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ORIGINAL PAPER

Laboratory Estimation of Rock Joint Stiffness and Frictional Parameters

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Abstract Numerical modeling of complex rock engineering problems involves the use of various input parameters which control usefulness of the output results. Hence, it is of utmost importance to select the right range of input physical and mechanical parameters based on laboratory or field estimation, and engineering judgment. Joint normal and shear stiffnesses are two popular input parameters to describe discontinuities in rock, which do not have specific guidelines for their estimation in literature. This study attempts to provide simple methods to estimate joint normal and shear stiffnesses in the laboratory using the uniaxial compression and small-scale direct shear tests. Samples have been prepared using rocks procured from different depths, geographical locations and formations. The study uses a mixture of relatively smooth natural joints and saw-cut joints in the various rock samples tested. The results indicate acceptable levels of uncertainty in the calculation of the stiffness parameters and provide a database of good first estimates and empirical relations which can be used for calculating values for joint stiffnesses when laboratory estimation is not possible. Joint basic friction angles have also been estimated as by-products in the small scale direct shear tests.

Keywords Joint normal stiffness · Joint shear stiffness · Normal stress · Joint basic friction angle

1 Introduction

An understanding of discontinuities in rock, specifically rock joints, is crucial to the sound construction of structures in and on rock such as foundations, dams, tunnels, underground chambers for hydro power and waste isolation and slopes. Hence, it is important that rock joints and fractures be adequately characterized and their properties quantified before making engineering decisions for construction in rocks. Rock masses have traditionally been described by rock mass classification systems such as the Rock Quality Designation (RQD) proposed by Deere et al. (1967), the Rock Mass Rating (RMR) by Bieniawski (1974), Q-system (Barton et al. 1974) and the Geological Strength Index (GSI) by Hoek et al. (1995), with multiple modifications to suit specific locations (Stille et al. 1982; Hoek et al. 2002; Romana 1985).

With the advent of powerful computers and numerical modeling capabilities, rock masses could be modeled more quantitatively by dissecting the rock mass properties into the intact rock and joint physical and mechanical properties. While some properties such as the rock density, Young's modulus, Poisson's

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ratio, uniaxial strength, cohesion, angle of internal friction and tensile strength of intact rock are easily estimated in the laboratory for the purpose of assigning to the numerical models, other properties may not be as easy to estimate. Often, numerical models make use of parameters which are obscure in definition or difficult to estimate in the laboratory, since their usage may describe the rock mass more efficiently during stress computation. Joint normal and shear stiffnesses are two such parameters which are not easily estimated in the laboratory and no formal guidelines are prescribed for their estimation. In this paper, we propose methods to estimate the joint normal and shear stiffnesses in the laboratory, and estimate basic friction angles, for smooth saw-cut joints or relatively smooth natural joints with relative ease and provide a framework for their subsequent usage in numerical models. The methodology may also be extended to rough natural joints without significant modification.

Engineering rock mechanics has usually described rock joint friction without much consideration to the complexities involved in the frictional response of rock joints/faults, since their importance is paramount when discussing earthquake mechanics and less so for engineering applications. A classical research on this topic is by Byerlee (1978), which introduces Byerlee's Law and assumes constant friction angles of 40° and 31° for the joint/fault gouge depending on whether the normal stress is less than or more than 200 MPa. While this serves as a good first estimate for the purpose of numerical modeling, other researchers argue that the frictional response may be a function of the rate of sliding (Dieterich 1992; Ruina 1983). A comprehensive review of the rate-and-state friction laws and their application to earthquake physics is discussed by Marone (1998). Even though they are discussed in conjunction with earthquake processes, some researchers (Helmstetter et al. 2004; Veveakis et al. 2007) suggest that landslides may follow similar mechanisms and it may be necessary to analyze them using rate-and-state friction models. Hence, readers are urged to exercise caution when applying laboratory estimated friction angles for analyses of landslides, since they may be treated as static events obeying classical rock joint frictional formulations or dynamic events following rate-and-state formulations. It may be necessary to carry out site-specific tests to discern the applicability of various friction laws to a specific landslide event.

2 Review of Literature on Joint Normal and Shear Behavior

Goodman (1974, 1976) suggested a hyperbolic relation to describe joint closure under normal compressive stress (Fig. 1). Bandis et al. (1983) presented a modification of Goodman's hyperbolic model, which was shown to provide a better fit to experimental data across the whole range of stress and closure values. It is necessary to estimate two parameters: (a) initial normal stiffness and (b) maximum joint closure to fit these hyperbolic models. Shehata (1971) used a semi-logarithmic empirical model to describe the same relation. Alternative models, based on Hertzian contact theory, have also been suggested to describe the non-linear stress deformation behavior (Sun et al. 1985; Swan 1983; Matsuki et al. 2001). Malama and Kulatilake (2003) proposed two functions: one

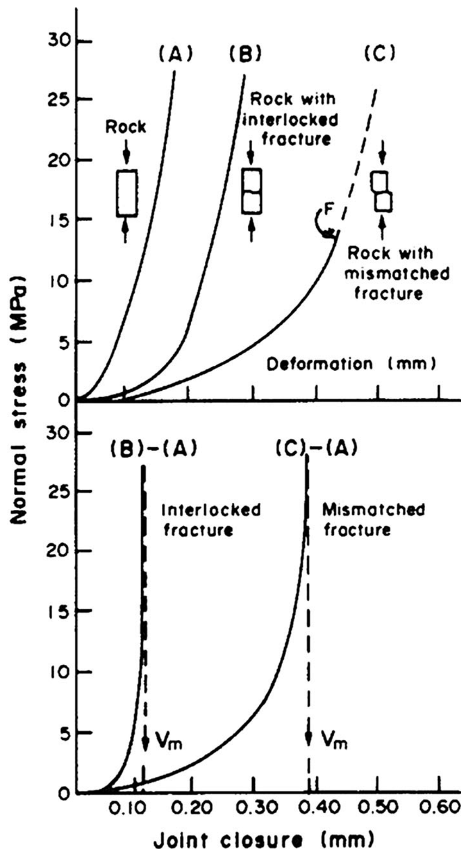

Fig. 1 Normal stress versus rock and joint deformation for intact rock specimen and specimen with single fracture for granodiorite (Goodman 1976). Here, the parameter V_m is the same as the parameter D_j introduced in Eq. 2

incorporating a single negative exponential function and the other incorporating a generalized negative exponential function to describe the same relation. By applying different functions for four natural rough rock joints, they have shown that the function that incorporates the generalized negative exponential function fits the data best. The function that incorporates the single negative exponential function and the hyperbolic function have given satisfactory results. The power function has given poor fits. Cyclic loading on the same rock sample causes accumulation of strain energy and the deformation profile of the sample may be influenced substantially (Wang and Park 2001) after each loading–unloading cycle. Sun et al. (1985) have performed cyclic loading to generate effective normal stress versus normal deformation curves. They state that while cyclic loading damages the asperities and micro-fractures, up to 70 % of the original stress-deformation curve was recoverable. Jing et al. (1994) also have studied the effect of cyclic loading on the normal stress-normal deformation curves on jointed rock samples. Results of all these studies have shown very clearly that the normal stiffness changes very significantly with the normal stress. Jing et al. (1994) have shown that the shear stiffness also depends on the normal stress. However, this dependency is not pronounced as for the normal stiffness. This paper focuses on providing procedures to estimate equations for both the normal stiffness and the shear stiffness as a function of normal stress.

The word joint normal stiffness was first coined by Goodman et al. (1968) for the purpose of defining a joint element for finite element computation of a jointed rock block. It has, since, become the most common parameter for describing joint mechanical behavior in mathematical models. The joint normal (K_n) and shear (K_s) stiffnesses may be defined as the ratio of normal (σ) and shear (τ) stresses acting on a joint to the respective normal (u) and shear (v) deformations. Since joints deform non-linearly, it is more realistic to express these relations in differential form as shown in Eq. (1).

$$d\sigma = K_n du \quad \text{and} \quad d\tau = K_s dv \tag{1}$$

Theoretically, the joint normal deformation (D_j) of a rock sample under uniaxial stress, for each value of normal stress, may be obtained by subtracting the normal deformation of an intact sample (D_i) from the normal deformation of a sample with a single joint (D_{rm}).

$$D_j = D_{rm} - D_i \tag{2}$$

Using Eq. (2), the joint normal stiffness can be determined for each normal stress value as the ratio of the normal stress to the joint normal deformation (D_j). The joint shear stiffness can be determined more directly through laboratory direct shear tests. However, extensive stiffness data is not available for joints in different rock types. Usually the joint normal deformation-normal stress relation is obtained by testing two rock samples coming from the same rock type subjected to uniaxial loading without and with the joint. However, it is important to note that the inherent heterogeneity in rock ensures that no two samples can be expected to give similar responses of the intact material behavior during uniaxial loading. This means the accuracy will be less if the normal deformation-normal stress relation for a joint is estimated using two samples as mentioned above. Malama and Kulatilake (2003) showed how to obtain the normal deformation-normal stress relation for a joint by performing a uniaxial test on only one sample having a horizontal joint. The same procedure is suggested in this paper in developing the equation between joint normal stiffness and normal stress. Rutqvist (1995) used well testing to determine the hydraulic normal stiffness of fractures in situ. Jiang et al. (2009) used the relation between transmissivity and depth to estimate fracture normal stiffness for large-scale rock masses, applicable for petroleum reservoirs. The latter two methods are useful for hydro-mechanical coupling and not particularly applicable to the context of modeling civil and mining constructions such as foundations, dams, tunnels, underground chambers for hydro power and slopes.

In this paper, we show how to estimate joint normal stiffness for smooth joints using simple laboratory based techniques and by avoiding cyclic loading. Also presented are the laboratory test results for joint shear stiffness and frictional properties.

3 Description of Experimental Methods

3.1 Estimation of Joint Normal Stiffness

To estimate the joint normal stiffness of a rock sample, we suggest a modified uniaxial compressive strength test. The intact rock sample was prepared as per the American Society for Testing and Materials (ASTM) standard D7012 (2014). A single saw-cut was introduced halfway through the length of the prepared cylindrical specimen. The ends of both smaller cylinders were ground and smoothened so as to eliminate any frictional or roughness effects. A masking tape was wrapped around the saw-cut joint to further ensure that no shearing or slipping occurred along the horizontal joint. This sample, now ready for testing, was uniaxially compressed to failure at a constant load rate. To estimate the stiffness properties of interfaces between two rock types, the samples were created mismatched, i.e., the top half of the sample belonging to one rock type and the bottom half to another. Reasonable effort was made to ensure that the samples were free from preexisting fractures, joints or voids. Figure 2 shows some of the prepared samples for the uniaxial compression test for samples with a single horizontal joint. During the compression, a curve of normal stress versus normal deformation was plotted. Figure 3 shows the total normal deformation of the sample expressed as a function of the applied normal stress. As can be observed from it, the curve contains an initial non-linear part due to the closure of the saw-cut joint and a linear part from the intact rock. A straight line parallel to the linear part and passing through the origin was drawn and designated as the theoretical intact deformation for that sample (orange line in Fig. 3). Then, the joint deformation was estimated for each normal stress value by subtracting intact rock deformation from the experimental rock mass deformation curve (Fig. 4). A best fit regression function for this curve has been found to be the exponential function as shown in Eq. (3), where A and B are empirical constants and σ and D_j have the usual meanings as previously defined elsewhere.

$$\sigma = \mathrm{A}e^{\mathrm{BD_j}} \tag{3}$$

Applying the natural logarithm function on both sides of Eq. (3) and subsequently differentiating with respect to σ yields

$$d\sigma/d\mathrm{D_j} = \mathrm{B}\sigma \tag{4}$$

The left hand side of Eq. (4) is, by definition, $\mathrm{K_n}$. Hence,

$$\mathrm{K_n} = \mathrm{B}\sigma. \tag{5}$$

3.2 Estimation of Joint Shear Stiffness

The small scale direct shear (SSDS) test is particularly useful for estimating the frictional properties of rock joints and faults in the laboratory. For our purposes, we follow ASTM standard D5607 (2008) to prepare

Fig. 2 Samples prepared for uniaxial compression test to estimate joint normal stiffness

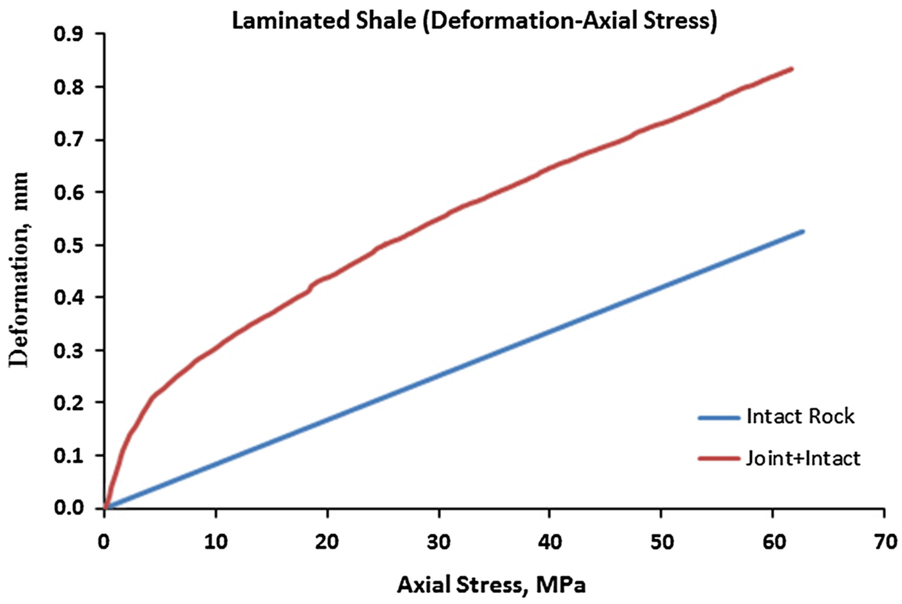

Fig. 3 Total normal deformation and intact rock normal deformation as functions of applied normal stress for sample LSS (sample number 6 in Table 1)

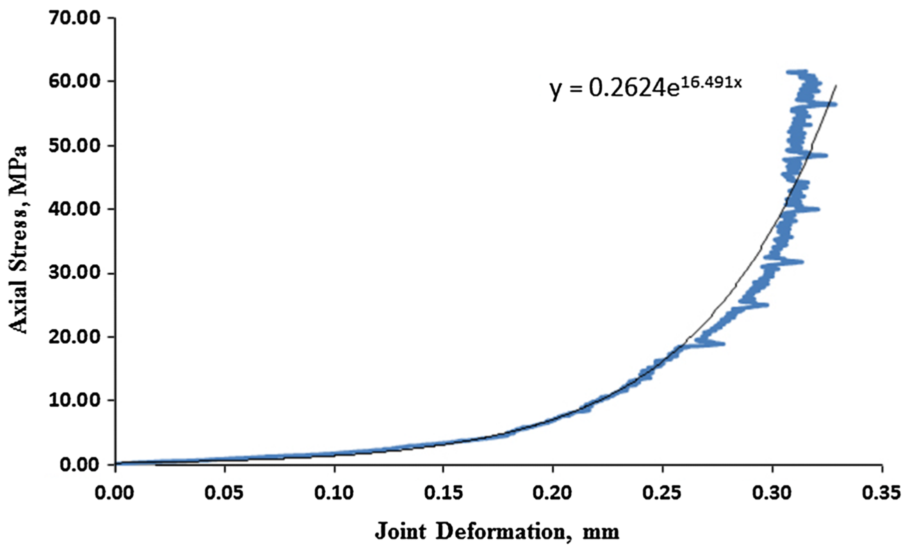

Fig. 4 Normal stress as a function of joint deformation and the fitted exponential curve for sample LSS (sample number 6 in Table 1)

the direct shear samples and to perform the tests. A labelled illustration of a shear box used for direct shear testing is given in Fig. 5 and the actual shear box used for the testing, a Humboldt 25SIXTYSHEAR, is shown in Fig. 6. Samples used have either natural joints or saw-cut joints. Each sample contains a top half and a bottom half, each embedded in a hydro-stone mold so as to be held securely within the frames in the shear box. During the experiment, a normal force is applied on the top frame while a constantly increasing horizontal shear force pushes the bottom frame, thus shearing the natural or saw-cut joint surface. A dial gauge connected to the shear box records the shear displacement of the bottom frame. During the experiment, a plot of shear stress versus shear displacement was generated as shown in Fig. 7. This process was repeated on each sample for four different normal stress values. Due to the linear nature of the shear stress versus shear displacement curve in most of the pre-peak range, the shear stiffness was

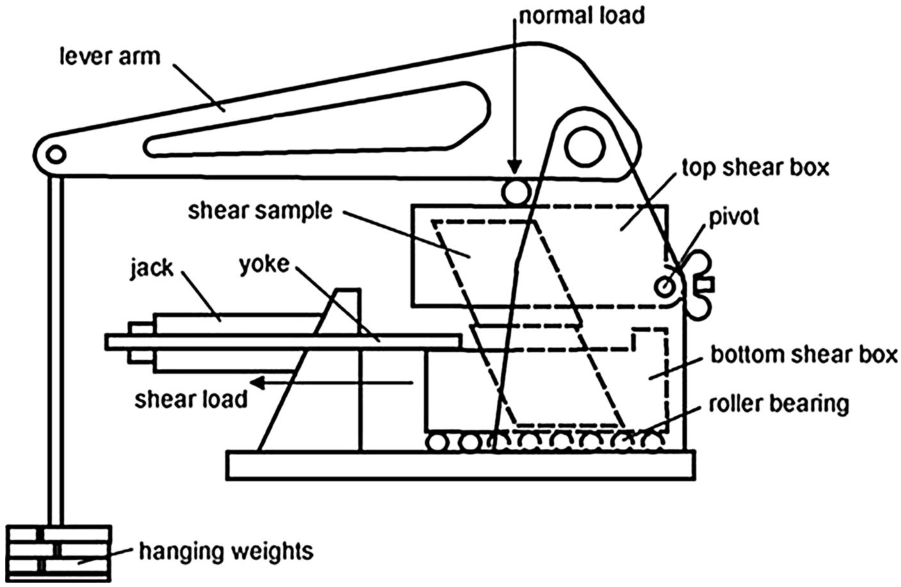

Fig. 5 Illustration of a shear box (Hoek and Marinos 2007)

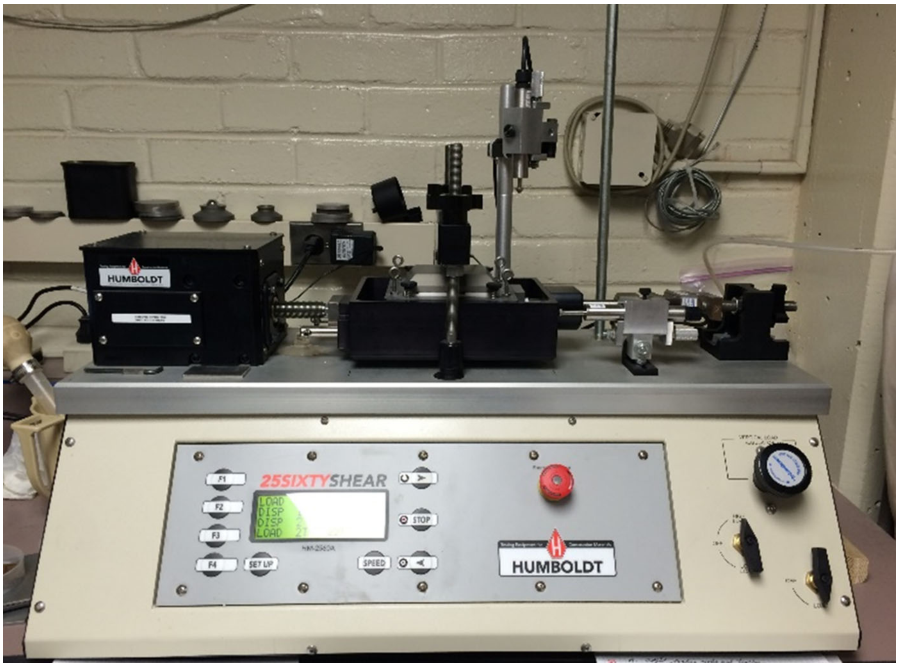

Fig. 6 The Humboldt 25SIXTYSHEAR shear box used for testing

simply estimated as the slope of the linear portion of the curve for each normal stress value. Plotting a curve of shear stiffness versus normal stress (Fig. 8) and forcing the linear curve through the origin gives a coefficient henceforth designated as F. Thus,

$$K_s = F\sigma \quad (6)$$

A curve of normal stress versus shear strength (Fig. 9) was used to estimate the basic friction angle (ϕ) for each sample using the classical Mohr–

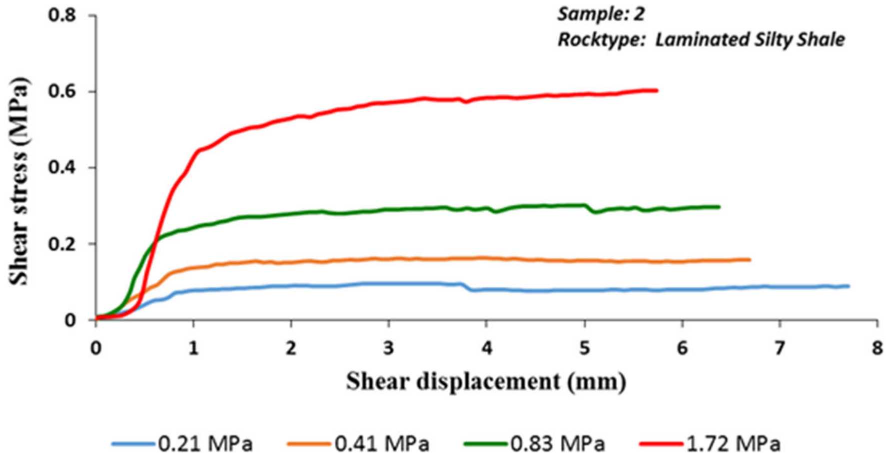

Fig. 7 Shear stress versus shear displacement for sample LSS2 (sample 2 in Table 2) at different normal stress levels

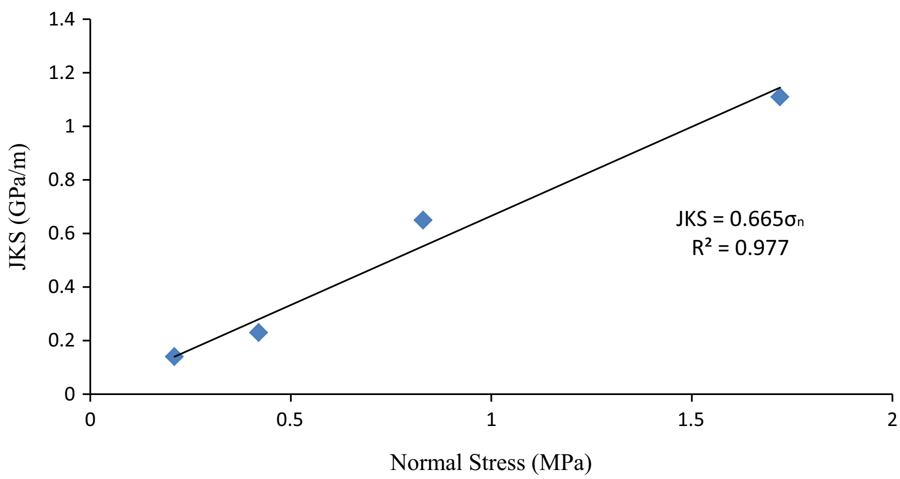

Fig. 8 Joint shear stiffness expressed as a linear function of applied normal stress for sample LSS2 (sample 2 in Table 2)

Coulomb strength criterion given in Eq. (7). Tests on natural joints can also be used to estimate peak friction angle, residual friction angle and joint cohesion (usually assumed to be zero since joint cohesion values are significantly lower than intact rock cohesion values).

$$\tau = \sigma \tan\phi. \qquad (7)$$

4 Discussion of Results

The results are obtained from samples consisting of four broad groups based on the geographical location from where they were collected. Coal measure rock samples (sandstones and shales) obtained from the United States of America, coal samples from China, limestones and mudstones (Devonian Rodeo Creek and Devonian Popovich in Tables 1 and 2) associated with a gold mine in USA, and dacite, conglomerates, mudstones and limestones from another gold mine in USA have been analyzed in the study. All groups of samples come from different depths, geographical locations and periods of formation.

4.1 Results Pertinent to Joint Normal Stiffness

Table 1 shows the results and statistical analysis for the Joint Normal Stiffness tests performed on different rock specimens as per the methodology detailed in Sect. 3.1. Tests have been performed on coal measure rocks (sample numbers 1–14) and rocks associated with gold (samples 15–36). Experiments where two types of samples have been tested (different top and

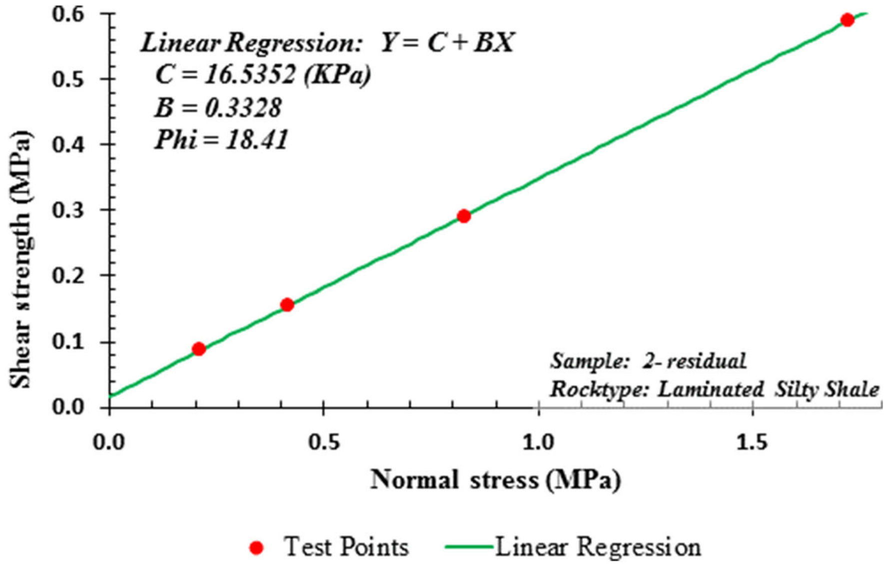

Fig. 9 Shear strength versus normal stress and calculated friction angle for LSS2 (sample 2 in Table 2)

bottom pieces) to determine interface properties are indicated as LSS/SST, SST/SS etc. This means that the top piece was the first sample (LSS, SST etc.) and the bottom piece was the second sample (SST, SS etc.). All tests associated with each rock type have been grouped together in obtaining values for statistical parameters. It can be seen from the low coefficient of variation under 20 % that this method has worked well for all the rock types except sandstone. Even for sandstone a medium level coefficient of variation value has resulted. Overall, the results show low coefficients of variation, indicating reliability in the test results. The constant B introduced in Eq. (5) (columns 4 and 5 in Table 1) is seen to consistently take values in the narrow range of 10–20 despite the samples having come from a variety of regions and origins.

4.2 Results Pertinent to Joint Shear Stiffness

The results and statistical analyses for the joint shear stiffness tests are tabulated in Table 2. Samples 1–17 are from coal and coal measure strata and the remaining samples are from gold mines. The samples tested have, for the most part, saw-cut joints, i.e., perfectly smooth joints created artificially by sawing an intact specimen in half. Some samples were available with smooth natural joints and have also been used for the study. Since the smoothness of the natural joints was comparable to the saw-cut joints, no discrimination has been made for calculating the mean, standard deviation and coefficient of variation on the basis of joint type. All samples were tested at four different normal stress values. Column 9 in Table 2 gives the value of the constant F introduced in Eq. (6) calculated from a best fit curve of K_s versus σ passing through the origin. It may be noted that different groups of samples have been tested under sets of slightly different effective normal stresses at the authors' discretion. Similar to results obtained for the joint normal stiffness tests, the shear stiffness coefficient F takes values inside a narrow range of 0.65–2.15 with dacite being the exception and having 2.68 as the mean F value. Since both normal and shear stiffness can be expressed as linear functions of the applied normal stress, a one-to-one relation can be established between the normal and shear stiffness. It can be done by simply dividing Eq. (5) by Eq. (6) (B/F ratio). This can be used to empirically estimate either parameter when the other is available. Table 3 shows the relation between the normal and shear stiffness for the different rocks tested. Due to the grouping procedure followed for the sandstone and shales for the estimation of joint normal stiffness, the parameter associated with shear stiffness, F, has also been averaged in a similar way in Table 3. It can be observed from Table 3 that the normal stiffness is consistently larger than the shear stiffness by 5–25 times. This provides a reasonable and usable guideline for estimating one stiffness parameter given the other.

Table 1 Results and statistical analysis for joint normal stiffness estimation

Sample number	Rock type	Sample name	K_n/σ (B)	Mean	Coefficient of variation
1	Sandstone	SST	13.207	21.89	0.31
2		LSS/SST	17.524		
3		SST/SS	30.386		
4		S/SST	26.461		
5	Shales	S	23.302	18.90	0.17
6		LSS	16.491		
7		SS	15.945		
8		LSS/S	18.881		
9		LSS/SS	15.742		
10		S/SS	23.023		
11	Coal	C16	20.123	18.18	0.17
12		C17	12.846		
13		C18	20.177		
14		C19	19.556		
15	Devonian Rodeo Creek	DRC1	10.272	10.02	0.09
16		DRC2	11.316		
17		DRC3	9.190		
18		DRC4	9.311		
19	Devonian Popovich	DP1	15.571	14.88	0.13
20		DP2	11.221		
21		DP3	14.278		
22		DP4	16.280		
23		DP5	14.789		
24		DP6	17.117		
25	Rodeo Creek/Popovich Interface	DRC/DP1	9.460	11.56	0.19
26		DRC/DP2	15.144		
27		DRC/DP3	10.339		
28		DRC/DP4	11.305		
29	Dacite	D1	18.993	20.68	0.08
30		D2	22.367		
31	Fragmental	F1	8.786	10.18	0.14
32		F2	11.569		
33	Mudstone	M1	17.582	15.14	0.16
34		M2	12.706		
35	Limestone	L1	15.474	14.93	0.04
36		L2	14.394		

SST sandstone, *LSS* laminated silty shale, *SS* sandy shale, *S* shale, *C* coal, *DRC* devonian rodeo creek, *DP* devonian popovich, *D* dacite, *F* fragmental, *M* mudstone, *L* limestone

It may be noted here that previous studies (Wang et al. 2012) have shown that rock masses are usually not significantly sensitive to changes in the stiffness values unless the changes are of the order of at least $10^{\pm 2}$ times the original value. For example, consider a hypothetical situation where two identical rock samples, each with a single joint with different normal and shear stiffnesses, are subjected to the same stress state. The samples would exhibit similar deformation characteristics if the difference between their

Table 2 Results and statistical analysis for joint shear stiffness estimation

Sample number	Rock type	Sample name	Joint type	K_s (GPa/m) at different normal stresses (σ expressed in MPa)				K_s/σ (F)	Mean	Coefficient of variation
				σ = 1.72	σ = 0.83	σ = 0.42	σ = 0.21			
1	Lam. Silty Shale	LSS1	N	1.07	0.60	0.21	0.10	0.58	0.68	0.20
2		LSS2	N	1.11	0.65	0.23	0.14	0.67		
3		LSS3	N	0.95	0.47	0.38	0.05	0.57		
4		LSS4	N	1.44	0.81	0.47	0.14	0.90		
5	Sand-stone	SST1	SC	2.91	1.37	1.1	0.36	1.92	1.32	0.26
6		SST2	SC	1.97	0.98	0.46	0.19	1.08		
7		SST3	SC	2.23	1.16	0.68	0.06	1.15		
8		SST4	N	1.52	0.71	0.46	0.36	1.14		
9	Sandy Shale	SS1	SC	1.79	1.10	0.52	0.29	1.25	1.23	0.29
10		SS2	SC	1.84	1.07	0.85	0.43	1.61		
11		SS3	SC	1.15	0.52	0.40	0.08	0.66		
12		SS4	SC	1.68	1.26	0.64	0.35	1.42		
13	Shale	S1	N	1.62	0.83	0.53	0.11	0.93	0.92	0.11
14		S2	SC	1.65	1.25	0.38	0.17	1.04		
15		S3	SC	1.10	0.59	0.43	0.17	0.80		
				σ = 1.00	σ = 0.75	σ = 0.50	σ = 0.25			
16	Coal	C26	N	0.553	0.473	0.348	0.246	0.61	0.70	0.13
17		C27	N	0.807	–	0.394	0.130	0.79		
				σ = 1.38	σ = 0.83	σ = 0.42	σ = 0.21			
18	Dev. Rodeo Creek	DRC1	N	1.02	0.66	0.40	0.18	0.77	1.08	0.28
19		DRC2	N	1.87	0.99	0.28	0.11	1.26		
20		DRC3	N	0.89	0.54	0.40	0.07	0.66		
21		DRC4	N	2.13	1.77	0.69	0.47	1.71		
22		DRC5	N	1.87	0.98	0.50	0.19	1.30		
23		DRC6	N	1.40	0.68	0.18	0.11	0.93		
24		DRC7	SC	1.28	0.86	0.25	0.09	0.93		
25		DRC8	SC	1.52	0.48	0.27	0.06	0.94		
26		DRC9	SC	1.31	0.86	0.36	0.08	0.96		
27		DRC10	SC	1.88	1.22	0.25	0.18	1.34		
28		DRC11	SC	2.16	1.46	0.64	0.11	1.60		
29		DRC12	SC	1.08	0.45	0.20	0.16	0.71		
30		DRC13	SC	–	1.01	0.42	0.11	1.15		
31		DRC14	SC	–	0.73	0.47	0.23	0.94		
32	Dev. Pop.	DP1	SC	1.38	1.28	0.53	0.16	1.15	1.05	0.30
33		DP2	SC	1.79	1.05	0.43	0.27	1.27		
34		DP4	SC	1.34	0.67	0.60	0.36	0.97		
35		DP5	SC	1.35	1.08	0.41	0.14	1.05		
36		DP6	SC	0.88	0.54	0.35	0.13	0.65		
37		DP8	SC	2.15	1.47	0.61	0.19	1.60		
38		DP9	SC	1.70	1.44	0.59	0.34	1.38		
39		DP10	SC	1.08	0.80	0.51	0.09	0.83		

Table 2 continued

				σ = 1.38	σ = 0.83	σ = 0.42	σ = 0.21			
40		DP11	SC	1.68	1.21	0.73	0.18	1.31		
41		DP12	SC	0.73	0.40	0.23	0.11	0.52		
42		DP13	SC	1.07	0.92	0.27	0.12	0.85		
43	Dev. Rodeo Creek/Pop. Int.	DR/DP1	SC	1.54	0.99	0.40	0.19	0.89	1.03	0.19
44		DR/DP2	SC	0.99	0.74	0.18	0.08	1.32		
45		DR/DP3	SC	–	1.01	0.42	0.11	0.85		
46		DR/DP4	SC	1.54	0.99	0.40	0.19	0.89		
47		DR/DP5	SC	1.10	0.72	0.39	0.14	1.21		
				σ = 1.38	σ = 0.69	σ = 0.34	σ = 0.17			
48	Dacite	DS1	SC	2.57	1.68	1.10	0.45	2.54	2.68	0.05
49		DS2	SC	2.42	1.74	1.18	0.60	2.82		
50	Frag.	FS3	SC	–	1.47	0.99	0.43	1.89	2.06	0.08
51		FS2	SC	2.17	1.34	0.83	0.50	2.22		
52	Muds.	MS1	SC	1.93	1.36	0.89	0.61	2.39	2.01	0.19
53		MS2	SC	1.83	1.06	0.49	0.37	1.62		
54	Lime.	LS1	SC	2.34	1.18	0.56	0.32	1.73	2.14	0.19
55		LS2	SC	2.29	1.53	0.96	0.59	2.54		
56	Lim./Dacite	L/DS1	SC	1.99	1.54	0.66	0.37	1.95	1.66	0.18
57		L/DS2	SC	2.18	1.03	0.43	0.19	1.36		

SST sandstone, *LSS* laminated silty shale, *SS* sandy shale, *S* shale, *C* coal, *DRC* devonian rodeo creek, *DP* devonian popovich, *DS* dacite shear, *FS* fragmental shear, *MS* mudstone shear, *LS* limestone shear, *N* natural joint, *SC* saw-cut joint

Table 3 Relation between joint normal and shear stiffness

Rock/formation type	Parameter B	Parameter F	B/F
Sandstone	21.89	1.32	16.6
Shale	18.90	0.96	19.7
Coal	18.18	0.70	26.0
Devonian Rodeo Creek (DRC)	10.02	1.08	9.3
Devonian Popovich (DP)	14.88	1.05	14.2
DRC/DP	11.56	1.03	11.2
Dacite	20.68	2.68	7.7
Fragmental	10.18	2.06	4.9
Mudstone	15.14	2.01	7.5
Limestone	14.93	2.14	7.0

corresponding stiffnesses is less than approximately 100 Pa/m. Hence, the inherent uncertainty present in the calculation of joint normal and shear stiffnesses in the experimental results presented in this manuscript may be deemed acceptable.

4.3 Joint Friction Angle

Table 4 shows the calculated basic friction angles and the associated statistical summary. Since saw-cut joints are perfectly smooth, no cohesion has been assumed for them. Similarly, due to the extremely low values of cohesion for natural joints-of the order of ‘kPa’ as compared to ‘MPa’ for intact rock, cohesion has not been presented in the table. The range of the basic friction angles for the thirteen rock types reported is between 17° and 31°, with friction coefficients varying between 0.313 and 0.597. Most rocks have acceptable statistical uncertainties under 25 % in the estimated friction angles. Only the

Table 4 Friction angle estimation from small scale direct shear experiments

Rock/formation type	Number of samples	Mean basic/residual friction angle (ϕ, °)	Coefficient of variation	Coefficient of friction (tan ϕ)
Shale	3	18.02	0.11	0.325
Sandstone	4	20.14	0.04	0.367
Sandy Shale	4	17.40	0.09	0.313
Laminated silty shale	4	17.53	0.06	0.316
Coal	4	30.84	0.06	0.597
Devonian Rodeo Creek (DRC)	17	27.65	0.14	0.524
Devonian Popovich (DP)	13	28.52	0.20	0.543
DRC/DP	5	26.48	0.21	0.498
Dacite	2	28.55	0.10	0.544
Fragmental	2	25.75	0.13	0.482
Mudstone	2	27.35	0.01	0.517
Limestone	2	22.25	0.23	0.409
Limestone/Dacite	2	22.10	0.35	0.406

limestone-dacite interface test sample appears to have a high coefficient of variation, exceeding 25 % (35 %). Hence, the tests provide a useful addition to literature in the form of residual friction angles for joints in coal measure rocks including shales, sandstones, coal, and some medium hard rocks including limestone, mudstone, tuff and dacite.

5 Conclusions

This study presents simple methods to estimate rock joint normal and shear stiffnesses in the laboratory. Joint normal stiffness has been estimated using a modified uniaxial compression test which requires no cyclic loading. Joint shear stiffness has been estimated through multi-stage small scale direct shear tests. The tests have been performed on rocks obtained from around the world, varying in origin, depth and associated mineral. Despite this, the variability in results falls within a narrow range due to which it can be used as a foundation for estimating joint stiffness parameters when none are available. It has been observed that the joint normal and shear stiffnesses are linear functions of the normal stress applied on the rock sample. The normal stiffness, with units GPa/m, has been found to typically be approximately 10–20 times the applied normal stress expressed in MPa. Similarly, the shear stiffness, expressed in GPa/m, is between 0.65 and 2.15 times normal stress expressed in MPa. This observation serves as a useful guideline when building numerical models which use stiffnesses to describe joints. This is because it allows for using dynamically changing values of stiffnesses rather than a single static value which becomes an underestimation at high normal stress values. Also, it allows one to select physically suitable values rather than arbitrary values when building models where laboratory testing is not possible. Finally, the stiffness and friction angle values serve as a useful addition to the existing literature on mechanical property values for joints and fractures in different rocks.

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