

# RISK ANALYSIS FOR ROCK SLOPES IN OPEN PIT MINES

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PART VI

## EXECUTIVE SUMMARY INTRODUCTION AND TECHNICAL SUMMARY BIBLIOGRAPHY

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## FOREWORD

This report was prepared by the Massachusetts Institute of Technology, Department of Civil Engineering, under USBM Contract number J0275051. The contract was initiated under the Advancing Metal and Nonmetal Mining Technology Program. It was administered under the technical direction of the Denver Research Center with Louis A. Panek acting as Technical Project Officer. Larry Rock was the contract administrator for the Bureau of Mines. This report is a summary of the work recently completed as a part of this contract during the period March 1, 1977 to October 31, 1979. This report was submitted by the authors on November 30, 1979.

The research leading to this report was only possible with the progressive and encouraging attitude of the responsible persons at the Bureau of Mines. J.W. Corwine and D.D. Bolstad were instrumental in preparing for and initiating the research. The Technical Monitor, L. Panek, contributed significantly, not only by providing supervision but by many valuable comments and questions. The mining and engineering industry provided support whose value cannot be overemphasized, mostly in the form of data on rock mass parameters but also through critical comments and through information on their design procedures. For reasons of not identifying data with particular projects we can only mention two of the many firms: Pincock, Allen and Holt (Tucson, Arizona) and Duvall Mines. Finally, we would like to point out that some of the students working on the research were only partially supported by research funds, the remainder came from Domestic Mining Mineral and Mineral Fuel Conservation Fellowships, U.S. Department of Education and from the Undergraduate Research Opportunities Program at M.I.T. The help and support of all the individuals, government agencies, and private firms is gratefully acknowledged.

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Executive Summary  
Introduction and Technical Summary  
Bibliography

TABLE OF CONTENTS

	<u>page</u>
EXECUTIVE SUMMARY	2
1. Goal of Research	2
2. Research Approach	3
3. Structure of Report	10
4. Conclusion	11
INTRODUCTION AND TECHNICAL SUMMARY	12
1. Goal of Research	12
2. Work Performed	13
3. Research Approach and Report Structure	14
4. Technical Summary of Research Results	15
5. Conclusions and Major Areas of Future Work	41
BIBLIOGRAPHY	43

## EXECUTIVE SUMMARY

1. Goal of Research

The goal of the research was to create a basis for risk analysis in pit mine slope design, and to develop practical procedures for the application of risk analysis. By designing rock slopes in open pit mines on the basis of risk, the natural uncertainties affecting pit slope geometries can be rationally treated. In this way decisions on mine planning and design can be made more internally consistent, and can be more straightforwardly related to economical objective functions and uncertainties.

To make applied risk analysis possible and practical, this research was organized in three tasks:

- 1) to empirically determine appropriate statistical descriptions of geologic variables;
- 2) to develop probabilistic techniques of rock slope stability analysis; and
- 3) to combine the statistical descriptions of geologic variables with probabilistic slope stability analysis in procedures of reliability analysis and in risk screening procedures for rock slopes in open pit mines.

The research was aimed at producing practically useable results, while providing a rigorous foundation for further refinements. Related developments, beyond the specific objectives of the proposal, have also been made, mainly in the area of rock mass deformability.

## 2. Research Approach

The research is divided into five parts that correspond to the above-mentioned objectives and additional work--I. Geologic Distributions, II. Limit Equilibrium Analyses for Rock Wedge Stability, III. Reliability Analyses of Rock Slope Stability, IV. Field Exploration, V. Probabilistic Models of Jointed Rock Mass Deformation and their Implications. Specifically, the following work was conducted in each part:

### Part I: Distribution of Rock Mass Properties

Clearly, discontinuities (joints, foliation surfaces, bedding planes, faults) in a rock mass govern to a large extent its engineering properties and thus the performance of any slope in the mass. Discontinuities are described by their geometric characteristics (i.e., attitude (orientation), spacing and "length" or persistence), as well as aperture and surface geometry, and by resistance characteristics (frequently in form of the standard geotechnical parameters friction and cohesion). The thrust of Part I was to collect in-situ data from exploration programs and the published literature, and to empirically establish distributional patterns for these variables. For this purpose geometric data representing about 20,000 joints and resistance data from about 2,000 tests were used. Empirical analyses are compared with theoretical considerations on the processes governing geological formations to ensure that distributional forms considered are consistent with present theoretical understanding.

Appropriate distributional forms for spacing and length of discontinuities were found. As far as attitude (orientation) is concerned none

of the existing nor a newly developed form are satisfactory under all circumstances, although the distributional form developed in this research satisfies more cases than all the others. Another major outgrowth of the work in Part I was the development of sampling procedures for discontinuity geometry. Although not actually part of the proposed research, it proved to be essential to enable the user to obtain distributions of joint geometry parameters by preparing and pursuing appropriate sampling plans. Particularly important in this respect is the consideration of possible biases, that are due to the fact that some joints are more readily sampled than others. In addition to the geometric characteristics of joints a major effort was devoted to resistance and deformability characteristics. The most striking observation in reviewing more than 2,000 test results is their great variability both regarding resistance (primarily expressed by friction angle  $\Phi$ ) and stiffness. Good statistical estimates of joint resistance can be obtained but only if the number of influencing factors is restricted. In addition some general trends, like the usually resistance reducing effect of moisture could be confirmed.

In addition to the main task in Part I, several related tasks were pursued. The most important of these was an investigation of the possibility of spatial correlation among parameters governing slope stability. Such correlation, were it to exist, would be critical for performance predictions. One data set showed evidence of such spatial correlation while the other clearly did not. Further work in this area is necessary.

The work on geologic distributions has thus resulted in characteristic distributional forms of joint properties, where they exist, or

representative ranges of such properties. Equally important are the sampling procedures by which these distributional forms can be obtained in practice. It is now possible to devise for a particular site a sampling plan, then collect data accordingly and obtain the specific distributional forms of geologic parameters. The latter will then be used with analysis procedures developed in this research to determine the reliability of rock slopes at this site.

## Part II. Limit Equilibrium Analysis for Rock Wedge Stability

A consequence of the discontinuous character of rock masses is that instability of rock slopes often occurs in the form of excessive movement of bodies that are bounded by discontinuities or by a combination of discontinuities and intact rock. It seems thus logical to apply the well known limit equilibrium analyses for wedges and blocks. These approaches suffer, however, from procedural simplifications like simplified shapes and limited consideration of external forces; more important, however, is the basic deficiency of limit equilibrium analysis--the rigid body assumption and the associated indeterminacy requiring force distribution assumptions. These assumptions often lead to substantial inaccuracies and to potentially unsafe conclusions.

Since the basic correctness of wedge stability analysis is essential, it was necessary to first improve these methods in a purely deterministic sense before using them in reliability analysis. This led to a stability analysis method for 2- and 3-plane wedges with a wide range of additional features regarding failure modes and external forces. The above-mentioned rigid body assumption excludes by definition the consideration of deformations

and thus stresses, which in turn leads to the often inaccurate force distribution assumptions. This deficiency was corrected by an approach that includes the effects of the in-situ stress field and of the stiffness of discontinuities. The significant differences and particularly the often lower factors of safety obtained with this approach as compared with the standard one, attest to the importance of this development. The final step in making limit equilibrium approaches more valid was achieved with the method of artificial supports which makes it possible to identify amongst all possible failure modes the critical one and to determine the corresponding factor of safety.

The development of more correct and generally valid wedge stability analyses included the creation of appropriate computer codes. In addition a simplified analysis for 2-plane wedges to be applied with a programmable pocket calculator was also produced. Although originally intended to be only a step toward reliability analysis, the work on limit equilibrium analysis turned out to be much more important. In addition to being essential for the reliability analysis it stands by itself and provides several major contributions to rock slope stability analysis.

### Part III. Reliability Analyses of Rock Slope Stability

The innate variability of natural materials and processes causes inaccuracy or uncertainty in the models representing the natural phenomena and uncertainty in estimating the parameters that are used in these models. This inevitable uncertainty has been recognized and considered traditionally by assigning factors of safety. Such an approach is unsatisfactory because the assumed mechanistic model is only correct for

a factor of safety of "1" and particularly because the factor of safety does not express uncertainty directly. A change in factor of safety, caused for instance by changes in slope geometry, can thus not be related to an increase or decrease in risk. It is also not possible, based on factors of safety, to decide if and what amount of additional exploration would be beneficial, i.e., would additional exploration lead to savings. Reliability analyses, in contrast, are well suited to a rational incorporation of uncertainty in design and exploration decisions. Such approaches should not only express the parameters probabilistically but they have to consider the stochastic character of the mechanism, i.e., their spatial variability.

It was thus necessary to first develop analysis procedures that can satisfactorily describe the stochastic features of slope failure mechanisms. This problem is very complex, but methods were developed that consider the spatial variability of the major contributors to slope instability--mechanisms involving the discontinuities. In addition to considering spatial variability, these analysis methods make use of the basic improvements in representing mechanics of slope failure that were developed in Part II. For reasons of clarity, kinetic and kinematic instability were treated separately but leading to approaches that are entirely compatible.

Building upon the probabilistic kinetic and kinematic analyses it was possible to develop reliability analysis methods. With one method the reliability, i.e., the probability of failure, of an individual 2-plane wedge in a slope can be determined. The other method is aimed at slopes with a single slope parallel set of joints. In each case it was

possible to develop an encompassing approach relying on computer codes and a simplified or summary approach relying on charts or pocket calculator computation. Although these reliability analyses represent only a first step, they invoke two of the most frequent conditions--2-plane wedges and single slope parallel joint sets. Specific solutions, procedures and programs for these rock slope problems, and the possibility to determine risk and further exploration are thus provided. It is important to note at this point that these methods of reliability analysis use as input parameters the geologic distributions identified, and determined with sampling procedures, developed in Part I.

#### Part IV. Field Exploration to Determine Rock Mass Properties

The change in probability of failure with increasing information was used to analyze the optimal strategy of allocating exploration effort for a given slope. Total optimization of exploration is a major research effort in its own right; the present work holds no claim to exhaustiveness. However, by concentrating on the one-slope problem, insight is gained into the overall problem.

The technique used in optimizing exploration is Bayesian decision theory in which statistical description of important geological parameters comes from the work under Part I, inference of the value of decreased uncertainty in design comes from the work under Part III, and decision theory is used to link these together.

In conjunction with the analysis of optimal exploration strategies for single slopes, a graphical risk screening procedure was developed to determine those slope sections within a pit for which increased exploration would beneficially reduce risk. This allows routine use of decision

theory methods in exploration.

From these descriptions it is clear that Parts I, II, and III of the research serve as a foundation for Part IV. For this reason, and as originally proposed, the major part of the work has focused on statistical description and reliability models.

Part V. Probabilistic Models of Jointed Rock Mass Deformation and Their  
Implication

The extensive treatment of geological distributions and probabilistic methods to analyze rock slope engineering problems revealed a number of interesting problems that are not directly in the line of the proposed research, whose solution would however be beneficial. In this sense substantial work was undertaken on obtaining foundation settlement predictions based on joint spacing and orientation distributions rather than using RQD-related correction factors. With the probabilistic deformation model it is possible to consider variability of joint spacing and attitude (further development will permit one to also consider the variability of joint stiffnesses and complicating effects due to interaction of several joint sets). It could be shown that the common empirical RQD-correction factor relations correspond to the model results as long as the joints are parallel. This correspondence breaks down when the variability in jointing geometry (notably orientation) is increased. The applicability of empirical relations becomes thus questionable if they are extended beyond the range of their base cases. On the other hand, the strength of the probabilistic model for rock mass deformation is that measurable geologic attributes of the rock mass can be used to analytically predict deformations. This is an improvement

over most current models requiring empirical or hypothetical parameters and over large in-situ testing which can be prohibitively expensive.

### 3. Structure of Report

The report is structured corresponding to the preceding description of the research. The main text consists of an 'Introduction and Technical Summary', and Parts I through V as discussed above, each Part containing a varying number of chapters. The Technical Summary provides a detailed chapter by chapter summary of developments and practical consequences of the material treated in Parts I through V. Each of the five parts commences with an introduction outlining the purpose of the research, the approach and detailed structure of the particular part.

Most important is the major principle that underlies the report structure: The individual parts and the chapters within them are building blocks that provide either the basis for the development in another block or that will be integrated in the reliability analysis and exploration approaches. Although there is thus a strong and systematic relation between chapters and parts, the report is written such that each chapter can be used individually. Most chapters provide a method or report results that can be practically applied by themselves. Each chapter thus starts with a section "summary and practical application (consequences)" to give potential users an idea on content and capabilities of the methods.

Supporting documentation is included in the appendices. Appendices A through M contain detailed data and analytical developments or background information. The necessary computer documentation has been included in the separate Appendix UM - Users' Manual.

#### 4. Conclusion

Encompassing approaches to consider the stochastic character of rock masses in sampling and representation of parameters and in stability analysis have been created. The approaches involve substantial theoretical development leading to practical procedures while simultaneously forming the basis for further development. A significant step toward reliability analysis for rock slopes has been made and some practically useable approaches are now available.

## INTRODUCTION AND TECHNICAL SUMMARY

1. Goal of Research

The goal of this research is to create a basis for risk analysis in pit mine slope design, and to develop practical procedures for the application of risk analysis. By designing rock slopes in open pit mines on the basis of risk, the natural uncertainties affecting pit slope geometries can be rationally treated. In this way decisions on mine planning and design can be made more internally consistent, and can be more straightforwardly related to economical objective functions and uncertainties.

To make applied risk analysis possible and practical, this research is organized in three tasks: to empirically determine appropriate statistical descriptions of geologic variables, to develop probabilistic techniques of rock slope stability analysis, and to combine probabilistic slope stability analysis and proper statistical descriptions in applicable models. The specific objectives for this research, as indicated in the proposal, are to:

1. Determine appropriate distributional forms for geological parameters important in slope stability analysis, and to apply Bayesian techniques to formally incorporate statistical sampling errors in those distributions.
2. Improve limit equilibrium analyses of rock slope stability and convert them to handle uncertain geological variables, thus yielding probability distributions over factor-of-safety from which probabilities-of-failure can be derived.
3. Evaluate changes in probabilities-of-failure that could result from increased exploration; and develop a simple screening procedure that can be used to determine those slopes with high instability and severe consequences of failure, that would

economically benefit from increased exploration and analysis.

The research is aimed at producing practically useable results, while providing a rigorous foundation for further refinements. Related developments, beyond the specific objectives of the proposal, have also been made.

## 2. Work Performed

This section is intended to provide the reader with an overview of the work performed. A more detailed review is given in Section 4 "Technical Summary".

The goals stated above and the research that will be described in this report can be viewed as an attempt to incorporate natural uncertainty in pit slope design. This has been achieved to a substantial extent. In addition, various problems that were not or only partially known prior to conducting this research but whose solution was essential were identified and treated. A solid and broad basis for risk analysis of rock slopes in open pit mines has thus been created, which made it possible to develop exploration strategy and risk screening procedures. In addition to accomplishing the research objectives various related subjects in the area of rock mass behavior were considered and advanced, notably an analysis method for jointed rock mass deformation.

Since risk analysis requires information on the stochastic character of rock mass parameters and appropriately structured methods of reliability analysis, it was in these two areas where the major research effort was concentrated. Geometric and strength parameters of rock masses in particular of rock joints (discontinuities) have to be represented in form of distributions, and sampling procedures to infer these distributions had to be developed. Such distributional forms were established for joint spacing (Exponential) and joint trace length (logNormal). Joint attitude distribution cannot be represented by a particular form, while joint-resistance and deformability parameters are often normally distributed indicating measurement effects rather than an inherent distribution.

Reliability analysis, in addition to requiring probabilistic description of geometric and strength parameters, has to be based on mechanically correct models of rock slope failure. This was achieved through approaches that include stress field and stiffness effects as well as through the entirely new method of artificial supports.

The just mentioned analysis methods are, however, only a step toward reliability analysis, particularly since they are deterministically oriented. They can be used in simulation procedures together with the probabilistically described rock mass parameters to obtain for instance distributions of safety factors. A more satisfactory approach, that is quite unique, is to consider the stochastic character of the failure mechanism, i.e. the fact that the mechanism changes with changing rock mass characteristics. This has been achieved with methods employing a combination of simulation and dynamic programming procedures together with appropriate mechanistic models. A particularly remarkable consequence of this approach is the fact that one of the major rock engineering problems--the persistence problem--can now be rationally treated.

The information on and representation of the stochastic character of rock masses and the reliability analyses form the basis for the final step of this research, the development of exploration strategies and risk screening procedures. It is now possible to decide if and what further exploration for specific slope sections is required.

### 3. Research Approach and Report Structure

The research and this report are divided into five parts that correspond to the objectives mentioned in Section 1 and the additional work:

- I. Geological Distributions.
- II. Limit Equilibrium Analyses for Rock Wedge Stability.
- III. Reliability Analysis of Rock Slope Stability.
- IV. Field Exploration to Determine Rock Mass Properties.
- V. Probabilistic Models of Jointed Rock Mass Deformation and their Implications.

The report consists of an introduction and the five parts, each containing a varying number of chapters, and commencing with an introduction that outlines the purpose of the research, the approach and detailed structure of the particular part.

Most important is the major principle that underlies the report structure: The individual parts and the chapters within them are building blocks that provide either the basis for the development in another block or that will be integrated in the reliability analysis and exploration approaches. Although there is thus a strong and systematic relation between chapters and parts, the report is written such that each chapter can be used individually. Most chapters provide a method or report results that can be practically applied by themselves. Each chapter starts thus with a section "summary and practical application (consequences)" to give potential users an idea on content and capabilities of the methods; basically the same information appears in the Technical Summary, Section 4 below.

Supporting documentation is included in the appendices. Appendices A through M contain detailed data and analytical developments or background information. Due to the large amount of computer documentation, this has been included in the separate Appendix UM - Users' Manual.

#### 4. Technical Summary of Research Results

The work performed and results obtained will be summarized by first discussing in general the research conducted within the particular Part and then describing (where applicable) the results of individual chapters within the Part.

#### 4.1 Part I: Geological Distributions

The thrust of Part I was to collect in-situ data from exploration programs and the published literature, and to empirically establish distributional patterns for these variables. Empirical analyses are compared with theoretical considerations on the processes governing geological formations to ensure that distributional forms considered are consistent with present theoretical understanding.

Clearly, discontinuities in a rock mass govern to a large extent its engineering properties and thus the performance of any slope in the mass. Discontinuities are described by their geometric characteristics (i.e., attitude (orientation), spacing and "length" or persistence), as well as aperture and surface geometry, and by resistance characteristics (frequently in form of the standard geotechnical parameters friction angle and cohesion). Work has been devoted to determining the appropriate distributional forms for attitude, spacing and length of discontinuities and to distributional properties of resistance and deformability parameters.

Bayesian statistical procedures were used to infer the appropriateness of parameter distributions and to incorporate parameter uncertainty for use in slope stability analysis (i.e., "predictive" distributions). Attempts were made to develop composite distributions formed from among the best fitting forms to obtain a more complete picture of aggregate uncertainties.

In addition to the main task in Part I, several related tasks were pursued. The most important of these was an investigation of the possibility of spatial correlation among parameters governing slope stability. Such correlation, were it to exist, would be critical for performance predictions.

In Chapter 2 (recall that Chapter 1 of each Part is an Introduction) joint trace length data from six (6) sites were analyzed to infer appropriate distributional forms. The total number of data analyzed was about ten thousand. These data were primarily sampled from large area surveys, and corrected for bias where appropriate. The conclusion from these analyses is that trace lengths, in general, are logNormally distributed. Evidence for Exponential distribution is very weak. This conclusion seems to hold across strikingly different geological formations. However, the implication of this finding for the distribution of three-dimensional joint size is confounded by biases in the way joints appear in outcrop; the actual size distribution can thus be Exponential or logNormal.

The same six sites provided data on joint spacing (intensity) that were analyzed in Chapter 3 to identify appropriate indices of intensity and their respective distributions. Direct observations and simulations of spacing were analyzed. These data strongly support the hypothesis that joint spacing is Exponentially distributed, regardless of orientation of the sampling line, and regardless of whether each joint set is considered separately or not. With certain exceptions, this conclusion seems independent of geology. The implication of this finding for modeling jointed rock is that joints may be considered to be randomly located in space.

Joint orientation (attitude) data was examined in Chapter 4. The data, which represents 22 individual joint sets from eight sites (a total of approximately 20,000 joints), were analyzed to find

appropriate distributional forms. These data do not strongly support any of the commonly used distributions as broadly applicable in practice. The distributions include the Uniform, Fisher, Bingham, and Bivariate Normal. In addition, a Bivariate Fisher distribution was developed in the context of this research, which in some but not all cases was satisfactory. For 18 of the 22 data sets analyzed, no analytical distribution could be found that passed  $\chi^2$  goodness-of-fit tests at 5%. The implication of this finding is that great care must be taken in applying analytical distributions to orientation data, and that non-parametric (i.e., distribution free) techniques of statistical estimation need to be developed.

In Chapter 5 autocorrelation of geometric joint properties is examined. Many or perhaps most spatial variables are expected to exhibit some form of spatial correlation. That is, properties at a point are not independent of properties at a nearby point. The spatial correlation of three joint parameters--trace length, strike (azimuth) and dip angle--was investigated at Site A where a large amount of data on these properties was available. However, no significant spatial correlation could be found even after extensive data manipulation. This was in contrast to the results of the only other quantitative study and could be explained by geologic differences. Since autocorrelation may have a strong effect on probability of failure, further research is necessary.

The investigation of resistance and deformability parameters in Chapter 6 starts with a review of mechanisms and models that are used to represent them. For purposes of developing statistical estimators only relatively simple models are appropriate where a large number of tests per parameter are available. The Coulomb  $C-\phi$  model was chosen for resistance representation and the stiffness model for deformability. Review and statistical evaluation of approximately 2,000 discontinuity shear tests has shown that good statistical estimates for use in

reliability analyses can be obtained for friction angles. This is, however, only correct as long as the number influencing factors is restricted. Only if test results for a specific discontinuity at a specific site, in a limited stress-range and using the same testing procedure are considered, is it possible to produce good estimates. Relaxation of the restrictions causes large scatter and weak statistical estimates. In standard practice, stiffnesses are much less frequently measured than friction angles and fewer data are thus available. The variability of stiffnesses, however, is even greater than that of friction angles. One can similarly conclude that statistical estimates of stiffness are only satisfactory as long as the above-mentioned restrictions are adhered to. The large number of data made it possible to draw general conclusions and to study trends in correlations between friction angle or stiffness and other rock properties. The most striking observation is the great variability of test results--most peak and residual friction angles for the entire data set lie between 20 and 40°. Stiffnesses range over two to three orders of magnitude. Peak and residual friction angles of the data set collected by MIT seem to be normally distributed. However, neither individual rock type nor discontinuity geometry show unique relations with friction angle; e.g. friction angle for the same rock type may occur over the entire above-mentioned range. However, stress range and moisture conditions produce trends that are consistent in most instances--friction angle decreases with increasing normal stress, and moisture reduces the friction angle except for smooth polished surfaces. Rock type and stiffness are not uniquely related, and discontinuity geometry ("roughness") and width have a stronger effect on stiffness than rock type.

#### 4.2 Part II. Limit Equilibrium Analysis for Rock Wedge Stability

A consequence of the discontinuous character of rock masses is that instability of rock slopes often occurs in the form of excessive

movement of bodies that are bounded by discontinuities or by a combination of discontinuities and intact rock. It seems thus logical to apply the well known limit equilibrium analyses for wedges and blocks. These approaches suffer, however, from procedural simplifications like simplified shapes and limited consideration of external forces; more important, however, is the basic deficiency of limit equilibrium analysis--the rigid body assumption and the associated indeterminacy requiring force distribution assumptions. These assumptions often lead to substantial inaccuracies and to potentially unsafe conclusions.

Since the basic correctness of wedge stability analysis is essential in reliability analyses it was necessary to first improve these methods in a purely deterministic sense. This led to a stability analysis for 2- and 3-plane wedges with a wide range of additional features regarding failure modes and external forces. The above-mentioned rigid body assumption excludes by definition the consideration deformations and thus stresses, which in turn leads to the often inaccurate force distribution assumptions. This deficiency was corrected by an approach that includes the effects of the in-situ stress field and of the stiffness of discontinuities. The significant differences and particularly the often lower factors of safety obtained with this approach as compared with the standard one, attest to the importance of this development. The final step in making limit equilibrium approaches more valid was achieved with the method of artificial supports which makes it possible to identify among all possible failure modes the critical one and to determine the corresponding factor of safety.

The development of more correct and generally valid wedge stability analyses included the creation of appropriate computer codes. In addition a simplified analysis for 2-plane wedges to be applied with a programmable pocket calculator was also produced. Although originally intended to be only a step toward reliability analysis, the work on limit equilibrium analysis turned out to be much more important. In

addition to being essential for the reliability analysis it stands by itself and provides several major contributions to rock slope stability analysis.

Specifically, the computerized stability analysis for wedges formed by two or three joint planes SWARS (Sliding Wedge Analysis for Rock Slopes) was developed in Chapter 2. The designer simply has to list the attitude of up to eight (or more) joint sets in the rock mass under consideration, the attitude, length and height of the slope and the inclination of the surface above the slope. The analysis procedure combines all the joints and the slope to determine all possible geometrically and kinematically feasible 2- and 3-joint wedges. The analysis is set up such that only wedges that fulfill the geometric and kinematic conditions are created, thus greatly reducing the following computations. The only separate kinematic test is that for size, the maximum extent of the wedge on the slope face has to be within the slope dimensions or within a specified smaller size. The kinetic analysis which is performed as limit equilibrium analysis on a rigid body includes a large number of practically important and often innovative features, point loads can act in any direction and can be applied at any point, in particular they do not have to act through the center of gravity. This is much more realistic than the usual center of gravity assumption of similar analysis methods. Bolt (anchor) design can now be based on the actual physical features rather than on an idealized pressure. This is particularly advantageous together with the aforementioned possibility to limit and thus specify the size of examined wedges. Bolts (anchors) can be specified differently for different wedges both excluding over-design as well as the danger of missing a smaller wedge within a large one. Water pressure can be uniform or hydrostatic. The stability analysis for each wedge is conducted for both peak and residual joint resistance parameters and results in a peak and residual factor of safety. Very important is the capability of SWARS to analyze a variety of modes of failure:

translational sliding on one or two planes, lifting off from all planes, and (for 2-joint wedges only) toppling about the edges and the lower apex. The output can be a listing of all kinetically analyzed wedges or simply those that have a factor of safety below a specified value. In addition the geometry, resultant force vector and mode of failure are presented. Limitations of SWARS are the water pressure and seismic force assumptions that, although realistic, require some judgement by the user. Also, some modes of failure are not considered. SWARS is a very effective and efficient design tool. The designer can simply describe the characteristics of the rock mass and can then concentrate on the engineering: varying slope geometry and stabilizing measures (reduction of external loads, application of bolts) to optimize slope stability.

Chapter 3 describes a simplified analysis method for 2-joint wedges for use with a programmable pocket calculator. The routine works with three cards on HP 97/67 calculators. The user can specify the attitude of two joints, the slope geometry (attitude of face and top of slope) and wedge size and have the program calculate the wedge volume and areas of the wedge planes. The second step determines the wedge weight and water forces based on a user-specified water level. At this point the user can also specify other external driving or resisting forces. All forces are then summed by the routine and the resulting force and failure mode is pointed out. Sliding failure on each of the planes, on both planes or lifting off are considered. Finally a factor of safety for the potential failure mode is computed based on the previously derived resultant force and the joint resistance, the latter is specified in form of Coulomb parameters. The analysis is not intended to replace the more comprehensive approach of SWARS but to be used in preliminary design or in the field to check the stability of specific wedges and design stabilizing measures.

One of the major problems associated with rigid body analysis is the necessity to make assumptions with regard to direction or even magnitude of forces to make unique solutions possible. The exclusion of moments is a frequent assumption; the common simplifying assumptions and the subject of Chapter 4 is the normality of reactions on the joint planes, an assumption that is often not explicitly stated. However, assuming normal reactions only can have severe and often unsafe consequences. More realistic force assumptions can be made if one considers the stiffness of joints or the state of stress in the slope. Quite obviously these are artificial means to introduce the effect of stresses in rigid body analysis methods. They are however justified since the wedge itself is still considered to be a rigid body, but acted upon by stresses (forces) determined via joint stiffness or related to the stress state in the slope. The standard normality assumptions for joint plane reactions imply that there is no shear stiffness (the ratios of normal stiffness to shear stiffness is infinite) and that the ratio of lateral to vertical stress "K" is 1. Stated in this way one becomes aware how far from reality one may be. Initial developments by other researchers for the stiffness and stress approaches were available but limited to simple geometries and frictional resistance only. In this chapter the stiffness and the stress approach are considerably expanded to be applicable to a broader range of 2-joint wedge geometries and particularly to include all the other capabilities of wedge stability analysis. These generalized approaches are used to again demonstrate the effect of the simplifying normality assumption. Using the stiffness approach one can show that, particularly for narrow wedges or wedges with one steep joint, considerable deviations in stability predictions result--the required friction angle (an expression of safety) can be up to  $30^{\circ}$  greater than for the normality assumption. A lateral stress ratio different from  $K = 1$  (implying non-normal reactions) is particularly significant if the wedges have relatively flat lying lines of intersection.  $K > 1$  underestimates the factor of safety, while  $K < 1$  overestimates it. The stress and

stiffness approach are related, both representing the effect of a more correct force assumption, i.e., the underlying stress distribution. It can be shown that the stress approach correctly models the wedge in its natural environment. A state of stress has developed in the slope through a series of historical changes and the present stability is governed by the present state of stress. The joint stiffnesses in contrast can only be used to determine stress changes due to additional loads, stiffness cannot provide information on the present state of stress. The stiffness approach can therefore be used to make predictions on the effect of additional loads. A method of analysis and computer program SWARS-2PM for 2-joint wedges was therefore developed, incorporating the generalized stress and stiffness approaches and applying them in the correct sequence--stress approach to analyze the effect of weight, stiffness approach for the effects of other forces. Since the method and program is basically an extension of SWARS-2P, it includes all the powerful features of the latter, making it a highly effective tool for slope design.

A more basic and thus potentially more far-reaching approach to reducing the negative effects of rigid body assumptions has been taken in Chapter 5 with the Method of Artificial Supports (MAS). A satisfactory representation of the actual reactions on surfaces supporting rigid bodies is achieved through introduction of artificial supports and reactions in these supports. Specifically, after a brief review of common wedge stability approaches the method of artificial supports is introduced. The three-support model, the simplest version of the MAS, and the corresponding computer program PF3 are developed and used to analyze the sliding modes of failure. It is at this point that rotational and translational failure can be compared, translational failure being a special case of rotational failure. A number of cases with different force applications and support geometries are investigated. Rotational failure is usually the critical mode, if driving

forces other than the weight become significant--a fact that so far has not been generally recognized. To examine the correctness of the three-support model an n-support model and associated analysis was developed. Specifically a 10-support model is applied to the previously considered 3-support cases, i.e., to cases with the same applied forms. It can be concluded that the 3-support model provides results of satisfactory accuracy if a sufficient number (between 5 and 10) of 3-support geometries are used to determine the critical coefficient of friction and the mean taken. The MAS and its applications are then extended to include the remaining modes of failure, which are sliding rotation about a fixed axis, toppling and translational sliding along a line of intersection. For the fixed point rotation an analysis method and computer program (ROTFA) were developed, while the toppling and intersection sliding analyses are extensions of previously known methods to include the MAS concept. This is followed by the description and example application of the algorithm for the complete wedge failure analysis. Equally important as the practical user-oriented analysis methods and programs are the entire concept and methodology. It seems that a new and promising way has been opened to a more satisfactory consideration of rigid body reactions and thus a more satisfactory limit equilibrium analysis.

#### 4.3 Part III. Reliability Analyses of Rock Slope Stability

The innate variability of natural materials and processes causes inaccuracy or uncertainty in the models representing the natural phenomena and uncertainty in estimating the parameters that are used in these models. This inevitable uncertainty has been recognized and considered traditionally by assigning factors of safety. Such an approach is unsatisfactory because the assumed mechanistic model is only correct for a factor of safety of "1" and particularly because the factor of safety does not express uncertainty directly. A change in factor of safety, caused for instance by changes in slope

geometry, can thus neither be related to an increase or decrease in risk nor is it possible to decide if and what amount of additional exploration would be beneficial. Reliability analyses, in contrast, are well suited to a rational incorporation of uncertainty in design and exploration decisions. Such approaches should not only express the parameters probabilistically, but they have to consider the stochastic character of the mechanism, i.e. their spatial variability.

It was thus necessary to first develop analysis procedures that can satisfactorily describe the stochastic features of slope failure mechanisms. This problem is very complex, but methods were developed that consider the spatial variability of the major contributors to slope instability--mechanisms involving the discontinuities. For reasons of clarity, kinetic and kinematic instability were considered separately but leading to approaches that are entirely compatible.

Building upon the probabilistic kinetic and kinematic analyses it was possible to develop reliability analysis methods. With one method the reliability, i.e. the probability of failure, of an individual 2-plane wedge in a slope can be determined. The other method is aimed at slopes with a single slope parallel set of joints. In each case it was possible to develop an encompassing approach relying on computer codes and a simplified or summary approach relying on charts or pocket calculator computation. In addition to providing specific solutions and procedures for these problems, and thus the possibility to determine risk and further exploration, a major contribution of these approaches is the treatment of joint persistence. Instead of having to assume a certain persistence it is now possible to use exploration data on joint geometry and to include the persistence effect rigorously.

The stochastic character of geometric and mechanical properties of rock masses can be considered in two basically different ways in

slope stability analysis:

1. A deterministic mechanism and analysis method is established. Distributions of geometric and mechanical properties are specified. Through simulation (Monte Carlo simulation) or closed form procedures (second moment analysis e.g.) distributions of resisting and driving forces and thus distributions of safety factors or probabilities of failure can be obtained.
2. The stochastic character is taken into account on the level of the mechanism, i.e., different mechanisms take place depending on the particular combination of geometric and mechanical properties. Usually, geometry and mechanical property distributions are specified and an appropriate number of 'geometric pattern - mechanical property' combinations are simulated. For each such combination or realization the suitable mechanisms are determined and the resisting and driving forces derived. By combining the results of all realizations one will arrive at distributions of resisting and driving forces or of other expressions that represent the stability of the rock mass.

The second approach is more satisfactory, because it represents what actually takes place and also because the other approach requires significant simplification (if second moment analysis is used) or because it can be inaccurate (simulation).

Therefore, approaches to stochastic representation of the mechanisms are developed in Chapter 2. They provide significant improvements, but should be considered not as the final answer but as a step toward complete stochastic approaches, because some simplifications are still included. The main problem addressed is the interaction between intact rock and joints, i.e., the "persistence problem." Under the usually encountered stress conditions in rock slopes small changes in

the portion of the failure surface located in intact rock can cause major differences in stability. These intact rock portions are located between joint segments and failure will occur through transitions from joint segment to joint segment if there is no continuous joint. Such (intact rock) transitions can take place "in plane" i.e., connecting joint segments within the same plane, or they occur "out of plane" (also called "en-echelon") connecting joint segments located in different joint planes. The distribution of joint spacing and "length" will thus have a major effect on the failure surface and resistance of a slope. By considering the stochastic character of joint spacing and length, a first solution to the persistence problem can be provided. (Joint orientation and mechanical properties are considered to be deterministic.) The stochastic character of joint orientation will be considered in a second step (in the following Chapter 3) as far as wedge analysis is concerned; for slope parallel joints the stochastic character of joint orientation is not yet considered. Specifically, the approaches consist of a "stochastic model of joint geometry" and a "mechanical model." The stochastic model of joint geometry produces joint patterns that correspond to the user-specified distributions. The model is three dimensional but due to the above-mentioned limitations regarding orientation the associated computer program is only given for a 2-dimensional version, i.e., joints are assumed to be parallel and only a single set is considered at any one time (it will however be shown in Chapter 3, that two non-parallel sets forming wedges can be combined to extend the stochastic approach to wedge stability analysis). The user specifies the spacing and length distributions that can be obtained with joint survey sampling as discussed in Parts I and IV of this report. Exponential distributions are used, which makes the procedure practically very attractive--the user needs to specify only the mean spacing and mean length. Also the Exponential distribution is entirely correct for spacing and a good approximation for joint lengths (size). The mechanical model does what has been described

initially: it determines the appropriate failure mechanisms for a particular combination of joint geometry and mechanical properties and derives the associated expressions for resistance. Two approaches, JOINTSIM and SLOPESIM, have been developed. JOINTSIM considers a rectangular block containing joint patterns as described above. The user also can specify the applied stress field either directly or as obtained from a larger scale analysis. The latter procedure is employed, for instance, in wedge analysis where JOINTSIM functions as a sub-analysis for each of the two joint sets forming the wedge. SLOPESIM in contrast considers an actual slope which contains a single slope parallel joint set, whose patterns are again described by the joint geometry model above. Both JOINTSIM and SLOPESIM contain a dynamic programming procedure and, integrated in this procedure, the resistance (derivation) model. The dynamic programming procedure starts with the realization of one particular joint pattern based on the joint geometry model and user-specified joint length and spacing distributions (by producing a number of realizations one obtains a set of joint patterns that is representative of the specified distributions). In JOINTSIM all possible paths from the right-hand to the left-hand side of the jointed block are produced and their resistance determined (by means of the resistance derivation model described below). The path of minimum resistance and the associated resistance is then obtained. This is done for each realization, resulting thus for all realizations in a distribution of resistance which will be used in the reliability analyses. In SLOPESIM all possible paths from each joint plane exit point (the point of intersection between a joint plane and the slope face) to the upper free surface of the slope are produced. Safety margins are computed for each path by means of the resistance derivation model, and the path of minimum safety margin for each exit point is derived. This is done for each realization and leads thus to a distribution of minimum safety margins for each slope interval once all realizations have been completed. This safety margin distribution will be used in the reliability analysis.

The resistance derivation models which are integral parts of JOINTSIM and SLOPESIM produce resistances for the segments of the failure path; in particular, the resistance of joint segments, of in-plane transitions through intact rock and out-of-plane transitions through intact rock. At the prevailing low stress levels failure through intact rock usually takes place in tension, a fact that has been considered in both the JOINTSIM and SLOPESIM approaches, but in a more generally applicable manner in the latter. The procedure described in this chapter makes it possible to take the stochastic character of rock mass behavior into account. These approaches are an essential part of the reliability analysis discussed later. In connection with these reliability analyses but also used independently they represent a significant advance in slope stability analysis. This is primarily so, because they make it possible together with the knowledge on joint geometry distributions and appropriate sampling procedures developed in this research, to treat persistence correctly.

Slope stability analysis can usually be divided into a kinetic and a kinematic analysis. Chapter 3 describes the DAYLITE procedure for probabilistic kinematic stability analysis of 2-joint wedges.

DAYLITE: is mainly intended to be used as a part of reliability analysis for wedges together with kinetic probabilistic analysis described previously. In addition it can be used as an independent kinematic analysis providing considerable improvement over the common deterministic analyses (these, even if they are based on statistical joint surveys, can be very inaccurate because they cannot consider all combinations of joint planes). Kinematic analysis for 2-joint wedges can be conducted by examining the attitude of the line of intersection (of the joints) relative to the slope, specifically to check if it daylights or not. To do this probabilistically, i.e. where joints are not deterministically defined but represented by orientation distributions, DAYLITE addresses the question: Given a direction, defined by its spherical coordinates  $\theta$  and  $\phi$ , in the critical zone

of stability, what is the probability that two joints will combine to form a wedge whose line of intersection points in the  $(\theta, \phi)$  direction?

The solution procedure involves three steps:

1. Defining and partitioning the critical zone of kinematic instability into cells.
2. Examining each cell to find pairs of joint planes that have the corresponding line of intersection.
3. Calculating the probability that the slope will contain the requisite pair of joints.

DAYLITE derives the intersection probabilities numerically. The kinematically critical zone for wedge instability is partitioned into equal area spherical rectangles or cells, and the probability that the line of intersection of two joint planes will be in that cell is computed. In addition, the central angle  $\psi$  of the wedge, and the associated angle,  $\beta_1$  the angle defining the orientation of the wedge relative to a horizontal, are computed. Thus, not only are all wedges identified but they are also categorized with respect to their geometry. The four parameters  $\phi$ ,  $\theta$ ,  $\psi$  and  $\beta_1$  completely define a wedge with respect to its shape and orientation in space. Accordingly, the analysis method DAYLITE produces three different probabilities:

$P[\theta_i, \phi_i]$  - the probability that a wedge (regardless of its shape) will have a line of intersection oriented in the  $(\theta_i, \phi_i)$  direction.

$P[\theta_i, \phi_i, \psi_j]$  - the probability that a wedge with a central angle of  $\psi_j$  (regardless of  $\beta_1$ ) will have a line of intersection oriented in the  $(\theta_i, \phi_i)$  direction.

$P[\theta_i, \phi_i, \psi_j, (\beta_1)_k]$  - the probability that a wedge with a central angle of  $\psi_j$  and a  $\beta_1$  of  $(\beta_1)_k$  will have a line of intersection oriented in the  $(\theta_i, \phi_i)$  direction.

DAYLITE makes it thus possible to compute the probability that a kinematically stable or unstable wedge will form, based on joint survey data in form of orientation distribution. The main purpose of determining such probabilities is to use them together with kinetic analysis for complete slope reliability analysis (as will be shown in Chapter 4). In addition, probabilistic kinematic stability analysis is useful by itself in giving a first indication on the probability that unstable wedges will form, since (as mentioned previously) the use of mean orientations to represent joint sets may be deceiving. Another use of DAYLITE is the preliminary optimization of slope geometries. By varying slope geometries for a given sample of joint orientation data one can determine the geometry that minimizes the probability of kinematic instability. This procedure can be also applied to more than two joint sets by combining with DAYLITE in turn any two sets.

In Chapter 4 a reliability analysis for 2-joint wedges has been developed that makes it possible to compute the probability of failure ( $\overline{P}_f$ ) of an individual wedge in a slope. In addition an approximative procedure is provided to determine the probability of failure  $P_f$  of a slope containing many wedges. The probability of failure  $\overline{P}_f$  depends on the probability that a wedge is kinematically and kinetically unstable and can be expressed as

$$\overline{P}_f = \sum_{\text{all } i} P[U_i/\theta_i, \phi_i] P[\theta_i, \phi_i].$$

$P[\theta_i, \phi_i]$  is the probability that the wedges with the line of intersection  $\theta_i, \phi_i$  are kinematically unstable (i.e. daylight) and  $P[U_i/\theta_i, \phi_i]$  is the probability that the wedges represented by  $\theta_i, \phi_i$  are kinetically unstable. The reliability analysis for  $\overline{P}_f$  proceeds thus in three steps--determination of 1)  $P[\theta_i, \phi_i]$  and 2)  $P[U_i/\theta_i, \phi_i]$  followed by 3) the summation.  $P[\theta_i, \phi_i]$  is determined with the procedure DAYLITE described in Chapter 3 which in addition to the kinematic probability of failure derives information on wedge shape

and size distribution for the particular  $\theta_i, \phi_i$  - pair.  $P[U_i/\theta_i, \phi_i]$  uses distributions of apparent persistence derived with JOINTSIM and user specified distributions of intact rock and joint resistance to obtain the kinetic probability of failure for a particular  $\theta_i, \phi_i$  pair. The computation is performed with SWARS-2MC, the Monte Carlo simulation version of the wedge analysis SWARS-2PM. The results are a number of factors of safety for the particular  $\theta_i, \phi_i$  pair and  $P[U_i/\theta_i, \phi_i]$  is simply the percentage of  $FS < 1$ . Finally the products  $P[\theta_i, \phi_i] * P[U/\theta_i, \phi_i]$  are summed to arrive at  $\overline{P_f}$ . The reliability analysis integrates thus the previously developed procedures for kinematic and kinetic probabilistic stability analyses DAYLITE and JOINTSIM. The integration is not fully automatic but has interfaces that require user action. This makes it possible to simplify or streamline the problem before going to the next step. Nevertheless, an algorithm and compatibility of output of the preceding step with the input of the following one makes the procedure computationally convenient. The probability of failure of an individual wedge  $\overline{P_f}$  can thus be obtained with a procedure that is well suited to practical application. In contrast the probability of the failure  $P_f$  of a slope containing many wedges is a very complex problem of system reliability. Also, it is necessary to know how parameters are spatially correlated and if and how the wedges are mechanically correlated. Not much information is available on the correlation and as a consequence system reliability approaches can only be approximations of the real situation. Such an approximation is given in this chapter; it makes an estimation of the upper bound of  $P_f$  practically achievable. At the same time it becomes quite clear that more rigorous approaches would be desirable but depend on better knowledge of the actual behavior, knowledge that is limited or nonexistent at the present time.

The simplified reliability analysis method presented in Chapter 5 derives the probability of failure of single 2-plane wedges in slopes with a horizontal upper surface. This is accomplished by computing

factors of safety for all possible or for representative combinations of joint orientations from two clusters and then deriving probability of failure as the percentage of factors or safety smaller than one. Although basically possible to conduct this procedure for various distributions of mechanical properties, this is not done, because it would defeat the purpose of simplified method. Mechanical properties that can be considered are: joint friction angle, intact rock cohesive strength and persistence, specified independently for each plane. A prismatic water pressure distribution with the same water level but different maximum values per plane can be given, the extremes of zero pressure and full hydrostatic head can be modelled in this way as can any intermediate distribution. Kinematic instability and the mode of sliding failure (along one or both planes) can be determined; a rigorous derivation of the factor of safety is however only provided for sliding on two planes, while for sliding on one plane it is approximately checked if  $FS > 1$  or  $FS < 1$ . The procedure makes use of pole diagrams to obtain wedge angle distributions, of an especially developed plot to determine if sliding occurs along the line of intersection or on one plane, and of a simple pocket calculator program to examine kinematic stability and to determine the factor of safety. In spite of the significant simplifications of both probabilistic and mechanical nature, probability of failure ( $P_f$ ) as derived by this analysis is useful. It makes it possible to quickly narrow down viable design options and to establish which design options should be investigated more thoroughly.

Chapter 6 presents a reliability analysis for slopes with a single slope parallel but otherwise randomly distributed set of joints, a simple but very frequently occurring case. SLOPESIM, which has been described previously, can be used to analyze the reliability of such slopes in a very encompassing manner, i.e. including practically all parameters that affect slope stability. An extensive parametric

study applying SLOPESIM was conducted to determine which parameters have the greatest effect on probability of failure. This provided substantial insight into slope stability, makes selection of parameters for practical application more effective and forms the basis for a simplified analysis procedure (to be developed in Chapter 7).

Specifically the influence of the following parameters is examined.

Strength parameters:	Intact rock cohesive strength	$C_r$
	Joint friction angle	$\Phi_j$
Joint and slope geometry:	Persistence (mean)	$\bar{K}$
	Spacing (mean)	$\bar{SP}$
	Joint length (mean)	$\bar{JL}$
	Joint inclination	$\alpha$
	Slope inclination	$\theta$
	Slope height	$h$

The effect of these parameters on reliability expressed in the form of a probability of failure  $P_f$  or a reliability index ( $\beta$ ) was studied by systematically varying one parameter while keeping the others constant. The major results show that in most situations the influence of strength parameters ( $C_r$ ,  $\Phi_j$ ) predominates; slopes in which  $\Phi_j$  and  $C_r$  are high ( $\Phi_j \approx \alpha$ ,  $C_r \geq 500$  ksf), tend to be reliable at all depths investigated (0-100') regardless of other parameters. Only when  $C_r$  and  $\Phi_j$  are low, can joint and slope geometry parameters have a large effect on reliability. Among the joint geometry parameters ( $\bar{K}$ ,  $\bar{JL}$ ,  $\bar{SP}$ ), mean joint plane persistence ( $\bar{K}$ ) appears to have the largest influence (to a depth of 100'). Increasing  $\bar{K}$  can lead to significant increases in  $P_f$  (decrease in  $\beta$ ) when  $C_r$  and  $\Phi$  are both low. Increasing mean joint length ( $\bar{JL}$ ) can also substantially increase  $P_f$  at any given depth in the slope. Mean joint plane spacing ( $\bar{SP}$ ) appears to play a less significant role. Joint planes in slopes in which  $\theta \approx \alpha$  or in which  $\alpha \approx \Phi_j$  tend to be highly reliable at any depth regardless of other input parameters. Joint planes with inclinations of roughly 40 to 50° tend to be least reliable. Increasing  $\alpha$  beyond this range towards  $\theta$  or decreasing  $\alpha$  towards  $\Phi_j$  increases joint plane reliability appreciably. Increasing the difference between slope angle ( $\theta$ ) and joint inclination ( $\alpha$ ) always decreases reliability

due to an increase in driving force. The slope depth  $h$  is naturally a highly significant parameter and probability of failure  $P_f$  is thus usually presented in the form of  $P_f(h)$  curves. These  $P_f(h)$  relations can be often approximated by  $P_1(h)$  where  $P_1$  is the probability of a joint plane being 100% persistent as a function of depth. When either  $C_r$  or  $\phi_j$  are high ( $\phi_j \approx \alpha$  or  $C_r \geq 500$  ksf), the  $P_1 \approx P_f$  approximation is applicable regardless of the values of other parameters. Even when both  $C_r$  and  $\phi_j$  are low ( $\phi_j \ll \alpha$  and  $C_r < 100$  ksf), the approximation ( $P_f(h) \approx P_1$ ) often holds for a number of parameter combinations. Through the development of SLOPESIM and the subsequent detailed parametric study presented in this chapter significant insight into rock slope reliability has been gained, and practically useful results have been obtained. In addition to establishing the relative influence of the strength and geometric parameters ( $C_r$ ,  $\phi_j$ ,  $\overline{SP}$ ,  $\overline{JL}$ ,  $\overline{K}$ ,  $\theta$ ,  $\alpha$ ,  $h$ ) situations can be defined in which accurate assessment of certain critical parameters is necessary. The slope designer can gear an exploration program towards their accurate measurement while deemphasizing less important parameters.

Chapter 7 is at the same time a simplified reliability analysis and a design approach for slopes with a single slope parallel set of joints. It allows the designer to specify an acceptable probability of joint plane failure  $P_a$  and a slope height, and to obtain the maximum slope angle for these conditions. The approach is particularly attractive because it considers the variability of other slope parameters, particularly the strength parameters. Three zones in the slope need to be considered. At shallow depth a potentially unstable zone exists whose extent ( $h'$ ) is beyond the control of the designer (i.e. unaffected by slope angle) and is dependent only on the geometric properties of the joints within the slope (joint length, joint plane persistence, and joint inclination). Mean joint length ( $\overline{JL}$ ) in particular has a large influence on  $h'$ , since  $h'$  is directly proportional to  $\overline{JL}$ . Within this zone ( $0 - h'$ ), monitoring

and design based on observed behavior is required. Stabilizing measures (e.g. bolting) will often be required in this zone. Below  $h'$  and above a critical depth ( $h_c$ ) lies the "zone of stability" in which  $P_f$ , the probability of joint plane failure, lies below the acceptable level of risk ( $P_a$ ). The "zone of deep instability" lies below  $h_c$ . For  $h > h_c$  reliability decreases exponentially, eventually reaching a value of zero ( $P_f = 100\%$ ). Reliability design of slopes must be based on minimizing the zones of instability (both shallow and deep) for the final slope design. Stability field diagrams developed in this chapter allow a designer to consider variability of up to 3 input parameters on a single diagram in order to ensure that the slope does not fall within the zone of "deep instability." They are particularly useful in determining "stable" combinations of  $\phi_j$  (joint friction angle) and  $C_r$  (intact rock strength) for different values of slope angle ( $\theta$ ). Ranges (e.g. from test data) of  $C_r$  and  $\phi_j$  can be plotted to obtain the maximum possible slope angle  $\theta$  for a given depth  $h$  and chosen probability of failure  $P_a$ .

#### 4.4 Part IV. Field Exploration to Determine Rock Mass Properties

The change in probability of failure with increasing information was used to analyze the optimal strategy of allocating exploration effort for a given slope. Total optimization of exploration is a major research effort in its own right; the present work holds no claim to exhaustiveness. However, by concentrating on the one-slope problem, insight is gained into the overall problem.

The technique used in optimizing exploration is Bayesian decision theory in which statistical description of important geological parameters come from the work under Part I, inference of the value of decreased uncertainty in design comes from the work under Part III, and decision theory is used to link these together.

In conjunction with the analysis of optimal exploration strategies for single slopes, a graphical risk screening procedure was developed to determine those slope sections within a pit for which increased exploration would beneficially reduce risk. This allows routine use of decision theory methods in exploration.

Specifically, in Chapter 2 sampling plans and Methods of Statistical inference for rock mass properties are developed using the empirical probabilistic description of Part I. As fairly standard sampling theory is readily applicable to resistance and deformation properties of discontinuities, there seemed little need to repeat such work. Therefore most of the sampling work is directed toward geometric properties of discontinuities; in particular, trace length, spacing, and orientation. This led to the development of procedures for sampling, and to mathematical relations for inferring in-situ properties from joint survey data. The major difficulty to be overcome in drawing inferences from joint survey data is statistical biases. The existence of strong biases in joint surveys means that common sampling plans are not representative of the population of properties being sampled for. Therefore, direct ("naive") estimates based on the uncorrected sample statistics can be greatly in error. The most important biases in joint surveys for geometric properties are: (1) orientation bias, (2) length bias, (3) truncation bias, and (4) censoring. Orientation bias arises because outcrops and borings have defined orientation. The joint survey is blind to joints sub-parallel to the measured outcrops and borings. Length bias arises because larger joints have a proportionally larger probability of appearing in the sample than smaller joints. Depending on the sampling plan this length bias can even be quadratic (i.e., probability of being sampled is proportional to the size squared). Truncation bias arises because small trace lengths are often left out of joint surveys (e.g., any trace less than 1.0 m). While important in special cases, truncation is on the whole less important than the other biases. Censoring bias can be severe, and leads to an unconservative

vative underestimating of joint "persistence." In drawing inferences from survey data, observations must be given weight in inverse proportion to their probability of entering the sample. That is, if a joint is observed which should have low probability of entering the sample, it is given more weight in subsequent inferences than a joint having higher probability. This is simply because observing a low probability joint implies more about the number of similar joints in the rock mass as a whole, than observing a higher probability joint does. Inference procedures are developed in this chapter for maximum likelihood estimation where possible, and for moment estimation otherwise. In both cases the sampling variances of the estimates are derived for measures of confidence. A central part of statistical estimation, and a necessary parameter for reliability models, is the confidence with which estimates are made. The inference procedures and methods to determine measures of confidence are in such a form that they can be applied in practice.

The issue addressed in Chapter 3 is the question of balancing exploration expenditures and plans on one hand against resulting benefits in slope design on the other hand. The approach to exploration strategy analysis (optimization) is through the "expected value of information" (EVI) of decision theory. This approach is developed for application to rock slope decision, and two cases are analyzed: the one-joint set (parallel to slope) case, and the general shear case. Both the strategy for exploration without reference to a particular slope, and for a specifically identified slope are considered. The development of EVI procedures for pit slopes is not sufficiently advanced that charts and tables can yet be prepared. However, the approach shows itself to be powerful, and can be applied to specific situations such as specific slope sections for which risk screening charts were developed.

#### 4.5 Part V. Probabilistic Models of Jointed Rock Mass Deformation and Their Implications

The extensive treatment of geological distributions and probabilistic methods to analyze related engineering problems revealed a number of interesting problems that are not directly in the line of the proposed research, whose solution would however be beneficial. In this sense substantial work was undertaken on obtaining foundation settlement predictions based on spacing and orientation distributions rather than using RQD-related correction factors.

Rock mass deformability is commonly predicted using correlations with RQD or other indices of rock mass quality, or by extrapolation from large in-situ tests. This situation is due in part to the natural variability of rock masses, and to the inability to adequately model the effects of discontinuity on deformation. Based on the probabilistic description of rock masses empirically developed in Part I a new approach was attempted in modeling for predictions of deformability. This approach decomposes the rock mass into spatially variable joints described by probability distributions, and intact rock blocks. Stiffnesses and elastic parameters are associated with the joints and intact blocks respectively, and deformations built up by adding the contributions of each. This model allows relaxation of many of the simplistic geometric assumptions which constrain parallel attempts to model deformation in rock masses. The model has been applied to four standard cases in order to test its results against previously existing models, and to check whether it replicates empirical correlations like that due to Deere. Simple cylindrical geometries were considered in two and three dimensions, having joint sets randomly positioned and oriented according to the probabilistic descriptions of Part I. These cases were treated as laterally unconfined, laterally restrained, and laterally confined by springs. After these calibration conditions were considered the model was used to replicate common correlations to RQD. Experiments with the model seem to show that spatial variability of joint geometry has a significant influence on the deformability of the

rock mass. When joints are taken as parallel, common empirical correlations are closely approximated by the model output. Deformations are summed over elastic deformation of the intact rock and movement along joints; RQD is calculated by the spacing distribution of joints (in Monte Carlo runs this is done for each realization, while in closed form runs this is done mathematically using the pdf of spacing, as described in Part I). However, the closeness to empirical correlation breaks down when variability of the jointing geometry is increased. This may carry implications for the extent to which the empirical correlations can be extended beyond their base cases. The strength of the probabilistic (stochastic) model for rock mass deformability is that measurable geologic attributes of the rock mass can be used to analytically predict deformations. This is an improvement over most current models requiring ad hoc or hypothetical parameters, and over large in-situ testing which can be prohibitively expensive.

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##### 5. Conclusions and Major Areas of Future Work

This research on risk analysis for rock slopes in open pit mines has thus created encompassing approaches to consider the stochastic character of rock masses in sampling and representation of parameters and in stability analysis. The approaches involve significant theoretical development leading to practical procedures while simultaneously forming the basis for further advancement.

Still, work remains to be done in the following areas for which the research has laid groundwork: With regard to geologic distributions, gaps remain to be closed in distributional forms for joint orientation, spatial correlation and knowledge on size and shape. The problem of systems reliability has only been touched so far, but will have to be considered much more thoroughly, together with the work on spatial correlations. Also, a great potential for further contributions exists in the risk analysis area. This is particularly so since present

economic mine planning considers the geotechnical aspects in a surprisingly simplified manner. Finally, the rock mass deformation analysis can be further expanded to include interaction with water flow and to make it applicable to specific geometries. In addition to these major aspects there are many details that would benefit from further work.

A significant step toward reliability analysis for rock slopes has been made and practically useable approaches are now available; in addition it can be shown that both the practical approaches and the underlying theoretical developments would benefit from some further research.

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