mathbf{O} & \mathbf{O} & \cdots & \mathbf{O}
\end{bmatrix},
$$

a 60x60 matrix of transitions with 3x3 zero matrices everywhere but the upper diagonal. If we define the 1x60 row vector of states

$$\mathbf{S}(t) = [X(0,t), Y(0,t), Z(0,t), \ldots, X(19,t), Y(19,t), Z(19,t)],$$

and the 1x60 row vectors of flows

$$
\begin{aligned}
\mathbf{B}(t) &= [\, f_x(t)b(t),\, f_y(t)b(t),\, f_z(t)b(t), 0,0,0,\ldots,0,0,0]\\
\mathbf{M}(t) &= [g_x(0,t)m(0,t), g_y(0,t)m(0,t), g_z(0,t)m(0,t), \ldots,\\
&\qquad g_x(19,t)m(19,t), g_y(19,t)m(19,t), g_z(19,t)m(19,t)],
\end{aligned}
$$

then the projections model can be represented in matrix form as

$$\mathbf{S}(t) = \mathbf{S}(t\text{-}1)\mathbf{T}(t) + \mathbf{B}(t) + \mathbf{M}(t)$$

with initial conditions $\mathbf{S}(0)$ as defined above.

Reference:

1. Aptech Systems Inc.. *GAUSS Mathematical and Statistical System version 10.0*. 2009 Maple Valley, WA: Aptech Systems Inc.