



PB95-105482

IC 9398

**BUREAU OF MINES
INFORMATION CIRCULAR/1994**

**Critique of Pillar Design Equations
From 1833 To 1990**

By C. Babcock



UNITED STATES DEPARTMENT OF THE INTERIOR

REPRODUCED BY:
U.S. Department of Commerce
National Technical Information Service
Springfield, Virginia 22161

U.S. Department of the Interior
Mission Statement

As the Nation's principal conservation agency, the Department of the Interior has responsibility for most of our nationally-owned public lands and natural resources. This includes fostering sound use of our land and water resources; protecting our fish, wildlife, and biological diversity; preserving the environmental and cultural values of our national parks and historical places; and providing for the enjoyment of life through outdoor recreation. The Department assesses our energy and mineral resources and works to ensure that their development is in the best interests of all our people by encouraging stewardship and citizen participation in their care. The Department also has a major responsibility for American Indian reservation communities and for people who live in island territories under U.S. administration.

REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188	
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.				
1.  PB95-105482	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED Information Circular (IC)		
4. TITLE AND SUBTITLE CRITIQUE OF PILLAR DESIGN EQUATIONS FROM 1833 TO 1990			5. FUNDING NUMBERS	
6. AUTHOR(S) CLARENCE O. BABCOCK				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) U.S. Bureau of Mines Denver Research Center Bldg. 20, Denver Federal Center Denver, CO 80225			8. PERFORMING ORGANIZATION REPORT NUMBER IC 9398	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)			10. SPONSORING/MONITORING AGENCY REPORT NUMBER	
11. SUPPLEMENTARY NOTES				
12a. DISTRIBUTION/AVAILABILITY STATEMENT			12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words) When a flat-lying mineral deposit such as a coal seam is mined, some coal is left to separate the roof and floor so that men and machines can safely mine the seam. The mineral left is in the form of mine pillars. If too much is taken, support problems and unsafe working conditions result. If too little is taken, the mining operation will be unable to compete economically with other mines. Therefore, it is important to select pillars of the optimal size to maximize mining recovery and minimize risk to miners and equipment. Later, if the pillar support is no longer needed and the surface can be allowed to subside the thickness of the coal seam, the pillar support can be removed to increase mining recovery, and initial pillar size is not as critical. Only a small part of the information available on pillar design can be presented here. The purpose of this report is to summarize some of the work done by the U.S. Bureau of Mines on pillar design, that reported in the literature from 1833 to the present time, and present rock mechanics test results that aid in understanding mine pillar behavior.				
14. SUBJECT TERMS			15. NUMBER OF PAGES 98	
			16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT	

GENERAL INSTRUCTIONS FOR COMPLETING SF 298

The Report Documentation Page (RDP) is used in announcing and cataloging reports. It is important that this information be consistent with the rest of the report, particularly the cover and title page. Instructions for filling in each block of the form follow. It is important to *stay within the lines* to meet optical scanning requirements.

Block 1. Agency Use Only (Leave blank).

Block 2. Report Date. Full publication date including day, month, and year, if available (e.g. 1 Jan 88). Must cite at least the year.

Block 3. Type of Report and Dates Covered. State whether report is interim, final, etc. If applicable, enter inclusive report dates (e.g. 10 Jun 87 - 30 Jun 88).

Block 4. Title and Subtitle. A title is taken from the part of the report that provides the most meaningful and complete information. When a report is prepared in more than one volume, repeat the primary title, add volume number, and include subtitle for the specific volume. On classified documents enter the title classification in parentheses.

Block 5. Funding Numbers. To include contract and grant numbers; may include program element number(s), project number(s), task number(s), and work unit number(s). Use the following labels:

C - Contract	PR - Project
G - Grant	TA - Task
PE - Program Element	WU - Work Unit Accession No.

Block 6. Author(s). Name(s) of person(s) responsible for writing the report, performing the research, or credited with the content of the report. If editor or compiler, this should follow the name(s).

Block 7. Performing Organization Name(s) and Address(es). Self-explanatory.

Block 8. Performing Organization Report Number. Enter the unique alphanumeric report number(s) assigned by the organization performing the report.

Block 9. Sponsoring/Monitoring Agency Name(s) and Address(es). Self-explanatory.

Block 10. Sponsoring/Monitoring Agency Report Number. (If known)

Block 11. Supplementary Notes. Enter information not included elsewhere such as: Prepared in cooperation with...; Trans. of...; To be published in.... When a report is revised, include a statement whether the new report supersedes or supplements the older report.

Block 12a. Distribution/Availability Statement. Denotes public availability or limitations. Cite any availability to the public. Enter additional limitations or special markings in all capitals (e.g. NOFORN, REL, ITAR).

DOD - See DoDD 5230.24, "Distribution Statements on Technical Documents."

DOE - See authorities.

NASA - See Handbook NHB 2200.2.

NTIS - Leave blank.

Block 12b. Distribution Code.

DOD - Leave blank.

DOE - Enter DOE distribution categories from the Standard Distribution for Unclassified Scientific and Technical Reports.

NASA - Leave blank.

NTIS - Leave blank.

Block 13. Abstract. Include a brief (*Maximum 200 words*) factual summary of the most significant information contained in the report.

Block 14. Subject Terms. Keywords or phrases identifying major subjects in the report.

Block 15. Number of Pages. Enter the total number of pages.

Block 16. Price Code. Enter appropriate price code (*NTIS only*).

Blocks 17. - 19. Security Classifications. Self-explanatory. Enter U.S. Security Classification in accordance with U.S. Security Regulations (i.e., UNCLASSIFIED). If form contains classified information, stamp classification on the top and bottom of the page.

Block 20. Limitation of Abstract. This block must be completed to assign a limitation to the abstract. Enter either UL (unlimited) or SAR (same as report). An entry in this block is necessary if the abstract is to be limited. If blank, the abstract is assumed to be unlimited.

Information Circular 9398

Critique of Pillar Design Equations From 1833 To 1990

By C. Babcock

**UNITED STATES DEPARTMENT OF THE INTERIOR
Bruce Babbitt, Secretary**

BUREAU OF MINES

Library of Congress Cataloging in Publication Data:

Babcock, C.

Critique of pillar design equations from 1833 to 1990 / by C. Babcock.

p. cm. — (Information circular; 9398)

Includes bibliographical references (p. 91).

1. Pillaring (Mining)—Mathematical models. 2. Rock mechanics—Mathematical models. I. Title. II. Series: Information circular (United States. Bureau of Mines); 9398.

TN295.U4 1993 [TN292] 622 s—dc20 [622'.28] 93-19066 CIP

CONTENTS

	<i>Page</i>
Abstract	1
Introduction	2
Pillar design equations	3
Bauschinger's theory for design of pillars of any shape	3
Bunting's tributary area method for pillar design	8
Pillar behavior concepts reported by Rice	10
Pillar design by Holland and associates	13
Grobbelaar's theory for strength of pillars	15
Case 1 - Comparison of relative strengths of square and circular pillars	15
Case 2 - Comparison of relative strengths of square and strip pillars	16
Case 3 - Comparison of relative strengths of square and rectangular pillars	18
Comparison of pillar sizes selected with equations of Wilson, Wardell, and Salamon and Oravec	18
Barron's pillar design considerations	21
Integrity index for pillar design	21
Comments on pillar design equations	23
Lu's integrity factor method of in situ pillar design	25
Longwall chain pillar design equation of Hsiung and Peng	27
Barrier pillar design	29
Babcock's method of assessing pillar stability	31
Rock mechanics concepts useful for pillar design	38
Measurement of physical properties of rock or coal in laboratory or in situ	38
Physical properties of intact specimens in laboratory	38
Measurement of physical properties in situ	39
When are physical property constants not constant?	39
Mohr-Coulomb stress-failure relationships	41
Use of force equilibrium to obtain Mohr's stress circle and Coulomb failure criteria relationships for rigid bodies	41
What is failure in rigid body?	44
Mohr-Coulomb stress-failure model	45
How does stress condition produce failure state?	45
Role of constraint on shearing strength	45
Relationship between vertical and horizontal stresses in underground coal seam	46
Estimation with Mohr-Coulomb stress-failure model of horizontal stresses needed for pillar survival	46
Estimation of unconfined shear strengths of coal in situ from permeability-versus-depth relationships	50
Effect of end constraint on compressive strength of test specimens in laboratory	51
Constraint implied for standard uniaxial tests between two steel platens in laboratory	51
Changes in compressive strengths of rock and coal specimens produced by end constraint in laboratory tests	53
Interpretation of pillar design equations with width-height ratio as fundamental variable in terms of end constraint	58
Bursting of model coal pillars in laboratory	61
Burst proneness of coal specimens relative to test geometry and end constraint conditions	66
Use of Mohr-Coulomb stress-failure model to describe coal burst behavior	67
Changes in roof above longwall face in coal mine	68
Calculation of mine pillar stresses from borehole pressure cell data	69
Role of end constraint on compressive strength	74
Other concepts of interest in pillar design	76
Strain energy in a mine pillar and its role in bursting	76
Geometric method for prediction of stresses in inclusions, orebodies, and mining systems	81
Apparent increase in mine pillar strength resulting from decreasing tributary area stress with decreasing recovery	82

CONTENTS—Continued

	<i>Page</i>
Volume of roof and floor rock that influences strength of coal mine pillar	82
Summaries and conclusions by sections	84
References	91
Appendix.—List of symbols	94

ILLUSTRATIONS

1. Compressive strength of rectangular pillars normalized to strength of pillar of cubic shape	5
2. Ratio of compressive strength per unit area of rectangular pillar to square pillar	5
3. Comparison of compressive strength per unit area of rectangular pillar to that of cubic pillar	6
4. Rectangular pillars of different shapes and sizes that have same average compressive strength	7
5. Tributary area method for chamber-and-pillar mining system	8
6. Chamber pillar width determined with equation 30	10
7. Tributary area method for room-and-pillar mining system	11
8. Mine pillar geometries used with equations of Grobbelaar	16
9. Comparison of percentage mining recovery using pillars selected with equations of Wilson, Wardell, Salamon and Oravec, for 6-ft coal seam	21
10. Comparison of percentage mining recovery using pillars selected with equations of Wilson, Wardell, Salamon and Oravec, for 10-ft coal seam	21
11. Air injection apparatus used by Barron to investigate integrity of pillars and ribs in coal mines	22
12. Barron's fracture index $ I $ across pillar at different times for mine B	22
13. Barron used same polynomial to describe the vertical pillar stress distributions for <i>A</i> and <i>B</i>	23
14. Barron used vertical slices and force equilibrium in his computer model for pillar design	23
15. Photoelastic type analysis of stresses in horizontal direction on pillar shows unsuitability of method used in figure 14 by Barron	24
16. Relative orientation of principal stress directions in mine pillar	25
17. Lu's integrity factor method of determining mine pillar stability	26
18. Creep in salt as function of initial setting pressure and time	26
19. Three stages of pillar loading in longwall mining system	27
20. Nomograph of mine inspector's formula for barrier pillars in Pennsylvania	28
21. Nomograph of barrier pillar legislation equation for Pennsylvania reported by Ashley	30
22. Nomograph of Ash-Eaton equation for flat-lying barrier pillars in Pennsylvania	31
23. Mohr-Coulomb stress-failure models	32
24. Mohr-Coulomb stress-failure models for Babcock's pillar stability method	33
25. Estimation of angle of internal friction for coal pillar in situ from angle of slabbing of rib	34
26. BPC arrays of two cells each used by Harmel, Lu, and Barron in longwall entry study	34
27. Changes in vertical and horizontal pillar stress with mining advance derived from Harmel, Lu, and Barron data	35
28. Increases and decreases in pillar stress over mining interval	35
29. Changing Mohr circle size with mining advance	37
30. Determination of Poisson's ratio in situ from ratio of horizontal to vertical pressure cell change	39
31. Poisson's ratio versus average axial stress for selected rocks	40
32. Young's modulus versus average axial stress for selected rocks	41
33. Rock specimen configurations for compression testing	42
34. Radial end constraint from solid steel rings, versus average axial stress, for specimens of limestone, marble, sandstone, and granite	43
35. Rigid body can be used to obtain Mohr's circles for three-dimensional state of stress	44
36. Rigid bodies can be used to obtain Coulomb failure condition	45

ILLUSTRATIONS—Continued

Page

37. Three components of shear strength and their relationship to compressive strength for Mohr-Coulomb stress-failure criteria	46
38. States of stress related to Mohr-Coulomb stress-failure conditions of figure 38	46
39. Mohr's circle construction used to relate vertical stress to confining horizontal stress	47
40. Horizontal stress needed to prevent failure in terms of vertical stress, unconfined shear strength, and angle of internal friction	48
41. Mining depth to which constraint is not required for unmined coal seam	48
42. Horizontal-vertical stress ratio needed for pillar survival when unconfined shear strength is zero	49
43. Depth to which cleats in coal pillar can stand open and constraint is unnecessary for pillar stability	50
44. Permeability of coal in three basins used to evaluate mining depth to which cracks remain open when seam is unmined	51
45. End constraint inferred for laboratory testing of rock or coal specimens between two steel platens in so-called uniaxial stress tests	52
46. Eight end conditions used in tests	58
47. Geometric basis for calculation of pillar design equation	59
48. Changes in relative strength of square pillars as function of geometry and of end constraint	60
49. Short coal specimen was made to burst violently by dynamically shattering two solid glass layers, one above and the other below the coal specimen.	62
50. Method of calculating velocities of segmented steel platen segments thrown horizontally by model coal pillar bursts in laboratory	62
51. Model coal burst in laboratory produced with test configuration of figure 50	63
52. Polar coordinate grid pattern on 8-ft-square horizontal surface used to collect segmented steel platen pieces thrown by bursting model coal pillars	64
53. Calculation of momentum and kinetic energy for model coal pillar bursts in laboratory	65
54. Tangential strains showing slipping of lower plastic platen with increasing load	65
55. Histograms of burst-proneness versus frequency summed over eight end conditions for six geometries ..	66
56. Histograms of burst-proneness versus frequency summed over six geometries for eight end conditions ..	66
57. Mohr-Coulomb stress-failure models	67
58. Dimensions, geometry, and boundary conditions of finite-element models of progressive fracture behavior occurring above longwall face	68
59. Directions of fracture lines within high shear stress area above longwall face	69
60. Safety factor contour lines of fractured roof over longwall face, model II, 5 mt prop capacity	70
61. Safety factor contour lines of fractured roof over longwall face, model II, 50 mt prop capacity	70
62. Cross-sensitivity of borehole pressure cell to transverse loading	71
63. Borehole deformations produced by biaxial stress field and borehole pressure cell responses they produce ..	71
64. Least squares regression lines fitted to experimental data from radial loading tests	72
65. BPC pressure and coal stress changes at York Canyon Mine with longwall face advance	73
66. Failure modes observed in laboratory for rock and coal specimens	74
67. Changes in radial stresses at ends of test specimens with length-diameter ratio of 2, produced by changing end conditions	75
68. Four different platen configurations used to study constraint during testing	76
69. Circumferential strains of rock disks with diameter-length ratio of 8.5	77
70. Effects of adding PTFE layers between specimen and platens on circumferential strains for limestone ..	78
71. Reduction in difference in specimen end and midlength strains using PTFE layers between specimen and platens	79
72. Change in role of geometry on compressive strength against steel platens to one where end friction is removed with PTFE layers	80
73. Strain energy in unit cube produced by principal stresses and principal strains	81
74. Locations of unit cubes used to define strain energy behavior of coal mine pillar	81

ILLUSTRATIONS—Continued

	<i>Page</i>
75. Apparent increase in pillar strength with increasing pillar size is actually result of decreased recovery and average vertical stress	83
76. Model glass pillars <i>A, B, C, D, E,</i> and <i>F</i> of same total area loaded in compression between two pieces of plastic and broken in tension	84
77. Zone <i>ABCD</i> contains stress anomaly produced by inclusion when plate was loaded vertically and horizontally on edges	85
78. For given total pillar area constraining volumes of rock in roof and floor that control pillar behavior decreases with decrease in pillar width	86

TABLES

1. Comparison of rectangular pillar sizes for Bauschinger's equation in form of equation 10 with tributary area results of equation 18	6
2. Strip pillar widths and entry widths corresponding to square pillars with same vertical stress	16
3. Comparison of pillar widths, calculated with equations of Wardell and Wilson	17
4. Comparison of average pillar stresses, calculated with equations of Wardell and Wilson	19
5. Parameters for straight-line fit to table 4 data, for 16-, 20-, and 24-ft room widths	20
6. Pillar widths, calculated with equations of Salamon and Oravec	20
7. Percentage recovery using pillars of sizes given in table 6	20
8. Ratio of horizontal pillar stress to vertical pillar stress needed for pillar stability for small value of cohesive strength	49
9. Ratio of horizontal pillar stress to vertical pillar stress needed for pillar stability for large value of cohesive strength	50
10. Average compressive strengths of rocks with and without solid ring end constraint	53
11. Average compressive strengths of rocks with and without radial end prestress constraint	53
12. Compressive strengths of model rock pillars without prestress and with two different prestress end constraints normalized to compressive strength of straight specimens with D:L of 1.0	54
13. Compressive strength of model rock pillars normalized to compressive strength of straight specimens with same D:L	55
14. Average radial constraint stress produced by rings above that produced by end friction as percentage of average compressive stress at failure	56
15. Average compressive strengths of four coals	56
16. Average relative compressive strengths for four specimens, one for each of four coals tested	57
17. Summary of test results and burst proneness of coals from 15 mines, in 11 coal seams, and in 6 States ..	64
18. Strain energy stored in mine pillar	79

UNIT OF MEASURE ABBREVIATIONS USED IN THIS REPORT

cm	centimeter	kPa	kiloPascal
cm/s	centimeter per second	lbf	pound (force)
deg	degree	lbf/in ²	pound (force) per square inch
ft	foot	lb/ft ³	pound per cubic foot
ft ²	square foot	lb/in	pound per inch
ft ³	cubic foot	m	meter
ft ³ /s	cubic foot per second	mD	millidarcy
g	acceleration due to gravity	MPa	megaPascal
gcm/s	gram centimeter per second	mt	metric ton
gcm ² /s ²	gram centimeter squared per second	μin/in	micro inch per inch
gm • cm/s	gram centimeter per second	μm/m	micrometer per meter
GPa	gigaPascal	pct	percent
in	inch	psi	pound per square inch
in • lbf	inch pound, strain energy	psi/ft	pound per square inch per foot
KN	kilo newton	s	second

CRITIQUE OF PILLAR DESIGN EQUATIONS FROM 1833 TO 1990

By C. Babcock¹

ABSTRACT

When a flat-lying mineral deposit such as a coal seam is mined, some coal is left to separate the roof and floor so that miners and machines can safely mine the seam. The mineral left is in the form of mine pillars. If too much is taken, support problems and unsafe working conditions result. If too little is taken, the mining operation will be unable to compete economically with other mines. Therefore, one must select pillars of the optimal size to maximize mining recovery and minimize risk to miners and equipment. Later, if the pillar support is no longer needed and the surface can be allowed to subside the thickness of the coal seam, then the pillar support can be removed to increase mining recovery, and initial pillar size is not as critical.

Only a small part of the information available on pillar design can be presented here. The purpose of this report is to summarize some of the work done by the U.S. Bureau of Mines and others on pillar design, which is reported in the literature from 1833 to the present, and present rock mechanics test results that aid in understanding mine pillar behavior. The variable names used are those of the authors cited. This results in multiple use of some variable names.

¹Mining engineer, Denver Research Center, U.S. Bureau of Mines, Denver, CO (retired).

INTRODUCTION

In spite of much effort since 1833 when Vicat (84)² gave an equation for the strength of rectangular test specimens loaded in compression, pillar design in mining has remained largely an art form. Much of what is known about such design has been based on laboratory testing of rock or coal specimens in compression in so-called uniaxial or triaxial tests. In the U.S. Bureau of Mines report that follows, selected significant ideas in pillar design are presented. Usually, until fairly recently, the geometry of the pillar has been the basis for design. In 1867, Bauschinger (22) produced a pillar design equation that anticipated much of the work that has been done since that time. Bunting (29) in 1911 was the first to indicate that the pillar size should be increased with mining depth and coal seam thickness, and that the tributary area method could be used for pillar design.

Many, if not most, of the pillar design equations can be expressed in the form (19)

$$\sigma / \sigma_{1:1} = A + B W^\alpha / H^\beta, \quad (1)$$

where σ and $\sigma_{1:1}$ are the compressive strengths of a pillar with any width (W) and height (H), and the compressive strength of a pillar when $W = H$, respectively. A , B , α , and β are constants usually chosen so that the equation fits data obtained experimentally in the laboratory or in the mine. If $\alpha = \beta = 1$, then both $\sigma/\sigma_{1:1}$ and $W:H$ are dimensionless ratios, and A and B are also dimensionless constants. Any consistent units of stress, say pound per square inch or megaPascal, and length, feet or meter, could be used. If α is not equal to β , B will have dimensions that make the term $B(W^\alpha/H^\beta)$ a dimensionless product. Equation 1 is a simplified version of Bauschinger's (22) equation, as modified by Johnson in his work with materials of construction in 1897. Johnson was the first to use the constants 0.778 and 0.222, which are obtained in tests of rock specimens against steel platens.

The results of specimen size on strength were measured in the laboratory in 1914. The results of this work were reported by Rice (80) in 1929. The compressive strengths of large 54-in cubes were only one-eighth that of small cubes per unit area. Rice also reported in 1935 (77) and 1939 (78) on the geologic structure and mining practice combinations that produced coal bumps in France, Germany, and the United States. He proposed methods of bump control and identified two kinds of bumps he called pressure and shock bumps.

Holland (51) and associates, beginning in 1941, observed that end friction increased the compressive strength

of cubic specimens in the laboratory and pillar strength in the mine. He also observed mining conditions that produced bumps similar to those reported by Rice.

Grobbelaar (44), in 1970, presented a pillar design equation for square, rectangular, strip, and circular pillars. Most of his equations depended upon the compressive strength of a unit cube made up of smaller elements, for which a statistical analysis was used.

Wilson (87), when he was with the National Coal Board of England in 1972, published a constrained-core concept for pillar design. His results for pillar design have been widely used since that time.

Wardell (84), also formerly of the National Coal Board of England, published an alternative to the Wilson approach in 1977. Total pillar area, determined by the Wardell approach, is typically twice as large for a given entry width and mining depth as that determined by Wilson.

In 1976, Salamon and Oravecz (81) of South Africa published pillar design equations based on pillar geometry that were more significant than similar work of many others because they compared the design equation predictions to the in situ behavior in 125 mines.

In 1978, Barron (20) published the results of a novel method to establish the structural integrity of a mine pillar to improve pillar design by using compressed air at a standard volume and pressure between two packers in a drill hole in situ in the pillar. The decay rate of the air pressure was taken to be inversely proportional to the support capacity of the pillar. The theory associated with this work is to the effect that when a relatively large part of the pillar is confined, as indicated by a small pressure loss rate, the cracks are closed and the pillar can support a greater load.

In 1983, Barron (21) of Canada published a two-volume report in which he presented two computer programs for pillar design. His approach made use of horizontal force equilibrium on thin vertical slices parallel to one another and to one rib of the pillar, in a manner similar to the work of Wilson (87). The analysis was complex and had the appeal of analysis with computers.

In 1984, Babcock and Bickel (17) presented a paper showing that, (contrary to expectations of many researchers that large, flat pillars were indestructible), 13 of 15 coals tested from 11 coal seams could be made to burst in laboratory testing when the horizontal constraint was dynamically lost during loading.

In 1985, Hsiung and Peng (57) presented a design method for chain pillars in a longwall mining system. The design goal was for pillar survival after mining was completed for one panel.

In 1986, Lu (63) of the Bureau, published a pillar design procedure in which he used two hydraulic borehole

²Italic numbers in parentheses refer to items in the list of references preceding the appendix.

pressure cells (BPC) and one cylindrical pressure cell (CPC) developed by Panek (73-74) to estimate the absolute stress state in the coal seam, and the physical properties determined from triaxial stress tests in the laboratory. His method makes use of creep behavior to estimate the state of absolute stress from an in situ drill hole without overcoring.

In 1986, Babcock (12) presented equations for translating the pressures in BPC into the corresponding rock or coal stresses. The common practice of using the cell pressure changes as the rock or coal stress changes should be avoided.

In 1990, Babcock showed (16) that if the state of stress is the same for cylindrical specimens of different diameter-length ratios, the strengths are all the same.

This report presents an in situ pillar design procedure that uses only the ratio of the horizontal-vertical stress change as measured with two BPC and the angle of internal friction. That is, the change in strength results not from the change in geometry, but from the change in stress with geometry. Constraint, not geometry, is the heart of the pillar strength problem.

Even today, 1990 the most common method of mine pillar design is probably for a mine operator to use pillars

of the size as those already in use in other mines in the mining district with the same amount of overburden and coal seam thickness. The pillar sizes in use are initially selected by a trial-and-error process. This comparison of pillars of about equal volume does not involve the large errors that occur when small-scale laboratory test results between steel platens are compared with large-scale mine pillars between roof and floor rock. The laboratory test specimens are three to eight times stronger per unit area than the mine pillars.

One promising method of pillar design for future consideration is to treat the pillar as a foundation of civil engineering that bears on *both* the floor *and* roof. If a foundation for a bridge or building fails, the structure may be damaged. In mining, if either the roof or floor over or under a mine pillar fails, the pillar may be damaged. The three structural elements, roof-pillar-floor, acting together determine the suitability of the pillar size chosen. The design should be for the roof-pillar-floor structural sequence, not for the pillar alone. Because the bridge, building, and pillar are, in general, statically indeterminate, foundation failure may or may not damage or destroy the structure. In any case, the pillar design should be for the roof-pillar-floor structural sequence, not for the pillar alone.

PILLAR DESIGN EQUATIONS

BAUSCHINGER'S THEORY FOR DESIGN OF PILLARS OF ANY SHAPE

In 1912, Bunting (29) reported that

"The relation between the crushing-strength and relative dimensions of Swiss sandstone has been studied very exhaustively by Professor Bauschinger, as stated by Professor Johnson, and as a result of these tests, he recommends for all shapes of cross-sectional areas and relative heights the formula:

$$p = (((A)^{0.5}/(u/4))^{0.5}) (a + b(A)^{0.5}/h), \quad (2)$$

where p , A , u , h , a , and b , are the compressive strength per unit area, the loaded area, the length of the pillar perimeter, the pillar height, and two constants, respectively."

Any consistent units of length may be used, for example inches for laboratory specimen tests and feet for mine pillars. The length terms cancel to produce dimensionless products. The units for p are the same as the units for a and b as will be shown later. If these are in pounds per square inch, p is also in pounds per square inch.

From the wording in the Bunting paper, it is not clear which author, Bauschinger (22) or Johnson, was responsible for equation 2. However, Gonnerman (42) reported that

"(The) formula (of Bauschinger) was based on (the) area and length of (the) specimen. Prisms and cylinders of geometrically similar forms have the same strength (p , per unit area). In prisms of (the) same length and sectional area, strength varies inversely as (the) square root of (the) circumference."

From this, it is obvious that equation 2 is the work of Bauschinger. Equation 2 is in terms of strength per unit area. For pillars of rectangular shape where the pillar is w units wide and some dimensionless constant, k , times the width (kw) long, Bauschinger's equation 2 has the dimensions

$$p = (((L^2)^{0.5}/(4L/4))^{0.5}) (a + b(L^2)^{0.5}/L) = (a + b). \quad (3)$$

That is, p has the same units as a and b , as stated previously. L denotes the length unit.

It can be shown that Johnson's equation 4 is related to the Bauschinger equation 2. Johnson assumed that the

compressive strength, p , is related only to the smallest dimension, b_1 , and the height, h . In that case, a square pillar where the length equals the width can be used. Substituting into equation 2 where $A = b_1^2$ and $u = 4b_1$ gives

$$p = a + bb_1/h. \quad (4)$$

For a given coal or rock, the coefficients a and b are the same for all rectangular test specimens of about the same volume, but must have the same units as p (for example, pound per square inch). If the relative compressive strengths of a given geometry are used (that is, a specimen of cubic shape) called $p_{1:1}$ as a reference strength, the relationship of Johnson is obtained. This is

$$\frac{\text{Strength of rectangular prism}}{\text{Strength of cube}} = p/p_{1:1} = a/(a+b) + (b/(a+b)) b_1/h. \quad (5)$$

From many laboratory tests against steel platens, the values of the coefficients are $a/(a+b) = 0.778$ and $b/(a+b) = 0.222$. In some later equations, the variables a and b were renamed so $a = 0.778$ and $b = 0.222$. Equation 5 becomes

$$\frac{\text{Strength of rectangular prism}}{\text{Strength of cube}} = 0.778 + 0.222 b_1/h, \quad (6)$$

where b_1 , and h are the smallest lateral dimension and height of the prism, respectively.

Johnson's contribution of normalizing the strength per unit area of any rectangular specimen to the strength of a cubic specimen has been an important one that is widely used today in rock mechanics. The result is that equation 6 is a dimensionless equation that can be used with consistent dimensions of any kind, because $p/p_{1:1}$, $a/(a+b) = 0.778$, $b/(a+b) = 0.222$, and b_1/h are all dimensionless ratios.

Later, Obert and Duvall (70) used an equation similar to equation 6 for tests on right circular cylindrical specimens (from diamond drill cores) as,

$$\sigma/\sigma_{1:1} = 0.778 + 0.222 D/L, \quad (7)$$

where σ , $\sigma_{1:1}$, D , and L are the compressive strength in the axial direction of a cylindrical specimen of any length, of a cylindrical specimen when the length is equal to the diameter, and the diameter and length of the specimen, respectively. Again, only dimensionless ratios $\sigma/\sigma_{1:1}$, 0.778, 0.222, and D/L are used. The values of a and b in the above equations change with end constraint conditions, as is shown in later sections on end constraint. The values of $a = 0.778$ and $b = 0.222$ were obtained with the end

constraint conditions provided by steel platens when specimens are tested in the laboratory. The corresponding values of these parameters for determining in situ pillar strengths vary widely when roof and floor rock, and not steel, are providing the end constraint (19). In equation 2, the specimen could be of any cross-sectional shape. In equation 6, the cross-sectional shape is rectangular, with the square cross-sectional area a special case. In equation 7 the cross-sectional shape is circular.

Equation 2 can be rewritten for a pillar of rectangular shape with the smallest width denoted by w and the longer dimension by kw , where k is some multiple of the w value. Then $A = kw^2$ and equation 2 becomes

$$p = ((4(kw^2)^{0.5}/(2w + 2kw))^{0.5})(a + b(kw^2)^{0.5}/h), \quad (8)$$

where $k \geq 1$. This reduces to

$$p = ((2(k)^{0.5}/(1 + k))^{0.5})(a + b(k)^{0.5}w/h). \quad (9)$$

If the strength per unit area, p , is normalized to the strength of a cubic specimen, $p_{1:1}$, and the $a = 0.778$ and $b = 0.222$ values of Johnson are used,

$$p/p_{1:1} = 0.778((2(k)^{0.5}/(1 + k))^{0.5}) + 0.222((2(k)^{1.5}/(1 + k))^{0.5})w/h, \quad (10)$$

when $k = 1$, $w/h = 1$, and $p/p_{1:1} = 1$.

If pillar design is based on replacing one pillar size and shape now in use that gives satisfactory support, with another pillar size and shape obtained from equation 10, the results should be satisfactory.

Both Bauschinger (22) and Johnson were unaware of the role of volume on pillar strength because only geometry and not volume is used in their equations.

It is now known that compressive pillar strength per unit area decreases with the volume. Compressive strengths obtained from small laboratory specimens are three to eight times stronger per unit area on the average than the larger specimens tested (80). The difference in strength between small laboratory test specimens and mine pillars may be even larger. This result is obtained because the average strength of selected test specimens from intact pieces of coal or rock does not represent the average strength of the larger pillar with many fractures in the form of cleats, joints, faults, unconsolidated bedding planes, etc. When two mine pillars of different shapes are considered and the pillar volumes are very large, the difference in strength between one in situ pillar and another in a given mine, on the basis of volume, can be ignored.

Three graphs, useful for pillar design, using equation 10 are given in figures 1, 2, and 3. In figure 1, the relative

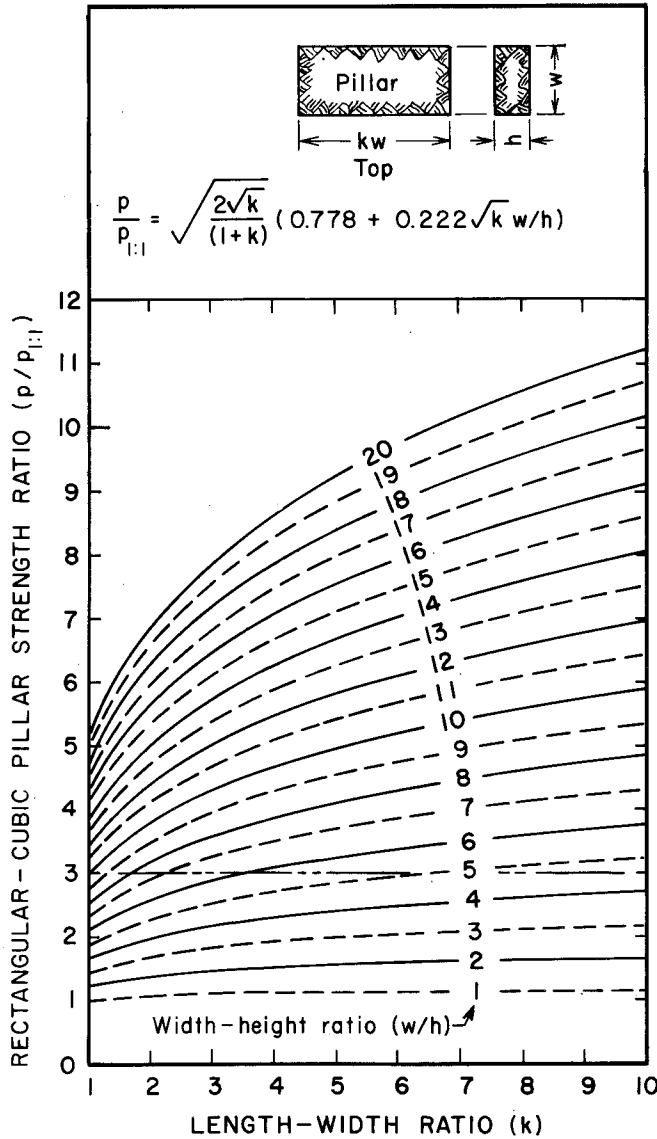


Figure 1.—Compressive strength of rectangular pillars normalized to strength of pillar of cubic shape as function of length-width ratio. Example: For broken line with $p/p_{1:1} = 3$, rectangular pillars with several different width-height ratios (w/h) and length-width ratios (k) have same compressive strength per unit area.

compressive strengths, $p/p_{1:1}$, per unit area, for pillars of rectangular shape to pillars of cubic shape are given as functions of the length-width ratio, k , and of the width-height ratio, w/h . For very long ($k = 10$) and squat ($w/h = 20$) pillars, the strength per unit area was 11.24 times that of a pillar of cubical shape. For square pillars ($k = 1$), the strength per unit area was increased 5.22 times over that of a cubical shape by reducing the w/h value to 20. For a long pillar ($k = 10$) with the height equal to the width ($w/h = 1$), the increase in length increased the strength per unit area only 1.12 times. The squat pillar geometry (large w/h ratios) is very effective in changing

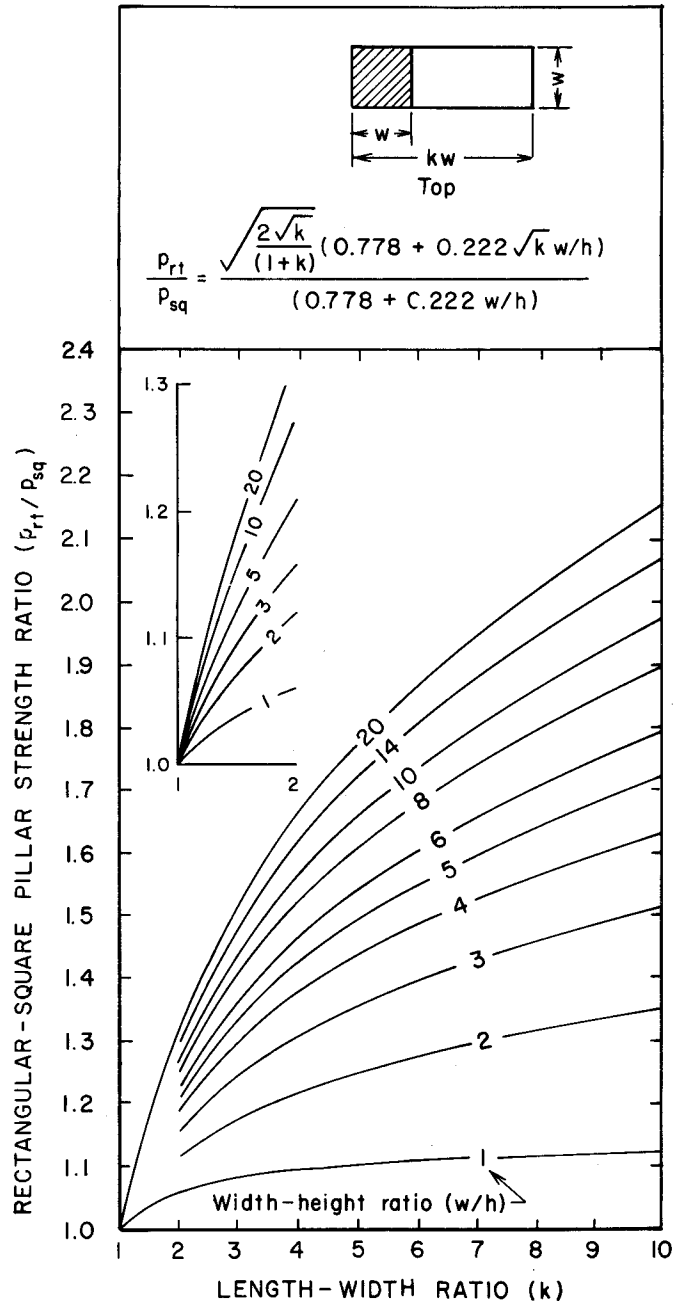


Figure 2.—Ratio of compressive strength per unit area of rectangular pillar to square pillar as function of the length-width ratio.

the pillar strength, because it brings the entire pillar mass near to the end constraint. If the loading was everywhere the same with no end friction, the value of $p/p_{1:1}$ would be 1 for all values of k and w/h (16). Figure 1 shows how both k and w/h acting together change the pillar strength per unit area.

Figure 2 gives the relative compressive strengths per unit area of rectangular pillars normalized to the strengths of square pillars with the same least width-height ratio,

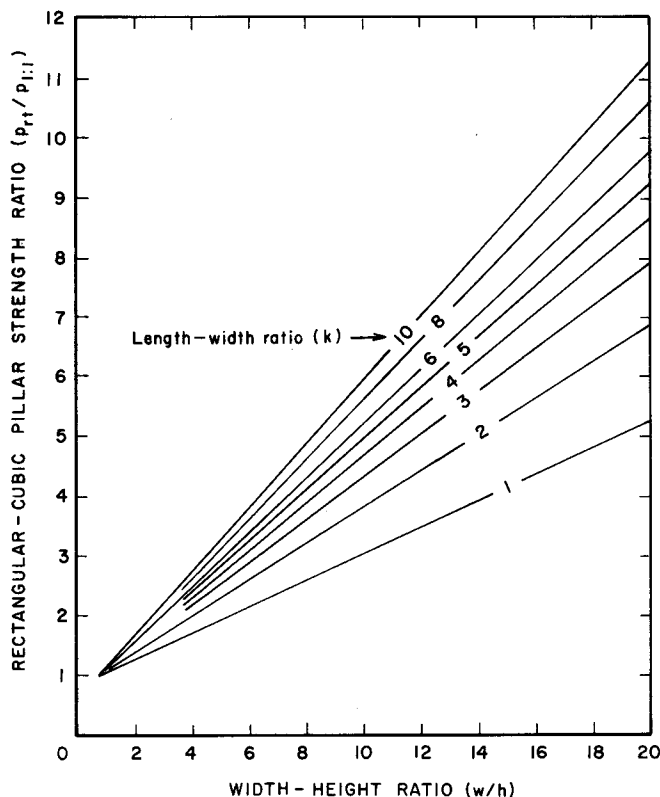


Figure 3.—Comparison of compressive strength per unit area of rectangular pillar to that of cubic pillar as function of width-height ratio.

p_r/p_{sq} . For the present purpose, the difference in volume effect is ignored. Here, the maximum increase in relative strength was 2.15 for $k = 10$, and $w:h = 20$, compared to a maximum increase in relative strength of 1.2 for $k = 10$, and $w:h = 1$. This shows that for a given pillar height, the k value changes the relative strength less than the $w:h$ ratio.

Figure 3 gives the relative compressive strengths per unit area of rectangular pillars normalized to the strength of cubic pillars with $w:h = 1$. The strengths are proportional to the $w:h$ ratio for a given value of k . That is, for a given k value, the change in strength is a linear function of $w:h$.

Figure 1 can be used to select several different pillars all of rectangular shape with the same strength per unit area according to equation 1-9. This procedure will allow a mine operator to change from one rectangular pillar geometry in use that gives satisfactory support to another rectangular geometry that should also give satisfactory support. Assume some value of $p/p_{1:1}$, say 3, in figure 1. The k and corresponding $w:h$ ratio values where the $p/p_{1:1} = 3$ line and the $w:h$ curves intersect are given in table 1. Equations 2 and 10 involve only the pillar geometry and

not its size. Assume the stable pillar in mine use is 60 by 60 by 6 ft in size. All other pillars will also be 6 ft high for the same coal seam. The pillar sizes in table 1 are calculated in this way: for $p:p_{1:1} = 3$, $w:h = 9$, and $k = 1.25$: since $h = 6$ ft, $w = 9 \times 6 = 54$ ft. The pillar length is k times w or $1.25 \times 54 = 67.5$ ft. The areas of the pillars increase with an increase in the k values, as shown in table 1. Any pillar load or strength per unit area must involve the tributary area of overburden carried by the pillar. While this is not stated by Bauschinger, or at least was not reported by Johnson or Gonnerman (42), the increase in pillar size with increasing k suggests that this notion was used by Bauschinger. However, his equation 2 does not involve entry size, and the strength per unit area does not increase with area. If the pillar size of Bauschinger is used (in the form of equation 10) with an entry width of 20 ft, the recoveries are all about 44 pct, as shown in table 1. The pillar geometries for the $(k, w:h)$ pairs of the same unit strength as the square pillar are shown in figure 4. These pairs are (1,10), (1.25, 9), (1.63,8), (2.29,7), (3.60,6), (6.80,5), and (9.40, 4.6).

Table 1.—Comparison of rectangular pillar sizes¹ for Bauschinger's equation in form of equation 10 with tributary area results of equation 18

Width-height ratio (w:h)	Ratio (k)	Width (w), ft	Length as multiple of width (kw), ft	Area (A), ft ²	Recovery (R), pct
MODIFIED BAUSCHINGER'S EQUATION 10					
10 ...	1.00	60	60.0	3,600	43.8
9	1.25	54	67.5	3,645	43.7
8	1.63	48	78.2	3,756	43.8
7	2.29	42	96.2	4,040	43.9
6	3.60	36	129.6	4,666	44.3
5	6.80	30	204.0	6,120	45.4
4.6 ...	9.40	27.6	259.4	7,159	45.4
TRIBUTARY AREA METHOD WITH EQUATION 18					
10 ...	1.00	60	60.0	3,600	43.8
9	1.25	53.9	67.4	3,633	44.6
8	1.63	48	78.2	3,754	43.9
7	2.29	42.2	96.6	4,076	43.8
6	3.60	36.7	132.1	4,848	43.8
5	6.80	31.9	210.8	6,724	43.9
4.6 ...	9.40	30.3	284.8	8,629	43.7

¹See figure 1.

Note.—Entry size is 20 ft, height is 6 ft.

Next consider the pillar sizes that would be chosen from tributary area relationships alone. If the overburden stress concentrations on square and rectangular pillars are denoted at p_{sq} and p_m , respectively, with entry widths R_1 and R_2 , the following relationships are

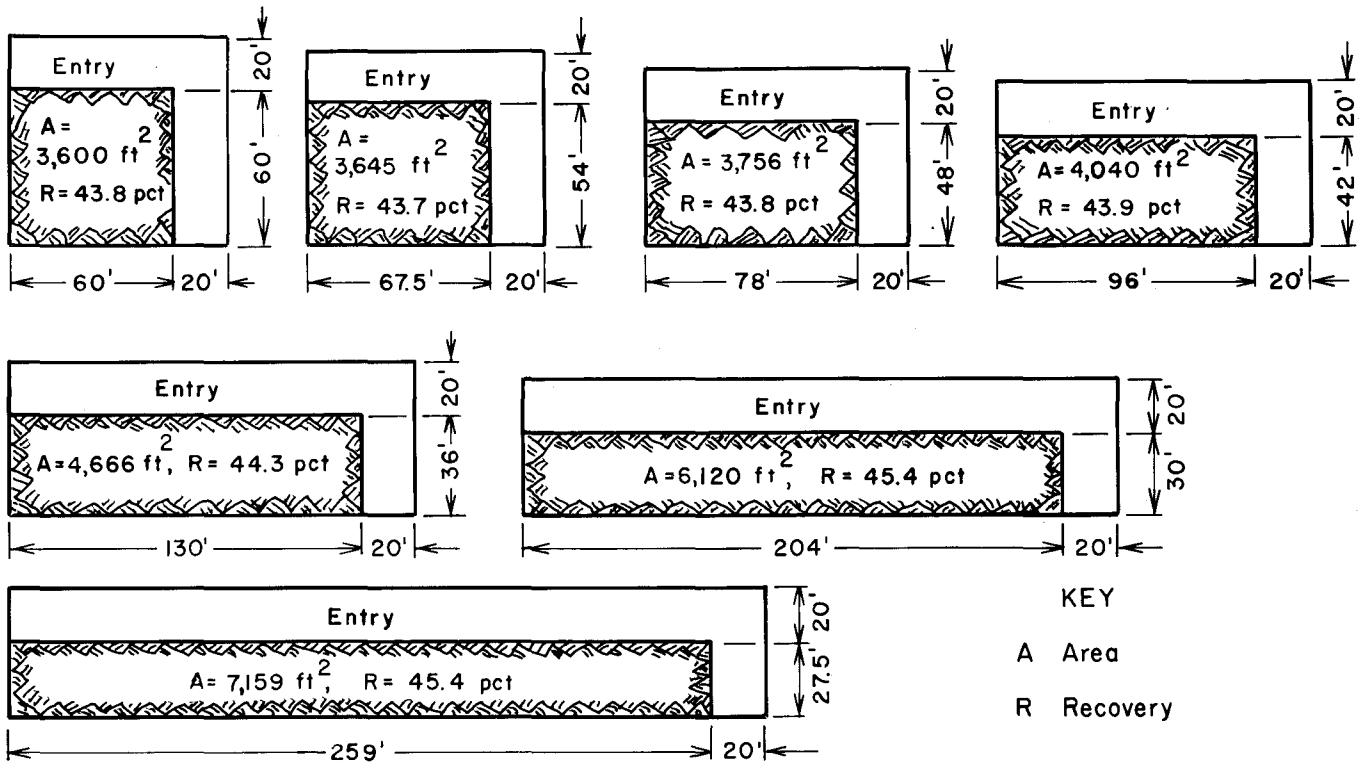


Figure 4.—Rectangular pillars of different shapes and sizes that have same average compressive strength according to Bauschinger's work as represented by equation 1-6 and table 1.

$$p_{sq} = (w_1 + R_1)^2 / w_1^2 \quad (11)$$

and

$$p_{rt} = (w_2 + R_2)(kw_2 + R_2) / (kw_2^2). \quad (12)$$

If $p_{sq} = p_{rt}$, then

$$(w_1 + R_1)^2 / w_1^2 = (w_2 + R_2)(kw_2 + R_2) / kw_2^2. \quad (13)$$

The stress concentration on a square pillar of width w_1 and room width R_1 can be expressed as

$$(w_1 + R_1)^2 / w_1^2 = \text{a known constant} = C, \quad (14)$$

for given values of w_1 and R_1 .

If a rectangular pillar has a least width of w_2 , a length of kw_2 , a room width of R_2 , and $p_{sq} = p_{rt}$, then

$$(w_2 + R_2)(kw_2 + R_2) / kw_2^2 = C \quad (15)$$

or

$$k(C-1)w_2^2 - R_2(k+1)w_2 - R_2^2 = 0. \quad (16)$$

The two roots are obtained from the quadratic equation

$$w = (R_2(1+k) \pm (R_2^2(k+1)^2 + 4k(C-1)R_2)^{0.5}) / 2k(C-1). \quad (17)$$

If R_2 is 20 ft, this equation becomes for all k values

$$w = (20(1+k) \pm (400(k+1)^2 + 1.245k)^{0.5}) / 1.556k. \quad (18)$$

The roots of this equation are both real, but one is positive (+) and the other is negative (-). Since a pillar cannot have a negative dimension, only the (+) root is of physical significance.

The sizes of the rectangular pillars that are equal in compressive strength to the square pillars of dimensions 60 by 60 by 6 ft presented in the second half of table 1 are derived as follows. The k values are taken from table 1 and the entry width is 20 ft. For $k = 1.25$, $w = 53.9$ ft from equation 10. The length is then $1.25 \times 53.9 = 67.4$ ft. This is almost exactly the same pillar size obtained with the modified Bauschinger equation 10. Notice that the

pillar dimensions for the top and bottom half of table 1 are nearly the same (within 2 ft) for k values of 3.6 or less. Bauschinger's equation 2 relates the unit strength to its shape. This strength per unit area, p , is independent of the pillar size. His results appear to have been derived from forces applied in the laboratory by a compression machine to specimens of different shapes. A mine pillar carries an overburden load related to its mining depth and percent recovery is apparently not considered. However, when pillar sizes selected with equation 10 are compared to the pillar size selected with the tributary area method and a 20-ft entry width is used in both cases, the results are almost the same. If a mine pillar of one shape is in use that gives an acceptable support, it can be replaced by another pillar of different shape using the equation of Bauschinger (10), or of the tributary area method equation (18) about equally well. Notice that recovery is about 44 pct for all the pillars in table 1. The concept of the tributary area method is preferred over the Bauschinger equation because it is so easy to use and remember. This might not be true if some pillar shape other than square or rectangular is used. In that case, the Bauschinger equation in its original form (2) may be worth the additional effort required in its use.

BUNTING'S TRIBUTARY AREA METHOD FOR PILLAR DESIGN

In 1912, Bunting (29) reported on a method of chamber-and-pillar mining for anthracite coal seams. While he does not show what he means by a chamber-and-pillar mining system, his equation 9 of reference 29 can be shown to represent what is now called a tributary area method of pillar design for a mining system with parallel entries (chambers) separated by parallel continuous pillars in the direction of the entries as shown in figure 5. He was perhaps the first to select the pillar width on the basis of an overburden load extending from the centerline of the entry on one side of the pillar to the centerline of the entry on the other side of the pillar. This conclusion is supported by his statement about mining practices up to the time of his report which was,

"... it has been quite usual to work both overlying and underlying veins with the same width of chambers and pillars, when the lower vein was two and one half times as thick, and twice as far below the surface as the upper." This shows that it was not a standard practice in 1912 to increase pillar size with mining depth. In 1911, Conner (32) reported on his work for the Councils and Board of School Control of the City of Scranton, PA, to investigate cave-ins over anthracite mines under

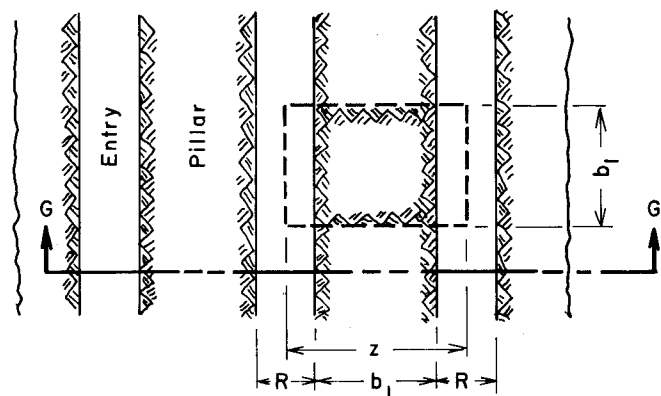
that city. He stated, "In our examination of the mines and maps, it was found that in nearly all beds, irrespective of the thickness or depth from the surface, the old empirical rule of leaving about one-third of the coal as pillars had been followed"

From these two references, in 1911 there was a lack of awareness, at least for anthracite mining in Pennsylvania, of the need to increase the pillar size with depth and seam thickness.

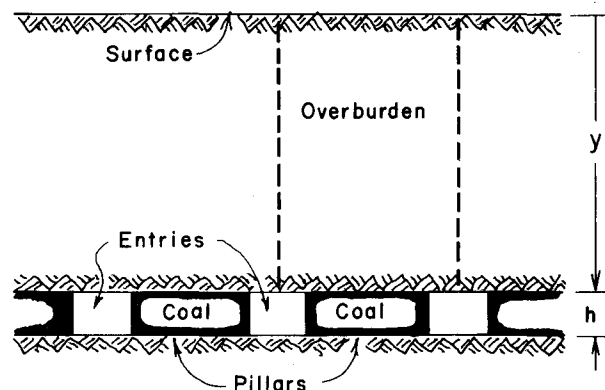
For specimens of rectangular shape, Gonnerman (42) reported that Johnson fitted the Bauschinger test data for Swiss sandstone with the equation

$$p = 5,500 + 1,565 b_1 / h, \quad (19)$$

where p (compressive strength), 5,500, and 1,565 are all in pound per square inch; b_1 is the smallest specimen width, in inches; and h is the specimen height, in inches. The specimen length was ignored. Bauschinger's work in 1867



Plan view of chamber-and-pillar system



Vertical section G-G of chamber

Figure 5.—Tributary area method for chamber-and-pillar mining system (29).

(22) was cited by Gonnerman (42). Johnson also normalized the compressive strength of Bauschinger's results for sandstone for rectangular specimens to the strength of cubic specimens with the equation

$$\frac{\text{Strength of prism}}{\text{Strength of cube}} = 0.778 + 0.222 b_1 / h. \quad (20)$$

The equation is dimensionally homogeneous and can be used with any stress and length units. Bunting (29) cited work by Johnson who fitted the test results for anthracite coal obtained in testing by Daniels (36) and Carpenter (30) with cubes 2 to 6 in on a side and on rectangles 2.25 to 12.25 in high to the equation

$$p = 1,750 + 750 b_1 / h, \quad (21)$$

where the variables are the same as in equation 19. Because b_1/h is a dimensionless ratio, any consistent units of length can be used. The variables p , and the coefficients 1,750 and 750 are all in pound per square inch. To apply the laboratory results to mining conditions, Johnson used a safety factor of 2.5 to obtain the equation

$$p = 700 + 300 b_1 / h, \quad (22)$$

where p , 700, and 300 are all in pound per square inch.

An assumed average pillar strength of 1,000 psi was used for mine pillar design purposes when a cube tested in the laboratory had an actual strength of 2,500 psi to provide what was considered a safety factor of 2.5. Since that time, differences as large as eight times have been observed on the basis of test volume alone (80). The equation in normalized form is

$$\frac{\text{Strength of prism of coal}}{\text{Strength of a cube of coal}} = 0.70 + 0.30 b_1 / h. \quad (23)$$

The terms of this equation are all dimensionless, the relative compressive strength of a prism of coal being compared to the compressive strength of a cube of coal. Note the similarity to equation 20 with somewhat different coefficients. The history of pillar design by many investigators since the work of Johnson in 1897 cited by Gonnerman (42) has been one of largely fitting equation 1 of the introduction with coefficients to satisfy the conditions observed in mining.

Equation 20 is similar to one reported in 1967 and popularized by Obert and Duvall (70) for use with cylindrical test specimens in the laboratory loaded between two steel platens. Their equation is

$$\sigma / \sigma_{1:1} = 0.778 + 0.222 D / L, \quad (24)$$

where σ is the average compressive strength, pound per square inch, of a right circular cylindrical specimen of rock, coal, or other solid material of a diameter to length domain of $0.5 < D:L < 3$; normalized to the strength, $\sigma_{1:1}$, of a cylindrical specimen of $D:L$ of 1. Since both ratios, $\sigma/\sigma_{1:1}$ and $D:L$ are dimensionless, any units of stress and length can be used.

In the tributary area method for chamber-and-pillar mining shown in figure 5, the weight of the overburden on a pillar is taken to result from the volume of rock that extends from the centerlines of the entries that create the pillar. In chamber-and-pillar mining, only parallel entries of width R , in feet, are used, and pillar length in the direction of the entries is arbitrary. Normally, this would be taken to be one unit long, but any length could be used, and Bunting apparently used a length equal to the pillar width, b_1 . The tributary area load carried by the pillar, enclosed by the broken line in figure 5, is of width z and length b_1 . Similar loads occur along each b_1 unit of length for all the pillars in the system. The total overburden force on the pillar, F_1 , in pounds, at mining depth y , least width b_1 , entry width R (all in feet), and a rock density of 144 lb/ft³ (as assumed by Bunting), is

$$F_1 = 144 z b_1 y = 144 (R + b_1) b_1 y. \quad (25)$$

The average pillar stress, p_1 , in pound per square inch, is obtained by dividing this force by the pillar area, A_1 , in square inches. The area is

$$A_1 = 144 b_1^2. \quad (26)$$

So,

$$p_1 = F_1 / A_1 = z y / b_1 = (R + b_1) y / b_1. \quad (27)$$

If this average stress, called p_1 , is made equal to the design stress when the strength for a cube was taken as 1,000 psi (as assumed by Bunting), his equation 9 (29) for the size of pillars in chamber-and-pillar mining results:

$$z y = 1,000 (0.70 + 0.30 b_1 / h) b_1. \quad (28)$$

Bunting shows three graphs in the work cited for which this equation is solved for the mining depth, y , in feet. This gives solutions for unequal increments of mining depth.

More useful results are obtained by solving for the pillar width, b_1 , as the dependent variable. Equation 28 can be rewritten as a polynomial in b_1 as

$$300 b_1^2 / h + 700 b_1 - y z = 0. \quad (29)$$

The two roots are obtained by the quadratic equation as

$$b_1 = (-700 \pm (490,000 + 1,200 zy/h)^{0.5}) h / 600. \quad (30)$$

The quantity under the radical sign (radicand) is always positive (+) and the square root is always positive (+) and larger than 700. Therefore, the two roots are real and different. One root is positive (+) and the other is negative (-). Only the (+) root has any physical meaning. If the solution in terms of the entry width, R , and the pillar width, b_1 , is desired, z can be replaced with the quantity $R + b_1$. Equation 30 is plotted in figure 6 for an entry width $R = 20$ ft, for a seam thickness domain of $4 < h < 11$, and for mining depth domain of $400 < y < 3,400$ ft. The dependent variable, b_1 , ranges from 0 to 100 ft. The broken line with arrows gives an example of the use of this graph. In this case, it is assumed that the seam height is 7 ft, the mining depth is 1,400 ft, and the required pillar width is 35 ft.

Bunting (29) also gave an equation for checker-board mining (now called room-and-pillar mining) that *does not* satisfy the tributary area requirements. The fact that his equation is almost the same as the tributary area equation suggests that he was attempting to use this relationship. His equation 10 is

$$yz = 1,000 (0.70 + 0.30 b_1 / h) b_1^2. \quad (31)$$

The correct tributary area equation for room-and-pillar mining from figure 7 is derived as follows. The force, F_2 , from the overburden applied to the pillar, in pounds, again assuming a density of 144 lb/ft³ is

$$F_2 = 144 z^2 y. \quad (32)$$

The pillar area, A_2 in square inches, which supports this force is

$$A_2 = 144 b_1^2. \quad (33)$$

The average pillar stress, p_2 in pound per square inch, is

$$p_2 = F_2 / A_2 = z^2 y / b_1^2. \quad (34)$$

Again, if this stress, p_2 , is made equal to the design stress when the strength of a cube is taken as 1,000 psi as assumed by Bunting, the result is

$$yz^2 = 1,000 (0.70 + 0.30 b_1 / h) b_1^2. \quad (35)$$

This equation which *does* satisfy the tributary area conditions, is different from the Bunting equation 10 (31) by a factor of z . Equation 35 is the correct equation to use

for square pillars when the tributary area method is to be used, and 1,000 psi is taken to be a reasonable estimate of compressive strength in situ. A compressive strength of 1,000 psi may be a reasonable strength for a strong coal in situ, such as the Wilberg coal in Utah for which laboratory strengths of 4,500 psi or more are commonly observed. A factor of 4.5 would exist between the compressive strength in the laboratory and in the mine.

PILLAR BEHAVIOR CONCEPTS REPORTED BY RICE

Rice, formerly the Chief Mining Engineer with the USBM, was an early prolific contributor to mine health and safety research, including pillar behavior. In 1929, Rice (80) reported on work done by the USBM from 1912 to 1914. The belated report was written in response to public interest in the test results. The work was done under the direction of Enzian with help from Davies and Rice. Rice reported, "If a coal pillar is left of sufficient size, so that the spalling or crushing of the edges will not

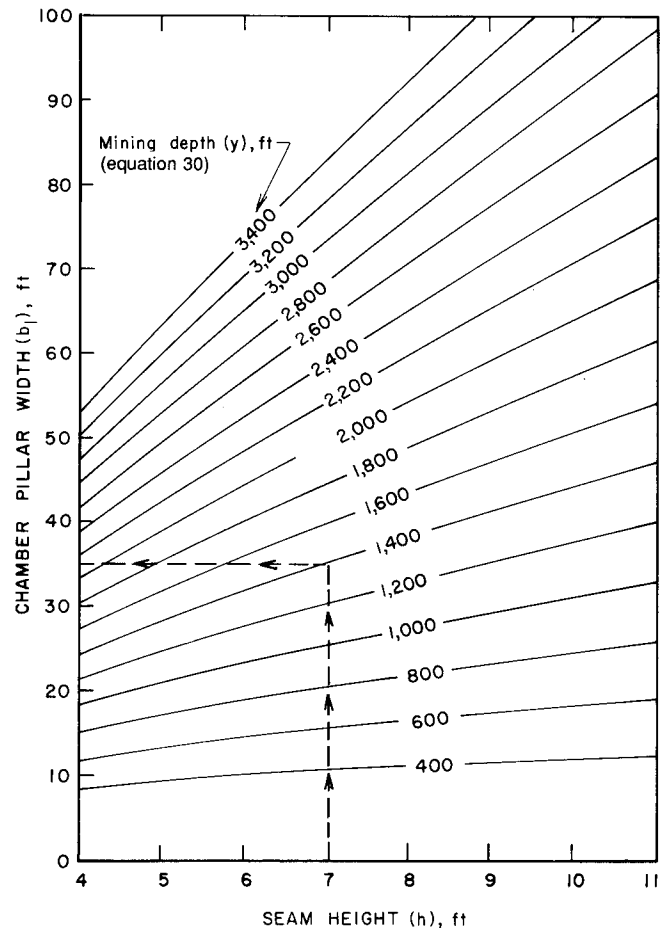


Figure 6.—Chamber pillar width determined with equation 30.

affect the interior, the average unit strength will be far higher than that developed in a compression machined by a specimen that is free to rupture on four sides." Further, he reported "In many instances, however, in both horizontal and dipping workings, the weakness of a "draw slate" over the top of a coal bed, a clay or shale parting, or a clay floor that squeezes out under pressure and drags off vertical pieces of coal at the pillar face causes failure to support the overlying strata, rather than the compressive weakness of the coal itself." He also reported on compression tests on coal cubes up to 54 in on a side made at the National Institute of Standards and Technology in a 10,000,000 lbf compression machine. In related tests, the strength decreased with an increase in size of the test cube. The strength per unit area was eight times as great for the smallest cubes as it was for the largest cubes tested. The awareness of decreasing strength with increasing specimen size was known at least as far back as 1914. It is conventional at this time (1990) to take the strength per unit area of in situ pillars as $1/3$ to $1/8$ the strength of specimens obtained in the laboratory, because of the decrease in strength with increasing volume. This decrease

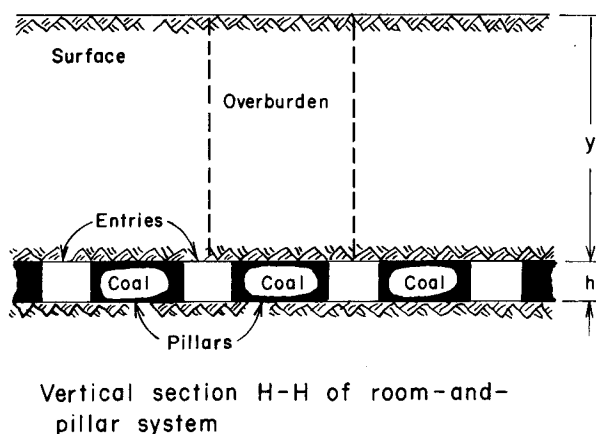
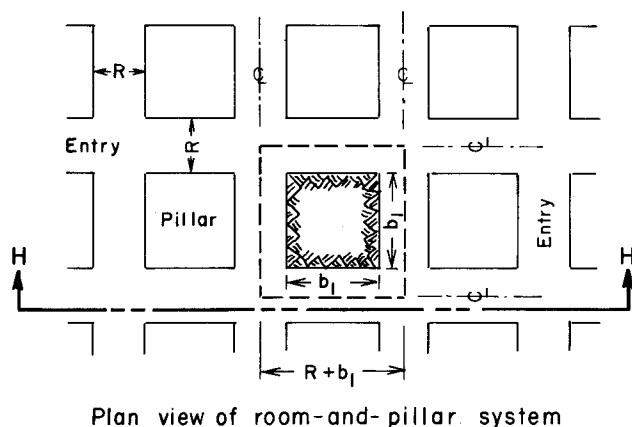


Figure 7.—Tributary area method for room-and-pillar mining system (29).

in strength is offset in varying degrees by the increase in pillar strength with an increase in the width-height ratio.

In 1935, Rice (76-77) gave "a brief review of suggested causes and remedial measures given in English, Canadian, and United States reports on bumps in coal mines, and then presented suggestions arising from his investigations of the bumps in the Cumberland mines, made at the request of mine operators, to determine the causes and remedies." Some of his comments condensed from the original follow. In a burst, miners may be killed or injured by impact, buried by coal, or flung by air pressure. Conditions favorable for bursts are strong, rigid roof and floor strata; strong coal (4,000 to 5,000 psi for a specimen with a width-height ratio of greater than 3); and a mining depth of greater than 1,000 ft. Bursts are produced by faulty mining methods, including using undersized pillars, leaving remnant pillars past the withdrawal line, leaving wedge-shaped pillars pointing in the inby direction (pointing toward the working face), and mining separate panels rather than using long retreat lines. Strong roof rock rarely fails during the bump event; however, weaker roof materials adjacent to the coal seam are commonly included in bump debris. Under strong roof, an open space 4 or 5 in high results between the crushed coal and roof after the bump. Unfilled cribs have been moved several feet laterally and left standing intact. A great blow is imparted to a pillar or group of pillars, and it may not be the pillar nearest the goane (now called gob) or worked-out territory that receives the greatest shock. Poor mining methods alone do not cause bursts, but when used in combination with the natural conditions listed, bursts occur. He proposed the following remedial measures:

1. Mine less than 20 pct of coal in first mining,
2. The retreat should mine all the pillars on straight and long retreat faces,
3. Use lines of rock-filled cribs adjacent to the retreat faces to cushion the roof strata or cause it to break.

Rice wrote a report on mining in Europe (78) in which he cites a French report by Jarlier (59), an authority on rock bursts, that

the roof layers of mines form domes over residual pillars as a result of the action of lateral forces. When the bending stresses in these deformed strata exceed their ultimate strength, a sudden fracture occurs above the dome, and the residual pillar receives a terrific shock, the potential energy of the roof being transformed into kinetic energy. This results either in a bursting of the coal pillar or in a sudden upheaval of the floor of the workings. Jarlier's theory explaining the occurrence of smaller rock bursts is based on the

concept of the Weber (86) pressure wave, according to which a dome is formed over the standing coal as well as over the residual pillars. When through the advance of the coal face the equilibrium of the highly stressed layers is disturbed, bumps follow.

Rice (78) also reported on German theories of coal bumps. "Weber (86) ... advanced the theory that as a result of mining operations hollow spaces are formed between the rock layers in the roof, and sudden failure of the enclosing roof strata causes the bumps." Also, "A number of German investigators hold theories similar to those of Jarlier, including Spackeler, Lindemann, and others."

Spackeler (83) assumes that the more powerful bumps are caused by bending of the hard main roof, which finally leads to a sudden rupture and liberation of great forces, as a consequence of the elastic rebound. In addition to this, the most varying effects may be produced at a considerable distance from the seat of the bumps. He holds that while stowing may not prevent the bending of the hard roof strata up to the breaking limit, it will nevertheless retard rupture and result in less frequent bumps.

Rice (78) concluded from his work that there are two distinct types of bumps, which he called pressure bumps and shock bumps.

Pressure bumps occur when the unit loading on the pillars exceed their bearing strength in places where the coal and roof and floor are strong. Under such conditions the pillars occasionally are ruptured violently, just as a small block of coal or rock in a testing machine will rupture suddenly and violently when the ultimate compressive strength is reached. These effects are similar to those of most rock bursts, as in the deep gold mines of South Africa and in the deep copper mines of Lake Superior.

Rice continued, "This type of bump rarely occurs in coal mines where the roof strata over the coal seam are generally thick shale of easily broken, laminated structure, where no thick, rigid stratum, like a massive sandstone or conglomerate, is present immediately over, or within 10 or 15 feet of, the coal, or where the floor is so soft that if the pillars are laid out too small a squeeze but not a sudden bump may result."

Further, "Shock bumps are thought to be due to the breaking of a massive, rigid stratum, which may be a

considerable height above the coal. The stratum acts as a beam or plate; subsidence of the immediate coal roof leaves it spanning a wide goaf. When it breaks it causes a powerful, hammerlike blow to be delivered to the immediate roof, which it transmits as a shock wave to the coal pillars."

"From his observations regarding bumps the senior author (taken from Rice (79)) concludes that in coal mines where natural conditions are conducive to their occurrence, they (bumps) are intensified by the following factors when room-and-pillar mining methods are used:

- (1) Pillars are too small.
- (2) Pillars project during withdrawal.
- (3) Pillars are narrowed to points by diagonal slicing of the pillar ends.
- (4) Pillars are pulled in separate panels, rather than by a long, straight retreat line.
- (5) All pillars are taken out."

Rice reported (78) "Some of the measures recommended by Spackeler (Germany) (83) for combating bumps are: 1. Mining to be pursued in a uniform plane with a straight working face; no remnant pillars to be left. 2. Systematic encouragement to breaking down of the roof strata, chiefly the main roof. 3. Uniform working of adjacent seams."

Rice also reported, "Gaertner (41) links bumps very definitely to gas outflow. This view is opposed by most other investigators." That Gaertner's views were still strong in Europe in 1964 is apparent from a symposium on gas outbursts held in France, where most papers emphasized the role of gas under pressure as the transport mechanism in bursting. In this symposium, Belin (24) reported that former U.S.S.R. researchers proposed a four-stage theory of outburst: (1) elastic energy crushes the coal, (2) free gas ejects the coal, (3) desorbed gas propels the coal, and (4) the outburst ends when the friable part of the coal is expelled.

Babcock (17) reported that bursts are caused by the sudden loss of pillar end constraint when (1) the pillar while constrained is loaded beyond its unconfined strength; (2) the constraint is dynamically lost and the pillar can no longer support the stored elastic strain energy; and (3) dynamic failure (bursting) results. That is, the bursting is produced by the sudden loss of *horizontal* constraint rather than by the sudden dynamic hammering by a changing *vertical* load as suggested by the conclusions of Rice (76), Jarlier (59), and Spackler (83). This subject is treated in the section on the bursting of model coal pillars in the laboratory. Unreported laboratory work by this author in which air pressures to 500 psi were applied did not transport coal broken by stress until the loading stresses were greatly reduced. At that time, the broken coal was rapidly dispersed. In mining, if no stress burst was produced, and

the vertical pillar loads were decreased with progressive crushing, a gas outburst could result. The coal is first crushed by stress and then transported by gas pressure when the constraint stress is reduced and cannot hold the broken coal in place.

PILLAR DESIGN BY HOLLAND AND ASSOCIATES

Professor Holland was a pioneer in the field of rock mechanics relative to coal mine design with his most significant contributions in the domain of concepts explaining coal bumps or bursts. He was a professor and/or dean first at Virginia Polytechnic Institute (VPI) and later at West Virginia University. A brief summary of some of his ideas follows.

In 1941, Holland (51) reported a decrease in compressive strength with an increase in test cube size in the laboratory for both soft and weak and hard and strong coal. The strength of small cubes of bituminous coal cubes varied as the square root of the ratio (width of specimen-thickness of specimen).

"The factor that causes the increase in strength as the L/H (lateral dimension/height for rectangular specimens) ratio increases must be the effect of the confining pressure due to friction between the faces of the coal specimen and the bearing blocks of the test machine. The ultimate bearing stress of coal or the stress under which it deforms by plastic flow or by squeezing is the maximum stress a pillar can develop."

In the work cited, Holland (51) also concluded that (1) larger pillars are less likely to fail suddenly than smaller pillars of the same total area; (2) pillars of strong coal should be larger than those of weak coal; (3) larger pillars with a relatively smaller confined perimeter are less likely to burst than smaller pillars; and (4) heavy pack walls to prevent high roof stress is the best way to prevent shock bumps. He further concluded, "In view of the data in the foregoing pages, it does not seem possible to predict the occurrence of bumps."

In 1954, Holland and Thomas (56) reported "Analyses of 117 occurrences of bumps in coal mines showed that 67.6 percent (2/3) are associated with pillar-line points. Slabbing and splitting pillars, development, and other unfavorable mining practices in abutment areas may be contributory or primary causes of bumps." Further, they stated "the cure for pillar-line bumps is to recognize areas that can be expected to become highly stressed and to project the mining plan and operations to eliminate such areas insofar as possible. If such areas cannot be eliminated, then the mining plans and operations should be projected

to avoid the necessity of disturbing these areas before the stresses are dissipated."

In 1955, Holland (52) reported in a footnote

"the key to this situation (minimum width of the pillar that will not fail abruptly) may lie in the coefficient of friction, under extremely high loading conditions, between the coal and the roof and floor or within the coal itself. The coefficient when the coal is loaded beyond a critical unit load, or following a very small movement of the coal relative to the top and bottom rocks, may suddenly decrease and permit *an instantaneous decrease of the constraint force, thereby permitting a pillar burst to occur* (Author's italics). In this connection the brown stains on the roof, common to all burst areas indicate relative motion between the coal and top rock under high stress conditions at the interface. More research is warranted upon this phase of bursts." Holland also related strain energy to rock bursts. He stated, "Coal and associated rock are highly elastic under load and when stressed have the ability to store elastic strain energy. When these materials are not constrained, the amount of energy stored varies approximately as the square of the stress in the rock or coal and inversely as Young's modulus of elasticity for the material."

Holland gave for the strain energy in a laterally constrained cube, the equation

$$W = (1/2E)(P_1^2 + 2(P_1/(m-1))^2 - (2/m)((2P_1^2/(m-1) + (P_1/(m-1))^2)). \quad (36)$$

The variables are W for the strain energy stored per unit volume, in • lbf, P_1 for the principal stress acting (vertical direction implied), lbf/in², E for the Young's modulus, lbf/in², and m for the Poisson's number, dimensionless. The value of m is the reciprocal of the Poisson's ratio. For example, a Poisson's ratio of 0.25 gives a Poisson's number, $m = 4$.

In 1956, Holland and Gaddy (54) presented the equation for the compressive strength of rectangular test specimens as,

$$S = K (W)^{0.5} / T \quad (37)$$

where S is the compressive strength in psi, W is the smallest width of model pillar, in, T is the thickness, in, and K is a coefficient depending upon the coal used and other factors. K dimensional units are lb/in^{1.5}.

In 1962, Holland (49-50) considered the concept that the pillar is a foundation on a soil-like material and related the failure of the floor to the angle of internal friction. Several of the tabulated values (angle of internal friction in degrees, ratio of width of failure zone to pillar width, ratio of bearing strength of floor to unconfined compressive strength) given were (0, 1, 2.6), (20, 2.53, 5.2), (40, 8.46, 17.6), respectively. This shows that the bearing strength of the floor increased rapidly with an increasing angle of internal friction.

In the light of the present knowledge (1990) that the roof-pillar-floor combination should be treated as a separate structural unit for analysis in the pillar design problem, this approach should be researched further. However, the pillar should be treated as being supported from both the top and bottom by the roof and floor and not by the roof alone.

Also in 1962, Holland reported (49) "Field tests on mine pillars in the previous 20 to 30 years support laboratory theories of pillar design." "It may be accepted that a very important stress criteria for the design of coal pillars is the average stress in the pillars." For mining recoveries of 60 pct or greater, use the average stress. For mining recoveries of 45 to 60 pct, use a design stress that is 1.1 times the average stress. For mining recoveries that are less than 45 pct, use a design stress that is 1.2 times the average stress.

In 1964, Holland (53) gave the relationship

$$S = C (W / T)^{0.5} \quad (38)$$

for laboratory tests on rectangular test specimens, where S is the compressive strength, psi; W is the least width of the specimen; T is the thickness, in; and C , psi, is a coefficient that changes with the coal tested. This equation was used for a range of $W:T$ values of three to seven. He reported that for $W:T$ of five or larger for weaker coals and for $W:T$ of seven or more for stronger coals, specimen failure was gradual rather than sudden. Holland also concluded that for specimens with $W:T$ about 9 or 10, the specimens do not fail elastically, but are forced in a condition resembling plastic failure and develop extremely high-unit compression strength. Babcock and Bickel (17) have shown that this need not be true if end friction is dynamically lost because specimens with a diameter-length ratio of 8.5 were made to burst for 13 of 15 coals tested. The elastic energy stored was responsible for the burst. Holland further concluded that several things lead to pillar failure, including the bearing strength of the roof and floor, roof strength over passageways, stress distribution in the pillar, bottom shale composition, and the presence of water.

In 1964, Holland (53), concluded that equation 37 predicts coal pillar unit ultimate strength for $W:T$ ratios lying

between 1 and 9 or 10 accurately enough for the prediction to be used as a criterion for design. This formula, based on laboratory test data, was verified by examining eight mines at depths of 50 to 1,000 ft, with coal seam thicknesses of 2.75 to 6.5 ft, pillar widths of 10 to 40 ft, opening widths of 16 to 50 ft, and recoveries of 40 to 70 pct. Pillars in one of the eight mines failed immediately after mining, but the pillars in the other seven mines remained intact for from 4 to 34 years. Meikle and Holland (65) reported in prior work that friction between the coal pillars and the top and bottom rock in mines and between platens in laboratory tests, is the factor directly responsible for the apparent increase in pillar and specimen unit strength as $L:T$ increases. Results determined for three different frictional end conditions were given by the equation

$$S = C (L/T)^n, \quad (39)$$

where S , C , L , T , and n are the compressive strength, pound per square inch; a proportionality constant that changes with the material tested, pound per square inch; the length of a rectangular specimen tested, inch; the thickness of the specimen, inch; and an exponent to be chosen to fit the equation to data tested, dimensionless. Holland had reported earlier (52) that $n = 1/2$. In the work cited here, the values of (C, n) were (5,193, 0.66), (4,229, 0.54), and (3,415, 0.61) for end conditions that were unlubricated, graphite lubricated, and graphite-grease lubricated, respectively. He concluded that while the strength was reduced 30 to 40 pct by changing the end friction, the value of n could be taken as $1/2$ for engineering purposes.

While Holland and associates recognized the effects of end friction on pillar strength, they still emphasized the $L:T$ relationships. The C term in equation 39, containing the effects of different coals and the end friction, cannot eliminate the $L:T$ effect that occurs as a separate variable term when C is a constant for a given coal-end friction condition. That is, equation 39 does not support the conclusion that with the complete removal of the end friction (section on role of end constraint on compressive strength), the compressive strength is independent of the specimen or pillar geometry.

Holland and Olsen (55) presented "the results of studies on sandstone, limestone, shale, and coal and the coefficient of friction between coal and the other three under wet and dry conditions. With the coefficients of friction thus determined and the use of results obtained earlier by Meikle and Holland (65), it seems probable that coal pillar strength may be more closely estimated." Holland and Olsen (55) used a shear apparatus, which displaced one rock parallel to the area of contact with coal when the rock and coal were held together with a normal force. They performed 386 tests on coal-rock combinations for

four coals and four rock combinations. The average coefficients of friction were 0.473 and 0.441 for dry and wet conditions, respectively. They concluded from statistical analysis of the data that "...the presence of water in and on the specimens tested had little or no measurable effect upon the coefficient of friction between the coal and rock."

Water can destroy, or greatly decrease, the bearing capacity of roof and floor rocks, especially for shales or siltstones when swelling clay minerals, or other rocks bonded with water-soluble cement, are present. Water also reduces the friction between the roof-pillar-floor rock sequence and can reduce end constraint and pillar strength.

This author concluded that the normal force of 69 lbf across a 2.5-in-square contact area (used by Meikle and Holland (65) for a normal pressure of 11 psi) gave questionable results when the normal pressures in the mines are in the range of 500 to 5,000 psi. Other tests with realistic normal stresses should be made before the conclusions are accepted as valid for pillar design.

GROBBELAAR'S THEORY FOR STRENGTH OF PILLARS (44)

In 1970, Grobbelaar published a book for the design of mine pillars. His approach was based on the force equilibrium of cubic elements, the height of which were equal to the mining height. In a complex analysis, he used about 132 equations and 39 figures with several functional curves per figure. His work used the geometry of the pillar with width, height, and length relationships, taking the height as the unit dimension. He concluded, "For non-elastic pillar materials such as sand, sandstone, coal, and rock salt, the correspondence between selected theoretical curves and practical observations is satisfactory. This shows that the derived expressions may provide a powerful basis for the design of mine pillars. Without reliable material constants and experience, the theory cannot be regarded as proved and it is recommended that its applicability be explored."

Grobbelaar simplified some of his equations for several pillar shapes. Given below in slightly altered form are his equations B.20, B.21, B.22, and B.23 for square, rectangular, strip, and circular pillars, respectively. He uses the height as the fundamental unit of measurement, so that his W becomes $W:H$ in the following equations. In slightly altered form his equations become

$$\sigma_{sq} / \sigma_u = (6 + W/H)/7, \text{ for square pillars, } (40)$$

$$\sigma_{rt} / \sigma_u = H(3(L/H + W/H)(4 + W/H)$$

$$- 4W(3 + W/H)/H)/14L,$$

$$\text{for rectangular pillars, } (41)$$

$$\sigma_{st} / \sigma_u = 6(1 + W/4H)/7, \text{ for strip pillars, } (42)$$

and

$$\sigma_{cp} / \sigma_u = 2(3 + D/2H)/7, \text{ for circular pillars. } (43)$$

In the equations, σ_{sq} , σ_{rt} , σ_{st} , σ_{cp} , σ_u , W , H , L , and D are the strength per unit area of the square, rectangular, strip, and circular pillars, strength of the unit cube, and the width, height, length, and diameter of the pillars, respectively. Since the ratios are dimensionless, any consistent units of measurement can be used. It is convenient to use pound per square inch and inches for laboratory-scale studies and pound per square inch and feet for mine-scale studies.

Since physical properties are an essential part of the equation use as indicated by Grobbelaar, these equations are of limited usefulness unless the statistical strength of the unit cube, σ_u , is known. However, the relative strengths of the pillars of the shapes given can be compared without a knowledge of σ_u .

Case 1. Comparison of Relative Strength of Square and Circular Pillars

For a square pillar of width, W , and a circular pillar of diameter, D , shown in figure 8.4 to have the same strength per unit area, equation 40 must equal equation 43. That is,

$$(6 + W/H)/7 = 2(3 + D/2H)/7, (44)$$

from which

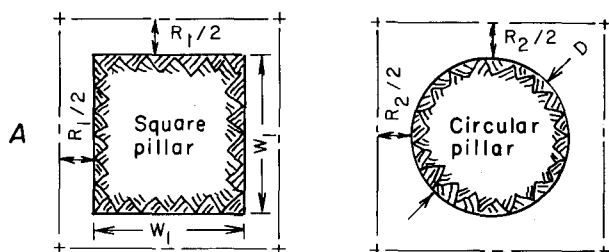
$$D = W. (45)$$

The strengths of the square and circular pillars per unit area are the same. The circular area is only $\pi D^2/4 = 0.785 D^2$ square units compared to the D^2 square units for the square pillar. The circular pillar is therefore 21.5 pct weaker than the square pillar with respect to the overburden it can carry at the same mining depth because the area is this much smaller. While the mining recovery is greater for the circular pillar, the stress is also greater. For a 50-ft pillar and a 20-ft entry, the average tributary stress concentrations for the square and circular pillars, respectively, are

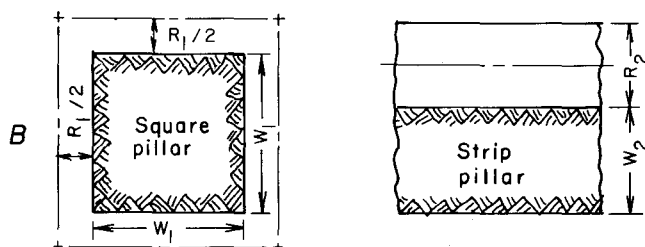
$$(20 + 50)^2 / 50^2 = 1.96 (46)$$

and

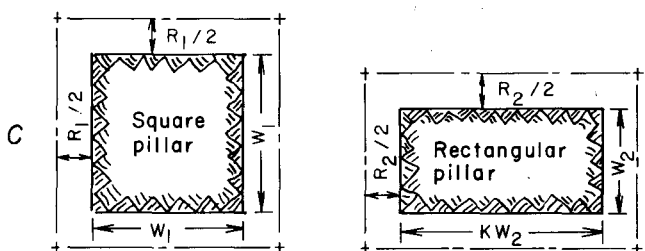
$$(20 + 50)^2 / 0.785 (50)^2 = 2.50. (47)$$



Case 1. Square vs circular pillars



Case 2. Square vs strip pillars



Case 3. Square vs rectangular pillars

Figure 8.—Mine pillar geometries used with equations of Grobbelaar. W_1 and W_2 are the widths of square and rectangular pillars, respectively; D is the circular pillar diameter; R_1 and R_2 are the entry widths used with square and rectangular pillars; and K is the length-width ratio for rectangular pillars (44).

That is, the average square pillar stress is 1.96 times the average overburden stress and the average circular pillar stress is 2.50 times the average overburden stress. The average stresses are 27.4 pct greater for the circular pillar than for the square pillar in this case, when the tributary support areas are considered.

The mining recoveries are

$$(70^2 - 50^2) / 70^2 = 49.0 \text{ pct for the square pillar, (48)}$$

and

$$(70^2 - 0.785(50)^2) / 70^2 = 59.9 \text{ pct}$$

for the circular pillar.

(49)

The author concludes that in contrast to the conclusions of Grobbelaar, there is no advantage to using the circular pillar over the square pillar, since the strengths are the same for the same mining recovery.

Case 2. Comparison of Relative Strengths of Square and Strip Pillars

For a square and strip pillar of the same height, H , shown in figure 8B to have the same strength per unit area, equation 40 must equal equation 42 so that

$$(6 + W_1/H) / 7 = 6(1 + W_2/4H) / 7, \quad (50)$$

where the square pillar width is W_1 and the strip pillar width is W_2 . From this

$$W_2 = 2W_1 / 3. \quad (51)$$

For the tributary stresses to be the same

$$\begin{aligned} \sigma_{sq} / \sigma_u &= (W_1 + R_1)^2 / W_1^2 = \sigma_{st} / \sigma_u \\ &= (R_2 + W_2) / W_2. \end{aligned} \quad (52)$$

Solving for R_2 in terms of R_1 and W_1 gives

$$R_2 = 2R_1(2 + R_1 / W_1) / 3. \quad (53)$$

Strip pillar entry widths, R_2 , corresponding to square pillar entry width of $R_1 = 20$ ft, for square pillars widths, W_1 , are given in table 2. The strip pillar widths, are $W_2 = 2W_1 / 3$. W_1 , W_2 , and W_3 are all in feet.

Table 2.—Strip pillar widths and entry widths corresponding to square pillars with same vertical stress, feet

Square pillar width (W_1)	Strip pillar width (W_2)	Strip pillar entry width (R_2)
10	7	53
20	13	40
30	20	36
40	27	33
50	33	32
60	40	31
70	47	31
80	53	30
100	67	29
125	83	29
150	100	28

NOTE.—Square pillar entry width (R_1) is 20 ft.

From this, it is apparent that the R_2 value is far too large for use in coal mining where the entry width is usually in the range 16 to 24 ft. While the pillar size may be all right, the roof span is much too large. Such small strip pillars and large rooms are used in salt and potash mining, but not in coal.

If the same entry widths are to be used for square and strip pillars, and the strip pillars are two-thirds as wide as the square pillars, the strip pillars must be at a greater depth than the square pillars if Grobbelaar's relationships are to be used. The depths for which strip pillars are used with entry widths $R_1 = R_2$ can be determined as follows from figure 8B. The average stress concentrations on the square pillars, S_1 , and on the strip pillars, S_2 , are

$$S_1 = (W_1 + R_1)^2 / W_1^2,$$

and

$$S_2 = (W_2 + R_2) / W_2 = (2W_1 / 3 + R_2) / 2W_1 / 3. \quad (54)$$

If S_1 is not equal to S_2 , the square and strip pillars should be used at different mining depths, d_1 and d_2 , both in feet, respectively. That is,

$$(W_1 + R_1)^2 / W_1^2 = X (2W_1 / 3 + R_2) / (2W_1 / 3), \quad (55)$$

where X is a proportionality factor. If $d_2 = X d_1$, then

$$d_2 = 2(W_1 + R_1)^2 d_1 / W_1 (2W_1 + 3R_2). \quad (56)$$

If $R_1 = R_2 = 20$ ft, and a square pillar of Wilson (87) size of table 3 is used at a depth of 1,000 ft, what would be the depth of a strip pillar of equal performance that was 2×37 ft/3 = 24.7 ft? From equation 56,

$$\begin{aligned} d_2 &= 2(37 + 20)^2 \times 1,000 / 37 (2 \times 37 + 3 \times 20) \\ &= 1,311 \text{ ft.} \end{aligned} \quad (56a)$$

The percentage mining recovery for the square pillars at 1,000 ft, Re_1 , and the strip pillars at 1,311 ft, Re_2 would be

$$Re_1 = (57^2 - 37^2) \times 100 / 57^2 = 57.9 \text{ pct,}$$

and

$$Re_2 = 20 \times 100 / 44.7 = 44.7 \text{ pct.} \quad (57)$$

Table 3.—Comparison of pillar widths, in feet, calculated with equations of Wardell (85) and Wilson (87)

Depth, ft	16-ft room width		20-ft room width		24-foot width	
	Wardell	Wilson	Wardell	Wilson	Wardell	Wilson
6-FT-THICK COAL SEAM						
200	21	20	23	24	26	28
400	31	23	33	27	36	31
600	39	27	41	31	44	35
800	45	30	47	34	50	38
1,000	50	33	53	37	55	42
1,200	54	36	57	41	60	45
1,400	59	40	61	44	64	48
1,600	62	43	65	47	68	51
1,800	66	46	69	50	72	55
2,000	69	49	72	53	75	58
2,500	75	57	80	61	83	66
3,000	84	64	87	69	90	73
10-FT-THICK COAL SEAM						
200	29	22	33	26	36	30
400	46	28	49	32	52	36
600	58	33	61	37	64	42
800	68	39	71	43	74	47
1,000	76	44	80	48	83	52
1,200	84	49	87	53	90	58
1,400	91	54	94	59	97	63
1,600	97	59	100	64	103	68
1,800	103	64	106	69	109	73
2,000	108	69	112	74	115	78
2,500	121	81	124	86	127	91
3,000	132	93	135	98	139	103

Case 3. Comparison of Relative Strengths of Square and Rectangular Pillars

For a square and rectangular pillar shown in figure 8C to have the same strength per unit area, equations 40 and 41 must be equal. That is,

$$(6 + W_1/H)/7 = H(3(L/H + W_2/H)(4 + W_2/H) - 4W_2(3 + W_2/H)/H)/14L. \quad (58)$$

W_1 and W_2 indicate the widths of the square and shorter width dimension of the rectangular pillar, respectively. If L is some multiple of W_2 , say k , then

$$L = k W_2, \quad (59)$$

so

$$(6 + W_1/H) = H(3(kW_2/H + W_2/H)(4 + W_2/H) - 4W_2(3 + W_2/H)/H)/2kW_2, \quad (60)$$

from which

$$W_2 = 2W_1/(3 - 1/k). \quad (61)$$

As for the strip entries, but to a lesser extent, the entries must be larger than for the square pillars if the same stress is to be applied to both. W_2 falls between W_1 and $2W_1/3$.

Because of the difference in tributary loads produced on pillars of square, rectangular, strip, and circular shape, it is physically impossible for practical purposes to equate the pillar results so that

$$\sigma_{sq} = \sigma_{rt} = \sigma_{st} = \sigma_{cp} \quad (62)$$

for the same mining depth and the same entry size. Because the mining recoveries are different for pillars at the same mining depth, the same stresses and compressive strengths occur at different mining depths. To do this would require that the entry width (usually 20 ft) relationships for coal could not be satisfied. However, for purposes of illustration, for entry widths of 20 ft and a square pillar size of 60 ft, the stress concentrations on the pillars and the mining recovery would be as follows.

For the square pillar, the stress concentration is

$$\sigma_{sq}/\sigma_u = 80^2/60^2 = 1.78, \quad (63)$$

and the recovery, Re , is

$$Re_1 = (1 - 1/1.78) \times 100 = 43.8 \text{ pct.} \quad (64)$$

For the rectangular pillar, if $L = 2$, the stress concentration is

$$\begin{aligned} \sigma_{rt}/\sigma_u &= (W_2 + R_1) \times (2W_2 + R_2) \\ &= (80 \times 140)/(60 \times 120) = 1.55, \end{aligned} \quad (65)$$

and the recovery is

$$Re_2 = (1 - 1/1.55) \times 100 = 35.5 \text{ pct.} \quad (66)$$

If the square pillar was chosen for a depth of 1,000 ft, the correct depth for the rectangular pillar to obtain the same stress and entry width would be $1.78 \times 1,000/1.55 = 1,148$ ft.

If a mine pillar of a given shape (square, rectangular, circular, or strip) gives satisfactory service and a change in pillar shape is planned, the sizes of new pillars with the same strength per unit area can be obtained by equations 40, 41, 42, and 43.

In summary, because the mining recoveries, and hence the tributary loads and compressive strengths, are different for pillars of different shapes (square, rectangular, circular, and strip), changing pillar shapes at a given mining depth is more complex than just using pillars of the same areas.

COMPARISON OF PILLAR SIZES SELECTED WITH EQUATIONS OF WILSON (87), WARDELL (85), AND SALAMON AND ORAVECZ (81)

Wilson gave the equation for pillar size, W , in feet, for a safety factor of 1.0 as

$$\begin{aligned} W &= (R/3 + 2HD \times 10^{-3}) + ((R/3 + 2HD \times 10^{-3})^2 \\ &\quad + (R^2/3 - 4H^2D^2 \times 10^{-6}))^{1/2}, \end{aligned} \quad (67)$$

where R , H , and D are the entry width, entry height, and the depth of the coal seam below the surface, respectively, all in feet. The safety factor, S_p , is defined to be

$$S_f = \frac{\text{average strength per unit area}}{\text{average stress per unit area}}. \quad (68)$$

Wardell proposed the equation

$$S = a/\sqrt{H} + b(W/H)^2 = 1,000/\sqrt{H} + 20(W/H)^2 \quad (69)$$

for the strength of square mine pillars. Here, the variable S is the compressive strength of the pillar in pound per square inch; a and b are coefficients; and W and H are the width and height of the pillar in feet, respectively. To be dimensionally homogeneous, " a " must have units of pound

per square inch per foot to one-half power and "b" must be in pound per square inch. Wardell gives tables of minimum pillar widths for coal seams with pillar heights of 4, 5, 6, 7, 8, 9, 10, and 12 ft, based on a safety factor of 1.5. The pillar widths are derived by using the safety factor relationship given by equation 68, where the average applied stress is determined from a tributary area correlation that he does not define mathematically. For example, a safety factor of 1.5 means that the pillar strength is 1.5 times the uniaxial crushing strength of the pillar in compression. A relationship that generates his tabulated values was derived by Babcock (18) as

$$(W + R)^2 \times S_f \times 1.2D/W^2 = 1,000/\sqrt{H} + 20(W^2/H^2), \quad (70)$$

where D is the depth below surface, R is the room width, and W and H are the width and height of the coal pillar, all in feet.

The equations 67 and 70 were used to calculate the minimum pillar sizes given in table 3, for a safety factor of 1.0, seam thicknesses of 6 and 10 ft, entry widths of 16, 20, and 24 ft, and depths of 200 to 3,000 ft. The average pillar stresses corresponding to the pillar sizes in table 3 are given in table 4. A vertical stress of 1.2 psi/ft of mining depth, included in equation 70, is assumed to obtain the tabulated values. The pillar sizes selected provide for a nearly linear increase in pillar stress with depth. These results can all be represented by straight-lined relationships as indicated by the parameters given in table 5. In this table, σ , m, and r are the unconfined or uniaxial strength, pound per square inch, the slope of the stress (ordinate) versus depth (abscissa) curve, and the correlation coefficient. If these results were a perfect fit to a straight line, r would have a value of 1.0. The values of 0.992 or better indicate that a straight line assumption is reasonable. Notice that by changing the intercept, σ , and the slope, entry widths from 16 to 24 ft can be fitted. These results could, therefore, be described by a Mohr-Coulomb linear model with good success.

Table 4.—Comparison of average pillar stresses, in pounds per square inch, calculated with equations of Wardell (85) and Wilson (87)

Depth, ft	16-ft room width		20-ft room width		24-foot width	
	Wardell	Wilson	Wardell	Wilson	Wardell	Wilson
6-FT-THICK COAL SEAM						
200	745	778	839	807	888	705
400	1,103	1,380	1,238	1,454	1,333	1,299
600	1,432	1,826	1,594	1,949	1,720	1,778
800	1,764	2,257	1,951	2,422	2,103	2,236
1,000	2,091	2,646	2,277	2,848	2,476	2,615
1,200	2,420	3,004	2,628	3,188	2,822	3,004
1,400	2,715	3,293	2,962	3,554	3,176	3,372
1,600	3,039	3,615	3,283	3,902	3,514	3,721
1,800	3,334	3,924	3,594	4,234	3,840	4,016
2,000	3,642	4,223	3,919	4,553	4,182	4,341
2,500	4,417	4,921	4,688	5,290	4,986	5,094
3,000	5,102	5,625	5,445	5,989	5,776	5,843
10-FT-THICK COAL SEAM						
200	578	716	619	751	667	778
400	872	1,185	952	1,268	1,025	1,333
600	1,172	1,587	1,270	1,709	1,361	1,778
800	1,465	1,909	1,577	2,061	1,684	2,191
1,000	1,758	2,231	1,875	2,408	1,994	2,563
1,200	2,041	2,534	2,178	2,732	2,310	2,878
1,400	2,323	2,823	2,471	3,012	2,614	3,204
1,600	2,606	3,103	2,765	3,308	2,919	3,514
1,800	2,883	3,375	3,052	3,594	3,216	3,814
2,000	3,164	3,642	3,334	3,873	3,506	4,104
2,500	3,846	4,302	4,046	4,558	4,241	4,791
3,000	4,526	4,945	4,746	5,219	4,950	5,473

Table 5.—Parameters for straight-line fit to table 4 data, for 16-, 20-, and 24-ft room widths

Parameter	16-ft room width		20-ft room width		24-foot width	
	Wardell	Wilson	Wardell	Wilson	Wardell	Wilson
6-FT-THICK COAL SEAM						
Unconfirmed strength (σ) . . psi . .	505	812	615	862	684	699
Slope of stress line (m)	1.560	1.682	1.639	1.809	1.734	1.796
Correlation coefficient (r)	0.999	0.993	0.999	0.992	0.999	0.994
10-FT-THICK COAL SEAM						
Unconfirmed strength (σ) . . psi . .	329	667	387	739	445	786
Slope of stress line (m)	1.411	1.476	1.469	1.553	1.524	1.636
Correlation coefficient (r)	0.999	0.996	0.999	0.995	0.999	0.994

The area of a single pillar determined with equation 67 of Wilson is about one-half of the area of a single pillar determined with the equation 70 of Wardell. In Mohr-Coulomb stress-failure plots in which the angle of internal friction is rather high (50° to 60°), a change in the unconfined shear strength of 50 to 100 psi will account for the apparent difference in pillar strength. It is uncertain as to which, if either, unconfined shear strength is correct. Therefore, the use of these equations depends largely on whether a more or less conservative pillar size is selected. Because one pillar is twice as large in area as the other, it is not twice as strong because the percentage recovery is not twice as great.

In 1976, Salamon and Oravecz reported equations for pillar strength based on an empirical fit of a width-to-height mathematical model to the data from 125 mines in South Africa. Three of their equations of interest (2.14), (2.15), and (3.4) are

$$\text{Strength} = 7,200 W^{0.46} / h^{0.66} \text{ kPa}, \quad (71)$$

$$\text{Strength} = 7,200 R^{0.59} / V^{0.067} \text{ kPa}, \quad (72)$$

and

$$\text{Safety factor} = S_f = 288 W^{2.46} / (H h^{0.66} (W + B)^2), \quad (73)$$

where W , h , B , R , and V are the pillar width, the pillar height, the entry width, all in meters, the ratio of the width to height, and the pillar volume, cubic meters, respectively. By changing the pillar volume from a laboratory size to one of mine pillar size, the volume, $V^{0.067}$, changes by a factor of 3. That is, according to this relationship, a mine pillar is one-third as strong per unit area as a laboratory specimen. To select a pillar size for a safety factor of 1.0, equation 73 will be used. Because it cannot be solved explicitly for W , an iterative approach can be used in which H , h , and B are selected to obtain by trial and error the wanted value of W , for $S = 1$. The value in meters was converted to feet to make a comparison to the work of

others in this report. The results for equation 73 for depths of 200 to 3,000 ft, mining heights of 6 and 10 ft, and entry widths, B , of 16, 20, and 24 ft are given in table 6. The percentage recoveries for these pillar sizes are given in table 7. To compare these results to those of Wilson and Wardell, see tables 3 and 4.

Table 6.—Pillar widths, in feet, calculated with equations of Salamon and Oravecz (81)

Depth, ft	6-ft-thick coal seam entry width, ft			10-ft-thick coal seam entry width, ft		
	16	20	24	16	20	24
200	12	14	17	15	17	19
400	18	21	29	24	27	31
600	25	29	33	33	38	43
800	32	37	41	43	50	55
1,000	39	45	50	55	62	69
1,200	47	53	59	69	76	83
1,400	55	61	68	83	92	99
1,600	64	71	78	99	108	116
1,800	74	82	90	115	126	134
2,000	85	92	101	135	145	155
2,500	115	122	133	192	202	215
3,000	149	159	170	261	273	283

Table 7.—Percentage recovery using pillars of sizes given in table 6

Depth, ft	6-ft-thick coal seam entry width, ft			10-ft-thick coal seam entry width, ft		
	16	20	24	16	20	24
200	81.6	83.0	82.8	76.6	78.9	80.5
400	72.0	73.8	70.1	64.0	67.0	68.2
600	62.8	65.0	66.5	54.6	57.1	58.8
800	55.5	57.9	60.2	46.9	49.0	51.5
1,000	49.7	52.1	54.3	40.0	42.8	45.0
1,200	44.3	47.3	49.5	34.1	37.3	39.8
1,400	40.0	43.3	45.4	32.5	32.5	35.2
1,600	36.0	39.1	41.5	25.9	28.8	31.3
1,800	32.4	35.4	37.7	22.9	25.5	28.1
2,000	29.2	32.5	34.7	20.1	22.8	25.0
2,500	22.9	26.2	28.2	14.8	17.2	19.1
3,000	18.5	21.1	23.2	11.2	13.2	15.0

A more effective way of making such a comparison is with mining recoveries as shown in figures 9 and 10 for 6- and 10-ft seams, respectively. Observe that the curves are all concave upward. This indicates that the average pillar stress increases with increasing depth and pillar size, from constraint. For the 6-ft mining height, the recoveries ranged from 83 to 21 pct for Salamon, 71 to 34 pct for Wardell, and from 70 to 40 pct for Wilson for depths of 200 to 3,000 ft, respectively. For the 10-ft mining height, the recoveries ranged from 79 to 13.5 pct for Salamon, 61 to 24 pct for Wardell, and from 67.5 to 31 pct for Wilson.

While these recoveries do not seem to vary widely, the 31 pct recovery for Wilson at 3,000 ft is 2.3 times that for Salamon with 13.5 pct. Without secondary recovery, 69 and 86 pct of the coal would be lost.

It is of interest to consider how suitable a model would be that assumed no increase in pillar strength with depth selected on a tributary basis only. In figures 9 and 10, the straight lines represent such relationships. The predicted pillar sizes from this line would all lie within the range of pillar sizes selected with the Salamon and Oravec, Wilson, and Wardell theories. The straight line functions have the form

$$\text{Recovery} = 73.5 - 1.75 \times d^* \text{ for a 6-ft mining height,}$$

and

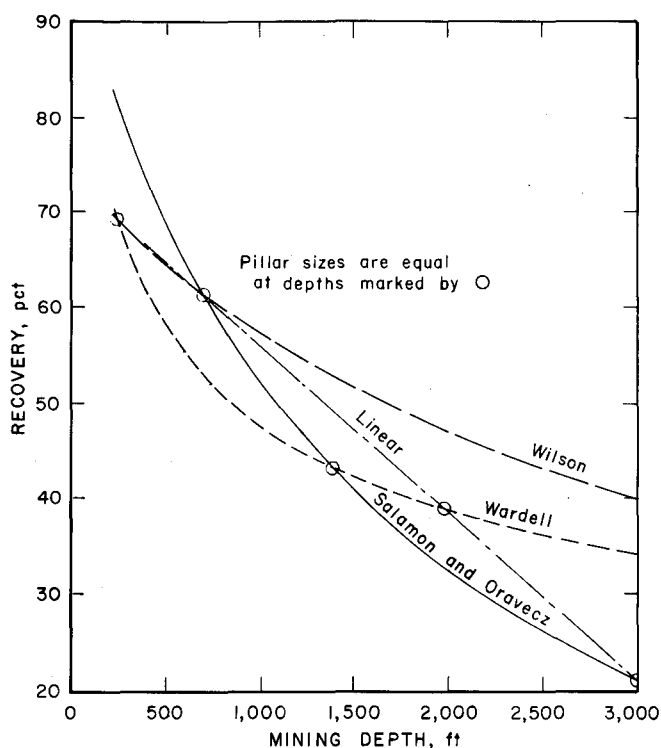


Figure 9.—Comparison of percentage mining recovery using pillars selected with equations of Wilson, Wardell, Salamon and Oravec, for 6-ft coal seam.

$$\text{Recovery} = 66 - 1.75 \times d^* \text{ for a 10-ft mining height,} \quad (74)$$

where d^* is the mining depth in hundreds of feet.

The comparison of the mining recoveries obtained with the pillar design equations of Wilson, Wardell, Salamon and Oravec, with a linear model by Babcock revealed that the linear model recoveries to mining depths of 3,000 ft fell within the range of the recoveries obtained with the pillar design equations of the four authors cited. At several different mining depths, the linear model gave the same recovery as one or more of the other theories.

BARRON'S PILLAR DESIGN CONSIDERATIONS

Integrity Index for Pillar Design

Barron (20), in 1978, proposed a novel method of pillar design using what he called an "Integrity Index," in which the index, $|I|$, was defined as "the percentage of the area of the 'solid central core' of the pillar to the total pillar area." He assumes that the ratio of the central pillar area that is not permeable to the total pillar area is a measure of the pillar integrity or strength. $|I|$ is, therefore, a dimensionless ratio. The method uses the rate of air pressure decay from a constant volume container into a sealed length of drill hole. Two packers are used to seal off the selected length of drill hole for testing. His apparatus is

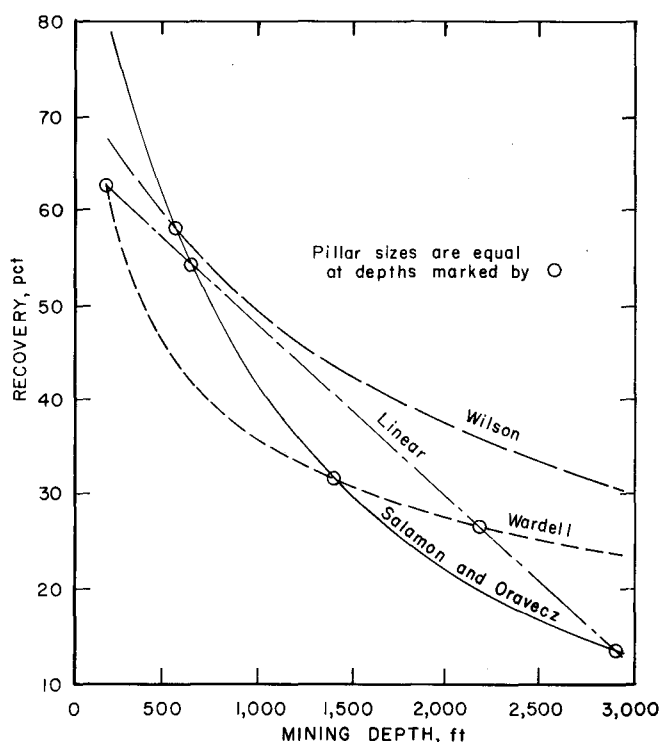


Figure 10.—Comparison of percentage mining recovery using pillars selected with equations of Wilson, Wardell, Salamon and Oravec, for 10-ft coal seam.

shown in figure 11. He states, "Distinction between fractured and stress-relieved coal (both of high permeability) and also between very competent and highly stressed coal (both of low permeability) is not possible other than by a subjective judgement based on the specific configuration." His index, therefore, shows the percentage of the pillar area that has low permeability for whatever reason. He also indicates that there exists "a high permeability zone at the pillar edges followed by a low permeability zone (averaging 6.6 ft inside the rib for the four mines used in his tests) with a central, relatively undisturbed, pillar core with intermediate permeability." This fits the expected vertical pillar stress distribution for the pillars between a horizontal row of holes in an elastic plate loaded vertically, and for the mine pillars described in the work of Wilson (87). That is, the pillar is most badly broken where both the vertical stress would be high for an intact solid body and the strength is least near the unconstrained pillar edge.

The figures presented by Barron (20) show that the permeability varies widely across the pillar and not in any consistent manner for the four mines tested. His mine B results are shown in figure 12. The results on November 18, 1974, were obtained shortly after the pillar was formed. Then four permeable zones, 1, 2, 3, and 4, were defined. Five months later, April 24, 1975, additional results were obtained. Barron (20) concluded, "The left side of the pillar had deteriorated and is now almost completely stress relieved (fractured) with the load being redistributed to give an increase in the peak stresses (overall higher $|I|$ values on the other side of the pillar)." The stresses were essentially unchanged 1 month later, May 28, 1975.

Considering how many assumptions are made by the many investigators relative to pillar failure, work of the type performed by Barron (20) should be expanded to give a realistic look at the actual in situ pillar behavior. It would be interesting to see in future work how the average permeability changes for pillars in the same seam under an increasing depth of cover under mountainous terrain. The permeability should reflect the fracture pattern of coal at shallow depths where the vertical stress is inadequate to close open cleats or cracks. At intermediate depths, some differential cleat or crack closure should occur where vertical pillar stress concentrations occur. At greater depths, most of the cracks should be closed for strong roof and floor rocks and open for weak roof and floor rock conditions. This would help separate the effects of (1) the extent of cleating in the coal, (2) the unbroken coal within the seam if such coal exists, and (3) how the cracks close with depth and horizontal constraint stress. Another approach that would more readily isolate the parts of the pillar that are being broken by stress rather than to reveal the geologic structure would be as follows. The drill hole is painted with a bonding material that is forced into the old cracks to make the hole initially more airtight. This could be cement, epoxy, or rock glue. As the pillar breaks

with changes in load, new cracks will form or old cracks will be reopened. This would reveal the active areas of pillar destruction with mining.

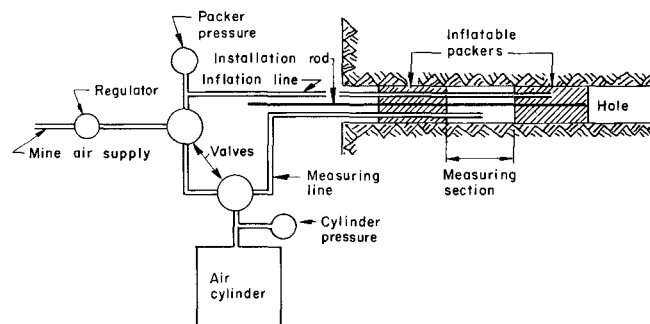


Figure 11.—Air injection apparatus used by Barron to investigate integrity of pillars and ribs in coal mines (20).

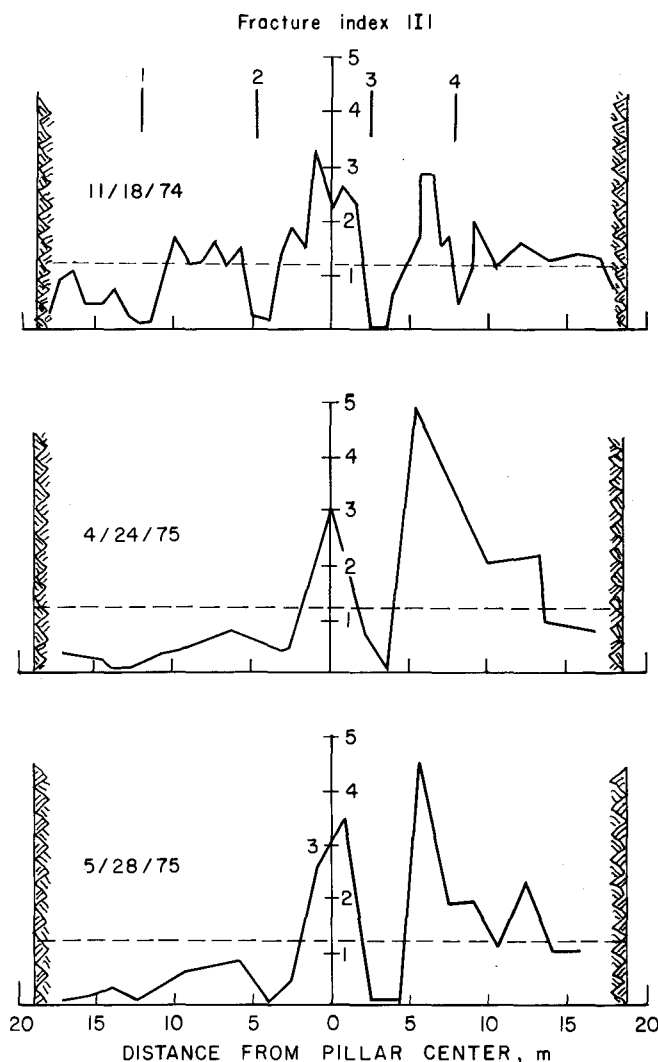


Figure 12.—Barron's fracture index $|I|$ across pillar at different times for mine B. Dashed line indicates $|I| = 1.18$ (20).

Comments on Pillar Design Equations

Barron (21) provided two computer programs for pillar design. One is for row pillars and the other for square or rectangular pillar arrays. This work refers to the earlier work of Wilson (87). The work is based on a complex analysis using more than 153 variables in over 120 equations and many serial assumptions. If there is a most important assumption it may be that the pillar is treated for purposes of analysis as though it is composed of parallel vertical slices parallel to one side of the pillar for which the horizontal forces are in equilibrium with the forces supplied by the roof and floor rock. Barron's equations are not given because of their complexity, but some comments on his work follow.

Barron replaced the vertical stress distribution at mid-height of a row of holes of uniform size and shape in a plate of a single material with an approximate equation with three parameters as shown in figure 13A. This provided for differences in hole shape and the resulting stress distributions. This equation was next assumed to apply to a three material system where the roof-coal-floor materials could all be different. This system is shown in figure 13B.

In the beginning, the pillars were assumed to be in the range $0.25 \leq W/H \leq 4$, but later pillars as wide as 180 ft were analyzed. Here W and H are the pillar width and height, respectively. The pillars were imagined to be made of vertical slices as shown in figure 14A. Force equilibrium was used to establish a relationship between the

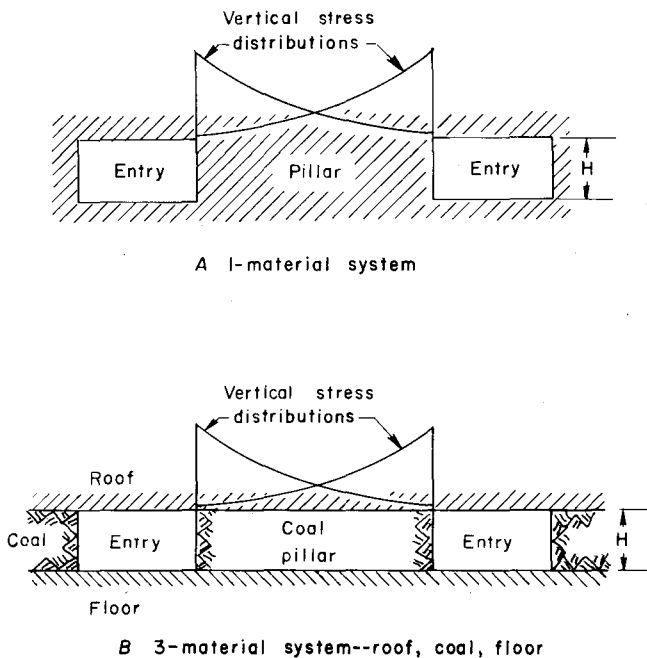


Figure 13.—Barron used same polynomial to describe vertical pillar stress distributions for A and B. A, One-material system; B, 3-material system—roof, coal, floor.

shearing forces at the top and bottom of the n th slice and the average horizontal forces on the sides of the slice as shown in figures 14B. P_i denotes the horizontal force, and dP_i denotes the change in horizontal force on the sides of the vertical slices, for the i th slice. The use of forces P_i and dP_i is equivalent to the use of average stress τ_i on the ends of the slice and σ_{ij} on the sides of the slice as shown in figure 14C. The first subscript, i , denotes the number of the slice on the left and the second subscript, j , the number of the slice on the right. The stress state within the slice was obtained by means of the Mohr-Coulomb rigid body relationships in terms of the angle of internal friction and the cohesive strength in shear that were different for the solid and broken coal. The Mohr circle stress envelope was used to model the pseudoductile behavior. Slice failure was assumed to occur from the edge of the pillar inward with a function assumed to make this distribution. Several conditions were defined for stable and

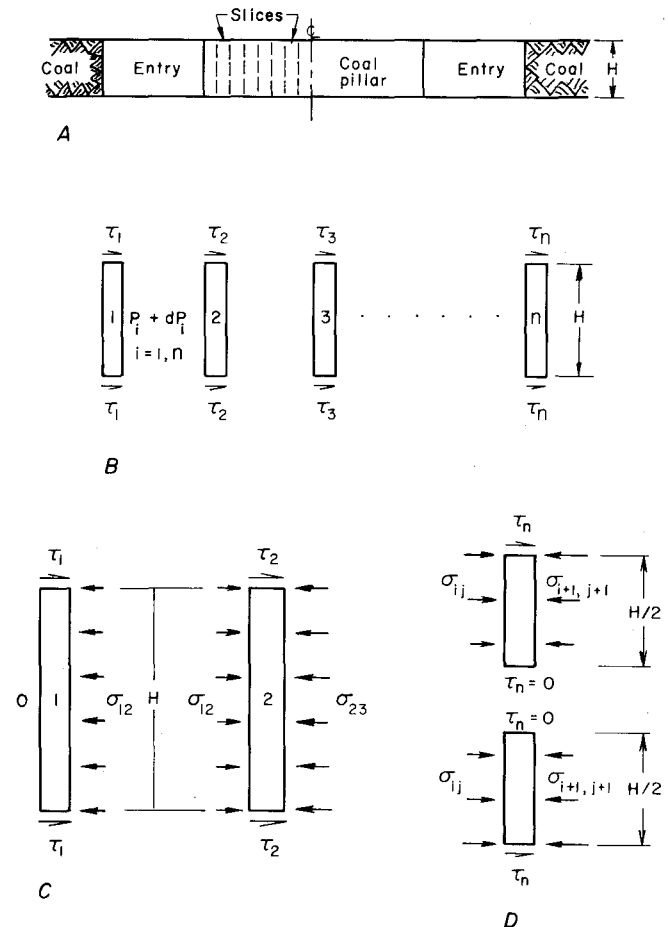
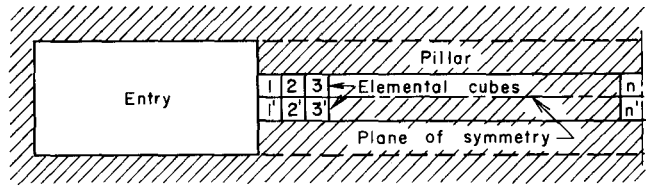


Figure 14.—Barron used vertical slices and force equilibrium in his computer model for pillar design. A, Assumption: Force equilibrium of imaginary vertical slices is basis of pillar stress analysis; B, forces on sides of slices balance forces on ends; C, force equilibrium condition implies uniform stress on sides of slices; D, pillar symmetry requires nonuniform stress state (21).

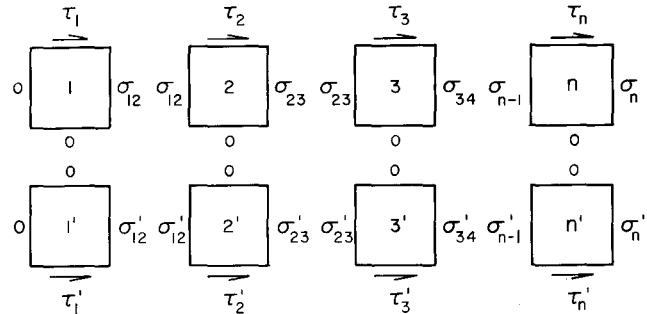
unstable pillars for both the brittle and pseudoductile behaviors. The pillar could be unfailed, partially failed, totally failed statically, and dynamically failed. Babcock has observed these behaviors in the laboratory by changing the end constraint conditions.

A critical assumption in Barron's work and one that limits the validity of the results, is that the state of stress on the slice is uniform from the top to the bottom as shown in figure 14C. This is shown in figure 12 of part 1 of his work (21). While the horizontal stress is uniformly zero on the free surface at the pillar rib, this is not true anywhere else within the pillar if shear stress exists on the top and bottom. This assumption indicates that the pillar behavior and failure are determined by the *average* stress in the horizontal direction. The *extreme* stress and not the *mean* (average) stress controls the pillar behavior with respect to failure. That is, the largest and smallest principal stresses create the Mohr-Coulomb stress-failure condition, not the volumetric stress represented by the center of the Mohr stress circle. The statistics of means are a very poor estimator of the statistics of extremes. That is, the mean behavior is the "expected behavior" and the extreme behavior is the "unexpected" behavior relative to a population distribution. The true nature of the pillar stress problem can be defined somewhat from the horizontal stress between mine entries revealed by the photoelastic method of analysis. This can be obtained experimentally as is done in photoelasticity (35, 39) from isochromatic fringes (not shown in figure 15). Two parallel rows of elemental cubes that touch each other along an axis of symmetry are the basis of the analysis in figure 15A. These are small in practice, but are shown on a larger scale for clarity. The upper row of cubes is numbered 1, 2, 3, ..., n and the lower row of cubes is numbered 1', 2', 3', ..., n'. The stresses on the cubes are shown in figure 15B.

Because of symmetry, the shearing stresses on the bottom of the upper row and on the top of the bottom row must be the same. This is an additional requirement to the equilibrium requirement that says the stresses must balance one another, but could be of opposite sign. A zero shear stress on both surfaces is the only stress that satisfies both conditions. These surfaces are acted upon by principal stresses. If the cubes are given a unit dimension, the forces and stresses will have the same numerical values. The stresses can then be summed for the force balance of the equilibrium condition. The stresses on the cube acting in the horizontal direction will be denoted by two subscripts, the first denoting the cube on the left and the second the cube on the right. For example, σ_{12} , denotes the stress acting between the first and second cubes above the $\tau = 0$ shear plane. For the shear stress, τ_1 , denotes the stress acting on the top or bottom of the first cube. For equilibrium, $\tau_1 + \sigma_{12} = 0$, $\tau_2 + \sigma_{23} = 0$, ..., $\tau_n + \sigma_{(n-1)n} = 0$. By finding the shearing stress values,



A Uniform plate with row of holes



B Summation procedure used to obtain horizontal stress

Figure 15.—Photoelastic type analysis of stresses in horizontal direction on pillar shows unsuitability of method used in figure 14 by Barron. A, Two rows of elemental cubes used to define horizontal stress; B, from symmetry: $\sigma_{ij} = \sigma'_{ij}$; $\tau_k = \tau'_k$; $i = 1, n$; $j = 1, n$; and $k = 1, n$.

τ_i , ($i = 1, n$) from isochromatic photoelastic fringes (not shown) the values of the horizontal stresses can be calculated. The same procedure is used for the cubes below the $\tau = 0$ shear plane. These quantities are denoted with the prime ($'$). The moments produced by the shearing stresses are second order infinitesimals, which are ignored in elastic theory because they are infinitely small compared to the forces on the unit cubes. Since the unit cube has side areas of unity (1) the stresses have the same values as the forces and stresses instead of forces can be summed in equilibrium.

The state of horizontal stress is not uniform from the top to the bottom of the slices, as assumed by Barron, but varies from the top to the middle and from the middle to the bottom. Such a distribution has some of the characteristics shown in figure 16. In figure 16A, the shearing stresses are acting inward as would be the case for strong roof and floor rock. In figure 16B, the shearing stresses are acting outward as would be the case for soft roof and floor rock. The distribution has principal stress values that are vertical or horizontal along the dotted lines of figures 16A and 16B and are rotated from these directions everywhere else. The principal stresses in figure 16C are everywhere vertical and horizontal.

The larger the mismatch of physical properties between the roof-coal-floor and the larger the W/H ratio, the more

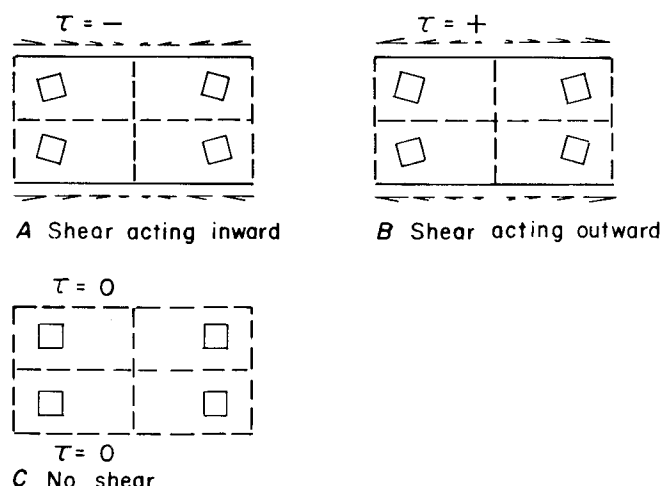


Figure 16.—Relative orientation of principal stress directions in mine pillar. A, Shear acting inward; B, shear acting outward; C, no shear.

the principal stress state is rotated by end shear. Only for the case where there is no end constraint (fig. 16C) does the principal stress state become vertical everywhere in the pillar as assumed by Barron. Barron assumes that the load is shed from a failed slice unto the adjacent slices or over the entire pillar core as yet unfailed. This is done with a function that can range from no dependence on the distance from the failed slice to one where the stress is inversely proportional to the square root of the distance. He gives no experimental evidence to support such assumptions. Later on, some problems arose with the occurrence of square roots of some negative arguments. This indicated to Babcock that somewhere assumptions were made that were physically unacceptable. One disturbing aspect of the results obtained was their sensitivity to the pillar slice thickness in obtaining the brittle fracture or pseudoductile zones and behavior in the pillar. This problem occurred even though the vertical stress profile was nearly identical for a range of pillar slices numbering 36, 180, and 540 in a pillar semiwidth of 90 ft. A slice width of less than 0.25 ft was recommended.

While the work of Barron is interesting and the existence of computer programs is appealing, in view of the many serial assumptions, the many variables used, and the questionable nature of some of the assumptions, it appears to Babcock that the work is largely of academic interest at this time. If the work could be extended to include a variable stress in the vertical direction and a local failing of the slices with a resultant stress gradient throughout the pillar, the results would be more believable. Also, the extreme dependence of nonelastic behavior on the width of the slice used when the vertical stresses are almost the same for all slice widths causes doubts about the meaningfulness of the results.

LU'S INTEGRITY FACTOR METHOD OF IN SITU PILLAR DESIGN

Lu (63), proposes a pillar design procedure that uses what he calls "pillar integrity factors." He defines these to be, "The quotient resulting from division of integrated total pillar strength across the critical dimension by the integrated pillar load across the same dimension. The integrity factor is dynamic, while the conventional safety factor is static." The pillar strength and overall load-carrying ability changes during the mining operation. By using three hydraulic pressure cell arrays, he measures the change in cell pressures with mining advance.

Using a procedure reported earlier in an unpublished report,³ he estimates the absolute stresses in situ in the vertical horizontal directions. From triaxial strength tests in the laboratory, he estimates the pillar strength for these states of stress. He defined his pillar stability factor as the ratio of the area under the strength curve, A_s , divided by the area under the vertical stress (load) curve, A_l , both taken across the entire pillar width. Since both involve the pillar width, the ratio is one of average strength divided by the average vertical stress values. This concept is illustrated in figure 17. He calls this ratio, $I = A_s/A_l$. If I is greater than 1, the pillar is stable. If I is less than 1, the pillar is unstable. The value of I changes with mining advance. The $I = 1.89$ value for figure 17 when the longwall face is at the cell location indicates that the pillar is stable at that time.

Lu uses two kinds of hydraulic pressure cells developed by Panek (73-74). Two of the borehole pressure cells (BPC's) and one cylindrical pressure cell (CPC) are used in the same drill hole, so that the three pressure cell pressures can be assumed to respond to stress at the same "place." One BPC responds mostly to stress changes in the vertical and the other BPC responds mostly to stress changes in the horizontal directions. The word "mostly" is used to indicate that both cells respond to a biaxial stress field change, as will be shown later in a section dealing with calculation of mine pillar stresses from borehole pressure cell data. Stated another way, the cell pressure changes are not the coal stress changes, and the use in this manner, along with the assumption that the coal stress will creep to equilibrium, are the most questionable elements of his work.

It is very desirable to be able to obtain the absolute state of stress with drill hole instruments. For most competent rocks, this is achieved by means of the overcoring process. A drill hole instrument can only measure changes

³Babcock, C., P. Lu, F. Horino, and R. Harmel. Field Test and Evaluation of USBM Borehole Pressure Cells for the WIPP Project. Final Report to DOE, Interagency Agreement DE-AI04-82AL20398, Oct. 1983, 41 pp.

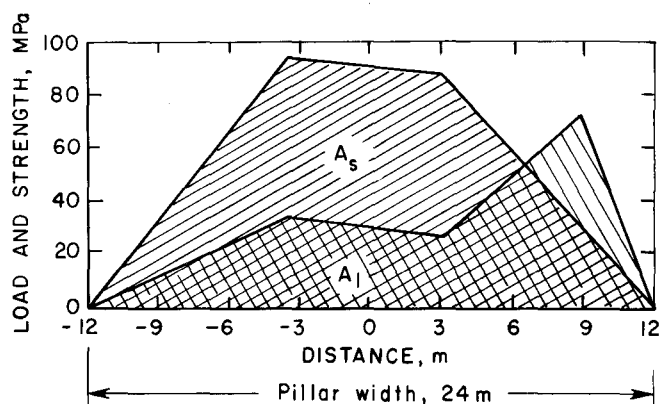


Figure 17.—Lu's integrity factor method of determining mine pillar stability. Integrity factor, $I = A_s/A_l = 1.89$. A_s = area under strength curve and A_l = area under load curve. I is essentially safety factor that changes with mining advance. Longwall mining face is at hydraulic cell location (63).

in the hole size and shape after installation. In competent rock, the state of stress prior to the instrument installation in the drill hole cannot be detected. Unfortunately, overcoring of a drill hole in coal to produce an intact core necessary for the success of this method is all but impossible. If the absolute state of stress values are to be obtained with drill hole instruments, some method using creep behavior of the hole boundary of the kind proposed by Lu is needed.

Obert and others at the USBM Applied Physics Research Laboratory (APRL), College Park, MD, performed tests with BPC's or CPC's in which several cells were set at different pressures to determine if the pressures at the end of the test period would all be the same. They were not, which indicated that the material tested exhibited both fluid and solid properties, because the equilibrium stress state was not hydrostatic. A rheological Kelvin model with a spring and dashpot in parallel can be used to model this behavior. The dashpot behavior is that of a viscous fluid, and the spring is that of an elastic solid. The rock had both fluid and solid properties.

Somewhat similar results were reported by Lu⁴ in work with Babcock. These results are shown in figure 18. Three setting pressures were used in each of three holes each containing the two BPC and one CPC cell arrays. The setting pressures for the three cells (two BPC's, one vertical and one horizontal, and one CPC) in each of the holes 1a, 2a, and 3a were 4,500, 3,500, and 2,500 psi, respectively. The equilibrium pressures for these cells before failure were different, increasing with the setting pressures. Some of these cell pressures caused failure of the salt with time. The salt around the BPC's of hole 1a failed before equilibrium was reached (that is, before the horizontal parts of the curves were reached). The state of

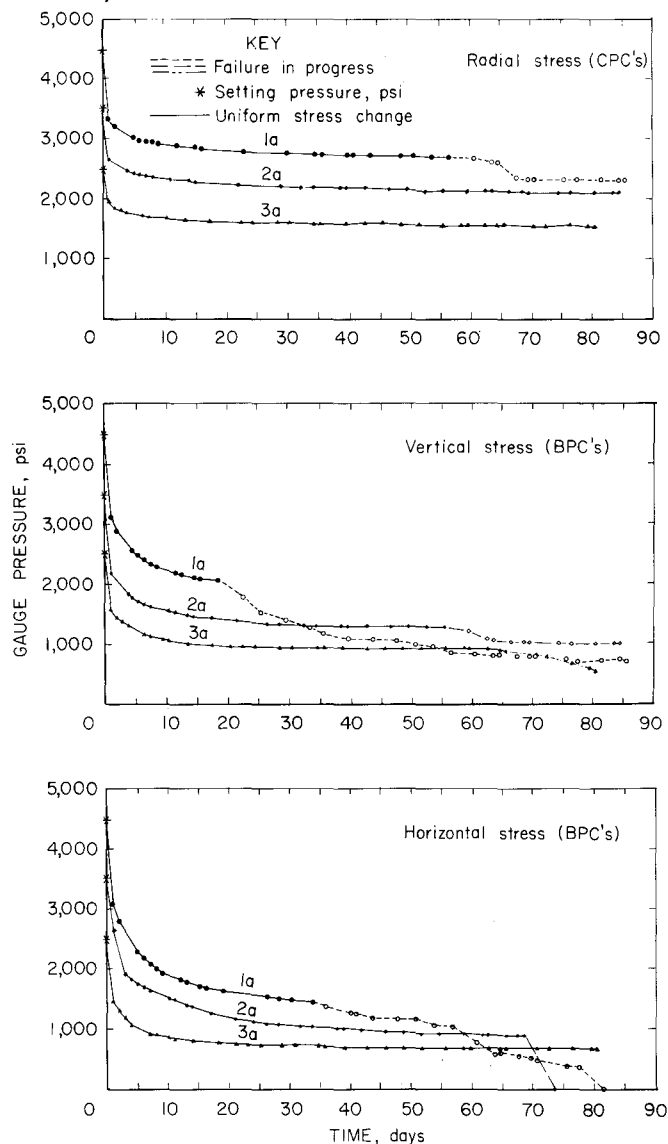


Figure 18.—Creep in salt as function of initial setting pressure and time. Results 1a, 2a, and 3a are for horizontal holes in the mine pillar at midheight, with those numbers. Three cell arrays were used per hole, one radial stress CPC, one vertical stress BPC, and one horizontal stress BPC¹.

stress in the salt calculated with Lu's equation 1 was very different for holes 2a and 3a in figure 18. For 2a, $\sigma_v = 2,139$ psi, and $\sigma_h = 1,328$ psi. For 3a, $\sigma_v = 1,590$ psi and $\sigma_h = 1,060$ psi. Here σ_v and σ_h are the vertical and horizontal stresses in the salt, respectively. Since the mine cover was about 900 ft, the vertical tributary stress would be about 1,100 psi. Considering that a stress concentration may exist next to the drift opening, the vertical stress for hole 3a was a reasonable estimate. The ability of the salt to support cell pressures was definitely related to the confinement of the salt. Since the results were dependent upon the setting pressures of the hydraulic cells and there

⁴Cited in footnote 3.

was no way to determine which setting pressure was "correct," the results are presently somewhat subjective.

If the coal or salt had a viscoelastic behavior in which the change in behavior was related to the *total* or absolute stress state, the stress state change could be projected to some reference state of stress (that is, 0 stress state), and the absolute stress state obtained for the time in question. If the creep behavior was only a part of the total stress state and the remainder was elastic or plastic as would be the case for elastic-viscoelastic, or plastic-viscoelastic materials, there is some doubt whether or not this would be possible. That is, the results that depended upon creep would not be a sufficient condition to identify the total stress tensor. In terms of a rheological spring-dashpot model to represent the elastic-viscous behavior, knowing the ultimate creep state (the dashpot behavior) would not disclose the elastic (spring) part of the behavior.

LONGWALL CHAIN PILLAR DESIGN EQUATION OF HSIUNG AND PENG (57)

The loading of longwall chain pillars is very different from the loading of pillars in a room-and-pillar mining system. Most of the pillar design information in this report is for room-and-pillar mining of coal in which the lateral extent in plan area is large compared to the mining depth, say 1.2 times the mining depth. In that case, the pillars in the middle of the mining area received the full overburden or tributary load. This occurs because the pillars are "far" from the solid unmined coal so that the cantilever support has disappeared.

In the case of the longwall mining system, there are three stages of pillar loading, as shown in figure 19A, 19B, and 19C. First, the pillar must support a relatively short roof-floor span in the horizontal direction normal to the entry length before panel mining as in figure 19A. Cantilever support from the unmined panels reduces the pillar load below that of the overburden load. Second, when the first panel has been mined as in figure 19B, the roof span is increased in length and the cantilever support is greatly reduced. The roof sags into the pillar area with the most load on pillar rows 2 and 3. These pillars must survive to protect the entries 1, 2, and 5, 6 from the left side so that the adjacent panel 2 can be mined. This is the critical stage for chain pillar design. Third, after panel 2 is mined in figure 19C, the pillar rows 3 and 4 are no longer needed and their survival is no longer necessary. The behavior could be important if multiple-seam mining is to be done, and remnant pillars could produce ground control problems for that mining.

Hsiung and Peng (57) reported on a comprehensive study of the design of longwall chain pillars to support the entry system after the mining of the first panel. They reported, "A chain pillar design formula under weak roof conditions was developed by statistically analyzing the results from three-dimensional finite-element parameter analyses." Also, "Case studies were performed to verify

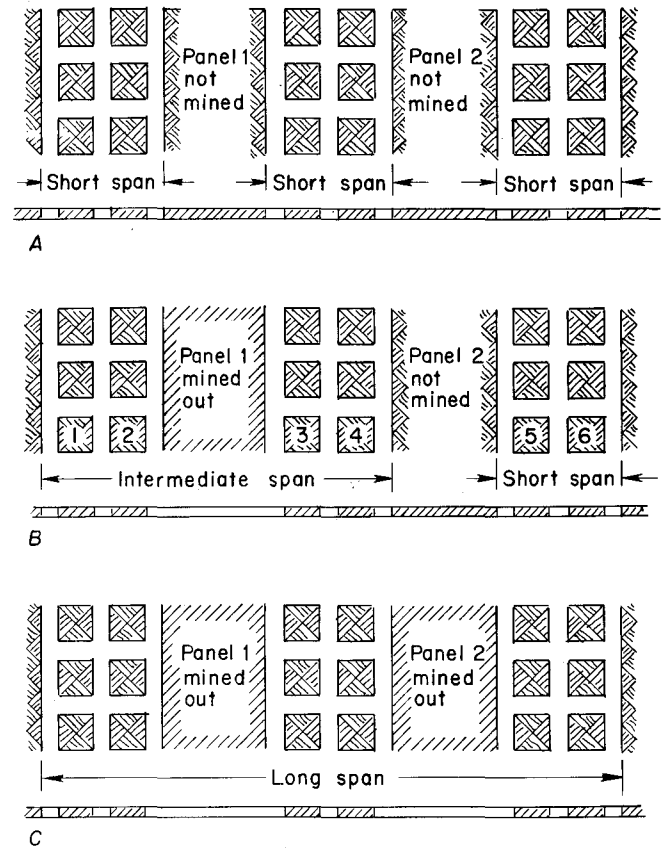


Figure 19.—Three stages of pillar loading in longwall mining system. A, Pillars not fully loaded by overburden. Short spans have cantilever support from unmined panels; B, pillar rows 1, 2, 3, and 4 receive additional overburden loads. Cantilever support is gone for rows 3 and 4.

the effectiveness of the formula developed." Their work also included the conversion of rectangular pillar sizes to square pillars of equivalent strength, the influence of high in situ horizontal stress on chain pillar stability, and the yielding behavior of the roof, coal seam, and floor members. Their work was summarized to a large extent by their equation 10:

$$\begin{aligned} \log W_p = & -0.004676 E_i/E_c & (a) \\ & -0.00404 E_m/E_c & (b) \\ & -0.0333 \log (E_f/E_c) & (c) \\ & -0.0789 \log \sigma_{cc} & (d) \\ & +0.5144 \log h & (e) \\ & +0.0494 \log (L_p/2) & (f) \\ & +0.1941 \log P_w & (g) \end{aligned} \quad (75)$$

The variables are width of pillar, W_p in feet; Young's moduli for the intermediate roof, coal pillar, main roof, and floor, E_i , E_c , E_m , and E_p , respectively, all in pound per square inch; the uniaxial compressive strength of coal, σ_{cc} , in pound per square inch; the overburden depth, h , in feet; the length of the longwall panel, L_p , in feet; and the panel width, P_w , in feet.

Equation 75 has both linear and logarithmic terms, which makes the physical interpretation somewhat more difficult than necessary, as will be shown below. Hsiung and Peng (57) provide an excellent nomograph for the graphical solution of this equation. This equation can reveal much about the physical meaning of the terms if written in the form

$$W_p = 10^a + b + c + d + e + f + g, \quad (76)$$

or as

$$W_p = (10^a)(10^b)(10^c)(10^d)(10^e)(10^f)(10^g). \quad (77)$$

In this form, the relative contributions to the pillar width, W_p , of each factor can be observed. The range of variables considered by Hsiung and Peng in their nomograph (their figure 20) were

$$0 \leq E_i/E_c \leq 15, \text{ for the (a) term,}$$

$$0 \leq E_m/E_c \leq 15, \text{ for the (b) term,}$$

$$1 \leq E_f/E_c \leq 15, \text{ for the (c) term,}$$

$$500 \leq \sigma_{cc} \leq 7,000 \text{ psi, for the (d) term,}$$

$$400 \leq h \leq 2,000 \text{ ft, for the (e) term,}$$

$$1000 \leq L_p \leq 5,000 \text{ ft, for the (f) term,}$$

and

$$200 \leq P_w \leq 700 \text{ ft, for the (g) term.} \quad (78)$$

The corresponding ranges of values of (a) through (g) were

$$0 \geq a \geq -0.07029; 0 \geq b \geq -0.0606; 0 \geq c \geq -0.039164;$$

$$-0.21295 \geq d \geq -0.30338; 1.33850 \leq e \leq 1.69805;$$

$$0.133329 \leq f \leq 0.16786; \text{ and } 0.44663 \leq g \leq 0.55223. \quad (79)$$

The corresponding ranges of the equation 77 terms are

$$1 \geq 10^a \geq 0.85057; 1 \geq 10^b \geq 0.86976; 1 \geq 10^c \geq 0.91377;$$

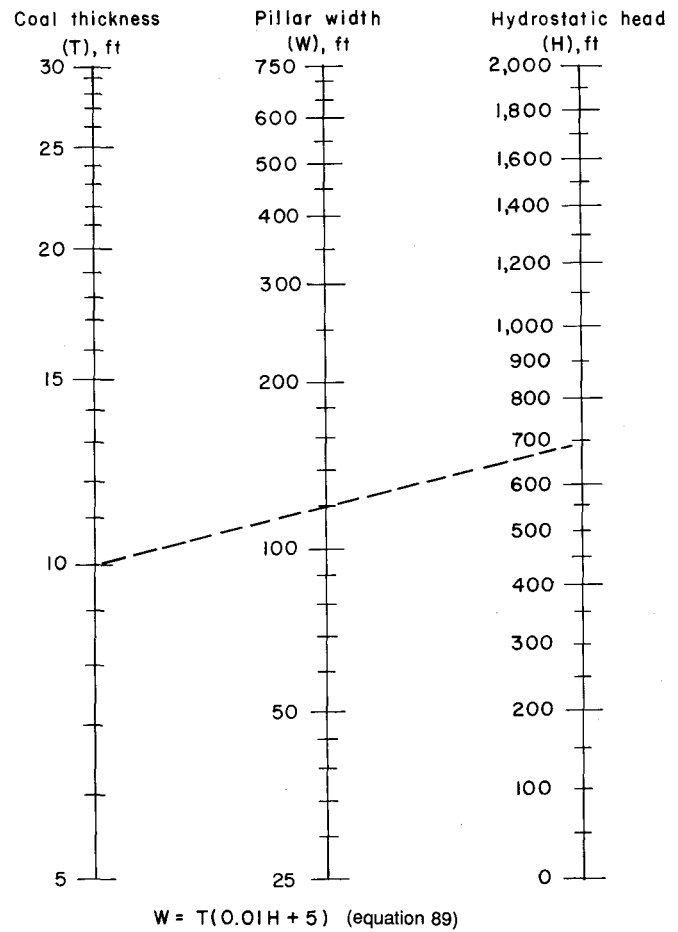


Figure 20.—Nomograph of mine inspector's formula for barrier pillars in Pennsylvania. Broken line defines example problem. If $T = 10$ ft, $H = 700$ ft, then $W = 120$ ft.

$$0.61242 \geq 10^d \geq 0.49730; 21.802 \geq 10^e \geq 49.894;$$

$$1.35934 \leq 10^f \leq 1.47183; \text{ and } 2.79660 \leq 10^g \leq 3.56640. \quad (80)$$

The smallest pillar size, W_p , results when

$$10^a = 0.85086; 10^b = 0.86976; 10^c = 0.91377; 10^d = 0.49730;$$

$$10^e = 21.802; 10^f = 1.35934; \text{ and } 10^g = 2.79660. \quad (81)$$

From equation 77,

$$W_p = (0.85086) (0.86976) (0.91377) (0.49730) (21.802) (1.35934) (2.79660) = 27.9 \text{ ft.} \quad (82)$$

The largest pillar size, W_p , results when

$$10^a = 1; 10^b = 1; 10^c = 1; 10^d = 0.61242; 10^e = 49.894; \\ 10^f = 1.47183; \text{ and } 10^g = 3.56640. \quad (83)$$

From equation 77,

$$W_p = (1) (1) (1) (0.61242) (49.894) (1.47183) (3.56643) \\ = 160.4 \text{ ft.} \quad (84)$$

Of interest is the ratio of the corresponding terms of equations 84 and 82. These terms are, respectively, 1/0.85086, 1/0.86976, 1/0.91377, 0.61242/0.49730, 49.894/21.802, 1.47183/1.35934, and 3.56643/2.79660. The product of these ratios is

$$(1.1753) (1.1497) (1.0944) (1.2315) (2.2885)$$

$$(1.08275) (1.27527) = 160.4/27.9$$

$$(a') (b') (c') (d') (e') (f') (g') = 5.75. \quad (85)$$

For the ranges of variables used in their nomograph, the relative changes of the W_p values can be assigned, 17.6 pct to (a'), 15.0 pct to (b'), 9.4 pct to (c'), 23.2 pct to (d'), 128.9 pct to (e'), 8.3 pct to (f'), and 27.5 pct to (g'). If the factors have a value of (1), they do not enter the problem of pillar width, W_p . The pillar size, W_p , increased with all the factors a' through g' . The factors, in decreasing order of importance, are e' , g' , d' , a' , b' , c' , and f' . These are the depth below the surface, d , in feet; the longwall panel width, P_w , in feet; the uniaxial compressive strength, σ_{cc} , in pound per square inch; and the length of the longwall panel, L_p , in feet. The other factors are of lesser importance and would disappear from the problem for a value of (1).

Hsiung and Peng conclude, "If the roof properties are not known, it is recommended that E_t/E_c and E_m/E_c are assumed to be zero. Since the parameter E_t/E_c in the formula is in a logarithmic form, it cannot be zero. Therefore, it is assumed to be one." For these conditions $10^a = 10^b = 10^c = 1$. In this case the physical properties do not appear in equation 77.

Consider the result of using $E_i = E_m = E_c = E_t = 1$. Then, for $10^a = 0.98929$, $10^b = 0.99074$, $10^c = 1$, the result is

$$(0.98929) (0.99074) (1) = 0.980129. \quad (86)$$

Making all the relative properties equal to 1 results in a difference in the results of less than 2 pct. When the intermediate roof, the main roof, and the floor are all strong ($E_i/E_c = E_m/E_c = E_t/E_c = 15$), the pillar width is reduced to a relative size of (0.85057)(0.86976)(0.91377)

= 0.67600, or 67.6 pct. The maximum size reduction because of end constraint is, therefore, 33.4 pct or 1/3.

For the conditions where $10^a = 1$ and $10^b = 1$, equation 10 of Hsiung and Peng can be written as

$$W_p = (h^{0.5144})((L_p/2)^{0.0494})(P_w^{0.1941})/ \\ (\sigma_{cc}^{0.0789})(E_t/E_c)^{0.0333}. \quad (87)$$

When $10^c = 1$, the term containing E_t/E_c disappears from equation 87. In laboratory testing of rock and coal specimens by Babcock, the stiff layer could be either on the top or bottom of the specimen and would produce end constraint and increased strength. Therefore, it seems that the term in equation 87 with the variable E_t/E_c could be used for either a strong roof or floor.

It is of interest to compare the pillar widths of Hsiung and Peng for chain pillars to those proposed by Wilson (87) and Wardell (85) for square pillars for room-and-pillar mining in the earlier section "Comparison of Pillar Sizes Selected With Equations of Wilson, Wardell, Salamon, and Oravec." If the intermediate roof, the main roof, and the floor are all strong, the pillar sizes selected are about equal.

BARRIER PILLAR DESIGN

Ash (4) describes a barrier pillar in the anthracite region of Pennsylvania as "a portion of the bed that is left unmined (1) along the property line or lines of adjoining mining properties, (2) between mines, or (3) between parts of mines." Further, that pillars "act as dams to prevent water impounded in a mine from breaking into an adjoining mine and causing loss of life, property, or both." Rice (79) indicates that other uses of barrier pillars are to provide separate ventilation, haulage, and escapeway systems, and to prevent a fire or explosion from penetrating from one mine to another.

Considerable confusion exists about how large a barrier pillar should be. This is apparent in the reports by Ashley (6) and Ash (3). During the discussion of his paper, in 1930, Ashley indicates that prior to his report, the width of barrier pillars for bituminous and anthracite coal mining, for a 3-ft-thick coal seam under 500 ft of hydraulic head should be 400 ft and 20 ft, respectively, under the prevailing rules. The confusion was also apparent from a five-page paper followed by 16 pages of comments. These included a comment to the effect that no barrier pillar could ever be large enough to retain completely the impounded water. Ash concluded, in 1954, after a lifetime of research with many associates and many papers on the subject that,

"A survey of the size, position, and physical condition of the component barrier pillars substantiates the uncertainty that exists in the minds of those concerned, not only as to the method by which the sizes of the barrier pillars were established, but also as to the stability of those that may be expected to act as dams. Where a barrier pillar must act as a dam, its failure may result in loss of life, the mine, or both, and it would be far better had the barrier pillar not existed and thereby given a false sense of security."

A barrier pillar design equation reported in 1928⁴ and cited by Ash was called "A Mine Inspector's Formula for Barrier Pillars." This formula was reported in at least three of Ash's publications (3-5) in an inconsistent way (that is, 0.001 was used instead of 0.01 in one report). Two of these references gave the formula as

$$W = (T \times 0.01 \times H) + 5T \quad (88)$$

which can be rewritten as

$$W = T (0.01H + 5) \quad (89)$$

where W = the width of the barrier pillar, ft;

H = the depth below drainage level, or hydrostatic head, ft;

and T = the thickness of the coal bed, ft.

A nomograph of equation 89 by Babcock is given in figure 20. A straight line is drawn between the given values of T and H to find W , the required barrier pillar width. In the example problem (fig. 20), T and H are given as 10 and 700 ft, respectively. The required pillar size is then 120 ft.

Equations 88 and 89 are not totally realistic because they specify a pillar width that is five times the coal seam thickness for all mining depths, if no water pressure exists ($H = 0$). This part of pillar design in anthracite mining had not changed even after the work of Bunting in 1911 (see "Bunting's Tributary Area Method For Pillar Design") indicated that the pillar size should increase with depth. Such a small pillar would be crushed at greater depths and the pillar would be permeable to waterflow at almost any water pressure.

Ashley (6), in 1930, reported, "The Legislature of the Commonwealth of Pennsylvania at its last session passed a new act dealing with barrier pillars, which may have a

wide interest in other states." They enacted legislation that specified the required pillar size for mining coal *in any seam* in words, which mathematically becomes

$$W = 20 + 4T + 0.1D, \quad (90)$$

where W is the barrier pillar width, feet; T is the seam thickness, feet; and D is the mining depth, feet. Babcock has given this equation in nomograph form in figure 21. In the example problem in this figure, the values of T and D are given as 10 and 1,000 ft, respectively, and the required pillar width is 160 ft. This act amended an earlier act of 1911 for barrier pillar design. Equation 90 is not totally acceptable because, in this case, no provision is made for the hydraulic head acting on the pillar.

Ash and Eaton (4) considered the barrier pillar as a dam and provided the following equation for a coal bed with a small dip. This is,

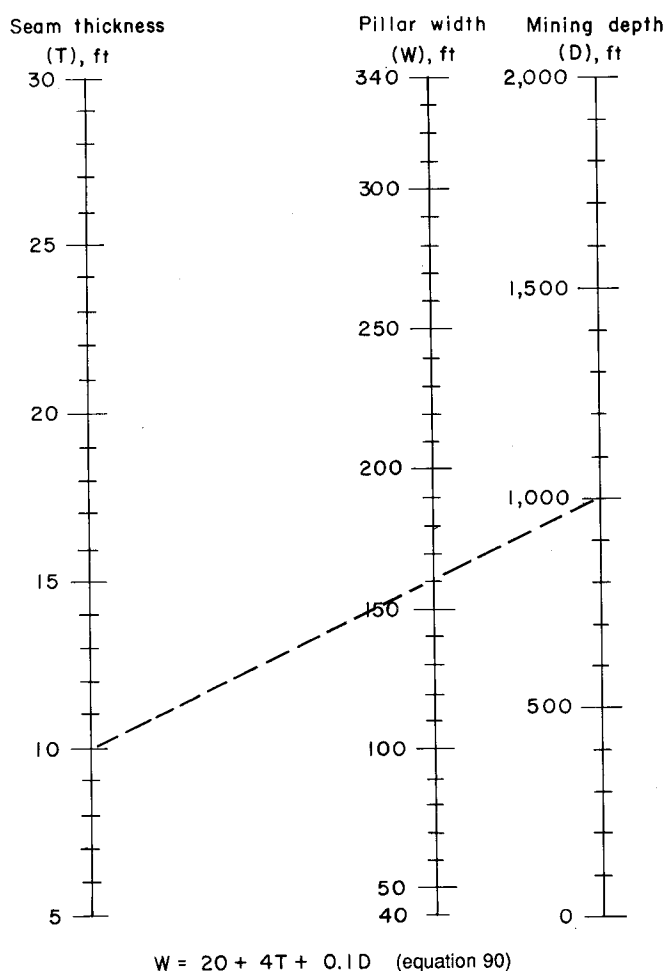


Figure 21.—Nomograph of barrier pillar legislation equation for Pennsylvania reported by Ashley. Broken line defines example problem. If $T = 10$ ft, $D = 1,000$ ft, then $W = 160$ ft.

⁴Coal Miner's Pocketbook, 12th ed., 1928, p. 645.

$$W = 50 + 0.426 D, \quad (91)$$

where W and D are the barrier pillar width and mining depth, respectively, both in feet. This equation is shown as a nomograph by this author in figure 22.

In summary, it seems that equation 88 may be unsafe, while equations 89 and 90 may be too conservative. Also, a rational barrier pillar equation should contain both mining depth and hydraulic head as variables. Gravity produces a nearly symmetrical pillar loading for flat-lying coal seams, while the hydraulic head produces a skewed loading. Research is needed on this loading problem. The pillar should increase in width with depth to avoid being crushed by the overburden and thereby losing its water barrier characteristics. The pillar should then also be structurally able to withstand skewed loading from water-pressure acting from one side to prevent catastrophic failure.

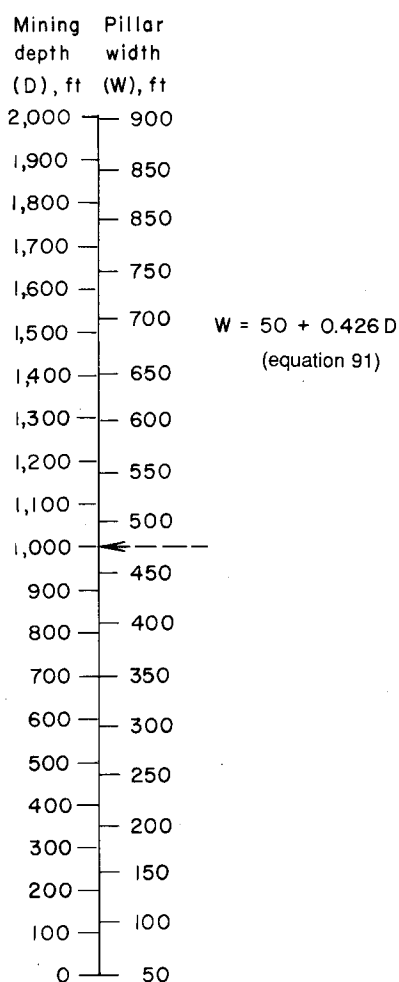


Figure 22.—Nomograph of Ash-Eaton equation for flat-lying barrier pillars in Pennsylvania. Broken line defines example problem. If $D = 1,000$ ft, then $W = 475$ ft.

BABCOCK'S METHOD OF ASSESSING PILLAR STABILITY

In theories of deformable bodies such as those of elasticity, plasticity, viscoelasticity, etc., the stresses and strains are related through constitutive relationships involving physical properties. If the physical properties are unknown or in doubt, or the assumptions on which the theory is based are unrealistic, the results are not very useful. Further, when this questionable state of stress is related to the failure condition, the results become even more uncertain.

A coal mine pillar is usually broken with cleats, joints, and bedding planes. Also, as many as nine physical properties (for orthotropic behavior) may be required to relate the stress and strain even for small pieces of intact coal (61). The scatter in the data may be such that a well condition matrix of physical properties does not result, and the solution of the set of equations obtained may not be possible. That is, the physical property matrix may have inconsistent relationships, and the system of equations cannot be solved.

Lu (63) reported on a method of using the creeping behavior of the hole boundary with time or mining advance to estimate the state of stress in a mine pillar. He used two types of hydraulic pressure cells developed by Panek (73), and Panek, Hornsey, and Lappi (74) for the purpose. The stress state measured was the vertical and horizontal stresses in a plane normal to a horizontal drill hole in a mine pillar.

To obtain the secondary principal stresses in a plane normal to the direction of a horizontal drill hole in a competent rock mass, an overcoring stress relief method, such as the procedure described by Bickel (25), is needed. The Lu method was shown to be sensitive to the setting pressure of the cells as described earlier in "Lu's Integrity Factor Method of In Situ Pillar Design." The overcoring method has never been made to work consistently in coal because of the difficulty of overcoring broken coal.

If a simple method of analysis can be derived that requires only a limited amount of physical property information and involves only the relative stress change in the pillar, a useful method of analysis will result.

A Mohr-Coulomb stress-failure model, based on equilibrium considerations, conveniently describes material behavior independent of the physical properties required by infinitesimal theories of elasticity, plasticity, viscoelasticity, discrete-block models, boundary-element models, and finite-element models. The solids treatable with the Mohr-Coulomb stress-failure model range from soil-like materials of civil engineering to rigid bodies as shown later in "Effect of End Constraint on Compressive Strength of Test Specimens in Laboratory." Also treatable with this model are deformable and broken solids. Since deformation is not considered, it is unnecessary to use the physical properties generally used to relate the strain to stress.

How the Mohr's stress circle is determined from equilibrium conditions alone for a rigid body, without involving physical properties, is given in "Mohr-Coulomb Stress-Failure Relationships."

In the Mohr-Coulomb stress-failure model the state of stress, represented by a Mohr's stress circle in normal stress, σ , shear stress, τ , space, increases in size to reach a Coulomb failure line. There is no mandatory way in which this must occur. Two ways that this could occur proposed by Kidybinski and Babcock (60) are given in figure 23. Babcock proposed the proportional loading approach given in figure 23A. The Coulomb failure line has the equation

$$\tau_{nf} = C - \sigma_{nf} \tan \phi, \quad (92)$$

where τ_{nf} , C , σ_{nf} , and ϕ are the shearing stress at failure, the unconfined shearing strength, the normal stress at failure, all in pound per square inch, and the angle of internal friction, in degrees. For proportional loading, the ratio of the principal stresses, σ_1/σ_2 , has a constant value. The locus of points on these stress circles closest to the failure line follow some straight line path defined by the equation

$$\tau_n = \sigma_n \tan(180 - \phi'), \quad (93)$$

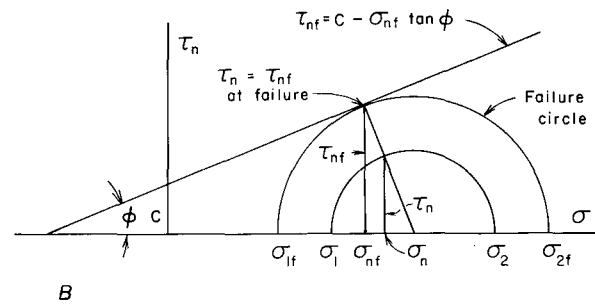
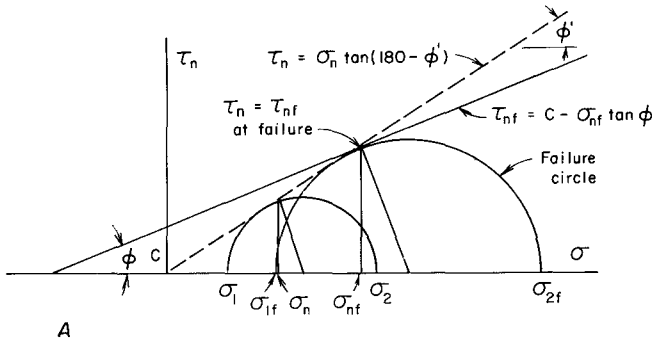


Figure 23.—Mohr-Coulomb stress-failure models. A, Proportional loading; B, sum of principal stresses is constant (67).

where τ_n , σ_n , and ϕ are the shearing stress, the normal stress, and the angle of the line defined by the locus of points for proportional loading, respectively.

If $\phi' > \phi$, as shown in figure 23A, failure will occur when the principal stress $\sigma_1 = \sigma_{1f}$ and the principal stress $\sigma_2 = \sigma_{2f}$. This will occur when the line of equation 92 intersects the line of equation 93. Then, the shearing stress at failure will be

$$\tau_n = \tau_{nf}. \quad (94)$$

Other scenarios are expected with this model. If $\phi' = \phi$ after $\phi' > \phi$, failure has already occurred, and the internal friction is controlling the behavior of a broken coal mass. If $\phi' < \phi$, failure is not expected.

Kidybinski proposed the loading where the sum of the principal stresses is a constant as shown in figure 23B. That is, $\sigma_1 + \sigma_2 = \text{a constant value}$. At failure,

$$\sigma_2 = \sigma_{2f}, \sigma_1 = \sigma_{1f}, \text{ and } \sigma_1 + \sigma_2 = \sigma_{1f} + \sigma_{2f}. \quad (95)$$

In both the figure 23 models the absolute stresses and the physical properties are needed to construct the Mohr's stress circles.

Next, consider the figure 24 Mohr-Coulomb models. In figure 24A, the unknown principal stress state in a mine pillar is taken to be that of the vertical stress, S_v , and horizontal stress, S_h , near the center of the pillar. The unknown condition is denoted by the broken semicircle used, and by the notation $S_v = ?$, and $S_h = ?$. This circle must touch or be below the Coulomb line as shown. The angle of the Coulomb failure line, ϕ , is known and the unconfined shear strength, $C > 0$, because even broken coal has some resistance to shear. Point A on the unknown circle is the closest point to the Coulomb failure line.

Next, consider figure 24B where a new set of coordinate axes, σ'_n and τ'_n , have their origin at the unknown point A. A proportional loading construction similar to that of figure 23A is also shown. The changes in the vertical stress, ΔS_v , and the horizontal stress, ΔS_h , near the center of the mine pillar are taken to be changes in the unknown stress $S_v = ?$ and $S_h = ?$. In the method being developed, only the relative values of these stress changes ΔS_v and ΔS_h will be needed. Therefore, the stress circle for the stress changes is also shown as a broken circle. The stress change is defined for proportional loading by the angle, ϕ' , as was done in figure 23A. Again, the three cases for the relationships between ϕ' and ϕ are considered.

To use the figure 24B model for considering pillar stability, experimental stress change data obtained from drill hole instrumentation installed in a mine pillar will be needed.

One of the most useful and robust rock mechanics instruments for measuring the stress changes in the coal

around a drill hole is the borehole pressure cell (BPC) of Panek (73). The cell pressure changes observed when using these cells are *not* the coal seam stress changes as are shown later in "Calculation of Mine Pillar Stresses From Borehole Pressure Cell Data." The cell pressures can be translated into coal stress changes, ΔS_v and ΔS_h , with equations 124 and 125. σ_v , σ_h , ΔS_h , and ΔS_v are the cell pressures in the vertical and the horizontal directions, and the coal seam stress changes in the vertical and horizontal directions, respectively. These equations given below as 96 and 97 are

$$\Delta S_h = (3/8m)(\text{Ln}((\sigma_h/\sigma_h^*)^3 (\sigma_v/\sigma_v^*)))$$

and

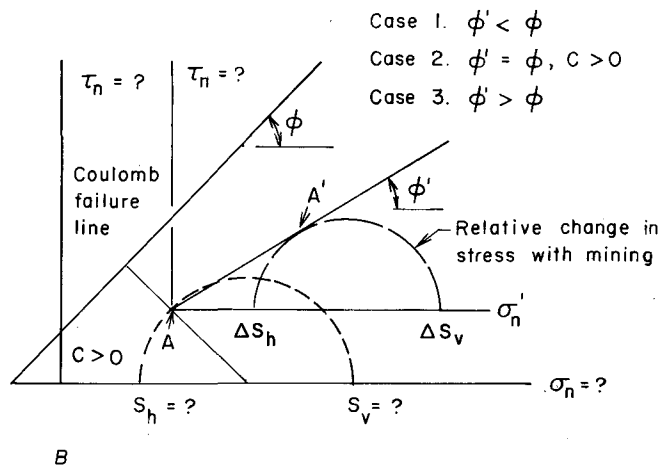
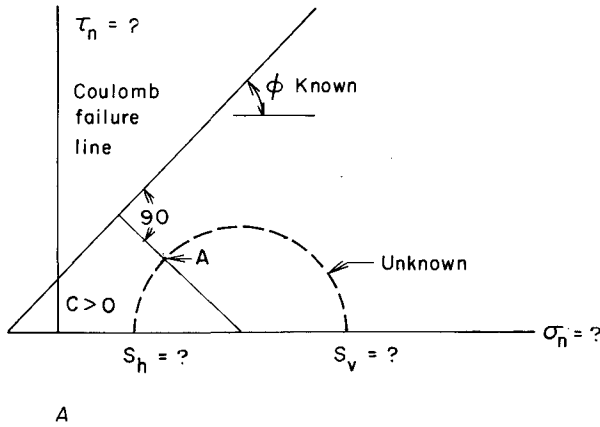


Figure 24.—Mohr-Coulomb stress-failure models for Babcock's pillar stability method. A, Question mark (?) denotes unknowns. S_v , S_h , σ_n , and τ_n are the vertical, horizontal, normal, and shearing stress components. ϕ , angle of internal friction, is known. C , the unconfined shear strength must be larger than 0. A is the point on the Mohr stress circle closest to the Coulomb failure line. B, Point A moves to point A' by proportional loading assumed in figure 23 along a line defined by ϕ' . For cases 1 and 2, no failure can occur. For case 3, failure is approaching.

$$\Delta S_v = (3/8m)(\text{Ln}((\sigma_v/\sigma_v^*)^3 (\sigma_h/\sigma_h^*))), \quad (96)$$

where m represents the physical properties, Young's modulus, E , and Poisson's ratio, ν , as defined by the function

$$m = 4.348 [(1 - \nu^2)/E]^{2/3}, \text{ in psi units}$$

and

$$m = 0.220 [(1 - \nu^2)/E]^{2/3}, \text{ in GPa units.} \quad (97)$$

Notice that physical properties E and ν occur in these equations. If some arbitrary value of m is used, the relative values of ΔS_v and ΔS_h can be obtained. Also, if only the ratio $\Delta S_h/\Delta S_v$ is used, the physical property terms cancel so that

$$\Delta S_h/\Delta S_v = (\text{Ln}((\sigma_h/\sigma_h^*)^3 (\sigma_v/\sigma_v^*))) / (\text{Ln}((\sigma_v/\sigma_v^*)^3 (\sigma_h/\sigma_h^*))). \quad (98)$$

The stress changes ΔS_h and ΔS_v may be either positive (+) or negative (-). If both are positive (+), the model of figure 24B can be used to describe the behavior as long as the stress changes are proportional. If the ratio is negative (-), that is, the cell pressure is increasing in one cell and decreasing in the other, the model of figure 23B can be used to describe the behavior. It seems unlikely that this kind of behavior could occur for an intact coal mass. It could occur for a broken coal mass.

The angle of internal friction, ϕ , can be estimated in situ in at least three ways. First, it can be obtained by means of a borehole shear tester (BST) developed by Dr. R. L. Handy of Iowa State University on USBM contract No. G0144021. Some of this work was reported in reference 45. The nature and use of this equipment has been reported by Haramy (46). Since the ratio of the shearing to the normal forces produced by pressures in hydraulic equipment is used to find ϕ , the absolute values of the normal and shearing stresses acting on the rock or coal and their physical properties are not needed. Second, if rib breaking occurs in the mine, the angle of breakage is related to the angle of internal shear as shown in figure 25. The angle decreases from the 45° angle for the condition where $\phi = 0$ at the rate of $\phi/2$, as can be verified from the Mohr's circle construction. In figure 25, the average angle is

$$\phi = (\phi_t + \phi_b)/2$$

and

$$\theta = (\theta_t + \theta_b)/2. \quad (99)$$

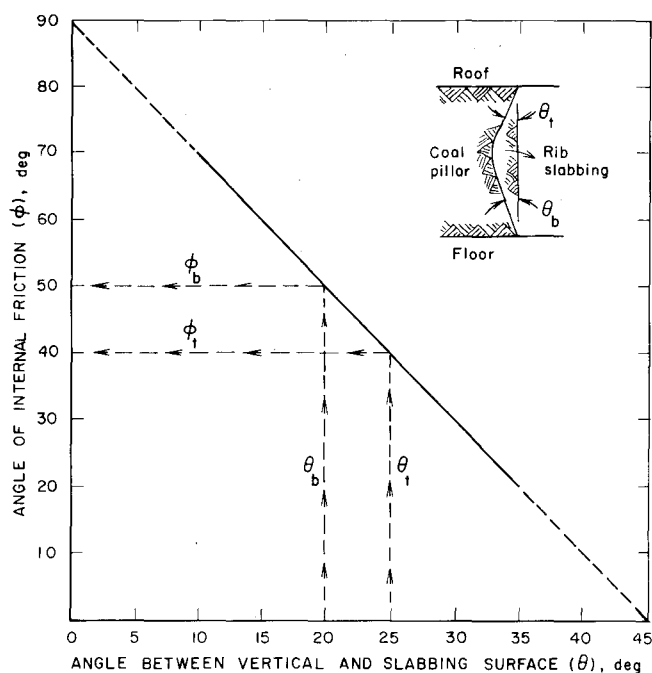


Figure 25.—Estimation of angle of internal friction for coal pillar in situ from angle of slabbing of rib.

A third way to estimate the angle of internal friction is to use the angle of repose of dry granular coal as is done in soil mechanics. This is the angle between a horizontal plane and the slope of the pile surface in a vertical plane.

There are borehole pressure cell data in the literature that can be used to evaluate some of the concepts presented here. Harmel, Lu, and Barron⁵ reported such data from borehole pressure cells used in a double entry longwall mining system at the Kaiser Steel York Canyon coal mine, near Raton, NM. Some of these data will be analyzed here with the present procedure. The physical test layout used in the work cited is shown in figure 26. Eight pairs of borehole pressure cells (BPC's) were installed in eight parallel horizontal holes as shown. One cell in each hole was oriented to respond primarily to the vertical stress and the other primarily to the horizontal stress. The cells were pressurized when the longwall face was far away from the cell location. In this report, this distance was -450 ft away and the face of the 7-Left longwall panel was being advanced in the direction of the location of the cells. The 8-Left panel remained unmined for the time used in this analysis. The chain pillar size was 42 by 100 ft. Entry width was 18 ft. The distances in feet are from the face to the cell location in the direction of face advance. The (-) denotes the face is approaching and the (+) denotes that the face has passed the cell location.

⁵Harmel, R., P. Lu, and L. Barron. Ground Control Study for Longwall Entry Design at York Canyon Mine, Raton, New Mexico—Two-Entry System. BuMines Progress Report 10034, Dec., 1982, Den. Res. Ctr., Denver, CO, 99 pp.

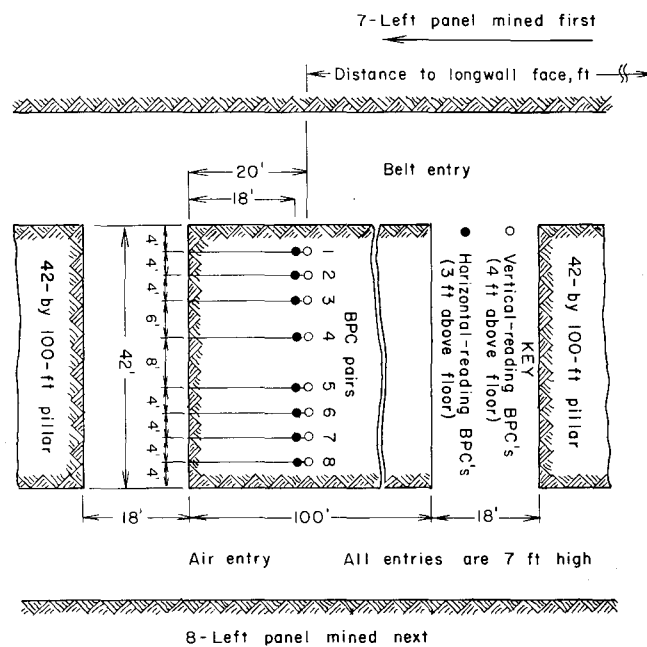


Figure 26.—BPC arrays of two cells each used by Harmel, Lu, and Barron in longwall entry study.

The changes in vertical and horizontal stresses from the time the longwall face was -472 ft away were converted into the changes in the coal stress by means of equations 96 and 97 by assuming some arbitrary values of Young's modulus, E , and Poisson's ratio in equation 97, for pound per square inch units, to provide units for the graphical plots of figures 27 and 28. The top part of the figure 27 is for the relative vertical stress change, ΔS_v , and the lower part of figure 27 for the relative horizontal stress change, ΔS_h , with mining advance. The curve numbers, 1, 5, 6, 7, 8, and 10, denote the location of the mining face when the data were taken. The mining sequence is for distances 1 (-450 ft), 5 (-25 ft), 6 (0 ft), 7 (+25 ft), 8 (+50 ft), and 10 (+372 ft). The reader can observe the change in relative stress with face advance by going from curves 1, 5, 6, 7, 8, to 10 in figure 27.

The vertical stress increased for the entire pillar from curves 1, 5, 6, and 7. That is, from the time the longwall face approached and passed the cell location by +25 ft, the vertical stress increased as indicated by these curves. While this increase in vertical stress was occurring, the change in the horizontal stress was less, but was more complicated. There was a slight increase in the horizontal stress near the center of the pillar. The horizontal stress decreased somewhat near the side of the pillar away from the mining. There was a slight decrease in the horizontal stress as the face approached and a slight increase as the face passed the cell location. When the face was +25 ft past the cell location, the rib near the mining showed an increase in horizontal stress that was two-thirds of the increase in the vertical stress value.

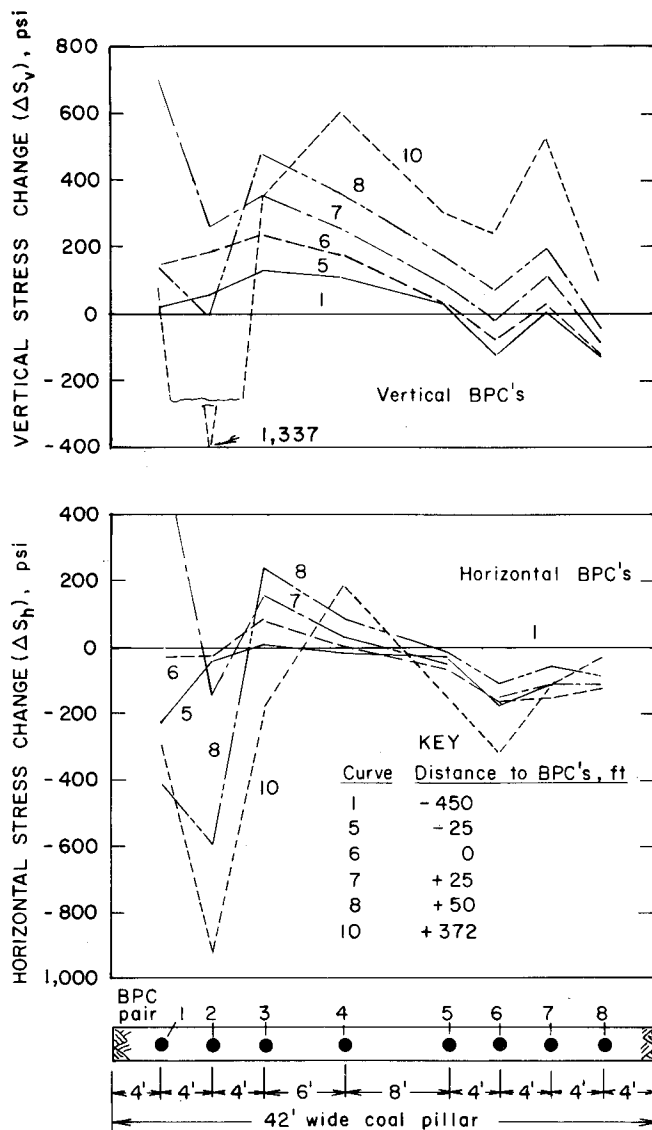


Figure 27.—Changes in vertical and horizontal pillar stress with mining advance derived from Harmel, Lu, and Barron data. Top, Relative vertical stress change; bottom, relative horizontal stress change.

When the face was +50 ft past the cell location for curve 8, there was a sudden decrease in both the vertical and horizontal stress changes. However, the horizontal stress was greatly reduced below the starting stress for curve 1, while the vertical stress change fell to the curve 1 value. At this time, the pillar was still able to support the overburden load on its left side near the mined-out panel, while the pillar had lost much of its strength in the horizontal direction. Finally, for curve 10, the vertical stress increased to its largest value since mining of 7-Left panel began, while the horizontal stress fell far below its premining value. The pillar had little horizontal strength at this time, but was still able to support an increased

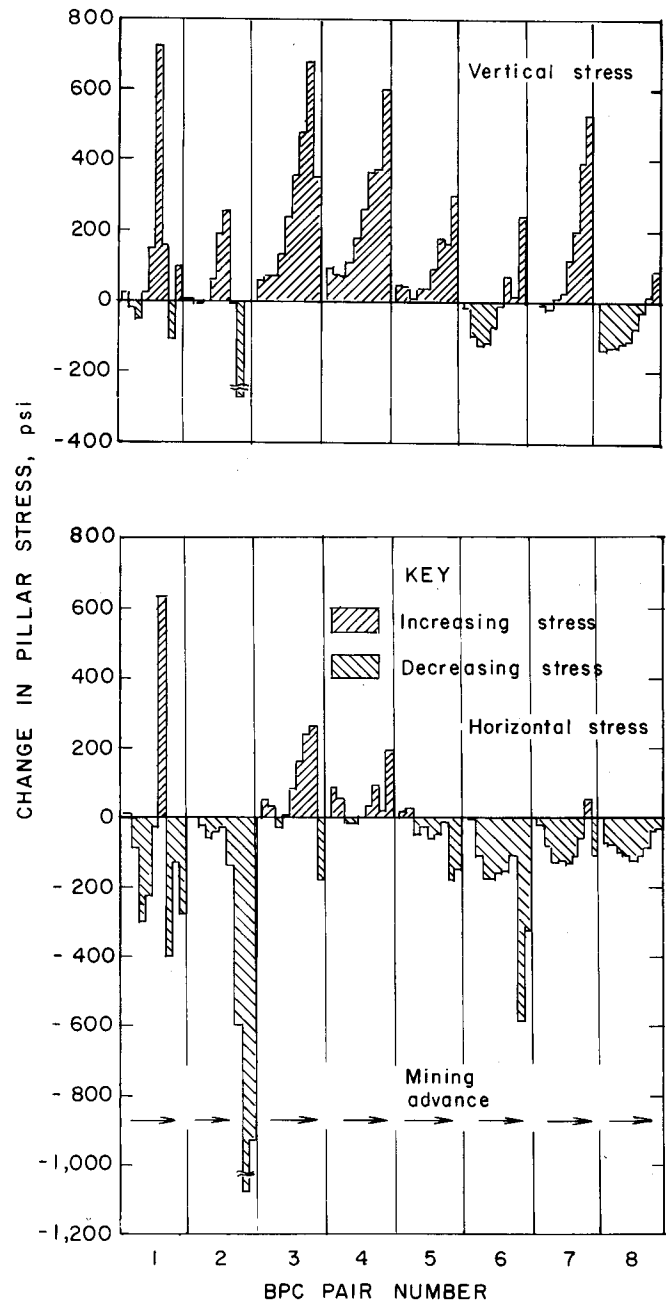


Figure 28.—Increases and decreases in pillar stress over mining interval.

overburden load in the form of an increased vertical stress. It is apparent that the pillar had become a yield pillar for one-third of the width on the side of the mined-out panel by the time the mining had passed the cell location by +372 ft.

Observe that, for the most part in figure 27, the horizontal stress *decreased*, while the vertical stress was increasing. Only about the center one-third of the pillar was constrained so that the horizontal stress was increased. For the most part, this constraint came from the roof and

floor near the center of the pillar, although the left edge of the pillar was constrained as shown by curve 7. The data of figure 27 can be shown in two other ways as was done in figures 28 and 29. In figure 28, the relative stress change for vertical and horizontal gauges are plotted from left to right for each of the 8 cell pairs (16 cells). The numbers in this figure are BPC cell numbers, not curve numbers as used in figure 27. The areas above the horizontal axis represent increases in pillar stress (+), and the areas below the axis represent decreases in pillar stress (-) during the mining operation. Note the relatively large (+) area for vertical stress and the (-) areas for horizontal stress. This shows that the vertical stress increased even though there was only a limited amount of horizontal confinement of the pillar.

The Harmel, Lu, Barron data in reduced form were next used to plot the relative Mohr's stress circle changes for pairs of vertical and horizontal cells as shown in figure 29. The curve numbers denote the mining advance as was done for figure 27. For cell pair 1, 4 ft from the edge of the pillar, the circle sizes and locations changed in a chaotic way. This indicates that the coal was probably broken and the stresses changed in an unpredictable manner. The stress change in both the vertical and horizontal directions (as indicated by the right and left sides of the Mohr stress circles, respectively) alternately increased and decreased in turn. The large diameter of 6 circle when the mining face was at the cell location indicates that a large shearing stress was supported by the coal at that time. After the face moved to the +25-ft location, the shearing stresses decreased according to the figure 23B model with the sum of the stress changes remaining constant from circles 6 to 7.

In figure 29 for cell pair 2, 8 ft from the edge of the pillar, the stress change was small as indicated by the small diameters and fixed locations of the circles 5, 6, and 7 until the face was +25 ft past the cell location. The shearing stress increased slightly as indicated by the increase in the circle diameters 8, 9, and 10 after that time. The average stresses decreased rapidly from 8 to 9 and then increased from 9 to 10. It does not look as though the coal around the cell pair 2 could support much shear stress and was probably badly broken.

In figure 29 for cell pair 3, 12 ft from the edge of the pillar, the stress change increased, as expected, for proportional loading from mining location 4 to 8. The angle of the tangent, ϕ' , increased slightly from circles 8 to 9 to have a value of $\phi' = 21^\circ$. After stress change circle 9, both the vertical and horizontal stress change decreased drastically, with an increase in the shearing stress, as indicated by the large increase in the 10 circle size. Circles 9 and 10 were for face locations of +155 ft and +372 ft past the cell location, respectively. Because the sudden shift in stress occurred long after the mining was past the cell location, it is concluded that the

condition $\phi' = \phi$ existed when the change from 9 to 10 occurred. That is, the internal friction was controlling the stress change, and the angle of internal friction was 21° . When the friction could no longer control the behavior, a large stress change adjustment occurred.

For cell pair 4, 18 ft from the left edge of the pillar, proportional loading occurred from mining position 4 through 10, with some unexplained behavior from 8 to 9. Again, the angle of stress change was $\phi' = 21^\circ$ for the tangent line to circles 5, 6, 7, 8, and 10. For cell pair 5, 26 ft from the left edge of the pillar, the tangent line for circles 6, 7, and 8, again had a slope of 21° . That is, $\phi' = 21^\circ$. After the face was +50 ft past the cell location (circle 8) to the time it was +155 ft past the cell location (circle 9), there was a sudden decrease in the horizontal stress, while the vertical stress did not change. From the time mining advanced from the +155-ft location to the +372-ft location, the horizontal stress was stable, while the vertical stress change increased as shown by circle 10.

For cell pair 6, 30 ft from the left side of the pillar, the tangent line to circles 6, 7, and 8 was again, $\phi' = 21^\circ$. The stress changes were small until the face was +50 ft past the cell location. Then, by the time the face had advanced to +155 ft for circle 9, the horizontal stress decreased drastically with only a small decrease in the vertical stress. The large circle diameter of 9 indicates that a large shearing stress developed during that time. By the time the face moved from +155 ft (circle 9) to +372 ft (circle 10), both the vertical and horizontal stresses increased without any change in the large shearing stress, which was maintained.

For cell pair 7, 34 ft from the left edge of the pillar, the tangent line to the stress change circles 6, 7, 8, and 9, had a slope of $\phi' = 13^\circ$, much less than the 21° for most of the orderly stress increase observed for the remainder of the pillar. From the time of circle 9 when the face was at the +155-ft location to circle 10 when the face was at the +372-ft location, the stress changed drastically with an increase in the vertical and a corresponding decrease in the horizontal stress. Both of the behaviors of figure 23 were observed for this cell pair. First, the proportional loading of figure 23A (and 23B) were observed for circles 6, 7, 8, and 9, and then the figure 23B model for a constant sum of the stressed was observed for the change from circle 9 to 10. That is, the Mohr's stress circle changed size about its center. Notice that there are other circles out of numerical sequence for the other cell pairs that also changed size about the circle centers. Had a reading been taken only at those times, it would have appeared that the model behavior of figure 23B had occurred. For example, the circles 1, 4, 6, and 7 of cell pair 1, circles 6 and 7 of cell pair 2, circles 8 and 10 of cell pair 5, circles 8 and 10 of cell pair 6, and the circles 9 and 10 of cell pair 7 exhibit this behavior.

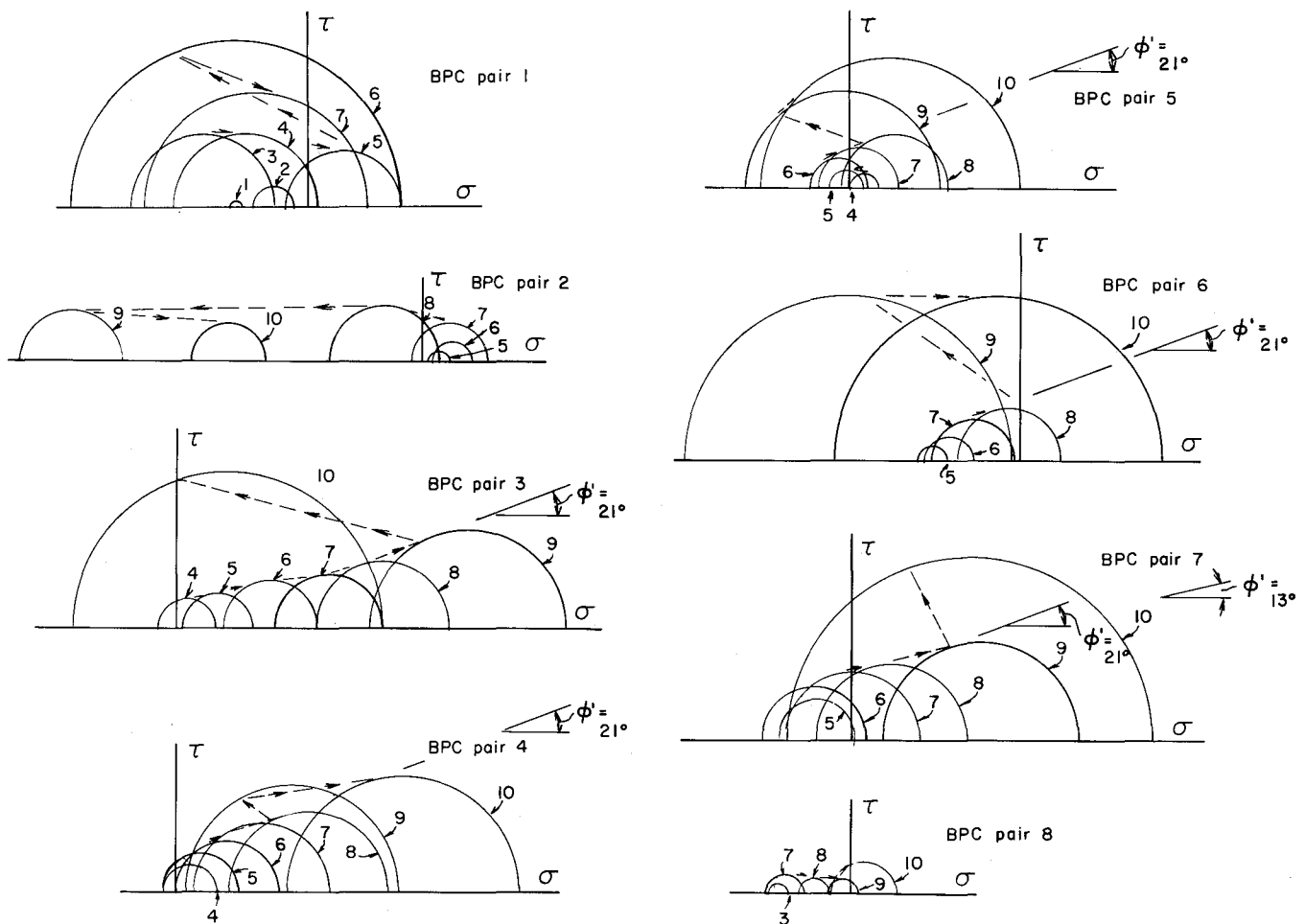


Figure 29.—Changing Mohr circle size with mining advance. Curve numbers correspond to mining advance distances in figure 27. σ = normal stress, and τ = shear stress.

For cell pair 8, 38 ft from the left side of the pillar (4 ft from the right side), very little stress change was observed. Whether this is the result of no change in stress or of a very weak rib condition is unknown. Because of the large change for the cell pair 7 only 4 ft away, the weak rib condition is indicated. That is, the right side of the pillar is being loaded, but the cell pair 8 does not show it.

The conclusions for the data considered are as follows. The pillar stability method based on equilibrium alone has considerable merit for analyzing the change in mine pillar stress with mining advance. No knowledge of actual physical properties is required. Only the ratio of the horizontal-vertical stress change, as measured by borehole pressure cells of the Panek design (73), is needed to obtain the relative change in the stress condition of the pillar. Units of measurement used to make the illustrations specific could be of any size and were based on assumed physical properties, the real values of which are unnecessary. While initially the angle of internal friction was

assumed to be known and that the unconfined shear strength was greater than zero, this information was also not necessary for the data studied. The pillar behaved as a broken coal mass for which the value of $\phi' = \phi$. If the coal pillar had been more intact and $\phi' > \phi$, then the angle of internal friction would have been necessary to understand the pillar behavior.

The stress changes noted were of several kinds. First, the proportional loading behavior of figure 23A and 24B was observed in 5 of the 8 cell pairs. This behavior occurred at least for a part of the pillar loading cycle for BPC cell pairs 3, 4, 5, 6, and 7. These cells were in the central part of the pillar width. The loading where the sum of the stress changes is constant (fig. 23B) also occurred. This behavior occurred for BPC cell pairs 1, 4, 5, 6, and 7 after the internal friction no longer controlled the stress changes. A sudden change from proportional loading to a more disorganized one indicated that the angle of internal friction was no longer controlling the behavior.

The angle of internal friction appeared to be $\phi = 21^\circ$. From this it was inferred that the coal pillar was broken and that the pillar strength was from internal friction. The

largest stress changes occurred long after the mining had passed the cell location.

ROCK MECHANICS CONCEPTS USEFUL FOR PILLAR DESIGN

MEASUREMENT OF PHYSICAL PROPERTIES OF ROCK OR COAL IN LABORATORY OR IN SITU

In experimental mechanics, changes in shape of a solid body are often measured and used to estimate the state of stress that exists in the body. To do this, some physical properties of the body are needed to couple the strains to the stresses. These coupling relationships are particularly simple if the body can be considered linearly elastic, homogeneous, and isotropic. In that case, two constants, the Young's modulus and the Poisson's ratio can provide the necessary relationships. Because a rock or coal mass often contains discontinuities such as cracks, joints, cleats, unbonded bedding planes, etc., a solid specimen of the mass is used for testing in the laboratory. These test results and properties for relatively small intact specimens and for large-scale mine pillars, roofs, and floors are very different. The problem can be considered in two parts: small specimens removed for testing and larger masses tested in situ.

Physical Properties of Intact Specimens in Laboratory

Much work has been reported in the literature on the physical properties of mine rock and coal in laboratory tests. A small sample of this work is cited in this report (7-11, 16-17, 22-23, 26, 29-30, 36, 51, 55, 61, 63, 65, 69-70, 72, 80, 84). One comprehensive study by Obert, Windes, and Duvall (72) reported in 1946 describes 15 physical tests for mine rock including longitudinal velocity of sound, torsional velocity of sound, Young's modulus, modulus of rigidity, Poisson's ratio, specific damping capacity, compressive strength, modulus of rupture, tensile strength, scleroscope hardness, abrasive hardness, impact toughness, apparent porosity, specific gravity, and grindability. The testing was in the laboratory on specimens obtained from diamond-drill core.

The physical properties of core-size specimens of intact rock or coal and the larger broken mass in situ are quite different as evidenced by a change in compressive strength reported by Rice (80) where small cubic specimens were eight times stronger per unit area than larger specimens of 54-in cubes. For mine-scale pillars, the difference is probably even greater. As long as additional structural features are added with an increase in volume, the average strength will usually continue to decrease. For example, faults or joints may be very large-scale structural features that are

contained by very large-scale masses, but are not present in smaller volume units.

Since the purpose of obtaining the physical properties for the theory of linear elasticity (Young's modulus and Poisson's ratio) is to relate the strains to the stresses that produce them, a step in the right direction is to use intact cores of rather large size (that is, 5.625 in diam) to obtain the state of stress in situ by overcoring. The physical properties are still obtained by laboratory methods, but on the same specimen for which the in situ stress state is to be obtained. Reproducibility of results between cores will indicate how suitable this size mass is to characterize the larger mass.

The overcoring method of obtaining the absolute state of stress was invented by Hast (47) of Sweden. He used a rigid inclusion gauge to determine the change produced in a single diameter by the overcoring process. A much softer gauge with a single-sensing element to measure the change in one drill hole diameter for each gauge was developed by Obert, Merrill, and Morgan (71). An improvement over this gauge and the one now in widespread use was developed by Merrill (66). The principles of operation of the overcoring procedure were reported by Merrill and Peterson (67). After the drill core is removed from the drill hole, the physical properties can be measured underground in the mine atmosphere, which often is humid so the core does not change its physical properties, or on the surface in a laboratory. In 1962, Fitzpatrick (40) reported on a device for obtaining the Young's modulus for plane stress when the core is loaded hydrostatically on its outer surface. In 1964, Obert (69) reported on a tri-axial method of obtaining elastic constants from stress relief cores at stress levels similar to those in a mine in situ.

Becker and Hooker (23), in 1967, reported on the effects of anisotropic rock properties on the rock stress. They concluded that large errors in the calculated stress state will result if an anisotropic rock is treated analytically as isotropic. Aggson (1), in 1977, reported on a method for analyzing overcoring data for nonlinearly elastic rock. The state-of-the-art with respect to overcoring procedures and data analysis have been reported by Bickel (25).

While the absolute state of stress can be obtained for many competent rock types where a drill overcore can be produced, this method does not work for mining coal that cannot be consistently overcored. Another procedure is needed for coal in situ and for other rocks where physical properties are strongly a function of the size of the test mass.

Measurement of Physical Properties In Situ

Panek, Hornsey, and Lappi (74) developed a method to measure the shear modulus, G , in situ, using a cylindrical pressure cell (CPC). The change in the cross-sectional area of the drill hole by a radial hydraulic pressure is related to G . The Young's modulus, E , and G for elastic, homogeneous, isotropic materials are related by the equation

$$E = 2(1 + \nu)G. \quad (100)$$

If the Poisson's ratio, ν , is assumed or determined from laboratory-type tests, E can be found. The value of G is an average value since the CPC is an integrating device acting around the entire circumference of the hole (i.e., 360°). If the Poisson's ratio is obtained by testing to obtain an average value, the resulting E will be an average value also.

Goodman, Van, and Heuze (43) developed a jack for the determination of the Young's modulus in situ in a drill hole. Hustrulid (58) reported that the results obtained were in error by a factor of 3. Part of the problem was the mismatch in curvature of the hole boundary and the gauge shoes. The area in contact between the cell and hole boundary can change drastically, changing the coupling and the gauge response.

Lu (62) presented a method to determine the in situ Young's modulus and the Poisson's ratio using three hydraulic pressure cells. He uses two BPC's (73) and one CPC (74) of Panek design for this purpose.

Two BPC's (73) can be used to determine the in situ Poisson's ratio by a method reported by Babcock (12). In this method, one BPC is orient to respond to stress changes in the vertical and the other to respond to stress changes in the horizontal directions in the interior of a mine pillar. These two cells could be those used in Lu's method cited above. If the cells are set at the same pressure, the ratio of the change in pressures can be used with figure 30 to obtain the in situ Poisson's ratio. This method should be used with an existing pillar, the shape of which is not being changed. The change in pillar stress is produced by the change in vertical stress from mining near the instrumented pillar. When the vertical stress is changed, it produces a change in the horizontal stress through the action of Poisson's ratio and the arching of the hole boundary (14). The method should work particularly well when the roof and floor rock are strong and provide horizontal constraint to produce a near plane strain condition near the BPC cell location. In the physical world, these relationships work well for materials in the elastic range, such as steel, structural aluminum, and for some rocks and minerals. Generally, the problem is more complex for rocks and minerals and varies with the rock type and state of stress. In some cases, the theory works well and, in other cases, it does not.

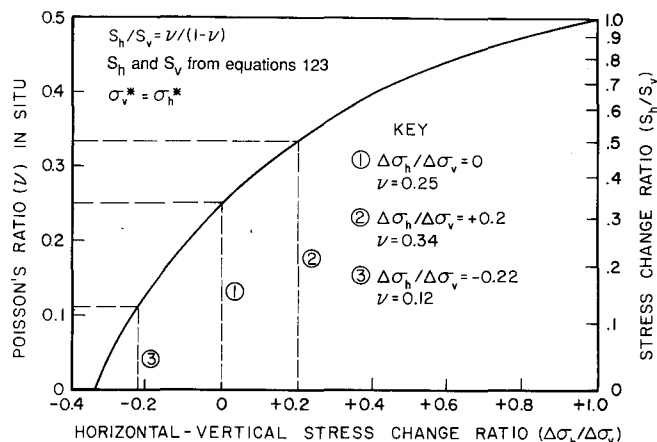


Figure 30.—Determination of Poisson's ratio (ν) in situ from ratio of horizontal to vertical pressure cell change when both had same initial setting pressure. Broken lines give three examples. Case 1, horizontal cell has no pressure cell change, while the vertical pressure changes. Poisson's ratio (ν) = 0.25. Case 2, cell pressure change ratio is +0.2. Poisson's ratio (ν) = 0.34. Case 3, cell pressure ratio is -0.22. Poisson's ratio (ν) = 0.11. This shows that cell pressure change ratio can be +, 0, or - for Poisson's ratio in range 0 to 0.5 (12).

If the three-hydraulic-pressure array is used for the Lu method (63), a second estimate of G , ν , and E can be obtained by setting the BPC's to the same setting pressure. The value of G is obtained as with the Panek, Hornsey, and Lappi (74) method, the Poisson's ratio is obtained by the author's method (12), and the value of E is calculated with equation 100. If unequal values of BPC pressures are used, the value of ν can be found from the equations 123 and 124.

When Are Physical Property Constants Not Constant?

It has been shown by Rice (80) and others since 1929 that the physical properties decrease with an increase in the test specimen size. Small laboratory specimens are made from intact pieces of rock or coal including diamond drill cores. As such they are biased in terms of increased strength because they have survived the drilling process or the process by which cleats or other discontinuities in the larger mass are formed. In addition to the effect of size and included structural features, the strength changes drastically with the state of stress and constraint. If laboratory test specimens were perfectly linearly elastic, homogeneous, isotropic properties can be assumed, the number of parameters relating stress to strain reduces to two for the entire body. One such pair of parameters are the widely used Young's modulus and Poisson's ratio, E and ν , respectively. Another pair are the Lamé's constants λ and μ widely used in the past. These idealized relationships produce a one-to-one correspondence between the states of strain and stress. That is, if the strains are known from

experimental testing, the stresses can be determined analytically. Unfortunately, no such perfect relationship exists in the physical world, even for small laboratory test specimens, and errors result in the estimation of the stresses from the strains. From the above, it is apparent that the use of "constants" to relate strain to stress is a highly questionable procedure for materials such as rock and coal encountered in mining.

Babcock reported (7) on the physical properties, E and ν , for laboratory testing of specimens of granite, limestone, marble, and sandstone. These properties were shown to vary widely with stress as shown in figure 31 and 32. Specimens with the geometries shown in figure 33 were used with length-to-diameter ratios of 3. These were instrumented with 90° strain rosettes to measure the axial strains, ϵ_a , and the tangential strains, ϵ_t , produced by axial loading in compression. The results shown in figures 30 and 31 indicate the following. The Poisson's ratios all increased with increasing stress by factors of about 10 for granite and sandstone (increasing from 0.05 to 0.50), and about 2 for limestone and marble (0.2 to 0.38 and 0.15 to 0.35, respectively). The Young's modulus approximately doubled for granite and sandstone and changed less drastically for limestone and marble. These changes were approximately the same for the straight specimens and specimens with steel rings on the ends.

The variation in E and ν reported above were from the nearly uniaxial stress state at midlength of the specimens. By way of comparison, the radial constraining stress within the steel rings is nearly a linear function of the axial stress applied as shown in figure 34. In the left column of panels in this figure, the theoretical constraint from equations 12 of reference 7 was calculated using the values of E and ν from figures 31 and 32. When the radial stress in the ring obtained experimentally was plotted against the average axial stress, the results in the right column of panels of figure 34 were obtained. The correlation coefficients for the theoretical and experimental results (left and right column of panels of figure 34) were better than 0.947 and almost 1.0, respectively, for a straight line fit.

The above testing was for competent rock for which diamond drilling core recovery was not a problem. In the case of coal, core recovery is often a problem because of structural discontinuities (cleats). In addition, coal is an anisotropic or orthotropic material as reported by Ko and Gerstle (61), which is characterized by nine parameters, not two. In the work cited, when the nine parameters were obtained experimentally, the scatter in the data would make an analytical solution with this input questionable. Again, since the physical properties vary from point to point, the interpretation of results would be difficult.

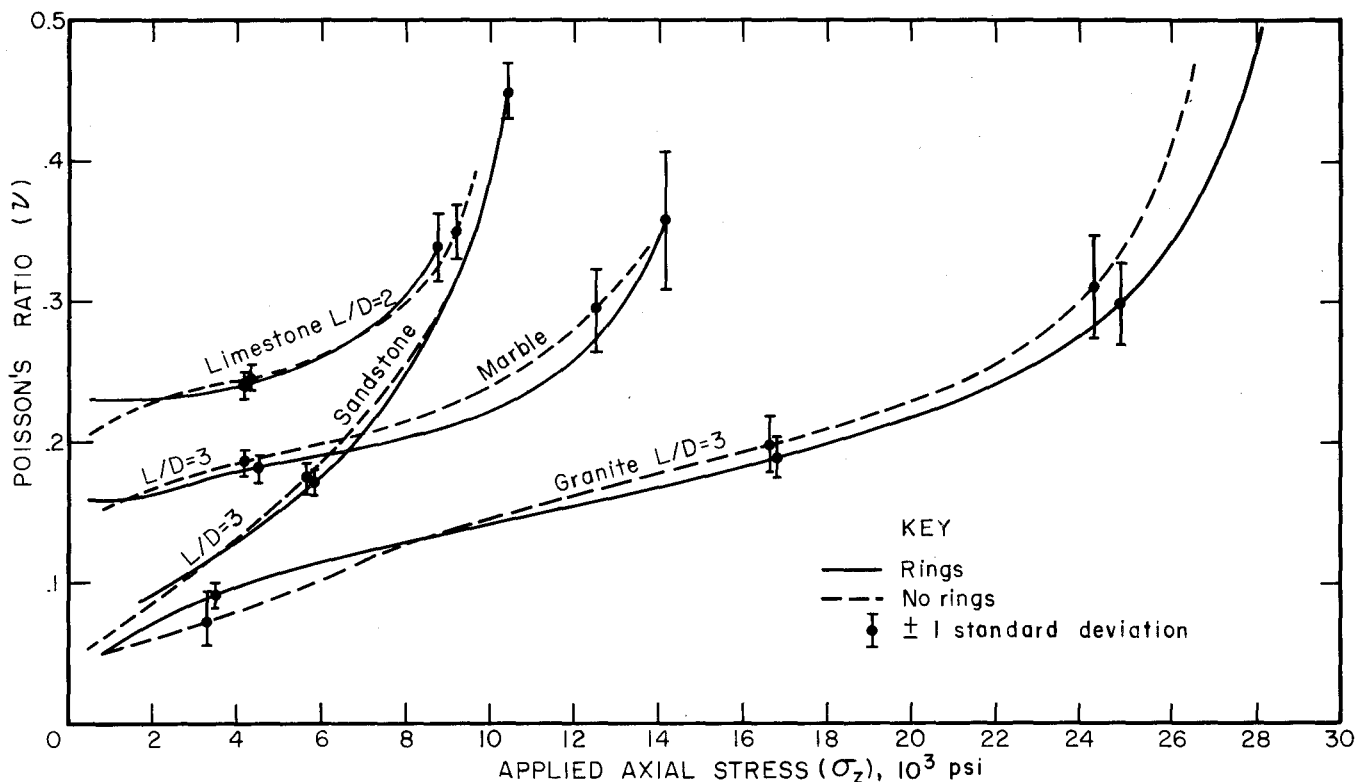


Figure 31.—Poisson's ratio versus average axial stress for selected rocks. L:D = length-diameter ratio (7).

It is concluded that elastic theories should be used with caution to relate observed experimental strains or displacements to stress in rocks and coal, particularly when these materials are stressed to failure. When the effects of volume and constraint are both considered, the difference in apparent strength could vary by two orders of magnitude (100 times).

MOHR-COULOMB STRESS-FAILURE RELATIONSHIPS

Use of Force Equilibrium To Obtain Mohr's Stress Circle and Coulomb Failure Criteria Relationships for Rigid Bodies

One of the most powerful and useful ways ever devised of representing stress and strain states in solid bodies is with Mohr's stress and strain circles.

It seems appropriate to acknowledge the contribution of Culmann to the Mohr's stress circle development. According to Hetenyi (48), Mohr (68) attributes the origin of the Mohr stress circle in plane stress to Culmann (34), and Hetenyi states, "there is a trend to call it the Culmann's circle." He further states, Mohr made a "detailed study of two- and three-dimensional stress and strain conditions at a point, with particular reference to yielding at failure of materials." Culmann presented in the reference cited the "first appearance of (a) diagram representing (the) state of plane stress, later called (the) Mohr's circle."

Both the Culmann-Mohr stress circle and the Coulomb failure criteria (33) can be derived from rigid body equilibrium relationships, as will be shown. These relationships have been applied to a wide variety of materials in the fields of soil and rock mechanics. Because deformation of the body is not considered, the bodies could include rigid solids; solids with elastic, plastic, viscoelastic, or

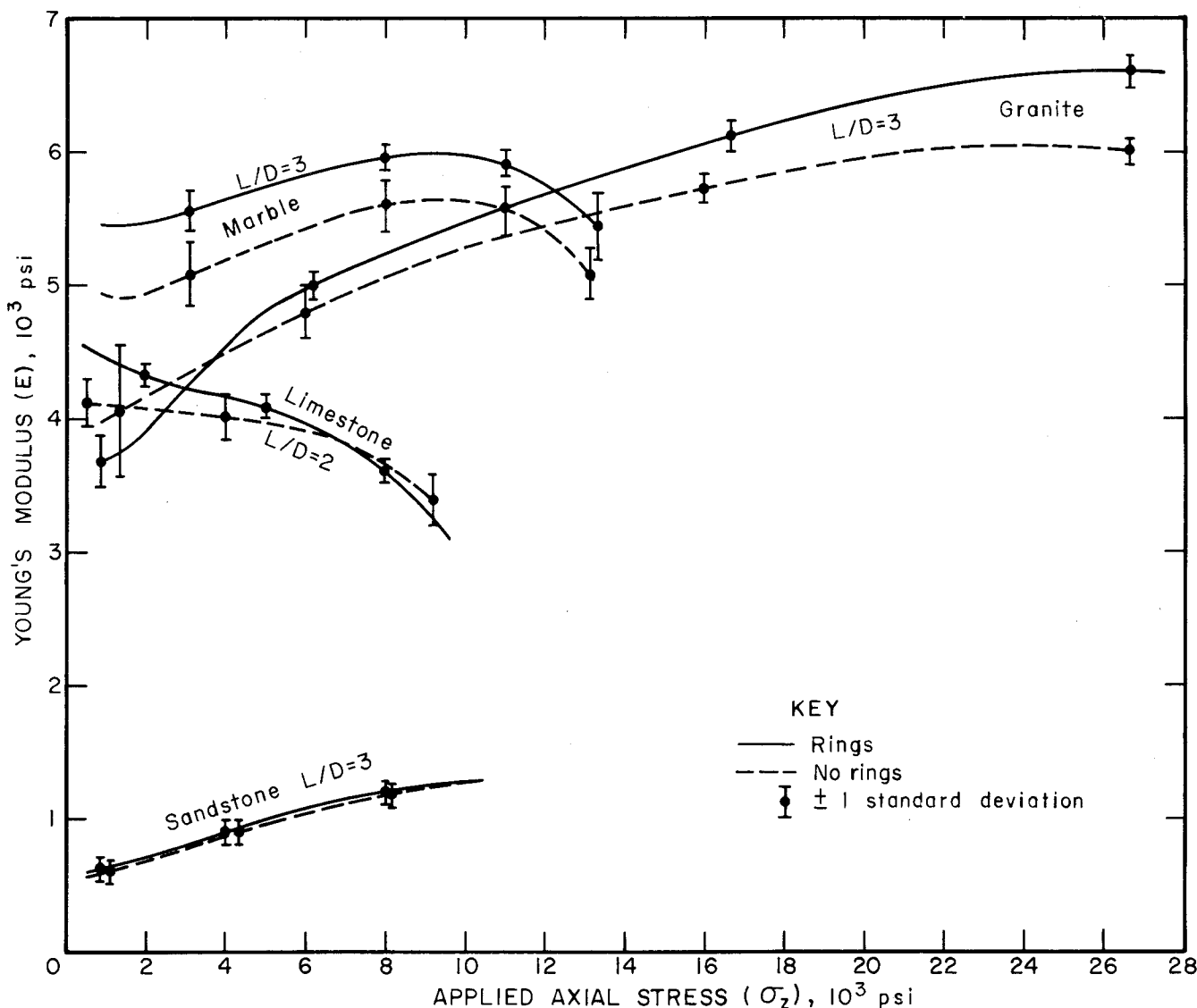


Figure 32.—Young's modulus versus average axial stress for selected rocks. L/D = length-diameter ratio (7).

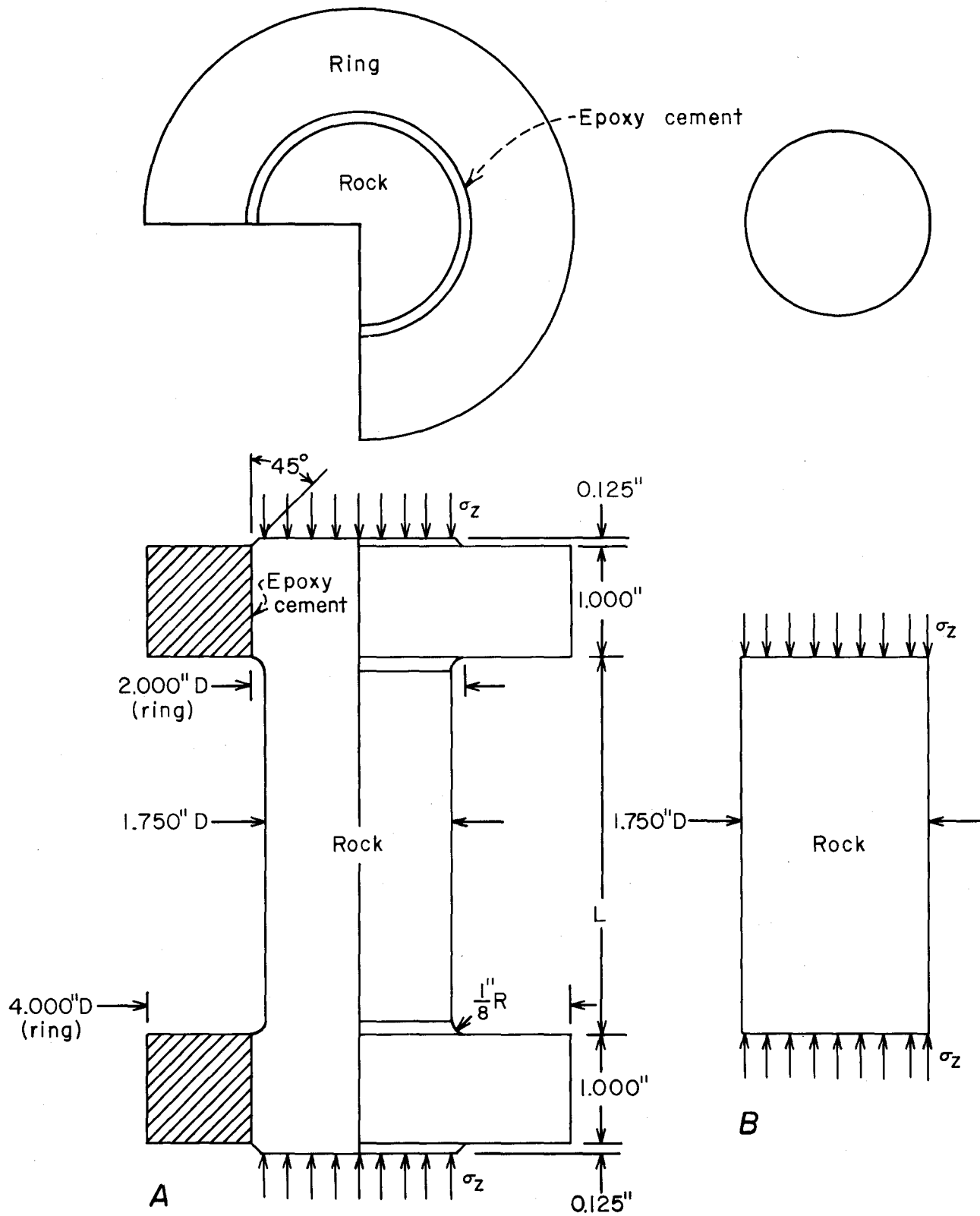


Figure 33.—Rock specimen configurations for compression testing. A, Test specimen with steel rings on ends for end constraint; B, straight test specimens (7).

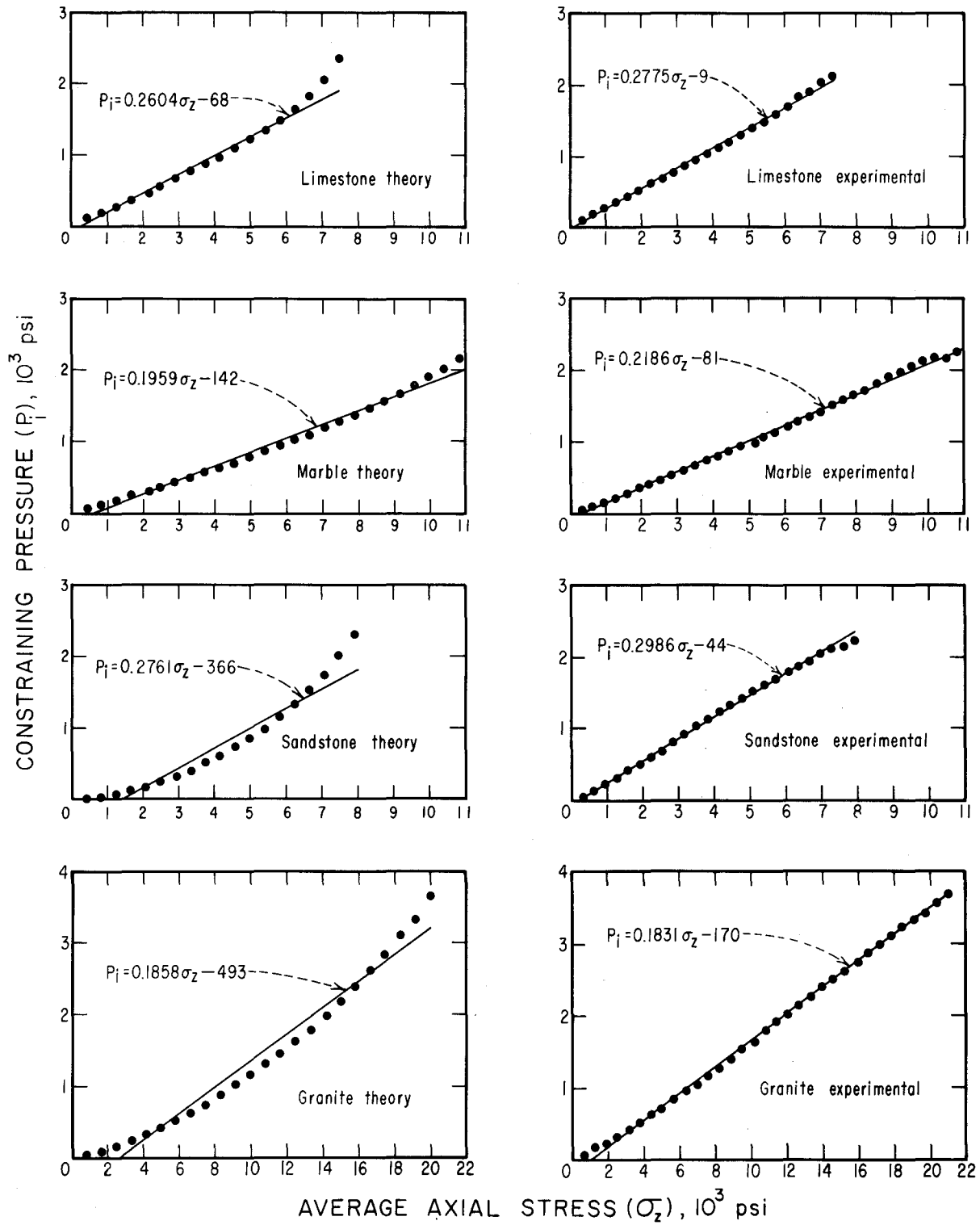


Figure 34.—Radial end constraint from solid steel rings (P_i), versus average axial stress (σ_z), for specimens of limestone, marble, sandstone, and granite (7).

viscoplastic properties; solids fractured to any extent; and cohesive or granular soils. This approach is particularly appealing for mine pillar design because it reveals that much of the problem of ground behavior can be defined with the limited amount of physical property and structural integrity data available to the designer as was done in section "Babcock's Method of Assessing Pillar Stability." The relationship is very powerful for stress. If the Mohr strain circle is needed, most of the generality is lost because of the necessary coupling (constitutive) relationships between stress and strain.

Consider the elemental cube of figure 35A with the three-dimensional state of principal stress in the x, y, and z directions. Figure 35B shows a xy plane section with a plane passing through the z axis and inclined to the other two axes at an angle θ . If the stresses σ_x and σ_y are not equal, there will be both a normal and a shearing stress, σ_n and τ_s , acting on this plane, which will be a function of

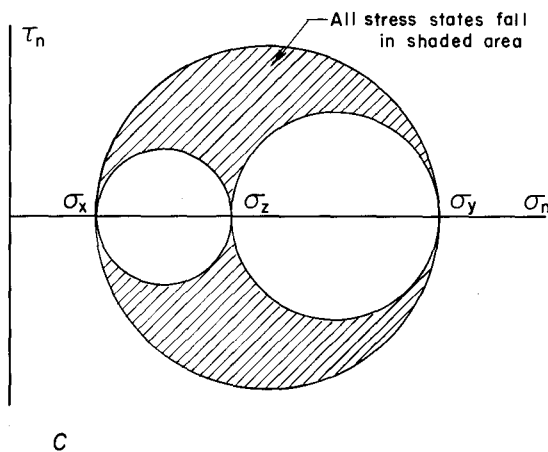
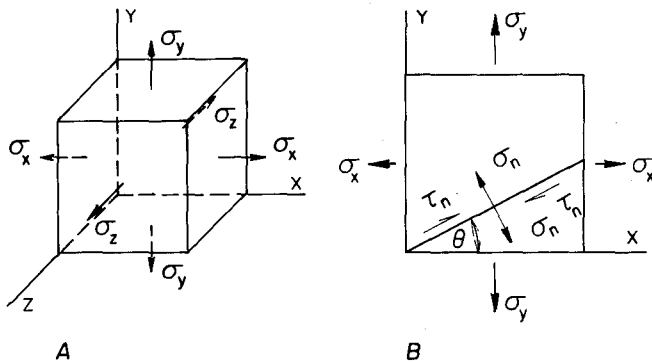


Figure 35.—Rigid body can be used to obtain Mohr's circles for three-dimensional state of stress. A, Rigid elemental cube with principal stresses applied; B, stress states on inclined surface used to construct Mohr's stress circle for rigid cube; C, three Mohr's circles used to represent three-dimensional stress state in rigid elemental cube.

the angle θ and the stresses σ_x and σ_y . If $\sigma_x = \sigma_y$, then the stress will be $\sigma_x = \sigma_y = \sigma_n$ and $\tau_s = 0$, for all planes. When both σ_n and τ_s exist, a summation of forces first in the normal and then in the shear direction can be made in terms of the principal stresses and the angle θ . When the inclined plane has a length and depth of unity so that the inclined area is one unit square, the stresses can be summed as forces per unit area (stress) to obtain

$$\sigma_n = \sigma_x \sin^2 \theta + \sigma_y \cos^2 \theta$$

and

$$\tau_s = ((\sigma_x - \sigma_y)/2) \sin 2\theta. \quad (101)$$

If these stresses are plotted with the σ on the abscissa and the τ_s on the ordinate, the Mohr's stress circle will result.

In the same way, by cyclic permutation between x, y, and z, two additional circles can be produced. These three circles for the three-dimensional state of stress are shown in figure 35C. The points that represent the possible states of stress within the cube all fall on the circles or fall in the shaded areas between the circles. So far, nothing has been said about how failure is related to the stress state or to deformation or strain.

Next, rigid bodies will be used to develop the Coulomb failure relationship. Consider the two rigid bodies that contact one another along a plane surface shown in figure 36A. If the bodies are held together by forces normal to the contact plane, they will resist being displaced in a direction parallel to the area of contact if friction exists between the areas. The amount of resistance to displacement is a measure of the frictional behavior. If the force is applied parallel to the surface, its magnitude when movement occurs is a measure of this friction. The friction, μ , is defined as the ratio of the shearing force to the normal force

$$\mu = F_s / F_n = A\tau_s / A\sigma_n = \tau_s / \sigma_n. \quad (102)$$

In a sense, this friction becomes a form of shear strength if any displacement represents failure of the system. Next, consider the possibility of a cohesion between the two bodies that is independent of the normal force acting. A frictional force is required to overcome this cohesive resistance to shear displacement. Defined as the unconfined shearing strength between two parts of the body, this parameter is shown in figure 36B as the value C. The effects of both the shearing strength produced by friction and by the unconfined shearing strength are given in figure 36B. The angle ϕ is the angle of internal friction, defined to be the arctan(μ).

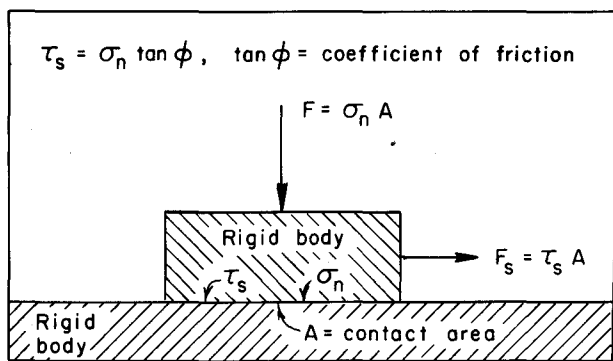
What Is Failure in Rigid Body?

So far the Mohr stress circle and the Coulomb failure condition have both been derived from rigid body

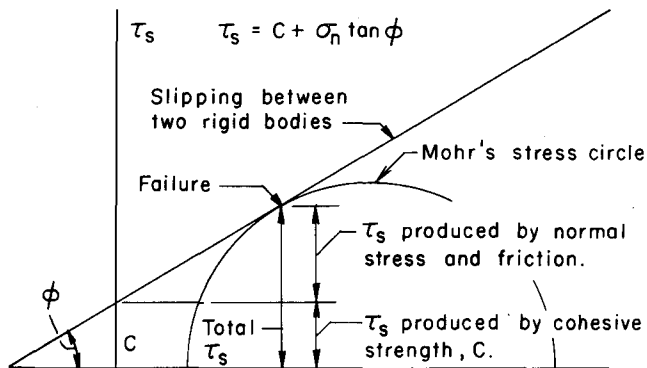
behavior. How can a rigid body fail? No failure is associated with the Mohr's stress circle alone. If one rigid body becomes two rigid bodies by a displacement between the two rigid parts, this could be taken as failure. Certainly, the rigid material of the body itself cannot be deformed if it is not broken.

Mohr-Coulomb Stress-Failure Model

In this model, failure is assumed to have occurred if a Mohr stress circle touches the Coulomb failure line in any way. Such failure conditions are shown in figure 36. No requirement is specified as to how this condition occurs, just that it does occur. The circle is tangent to the straight



A



B

Figure 36.—Rigid bodies can be used to obtain Coulomb failure condition. A, Two rigid bodies held together by normal force (F_n) produce average normal stress (σ_n) that can resist shearing force (F_s) and average shearing stress (τ_s) that tends to produce sliding. ϕ is angle of internal friction; B, rigid body analog of Coulomb failure condition for deformable bodies. Failure is displacement between two rigid bodies; C, unconfined shearing strength that may be produced by adhesive bonding between two bodies (19).

line at this point. The Coulomb line is the locus of such possible failure conditions.

How Does Stress Condition Produce Failure State?

Many generalized scenarios exist that illustrate how a particular stress state produces failure. Three such scenarios are given in reference 60. An additional example, proposed by this author, was described in figure 23. Several different experimental results for changing Mohr circle size are described in "Babcock's Method of Assessing Pillar Stability" for static failure and in "Bursting of Model Coal Pillars in the Laboratory" for dynamic (burst) behavior.

Role of Constraint on Shearing Strength

Failure occurs if the Mohr's stress circle touches the Coulomb failure line in figure 37. Any point of the Coulomb failure line is a possible failure condition. The shear strength at failure can be characterized (9) by means of the three circles shown in the figure in which the shear stress, τ at failure, has components $\tau = a$, $\tau = a + b$, and $\tau = a + b + c$.

The component "a" is the only "true" shear strength, and this comes from the physical properties of the material. This state of stress is shown on an elemental cube in figure 38A. Here, one extreme principal stress, σ_a , is shown acting vertically. The other extreme principal stress, σ_a^* , in tension, acts horizontally. The intermediate principal stress is between σ_a and σ_a^* . For this condition to occur, the point of failure in figure 38 must fall on the T axis. The ratio of these two stresses changes with the angle of internal friction, ϕ . The plane of failure on which $T = a$ is acting has only this shearing stress. There are no normal stress components acting across the failure plane.

The component "b" of shearing strength of figure 38 is produced by normal components of stress across the failure surface, σ_n in figure 39B. The extreme values of principal stresses acting on this elemental cube are σ_{a+b} in the vertical direction and $\sigma_{a+b}^* = 0$ in one horizontal direction. The third principal stress is between these two principal stress values. The addition of the normal stress σ_{abn} increases the shear strength by an amount "b" to the $T = a + b$ value. This increase in shearing strength results from constraint across the failure plane produced by the vertical stress σ_{a+b} .

The components of shear strength "a" and "b" are somewhat more characteristic than the shear strength components "c." The additional shearing strength component "c" results when any triaxial principal stress state moves the Mohr's circle from the origin. The shear strength $T = a + b + c$ in figure 37C is produced by both the unconfined shear strength provided by the physical properties "a" and

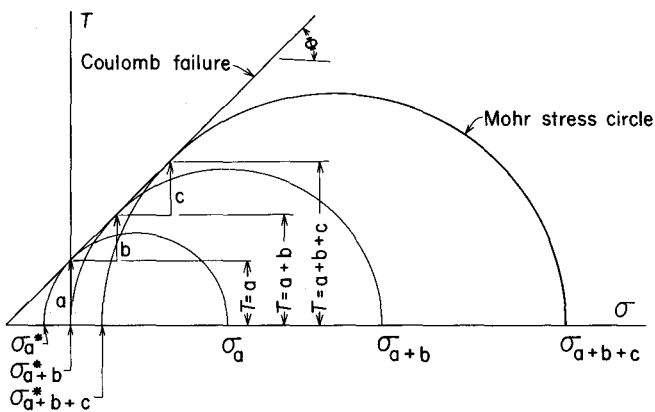


Figure 37.—Three components of shear strength, a , b , and c , and their relationship to compressive strength for Mohr-Coulomb stress-failure criteria. ϕ is angle of internal friction, and σ , τ are normal and shearing stresses, respectively (9). σ_a , σ_{a+b} , and σ_{a+b+c} are minimum principal stresses, and σ_a^* , σ_{a+b}^* , and σ_{a+b+c}^* are maximum principal stresses. Tension is +.

by normal stresses σ_n . These normal stresses are produced in part by the vertical principal stress σ_{a+b+c} and in part by the horizontal principal stress σ_{a+b+c}^* in figure 37C. An infinite number of different "c" values corresponding to an infinite number of stress states are possible. Dynamic changes in the "c" value of shear strength are related to burst and earthquake behavior.

Shear failure for any of the stress states 38A, 38B, or 38C if it occurs, will separate the elemental cube of 38D into two parts. The failure direction $\phi/2 + 45^\circ$ is obtained from the Mohr's circle construction. The angle on the circle construction is twice that on the solid (i.e., the principal stresses are 180 deg apart on the circle, but only 90° apart on the solid). The angle on the circle is $180^\circ + \phi$ counterclockwise from the principal stress direction, σ_{a+b+c} . The fracture plane is, therefore, at one-half this angle on the solid from the same principal stress direction.

The shear strength "a" is the physical property strength in unconfined shear with no normal stresses acting across the failure plane. The shear strength components "b" and "c" are produced by constraint across the failure surface. Given enough constraint strength in compression, the solid becomes indestructible. For example, a hydrostatic compressive stress can approach an infinite value without breaking the solid. The solid does not become infinitely strong since it will still break with a shearing stress of "a" once the constraint is removed.

It is important to remember how much strength is physical property strength and how much is constraint strength. The constraint strength can always be lost, and if this occurs dynamically, a coal or rock burst or an earthquake can occur. What was strong enough to survive with constraint can become suddenly far too weak without the

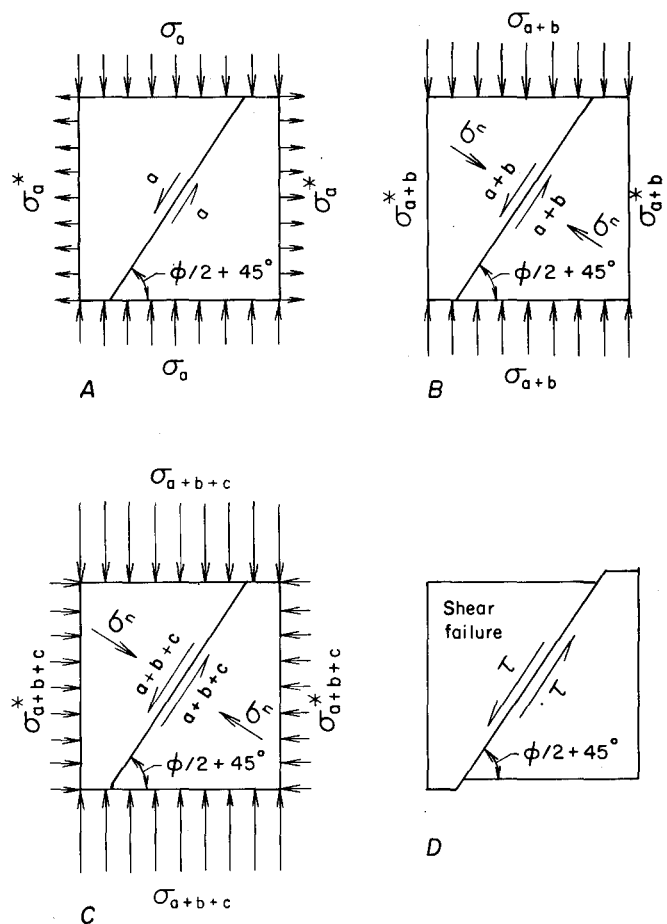


Figure 38.—States of stress related to Mohr-Coulomb stress-failure conditions of figure 38. A, Unconfined shear stress at failure, $\tau = a$; B, uniaxial stress at failure, $\tau = a+b$; C, biaxial stress at failure, $\tau = a+b+c$; D, shear displacement for failure for $\tau = a$, $\tau = a+b$, or $\tau = a+b+c$.

help of the constraint. The constraint part of the shearing strength can be lost dynamically, while the physical property part cannot; this is the major factor in dynamic breaking.

RELATIONSHIP BETWEEN VERTICAL AND HORIZONTAL STRESSES IN UNDERGROUND COAL SEAM

Estimation With Mohr-Coulomb Stress-Failure Model of Horizontal Stresses Needed for Pillar Survival

Horizontal constraint of a mine pillar by the roof and floor rock is a major factor in pillar strength. Coal seams contain many cleats, which alter the physical properties. If the cleats are open, the end constraint is not effective in increasing the pillar strength. If the cleats are closed, the opposite is true. The problem of pillar strength can be

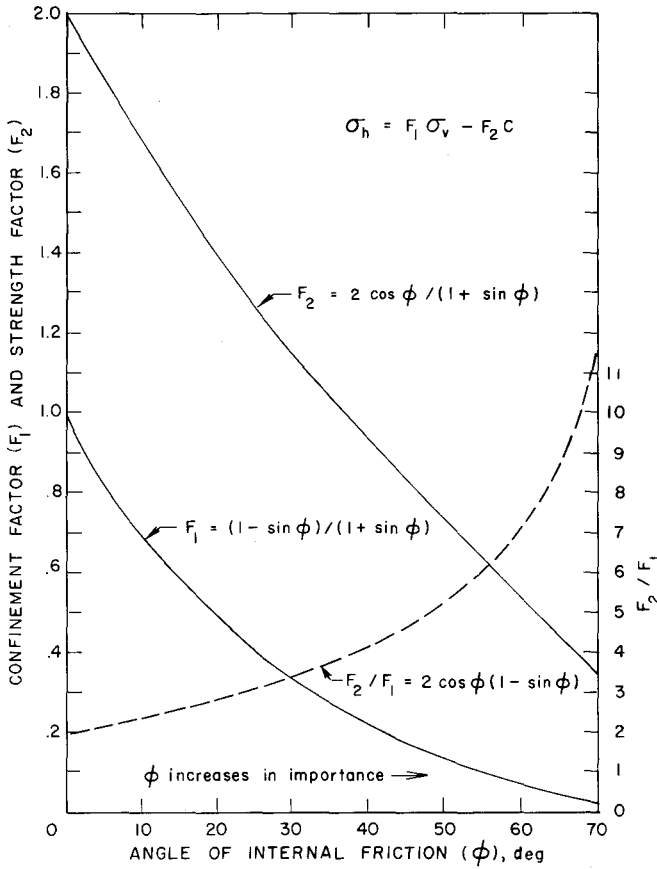


Figure 40.—Horizontal stress (σ_h) needed to prevent failure in terms of vertical stress (σ_v), unconfined shear strength (C), and angle of internal friction (ϕ).

$$\sigma_h = 1.2d(1 - \sin \phi)/(1 + \sin \phi) - 2C \cos \phi/(1 + \sin \phi). \quad (107)$$

Cleats in a flat-lying coal seam are usually vertical. There is no horizontal stress across open vertical cleats or cracks, so for $\sigma_h = 0$, these fractures can stand open to the depth, d , in an unmined coal seam, defined by

$$d = C \cos \phi / (0.6(1 - \sin \phi)). \quad (108)$$

No horizontal constraint is necessary to this depth. A plot of these results is given in figure 41. In the example given for an unconfined shear strength of the coal of $C = 300$ psi and an angle of internal friction $\phi = 50^\circ$, cracks in the unmined coal seam can stand open to a depth of 1,350 ft. Consider the situation where the unconfined shear strength, C , is 0 in equation 103. This occurs if the coal is broken. The total strength results from constraint across the failure surface. Equation 104 solved for σ_h/σ_v becomes, with $C = 0$

$$\sigma_h/\sigma_v = (1 - \sin \phi)/(1 + \sin \phi) = F_1. \quad (109)$$

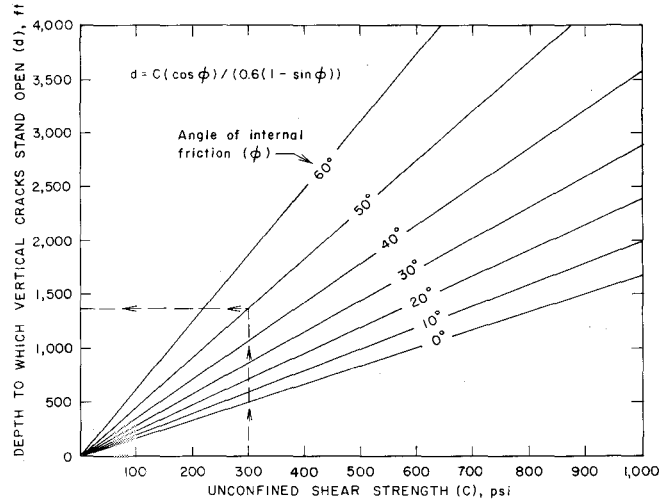


Figure 41.—Mining depth, in feet, to which constraint is not required for unmined coal seam. Example: if $C = 300$ psi and $\phi = 50^\circ$, then $d = 1,350$ ft.

In this case, the horizontal stress is a linear function of the vertical stress. That is, the horizontal constraint stress must be proportional to the depth of the pillar below the surface if the pillar is to survive. The proportionality factor is related to the angle of internal friction, ϕ . The larger the angle, the smaller the amount of horizontal constraint needed for pillar survival. This ratio for various values of ϕ is given in figure 42. If σ doubles for a recovery of 50 pct, the value of σ_h doubles, as well. The ratio is the same for all recoveries. In the example given, if $\phi = 40^\circ$, then a stress ratio, S_h/S_v ratio greater than 0.215 is needed to stabilize the pillar. The pillar is assumed to have no unconfined shear strength and $C = 0$. The rib for a broken pillar cannot stand because it is unconfined. A value of C greater than $C = 0$ must exist if the rib is vertical and the estimate of strength is on the conservative side.

Consider figure 43 for which mining recovery, R , changes the effective depth of the seam, d_e , to

$$d_e = C(1 - R) \cos \phi / 0.6(1 - \sin \phi). \quad (110)$$

Here, d_e is the depth to which cracks stand open in a mine pillar and no horizontal constraint is needed; ϕ is the angle of internal friction; C is the unconfined shear strength, psi; and R is the mining recovery expressed as a decimal (i.e., 0.50 = 50 pct). This is also the depth for which the pillar ribs stand intact because the principal stresses normal to the rib surface are zero. That is, the rib behavior is like a crack behavior since neither has a supporting normal stress acting across its free surface.

There are six vertical scales in figure 43. The one on the left is for a solid coal seam with no mining recovery. The five scales on the right are for mining recoveries of

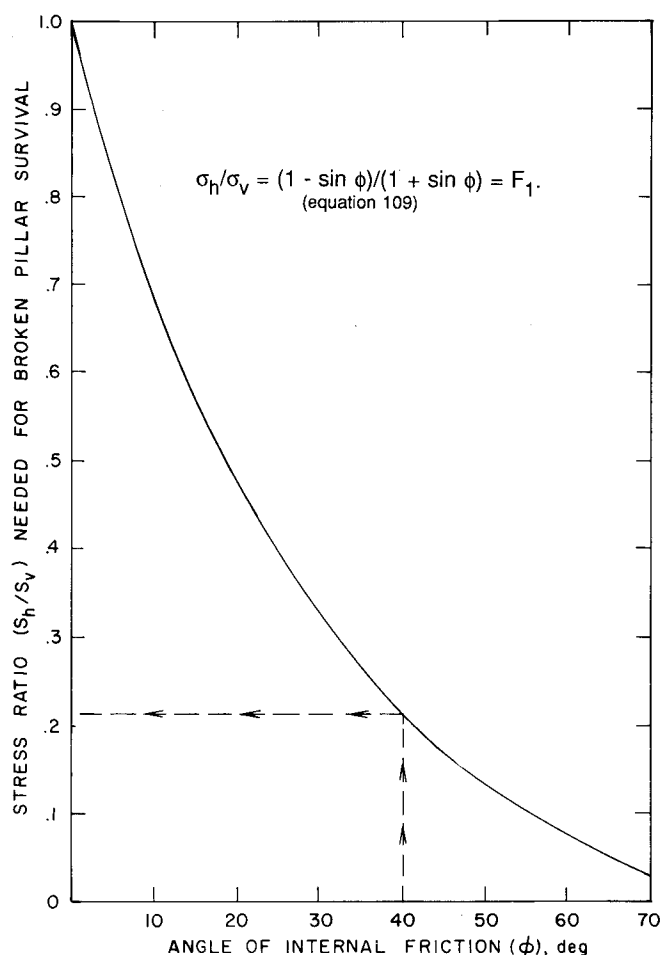


Figure 42.—Horizontal-vertical stress ratio needed for pillar survival when unconfined shear strength, $C = 0$. Example: for $\phi = 40^\circ$, S_h/S_v must be greater than 0.22 for stable pillar.

40, 50, 60, 70, and 80 pct. The curve parameters are for the C values. The small black circles on the plot are from laboratory testing of Colorado and Utah coals for the angle of internal friction and C values. In these tests, $26^\circ \leq \phi \leq 53^\circ$ and $440 \leq C \leq 1,500$ psi. Laboratory strength values are often taken to be three to as much as eight times larger (80) than the in situ values. The in situ values could range from 90 to 300 psi, if a strength difference of 5 is used. From the coal bed permeability studies cited in "Mohr-Coulomb Stress-Failure Relationships" (64), the value of C in situ was estimated to be about 270 psi.

An example problem will illustrate the use of figure 43. If $C = 270$ psi and $\phi = 47^\circ$, the unmined coal seam could have open cracks to depths of 1,100 ft. The depth to which open cracks occur and to which no roof and floor constraint is needed for pillar survival would be 660, 550,

425, 330, and 220 ft, respectively, for mining recoveries of 40, 50, 60, 70, and 80 pct, respectively. Coal pillars mined to these depths would not need constraint from the roof and floor rock to survive. At greater depths such constraint would be necessary.

The results given in tables 8 and 9 show how a small ($C = 100$ psi) and relatively large ($C = 800$ psi) unconfined shear strength (C) changes the amount of horizontal constraint necessary for a stable pillar. The value d^* is defined as

$$d^* = d/(1 - R), \quad (111)$$

where d is the depth of the coal seam being mined below the surface, and R is the fractional recovery by mining (i.e., 50 pct = 0.50). The depth d^* is the depth at which an unmined coal seam would have the same tributary area vertical stress as the pillar has at a depth of d . In table 8, the cohesive strength in unconfined shear, " C ," is 100 psi. The small number of 0.00 values near the top of the table indicate that only small depths can be mined without some horizontal pillar confinement. For angles of internal friction of 30° or larger, a horizontal stress of less than 30 pct (0.30) of the vertical stress is required for pillar stability away from the pillar edges. For large angles of internal friction, for instance, 50° or more, the horizontal stress required even for small " C " values (i.e., " C " = 100 psi) is only 13 pct (0.13), or less of the vertical stress. This means that practically no horizontal confinement is required, and most of the constraint comes from the vertical stress loading across the failure surface.

Table 8.—Ratio of horizontal pillar stress to vertical pillar stress (σ_h/σ_v) needed for pillar stability for small value of cohesive strength; $C = 100$ psi

Depth parameter (d^*), ft	Friction angle (ϕ), deg						
	0	10	20	30	40	50	60
COHESIVE STRENGTH (C) = 100 psi							
200	0.17	0.00	0.00	0.00	0.00	0.00	0.00
400	.58	.36	.20	.09	.03	.00	.00
600	.72	.47	.30	.17	.09	.03	.00
800	.79	.53	.34	.21	.12	.06	.02
1,000 . . .	.83	.56	.37	.24	.14	.07	.03
1,200 . . .	.86	.59	.39	.25	.15	.08	.03
1,400 . . .	.88	.60	.41	.26	.16	.09	.04
1,600 . . .	.90	.62	.42	.27	.17	.10	.04
1,800 . . .	.91	.63	.43	.28	.18	.10	.05
2,000 . . .	.92	.64	.43	.29	.18	.10	.05
2,500 . . .	.93	.65	.44	.30	.19	.11	.06
3,000 . . .	.94	.66	.45	.30	.19	.11	.06
>3,000 . .	1.00	.70	.49	.33	.22	.13	.07

Table 9.—Ratio of horizontal pillar stress to vertical pillar stress (σ_h/σ_v) needed for pillar stability for large value of cohesive strength; $C = 800$ psi

Depth parameter (d^*), ft	Friction angle (ϕ), deg						
	0	10	20	30	40	50	60
200	0.00	0.00	0.00	0.00	0.00	0.00	0.00
400	.00	.00	.00	.00	.00	.00	.00
600	.00	.00	.00	.00	.00	.00	.00
800	.00	.00	.00	.00	.00	.00	.00
1,000 ...	.00	.00	.00	.00	.00	.00	.00
1,200 ...	.00	.00	.00	.00	.00	.00	.00
1,400 ...	.05	.00	.00	.00	.00	.00	.00
1,600 ...	.17	.01	.00	.00	.00	.00	.00
1,800 ...	.26	.08	.00	.00	.00	.00	.00
2,000 ...	.33	.15	.02	.00	.00	.00	.00
2,500 ...	.47	.26	.12	.03	.00	.00	.00
3,000 ...	.56	.33	.18	.08	.01	.00	.00
>3,000 ..	1.00	.70	.49	.33	.22	.13	.07

Next consider table 9 for a cohesive strength in shear of $C = 800$ psi. While this appears large, cohesion values of $C = 1,240$ psi, or more, for intact coal specimens have been observed in the laboratory. Notice the large area of 0.00 values in table 9, indicating that no horizontal constraint is necessary for pillar survival. For angles of internal friction as small as 30° , a horizontal stress that is

only 8 pct (0.08) of the vertical stress is required for pillar stability away from the pillar edge where the opening stress concentration may exist. That is, static equilibrium is reached at some depth inside the pillar rib. For example, if a recovery of 50 pct is used, the d^* value would be 3,000 for a depth of 1,500 ft. The $>3,000$ at the bottom of tables 8 and 9 indicates the limits of horizontal constraint required at great depth assuming elastic behavior still applies. The mining depth, d , for a chosen recovery R , can be found for a given d^* value by solving equation 111 for d .

Estimation of Unconfined Shear Strengths of Coal In Situ From Permeability- Versus-Depth Relationships

Permeability of coal seams versus depth below the surface can be used for predicting the in situ unconfined shear strength and the depth to which coal can be mined when no horizontal constraint stress is present or needed.

If fractures in a coal seam can stand open, no horizontal constraint stress is necessary for pillar survival. When the fractures close, constraint stress will act across the fractures. If the extent of open fractures in the coal can

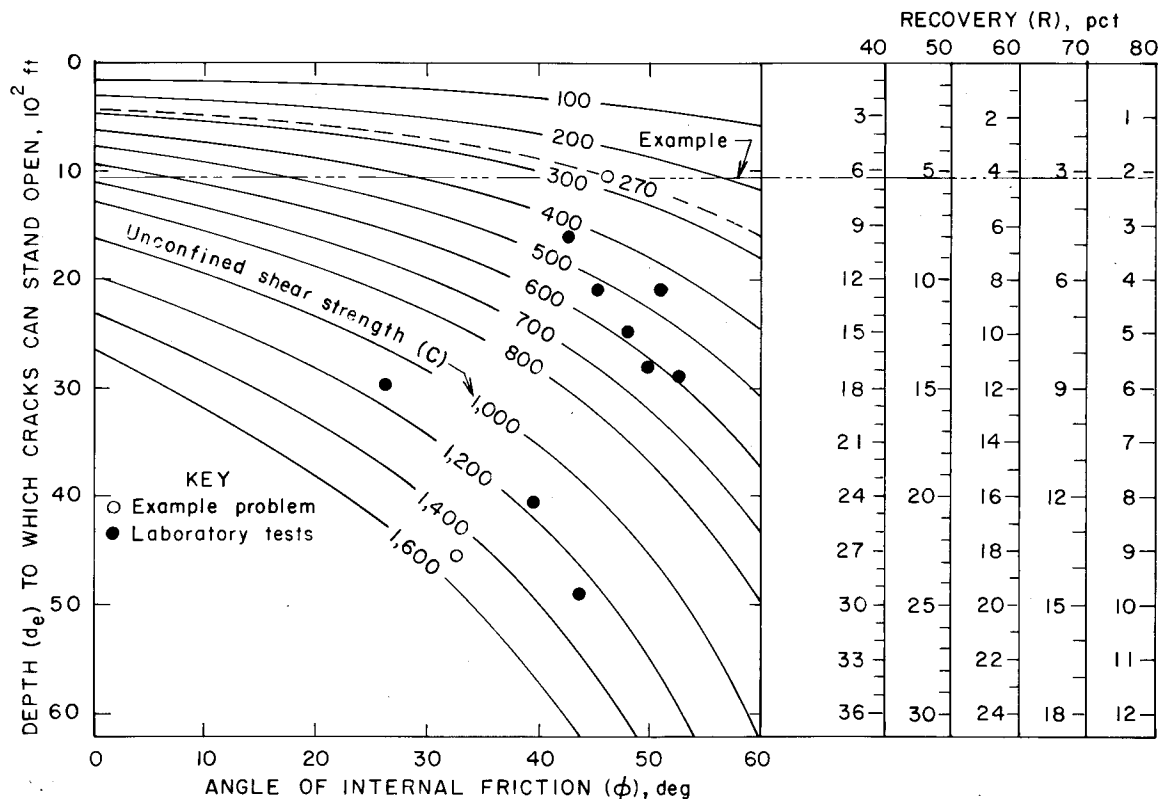


Figure 43.—Depth (d_e) to which cleats in coal pillar can stand open and constraint is unnecessary for pillar stability. Example: Depths to which cracks stand open for $C = 270$ psi, for in situ conditions for mining recoveries of 0, 40, 50, 60, 70, and 80% are 1,100, 640, 525, 420, 315, and 210 ft, respectively.

be defined as a function of the mining depth, the need for constraint with increasing mining depth will become apparent.

One area of research in which the extent of fracturing in coal is observed for other purposes than mining is that of recovery of gas from coal seams, by using the permeability of the seam as the significant variable. McKee (64) shows relationships between permeability and depth for the San Juan Basin, the Piceance Creek Basin, and the Black Warrior Basin in figures 7, 8, and 9 in the work cited. These coal basins are in Colorado-New Mexico, western Colorado, and Alabama, respectively. These figures have been combined into figure 44 of this report. The curves shown are of the same general shape and are based on a permeability theory fitted to experimental data by McKee. The data are not shown in the present plot. To a depth of 450 ft, the permeability is not a function of the seam depth. This means that the cracks are not being closed by the overburden stress to a depth of 450 ft. The unconfined shear strength in the Mohr-Coulomb stress-failure relationship can be based on this information. From a depth of 450 ft to depths of 1,200, 3,500, and 4,300 ft for the Black Warrior Basin, San Juan Basin, and the Piceance Creek Basin, respectively, the permeability changed rapidly with depth. In this range of depths, the cracks closed until closure was complete. At depths greater than 1,200, 3,500, and 4,300 ft, the values of permeability were proportional to the depth. Because the cracks are largely in the form of cleats, which are typically vertical in a horizontal coal seam, the crack closure relates to the horizontal confining stresses. In the zones 450 ft to either 1,200, 3,500, or 4,300 ft, the horizontal stresses increased more rapidly than the depth.

EFFECT OF END CONSTRAINT ON COMPRESSIVE STRENGTH OF TEST SPECIMENS IN LABORATORY

Constraint Implied for Standard Uniaxial Tests Between Two Steel Platens in Laboratory

Investigators testing rock or coal specimens between two steel platens in the laboratory commonly assume that such tests are "unconfined" (in some sense uniaxial) tests. When the specimen has an L-D ratio of 2, as in the American Society for Testing Materials (A.S.T.M.) tests (2), the state of stress at midlength is nearly such a uniaxial or unconfined stress state. Because the ends of the test specimen are against steel platens, this is untrue for most of the specimen length. Brady and Blake (28), using a finite-element method of stress analysis, reported on the state of stress in a test specimen. What is uniaxial (one axis) is the force applied to the specimen by the compression machine. For the most part, the state of stress within the specimen is triaxial and is biaxial at the free

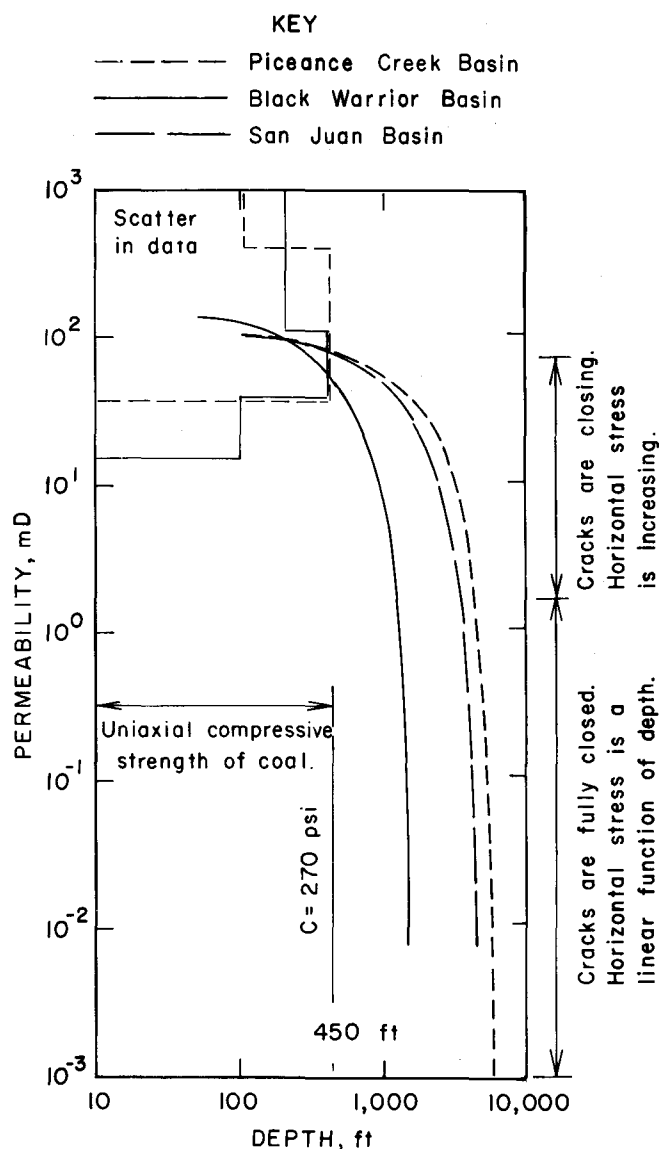


Figure 44.—Permeability of coal in three basins used to evaluate mining depth to which cracks remain open when seam is unmined (64).

surfaces. In spite of this, the compressive strength obtained in such tests is approximately the correct uniaxial stress strength. This is not true for test specimens of L-D of less than 2 as shown in "Role of End Constraint on Compressive Strength."

As was shown in equation 7, the strength in compression of a cylindrical specimen against steel platens according to the equation of Obert and Duvall (70) is

$$\sigma / \sigma_{1:1} = 0.778 + 0.222 D/L,$$

where σ , $\sigma_{1:1}$, D and L are the compressive strength of a cylindrical specimen of any length, the compressive strength of a specimen when the length is equal to the

diameter, and the diameter and length of the specimen, respectively.

Interesting results were obtained by Babcock, Morgan, and Haramy (19) when the Mohr-Coulomb stress-failure relationships were plotted as shown in figure 45. The failure circles shown intersect the σ stress axis in two places. The right-hand intersections correspond to the average axial stresses applied. The left-hand intersections correspond to the average constraint stresses applied to the specimen by the end shear against the steel platens. Notice that the σ scale units (the upper scale) are normalized to the stress at failure of the D-L = 1 specimen (the dotted circle). That is, this scale has units $\sigma/\sigma_{1:1} = 1, 2, 3, 4, \text{ and } 5$. If equation 7 is solved for D-L and the values of the stress ratio are substituted into it the second scale

1, 2, 3, ..., 19, results. The interpretation of the result is that a relative strength of 5 occurs according to equation 7 for a specimen with a D-L ratio of 19. The movement of the stress circle to the right with an increasing D-L ratio indicates that the radial constraint also increases. The amount of this constraint depends upon the angle of internal friction, ϕ . For example, for a D-L of 10, and an angle of internal friction of 40° the radial constraint at failure would be 16 pct, as shown in the example of figure 45. As the angle of internal friction increases, the amount of radial constraint decreases.

The relative compressive strength for a specimen with an increasing D-L is known to increase as defined by equation 7. The specimen against steel platens has some unknown end constraint. What this end constraint is can

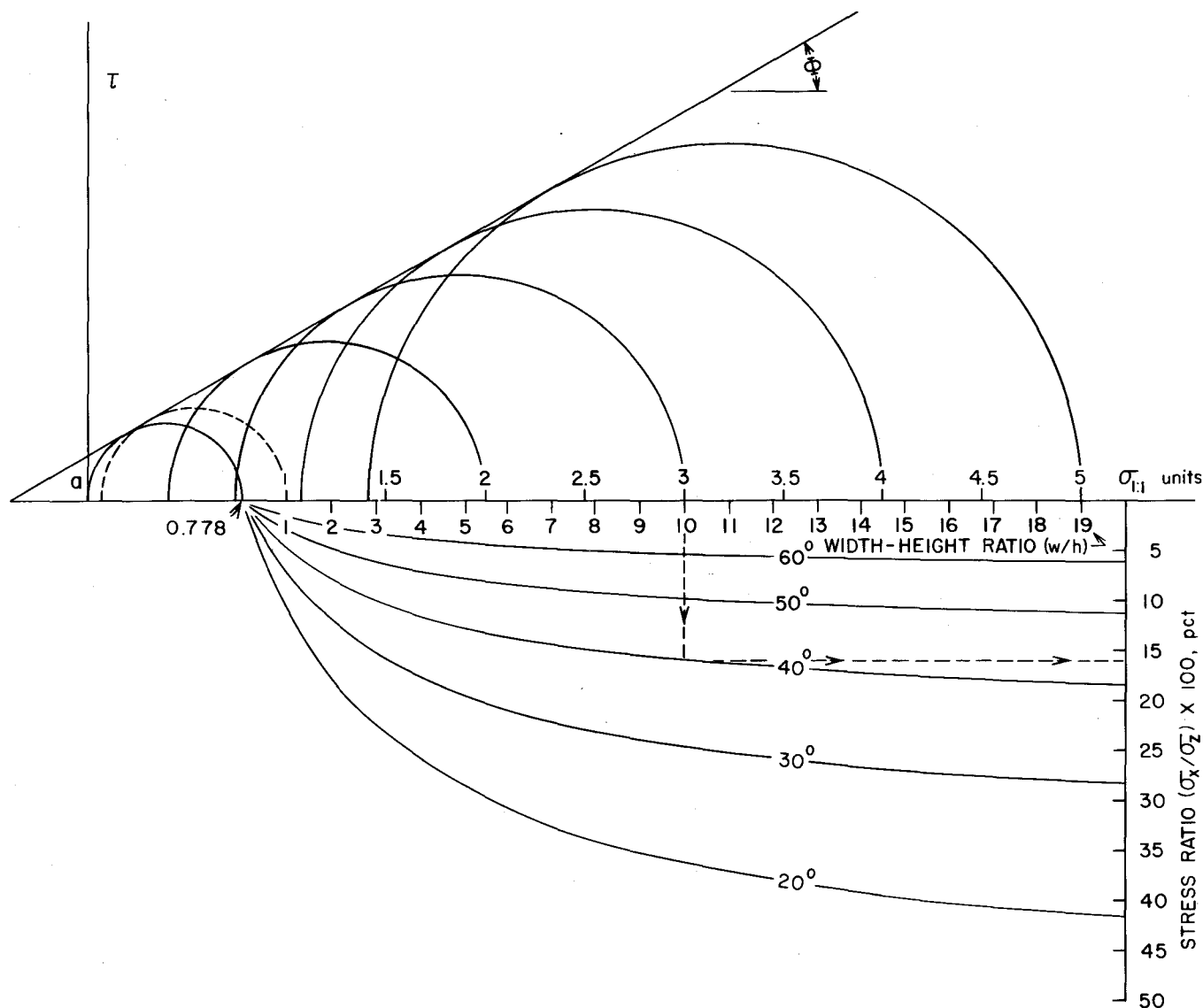


Figure 45.—End constraint inferred for laboratory testing of rock or coal specimens between two steel platens in so-called uniaxial stress tests. Dashed line is an example indicating that average end constraint stress (σ_x) is 16% of average axial stress (σ_z) when angle of internal friction (ϕ) of test specimen is 40° . σ_x increases as ϕ decreases (9).

be estimated by figure 45. As the Mohr stress circles are drawn to touch the Coulomb failure line, the circle moves from the origin. This stress value represents the average radial constraint provided by the steel platens. This constraint changes with the angle of internal friction, ϕ .

Changes in Compressive Strengths of Rock and Coal Specimens Produced by End Constraint in Laboratory Tests

Four separate studies have been made by the author to determine the effects of end constraint on rock or coal specimen compressive strength. In 1968 (7), model rock pillars of limestone, marble, sandstone, and granite with length-diameter ratios of 3, 2, 1, and 0.5 were broken in uniaxial compression with and without solid steel rings bonded to the specimen ends, as was shown in figure 34. The steel rings gave a passive end constraint that was approximately the same as would have been supplied by the roof and floor of the same material of infinite extent to a mine pillar. The strength data are given in table 10.

In 1969 (10-11), model pillars of limestone, marble, sandstone, and granite with diameter-length ratios of 0.333, 0.5, 1, 2, and 4 were broken in uniaxial compression with and without adjustable steel rings with radial prestress compressional stresses of 3,000 and 5,000 psi applied before axial loading. The compressive strengths are given in table 11.

The relative compressive strengths of the above test results are summarized in tables 12 and 13. The test specimen with a diameter equal to the length is the common basis for comparing strengths, as shown by equation 24. The relative strength ranged from 2.57 to 0.80 times the reference specimen strength against two steel platens. The differences resulted from the change in stress state, as will be shown in "Role of End Constraint on Compressive Strength."

Another way to normalize the compressive strength is given in table 13 where the strength for each D-L ratio for the straight specimen is taken as 1.0. Here, the increase in strength ranged from 1.58 to 0.88. These changes resulted from the differences in end constraint alone.

The average radial constraint stresses produced by rings as a percentage of the average compressive stress at failure are given in table 14. These values determined experimentally from the elastic behavior of the steel rings ranged from 13.0 to 82.9 pct.

In 1987 (8), the effect of using eight different end conditions on test specimens of four different coals, three from Colorado and one from Utah, were reported. Five geometries G_1 through G_5 with D-L had ratios of 1, 2, 3.5, 5, and 8.5, respectively. The end conditions used are given in figure 46 and are labeled E_1 through E_8 . The D-L ratios tested were 1:1, 2:1, 3.5:1, 5:1, and 8.5:1. The average strength for four specimens are given in table 15. In figure 46, the letters denote the steel platens of 9-in diameter of fixed type (S), the coal specimen (C), a

powdered clay layer 0.050-in thick (Cl), a powdered coal layer 0.050-in thick (Cd), a wet coal specimen (Wc), a sandstone disk 4 in. in diameter and 0.500-in thick (Ss), a PTFE layer 3/32-in thick (T), and a slate disk 4 in. in diameter and 0.500-in thick (Sl). The compressive strengths are given in table 15. These are normalized in three ways in table 16. In table 16 (top), the strength of the D-L of 1.0 against steel platens (E_1) had a reference strength of 1.0. The strengths ranged from 0.35 to indestructible in these tests. In table 16 (middle), the strengths were normalized to the D-L = 1 specimen for each end condition. Strengths ranged from 1.0 to indestructible. In table 16 (bottom), the strengths were normalized to the against-steel-platen conditions (E_1). In only 3 out of 40 tests the relative strength was greater than against the steel platens. The values 1.06 and 1.11 may not be statistically different from 1.0. Only a single specimen was tested for each of the four coals so the reproducibility within each coal could not be determined.

Table 10.—Average compressive strengths of rocks with and without solid ring end constraint, pounds per square inch (7)

Diameter-length ratio (D/L)	2	1	0.5	0.333
STRAIGHT SPECIMENS—NO RINGS				
Limestone	9,879	9,164	8,574	8,732
Marble	17,065	15,243	14,037	13,730
Sandstone	12,637	11,551	10,187	9,963
Granite	33,289	32,258	28,008	26,758
SOLID RINGS—NO PRESTRESS				
Limestone	10,620	9,896	9,060	8,848
Marble	23,118	16,075	14,220	14,137
Sandstone	12,914	12,303	10,778	10,478
Granite	38,470	34,794	27,883	27,534

Table 11.—Average compressive strengths of rocks with and without radial end prestress constraint, pounds per square inch (11)

Diameter-length ratio (D/L)	4	2	1	0.5	0.333
STRAIGHT SPECIMENS—NO RINGS					
Limestone . .	10,507	8,353	7,970	7,987	7,762
Marble	29,604	16,165	13,927	13,178	13,211
Sandstone . .	15,416	11,822	10,799	9,667	9,834
Granite	48,834	37,953	31,863	29,442	28,402
ADJUSTABLE RINGS—3,000-psi PRESTRESS CONSTRAINT					
Limestone . .	16,539	11,364	9,509	8,286	8,269
Marble	35,532	23,410	17,562	14,226	14,176
Sandstone . .	24,325	16,073	12,695	10,083	9,4348
Granite	69,010	47,920	37,803	30,482	28,119
ADJUSTABLE RINGS—5,000-psi PRESTRESS CONSTRAINT					
Limestone . .	16,364	11,605	9,559	8,810	8,311
Marble	35,748	25,066	17,920	14,293	14,010
Sandstone . .	24,142	15,175	11,913	9,634	8,652
Granite	77,204	50,873	37,179	29,700	29,234

¹D/L ratio for granite is 3.5.

Table 12.—Compressive strengths of model rock pillars without prestress and with two different prestress end constraints normalized to compressive strength of straight specimens with D:L of 1.0 (10)

Diameter-length ratio (D:L)	¹ 4	2	1	0.5	0.333
LIMESTONE					
Straight specimen:					
No prestress tests	ND	1.08	² 1.00	0.94	0.95
Prestress tests	1.32	1.05	² 1.00	1.00	.97
Solid rings, no prestress	ND	1.16	1.08	.99	.97
Adjustable rings:					
3,000-psi prestress	2.08	1.43	1.19	1.04	1.04
5,000-psi prestress	2.05	1.46	1.20	1.11	1.04
MARBLE					
Straight specimen:					
No prestress tests	ND	1.12	² 1.00	0.92	0.90
Prestress tests	2.13	1.16	² 1.00	.95	.95
Solid rings, no prestress	ND	1.52	1.06	.93	.95
Adjustable rings:					
3,000-psi prestress	2.55	1.68	1.26	1.02	1.02
5,000-psi prestress	2.57	1.80	1.29	1.03	1.01
SANDSTONE					
Straight specimen:					
No prestress tests	ND	1.09	² 1.00	0.88	0.86
Prestress tests	1.43	1.10	² 1.00	.90	.91
Solid rings, no prestress	ND	1.12	1.07	.93	.91
Adjustable rings:					
3,000-psi prestress	2.25	1.49	1.18	.93	.87
5,000-psi prestress	2.24	1.41	1.10	.89	.80
GRANITE					
Straight specimen:					
No prestress tests	ND	0.03	² 1.00	0.87	0.82
Prestress tests	1.53	1.19	² 1.00	.92	.89
Solid rings, no prestress	ND	1.19	1.08	.86	.85
Adjustable rings:					
3,000-psi prestress	2.17	1.50	1.19	.96	.88
5,000-psi prestress	2.42	1.60	1.17	.93	.92

ND Not determined.

¹D:L for granite is 3.5.

²Reference compressive strength of 1.0 to which all other strengths are normalized.

**Table 13.—Compressive strength of model rock pillars normalized
to compressive strength of straight specimens
with same (D:L) (10)**

Diameter-length ratio (D:L)	¹ 4	2	1	0.5	0.333
LIMESTONE					
Straight specimen:					
No prestress tests	ND	² 1.00	² 1.00	² 1.00	² 1.00
Prestress tests	² 1.00	² 1.00	² 1.00	² 1.00	² 1.00
Solid rings, no prestress	ND	1.08	1.08	1.06	1.01
Adjustable rings:					
3,000-psi prestress	1.57	1.36	1.19	1.04	1.07
5,000-psi prestress	1.56	1.39	1.20	1.10	1.07
MARBLE					
Straight specimen:					
No prestress tests	ND	² 1.00	² 1.00	² 1.00	² 1.00
Prestress tests	² 1.00	² 1.00	² 1.00	² 1.00	² 1.00
Solid rings, no prestress	ND	1.36	1.06	1.01	1.03
Adjustable rings:					
3,000-psi prestress	1.20	1.45	1.26	1.08	1.07
5,000-psi prestress	1.21	1.55	1.29	1.09	1.06
SANDSTONE					
Straight specimen:					
No prestress tests	ND	² 1.00	² 1.00	² 1.00	² 1.00
Prestress tests	² 1.00	² 1.00	² 1.00	² 1.00	² 1.00
Solid rings, no prestress	ND	1.02	1.07	1.06	1.05
Adjustable rings:					
3,000-psi prestress	1.58	1.36	1.18	1.04	.96
5,000-psi prestress	1.57	1.28	1.10	1.00	.88
GRANITE					
Straight specimen:					
No prestress tests	ND	² 1.00	² 1.00	² 1.00	² 1.00
Prestress tests	² 1.00	² 1.00	² 1.00	² 1.00	² 1.00
Solid rings, no prestress	ND	1.16	1.08	1.00	1.04
Adjustable rings:					
3,000-psi prestress	1.41	1.26	1.19	1.04	.99
5,000-psi prestress	1.58	1.34	1.17	1.01	1.03

ND Not determined.

¹D:L for granite is 3.5.

²Reference compressive strength of 1.0 to which all other strengths are normalized.

**Table 14.—Average radial constraint stress produced by rings
above that produced by end friction as percentage
of average compressive stress at failure (10)**

Diameter-length ratio (D:L)	¹ 4	2	1	0.5	0.333
LIMESTONE					
Solid rings, no prestress	ND	23.0	22.2	21.1	20.7
Adjustable rings:					
3,000-psi prestress	62.5	50.0	50.6	54.4	53.8
5,000-psi prestress	69.9	67.7	72.5	78.0	82.9
MARBLE					
Solid rings, no prestress	ND	20.0	16.5	17.1	16.8
Adjustable rings:					
3,000-psi prestress	52.3	41.6	33.7	38.0	35.7
5,000-psi prestress	54.5	48.5	46.8	52.7	53.7
SANDSTONE					
Solid rings, no prestress	ND	20.5	20.8	22.5	21.9
Adjustable rings:					
3,000-psi prestress	34.6	41.4	42.2	53.5	52.0
5,000-psi prestress	43.1	56.5	59.4	71.8	77.3
GRANITE					
Solid rings, no prestress	ND	16.0	13.9	13.0	13.5
Adjustable rings:					
3,000-psi prestress	27.8	24.3	21.2	21.5	22.2
5,000-psi prestress	30.3	31.8	27.3	29.5	29.8

ND Not determined.

¹D:L for granite is 3.5.

**Table 15.—Average compressive strengths of four coals, three
from Colorado and one from Utah,
pounds per square inch (8)**

End condition ¹	Diameter-length ratio (D:L)					Average
	G ₁	G ₂	G ₃	G ₄	G ₅	
	1	2	3.5	5	8.5	
E ₁	6,506	8,683	9,303	12,319	17,705	10,903
E ₂	2,310	5,117	6,859	13,060	21,576	9,784
E ₃	3,004	4,718	7,427	8,516	20,155	8,764
E ₄	4,414	6,879	9,418	11,883	20,155	10,550
E ₅	5,717	7,431	8,993	10,557	14,101	9,360
E ₆	3,302	5,762	6,348	6,247	8,762	6,084
E ₇	4,234	4,639	5,114	5,569	6,199	5,151
E ₈	3,502	4,827	8,361	11,732	19,706	9,626
Average . .	4,124	6,007	7,728	9,985	16,045	8,778

¹See figure 47

Table 16.—Average relative compressive strengths
for four specimens, one for each
of four coals tested (8)

End condition ¹	Diameter-length ratio (D:L)				
	G ₁	G ₂	G ₃	G ₄	G ₅
	1	2	3.5	5	8.5
NORMALIZED TO E ₁ , G ₁ VALUE					
E ₁	1.00	1.33	1.43	1.89	² 2.72
E ₂	.35	.79	1.05	2.01	(³)
E ₃	.46	.73	1.14	1.31	(³)
E ₄	.68	1.06	1.45	1.83	(³)
E ₅	.88	1.14	1.38	1.62	⁴ 2.17
E ₆	.51	.89	.98	.96	1.34
E ₇	.65	.71	.79	.86	.95
E ₈	.54	.74	1.29	1.80	⁵ 3.03
NORMALIZED TO E _i , G ₁ VALUE: i = 1, 8					
E ₁	1.00	1.33	1.43	1.89	² 2.72
E ₂	1.00	2.22	2.97	5.65	(³)
E ₃	1.00	1.57	2.47	2.83	(³)
E ₄	1.00	1.56	2.13	2.69	(³)
E ₅	1.00	1.30	1.57	1.85	⁴ 2.47
E ₆	1.00	1.74	1.92	1.89	2.65
E ₇	1.00	1.10	1.21	1.32	1.46
E ₈	1.00	1.38	2.39	3.34	⁵ 5.63
NORMALIZED TO E ₁ , G _j : j = 1, 5					
E ₁	1.00	1.00	1.00	1.00	1.00
E ₂	.35	.59	.74	1.06	(³)
E ₃	.46	.54	.80	.69	(³)
E ₄	.68	.79	1.01	.96	(³)
E ₅	.88	.86	.97	.86	⁴ .80
E ₆	.51	.66	.68	.51	.50
E ₇	.65	.53	.55	.45	.35
E ₈	.54	.56	.90	.95	⁵ 1.11

¹See figure 47.

²Three of four specimens failed.

³None of the four specimens failed.

⁴Two of four specimens failed.

⁵One of four specimens failed.

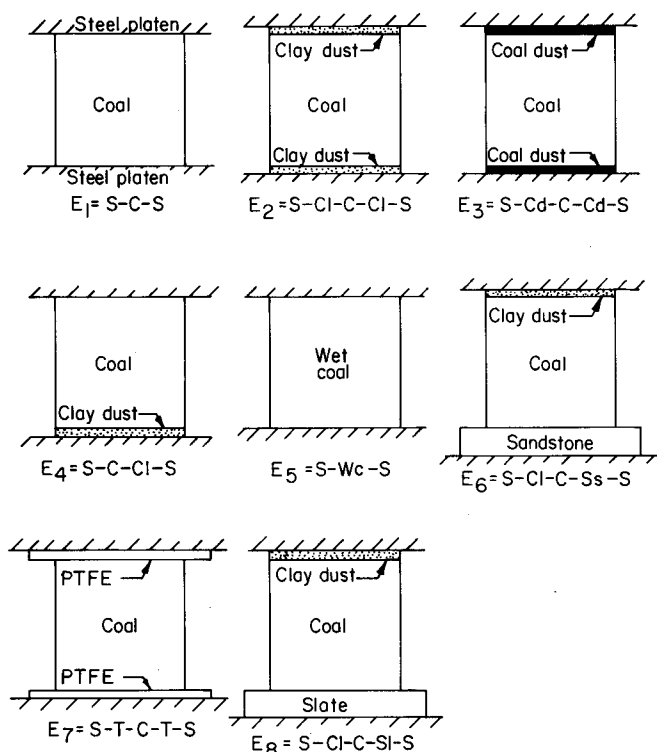


Figure 46.—Eight end conditions used in tests. Clay and coal dust layers are -35 mesh and 0.035-in thick. Sandstone and slate platens are 0.050-in thick and 4-in. in diameter. Steel platens are nonrotating heads (8).

In a final study in 1990 (16), it was shown that the state of stress and not the geometry (D-L ratio) was responsible for the change in strength with the D-L ratio. Specimens of coal, limestone, marble, and sandstone, with D-L ratios of 1, 2, 3.5, 5, and 8.5 were made to fail in compression at about the same stress for each rock type by removing end frictional effects with layers of PTFE of selected thickness.

Interpretation of Pillar Design Equations With Width-Height Ratio as Fundamental Variable in Terms of End Constraint

A pillar design equation in which the width and height are the primary variables has had a widespread appeal for 156 years since the work of Vicat in 1833 (84). This type equation is unsatisfactory because it does not explain the wide variety of different strengths and behaviors observed in the laboratory and in the mine. Test results in the laboratory between two steel platens do not model the variations in strength that are produced by changing the end constraint conditions. An equation of this type, which does explain the observed behavior, is developed in this section. This behavior includes the effects of changing the end constraint, the disappearance of the effects of geometry on strength when end constraint is removed, and

the production of coal bursts when end constraint is suddenly lost.

Many mine pillar design equations in the literature (19) are of the form

$$\sigma/\sigma_{1:1} = A + B (W^\alpha/H^\beta), \quad (112)$$

as shown in equation 1 of the introduction. The most commonly used equations of this type are those of Johnson⁶ for specimens or pillars of rectangular shape and Obert and Duvall (70) for specimens of cylindrical shape. These are

$$\begin{aligned} \sigma/\sigma_{1:1} = 0.778 + 0.222 W/H = 0.778 \\ + 0.222 D/L, \end{aligned} \quad (113)$$

where σ is the compressive strength for any W/H or D/L ratio, $\sigma_{1:1}$ is the compressive strength when W = H or D = L, and W and H are the width and height of cylindrical specimens made from diamond drill cores. A and B are coefficients of a rectangular specimen or mine pillar, and D and L are the diameter and length used to fit experimental laboratory data to mine results. These equations fit the experimental data well when rock or coal specimens are tested between two steel platens in compression in the laboratory. They are not very satisfactory for predicting the results when different end conditions are used in laboratory tests or for mine pillar design where the end constraint is provided by roof and floor rock. This is so because these equations contain, as unspecified factors, the constraint supplied by the steel platens. If the effects of other amounts of end constraint can be defined, a more useful and powerful relationship will result. The many different values of A, B, α , and β observed in the literature (19) are attempts to curve fit equation 112 to the experimental mine data observed.

Because the ratio W/H or D/L occurs in the equations, it is commonly assumed that the change in relative strength results from a difference in geometry. It is shown in "Role of End Constraint on Compressive Strength" that the changing stress with geometry and not the geometry itself is responsible for this effect.

Consider the solids of rectangular and cylindrical shapes shown in figure 47. The rectangular solid has a square cross-section area with dimension, W, and a height, H. The cylindrical solid has a circular cross-section area with a diameter, D, and a length, L. The shaded areas are, in general, constrained during testing by the loading platens. These are denoted by A_c . The unshaded areas on the sides of the solids are unloaded in so-called uniaxial testing, and these are denoted by A_u . The ratio of the

⁶Johnson's Materials of Construction, 1897.

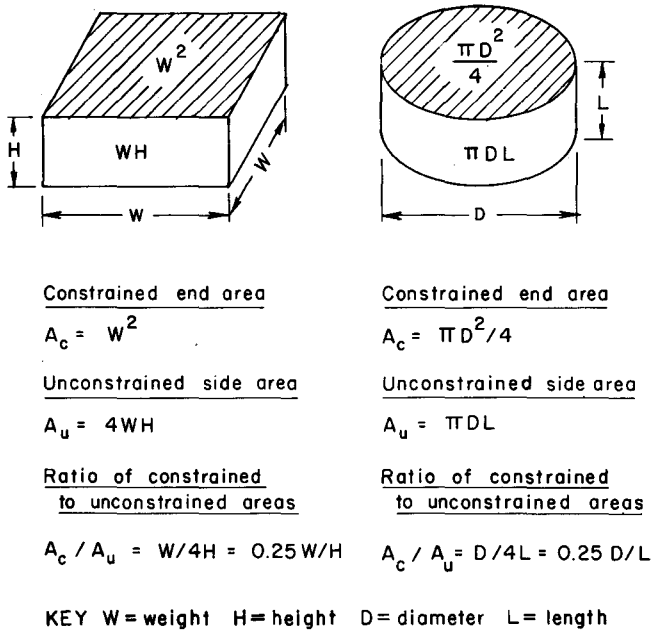


Figure 47.—Geometric basis for calculation of pillar design equation using pillar geometry and end constraint condition.

constrained to the unconstrained areas, A_c/A_u , are given in figure 47. The ratios are the same for both rectangular and cylindrical solids, and these both have values of 0.25 W/H or 0.25 D/L . Because steel is not perfectly rigid, it allows some expansion of the test specimen during loading. This could account for the difference between the 0.250 coefficient here and the 0.222 coefficient for the previous work. For perfect constraint (that is, no radial strain can occur) the equation would be of the form

$$\sigma/\sigma_{1:1} = 0.750 + 0.250 (W/H) = 0.750 + 0.250 (D/L). \quad (114)$$

The boundary condition $\sigma/\sigma_{1:1} = 1$, for $W/H = D/L = 1$, is satisfied. If the steel supplies 88.8 pct of the perfect constraint ($0.222/0.250 = 0.888$) value, an end constraint factor, F , could be added to make this correction. For other end constraint conditions, the value of F would be different. To satisfy the condition $\sigma/\sigma_{1:1} = 1$ for all values of F , the equation can be written in the form

$$\sigma/\sigma_{1:1} = 0.750 (1 + 0.333(1 - F)) + 0.250 F (W/H \text{ or } D/L). \quad (115)$$

F is taken as positive (+) for end friction acting inwards and negative (-) for end friction acting outwards.

What is the range of F ? If the specimen is totally restrained by the platen so that it cannot expand, the radial stress will be related to the axial stress by the

relationship $F = \nu/(1 - \nu)$, where ν is the Poisson's ratio of the pillar or specimen material. From figure 31, the Poisson's ratio at the center of the length of the test specimen at failure was 0.5 for granite, 0.4 for limestone, 0.37 for marble, and 0.45 for sandstone. The Poisson's ratio was increasing rapidly with axial stress at failure. An upper bound of 0.5 is reasonable because at this value the solid is incompressible. Another way of arriving at this value is from frictional considerations. The maximum coefficient of friction against dry steel for rough end conditions is about 1.0. This result would occur if end friction prevents all lateral expansion and Poisson's ratio is 0.5. Also, this would occur if sliding occurs along a steel plane with a dip of 45° or 100 pct. In the case of a circular area against steel, the constraint acts radially inwards towards the center. For radial constraint inwards, the value of F falls in the range $0 \leq F \leq 1$. $F = 0$ denotes a true uniaxial compression where there is no radial end constraint.

If soft end conditions are used between the platens and specimen, or the roof and floor are soft, the frictional stress will act radially outward. Now the opposite condition will produce the maximum end constraint factor, F . If the Poisson's ratio of the specimen is zero and for the end material is 0.5, the value of F will have an extreme value of -1.0. If the Poisson's ratio of the specimen is 0.5, there will be no end friction, and F will be equal to zero. The range for F is then $-1 \leq F \leq 0$. The total range of F is then $-1 \leq F \leq 1$. The values -1 and +1 are lower and upper bounds for F , respectively.

A substitution of any F value into equation 115 for W/H or D/L of 1 will reveal that the equation 115 is satisfied for $W/H = 1$.

$$\text{Let } A = 0.750 (1 + 0.333(1 - F))$$

and

$$B = 0.250F. \quad (116)$$

When the value of F is 0.888, A is 0.778 and $B = 0.222$. This shows that equation 115 contains equations 3 and 4 as special cases. Figure 48 shows how the relative pillar compressive strength $\sigma/\sigma_{1:1}$ changes with W/H as a function of the end constraint F . For $F = +1$ for perfect end constraint acting radially inwards, a pillar with an W/H of 10 would be 3.25 times stronger per unit area than the reference cube with a relative strength of 1. For $F = +0.888$, the strength would be three times stronger per unit area as has been reported by many investigators in laboratory tests where steel platens were used. If $F = 0$, the relative strength would be independent of W/H , as reported later. Since the strength of rock-like materials in tension is usually 1/10 to 1/20 of the strength in compression, the value $\sigma/\sigma_{1:1} = 0.1$ is taken as the practical limit of the figure 48 graph when the specimen

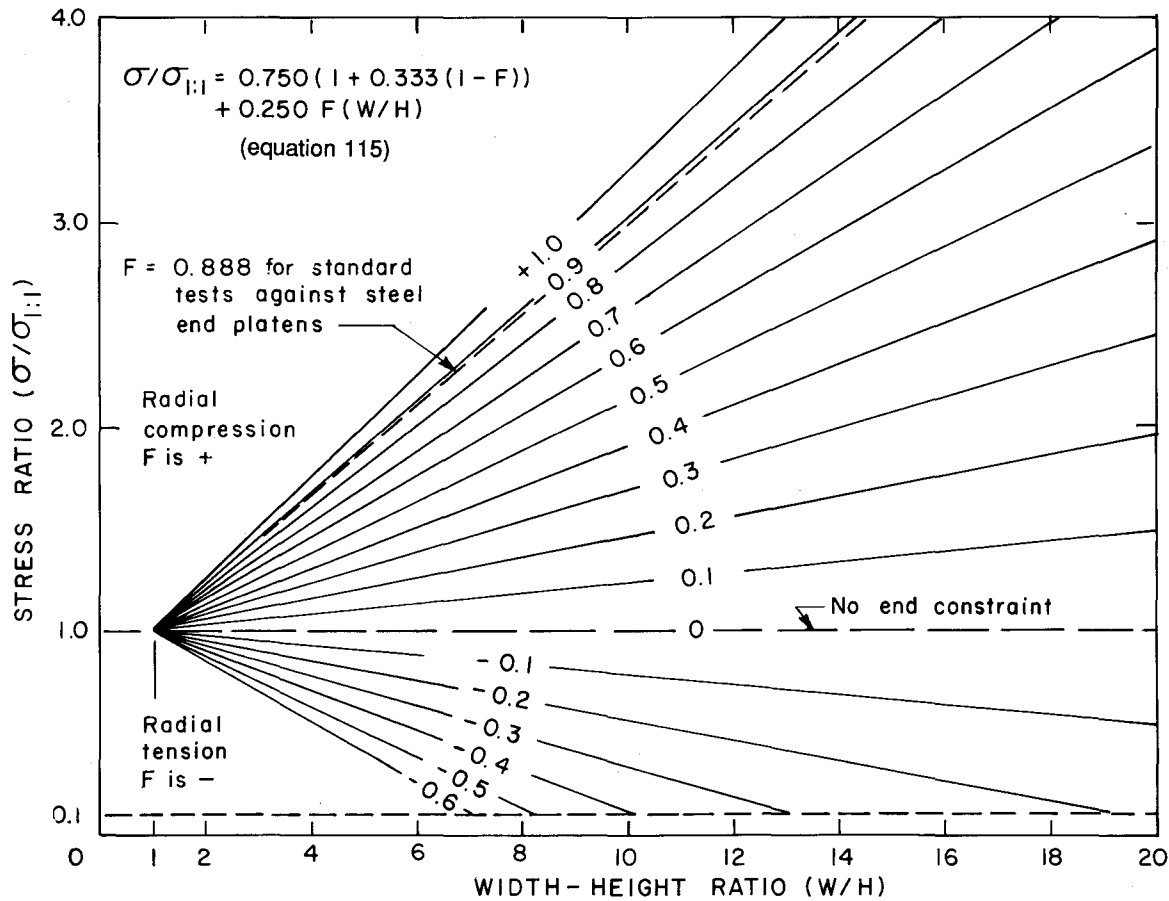


Figure 48.—Changes in relative strength of square pillars as function of geometry and of end constraint. F = coefficient of friction between ends of specimen and loading platens.

is loaded in radial tension normalized to the strength in compression. If the tensile strength is 10 pct of the compressive strength, the pillars would fail in tension for the W/H geometries used on the graph abscissa. For example, a pillar with a W/H of 10 would fail for a value of $F = -0.4$. As the W/H is decreased, the value of F must be more negative to produce failure. A given negative value of F was enough to break the large pillars, but was not negative enough to break the smaller pillars. Notice that the relative strengths of the long pillars (W/H is small) does not change much with the value of F in figure 48, staying in the range $0.75 \leq \sigma/\sigma_{1:1} \leq 1.25$, as $W/H \rightarrow 0$.

The appeal of equation 115 is that it can predict certain behavior that could not be predicted with the previous equation. First, the role of end constraint on strength can be estimated. Second, the strength becomes independent of specimen geometry when a state of true uniaxial stress is applied. Third, the bursting of mine pillars with the sudden loss of end constraint can be defined. Four examples will be given to illustrate these features. If the constraint is one-half of that supplied by steel,

$F = 0.5 \times 0.888 = 0.444$. In this case, the relative strength of a specimen with a D/L of 5 would be

$$\begin{aligned} \sigma/\sigma_{1:1} &= 0.750(1 + 0.333 \times (1 - 0.444)) \\ &\quad + 0.250 \times 0.444 \times 5 = 1.444. \end{aligned} \quad (117)$$

For the true uniaxial compression condition, there is only a uniform axial stress normal to the end surfaces and the friction factor, $F = 0$. In this case, the value of

$$\sigma/\sigma_{1:1} = 1, \text{ for all } W/H \text{ or } D/L. \quad (118)$$

In the third case of bursting, consider a specimen with D/L of 8.5 that is loaded between steel platens ($F = +0.888$) to 85 pct of its compressive strength. The relative strength (100 pct) is

$$\begin{aligned} \sigma/\sigma_{1:1} &= 0.750(1 + 0.333(1 - 0.888)) \\ &\quad + 0.250 \times 0.888 \times 8.5 = 2.665. \end{aligned} \quad (119)$$

The applied load would be 85 pct of this value or 2.265. If all end friction was lost dynamically, the new strength of the pillar would suddenly become equal to 1.0. That is, a pillar with a load of 2.265 will have a strength of 1.0 and bursting will occur, because it is overloaded 2.265 times.

In the fourth case of static crushing, equation 115 can be solved for the value of F . This becomes

$$F = (1 - \sigma/\sigma_{1:1}) / 0.250 (1 - D/L). \quad (120)$$

For a pillar with a W/H of 8.5 and a relative strength of 2.665 and an 85 pct load, what would the value of F become for static crushing to occur? For $\sigma/\sigma_{1:1} = 2.265$, the value of F would be 0.675. That is, if F decreases from 0.888 to 0.675, static crushing and not bursting would occur. At that time, the strength and load would exactly match. If F is greater than 0.675, slipping and a loss of constraint, but not static failure may occur. Slipping behavior and periodic loss of end constraint that lead to bursting was observed in the laboratory and shown in figure 54.

BURSTING OF MODEL COAL PILLARS IN LABORATORY

Long specimens of brittle solids, including coal, break violently when loaded against steel platens with a uniaxial compressive force that produces a triaxial compressive stress state within the specimen. Short specimens (that is, D/L of 8.5) usually cannot be broken at all between two steel platens. Such specimens can be broken violently (made to burst) if suitable end conditions are used. This demonstrates that the end constraint controls the burst behavior.

In mining, a necessary element in the pillar burst problem seems to be the presence of a strong rock layer in the mine roof close to or contacting the coal seam as reported by Rice (78). Apparently, this layer constrains the pillar in the horizontal direction so that it can support vertical stresses much greater than its unconfined compressive strength. With a sudden loss of horizontal confinement, the unconfined compressive strength is dynamically exceeded and bursting occurs.

How can these conditions be modeled in the laboratory? In initial experiments, a single layer of glass was used to represent the strong rock layer, with the expectation that the glass would be loaded in tension by the expansion of two pieces of plastic and by the coal. The much stiffer glass, with a Young's modulus of 10×10^6 psi, would deform laterally much less than the plastic or coal each with a Young's modulus of less than 0.5×10^6 psi. This leads to a sudden shattering of the glass layer by tensile stresses. The sudden loss of the radial constraint provided by the glass layer causes the specimen to burst.

Such a model coal pillar burst is shown in figure 49. Figure 49A shows the configuration used for testing. The four-element sandwich, from the top down was (1) a 3-in square by 0.25-in-thick plastic layer, (2) a 2.375-in square double strength window glass layer, (3) a coal disk of 2.125-in diam and 0.25-in thick, and (4) a 3-in square by 0.25-in-thick plastic layer. Figure 49B shows the results of the bursting.

It was desirable to make a comparison of the burst behaviors of a number of coals. Unfortunately, the behavior of the glass layer was as much a factor in the results obtained as the coal behavior. New glass with large residual strains breaks readily, while old glass in which the residual stresses have relaxed, does not. In addition, the size distributions of the glass pieces produced by the breaking were nonuniform (probably log normal because this is the most common crushing distribution) and would be difficult to use to calculate the burst behavior. It was observed in testing that the glass layer broke shortly before the bursting occurred in some cases. This meant that the coal burst could be produced by loading with a segmented platen—the broken glass layer. Therefore, a segmented steel platen with pieces of uniform size, 1/4 in square by 0.050 in thick, were used in place of the glass layer as shown in figure 50. In these tests, specimens of coal with a diameter of 2.125 in and a D/L ratio of 8.5 were used.

The coal specimen with the segmented steel platen covering the top was loaded in a compression machine between two pieces of methyl methacrylate (plastic), 3 in square and 0.25 in thick.

Test results for 15 coals from 11 coal seams from six States loaded with segmented steel platens were reported by Babcock and Bickel (17). Thirteen of the fifteen coals tested were made to burst. One such burst is shown in figure 51. A specimen with a diameter of 2.125 in was made to scatter over an area that was up to 2,600 times larger than the initial area of the specimen. Tests were performed above a horizontal surface 8-ft square that had a polar coordinate system marked on it. The bursts occurred about 3.5 in above the horizontal surface. The force of gravity brought the segmented platen pieces in contact with the horizontal surface in about 0.135 s. The array of steel platen segments was recorded on polar coordinates plots such as the one shown in figure 52. The specimen was of 2.125-in diameter before testing, and this was distributed over an area 8-ft square by the burst. The radial distances and directions the segments traveled before coming to rest on the horizontal surface were used to estimate the radial horizontal vector velocities of the platen segments. From the segment masses and velocities, the sums of the momentum and kinetic energy of each specimen tested were determined. The kinetic energy, $K.E.$, which is a scalar quantity, gave the magnitude of the burst. The momentum, $\bar{M}$, which is a vector quantity, gave

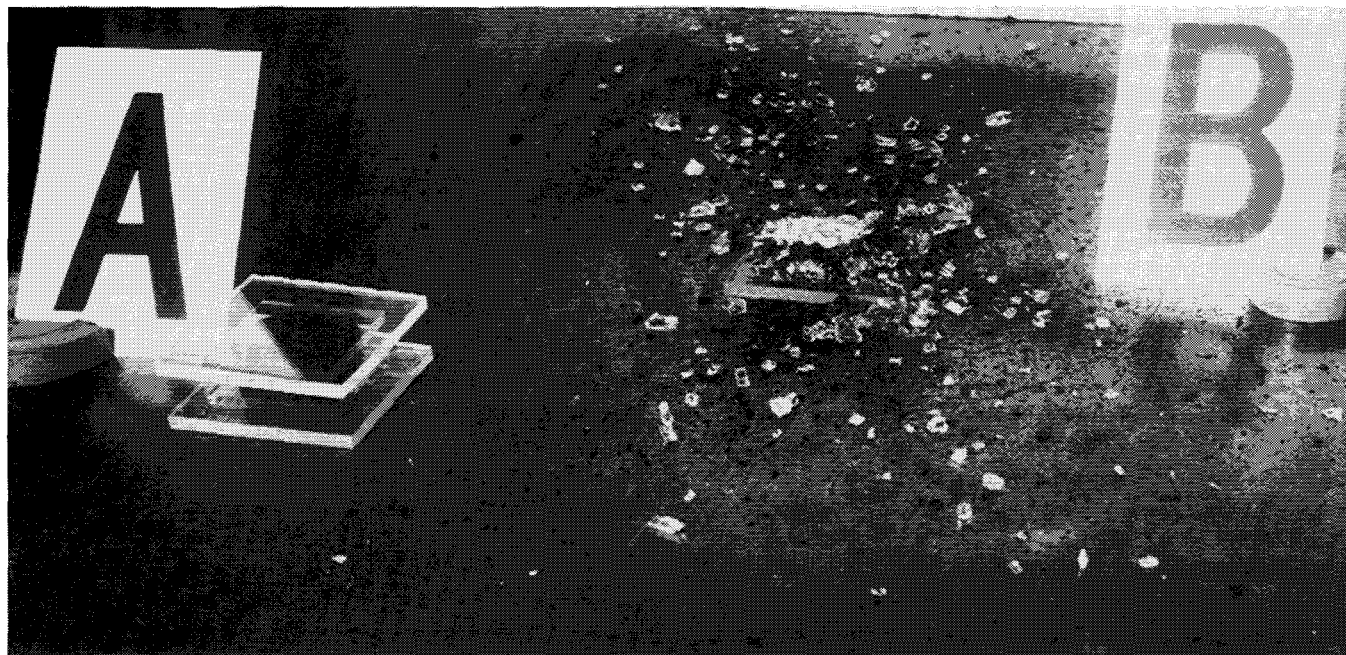


Figure 49.—Short coal specimen (diameter-length ratio of 8.5) was made to burst violently by dynamically shattering two solid glass layers, one above and the other below the coal specimen. A, Test configuration, with two plastic layers loading glass layers; B, plastic layers were loaded in compression by two 9-in-diam steel platens to produce burst shown.

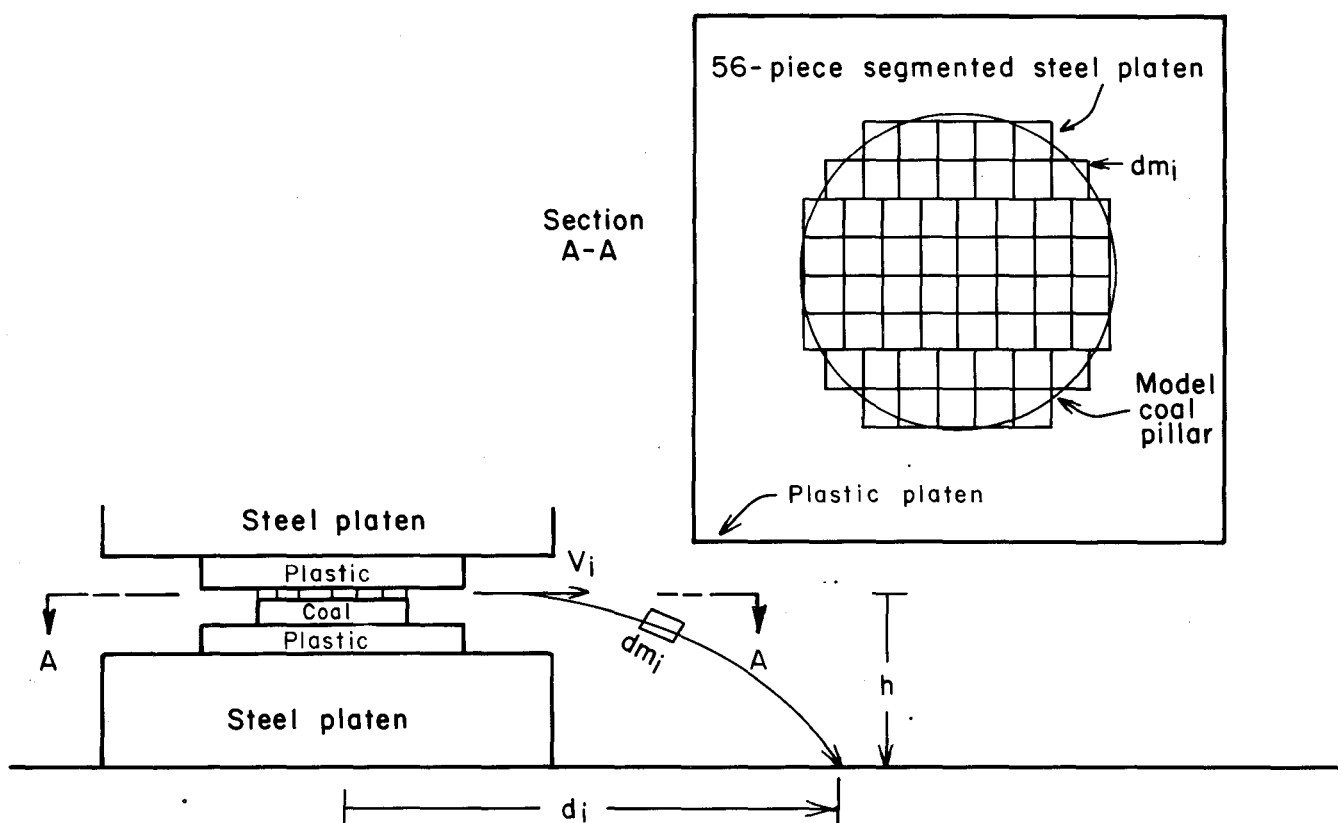


Figure 50.—Method of calculating velocities of segmented steel platen segments thrown horizontally by model coal pillar bursts in laboratory. V_i = velocity of i th segment, cm/s; m_i = mass of i th segment, g; h = height of fall, cm; d_i = distance i th segment is thrown by burst, cm (17).

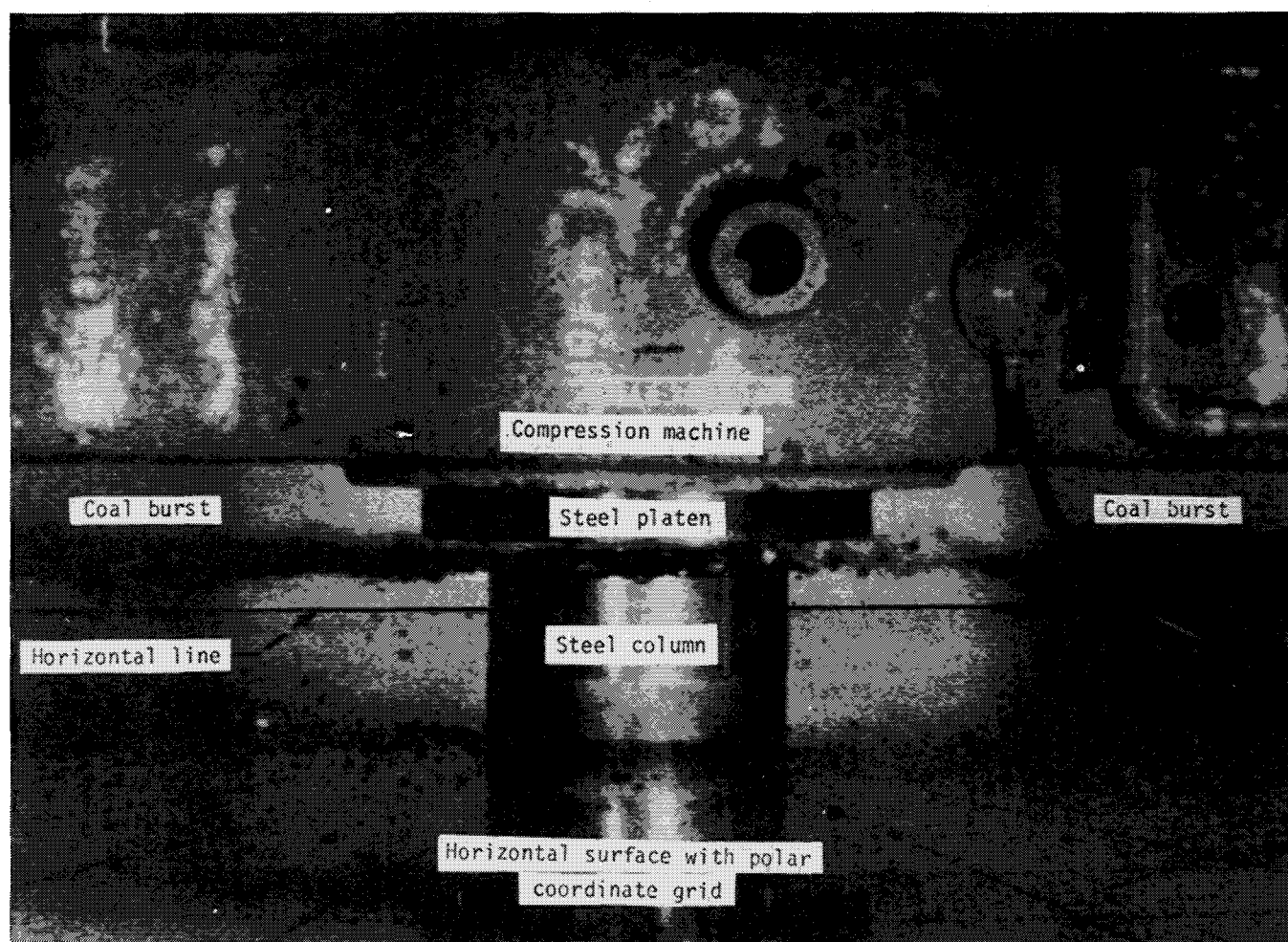


Figure 51.—Model coal burst in laboratory produced with test configuration of figure 50 (17).

the direction of the burst. An example of the procedure used to calculate the data is given in figure 53.

Figure 53A shows six segmented steel platen pieces each with a mass, m , and vector velocities $\bar{v}_1$, $\bar{v}_2$, $\bar{v}_3$, $\bar{v}_4$, $\bar{v}_5$, and $\bar{v}_6$.

Figure 53B shows the total momentum for the six pieces, $\bar{M}$, when

$$\bar{M} = m(\bar{v}_1 + \bar{v}_2 + \bar{v}_3 + \bar{v}_4 + \bar{v}_5 + \bar{v}_6). \quad (121)$$

$\bar{M}$ is the vector sum of the momentum of each of the separate masses. If the vector sum is zero, $\bar{M} = 0$.

Figure 53C shows the K.E., associated with the example problem. This is

$$\text{K. E.} = 0.5m(v_1^2 + v_2^2 + v_3^2 + v_4^2 + v_5^2 + v_6^2). \quad (122)$$

The sum of the square areas constructed on the sides of the vectors in figure 53C is proportional to the K.E. $\bar{M}$ could be zero for a given burst, but K.E. could not be. If

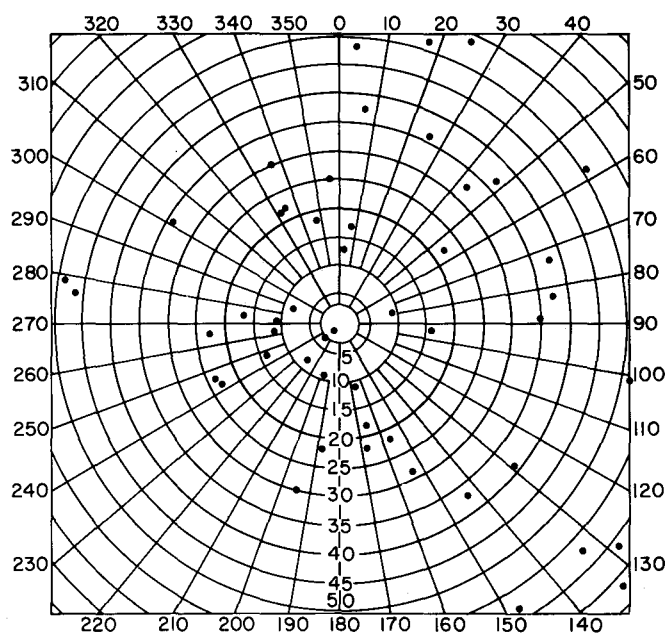
$\bar{M}$ is not zero, the bursting is directional. Use may be made of this fact to mine in a direction for which the momentum is a minimum to reduce the safety hazard.

The end constraint condition leading up to the bursting is shown in figure 54. Here, the periodic dynamic loss of end friction through slipping was observed by means of a strain-gauge-instrumented plastic platen used on the bottom of the coal specimen. As the vertical stress was increased, the horizontal strain increased spasmodically, indicating a sudden dynamic loss of end constraint. The total radial end constraint was not lost, because friction regained control after each occurrence of slipping. The slipping zone was defined by two limits called the stable and unstable limits. When the load reached a value for which the strain was at the unstable limit, slipping occurred. The slipping continued until it reached the stable limit, at which time the end friction again recovered control. This process was repeated eight times in the example given in figure 54 until the unstable and stable limit lines crossed. At that time bursting occurred.

The results for the 15 mines tested (17) are summarized in table 17. This shows the depth below the surface in meters, which ranged from 61 to 853 m. The "Mine burst now?" column indicates that there were no bursts in the mining above 274 m in depth. Five or six mines below the 274-m depth had bursting at the time of the report (1984). All of the coals from mines above 274 m burst in laboratory testing. Four of six coals from mines below 274 m burst in model tests. The conclusion was reached that most coals could be expected to burst given a depth of at least 344 m and suitable roof and floor conditions. It is not clear at this time why roof and floor end constraint should be dynamically lost, but sudden structural failure of either member or the loss of frictional constraint could produce a burst.

The sudden loss of end constraint for other materials than coal also occurs, as evidenced by the dynamic slipping of the ends of sandstone specimens loaded in compression in the work cited (17).

In additional tests by this author not reported elsewhere, 500 psi air pressure was applied to the ends of test specimens of 2 of the 15 coals that did not burst under compressive loading stress. Bursting by air transport after the specimens were crushed in compression loading was achieved only after the vertical loading stress was greatly reduced. The ability of a compressed gas to transport the broken coal is widely known. What is not so widely known is that until the constrained broken coal loses some restraint, the compressed gas is ineffective in causing a burst (gas outburst).



Sample data sheet for segmented steel platen array after burst.

Figure 52.—Polar coordinate grid pattern on 8-ft-square horizontal surface used to collect segmented steel platen pieces thrown by bursting model coal pillars. Polar coordinates 0 to 360° are on boundary (17).

Table 17.—Summary of test results and burst proneness of coals from 15 mines, in 11 coal seams, and in 6 States (17)

Depth, m	Mine number	State	Mine bursts now?	Model bursts	Kinetic energy (K.E.)	Kinetic energy (K.E.) burst rank	Momentum ¹ M, (gm cm/s)	Momentum, M burst rank
61	14	PA	No	Yes	223	5	958	7
85	10	IL	No	Yes	38	11	355	11
91	1	UT	No	Yes	164	8	1,268	4
91	3	CO	No	Yes	69	10	719	10
91	9	CO	No	Yes	259	3	741	9
183 ...	7	CO	No	Yes	281	1	1,739	2
253 ...	13	WV	No	Yes	1	13	905	8
274 ...	12	WV	No	Yes	273	2	65	13
274 ...	8	CO	No	Yes	124	9	1,018	6
344 ...	11	WV	Yes	No	0	(²)	0	(²)
427 ...	2	UT	Yes	Yes	214	6	1,177	5
503 ...	4	CO	No	No	0	(²)	0	(²)
541 ...	6	CO	Yes	Yes	203	7	1,328	3
671 ...	15	AL	Yes	Yes ³	1	12	194	12
853 ...	5	CO	Yes	Yes	237	4	2,300	1

¹Estimates of momentum and kinetic energy.

²Did not break in laboratory tests.

³Weak bursts sometimes occur.

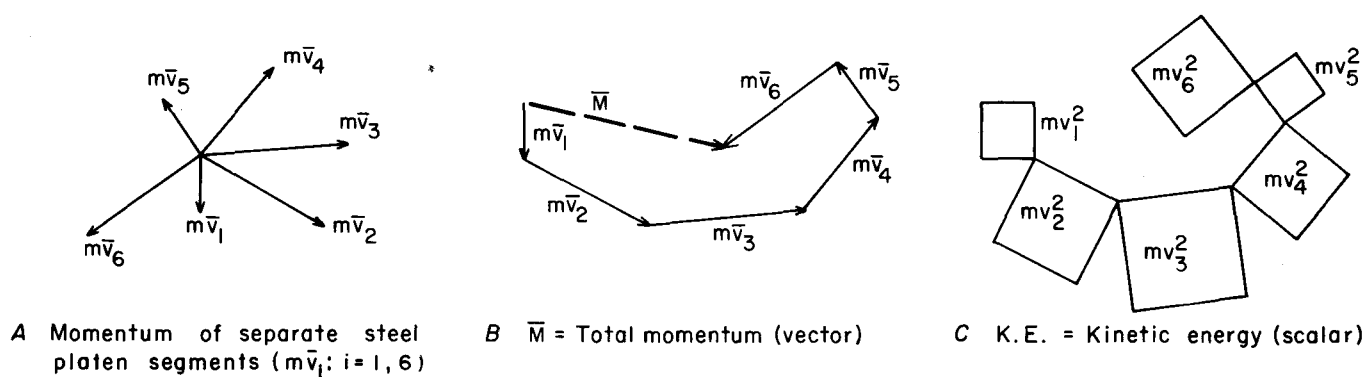


Figure 53.—Calculation of momentum and kinetic energy for model coal pillar bursts in laboratory. In sample problem six, steel platen segments each have a mass (m), g. A, Each segment has a momentum characterized by a velocity ($\vec{v}$), cm/s. B, Total momentum ($\vec{M}$), g-cm/s, is the vector sum of the momenta of the segments. C, Total kinetic energy (K.E.), g-cm²/s², of segmented platen equals sum of areas shown. The length of side of square = velocity, and each area is weighted by the mass (m) (17).

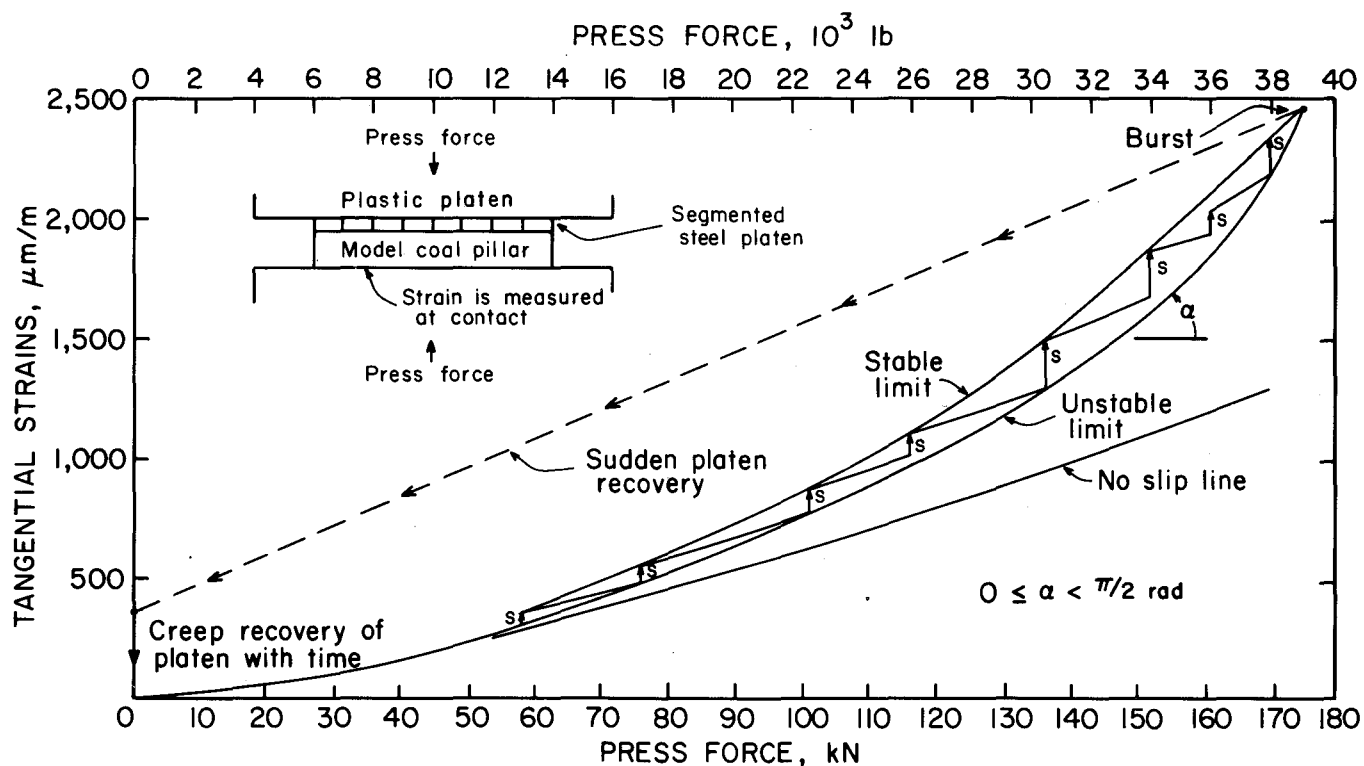


Figure 54.—Tangential strains showing slipping (S) of lower plastic platen with increasing load. Each time force-strain point fell on unstable limit line, slipping occurred. When force-strain point reached stable limit line, slipping was arrested by radial friction stresses. When unstable and stable limit lines intersected, end friction could no longer arrest slipping, and bursting occurred (17).

Burst Proneness of Coal Specimens Relative To Test Geometry and End Constraint Conditions

In the preceding study (17), the momentum and kinetic energy of bursting coal specimens were determined in the laboratory to quantify the burst behavior. A simpler method was used in another study (8) when such parameters were difficult to obtain. The percentage of the initial specimen weight that was thrown off a 30- by 30-in compression machine table was used as an indicator of burst behavior. The loss in specimen weight from the table ranged from 0 to 90 pct. The results from reference 8 are given in figures 55 and 56.

Figure 55 shows six histograms of burst-proneness versus frequency summed over eight end condition for six geometries. The average burst-proneness appeared to change with geometry, but was actually changed with the stress state as shown in "Role of End Constraint on Compressive Strength." One histogram is given for each of six sets of D/L specimens tested including 0.5, 1, 2, 3.5, 5, and 8.5. The percentages of the initial specimen weights

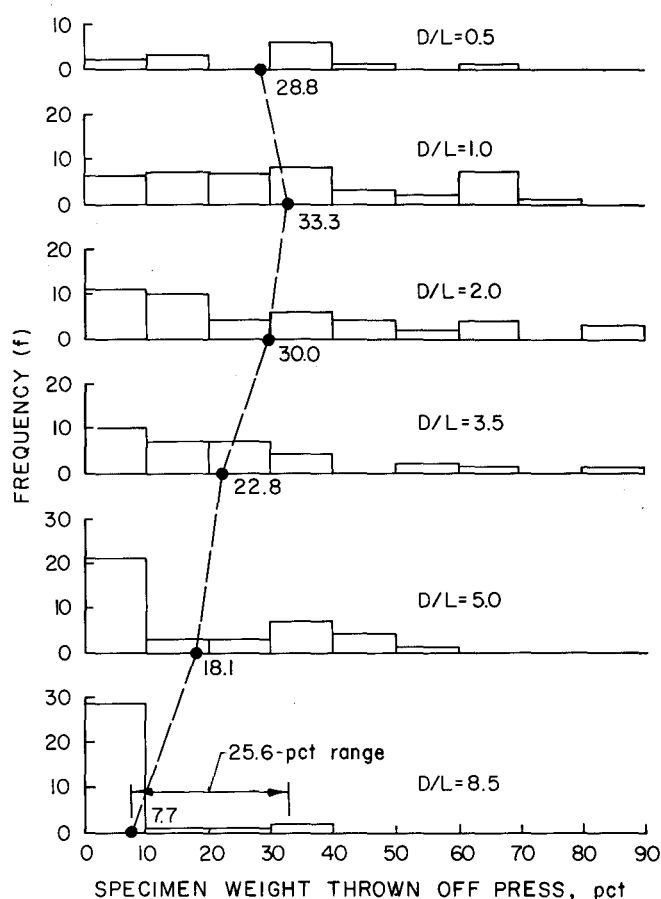


Figure 55.—Histograms of burst-proneness versus frequency summed over eight end conditions for six geometries. D/L = diameter-length ratio (8).

thrown from the press table are grouped in cells 10 pct units wide; that is, 0 - 9.9, 10 - 19.9, 20 - 29.9 pct, etc. The vertical scale, f , denotes frequency of test results in the various cells. For example, for the D/L = 1.0 tests, six specimens broke, with 0 to 9.9 pct thrown off the press table. Most of the specimens with a D/L ratio of 1.0 had an average burst proneness of 33.3 pct. That is, this geometry, with its associated state of stress, is the most burst prone of the geometries and states of stress tested. The averages (black dots) of the histograms are connected to show the trend between histograms. The range of burst proneness was from 0 to 89.9 pct. That is, from 0 to 89.9 pct of the initial coal specimen weight was thrown from the table when the specimens were broken in uniaxial force compression.

Several features of figure 55 are of interest. Notice that the frequency, f , and the percent loss are relatively uniform for D/L values of 0.5, 1.0, and 2.0. That is, some end conditions produced large bursts, while other end conditions produced little bursts. For short specimens, little bursting occurred, and this was not changed much by the

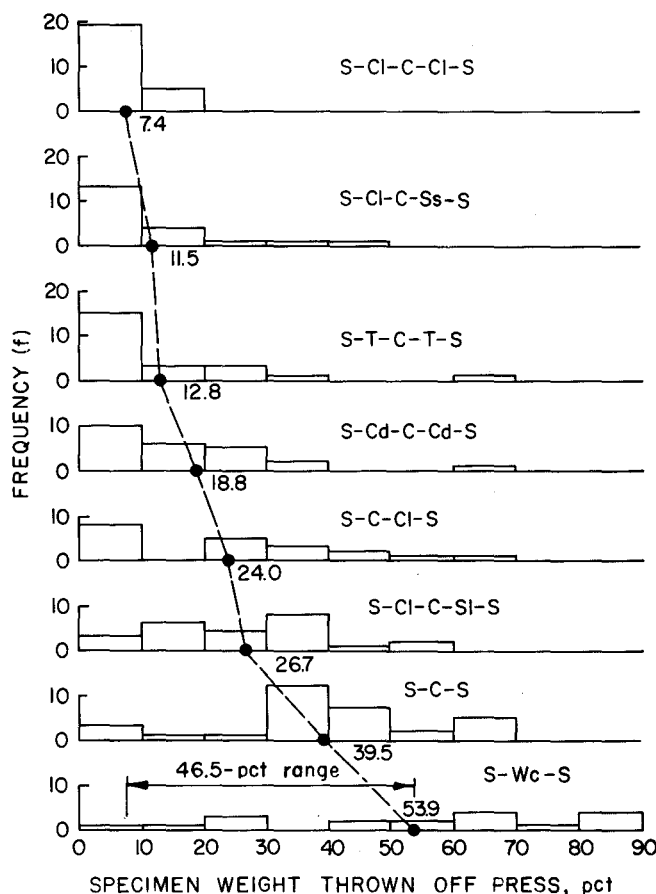


Figure 56.—Histograms of burst-proneness versus frequency summed over six geometries for eight end conditions. The symbols C, Cd, CI, S, SI, Ss, T, and Wc denote coal, coal dust, clay-dust, steel, slate, sandstone, PTFE, and wet-coal end conditions for specimen tests, respectively (8).

end conditions. In general, the bursting decreased with an increase in the D/L.

Figure 56 shows eight histograms of burst proneness versus frequency summed over six geometries for eight end conditions. The different end conditions are represented by the three to five letter or letter groups. Here, S, Cl, C, Ss, T, Cd, Sl, and Wc represent steel platen, clay layer 0.050-in thick, coal specimen, sandstone disk 4-in diam and 0.5-in thick, PTFE layer 1/32-in thick, coal dust layer 0.050-in thick, slate disk 4-in diam and 0.5-in thick, and a coal specimen soaked in water overnight. For example, S-Cl-C-SL-S represents a vertical test column with a steel platen on the top; a layer of clay next; a coal test specimen next; a slate layer beneath the coal specimen; and a steel platen on the bottom. Notice that the averages denoted by solid black dots change from 7.4 to 53.9 pct, a range that is about twice the range of the figure 56 range of 7.7 to 33.3 pct. This shows that the change in end conditions produced a bigger scatter in the burst behavior than the D/L. The burst proneness again ranged from 0 to 89.9 pct.

Several features of figure 56 are of interest. For soft end conditions, with clay layers on each end, the percent burst averaged only 7.4 pct for all geometries. That is, the geometry played very little part in the burst behavior. Other soft layer tests produced burst averages of 12.8 pct and 18.8 pct. When a soft layer was used on one end and a hard layer used on the other end, loss averages of 11.5 and 26.7 pct were obtained. The use of one hard layer and one soft layer gave a variety of results. When the end constraints were hard, the loss percent increased. The maximum loss, 53.9 pct, occurred when two steel end platens (hard) were used to test a wet coal specimen.

It is concluded that in mining where the pillar geometry is relatively fixed, most of the burst behavior variability can be related to the variability of the constraint provided by the roof and floor rock.

Use of Mohr-Coulomb Stress-Failure Model To Describe Coal Burst Behavior

In reference 17 a Mohr-Coulomb stress-failure model was developed to explain burst behavior. The dynamic bursting of a coal pillar can be explained as the result of the dynamic loss of horizontal constraint stress from the roof and/or floor rather than the dynamic increase in vertical pillar stress from mining.

In the Mohr-Coulomb stress-failure model, failure occurs in shear when the stress state presented by the Mohr's stress circle grows in size until it touches the failure line. This is usually taken to occur when the most compressive stress (the vertical stress in mining in most cases) increases with mining recovery. Since mining is usually a slow process where coal is removed by cutting

machines, this explanation of how a burst is produced is not very likely. In the past, when mining was done by blasting, dynamic loading and bursting in this manner may have occurred. Today, a more likely mechanism to explain bursting is one in which the loss of horizontal constraint occurs dynamically. In this case, the stress circle size increases dynamically, and if it dynamically crosses the Coulomb failure line, bursting occurs. Physically, the energy release is dynamic, and the coal pillar cannot handle the strain energy stored in it in a static manner. A coal burst results.

Consider the Mohr-Coulomb relationships for static and dynamic failures shown in figure 57. In figure 57A, a Mohr's stress circle 1 represents a stable stress state for a vertical stress, S_v , and a horizontal stress, S_h , that are the right and left points of intersection between the circle and

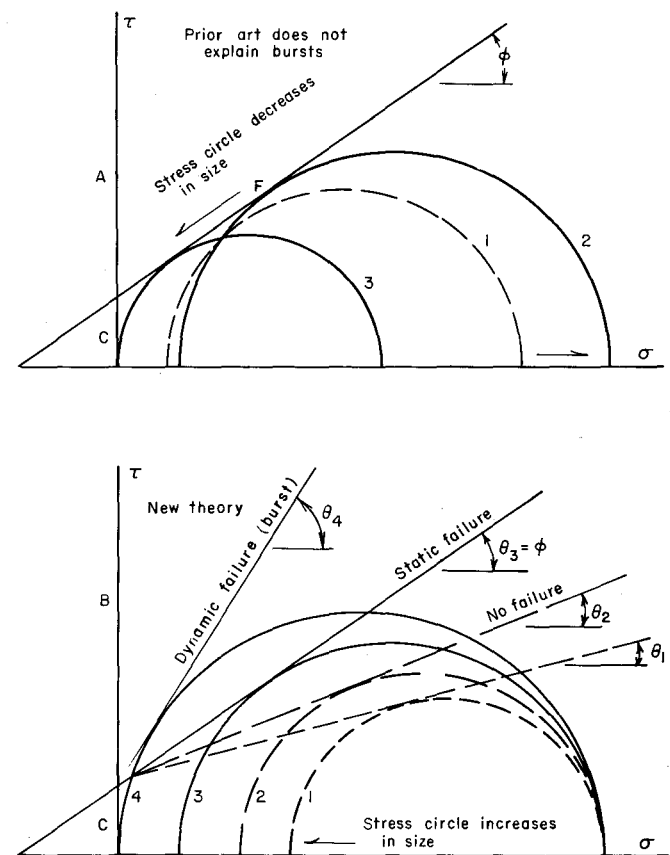


Figure 57.—Mohr-Coulomb stress-failure models. A, In conventional model, *increasing* stress produces static failure. Stress circle 1 before failure moves to stress circle 2 to touch the Coulomb failure line at point F. Circle 2 then decreases in size to circle 3. B, In new model, *decreasing* stress produces failure. Circle 1 with angle θ_1 is stable and no failure has occurred. Left side of circle represents horizontal pillar stress and right side, vertical pillar stress. If horizontal stress decreases dynamically through sudden loss of confining frictional stress from roof and floor, circles 2, 3, or 4 with angles θ_2, θ_3 , or θ_4 result. Circle 2 is stable, circle 3 produces static failure, and circle 4 produces dynamic failure (coal burst) (17).

the normal stress axis, σ , respectively. If this circle grows in size because of mining in a coal seam, circle 2 could result. Since this circle touches the Coulomb failure line, static failure has occurred. Because the coal has been broken, vertical stress decreases and some circle 3 of a reduced diameter results. Any following stress circles would touch the Coulomb failure line and would increase or decrease in size depending upon whether the stress was increasing or decreasing. All this behavior is of a static nature.

In the case of a coal pillar burst, the scenario is different, as shown in figure 57B. In this case, circle 1 is a stable stress state as before. The cohesive strength in shear, C , and the angle, θ_1 , define this condition. This angle is less than the angle of internal friction, ϕ . If a loss of end constraint occurs (that is, a sudden loss of end friction) between the roof and floor and the coal pillar so that Mohr's circle 2 occurs, the pillar will become stable again, because θ_2 is still less than ϕ . If slipping occurs so that the stress circle 3 results, the coal pillar will be statically crushed without violence. If the slipping occurs so that the resulting Mohr's circle becomes dynamically larger than the static failure circle, as for circle 4, dynamic failure (bursting) occurs. In this case, the angle, θ_4 , needed for stability is larger than the friction angle, ϕ , which can no longer control the behavior.

Since it is uncertain in mining if and when constraint will be lost and how much, it is difficult to tell if a pillar will be safe, will crush quietly, or will burst violently. Recent support for the notion that the horizontal stress is the parameter to be measured to define the pillar stability comes from the work of Descour and Miller (37). They conclude, "Microseismic data and other observations suggest that a nonuniform horizontal stress distribution in the roof and floor is probably responsible for most of the bounce and roof instability problems." The measurement of the sudden loss of horizontal confining stress may be the best method to evaluate the stability of a pillar, whether done statically with BPC's or with microseismic instrumentation.

CHANGES IN ROOF ABOVE LONGWALL FACE IN COAL MINE

Infinitesimal theories of elasticity are known to be poor estimators of finite behavior. This is so because the infinitesimal theory assumes that the reference shape of the body does not change. That is, either the beginning geometry or the final geometry can be used. In finite (large deformation) problems, an incremental change in geometry is needed to obtain the true stress-strain values.

The large deformations that occur in mine structures are poorly modeled by a single computer run with an infinitesimal linear elastic theory such as a finite-element analysis. The progressive change in shape must be considered to model the true behavior. A single computer

run using elastic constants, the Young's modulus, and Poisson's ratio allows only a very small change in the shape of the mine structure and provides for no change in these constant values. By making sequential computer runs in which the elastic constants are changed after each run according to observed rock and coal property changes, large-scale deformations, fractured zones, safety factors, and stress states can be modeled that are consistent with the observed large-scale mine deformations.

Dr. Kidybinski was a 6-month visitor at the USBM Denver Research Center in 1971. Kidybinski and Babcock (60) made finite-element analyses of the conditions above a longwall face of a coal mine near Katowice, Poland. The mine geometry considered is shown in figure 58. The

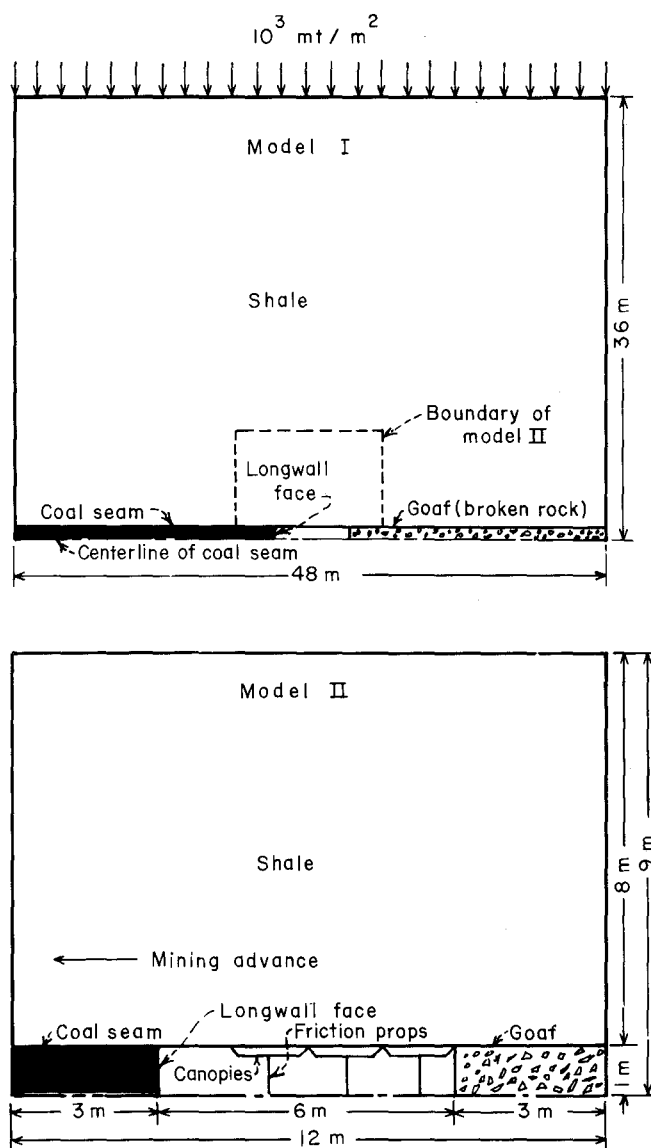


Figure 58.—Dimensions, geometry, and boundary conditions of finite-element models of progressive fracture behavior occurring above longwall face (60).

cost of running a finite-element model is related to the number of elements used. To obtain accurate results, the element size must be small where the stress gradient is large. Such conditions occur along the boundary of the mine workings, where the boundary changes direction rapidly as at the corners. This condition also occurs where two adjacent materials have very different physical properties as for the roof-coal and the coal-floor interfaces. Relatively large elements were used in model I to define the boundary conditions for model II. Model II was then used with a higher density of finite elements for better representation of the geometry of the problem. Kidybinski had observed cracking of the roof rock as the face was advanced by means of small drift-type openings driven in the roof rock above a longwall face. The cracking is shown in figure 59. Notice that the cracks are not 90° apart as would be required if failure occurred on the maximum shear stress surfaces. Instead, failure occurred on other surfaces at an angle defined by the failure point on the Mohr's stress circle where it touches the Coulomb failure line. The angles between the crack directions are therefore 90° plus or minus the angle of internal friction, ϕ . The direction of cracking could be used to estimate, ϕ , as was done in "Babcock's Method of Assessing Pillar Stability."

The mine opening closure observed was much greater than that generated by a single finite-element computer run with infinitesimal strains and constant physical properties. To obtain the finite deformations observed in Poland, a series of seven runs were made with model II of figure 58. After each run, the physical properties were changed for the failed elements according to progressive failure results for stiff compression testing reported by Bieniawski (26). When loaded in a stiff testing machine, the rock or coal retains some strength even though it is fractured. The stiffness with respect to deformation is progressively reduced as the extent of the fracturing is increased. In effect, the bulk physical properties are changed with increasing stress levels. The series of seven finite-element models produced result in agreement with those observed in the mine. Roof support in the form of vertical forces of 5 or 50 mt near the face gave the results of figure 60 and figure 61, respectively. The curve parameters are the safety factor values defined to be the shearing stress divided by the shearing strength. Any number less than 1.0 has failed according to the criteria used. Notice that the most badly fractured zone (a safety factor of 0.2) for a prop load of 50 mt in figure 61 was in the same place and was about the same size as the fracture pattern of figure 59. An increase in the force from 5 to 50 mt decreased the size of this zone appreciably. The most badly broken zone is above the zone supported by the 5 and 50 mt forces. The roof above the loading forces would be unbroken for a loading force of 50 mt. However, the

remainder of the roof over the mined-out area would be broken with a safety factor of less than 1.0. Today, shields capable of exerting forces of 500 st or more are in use compared to the 50 mt force supports used in the mine studied by Kidybinski prior to 1971. The broken zone could be expected to be reduced further or perhaps even eliminated when such large forces are supplied by the shields in use today. The roof above the solid coal is intact and unbroken by the mining process.

The stress-failure variation used (60) is described in "Babcock's Method of Assessing Pillar Stability." In this variation the Mohr's stress circle was assumed to result from a proportional loading, and failure occurred when and if the resulting circle touched the Coulomb failure line. The safety factor was defined to be the shear stress divided by the shear strength at any point in the coal-rock mass.

CALCULATION OF MINE PILLAR STRESSES FROM BOREHOLE PRESSURE CELL DATA

For mine design purposes, it is desirable to know the state of absolute stress or the change in the stress with mining advance or with time. In hard rock, the overcoring method with a borehole deformation gauge (25) gives the best results for absolute stresses. In difficult to core materials like coal, the method works poorly, if at all. One method to obtain the absolute state of stress that may be made to work some day is to use the creep behavior of the coal in the manner proposed by Lu⁷. He uses three hydraulic pressure cell arrays of Panek design (73-74). Two of these cells are borehole pressure cells (BPC's), and the

⁷Babcock, C., Lu, P., F. Horino, and R. Harmel. Field Test and Evaluation of USBM Borehole Pressure Cells for The Wipp Project. Final Report to DOE Interagency Agreement DE-AI04-82AL20398, Oct. 1983, 41 pp.

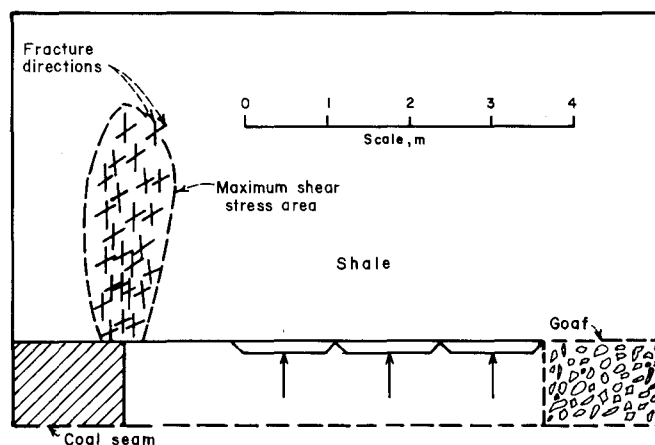


Figure 59.—Directions of fracture lines within high shear stress area above longwall face (60).

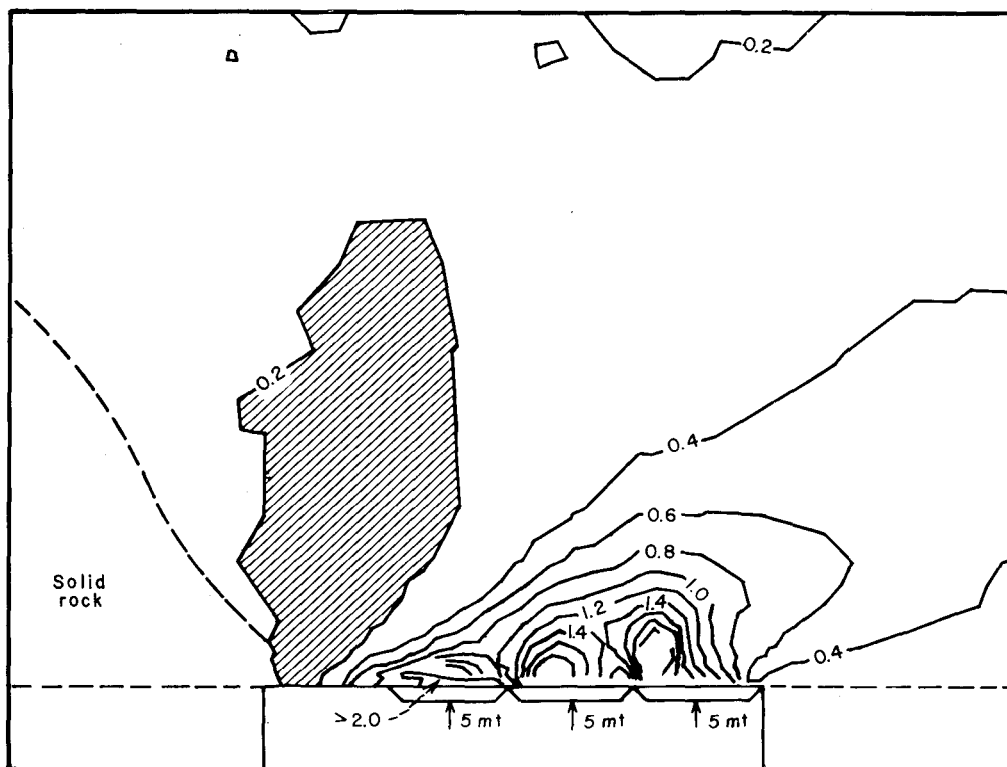


Figure 60.—Safety factor contour lines of fractured roof over longwall face, model II, 5 mt prop capacity (60).

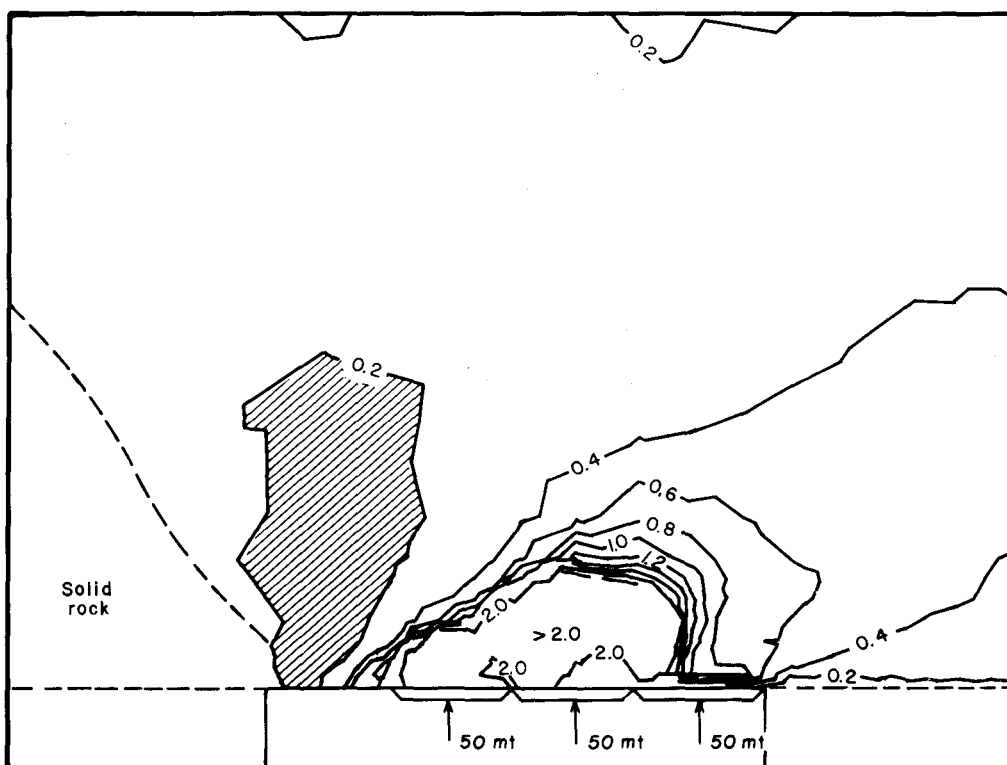


Figure 61.—Safety factor contour lines of fractured roof over longwall face, model II, 50 mt prop capacity (80).

third cell is a cylindrical pressure cell (CPC). In work with Babcock and others, the results obtained by the Lu method was dependent upon the setting pressure of the cells. If this dependency can be removed, the Lu procedure may be the method to use for absolute stresses in coal.

In the meantime, a knowledge of the change in stress with mining or with time is useful information as was shown in the "Babcock's Method of Assessing Pillar Stability" and "Calculation of Mine Pillar Stresses from Borehole Pressure Cell Data" (12). Panek (75) developed the borehole pressure cell in 1957. Babcock was involved in the initial work with this cell in iron ore. In the past, the cell pressure changes were often taken to be the rock or coal stress changes. This procedure was shown to lead to serious errors in the results (12). In 1970, Sellers (82) reported on a method to analyze BPC data. He assumed that the BPC was a circular inclusion bonded to the hole boundary with a cross sensitivity to transverse loading of 6 pct. This is unrealistic for two reasons. First, the cell has a smaller diameter than the hole to allow placement. When the cell is expanded to contact the wall of the hole, it is no longer circular, but is rectangular with semicircular ends. Such a shape has been named ovaloidal by Obert and Duvall (70). The contact areas of the cell against the hole boundary are on opposite ends of the long axis of the ovaloid. It does not contact the hole boundary for most of the hole perimeter, and therefore, responds indirectly to changes in the hole boundary shape. The BPC is a prop gauge that acts across a selected diameter of the drill hole. Second, the response to transverse loading is not the 6 pct assumed by Sellers, but 33.3 pct as shown in figure 62. In work by Lu (62), he assumed a cross sensitivity of 18.5 pct on an unrealistic basis of comparing the edge area of the cell to the flat area of the cell. The stiffnesses are not related in this way. The principles of these stiffnesses are presented in reference 13. When loaded in the direction normal to the flat dimension of the BPC, the overall stiffness is obtained by combining the reciprocals of the individual stiffness. When loaded in the transverse direction

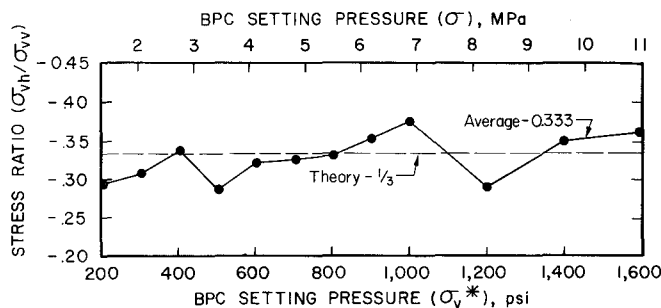


Figure 62.—Cross-sensitivity of borehole pressure cell to transverse loading (12).

parallel to the flat dimension of the cell, the stiffnesses add as the sum of the separate stiffnesses. A thin, soft layer, such as the fluid under confinement, drastically reduces the stiffness in the direction normal to the fluid layer, but only slightly in the direction parallel to the thin layer.

The author's method of BPC data analysis will be presented next (12). The behavior of the BPC is shown in figure 63. In figure 63A, the stresses around a drill hole in the vertical and horizontal directions S_v and S_h , respectively, are shown. The deformations of the circular hole by the S_v and the S_h stresses are (a, -a/3), and (b, -b/3), respectively, as shown in figure 63B. That is, the transverse hole boundary displacements are -1/3 of the load direction displacements. The minus sign denotes that the transverse radius becomes larger when the radius of the load direction becomes smaller. Figure 63C shows a vertical cross section through a horizontal hole in a pillar with the BPC expanded against the area A_v at the top and bottom of the hole. Figure 63D shows a vertical cross section through a horizontal hole in a pillar with the BPC

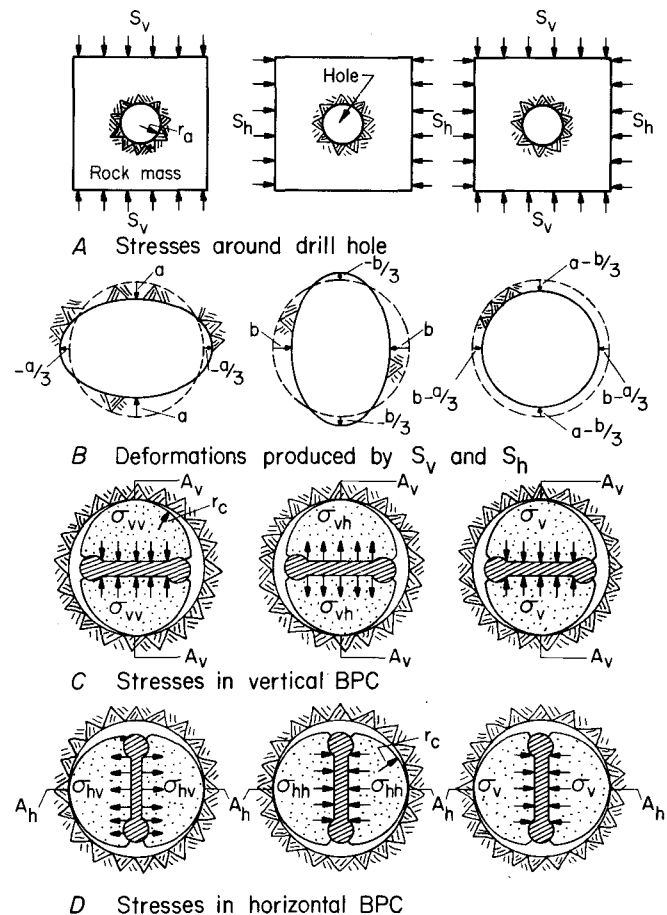


Figure 63.—Borehole deformations produced by biaxial stress field and borehole pressure cell responses they produce (12).

expanded against the area A_h at the sides of the hole. The cell in figure 63C is loaded directly by vertical loading and indirectly by the transverse loading. The cell in figure 63D is loaded directly by horizontal loading and indirectly by the transverse loading. By indirect, the wall of the hole is free to displace inwardly in the transverse direction without directly changing the pressure in the cell. This displacement causes the diameter of the hole in the load direction to change and to unload the cell. That is, compressional rock stress in the direction of expansion of the cell loads the cell, while compressional stress in the transverse direction unloads the cell. The cell in figure 63C responds to horizontal pressure changes when the horizontal hole diameter is changed as in the manner of an open hole (63B). The cell in figure 63D responds to vertical pressure changes when the vertical hole diameter is changed as in the manner of an open hole in figure 63B.

A stress state can be expressed in two parts: the volumetric or hydrostatic, and the deviatoric. In terms of the Mohr's circle behavior, the first locates the center of the stress circle and the second determines the size of the circle. Figure 64 shows the BPC response to the radial stress acting on a cylindrical core in which the BPC was placed

in a diamond drill hole concentric with the core axis. Notice that the BPC response to the applied radial pressure was a function of the setting pressure and to the physical properties of the core tested. When the core with BPC was tested with a uniaxial load in a biaxial test frame, the response to applied pressure was one-third as much in the transverse direction as in the direction of the applied stress as shown in figure 62. This ratio (-1/3) exactly agrees with the changes in the open hole radii shown in figure 63B. When the transverse load is exactly one-third the load normal to the flat dimension of the metal bladder, the indicated pressure change in the cell is zero. This is a very good reason for not using the cell pressure as the rock stress. The two can be very different. The equations 123 and 124 corrected for an algebraic mistake in reference 12 for determining the change in stress from the BPC readings where 1/4 becomes 3/8 are

$$\Delta S_v = (3 / 8m) \text{Ln}((\sigma_v/\sigma_v^*)^3 (\sigma_h/\sigma_h^*))$$

and

$$\Delta S_h = (3 / 8m) \text{Ln}((\sigma_h/\sigma_h^*)^3 (\sigma_v/\sigma_v^*)) \quad (123)$$

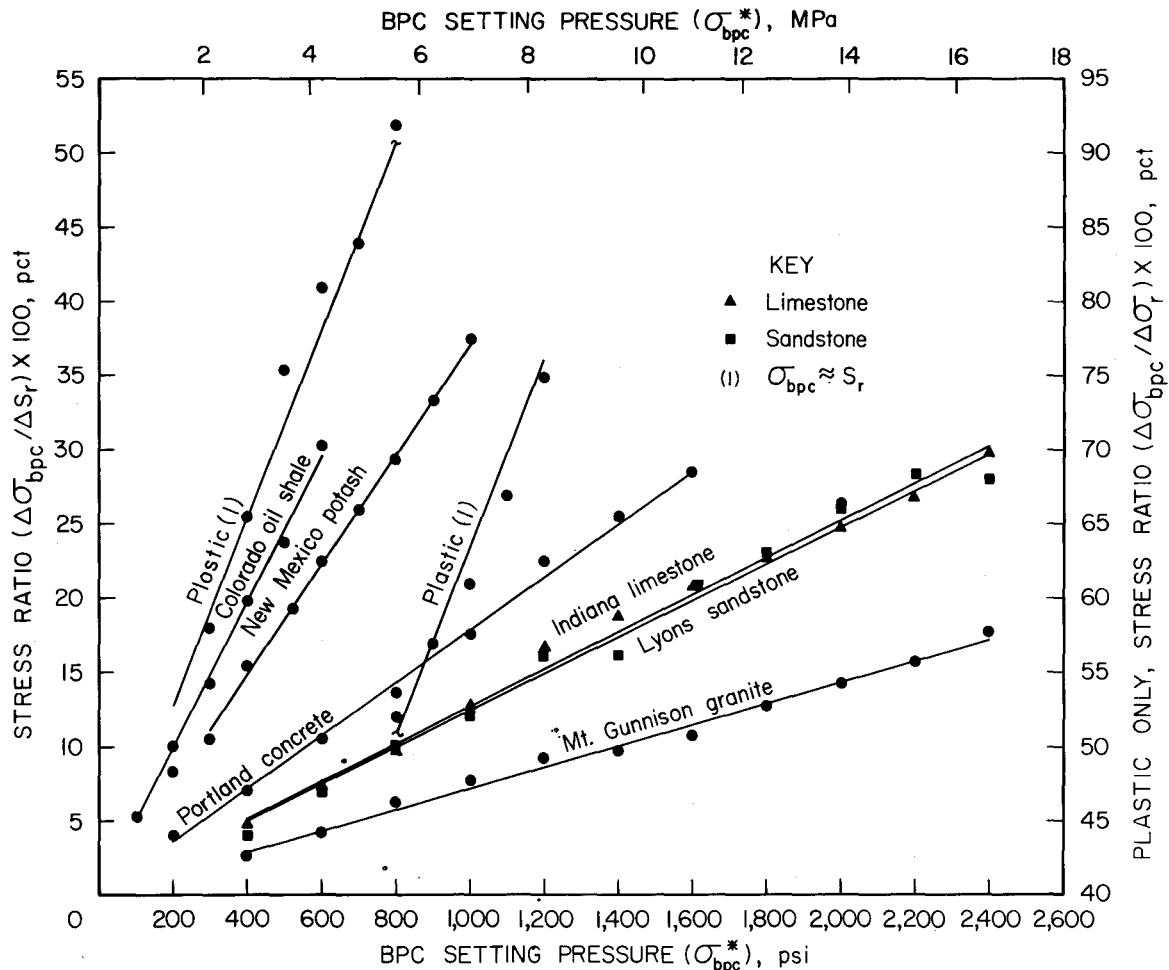


Figure 64.—Least squares regression lines fitted to experimental data from radial loading tests (12).

where the values ΔS_v , ΔS_h , σ_v , σ_v^* , σ_h , and σ_h^* are the changes in the vertical and horizontal pillar stresses, the vertical cell pressure, the vertical cell setting pressure, the horizontal cell pressure, and the horizontal cell setting pressure, respectively. Another change was the replacement of S_v and S_h with ΔS_v and ΔS_h , to denote that the coal stresses calculated from the cell pressures are stress changes after the installation of the BPC's. $\ln$ is the natural log and m is a physical property constant for selected rocks determined from the slopes of the lines in figure 64. The values of m were later shown to be related to the Young's modulus, E , and to the Poisson's ratio, ν , by the equations

$$m = 4.348 ((1 - \nu^2)/E)^{2/3}, \text{ in psi units}$$

and

$$m = 0.220 ((1 - \nu^2)/E)^{2/3}, \text{ in GPa units.} \quad (124)$$

Since the ratios σ_v/σ_v^* and σ_h/σ_h^* are dimensionless, the units for m determine the units for stress.

An example of the use of equations 123 and 124 follows. Assume that the BPC setting pressures are $\sigma_v^* = \sigma_h^* = 1,000$ psi in a coal seam. The physical properties are $E = 0.5 \times 10^6$ psi and $\nu = 0.25$. After a period of

time and mining advance, the gauge pressures become $\sigma_v = 1,825$ psi and $\sigma_h = 950$ psi. What are the stress changes in the coal that correspond to these BPC fluid pressure changes? From equation 124, $m = 0.0006611$. The vertical and horizontal stress changes in the coal pillar are $\sigma_v = 663$ psi, and $\sigma_h = 169$ psi. That is, both coal stresses would have been *increasing* while one cell pressure was *increasing* and the other cell pressure was *decreasing*. Use of the cell pressure changes as coal stress changes would have resulted in errors of 24 pct for the vertical stress and 130 pct for the horizontal stress with the wrong sign.

An application of the above equations for mine design purposes is given in figure 65 where the cell pressure change data obtained by Lu, Kneisley, and Harmel⁸ is compared to the data reduced with equations 124 and 125. The σ_v and σ_h values are the BPC pressure changes reported. Notice that the horizontal coal pillar stress change, ΔS_h , is about one-third the vertical stress change, ΔS_v , for the reduced data, even though the horizontal cell pressure, σ_h , is near zero. This occurs because the BPC responds to both stresses of the biaxial stress field. When

⁸Lu, P., R. O. Kneisley, and R. M. Harmel. Ground Control Study for Longwall Entry Design at York Canyon Mine, Raton, New Mexico. BuMines Den. Res. Ctr. Progress Report 10023, 1979, 62 pp.

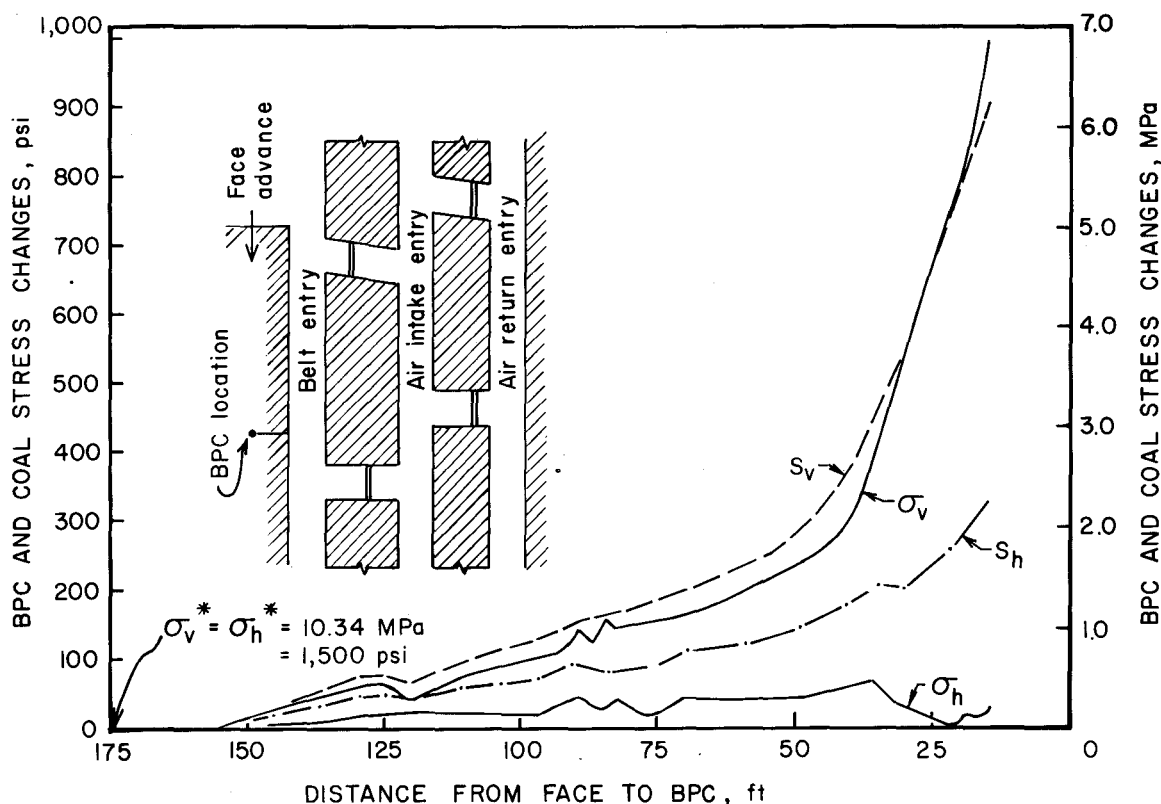


Figure 65.—BPC pressure and coal stress changes at York Canyon Mine with longwall face advance. σ_v and σ_h = vertical and horizontal pressure changes. S_v and S_h = vertical and horizontal pillar stress changes.²

the horizontal stress in the pillar is about one-third the vertical stress, the contributions cancel and the horizontal cell indicates that no stress change has occurred in the horizontal direction.

Results from equations 123 and 124 show this to be untrue. Figure 65 results for ΔS_v and ΔS_h are different by a factor of 1.5 from those given in reference 12 after the coefficient in equation 123 was changed from one-fourth to three-eighths. In figure 65, the vertical cell pressure, σ_v , is very similar to the coal stress change, ΔS_v . However, the horizontal cell pressure, σ_h , is very different than the coal stress change, ΔS_h . This is a partial explanation why since 1957 when the BPC was first developed it has been extensively used to estimate the vertical pillar stress change, but only sometimes used to estimate the horizontal pillar stress change.

ROLE OF END CONSTRAINT ON COMPRESSIVE STRENGTH

For 156 years the geometry of a test specimen in the laboratory and of a mine pillar have been used as a fundamental variable in predicting the strength of mine pillars. In a recent work, Babcock (16) showed that the change in stress with geometry produced by end constraint and not the geometry as such is the independent variable.

Many different failure modes of rock and coal specimens produced by changing the end constraint conditions were observed and photographed in the laboratory. These can be classified as shown in figure 66. The failure modes shown in A, B, and C were for long coal specimens with diameter-to-length ratios of 0.385. The failure modes in D, E, F, G, and H were for short coal specimens with diameter-to-length ratios of 8.5.

When two steel platens were used in contact with the specimen ends, the double cone failure mode of figure 66A occurs. The radial end constraint is inward and the material within the cone has a triaxial state of stress that prevents its failure. When two soft layers of pentatetrafluoroethylene (PTFE) are used on the ends of the specimens to remove the end constraint, the failure is in the form of vertical cracks that produce columnar breaking as in figure 66B. If the load is removed after this failure occurs, the broken pieces will stand in place. If loading continues, long column buckling will occur, and the already broken pieces will then burst. If one steel platen is used on the bottom and a layer of PTFE is used on the top, the bottom will produce a solid cone and the top vertical breaking as in figure 66C.

When a hard slate platen was used on the bottom and a layer of soft clay was used on the top of the short specimen of figure 66D, it did not break at all, even though it was loaded to 21,400 psi when the unconfined compressive strength was much less. The edge of the coal disk in

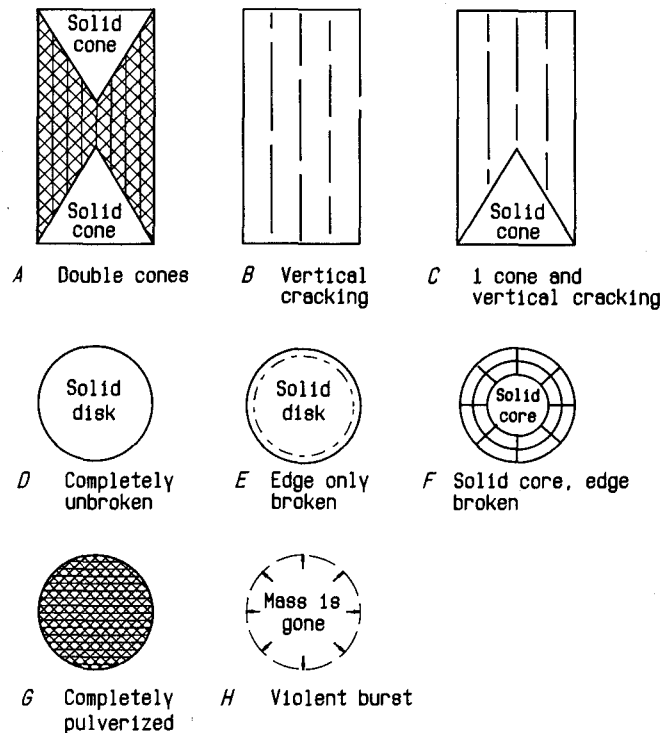


Figure 66.—Failure modes observed in laboratory for rock and coal specimens (16).

figure 66E was broken for a short distance when layers of powdered clay (0.050 in thick) were used on the top and bottom of the specimen. When thin layers of PTFE were used on the ends of the specimen, the breaking of the specimen extended inward toward the center as shown in figure 66F. If the end layers of PTFE were made thicker, the coal specimen was completely pulverized as in figure 66G. If the coal mass was constrained radially inward until the vertical stress was several times the unconfined specimen strength and the end constraint was then dynamically lost, bursting of coal specimen results. This occurred (17) for 13 of 15 different coals tested with a D-L of 8.5 as depicted in figure 66H.

The conclusion to be drawn from the above is that the specimen has an infinitely large number of compressive strengths depending upon the state of stress applied.

The change in the radial stress applied to cylindrical specimens of L-D of 2 by various end conditions was reported by Brady (27). He used the finite-element method of stress analysis to produce the curves shown in figure 67. Here the six curves are taken one each from six of his figures. In figure 67 r , a , σ_m , σ_o , E_p , and ν_i are the radius to any point on the end surface of the cylindrical specimen; the radius of the cylinder; the stress acting on the end surface in the radial direction; the stress acting normal to the end surface produced by the displacement of the end plane parallel to itself; and the Young's modulus and Poisson's

KEY

Insert number	1	2	3	4	5	6
L/D	0.075	0.050	0.025	0.075	0.075	0
E_i , psi	4×10^3	4×10^3	4×10^3	2×10^5	12×10^6	*
ν_i	0.45	0.45	0.45	0.24	0.24	*

* no inserts, specimen is against steel end plates:
 $E = 30 \times 10^6$ psi, $\nu = 0.30$

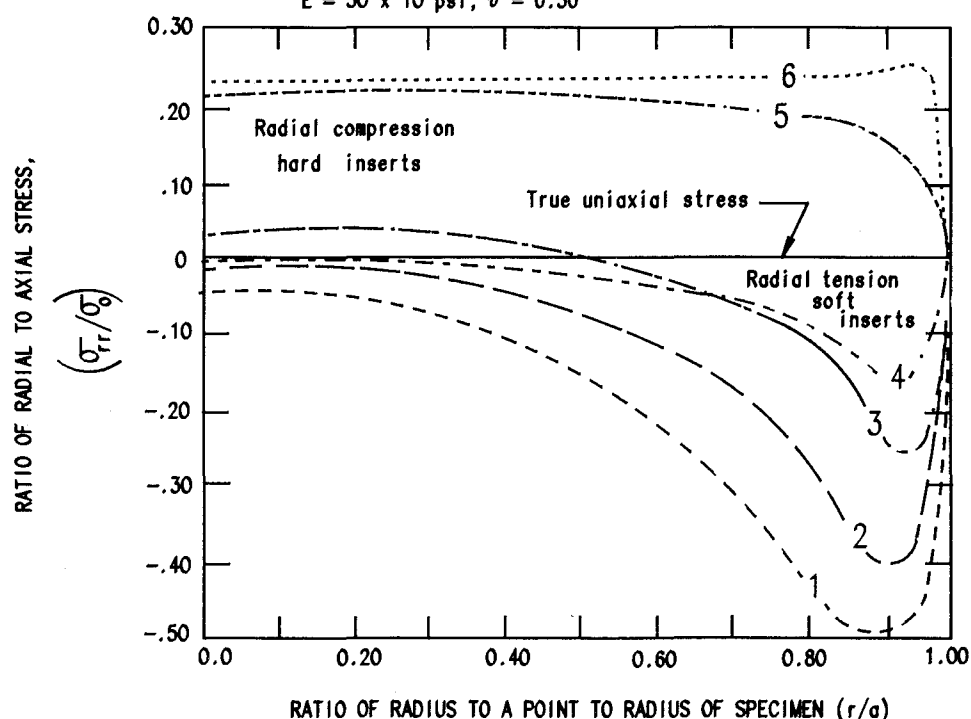


Figure 67.—Changes in radial stresses at ends of test specimens with length-diameter ratio (L:D) of 2, produced by changing end conditions (27).

ratio of the material loading the end surfaces, respectively. Notice that as the end materials (inserts) were changed from 1 through 6, the stress ratio σ_r/σ_a changed from minus (-) to plus (+). Curve 1, for a soft end material, is for a large radial tension near the edge of the disk. Curve 6 for a relatively hard material is for a large nearly uniform radial compression. That is, the end shear stresses act outwards for curve 1 and inward for curve 6 for axial compression of a cylindrical specimen with an L-D of 2. The specimen had a Young's modulus and Poisson's ratio of 2×10^6 and 0.24, respectively. Notice that for some condition between curves 3 and 5, the radial stress disappears. That is, for the correct choice of end material and material thickness, a true uniaxial stress state can be produced in the specimen. The American Standards for Testing Materials (ASTM) standard unconfined compressive test for strength (2) has a radial constraint of the ends that is greater than that of curve 6 because steel with a Young's modulus, E , of $E = 30 \times 10^6$ psi gives greater end

constraint than the curve 6 material where $E = 12 \times 10^6$ psi.

Figure 68 shows four different platen configurations denoted as I, II, III, and IV, used to study the effect of end constraint (16). For platens I, a test disk, B, was loaded by two cylinders of material C. These were loaded in turn by the compression machine platens A. By changing the material C, the effects of a mismatch in physical properties between B and C could be studied. For platens II, a disk material B was loaded by two steel cylinders D of the same diameter to measure the effect of a difference in physical properties alone between the platens and specimen. For platens III, a 3.5-in-diam spherical head was used to load the 2.125-in-diam specimen. These conditions satisfy the ASTM standards that allow the use of a platen up to two times the specimen diameter. Here E and F are hardened steel platens and B is the test material. For platens IV, a 9-in-diam fixed head was used to load a 2.125-in-diameter specimen.

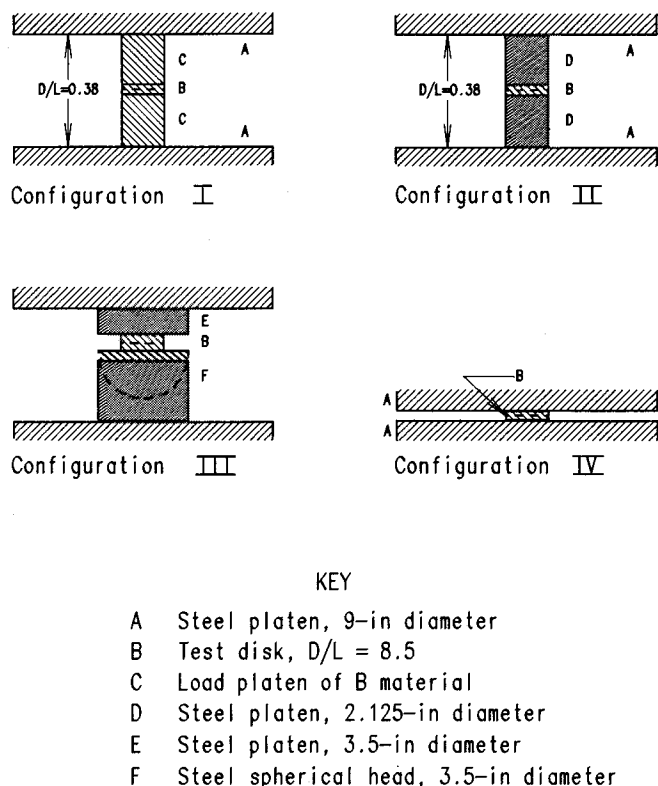


Figure 68.—Four different platen configurations used to study constraint during testing (16).

Figure 69 shows the circumferential strains produced by the axial loading of the specimen with platens I, II, III, and IV. The disk was loaded with two cylinders of the same material for platens I. Here the strains are those produced by the Poisson's ratio expansion because the end constraint is far away and there is no mismatch in physical properties between the three cylinders of the test column. The strains produced when platens II were used are less than for the Poisson's ratio expansion, but greater than for the platens III and IV. Therefore, if steel platens in contact with the test specimen are to be used in determining the compressive strength of specimens loaded with a uniaxial force, they should be of the same diameter as the test specimen to reduce the end constraint. Platens III and IV both provide strong radially inward end constraint which greatly increases the compressive strength, especially for the shorter test specimens.

Figure 70 shows the effect of changing the thickness of a soft layer of PTFE used on each end of the test specimen of limestone. The solid line is the desired uniaxial stress test condition where the end friction has been removed. A layer thickness of about 0.020 in produces the desired effect.

Figure 71 shows the circumferential strains at the ends and at the midlength of the test specimens of limestone, marble, and sandstone when loaded to failure. In 714,

when the specimen is against two steel platens III, the end strains are significantly less than the strains at midlength for the long specimens. In figure 71B, the end strains were made equal to the center strains at failure by monitoring all the test specimens with strain gauges when they were loaded to failure.

Figure 72 shows the average compressive strengths, in pound per square inch, when the specimens were loaded against steel platens III, in A, B, C, and D. These average compressive strengths were changed to A', B', C', and D' when the end friction was removed. Here the letters and their meanings are: S-SS-S (A) for steel platen-sandstone-steel platen; S-M-S (B) for steel platen-marble-steel platen; S-LS-S (C) for steel platen-limestone-steel platen; and S-C-S (D) for steel platen-coal-steel platen. Also, P-SS-P (A') for PTFE layer-sandstone-PTFE layer; P-M-P for PTFE layer-marble-PTFE layer (B'); P-LS-P for PTFE layer-limestone-PTFE layer (C'); and P-C-P for PTFE layer-coal-PTFE layer (D'). Notice that the results for strengths in figure 72 for limestone were nearly independent of the D-L ratio, decreased slightly for marble with an increase of the D-L ratio, and increased slightly for coal with an increase in the D-L ratio. The results for sandstone were independent of the D-L ratio for D-L values of 3 and above, but decreased greatly for smaller D-L ratios. The author believes the sampling procedure was not random, with longer core sections more often used for the long test specimens than for the shorter specimens.

It is concluded that increase in strength of test specimens in the laboratory with an increasing D-L ratio reported for so many years is actually the result of the state of stress in the specimen and not the geometry (D-L ratio). As the specimen becomes shorter, the effectiveness of the end friction in constraining the test specimen against failure is increased until a geometry is reached where the specimen cannot be broken at all.

OTHER CONCEPTS OF INTEREST IN PILLAR DESIGN

Strain Energy in a Mine Pillar and Its Role in Bursting

A more general and useful relationship for strain energy than equation 36 by Phillips cited by Holland (52) can be obtained as follows from Hooke's law. The total strain energy, W , in the cube for x, y, z principal stress space shown in figure 73 is

$$W = (1/2) \epsilon_x \sigma_x + (1/2) \epsilon_y \sigma_y + (1/2) \epsilon_z \sigma_z. \quad (125)$$

Assuming isotropic, homogeneous, linearly elastic behavior, Hooke's law relationships are

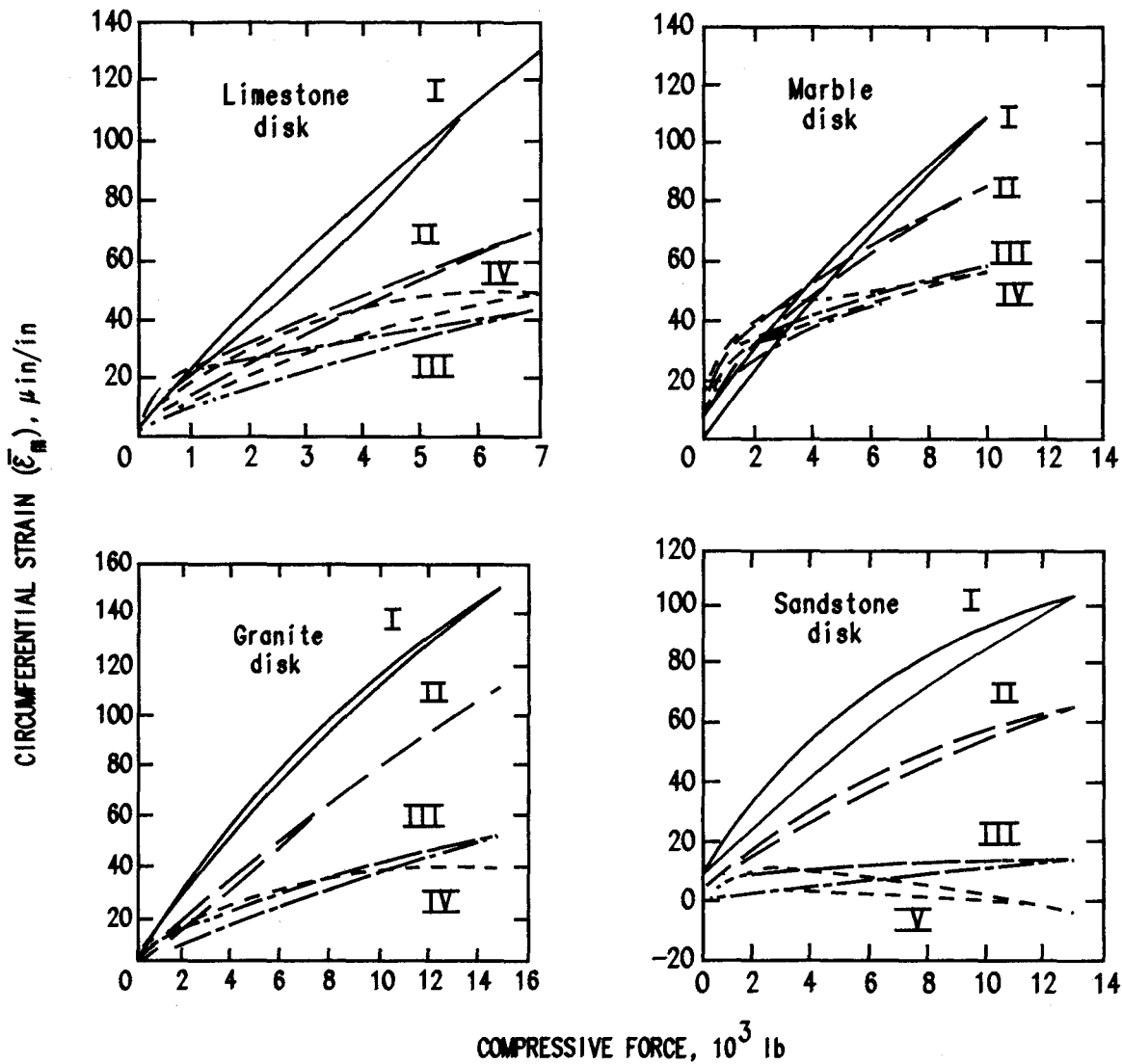


Figure 69.—Circumferential strains of rock disks with diameter-length ratio (D:L) of 8.5. Platen configurations I, II, III, and IV are given in figure 20.3 (16).

$$\epsilon_x = (1/E) (\sigma_x - \nu(\sigma_y + \sigma_z)),$$

$$\epsilon_y = (1/E) (\sigma_y - \nu(\sigma_x + \sigma_z)),$$

$$\epsilon_z = (1/E) (\sigma_z - \nu(\sigma_x + \sigma_y)). \quad (126)$$

ϵ_x , ϵ_y , and ϵ_z are the principal strains for the unit cube, σ_x , σ_y , and σ_z are the principal stresses for the unit cube, E is the Young's modulus for the coal or rock, and ν is the Poisson's ratio. If equations 126 are substituted into equation 125 the result is

$$W = (1/2E) (\sigma_x^2 + \sigma_y^2 + \sigma_z^2 - 2\nu \sigma_x \sigma_y - 2\nu \sigma_x \sigma_z - 2\nu \sigma_y \sigma_z). \quad (127)$$

Consider the three elemental cubes shown with solid lines, (1) at the corner of the pillar, (2) at the center of the pillar, and (3) at the center of the rib denotes as 1, 2, 3, in figure 74.

Case 1. - For cube 1 at the corner of the pillar, two principal stresses in the horizontal directions are both zero. That is, $\sigma_x = 0$, and $\sigma_y = 0$. Equation 127 becomes for cube 1

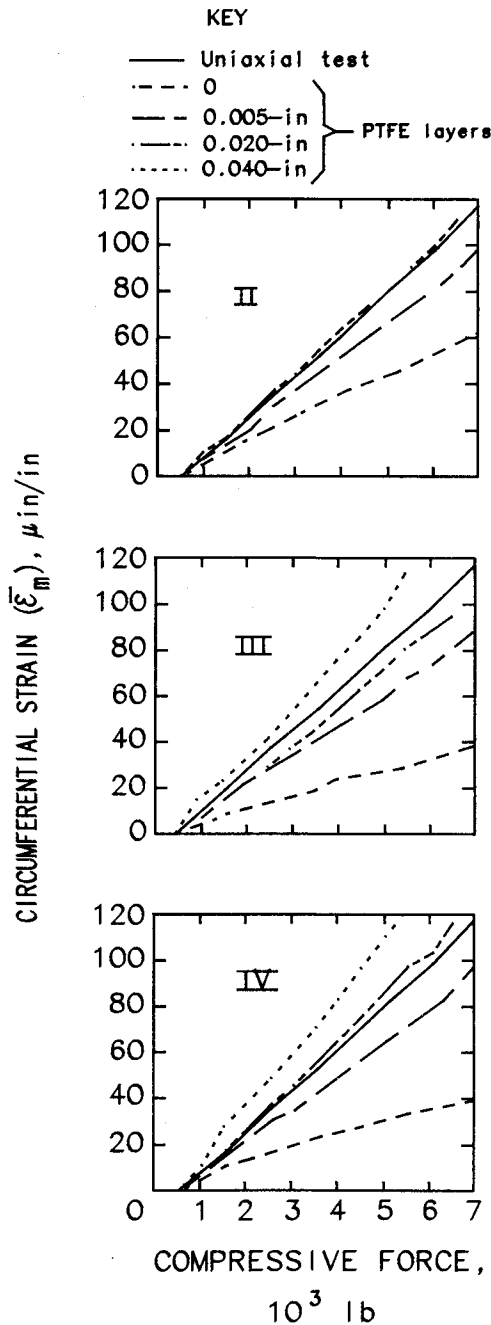


Figure 70.—Effects of adding PTFE layers between specimen and platens on circumferential strains for limestone. Platen configurations II, III, and IV are given in figure 20.3 (16).

$$W_1 = (\sigma_z^2)/2E. \quad (128)$$

This is equal to the word equation given by Holland in "Pillar Design by Holland and Associates."

Case 2. - For cube 2 at the center of the pillar, if the cube is totally constrained by the roof and floor, $\epsilon_x = \epsilon_y = 0$ and $\sigma_x = \sigma_y = \nu/(1 - \nu)$.

Then

$$W_2 = (\sigma_z^2/2E) ((1 + \nu)(1 - 2\nu)/(1 - \nu)). \quad (129)$$

This equation gives the same numerical results as Phillip's equation 36.

Case 3. - For cube 3 at the center of the pillar length at the rib, the stress normal to the rib is zero and the strain in the direction of the pillar length is also zero for perfect constraint in that direction. That is, $\sigma_x = 0$ and $\epsilon_y = 0$. For these conditions, equation 127 becomes

$$W_3 = (\sigma_z^2/2E) (1 - \nu^2). \quad (130)$$

If Poisson's ratio is zero, equations 128, 129, and 130 all give the same results, that the strain energy is everywhere a maximum through the pillar, and bursting is impossible. The Poisson's ratio expansion when restricted by the roof and floor produces the horizontal stress that reduces the strain energy in the pillar below its maximum value for the given vertical stresses applied. Again, the strain energy in a cube at the corner of the pillar is W_1 , the strain energy in the center of the pillar is W_2 , and the strain energy in the cube at the center of the rib is W_3 . If the stress distribution is known, the strain energy could be calculated for any location in the pillar as well with equation 127. This is beyond the scope of the present work.

A picture of how the strain energy is stored in a pillar at three locations of interest as a function of the Poisson's ratio can be obtained as follows. Because the W_1 value for the unconstrained condition at the corner of the pillar is a maximum value, if the remainder of the pillar is in triaxial compression, this value can be taken as the 100 pct strain energy value. The pillar would have this value everywhere if it were totally unconstrained by the roof and floor. Since it is constrained in most cases, the strain energy stored as static energy is less than this maximum value. If the constraint is suddenly lost, the difference in strain energy between the constrained and unconstrained states is suddenly added dynamically, and the author believes that the sudden change in strain energy produces the coal burst if one occurs.

There is a difference in strain energy throughout the pillar only if the Poisson's ratio is greater than zero. The strain energy increases with end constraint and with an increase in the Poisson's ratio. This relationship is shown in table 18. In the top part of the pillar, the percentage of the total strain energies present at locations, W_1 at the corner or the pillar, W_2 at the middle of the rib, and W_3 at the center of the pillar when the pillar is constrained, are given. The additional strain energy, in the percentage of the total, that will be added if the end constraint is suddenly removed is given in the second part of the table 18. For example, for $\nu = 0.35$, only 62 pct of the

Table 18.—Strain energy stored in mine pillar, percent of total

Poisson's ratio (ν)	0	0.10	0.20	0.25	0.30	0.35	0.40	0.45
STORED IN CONSTRAINED PILLAR								
Corner of pillar (W_1)	100	100	100	100	100	100	100	100
Center of pillar (W_2)	100	98	90	83	74	62	47	26
Center of rib (W_3)	100	99	96	94	91	88	84	80
ADDED BY SUDDEN LOSS OF END CONSTRAINT								
Corner of pillar (W_1)	0	0	0	0	0	0	0	0
Center of pillar (W_2)	0	2	10	17	26	38	53	74
Center of rib (W_3)	0	1	4	6	9	12	16	20

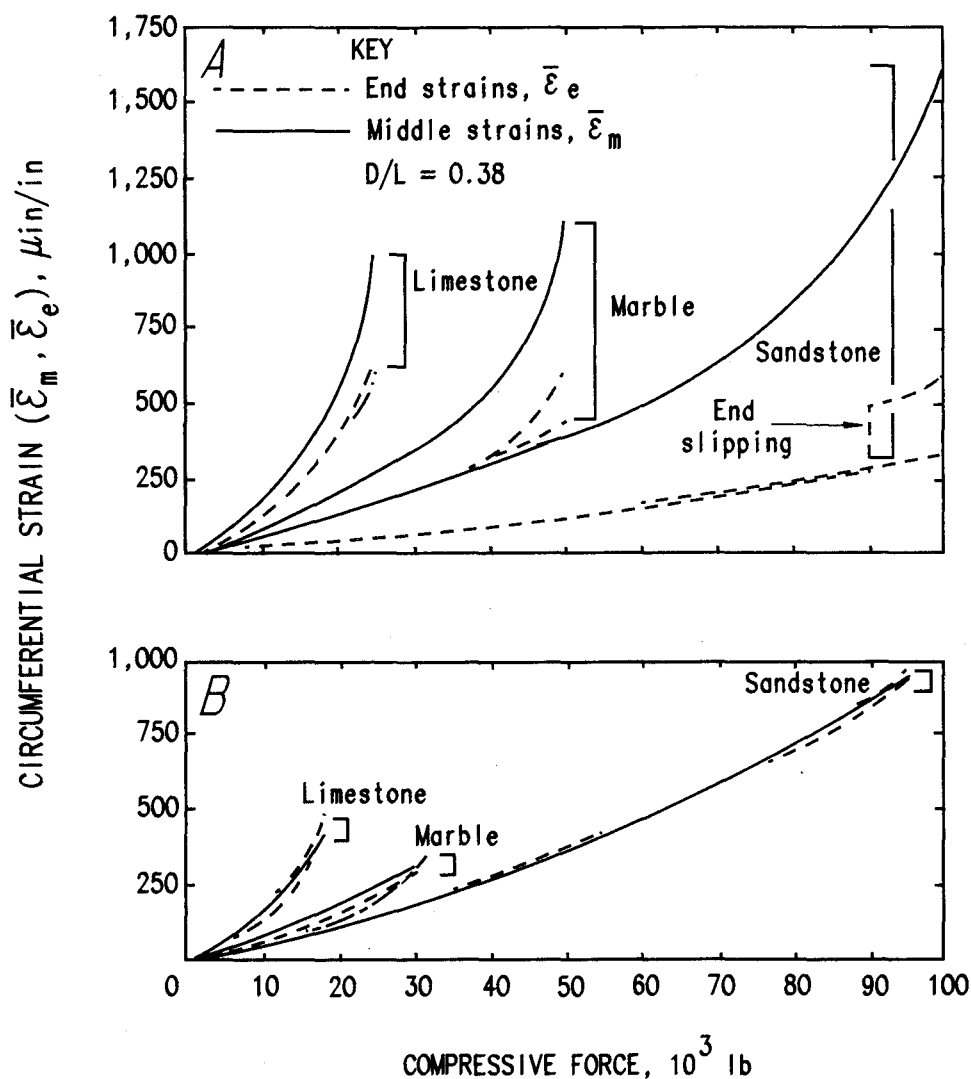


Figure 71.—Reduction in difference in specimen end and midlength strains using PTFE layers between specimen and platens. A, End constraint from steel platens reduces end strain over midlength strains; B, PTFE layers remove end constraint so that end and midlength strains are nearly alike (16).

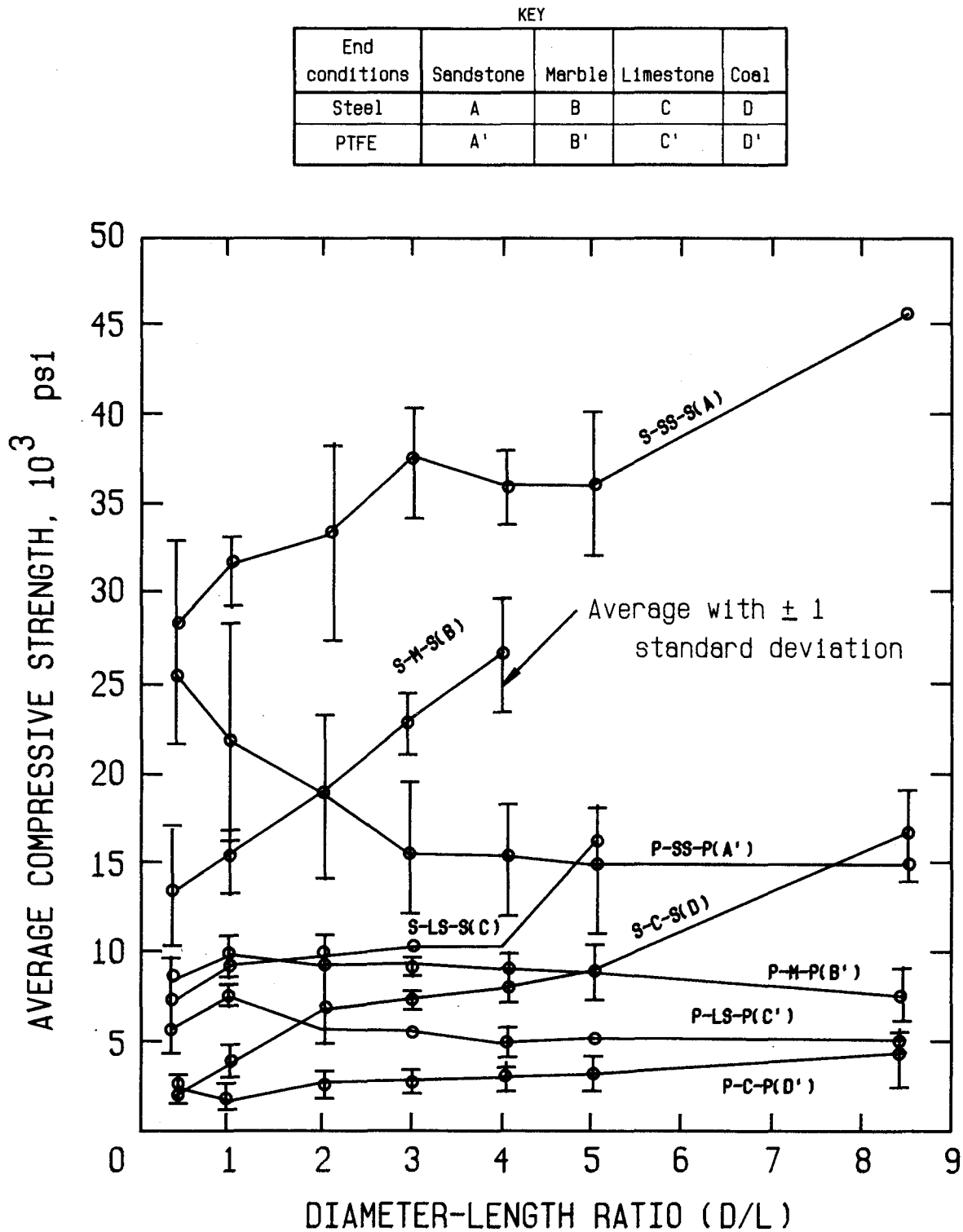
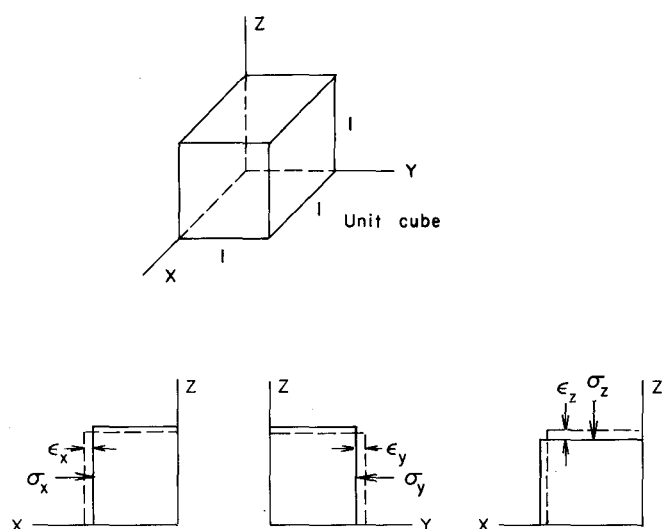


Figure 72.—Change in role of geometry on compressive strength against steel platens to one where end friction is removed with PTFE layers (16).



$$\text{Strain energy (W)} = \frac{\sigma_x \epsilon_x}{2} + \frac{\sigma_y \epsilon_y}{2} + \frac{\sigma_z \epsilon_z}{2}$$

where σ_x , σ_y , σ_z = principal stresses

and ϵ_x , ϵ_y , ϵ_z = principal strains

Figure 73.—Strain energy in unit cube produced by principal stresses and principal strains.

strain energy at the unconfined corner is present at the center of the constrained pillar for the same vertical stress. An additional 38 pct of the total available could be dynamically added if the end constraint was suddenly removed. As the Poisson's ratio increases, the burst potential increases as well. The percentage of the total strain energy possible for bursting would be 53 pct for $\nu = 0.40$ and 74 pct for $\nu = 0.45$. If the cube in the center had tensile stresses on its lateral faces, the strain energy would be a maximum at the pillar center and not at the corner as shown in figure 74. The ratios, R, of the total unconfined strain energy values (100 pct) divided by the static strain energy percentages at the center of the pillar, W_2 , in (ν , R) pairs are (0, 0), (0.10, 1.02), (0.20, 1.11), (0.25, 1.21), (0.30, 1.35), (0.35, 1.61), (0.40, 2.13), and (0.45, 3.85). This means that if, for example, the Poisson's ratio is 0.35, the strain energy of the pillar could be increased by a sudden loss of end constraint by up to 1.61 times. Since the constraint varies from zero at the rib increasing inward, the increase for the entire pillar area would be less than 1.61 times. The unit cubes in decreasing order of strain energy are 1, 3, and 2 if a uniform tributary type vertical loading is assumed. If there is an increase in vertical stress as would be the case from elastic theory near the edge of the pillar, the ordering would be the same. The dynamic strain energy from the central part of the pillar available to produce bursting if the end constraint from the roof and floor are lost would be decreased. The worst conditions for bursting would occur

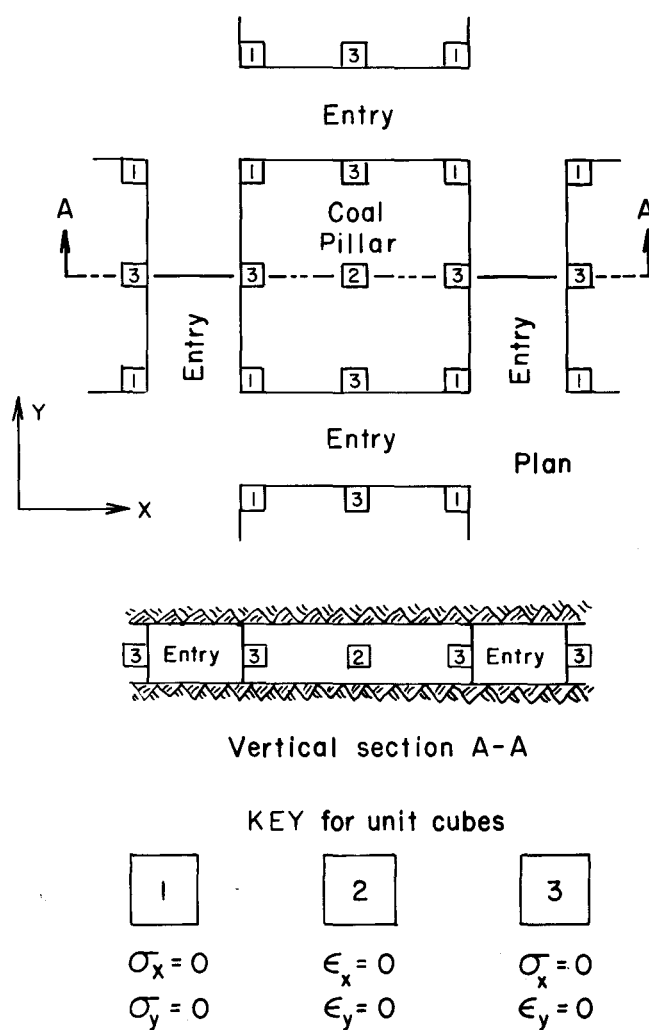


Figure 74.—Locations of unit cubes used to define strain energy behavior of coal mine pillar.

(1) if the pillar loading is uniform, (2) the Poisson's ratio is large, (3) the end constraint from the roof and floor is large, and (4) the end constraint is suddenly lost.

Geometric Method for Prediction of Stresses in Inclusions, Orebodies, and Mining Systems

Babcock (15) presented a method to reduce the complexity of designing room-and-pillar mining systems by treating them as anisotropic soft inclusions in the surrounding ore or host rock. Theoretical models were derived for the calculation of the effective Young's moduli for these systems. The predicted behavior of the plate-type elements used in these models agreed well with test results from 21 plates each with 49 holes spaced on a 7 by 7 Cartesian grid. The plates behaved as anisotropic solids, the effective Young's moduli of which could be predicted from the scaled hole size and spacing. Actual hole shape

was insignificant. The effective Young's moduli for simple mining systems were calculated in sample problems. This approach should provide information about field stress in the mining region and stress distribution in the surrounding ore or host rock when the actual mining configuration is replaced by an anisotropic inclusion in a finite-element analysis. The average inclusion stress rather than the tributary area stress can be applied to the actual mining system of pillars and openings.

In another study, Babcock (13) reported on a method of analysis in which an elastic inclusion, orebody, or mining system is replaced for purposes of analysis with an imaginary effective inclusion of rectangular shape that encloses the region. The theory was used to predict the average stresses in 48 finite-element models of single transversely isotropic inclusions of ovaloid and rectangular shape.

A three-dimensional Cartesian system of rectangular mine openings in a massive orebody produces the anisotropic behavior that results because of the directionality of the openings. If, in addition, as is often the case with orebodies, there are physical property differences in the orebody itself, the problem becomes even more complex. For example, a coal layer with orthotropic properties between a roof and floor rock with different properties would have directional stiffness that results in part from the pillar layout if the pillar dimensions are different in one direction than the other and in part from the coal properties themselves. If the long pillar dimensions are not in the orthotropic material directions, the analysis problem becomes very difficult. Methods such as the structural stability approach reported in chapter 11 would be a practical alternative to numerical or computer analysis in which far too much must be assumed.

Apparent Increase in Mine Pillar Strength Resulting From Decreasing Tributary Area Stress With Decreasing Recovery

One problem in determining pillar strength in situ is, as the pillar size changes, the load applied to it changes with the tributary area supported by it and not as the change in the pillar size by itself.

Large mine pillars are often stable while smaller pillars are not. From this it is often assumed that the large pillars are stronger per unit area than the smaller pillars. This may not be the case. Pillar sizes and mining recoveries are intimately tied together for entry widths of a given size. As the size of the pillars is decreased, the percentage recovery increases and the average stress over the pillars increases as well, as shown by Babcock (9). This increased stress and not the actual strength per unit area may cause this apparent effect, as shown in figure 75. The pillar of width W has an area of W^2 . This area supports the

overburden load from an area $(R + W)^2$. To make a comparison of the relative strengths of the mine pillars to test specimen strengths in laboratory tests, a mine pillar of cubic shape will be used for reference. In a 10-ft-thick coal seam, this would be a pillar with a width, $W = 10$ ft. The apparent strength change for the mine pillar is called S_a in figure 75. This apparent strength change results from recovery alone. By way of contrast, the relative strength of specimens is given by the straight line denoted by the ratio $\sigma/\sigma_{1:1}$. This increase in strength results from the end constraint provided to the specimen by the steel loading platens. Notice that the slopes of the two lines are about equal for pillars larger than 50 ft. From this it might be implied by some that the two relationships are modeling the same thing, pillar strength, when they are not.

Smaller pillars may be stronger on an area basis than larger pillars when loaded by weak roof and floor rock. This is shown by figure 76 from reference 9 in which equal total area of square glass pieces (1, 4, 9, and 16) were loaded in compression with the forces shown between two pieces of plastic. A single model pillar (fig. 76A) with a given area, A , was broken extensively by a force of 65,500 lb. The two arrays of 4 and 9 pillars in figures 76B and 76C were broken only slightly with a force of 75,000 lb and a third array of 16 pillars was not broken at all by a 75,000-lb force. All total areas were equal. That is, $A_1 = A_4 = A_9 = A_{16}$, where the subscript denotes the number of pieces in the array.

The compressive strengths of arrays of pillars are determined by the end constraint provided by the roof and floor as shown in "Role on End Constraint on Compressive Strength."

In terms of tributary area considerations alone, if end constraint is ignored, the most breaking would occur for figure 76D, and the least for 76A. Figures 76E and 76F show that the larger single pillar was broken much more extensively than the nine-pillar array for a larger load of 95,000 lb.

Volume of Roof and Floor Rock That Influences Strength of Coal Mine Pillar

It was shown in an earlier work (13) that the disturbance in the stress field in the material surrounding an inclusion for a condition of infinitesimal strains extends a distance into the material that is equal to the width of the inclusion. The stress anomaly produced by the inclusion in figure 77 extended over the area bounded by the straight lines AB, BD, AC, and CD drawn around the area enclosing the inclusion.

The inclusion behavior was accurately predicted from the behavior of an inclusion of elliptical shape in plane stress and plane strain in work by Babcock (13). The

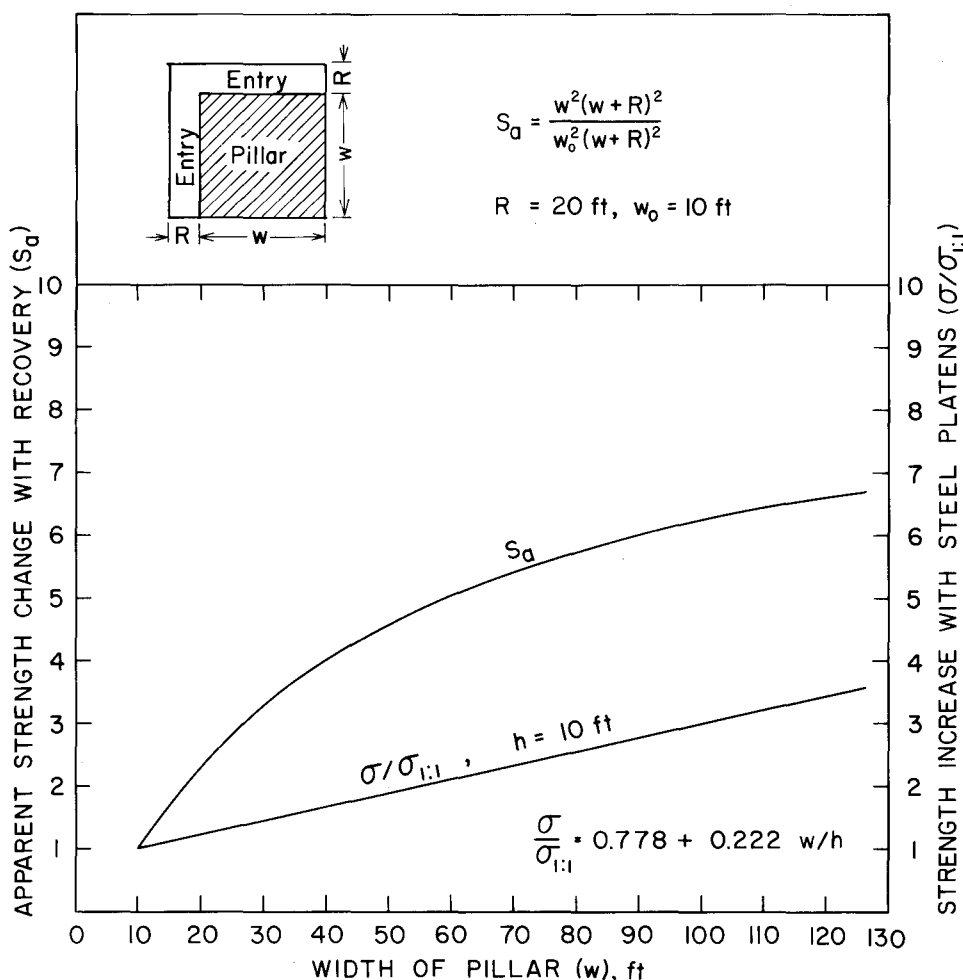


Figure 75.—Apparent increase in pillar strength with increasing pillar size is actually result of decreased recovery and average vertical stress. R = entry width, ft; h = seam height, ft; w_0 = width of cubic pillar, ft.

inclusion had a full range of width-to-height ratios and material stiffness from very soft to very hard. It has also been observed from the experimental behavior of a photoelastic plate loaded normal to a rectangular edge with small displacements that the disturbed zone penetrates into the plate a distance about equal to the length of loaded boundary. Finally, the St. Venant's principle (38) could be taken to indicate that the disturbance is local in nature. It was implied that the disturbed zones in the roof and floor above and below a mine pillar would be related to the minimum pillar dimension. This approach to pillar strength and its dependency upon the strength of the roof and floor rock was presented in reference 9. In the case of square mine pillars loading the roof and floor, the volume of roof and floor rock that provides support to mine pillars increases with pillar size, and the pillar strength is dependent upon the strength or weakness of this rock volume. When it is strong, the pillar is strong, and when it is weak, the pillar is weak. To make the results somewhat more general, assume that the

disturbance is proportional to rather than equal to the least width of the pillar by a factor, K . Babcock (9) considered the four square pillar sizes shown with dimensions W , $W/2$, $W/3$, and $W/4$ in figure 78. To compare the effects of an equal pillar area, assume arrays of 1, 4, 9, and 16 pillars, respectively, with the total load carrying areas of

$$\begin{aligned}
 W^2 &= 4(W/2)^2 = 9(W/3)^2 = 16(W/4)^2 \\
 &= W^2 \quad = W^2 \quad = W^2 \quad (131)
 \end{aligned}$$

The corresponding volumes of roof and floor rock disturbed by the pillars extend above and below the coal seam distances KW , $KW/2$, $KW/3$, and $KW/4$, respectively. That is, these volumes of rock exert some influence on the behavior of the coal pillars. The volumes of rock above the pillars are (area $\times$ height): $KW(W)^2$, $(KW/2)(W/2)^2$, $(KW/3)(W/3)^2$, and $(KW/4)(W/4)^2$, respectively. Because there are 1, 4, 9, and 16 pillars for

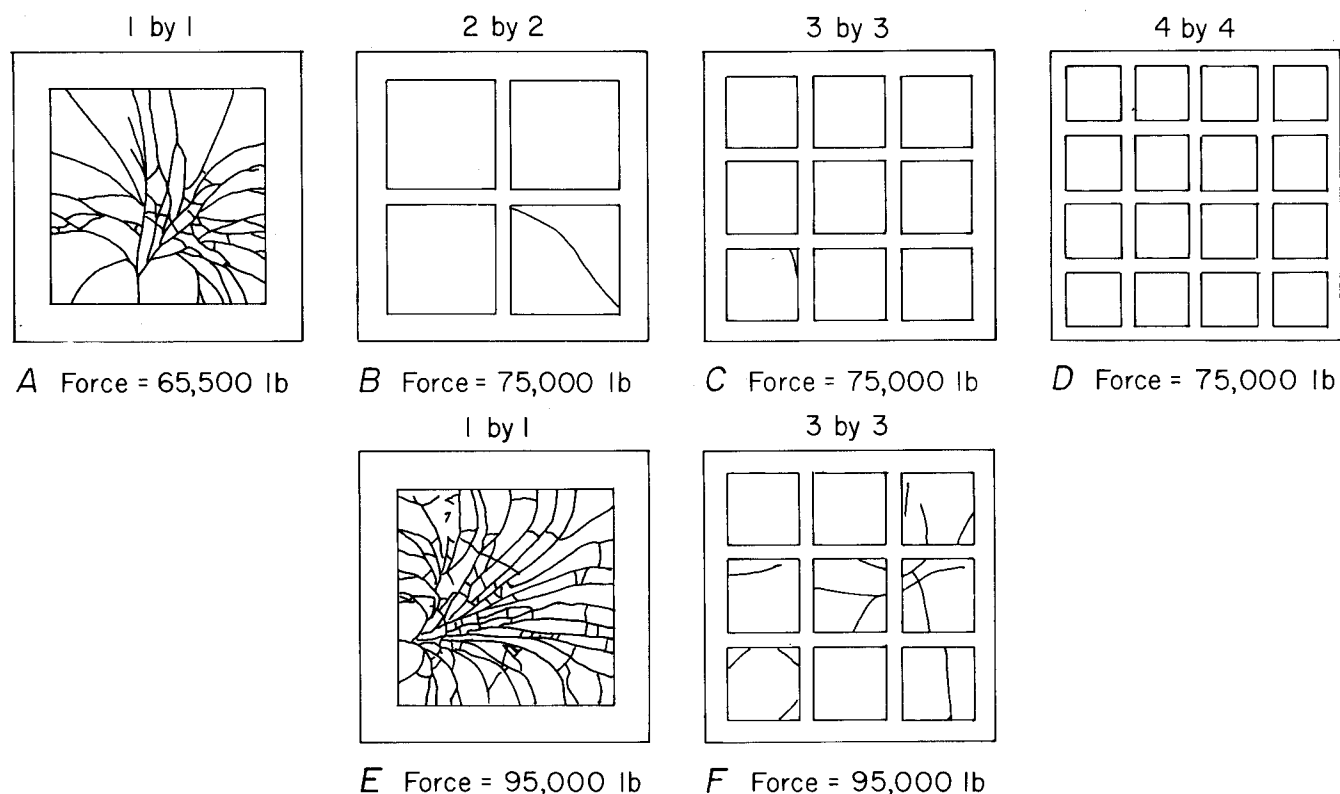


Figure 76.—Model glass pillars A, B, C, D, E, and F of same total area loaded in compression between two pieces of plastic and broken in tension. System of small pillars is stronger than single large pillar when roof and floor are soft and produce tensile stresses. Change in tributary load is not considered (9).

each of the four sizes to give the same total load area, the total volumes above the pillars in the mine roof rock are KW^3 , $KW^3/2$, $KW^3/3$, and $KW^3/4$. A similar volume exists for the floor rock; the total roof-floor volumes exerting an influence on the pillar behavior are double these values. In figure 78, these total volumes are denoted V_1 , V_4 , V_9 , and V_{16} , where the subscript denotes the number of pillars in the array.

The volume of roof and floor rock above and below a mine pillar that determines the strength per unit area of the pillar is proportional to the pillar width. The relative volumes are $1 : 1/2 : 1/3 : 1/4$. Therefore, when the roof and floor rock are strong, the largest pillars are strongest per unit area and the smallest pillars are the weakest per unit area. The converse is true for weak roof and floor rock. In that case, the smaller pillars are strongest per unit area and the largest pillars are the weakest. Strong and weak are relative terms. For the present purposes, a

roof and/or floor are said to be strong if the pillar is constrained horizontally and weak if not so constrained as revealed by horizontal pillar stresses measured with borehole pressure cells (BPC's). That is, if the ratio of horizontal-vertical pillar stress change is positive (+), the roof and floor are strong, and if negative (-), the roof and floor are weak.

If either the roof or floor rock is weak and the other is strong, the one controlling the behavior determines the pillar strength.

In the case of rectangular pillars, the smallest dimension determines the distance that the stress disturbance will extend into the roof and floor. If the same procedure that was just used for square pillars is repeated for rectangular pillars, the ratios of the volumes of roof and floor rock involved in loading the pillars are the same as for the square pillars.

SUMMARIES AND CONCLUSIONS BY SECTIONS

Bauschinger's Theory for Design of Pillars of Any Shape

Bauschinger reported in 1867 (22) on the strength of Swiss sandstone with an equation 2 that anticipates much

of the pillar design work since that time. His emphasis was on the test specimen or mine pillar geometry. He was not aware of the change in strength with test volume and the effects on strength of end constraint. His equation

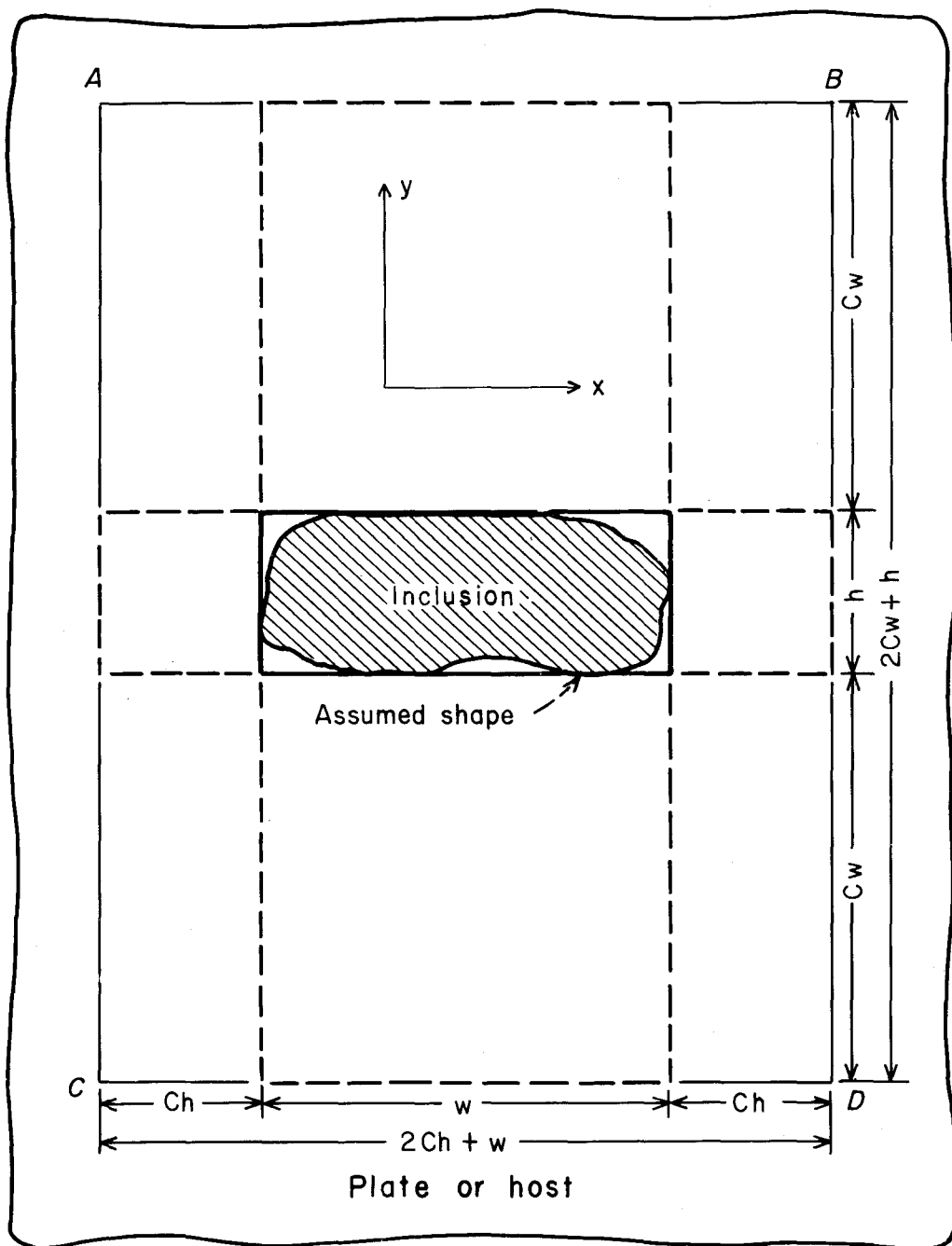


Figure 77.—Zone ABCD contains stress anomaly produced by inclusion when plate was loaded vertically and horizontally on edges. Constant (C) = 1 in experimental tests (13).

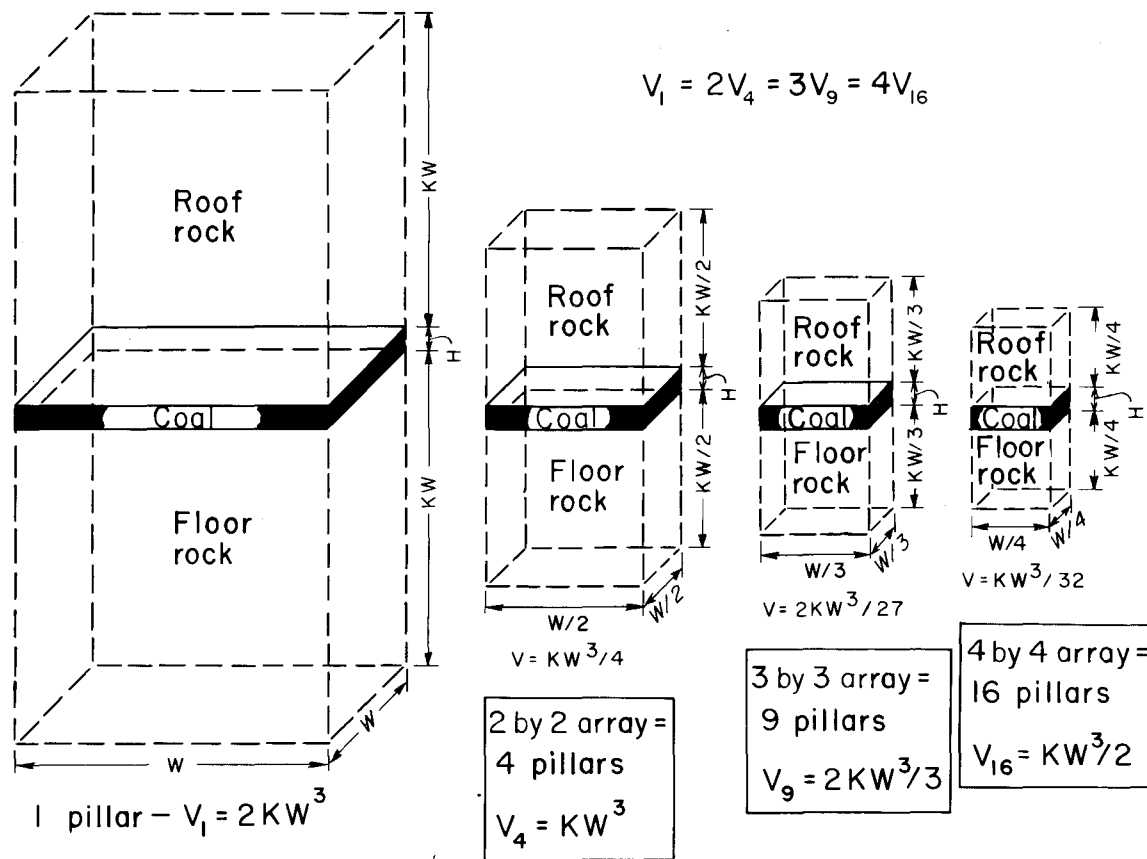


Figure 78.—For given total pillar area, constraining volumes ($V = 1, 4, 9$, and 16) of rock in roof and floor that control pillar behavior decreases with decrease in pillar width. H = seam height; W = pillar width; K = constant = 1 from laboratory testing. Subscript denotes size of pillar array (9).

related the compressive strength per unit area, the loaded area, the length of the pillar perimeter for a pillar of any shape, pillar length, and two physical constants. His equation in this report was treated for a pillar of rectangular shape and the results are shown to be very close to those obtained by using a tributary area method of loading.

Bunting's Tributary Area Method For Pillar Design

Bunting's (29) equation for chamber-and-pillar mining in anthracite coal in Pennsylvania made use of what is now called the tributary area method of pillar design. His equation for chamber-and-pillar mining resembles the equation for tributary area mining, but is incorrect if this was his intention. He observed that the same size pillar was used for mining in anthracite coal regardless of the mining depth and coal seam thickness. This indicated a general unawareness of the need in 1912 of an increase in

pillar size with depth and with seam thickness. He reported on the work of Bauschinger (22), Carpenter (30), Daniels (36), and Johnson.⁹ A major contribution to rock mechanics in Johnson's equation was the concept of normalizing the strength of a rectangular specimen to that of a unit cube for reference. This form of equation, in which the strength is normalized and the geometry is used as a primary variable, has been used by most pillar designers since that time.

Pillar Behavior Concepts Reported By Rice

Rice reported (80) the decrease in compressive strength per unit area with volume when the size of a test cube was increased from 2.5 to 54 in. This decrease was by a factor of 8.2 times. His criteria (77-78) for conditions that lead

⁹Johnson's Materials of Construction. 5th ed., 1919, p. 119.

to coal bursts in the United States, Canada, and Europe and his recommended procedures for controlling or preventing bursts are still in widespread use today.

Pillar Design By Holland and Associates

Holland (52), and Holland and Olsen (55) recognized the role of end friction as confinement for increasing the compressive strength of test specimens and mine pillars with increasing width-to-height ratios. He indicated that a sudden decrease in end friction between the roof and floor could lead to coal pillar bursts. His observations for the conditions necessary for bursting to occur were similar to those of Rice. With a number of associates he presented several pillar design equations, all of which were based on specimen or mine pillar geometry. Holland and Olsen reported that the friction between selected rocks, when either wet or dry, was almost constant. He reported (52) that the unconstrained edges of the pillar contained most of the strain energy per unit volume that caused the pillar to burst. The constrained core of the pillar contained less strain energy per unit volume.

Grobbelaar's Theory For The Strength of Pillars

Grobbelaar (44) presented pillar design equations which he simplified to handle pillars of square, rectangular, strip, and circular shapes. This author makes comparisons of the pillar sizes selected by using these equations. Because of the different tributary loads carried by pillars of different shapes, but the same entry widths, pillars of equal strength, but of different shape should not be used on the same mining level. The mining recoveries produced by the various pillar shapes reveal the economics of their use.

Comparison of the Pillar Sizes Selected With the Equations of Wilson, Wardell, Salamon and Oravecz

Comparisons are made of the pillar sizes selected by the equations of Wilson (87), Wardell (85), and Salamon and Oravecz (81). Tables of pillar sizes are given for seam thicknesses of 6 and 10 ft, for mining depths of 200 to 3,500 ft, and for entry widths of 16, 20, and 24 ft. Wilson's pillars are in general about one-half in area the size of Wardell's pillars. Both works imply that the unconfined shear strength is very small and this value for Wardell is about one-half that for Wilson. This factor alone accounts for the difference in pillar sizes specified. Stated another way, if the Mohr-Coulomb results are plotted with two scales, the units of one being twice those of the other, the same failure conditions would produce two different strengths, one of which is twice the other.

The pillar design equations of Salamon and Oravecz are given and the results presented for selected entry width, seam thickness, and depth. Their work, one of many in which theory relates pillar strength to the geometry variables raised to some power, is given because the theoretical results were compared with the results obtained for 125 mines in South Africa. On the average, the pillar sizes recommended fall within the range of sizes selected by Wilson and Wardell.

Barron's Pillar Design Considerations

Barron's (20) "Integrity Index" for pillar design implies that the soundness of a mine pillar can be determined by the permeability of the pillar mass. This was determined by the rate of decay of the pressure from a standard volume of pressured air introduced into the drill hole between two packers used to define a length of hole to be tested. The ratio of the pillar core area that was relatively nonpermeable to the total pillar area expressed as a percentage he called the "Integrity Index."

Barron (21) provides two computer programs for pillar design. He used several simple serial assumptions that resulted in complex relationships. While the appeal of the programs to computer users may be great, the appropriateness of the results should be compared to the in-mine conditions for validation. Some questions arise because of the pseudo-plastic results presented. The vertical stress distribution is not strongly different when the number of slices is changed from 38 to 540, but the plastic flow behavior is very sensitive to the slice width. The use of average stresses in parallel vertical slices used is a major weakness in the method used.

Lu's Integrity Factor Method of In Situ Pillar Design

Lu (63) used hydraulic pressure cell pressure changes in drill holes to estimate the change in the state of absolute stress in a coal seam. He combined this with an estimate of the triaxial strength of the coal obtained in laboratory testing to produce what he called a "Pillar Integrity Factor." This factor is the ratio of the pillar strength divided by the pillar stress. Both the strength and the stress were obtained for a vertical section parallel to one rib through the pillar by integration over the section. This factor, which changes with mining advance in longwall mining, is essentially a safety factor that changes with mining advance.

Longwall Chain Pillar Design Equation of Hsiung and Peng

The design of chain pillars in a longwall system is different than for pillar design for a room-and-pillar

system. Chain pillars have three stages to their life. First, during the development stage, the mined out area is long in one dimension compared with the other and the overburden may not be fully supported by the pillars, especially for two- and three-entry systems. Second, when the first panel is mined, the pillars are loaded from a long unsupported span to one side. Third, the pillars are loaded by two long spans from both sides after both panels are mined.

Hsiung and Peng (57) give an equation for the design of chain pillars and a nomograph for its solution for the critical pillar size needed to survive the mining of the first panel. This equation is given in different form by this author to emphasize the physical meaning of their work.

Barrier Pillar Design

Ashley (6), Ash (3), Ash and Eaton (4), Ash, Eaton, Hughes, Romisher, and Westfield (5), and Rice (79) reported on mine barrier pillar design. Three equations that have been commonly used are presented by Babcock as nomographs to help select barrier pillar size. There has been confusion in the size of barrier pillars to be used. The Code of Federal Regulations (CFR) Part 75 are mandatory for mining beneath water bodies and the decision as to barrier pillar size must be made, or at least approved of by the Coal Mine Safety District Manager in the district in which the mine is located.

Babcock's Method of Assessing Pillar Stability

A simple in situ pillar stability evaluation method is presented by the author. Often more information than that available is necessary to solve a ground control problem. The result is that much of the problem is assumed and the solution becomes only of academic interest. The object here was to define if a mine pillar is stable from a limited amount of information that is readily available. The information needed is the ratio of the horizontal to the vertical stress change and the angle of internal friction. Nothing need be known about the physical properties, the Young's modulus, or Poisson's ratio, but these properties were assumed to allow the plotting of the results. Only the relative values of stress change were needed for the intended purpose. The stress change ratio is obtained from borehole pressure cells. The angle of internal friction was assumed to be known, but was unnecessary for the data considered because this could be implied from the results obtained. In cases where the coal is not extensively broken, a knowledge of the angle of internal friction would be of use. Its value can be obtained from borehole shear tester results, from the angle of rib slabbing where it occurs, or from shear tests in the

laboratory. The field data of Harmel, Lu, and Barron were used to demonstrate the method.

Measurement of Physical Properties of Rock or Coal in Laboratory or In Situ

In most experimental testing, the changes in shape of a solid body are measured, and these results are converted to the corresponding states of stress. This conversion is made using constitutive relationships that tie the strains to the stresses. The validity of these relationships determines whether or not the estimated stresses are realistic. To carry the process further, the stress states are then used to imply the failure conditions with other relationships. Of particular interest are the relationships relating elastic strains to stresses. By assuming linearity and symmetry, the number of elastic constants is reduced from 36 to 2, the Young's modulus, E , and Poisson's ratio, ν , or the Lamé constants, λ and μ .

In laboratory testing (7), the physical properties, E and ν were not constant, but varied widely when loaded by a uniaxial force to failure. The ν changed by a factor of 10 and the E changed by a factor of 2 for limestone, sandstone, marble, and granite. These results indicate that there is not a "single-valued" equation between strains and stresses for the rock types tested. Therefore, some caution is warranted when using elastic theories in models used to predict failure, for closed-form mathematical, finite-element, boundary-element, or discrete-block models.

Some general considerations are discussed relative to the measurement of physical properties of rock or coal in the laboratory or in situ. This included the development of the borehole deformation gauge by Hast (47), Obert, Merrill, and Morgan (71), and Merrill (66); the measurement of the shear modulus, G , by Panek, Hornsey, and Lappi (74); the use of creep behavior by Lu (63) to measure the physical properties, and the use of the horizontal to vertical stress change by Babcock (12) to determine the in situ Poisson's ratio.

Mohr-Coulomb Stress-Failure Relationships

A relationship widely used for all kinds of materials, both broken and solid, is that between the Mohr stress circle (68) and the Coulomb (33) failure condition. Since both are derived from force equilibrium conditions, they can be applied equally well to materials with a wide range of physical properties. Their use initially was in soil mechanics where the mass could be cohesive or not. These can be applied equally well for rigid bodies where no strain or deformation occurs, except for a displacement along the failure surface. They have also been applied to

deformable bodies of all kinds. If only stress or force is used and deformation or strain is not, this model has much to recommend it. Most of the complexity of the infinitesimal theories come from the conjecture of the stress state from experimentally determined strains.

When the Mohr-Coulomb stress model is used, the solid fails in shear. This shear stress has three components (9). The first component is the result of the unconfined shear strength resulting from cohesion of the mass. The second component of shear strength occurs because a normal stress component that acts across the failure plane is supplied by the vertical uniaxial stress. The third component of shearing strength occurs because of an additional normal stress component that acts across the failure plane that is supplied by the horizontal biaxial or triaxial stress field. Any material can have any strength if the constraint from this component produces a nearly hydrostatic compressive stress state.

Relationship Between Vertical and Horizontal Stresses in an Underground Mine

A geometric construction, using the Mohr-Coulomb stress-failure model, was used to relate the vertical and horizontal stresses through the angle of internal friction. The horizontal stress component needed to prevent failure for a given vertical stress was defined in terms of mining depth and mining recovery. The horizontal constraint necessary was defined in terms of the vertical stress and the unconfined shearing strength. The unconfined shearing strength was 2 to 10 times as effective as the normal stress component of the vertical stress acting across the failure surface in preventing failure in shear. This varied with the angle of internal friction. The depth to which cracks can stand open in a coal seam was defined.

If the coal is strong and connected, no constraint is needed to some specific mining depth. This depth can be inferred from the permeability of the coal versus depth. Such information has been supplied for three coal basins by McKee, Bumb, Way, Koenig, Reverend, and Brandenburg (64), who studied the permeability of coal seams relative to obtaining gas from coal deposits. The permeability was great to depths of about 450 ft, after which it decreased drastically, but at different rates for the three coal basins reported. This indicates that the fractures in coal are open to that depth. The coal can stand with the cracks "open," with free surfaces across which no normal stresses are acting. To these depths, no horizontal constraint is needed. With increasing depths and crack closure increasing, horizontal constraint stresses are needed for pillar survival.

Effect of End Constraint on Compressive Strength of Test Specimens in Laboratory

Rock and coal specimens tested for compressive strength in the laboratory between two steel platens increase in strength with the diameter-to-length ratio. If the angle of internal friction for the specimen is known, the average constraint supplied to the ends of the specimen can be deduced using the Mohr-Coulomb stress-failure model (19).

Past work by the author (19) on the effects of end constraint on the compressive strength of rock or coal specimens in the laboratory was summarized. In one study, solid steel rings provided passive constraint during axial loading about equal to the support supplied by the roof and floor rock (7). In a second study, active radial end constraint of 3,000 or 5,000 psi was supplied before axial loading to simulate the effects of tectonic stresses on pillar strength (10-11). In a third study, eight different end conditions were used with coal specimens to establish the variations of compressive strength that such conditions would produce (8). In a fourth study, it was shown that the state of stress and not the geometry was responsible for the change in compressive strength with the diameter-to-length ratio (16). Given the same state of homogeneous stress, the strengths of selected geometries were made approximately equal.

Interpretation of Pillar Design Equations With Width-To-Height Ratio as Fundamental Variable in Terms of End Constraint

The typical pillar design equation in use for 160 years uses the pillar geometry as the fundamental variable in the selection of pillar size. As appealing as this approach is, and the literature is full of this type equation, it does not define the role of end constraint on pillar strength that is now known to be of paramount importance. An equation of this type was produced to include the effects of end constraint and this can explain the wide variety of pillar behaviors from crushed to bursting pillars by changing the end constraint factor. Because of the widespread appeal of equations of this type, one that provides for bursting and crushing behavior should be of interest.

Bursting of Model Coal Pillars in Laboratory

It was shown that 13 of 15 coals tested with a geometry that is generally thought not to burst could be made to burst violently by the sudden loss of radial end constraint in the laboratory (17). The kinetic energy and the

momentum of the model bursts were measured, and the coals from 11 coal seams from six States were ranked according to burst proneness. The kinetic energy, a scalar, is useful for measuring the general burst proneness of the coal. The momentum, a vector, is useful for measuring the directionality of the burst. If a coal has a directional breaking, the momentum is not zero, and it may be possible to mine in a direction that will reduce the likelihood of bursting. If the vector sum of a burst is zero, the center of the pillar mass before and after the burst is the same. If a burst occurs, the kinetic energy will always have some value greater than zero. It was concluded from this work that most of the coals in the United States can be expected to burst if the depth is great enough (at least 1,000 ft), and this expectation increases with mining depth. The roof and floor supply the end constraint conditions, and bursting can occur if enough of this constraint is dynamically lost for pillars that are overloaded for uniaxial stress conditions.

Another method of relating coal specimen burst behavior to end conditions and to specimen geometry was reported (8). The loss of initial specimen weight from the compression machine platform 30 in square was used as a convenient way to quantify burst behavior. Histograms were presented of percentages of specimen weight thrown from the table versus the frequency of that occurrence, *f*. These showed that when tests were summed over the eight different end conditions, the range of bursting assigned to seven different geometries was from 7.7 to 33.3 pct (a 25.6-pct difference). When summed over the seven different geometries, the range of bursting assigned to eight end conditions was from 7.4 to 53.9 pct (a 46.5-pct difference). For individual specimens, the range of bursting was from 0 to 90 pct. It was concluded that the end constraint produced twice as wide a range of bursting as the change in geometries. This change was the result of the associated stress states.

In the Mohr-Coulomb model of stress-failure, the circle must somehow change its size and location to produce the failure condition. The usual concept of increasing the vertical pillar stress by increasing the mining recovery produces a static failure because of the slow rate of mining. Because a burst is a dynamic behavior, the slowly increasing vertical stress is unlikely to produce bursting. A more realistic model of bursting behavior is obtained by changing the Mohr circle size by the sudden loss of the horizontal constraint stress on the ends of the pillars. This can occur if the frictional stress is lost for any reason, or if bonding of the coal layer to the roof and/or floor is lost (17).

Changes in Roof Above Longwall Face in Coal Mine

Large strain displacements observed by Kidybinski, above a longwall roof in Poland were modeled using the finite-element method of infinitesimal stress analysis by Kidybinski and Babcock (60), along with a progressive failure model. After each of seven computer solutions, the physical properties were changed for those elements for which failure stresses had occurred. Fracture zones in the roof and finite closure between roof and floor observed in Poland were created in the computer model.

Calculation of Mine Pillar Stresses From Borehole Pressure Cell Data

Hydraulic borehole pressure cells (BPC's) of Panek design (73-75) can be used in pairs in a coal mine pillar to measure changes in the vertical and horizontal stresses with mining advance or with time. The cell pressure changes are not the required coal stresses, but the stress changes can be used to obtain the coal stress changes by using equations reported by Babcock (12).

Role of End Constraint on Compressive Strength

Several different failure modes of rock and coal specimens were produced by changing end constraint conditions and specimen geometries. These ranged from bursting to crushing for short specimens (D-L of 8.5). The states of stress and not the geometry as such were responsible for these differences in behavior and strength. Testing with four different platen configurations produced very different results and this is certainly a factor in the strength results obtained. The changes in strain were monitored with strain gauges, and this behavior was used to control the test results. The elimination of the end friction by means of PTFE layers of selected thickness produced strength results that were nearly independent of the specimen test geometry.

Other Concepts of Interest in Pillar Design

In a mine at a specific depth, the larger pillars are stronger per unit area than smaller pillars for at least two reasons. First, the larger pillar benefits more from end constraint than the smaller pillars when the roof and floor rock are strong. Secondly, the mining recovery decreases as the pillar size is increased for a given entry width. The stronger pillar may be the result in part of the reduced

tributary area stress rather than a greater strength per unit area. There are similarities between the apparent increase in strength because of decreased mining recovery and the increase which results from end constraint between two steel platens in the laboratory (9).

The volume of roof and floor rock that reacts to the loading of mine pillars is twice the smallest dimension of the pillar cubed (9). If, instead of one large pillar, 2 by 2, 3 by 3, or 4 by 4 arrays of smaller pillars with the same total area as the one large pillar are used, the total volume of roof and floor rock is decreased proportionally to the

pillar width. That is, the relative volumes are 1, 1/2, 1/3, and 1/4, respectively, for the 1, 4, 9, and 16 pillar arrays. The result is that large pillars are stronger per unit area when the roof and floor rocks are strong and weaker when these rocks are weak. Because of the difference in recoveries and average tributary loads, this is not the only factor in pillar size selection. Pillar strengths can be controlled for several different roof and floor rock conditions by the proper selection of pillar size, small pillars being used for weak roof and floor conditions and large pillars being used for strong roof and floor conditions.

REFERENCES

1. Aggson, J. R. Test Procedures for Nonlinearly Elastic Stress-Relief Overcores. BuMines RI 8251, 1977, 9 pp.
2. American Society for Testing and Materials. Standard Test Method for Unconfined Compressive Strength of Intact Rock Cores Specimens. D2938-86 in 1987 Annual Book of ASTM Standards: Volume 04.08, Soil and Rock; Building Stones; Geotextiles. Philadelphia, PA, 1987, pp. 482-484.
3. Ash, S. H. Barrier Pillars in Wyoming Basin, Northern Field, Anthracite Region of Pennsylvania. BuMines B 538, 1954, 251 pp.
4. Ash, S. H., and W. L. Eaton. Barrier Pillars in the Anthracite Region of Pennsylvania. Trans. Soc. Min. Eng. AIME, v. 177, 1948, pp. 62-81.
5. Ash, S. H., W. L. Eaton, K. Hughes, W. M. Romischer, and J. Westfield. Water Pools in Pennsylvania Anthracite Mines. BuMines TP 727, 1949, 78 pp.
6. Ashley, G. H. Barrier Pillar Legislation. Trans. Soc. Min. Eng. AIME, v. 88, 1930, pp. 76-97.
7. Babcock, C. O. Changes in Breaking Strength of Model Rock Pillars Resulting From End Constraint. BuMines RI 7092, 1968, 19 pp.
8. _____. Compressive Strength, Burst Proneness, and Failure Modes of Model Coal Pillars Related to the End Constraint and Geometry. Paper in Rock Mechanics: Proceedings of the 28th U.S. Symposium, Univ. of AL, June 1987, pp. 173-180.
9. _____. Constraint Is the Prime Variable in Pillar Strength. Paper in Proceedings of the Fourth Conference on Ground Control, WV Univ., July 1985, fig. 3, p. 108.
10. _____. Effect of End Constraint on the Compressive Strength of Model Rock Pillars. Trans. Soc. Min. Eng. AIME, v. 244, 1969, pp. 357-363.
11. _____. Effect of Increasing End Constraint on the Compressive Strength of Model Rock Pillars. BuMines RI 7298, 1969, 18 pp.
12. _____. Equations for the Analysis of Borehole Pressure Cell Data. Paper in Rock Mechanics: Key to Energy Production, 27th Symposium on Rock Mechanics, Univ. of AL, 1986, ch. 36, pp. 233-240.
13. _____. A Geometric Method for the Prediction of Stresses in Inclusions, Orebodies, and Mining Systems. BuMines RI 7838, 1974, 35 pp.
14. _____. A New Method of Analysis To Obtain Exact Solutions for Stresses and Strains in Circular Inclusions. BuMines RI 7967, 1974, 40 pp.
15. _____. Plates With Rows of Holes Considered as Anisotropic Soft-Inclusion Models of Three-Dimensional Room-and Pillar Mining Systems. BuMines RI 7436, 1970, 28 pp.
16. _____. True Uniaxial Compressive Strengths of Rock or Coal Specimens Are Independent of Diameter-to-Length Ratios. BuMines RI 9316, 1990, 31 pp.
17. Babcock, C. O., and D. Bickel. Constraint - The Missing Variable in the Coal Burst Problem. Paper in Proceedings 25th Symposium on Rock Mechanics, Northwestern Univ., Evanston, IL, July 1984. Port City Press, Baltimore, MD, 1984, (for AIME), pp. 639-647.
18. Babcock, C. O., and V. E. Hooker. Results of Research To Develop Guidelines for Mining Near Surface and Underground Bodies of Water. BuMines IC 8741, 1977, 17 pp.
19. Babcock, C. O., T. Morgan, and K. Haramy. Review of Pillar Design Equations Including the Effects of Constraint. Paper in Proceedings First Annual Conference on Ground Control in Mining, WV Univ., Morgantown, WV, 1981, pp. 23-34.
20. Barron, K. An Air Injection Technique for Investigating the Integrity of Pillars and Ribs in Coal Mines. Int. J. of Rock Mech. Min. Sci. & Geomech. Abstr., v. 15, 1978, pp. 69-76.
21. _____. An Analytical Approach to the Design of Pillars in Coal. Final Report, 2 parts, D.S.S. File 18 Sq 23440-0-9106, D.S.S. Contract Serial 1SQ80-00161, CANMET Project 334-101, Can. Cent. for Miner. and Energy Technol., Energy, Mines and Resour., Ottawa, Canada, 1983, 400 pp.
22. Bauschinger, J. Mitteilungen aus dein Mechanisch-Technischen Laboratorium der K. Technischen Hochschule in Muchen (Communications of the Mechanical-Technical Laboratory of the Technical College in Munich), v. 6, 1867.
23. Becker, R. M., and V. E. Hooker. Some Anisotropic Considerations in Rock Stress Determinations. BuMines RI 6965, 1967, 23 pp.
24. Belin, J. General Report on Studies and Research on Coal and Gas Outbursts, Paper in Symposium on Coal and Gas Outbursts, Part I, Presented at Nimes, France, 1964, UN Econ. Comm. for Europe, UN Publ., Sales No. 66, I/E/Mim. 38, pp. 42-43.
25. Bickel, D. L. Overcoring Equipment and Techniques Used in Rock Stress Determination (An Update of IC 8618). BuMines IC 9013, 1985, 27 pp.
26. Bieniawski, Z. T. Deformation Behavior of Fractured Rock Under Multiaxial Compression. CSIR Rep. MEG 708, Sept. 1968.
27. Brady, B. T. Effects of Inserts on the Elastic Behavior of Cylindrical Materials Loaded Between Rough End-Plates. Int. J. Rock Mech. Min. Sci. & Geomech. Abstr., v. 8, 1971, pp. 357-369.
28. Brady, B. T., and W. Blake. An Elastic Solution of the Laterally Constrained Circular Cylinder Under Uniaxial Loading. Paper in Tenth U. S. Symposium on Rock Mech., Univ. of Texas (Austin), 1968, ed. by K. E. Gray, pp. 199-214.
29. Bunting, D. Chamber-Pillars in Deep Anthracite-Mines. Trans. Soc. Min. Eng. AIME, v. 42, 1912, pp. 236-245.
30. Carpenter, R. C. Sibley J. Eng., v. 16, No. 3, Dec. 1901, p. 105.
31. U.S. Code of Federal Regulations. Title 30—Mineral Resources; July 1988.

32. Conner, E. Mine-Caves Under the City of Scranton. Paper in Trans. Soc. Min. Eng. AIME, v. 42, 1911, pp. 246-263.
33. Coulomb, C. A. Essai sur une Application des Regles des Maximis et Minimis a Quelques Problemes de Statique Relatifs a l'Architecture (Tests for the Application of Maxima and Minima Rules and Statistical Problems Relative to Architecture). Mem. Acad. R. Pres. Divers Savants (Paris), v. 7, 1773.
34. Culmann, C. Die Graphische Statik (The Graphical Statics). 1st ed., Zurich, 1866, p. 226. Publisher unknown.
35. Dally, J. W., and W. F. Riley. Experimental Stress Analysis. McGraw-Hill, 1965, pp. 236-238 and 274-278.
36. Daniels, J. Eng. Min. J., v. 84, No. 6, Aug. 10, 1907, p. 263.
37. Descour, J. M., R. J. Miller, and R. D. Munson. Coal Mine Bump Monitoring. BuMines contract J0245009, Colorado School of Mines, Earth Mechanics Inst., Golden, CO, Dec. 1986, 71 pp.
38. Durelli, A. J., E. A. Phillips, and C. H. Tsao. Introduction to the Theoretical and Experimental Analysis of Stress and Strain. McGraw-Hill, 1958, p. 102.
39. Durelli, A. J., and W. F. Riley. Introduction to Photo Mechanics. Prentice-Hall, 1965, pp. 184-185.
40. Fitzpatrick, J. Biaxial Device for Determining the Modulus of Elasticity of Stress-Relief Cores. BuMines RI 6128, 1962, 13 pp.
41. Gaertner, A. Mining as a Cause of Outbursts. Glückauf, Jan.-Feb. 1931, pp. 149-181.
42. Gonnerman, H. Effect of Size and Shape of Test Specimens on Compressive Strength of Concrete. Struct. Mater. Res. Lab. Publ., Bull. 16, Lewis Inst., Chicago, Oct. 1925, 18 pp.
43. Goodman, R. E., T. K. Van, and F. W. Heuze. The Measurement of Rock Deformability in Bore Holes. Paper in Proceedings Tenth Symposium on Rock Mechanics, Univ. of Texas, Austin, 1968, pp. 523-556.
44. Grobbelaar, C. A Theory for the Strength of Pillars. Pillarco, Pretoria, So. Afr., June 1970, 117 pp.
45. Handy, R. L., J. M. Pitt, L. E. Engle, and D. E. Klockow. Rock Borehole Shear Test. Paper in Proceedings 17th U. S. Symposium on Rock Mechanics, Snowbird, UT, Aug. 25-27, 1976, pp. 4B6-1-4B6-11.
46. Harnam, K. Borehole Shear Tester: Equipment and Technique. BuMines IC 8867, 1981, 19 pp.
47. Hast, N. The Measurement of Rock Pressure in Mines. Sveriges Geologiska Undersokning, Arsbok, Serie C: Avhandlingar och lippsatser. 560, Stockholm, 1958, 183 pp.
48. Hetenyi, M. Handbook of Experimental Stress Analysis, Wiley, 1950, p. 394.
49. Holland, C. T. Design of Pillars for Overburden Support, Part I. Min. Congr. J., Mar. 1962, v. 48, No. 3, pp. 24-28, 32.
50. _____. Design of Pillars for Overburden Support, Part II. Min. Congr. J., Apr. 1962, v. 48, No. 4, pp. 66-71.
51. _____. Physical Properties of Coal and Associated Rock as Related to Causes of Bumps in Coal Mines. Trans. Soc. Min. Eng. AIME, v. 149, 1941, pp. 75-93.
52. _____. Rock Bursts or Bumps in Coal Mines. Colliery Eng. (London), Apr. 1955, pp. 145-153.
53. _____. The Strength of Coal in Mine-Pillars. Paper in Proceedings of the 6th Symposium on Rock Mechanics, Univ. of MO-Rolla, 1964, pp. 450-466.
54. Holland, C. T., and F. L. Gaddy. Some Aspects of Permanent Support of Overburden on Coal Beds. Paper in Proc. WV Coal Mining Inst., 1956.
55. Holland, C. T., and D. A. Olsen. Interfacial Friction Moisture and Coal Pillar Strength. Trans. Soc. Min. Eng. AIME, v. 241, 1968, pp. 323-328.
56. Holland, C. T., and E. Thomas. Coal-Mines Bumps: Some Aspects of Occurrence, Cause, and Control, BuMines B 535, 1954, 37 pp.
57. Hsiung, S. M., and S. S. Peng. Chain Pillar Design for U.S. Longwall Panels. Min. Sci. & Tech., v. 2, 1985, pp. 279-305.
58. Hustrulid, W. A. An Analysis of the Goodman Jack. Paper in Proceedings of 17th U.S. Symposium on Rock Mechanics, Univ. of Utah, Salt Lake City, UT, 1976, pp. 4B10-1 to 4B10-8.
59. Jarlier, M. Instantaneous Outbursts of Gases and Solids. Rev. Ind. Miner., 1931, p. 17.
60. Kidybinski, A., and C. Babcock. Stress Distribution and Rock Fracture Zones in the Roof of Longwall Face in a Coal Mine. Rock Mech., v. 5, No. 1, May 1973, pp. 1-19.
61. Ko, H., and K. H. Gerstle. Project Co-Directors. BuMines Grant G0110894, Univ. of CO, Denver, CO, Report for third year of five year plan, 1973, 186 pp.
62. Lu, P. In Situ Determination of Deformation Modulus and Poisson's Ratio of a Rock Mass with Hydraulic Borehole Pressure Cells. Paper in Rock Mechanics: Proceedings of the 28th U. S. Symposium. Univ. of AZ, Tucson, AZ, 1987, pp. 273-281.
63. _____. Triaxial-Load Measurement for Mine Pillar Stability Evaluations. Paper in Rock Mechanics: Key to Energy Production, Proc. 27th U.S. Symposium on Rock Mechanics, Univ. of AL, Tuscaloosa, AL, ch. 55, 1986, pp. 379-385.
64. McKee, C. R., A. C. Bumb, S. C. Way, R. A. Koenig, J. M. Reverend, and C. F. Brandenburg. Using Permeability-vs-Depth Conditions To Assess the Potential for Producing Gas from Coal Seams. Q. Rev. of Methane From Coal Seams Technol., Gas Res. Inst., Pub. by the CO Sch. of Mines Press, v. 4, No. 1, 1986, pp. 15-26.
65. Meikle, P. G., and C. T. Holland. The Effect of Friction on the Strength of Model Coal Pillars. Trans. Soc. Min. Eng. AIME, v. 232, 1965, pp. 322-327.
66. Merrill, R. Three-Component Borehole Deformation Gage for Determining the Stress in Rock. BuMines RI 7015, 1967, 38 pp.
67. Merrill, R. H., and J. R. Peterson. Deformation of a Borehole in Rock. BuMines RI 5881, 1961, 32 pp.
68. Mohr, Otto. Welche Unstaende bedingen die Elastizitetsgrenzen und den Bruch lines Materials? Abhandlungen Auf dem Gebiet der Technischen Mechanik, Abhandlung 5 (Which Conditions Serve the Elasticity Boundary Sets and the Fracture Line Materials? Proceedings in the Field of Technical Mechanics. Proceedings 5.) Berlin, 1906, pp. 189-219.
69. Obert, L. Triaxial Method for Determining the Elastic Constants of Stress Relief Cores. BuMines RI 6490, 1964, 22 pp.
70. Obert, L., and W. Duvall. Rock Mechanics and the Design of Structures in Rock. Wiley, 1967, p. 332.
71. Obert, L., R. H. Merrill, and T. A. Morgan. Borehole Deformation Gage for Determining the Stress in Mine Rock. BuMines RI 5978, 1962, 11 pp.
72. Obert, L., S. L. Windes, and W. I. Duvall. Standardized Tests for Determining the Physical Properties of Mine Rock. BuMines RI 3891, 1946, 67 pp.
73. Panek, L. A. Measurement of Rock Pressure With a Hydraulic Cell. Trans. Soc. Min. Eng. AIME, v. 220, 1961, pp. 287-290.
74. Panek, L. A., E. E. Hornsey, and R. L. Lappi. Determination of the Modulus of Rigidity of Rock by Expanding a Cylindrical Pressure Cell in a Drillhole. Paper in Proceedings 6th Symposium on Rock Mechanics, Univ. of MO-Rolla, 1964, pp. 427-449.
75. Panek, L. A., and J. A. Stock. Development of a Rock Stress Monitoring Station Based on the Flat Slot Method of Measuring Existing Rock Stress. BuMines RI 6537, 1964, 61 pp.
76. Rice, G. S. Bumps in Coal Mines-Theories of Causes and Suggested Means of Prevention or of Minimizing Effects. Trans. Soc. Min. Eng. AIME., v. 108, 1935, pp. 11-39.
77. _____. Bumps in Coal Mines of the Cumberland Field, Kentucky and Virginia—Causes and Remedy. BuMines RI 3267, 1935, 36 pp.
78. Rice, G. S., and I. Hartmann. Coal Mining in Europe: A Study of Practices in Different Coal Formations and Under Various Economic and Regulatory Conditions Compared With Those in the United States. BuMines B 414, 1939, p. 250.
79. _____. Recommendations of the United States Bureau of Mines on Certain Questions of Safety as of February 3, 1933, by the Mine Safety Board. BuMines IC 6732, 1933, 43 pp.
80. _____. Tests of Strength of Roof Supports Used in Anthracite Mines of Pennsylvania. BuMines B 303, 1929, 44 pp.

81. Salamon, M. D. G., and K. I. Oravecz. Rock Mechanics in Coal Mining. Chamber of Mines of So. Afr., Dec. 1976, 119 pp.

82. Sellers, J. B. The Measurement of Rock Stress Changes Using Hydraulic Borehole Gages. *Int. J. Rock Mech., Min. Sci.*, v. 7, 1970, pp. 423-435.

83. Spackeler, G. Investigations of Rock Bursts, *Ztschr. Berg-Hütt. Salinenwesen*, v. 79, 1931, pp. 195-230.

84. Vicat, L. J. (Researches on Physical Phenomena Which Precede and Accompany Rupture or Deformation of a Certain Class of Solids.) *Ann. Ponts et Chaussees*, Part II, 1833, p. 201.

85. Wardell, K., and Partners (Newcastle, United Kingdom). Guidelines for Mining Near Surface Waters (contract H0252021). BuMines OFR 30-(2)-77, 1977, 50 pp.; NTIS PB 264 729.

86. Weber, G. Additional Contribution to the Question of Earth Tremors and Damage Caused by Mining Operations. *Ztschr. Oberschles. Berg-Hütt. Ver.*, v. 70, Feb. 1931, pp. 60-64.

87. Wilson, A. H., and D. P. Ashwin. Research Into the Determination of Pillar Size, Part I. An Hypothesis Concerning Pillar Stability. *Min. Eng.*, (London), v. 131, pt. 9, June 1972, pp. 409-417.

APPENDIX.—LIST OF SYMBOLS

A	constant in equations 1, 112, dimensionless; area of test specimen, equation 2, in ² ; point on Mohr's circle closest to Coulomb failure line, figure 24; contact area between two rigid bodies held together by normal forces, figure 36, in ² ; curve in figure 72 for sandstone tested with steel end platens.		
ABCD	imaginary boundary of rectangular shape that encloses the stress anomaly in the vicinity of an inclusion of different physical properties from the host material, figure 77.		
A _c	constrained end area of test specimen, figure 47, in ² .		
A'	new location of point A with pillar stress change, figure 24; strength vs diameter-length ratio curve for sandstone with PTFE end conditions, figure 72.		
a	constant in equations 2-5, 8-9, psi; constant in equation 69, psi-ft ^{0.5} ; exponent to base 10, equations 76-80, 83; unconfined shearing strength component, psi, figure 37, 38, and 45; radial displacement of open hole boundary in direction of vertical uniaxial applied stress, figure 63, in; outer radius of cylindrical test specimen, figure 67, in.	b ₁	smallest lateral width of rectangular test specimen, equations 4-6, 19-23, in; width of pillar in chamber-and-pillar mining, figure 5, ft.
a'	term in equation 85.	β	exponent of pillar height term, introduction, equations 1 and 112, dimensionless.
α	exponent of pillar width, equations 1, 112, dimensionless; slope of unstable limit line, figure 54.	B'	curve in figure 72 for marble specimens tested with PTFE end conditions.
A _b	bearing area of BPC on horizontal part of hole boundary, figure 63.	BPC	hydraulic pressure cell, Panek design (74, 76), figures 62-65.
A ₁	area under load curve in figure 17, m × MPa; tributary pillar load area, equation 26.	C	constant in equations 14-17, dimensionless; coefficient in equations 38-39, psi; unconfined shear strength in equations 92, 103, 105, 107, 110, figure 27; denotes coal specimen in figures 46, 56, and 72; platen of same diameter and material as test specimen in figure 68; curve in figure 72 for limestone tested against steel platens; proportionality constant in figure 77.
A _s	area under strength curve in figure 17, m × MPa.	c	component of total shearing strength produced by horizontal stresses acting across shear failure surface in figure 38, psi.
A _u	unconstrained side area of test specimen, figure 47, in ² .	Cd	coal dust layer 0.050 in thick between steel platens and coal specimen, figures 46, 56.
A _v	bearing area of BPC on vertical part of hole boundary, figure 63.	Cl	clay dust layer between steel platens and test specimen, figures 46, 56.
A ₁	pillar area for chamber-and-pillar mining system, equations 26, 27, in ² ; area under load curve, figure 17.	CPC	hydraulic cylindrical pressure cell, Panek design (75), p. 25, figure 18.
A ₂	pillar area for room-and-pillar mining system, equation 33, 34; in ² .	C'	curve in figure 72 for limestone specimens tested with PTFE end conditions.
B	coal mine entry width, equation 73, m; constant in introduction, equations 1 and 112, factor in equation 116, dimensionless; curve in figure 72 for marble specimens tested against steel platens; disk specimen in figure 68.	D	diameter of test specimen of cylindrical shape loaded axially, equations 7, 24, 43, 113-115, 118, 120; diameter of circular pillar, equations 44, 45, ft; mining depth, in equations 67, 70, 90, 91, ft; curve in figure 72 for coal tested against steel platens; denotes platen in configuration II of figure 68 where platen material is of the same diameter but of different physical properties than the test sample.
b	constant in equations 2-5, 8-9, power of 10 in equations 76-83, psi; constant in equation 69, psi; component of total shearing strength in figures 37, 38 that results from uniaxial	D'	curve in figure 72 for coal specimen tested against PTFE surfaces.
		d	depth to which fractures in an unmined coal seam can stand open, equations 108, 111, figure 41, ft.
		d ₁	horizontal distance ith segment of segmented platen was thrown in model coal burst, figure 50, cm.
		dP _i	change in force on opposite vertical sides of a slice of mine pillar used by Barron in figure 14.
		d _e	effect depth to which cleats in a coal seam can stand open in a mine pillar, equation 110, figure 43, hundreds of ft.

d_{mi}	mass of i th element of segmented steel platen, figure 50, gm.	F_n	force normal to plane of sliding between two rigid bodies, equation 102, lbf.
d^*	mining depth, equation 74, hundreds of ft; equivalent depth of coal seam when stress concentration from mining is considered, equation 111, tables 8, 9, ft.	F_s	shearing force needed to displace one rigid body relative to another when held together by a normal force F_n , equation 102, figure 36, lbf.
d_1	mining depth where square pillars are used, equation 56, ft.	ϕ	angle of internal friction, equations 92, 99, 103-110, figures 23-24, 25, 36-43, 45, 57, tables 8-9.
d_2	mining depth where strip pillars are used, equations 56 and 56a, ft.	ϕ'	straight line followed by point A' for proportional loading, figures 23-24.
E	Young's modulus, equations 36, 97, 100, 124, 126-130, psi.	ϕ_b	angle of internal friction from rib slabbing direction, bottom of slab, figure 25, deg.
E_c	Young's modulus of coal, equations 75, 78, 87, psi.	ϕ_t	angle of internal friction from rib slabbing direction, top of slab, figure 25, deg.
E_i	Young's modulus of immediate roof rock, equations 75, 78, 87, psi; i th end condition, tables 15-16 ($i = 1, 8$), figure 46; Young's modulus of insert between test specimen and steel platens, figure 67, psi.	G	shear modulus, equation 100, psi.
E_f	Young's modulus of floor rock, equations 75, 78, 87, psi.	G_j	denotes the j th geometry, tables 15, 16; $j = 1, 5$.
E_m	Young's modulus of main roof rock, equations 75, 78, psi.	H	height of mine pillar or entry height, equations 1, 40-44, 50, 58, 60, 67, 69, 70, 73, figures 13-14, 20, 48, pp. 15-16, 18-19, ft; height below drainage level, or hydrostatic head in equations 88, 89, ft; height of square pillar, figure 47, ft.
ϵ_a	axial strain, p. 40, $\mu\text{in/in}$.	h	pillar heights determined from laboratory specimen tests, pp. 3-10, in; pillar heights, equations 1-2, 4-6, 7-9, 19-23, 28-31, 35, equations 71, 73, m; overburden depth, equation 75, 87, ft; height of test specimen, figure 45; height of model coal pillar burst above collection surface, figure 50; height of inclusion figure 77, cm.
ϵ_e	average circumferential strains at the ends of a test specimen, figure 71, $\mu\text{in/in}$.	I	Lu's integrity factor, pp. 25-26, dimensionless ratio.
ϵ_m	average circumferential strain at mid-length of test specimen, figures 70-71, $\mu\text{in/in}$.	$ I $	Barron's integrity index, figure 12, pp. 21-22, dimensionless ratio.
ϵ_t	tangential strain, p. 40, in/in.	K	coefficient in equation 37, for test specimen strength, $\text{lb/in}^{1.5}$, proportionality constant, figure 78.
$\epsilon_x, \epsilon_y, \epsilon_z$	principal strains for a unit cube, equations 125-126, figure 73, $\mu\text{in/in}$.	k	length of a rectangular pillar as a k multiple of the pillar width, w , equations 8-18, 59-61, figures 1-3, pp 4-7.
F	normal force holding two bodies together, figure 36, lb; frictional force producing end constraint, equations 115-116, 120, lb; failure point of contact between Mohr-Coulomb stress circle and failure line in figure 57; spherical head platen in configuration III, figure 68.	$K.E.$	kinetic energy, equation 122, pp. 61-65, $\text{gm}(\text{cm/s})^2$.
f	exponent of the base 10, equation 83; number of specimens bursting with specimen weight loss from press platform, figures 55, 56.	L	length of test specimen, equations 3, 7, 24, 39, 41, 58-59, 113-115, 118, 120; figures 31-33, 47, 55, 67, 71, 72; tables 10-16, pp. 3-4, 9, pp. 13-14, pp. 51-61, in; length of rectangular specimen, pp. 15-18, 61-67, 74-75, in.
F_1	overburden force on pillar in chamber and pillar mining, for length b_1 , equations 25, 27, lb; coefficient of vertical stress term in equation 105, dimensionless; horizontal-vertical earth stress ratio, equation 109, when the unconfined shear strength, $C = 0$, psi.	L_n	natural logarithm, base e , equations 98, 123.
F_2	overburden force on pillar in room-and-pillar mining system, equations 32, 34, lb; coefficient of unconfined shear strength, equation 105, dimensionless; physical property factor, equation 106, and strength factor, figure 40.	L_s	limestone, figure 72.
		L_p	length of longwall mining panel, equation 78, pp. 27-28, ft.
		M	total momentum for model coal pillar burst, equation 121, figure 53, table 17, gm cm/s .

- m Poisson's number, the reciprocal of Poisson's ratio, equation 36; physical property coefficients, equations 96-97, 121-124, psi or GPA units; slope of stress line, table 5; elemental mass of $e a c h$ segment of segmented platen, figure 50, 53, gm; unit of length, meters, figures 9, 12, 17, 58; tables 5, 17.
- μ coefficient of friction, equation 102.
- n exponent of equation 39, dimensionless; n th element of sum, figure 15.
- n' n 'th element of sum, figure 15.
- p compressive strength per unit area for pillar of any shape, equations 2; for pillars of square shape, equation 4; for pillars of rectangular shape, equations 3, 5, 8, 9, 10, 20-22; and figures 1-3; units consistent with constants a and b ; psi.
- P PTFE plastic end conditions, figure 72.
- P_i horizontal force on an imaginary vertical slice of a mine pillar, figure 14, used by Barron; radial constraining pressure supplied by solid steel rings to the ends of test specimens, figure 34, psi.
- P_{rt} tributary area stress concentration ratio for rectangular pillar, equation 12, figures 2-3, dimensionless.
- P_{sq} tributary area stress concentration ratio for square pillar, equation 11, 13, figure 2, pp. 5-7, dimensionless. PTFE plastic used to reduce end friction between steel platens and test specimens, figure 72, p. 74.
- P_w longwall mining panel width, equations 75, 78, 87; pp. 27-28, ft.
- P_1 principal uniaxial stress applied to unit cube, equation 36, psi.
- p_1 average pillar stress from tributary area loading, chamber-and-pillar mining, equation 27, psi.
- $P_{1:1}$ average specimen strength of a unit cube used for reference, equations 5, 7, 10; figures 1, 3, pp. 4-6, psi.
- p_2 average pillar stress from tributary area loading, room-and-pillar mining, equation 34, psi.
- R entry width, equations 25, 27, 67, 70, figures 5, 75, ft; ratio of pillar width-height, equation 72; mining recovery, figures 43 (pct), 110 and 111 (decimal), and table 1 (pct); figure 75; figure 5, ft; entry width, pp. 18-20, ft; radius of filet, figure 33, in; ratio of strain energies in mine pillar at selected locations, figure 74.
- r radius to any point on the end surface of a cylindrical specimen, figure 67, in; correlation coefficient, table 5.
- R_1 entry width with square pillars, equations 11-14, 52-56, 65, figure 8, pp. 16-18, ft.
- R_2 entry width, case 3, figure 8, equations 52-56, 65; table 2; entry width with rectangular pillars, equations 12-17, ft; entry width with circular, rectangular, and strip pillars, table 2, pp. 16-18, ft.
- Re_1 recovery when mining with square pillars, equations 57, 64, pp. 17-18, pct.
- Re_2 recovery when mining with strip pillars, equation 66, pp. 17-18, pct.
- S strength of rectangular test specimen, equations 37-39; figures 46, 54; pp. 13-14, psi; strength of square mine pillar, equation 69, psi; denotes slip of specimen during compressive loading, figure 54; denotes that steel platens were in contact with the ends of test specimens, figures 46, 56, and 72.
- S_a apparent strength change of mine pillar resulting from change in mining recovery with tributary load method, figure 75.
- S_f safety factor, equations 70, 73.
- S_h unknown horizontal pillar stress, both magnitude and units, figures 24, 30, 42, 63, 65; horizontal pillar stress after installation of hydraulic pressure cells, figures 30, 63; horizontal pillar stress, figure 42, psi.
- ΔS_h change in horizontal stress, figures 24, 30, 42, 63, 65, psi.
- S_1 slate layer between steel platens and test specimen, figures 46, 56.
- S_1 average stress concentration on square pillar, equation 54, dimensionless ratio.
- S_2 average stress concentration on strip pillar, equation 54, dimensionless ratio.
- S_s sandstone disk bearing material for end constraint, figures 46, 56.
- S_v unknown vertical pillar stress, both magnitude and units, figures 24, 30, 42, 63, 65; change in vertical pillar stress after installation of hydraulic pressure cells, figure 30.
- ΔS_v change in vertical stress, equations 96, 98, 123, figures 24 and 27, psi.
- σ uniaxial compressive strength of test specimen of circular or square cross-sectional area; equations 1, 7, 24, 112-120; normal component of stress on surface, psi, figure 57; normal stress in the construction of the Mohr's stress circle, figures 23, 29, 57, 62; unconfined compressive strength, table 5.
- σ_a compressive principal stress when only shearing stress is acting on failure surface, equations 1, 7, 24, 112-120; figure 38, psi.
- σ_a^* tensile principal stress when only shearing stress is acting on failure surface, figure 38, psi.
- σ_{a+b} compressive principal stress for Mohr's circle when shearing stress at failure is $\tau = a+b$, figure 38, psi.

σ_{a+b}^*	least compressive stress = 0, when shearing stress at failure is $\tau = a+b$, figure 38, psi.	σ_{nf}	normal stress at failure, figure 23.
σ_{a+b+c}	most compressive principal stress for Mohr's circle when shearing stress at failure is $\tau = a+b+c$, figure 38, psi.	σ_o	axial stress on ends of cylindrical test specimen, figure 67, psi.
σ_{a+b+c}^*	least compressive principal stress for Mohr's circle when shearing stress at failure is $\tau = a+b+c$, figure 38, psi.	σ_{rr}	stress acting on end surface of cylindrical specimen loaded axially, figure 67, psi.
$\Delta\sigma_{BPC}$	change in BPC pressure, figure 64, psi.	σ_{rt}	compressive strength on rectangular pillar, equations 41, 62, 65, psi.
$\Delta\sigma_{BPC}^*$	change in setting pressure of BPC, figure 64, psi or MPa.	σ_{sq}	compressive strength of square pillar, equations 40-52, 62, 63, psi.
σ_{cc}	uniaxial compressive strength of coal, equation 75, psi.	σ_{st}	compressive strength of strip pillar, equations 42, 52, 62, psi.
σ_{cp}	axial stress in circular pillar, equation 62, psi.	σ_u	uniaxial compressive strength of unit cube, equations 40-43, 52, 63, 65, psi.
σ_1	least compressive principal stress, figure 23, psi.	σ_v	absolute vertical earth stress from BPC measurement, Lu method, p. 26; Babcock's method for pillar stability, equations 96, 98; Mohr-Coulomb stress-failure relationships, equations 103-105, 109; ground stress change from borehole gauge readings, equation 123, figure 30; Mohr's circle construction, figures 39, 42; vertical earth stress, pp. 47-50; stress in vertical BPC, tables 8-9.
$\sigma_{1:1}$	compressive strength of unit cube or cylinder with length-diameter = 1, used as a reference strength for other geometries, psi; equations 1, 7, 112-115, 117-120; figure 45.	σ_v^*	vertical BPC setting pressure, equations 96, 98, 123, p. 26, 33, psi.
σ_2	most compressive principal stress, figure 23, psi.	$\Delta\sigma_v$	change in vertical BPC pressure to determine Poisson's ratio in situ, figure 30, psi.
σ_{lf}	least compressive stress at failure, figure 23, psi.	σ_{vh}	vertical BPC pressure change with change in applied horizontal earth stress, figures 62-63, psi.
σ_{2f}	most compressive principal stress at failure, figure 23, psi.	σ_{vv}	vertical BPC pressure change with change in applied vertical ground stress, figures 62-63, psi.
σ_h	horizontal earth stress, pp. 47-50, psi; horizontal pressure in BPC, pp. 33, psi; absolute horizontal ground stress from BPC, CPC measurements by method of Lu, p. 26, psi; tables 8-9, psi.	σ_x	principal stress in x direction, figures 35, 45, equations 125-127, psi.
σ_h^*	setting pressure for horizontal BPC, psi, p. 33, figure 30.	σ_y	principal stress in y direction, figure 35, pp. 76-81, psi.
$\Delta\sigma_h$	incremental change in stress in the horizontal direction, figure 30, psi.	σ_z	principal stress in z direction, figures 35, 45, psi; axial stress, figures 32-34, equations 125-130, psi.
σ_{hh}	change in horizontal BPC pressure, psi, produced by a change in the horizontal rock stress, figure 63.	T	thickness of rectangular test specimen, pp. 13-14, in; thickness of coal seam, figures 20, 21, ft; equation 88-90, ft; PTFE layer, figure 46, 56.
σ_{hv}	change in horizontal BPC pressure, psi, produced by a change in the vertical rock stress, figure 63.	τ_i	shearing stress on ith element, figure 15, psi.
σ_{ij}	normal stress acting between ith and jth cubes in photoelastic-type analysis to obtain horizontal pillar stress, figure 15, psi.	τ_i'	shearing stress on i'th element, figure 15, psi.
σ_{ij}'	normal stress acting between ith' and jth' cubes in photoelastic-type analysis to obtain horizontal pillar stress, figure 15, psi.	τ_n	shearing stress on nth element, figure 15, psi; shearing stress on the failure surface, equations 93, 94, figures 23-24, 35, 39; shearing stress on the horizontal surface in figure 14, psi.
$\sigma_{i+1, j+1}$	horizontal stress in i + 1, J + 1 slice of figure 14.	τ_{nf}	shearing stress at failure, equations 92, 94, figure 23, psi.
σ_n	normal stress acting across any internal surface of an elemental cube, figure 35, psi; normal stress across contact area between two rigid bodies, figure 36, psi.	τ_s	shearing stress between two rigid bodies, equations 101, 102, figure 36, psi.
σ_n'	normal stress axis for stress change from point A, figure 24.	θ	average rib slab angle, equation 99; angle from principal stress direction x to inclined plane, figure 35, deg.
		θ_b	angle between vertical and slabbing surface, at bottom of rib, figure 25, deg.

θ_t	angle between vertical and slabbing surface, at top of rib, figure 25, deg.		
θ_1	angle of tangent line to Mohr stress circle 1 for stable pillar, figure 57, deg.		
θ_2	angle of tangent line to Mohr stress circle 2 for stable pillar, figure 57, deg.	w	figure 73; width of barrier pillar, figures 20-22, equations 88-91, ft; dimension of cube of roof and floor rock that constrains coal pillar, figure 78, ft ³ .
θ_3	angle of tangent line to Mohr stress circle 3 for statically crushing pillar, figure 57, deg.		
θ_4	angle of tangent line to Mohr stress circle 4 for bursting pillar, figure 57, deg.	W_1	smallest width of rectangular model pillar, equations 8-10, figures 1-3, in; width of test specimen, figure 47; width of inclusion in plane strain, figure 77, any length units.
u	length of perimeter of test specimen, equation 2, in; coefficient of friction, equation 102.		
ν	Poisson's ratio, equation 97, 100, table 18, dimensionless.	W_2	width of square pillar, equations 50-60, figure 8, table 2, ft; strain energy in unit cube at corner of mine pillar, figure 74, in-lb.
ν_i	Poisson's ratio of inserts, figure 67, dimensionless.		
v_i	scalar velocity of ith segment of platen, cm/s, figure 53.	W_3	least width of rectangular pillar, figure 8, ft; width of strip pillar, figure 58, table 2, ft; strain energy in unit cube at center of mine pillar, figure 74, in-lb.
$\bar{v}_i$	vector velocity of ith segment of platen, cm/s, figure 53.	W_c	strain energy in unit cube at center of side of pillar, figure 58, in-lb.
V	mine pillar volume, equation 73, m ³	w_o	denotes wet coal used in tests, figure 46, 56.
V_i	volume of roof and floor rock above and below a mine pillar for $i = 1, 4, 9$, and 16 to indicate the number of pillars in each roof and floor array for a given pillar area, ft ³ ; figure 78.	W_p	cubic pillar of 10-ft width used for reference strength in figure 75.
W	width of mine pillar, equation 1; strain energy stored per unit volume, equation 36, 125-127,	x	width of pillar, equations 75-77, 82, 84, 87.
		y	cartesian coordinate axes, figure 35.
		z	cartesian coordinate axes, figure 35; depth of cover over coal seam, figures 5, 7, ft
			cartesian coordinate axes, figure 35; width of tributary area load supported by chamber pillar, figure 5, ft.