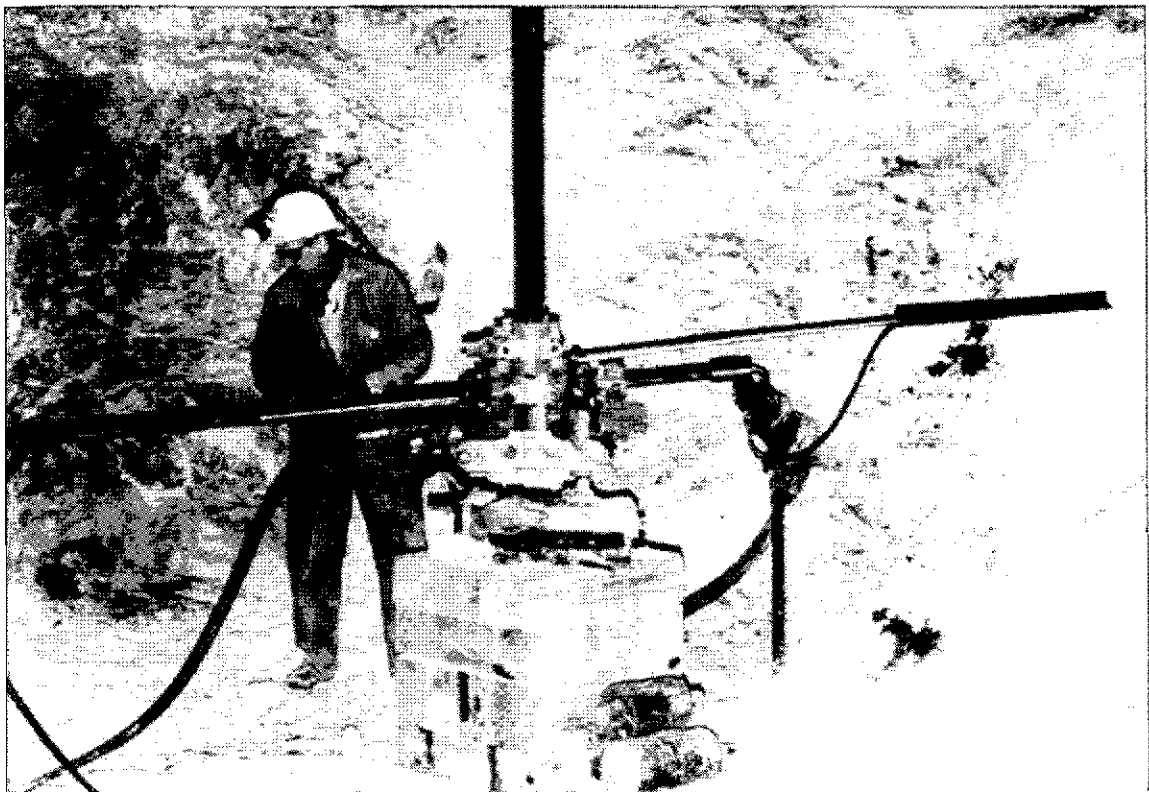


Rock Stress Determinations From Overcoring—An Overview

By David L. Bickel



UNITED STATES DEPARTMENT OF THE INTERIOR
BUREAU OF MINES



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Cover Photograph: Overcoring drill setup for taking measurements in a horizontal hole in a trona mine.

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**With an Appendix on Derivations for the Deformation
of a Hole in a Plate by Wilbur I. Duvall and
Appendixes on Computer Codes by Dennis R.
Dolinar and Meng-Cherng Sun**

**UNITED STATES DEPARTMENT OF THE INTERIOR
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UNIT OF MEASURE ABBREVIATIONS USED IN THIS BULLETIN

deg	degrec	min	minute
ft	foot	μ in	microinch
ft·lbf	foot pound (force)	μ in/°F	microinch per degree Fahrenheit
h	hour	μ in/in	microinch per inch
hp	horsepower	μ in/in/°F	microinch per inch per degree Fahrenheit
in	inch	$\mu\epsilon$	microstrain
in/ft	inch per foot	pct	percent
in/min	inch per minute	psi	pound (force) per square inch
lb	pound	rpi	revolution per inch
lb/ft ³	pound per cubic foot	rpm	revolution per minute
m	meter		

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ROCK STRESS DETERMINATIONS FROM OVERCORING—AN OVERVIEW

By David L. Bickel¹

With an Appendix on Derivations for the Deformation of a Hole in a Plate by Wilbur I. Duvall
and Appendixes on Computer Codes by Dennis R. Dolinar and Meng-Cherng Sun

ABSTRACT

This bulletin describes the U.S. Bureau of Mines three-component borehole deformation gage and the overcoring techniques used in rock stress determination. Thirty years of experience with the borehole gage has shown that it is an accurate instrument for measuring deformation of a borehole. The theoretical basis relating the deformation of a borehole to the in situ field stress is presented. Rock anisotropy, variation of topography on the stress field, and nonlinearity of the rock can be accounted for in calculating the in situ state of stress. Drilling equipment, accessories, and instrumentation necessary for the overcoring are presented, as well as the design, fabrication, operation, and calibration of the gage; site selection; and proper overcoring procedures. Biaxial and triaxial testing of the stress-relief core to determine the elastic properties of the rock are discussed. Data reduction, stress calculations, interpretation of results, and computer codes are included to provide a complete working knowledge for determining surface or underground in situ rock stress. A comparison of alternative methods for determining the absolute in situ field stress is also presented.

¹Physical scientist, Denver Research Center, U.S. Bureau of Mines, Denver, CO.

INTRODUCTION

Knowledge about the natural state of stress (referred to as in situ stress) in a rock mass is an important factor in practical mine design. This information can be very useful in assisting mine planners to anticipate the potential for a host of ground control problems occurring in and around underground openings. Problems include, but are not limited to, rock bursts, pillar spalling, roof buckling, and floor heave. These problems generally occur when the induced stress levels near an excavation exceed the strength of the rock mass. The U.S. Bureau of Mines has developed a technology to accurately measure this state of in situ stress through the use of stress-relief overcoring.

In general, both vertical and horizontal stresses increase with depth of overburden. However, the magnitude and direction of the horizontal stress can be affected by additional factors. These factors may include the nature of the geologic structures, tectonic forces existing within the earth's crust, residual stress (remaining internal stress after the boundary stresses are removed), and thermal stress. Regardless of the origin, overcoring can determine the magnitude and orientation of the stress components existing within the rock mass. In the overcoring method, physical detachment of a body of rock from the rock mass relieves stress in that body. The detached body of rock deforms and this deformation is measurable. Deformation measurements combined with the elastic properties of the host rock are used to calculate the absolute in situ state of stress.

The Bureau's techniques to determine the in situ rock stress were previously reported by Hooker and Bickel (1).² An update of this report was published by Bickel (2).

In addition to the Bureau's overcoring stress-relief method, other in situ stress determination devices and techniques have been developed over several decades. These devices and techniques include the flat jack, the borehole pressure cell, the cell developed by the South African Council for Scientific and Industrial Research (CSIR) or the cell developed by the Australian Commonwealth Scientific and Industrial Research Organization (CSIRO) and rosette strain gage (doorstopper technique), the surface rosette technique, and the hydraulic fracturing technique. A majority of the stress-measuring techniques for rocks use stress relief and are dependent upon the measurement of deformation of a borehole from which

rock stress is computed. Many of these techniques and devices have gained acceptance throughout the mining industry to varying degrees. While the techniques for obtaining the measurements are relatively simple, the interpretation of the data for some of the methods is often controversial.

This bulletin presents a comprehensive summary of the Bureau's research and development conducted in the area of in situ stress determination by the overcoring method, including the vast amount of field stress values obtained in various rock types and locations in the United States.

This research and development was done as part of the Bureau's program to improve mining safety. Chapter 1 covers the theoretical basis for determining the magnitude and orientation of the applied stress in rock as a function of borehole deformation. Anisotropic relationships are also provided. Chapter 2 covers the various Bureau-developed borehole deformation gages (BHG's). Chapter 3 gives a detailed description of the BHG calibration and operation procedures, and the required associated drilling equipment and accessories. Chapter 4 covers the testing procedures for determining the physical properties of the stress-relieved overcores and calculating the anisotropic parameters and elastic constants. Chapter 5 gives instructions for determining the two- and three-dimensional states of stress. Chapter 6 covers alternative methods for rock stress determination. Chapter 7 documents and describes more than three decades of rock stress determinations by the Bureau throughout the United States.

Appendix A covers the derivations for the deformation of a hole in a plate. Appendix B shows diagrams of the three-component (3-C) BHG and essential accessories. Appendix C provides the computer code for plotting the deformation measurements. Appendix D provides the computer code for calculating the least squares ellipse from biaxial test data. Appendix E provides the computer code for determining the isotropic and anisotropic secondary principal stresses. Appendix F provides the computer code for determining the least squares method of secondary principal stresses. Appendix G provides the computer code for determining the three-dimensional state of stress.

BACKGROUND

The concept for measuring the diametral deformation of a borehole to determine the absolute value of stress acting in the rock around the borehole dates back to the

1930's. Mathematical expressions (3) for calculating the applied stress on a plate by measuring the deformation of the hole within the plate (4) were developed. This work would provide the theoretical basis for calculating the stress in rock from the deformation of a borehole.

²Italic numbers in parentheses refer to items in the list of references preceding the appendixes at the end of this bulletin.

In 1933, absolute strain measurements were taken by drilling a ring of overlapping holes around a mechanical strain gage rosette (5). The Bureau obtained a strain-relief measurement in 1938 by cutting a mine pillar free from the roof (6). In 1949, a strain-relief measurement was taken by overdrilling a resistance strain gage rosette mounted on the wall of a tunnel (7). A stress-relief measurement by overcoring a 1-in-diam borehole in which a cell had been placed was reported in 1958 (8). This method records rock pressure using the principle of magnetostriction. The cell used consisted of a nickel alloy spool and winding that were confined by a screen of perm-alloy. The cell is a "rigid" inclusion-type instrument. If pressure applied to the coil is altered when an alternating current is passed through the coil, there will be a change in potential across the coil. By applying known loads to the cell, a calibration curve is produced. This curve is then used to convert the cell readings to the rock pressure.

In response to the problem of measuring and evaluating stress in rock around underground openings, the Bureau initiated research to perfect an instrument and techniques

for measuring the absolute stress from borehole deformation measurements. The Bureau designed a "soft" inclusion gage (9) to avoid the calibration inherent in a rigid-type cell (8, p. 24).

The first gage developed was a single-component BHG (fig. I.1), designed with one cantilever beam, to fit inside an EX-size (1½-in-diam) borehole. The movable piston and fixed stud in the BHG contacts the wall of the borehole. Sufficient bending of the cantilever to provide the contact force for the piston is accomplished by selecting the proper length piston and stud from the various lengths of available pistons and studs. As the borehole deforms during overcoring, the piston actuates the cantilever on which four resistance strain gages are mounted to form a four-arm (Wheatstone) bridge. The bending strain of the cantilever (transducer) is converted by the strain gages into an electrical signal that is read on a conventional strain indicator. The BHG is calibrated so that a change in measured strain corresponds to a known piston displacement. The single-component BHG has a sensitivity of 50 μ in of piston displacement.

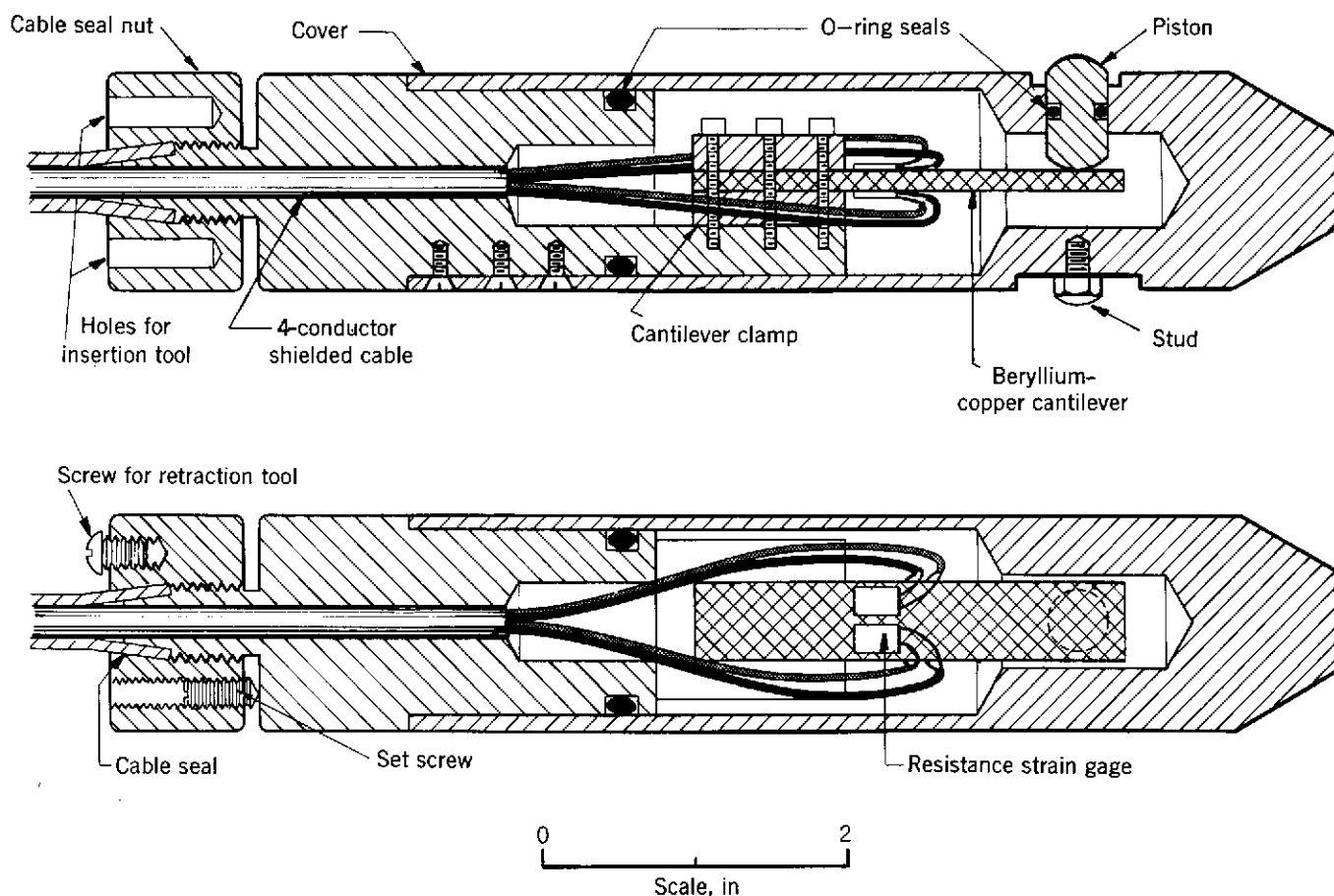


Figure I.1.—Single-component BHG.

To compute the state of stress in a plane normal to the borehole axis, deformation measurements across three different diameters are required. When the single-component BHG is used, three overcoring processes are needed (fig. I.2). Unfortunately, changes in rock properties, joints, and other discontinuities substantially affected the reliability of the measurements. This disadvantage led to the development of the 3-C BHG.

The 3-C BHG (figs. I.3-I.4), designed in the early 1960's (10), has six cantilevers to measure three diametral deformations, 60° apart, in a plane during one overcoring process. A resistance strain gage is bonded to each side of the cantilever. Opposite cantilevers are paired and the four strain gages are connected to form a Wheatstone bridge. Each pair of cantilevers (component) measures deformation across a diameter of the borehole. Strain gages are matched according to resistivity so that the BHG temperature sensitivity is less than $2 \mu\text{in}/^\circ\text{F}$. Sensitivity of the 3-C BHG is $1 \mu\text{in}$ across each diameter.

Variations in the pilot borehole diameter are compensated for by using shim washers to adjust piston length. To prevent wear of the piston from the borehole wall, a tungsten carbide insert is used.

Bolting the transducer to the gage body induces stress in the cantilever, causing excessive amounts of creep or drift (fig. I.5) to occur when the transducer is placed under constant load. (Creep is an indicated change of stress on the strain indicator without any movement in a deflected transducer.) Development of a taper-mount anchoring of the transducer (fig. I.6) significantly reduced the amount of creep. The tapered end of the transducer (fig. I.7) is pulled into a matching socket in the gage body and

secured by tightening a nut on the threads of the transducer. If deformation measurements are not taken for 10 min after the BHG is placed in the borehole, corrections for creep are not necessary (11). When the absolute stress (state of stress) is measured, long-term stability (weeks) for the instrumentation is not needed, but it must be stable during the time of the measurement. For the BHG, the time of measurement is usually 20 to 40 min. The gage, however, is stable under load for at least a month, then a zero shift or set may occur.

The addition of a "safety plug" (fig. I.6) that extends between the ends of the transducers prevents excessive bending, permanently deforming the transducers. The plug limits cantilever deflection to 0.023 in.

When fractured rock is overcored, an ideal 18-in length of core may not be possible. A modified BHG case to fit the standard BHG (11-12) allows for gage placement at any distance inside the pilot borehole, which permits overcoring shorter lengths of core. This gage is termed the "reversed borehole gage."

The process of borehole deformation during stress relief has been studied using linear finite-element methods (11-12). The finite-element model simulates both overcoring at a free surface (e.g., surface of a mine opening) and overcoring at depth in a borehole. From the finite-element results, if the first overcoring starts at a free surface, the complete stress relief is measured. However, if overcoring starts inside a 6-in-diam borehole and the BHG is positioned less than 6 in inside the pilot borehole, compensation must be made to the borehole deformation measurements (11).

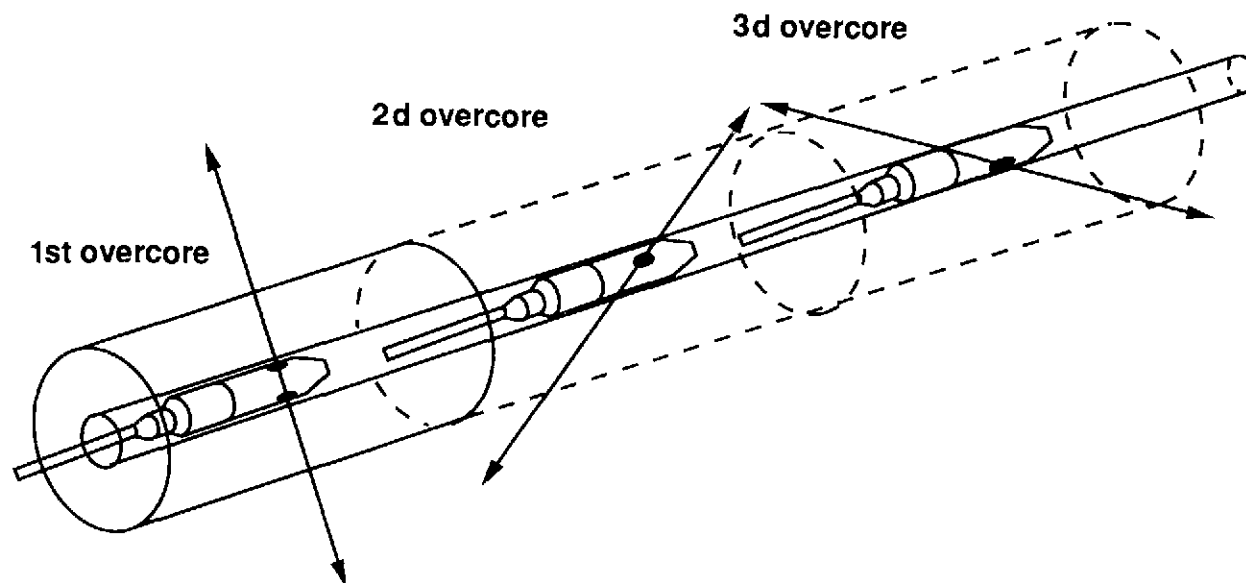


Figure I.2.—Gage position for three overcoring stress reliefs using single-component BHG.

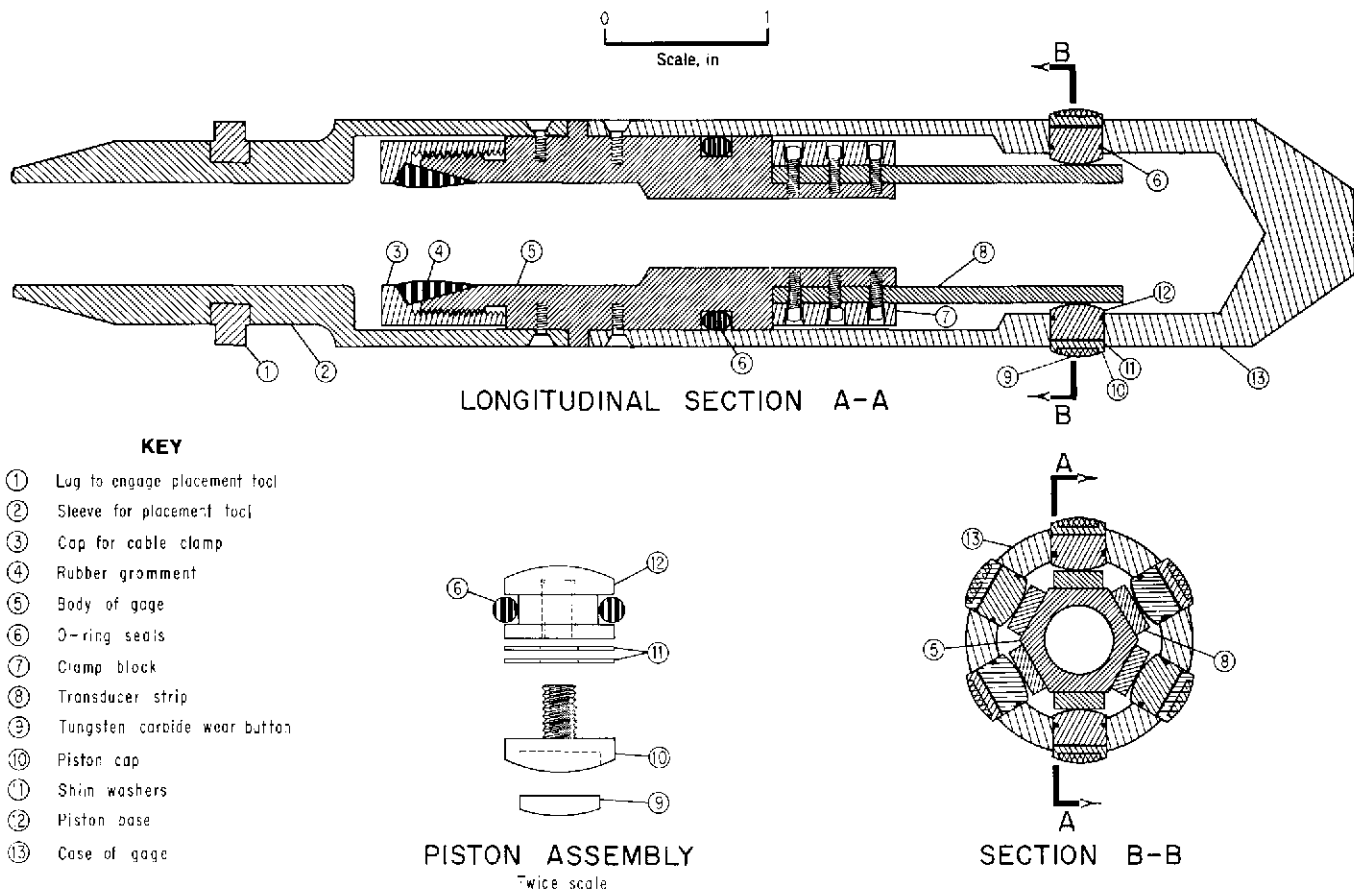


Figure 1.3.—Longitudinal section of early version of 3-C BHG.

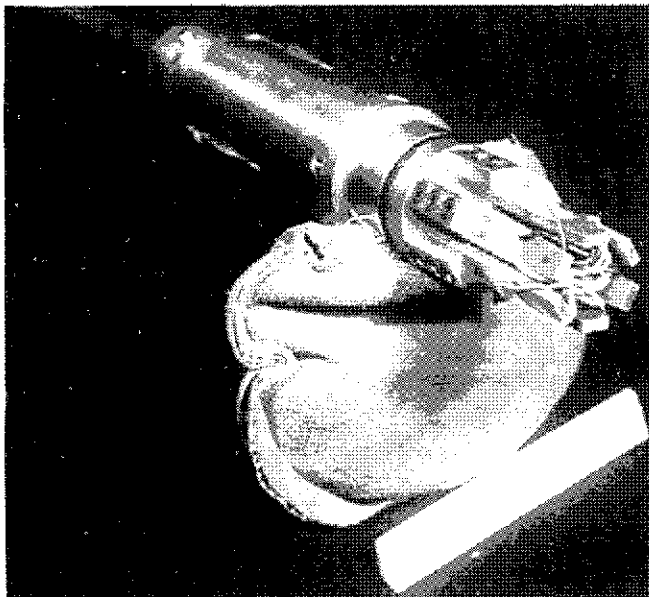


Figure 1.4.—Three-component BHG.

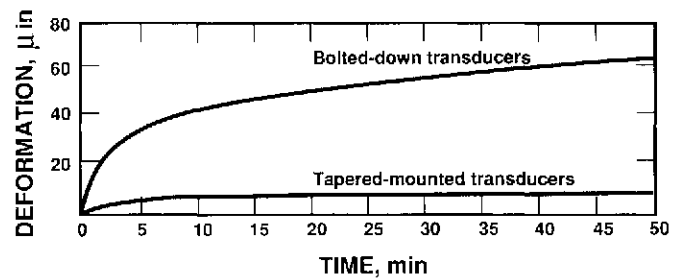


Figure 1.5.—Plot of drift in deformation reading versus time for two different transducer mounts.

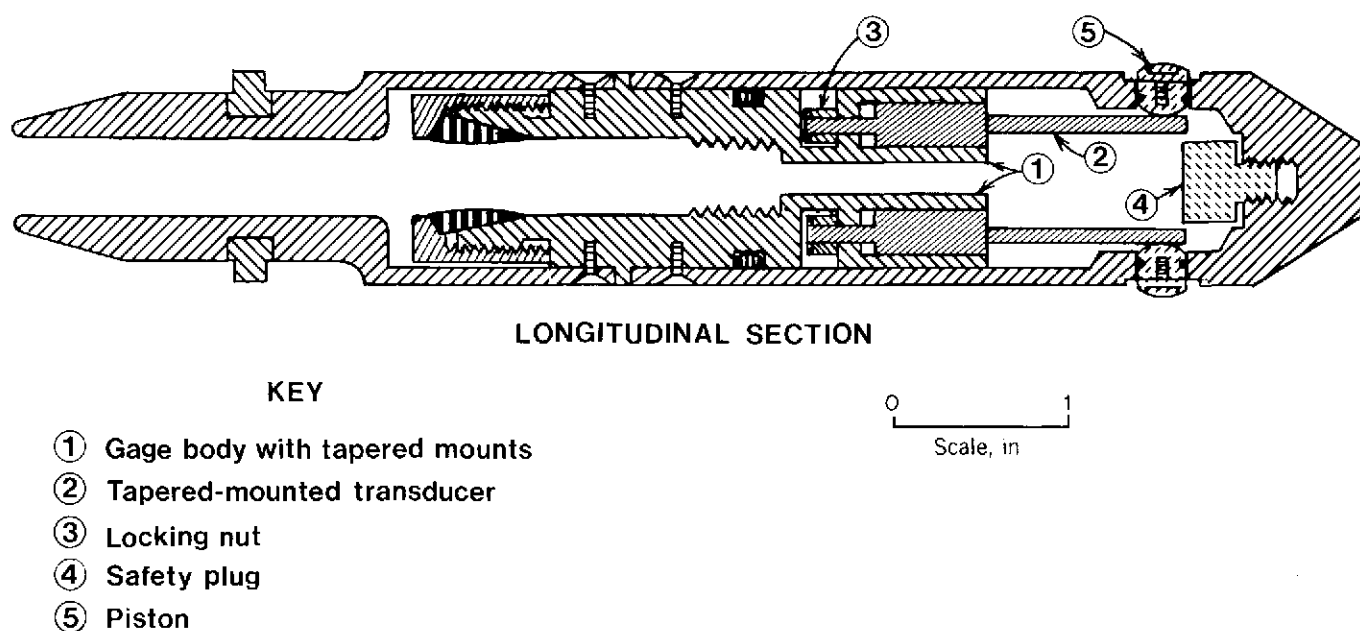


Figure I.6.—Current version of 3-C BHG with tapered-mounted cantilevers.

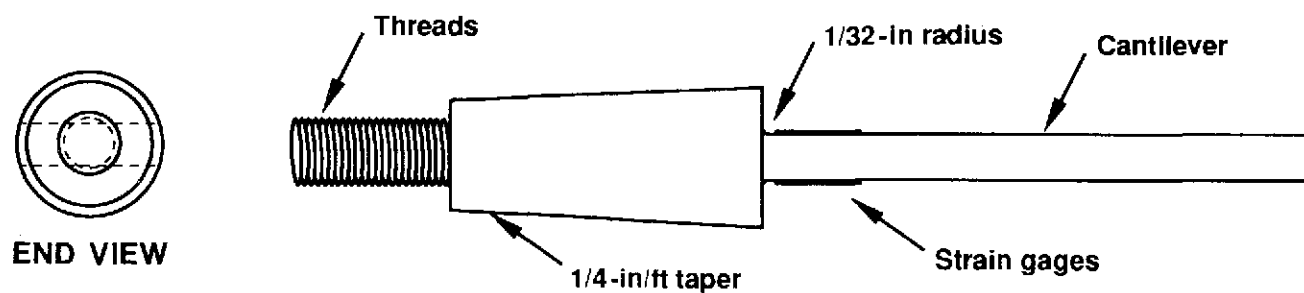


Figure I.7.—Tapered-mounted transducer.

The BHG is used in conjunction with the biaxial pressure chamber (13) (fig. I.8) and the triaxial pressure chamber (14) (fig. I.9) to determine the elastic modulus, the degree of anisotropy in the rock (2, 15), and the elastic constants of the rock (16-17). Fabrication of a two-component (2-C) BHG³ enhances laboratory triaxial testing of the core. Machining the transducers and gage body from a solid piece of metal provided for a creep-free gage.

During the 1970's, additional improvements of the 3-C BHG increased the versatility of the gage. For example, an extended placement end of the gage (11) enhances the retrieval of the gage from a vertical downhole in incompetent rock. A miniature 3-C BHG (12) for use in a 1-in-diam borehole reduces drilling costs and permits drilling

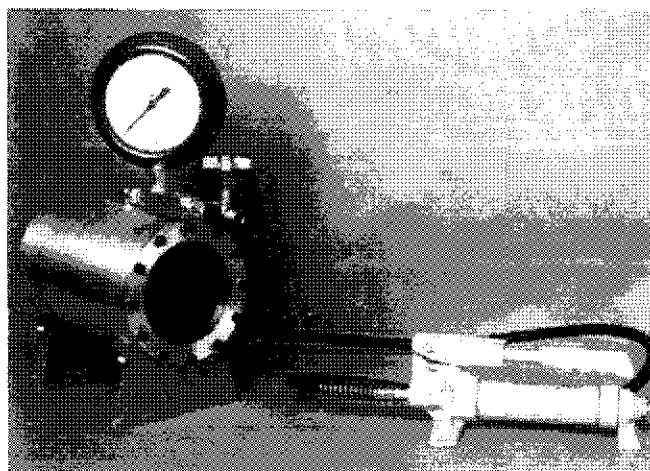


Figure I.8.—Biaxial pressure chamber.

³Information on this gage has not been previously published.

shorter cores in fractured material. However, reduced gage diameter requires smaller cantilevers, which limits the operating range for this gage. Further, the designed and fabricated four-component (4-C) BHG⁴ measures four diametral deformations 45° apart and provides redundancy of data to add confidence to the accuracy of the deformation measurements. Either all four deformation measurements or any combination of three measurements can be used to compute the stresses. Development of a six-component (6-C) BHG⁵ enhances laboratory testing of stress-relieved core by providing six diametral deformation measurements (30° apart) during one cycle. Reducing the weight of the field gage by using titanium instead of stainless steel for the gage body, case, and placement end increases the depth that can be overcored in a vertical uphole. For added strength, the titanium case and placement end are threaded to the gage body.

The mining industry has accepted the BHG as a standard for measuring the absolute state of stress. The gage and related equipment are commercially manufactured and available. Numerous national and international consultant companies provide in situ stress measurement services using the Bureau-developed 3-C BHG. Finally, the overcoring stress-relief technique using the Bureau's 3-C BHG has been included in the American Society for Testing and Materials (ASTM) industry standards annual book (18, pp. 1084-1102).

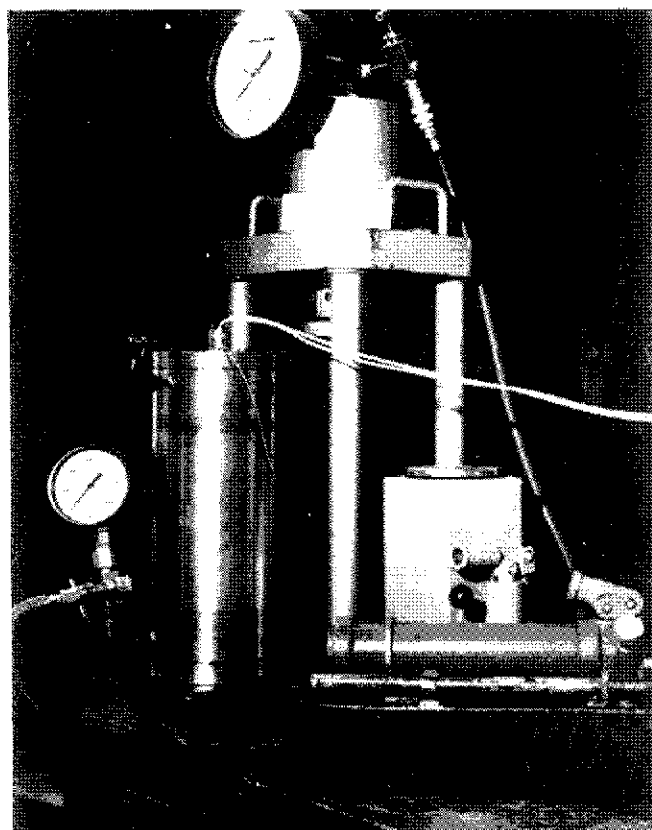


Figure 1.9.—Triaxial pressure chamber, press, and pump.

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The author wishes to acknowledge the contributions made by the following researchers at the Bureau's Denver Research Center, Denver, CO. Louis A. Panek, mining engineer, developed the method to calculate the three-dimensional stress components from borehole deformation measurements and designed hydraulic borehole pressure cells. Robert H. Merrill, physicist, proved that a relationship exists between an elastic isotropic plate and the deformation of a BHG, and designed the single-component BHG and the first 3-C BHG. Wilbur I. Duvall, physicist, derived equations for stress relief by center hole drilling and provided a least squares solution for determining the secondary principal stresses. Leonard Obert, physicist,

developed a triaxial method to determine the elastic constants of stress-relieved core and the relationship between core dishing and the applied stress in rock, and contributed to the design of the BHG. Robert M. Becker, mining engineer, developed the anisotropic equations for the stress-relieved core and applied the anisotropic rock properties in the stress calculations. Verne E. Hooker, geophysicist, provided testing procedures to determine the elastic constraints of the stress-relieved core and improved the overcoring field equipment and the procedures for obtaining the field deformation measurements. James A. Aggson, physicist, developed test procedures for nonlinearly elastic stress-relieved cores and contributed to the least squares solution for determining the secondary principal stresses.

⁴See footnote 3.

⁵See footnote 3.

CHAPTER 1.—DEFORMATION OF BOREHOLE IN ROCK

INTRODUCTION

Theoretical (3-4, 19-20), laboratory (21-22), and numerous field studies (7, 11, 17, 23-46) have shown that by measuring borehole diametral deformation resulting from a change in the applied stress, the magnitude and direction of the applied stress can be calculated once the deformation-applied stress relationship is known. Merrill and Peterson (21) showed that the deformation of a borehole in a rock model subjected to a known applied stress was in good agreement with the theoretical deformation of a hole in an elastic, isotropic plate.

To calculate the in situ stress in two dimensions, three diametral deformations are needed from a single drill hole (borehole). The calculations provide the secondary principal stresses⁶ in the plane perpendicular to the borehole. If diametral deformations are taken at various depths in the borehole, a stress profile can be obtained. A comparison of these results can disclose what effects physical property changes, joints, or other discontinuities have on the stress field and also how the stresses vary around mine structures and openings. Often there is a need to combine several sets of borehole deformation measurements (U_1 , U_2 , and U_3) to obtain the average secondary principal stresses P and Q for the average depth measured in the borehole. This is best accomplished by a least squares solution (47). The least squares analysis provides a means of determining the multiple correlation coefficient (R), the variance, and the scatter for a set of deformation measurements. From this information, confidence limits can be calculated for the average P and Q .

To calculate the magnitude and direction of the stresses in three dimensions (stress ellipsoid), at least two diametrical deformations are needed from each of three nonparallel (preferably orthogonal) boreholes. Procedures have been developed (9) for calculating the least squares estimate of the three-dimensional stress components and their standard errors (S_E).

Existing methods for determining rock stresses from borehole deformation measurements (8) have been generally based on isotropic relationships. Since rock is anisotropic in varying degrees, large errors may occur in evaluating the state of stress if the effects of rock anisotropy are ignored (16). The determination of rock stress from borehole deformation measurements in anisotropic rock has been derived from the plane elasticity displacement solution for a circular hole oriented along one elastic axis of an orthotropic medium (48).

⁶The maximum secondary principal stress (P) is the largest compressive or smallest tensile stress and the minimum secondary principal stress (Q) is the smallest compressive or largest tensile stress acting in a plane normal to the axis of the borehole.

THEORY

The derivations for the deformation of a hole in an isotropic plate (for plane stress and plane strain conditions) subjected to uniaxial and biaxial stress fields are presented in appendix A, part A. The more important equations for the biaxial cases are as follows (21).

The deformation of the hole in a biaxial stress field⁷ and in plane stress is given by

$$U = \frac{d}{E} [(S + T) + 2(S - T) \cos 2\theta], \quad (1.1)$$

where U = deformation of hole (change in length of diameter),

d = diameter of hole,

S, T = applied stresses perpendicular to axis of borehole,

θ = angle measured positive in counterclockwise direction from S to r (radial distance) (fig. 1.1),

and E = modulus of elasticity.

⁷Compressive stresses are positive and tensile stresses are negative.

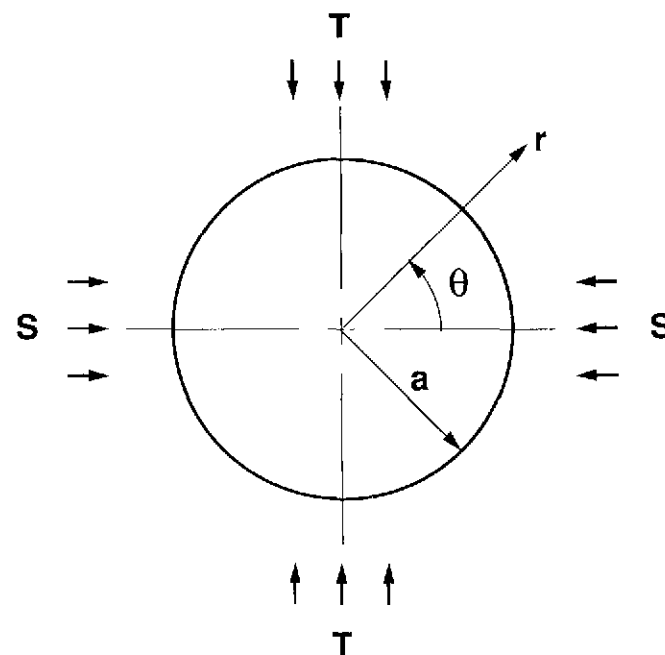


Figure 1.1.—Cross section of hole in plate.

When $\theta = 0^\circ$,

$$U = \frac{d}{E} (3S - T). \quad (1.2)$$

When $\theta = 90^\circ$,

$$U = \frac{d}{E} (3T - S). \quad (1.3)$$

Figure 1.2 is a plot of deformation versus the angle θ for one quadrant.

For a biaxial stress field in plane strain, the deformation is

$$U = \frac{d}{E} (1 - \nu^2) [(S + T) + 2(S - T) \cos 2\theta], \quad (1.4)$$

where ν = Poisson's ratio.

The difference between the condition of plane stress and plane strain is the quantity $(1 - \nu^2)$. When Poisson's ratio ν is between 0.25 and 0.30 (which is the case for most rocks), the difference in computed stress is about 6 to 10 pct (21).

If the deformation across three diameters and the modulus of elasticity E and Poisson's ratio ν are known, the magnitude and direction of the stresses S and T can be computed. For measured deformations across diameters 60° apart (60° rosette), the relationships for plane strain (fig. 1.3) are

$$S + T = \frac{E}{3d(1 - \nu^2)} (U_1 + U_2 + U_3), \quad (1.5)$$

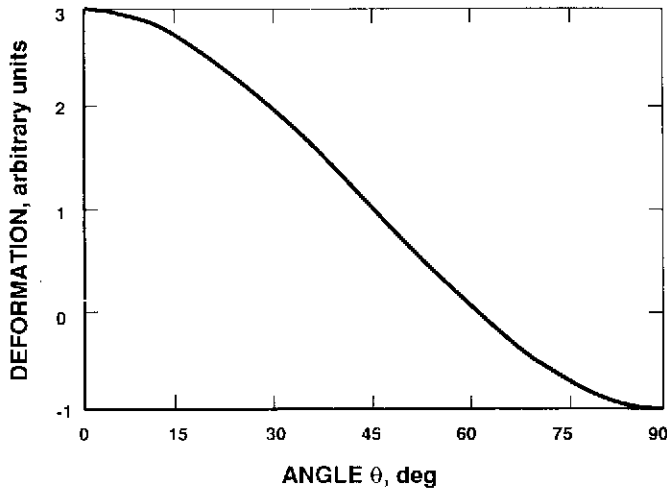


Figure 1.2.—Borehole deformation versus angle θ .

$$S - T = \frac{\sqrt{2}E}{6d(1 - \nu^2)} \left[(U_1 - U_2)^2 + (U_2 - U_3)^2 + (U_1 - U_3)^2 \right]^{1/2}, \quad (1.6)$$

$$\text{and} \quad \tan 2\theta_1 = \frac{\sqrt{3} (U_2 - U_3)}{2U_1 - U_2 - U_3}, \quad (1.7)$$

where U_1, U_2, U_3 = deformation measurements across diameters 60° apart, μin ,

and θ_1 = angle measured positive in counterclockwise direction from S to U_1 (fig. 1.3).

For deformation measurements across diameters 45° apart (45° rosette), the relationships for plane strain are

$$S + T = \frac{E(U_1 + U_3)}{2d(1 - \nu^2)}, \quad (1.8)$$

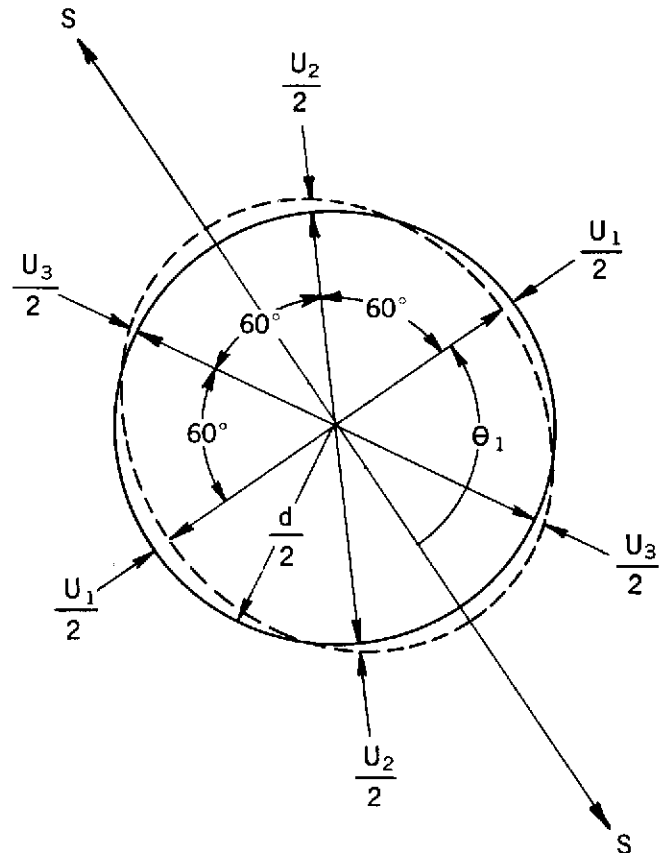


Figure 1.3.—Cross section for 60° deformation rosette.

$$S - T = \frac{E \left[(U_1 - U_2)^2 + (U_2 - U_3)^2 \right]^{1/2}}{2d\sqrt{2}(1 - \nu^2)}, \quad (1.9)$$

$$\text{and} \quad \tan 2\theta_1 = - \frac{2U_2 - U_1 - U_3}{U_1 - U_3}. \quad (1.10)$$

For deformation measurements across four diameters 45° apart (45° rosette), the relationships for plane strain are

$$S + T = \frac{E}{4d(1 - \nu^2)} (U_1 + U_2 + U_3 + U_4), \quad (1.11)$$

$$S - T = \frac{E}{4d(1 - \nu^2)} \left[(U_1 - U_3)^2 + (U_2 - U_4)^2 \right]^{1/2}, \quad (1.12)$$

$$\text{and} \quad \tan 2\theta_1 = \frac{U_2 - U_4}{U_1 - U_3}, \quad (1.13)$$

where U_1, U_2, U_3, U_4 = deformation measurements across diameters 45° apart, $\mu\text{in.}$

The angle θ could have two values 90° apart. The correct angle can be determined by using the following rules: For the 60° rosette,

1. If $U_2 > U_3$, θ_1 lies between 90° and 180° or 0° and -90°.
2. If $U_2 < U_3$, θ_1 lies between 0° and 90°.
3. If $U_2 = U_3$, and if
 - a. $U_1 > U_2 = U_3$, $\theta_1 = 0^\circ$;
 - b. $U_1 < U_2 = U_3$, $\theta_1 = 90^\circ$.

For the 45° rosette,

1. If $U_2 > \frac{U_1 + U_3}{2}$, θ_1 lies between 90° and 180° or 0° and -90°.
2. If $U_2 < \frac{U_1 + U_3}{2}$, θ_1 lies between 0° and 90°.
3. If $U_2 = \frac{U_1 + U_3}{2}$, and if
 - a. $U_1 > U_3$, $\theta_1 = 0^\circ$;
 - b. $U_1 < U_3$, $\theta_1 = 90^\circ$.

These angles can be checked by plotting the vectors for secondary principal stresses S and T relative to U_1 .

DEFORMATION AND STRESSES IN TWO DIMENSIONS

From the elastic plate theory, the maximum secondary principal stress (P) and the minimum secondary principal

stress (Q) in the plane perpendicular to a borehole can be calculated from three diametral deformations of the borehole and the physical properties of the rock. For the in situ stress determinations, P and Q are substituted for stresses S and T . For plane strain and a 60° rosette, equations 1.5 and 1.6 become

$$P + Q = \frac{E}{3d(1 - \nu^2)} (U_1 + U_2 + U_3) \quad (1.14)$$

$$\text{and} \quad P - Q = \frac{\sqrt{2}E}{6d(1 - \nu^2)} \left[(U_1 - U_2)^2 + (U_2 - U_3)^2 + (U_1 - U_3)^2 \right]^{1/2}. \quad (1.15)$$

Combining and expressing equations 1.14 and 1.15 in terms of P and Q ,

$$P = \frac{E}{6d(1 - \nu^2)} \left\{ (U_1 + U_2 + U_3) + \frac{\sqrt{2}}{2} \left[(U_1 - U_2)^2 + (U_2 - U_3)^2 + (U_1 - U_3)^2 \right]^{1/2} \right\} \quad (1.16)$$

$$\text{and} \quad Q = \frac{E}{6d(1 - \nu^2)} \left\{ (U_1 + U_2 + U_3) - \frac{\sqrt{2}}{2} \left[(U_1 - U_2)^2 + (U_2 - U_3)^2 + (U_1 - U_3)^2 \right]^{1/2} \right\}, \quad (1.17)$$

and θ_1 is obtained from equation 1.7.

Least Squares Solution in Two Dimensions

In an isotropic elastic rock, the diametral deformation of a borehole in a two-dimensional stress field (49) is given by

$$\begin{aligned} U = \frac{d}{E} & \left[1 + 2(1 - \nu^2) \cos 2\theta \right] S_x \\ & + \frac{d}{E} \left[1 - 2(1 - \nu^2) \cos 2\theta \right] S_y \\ & + \frac{d}{E} \left[4(1 - \nu^2) \sin 2\theta \right] T_{xy}, \end{aligned} \quad (1.18)$$

where S_x = applied normal stress in x direction,
 S_y = applied normal stress in y direction,
 τ_{xy} = applied shear stress in x-y plane,
 and θ = angle measured positive in counter-clockwise direction from X axis.

When more than three diametral deformations are measured in a borehole, a least squares analysis can be used to calculate the average rock stress components (47). To simplify equation 1.18, let

$$b_1 = S_x,$$

$$b_2 = S_y,$$

$$b_3 = \tau_{xy},$$

$$X_1 = \frac{d}{E} [1 + 2(1 - \nu^2) \cos 2\theta],$$

$$X_2 = \frac{d}{E} [1 - 2(1 - \nu^2) \cos 2\theta],$$

$$\text{and } X_3 = \frac{d}{E} [4(1 - \nu^2) \sin 2\theta], \quad (1.19)$$

then equation 1.18 becomes

$$U_j = b_1 X_{1j} + b_2 X_{2j} + b_3 X_{3j} + e_j, \quad (1.20)$$

where j = the different diametral deformations

and e_j = error in each of the equations.

The three stress components b_1 , b_2 , and b_3 are given by

$$b_1 = c_{11} \sum X_{1j} U_j + c_{12} \sum X_{2j} U_j + c_{13} \sum X_{3j} U_j,$$

$$b_2 = c_{21} \sum X_{1j} U_j + c_{22} \sum X_{2j} U_j + c_{23} \sum X_{3j} U_j,$$

$$\text{and } b_3 = c_{31} \sum X_{1j} U_j + c_{32} \sum X_{2j} U_j + c_{33} \sum X_{3j} U_j, \quad (1.21)$$

$$\text{where } c_{11} = \left[\left(\sum X_{2j}^2 \right) \left(\sum X_{3j}^2 \right) - \left(\sum X_{2j} X_{3j} \right)^2 \right] / D,$$

$$c_{22} = \left[\left(\sum X_{1j}^2 \right) \left(\sum X_{3j}^2 \right) - \left(\sum X_{1j} X_{3j} \right)^2 \right] / D,$$

$$c_{33} = \left[\left(\sum X_{1j}^2 \right) \left(\sum X_{2j}^2 \right) - \left(\sum X_{1j} X_{2j} \right)^2 \right] / D,$$

$$c_{12} = c_{21} = \left[\left(\sum X_{1j} X_{3j} \right) \left(\sum X_{3j} X_{2j} \right) - \left(\sum X_{3j}^2 \right) \left(\sum X_{1j} X_{2j} \right) \right] / D,$$

$$c_{23} = c_{32} = \left[\left(\sum X_{2j} X_{1j} \right) \left(\sum X_{1j} X_{3j} \right) - \left(\sum X_{1j}^2 \right) \left(\sum X_{2j} X_{3j} \right) \right] / D,$$

$$\text{and } c_{31} = c_{13} = \left[\left(\sum X_{1j} X_{2j} \right) \left(\sum X_{2j} X_{3j} \right) - \left(\sum X_{2j}^2 \right) \left(\sum X_{1j} X_{3j} \right) \right] / D.$$

The solution for b_1 , b_2 , and b_3 is obtained from the determinant (D):

$$D = \left(\sum X_{1j}^2 \right) \left(\sum X_{2j}^2 \right) \left(\sum X_{3j}^2 \right) + 2 \left(\sum X_{1j} X_{2j} \right) \left(\sum X_{2j} X_{3j} \right) \left(\sum X_{3j} X_{1j} \right) - \left(\sum X_{1j}^2 \right) \left(\sum X_{2j} X_{3j} \right)^2 - \left(\sum X_{2j}^2 \right) \left(\sum X_{3j} X_{1j} \right)^2 - \left(\sum X_{3j}^2 \right) \left(\sum X_{1j} X_{2j} \right)^2. \quad (1.22)$$

Statistical Interpretations

The standard error S_E of the data about the regression (50) for equation 1.20 is

$$S_E = \left[\frac{\sum e_j^2}{n - 3} \right]^{1/2}, \quad (1.23)$$

where n = number of measurements,

and the error sum of squares is given by

$$\sum e_j^2 = \sum U_j^2 - b_1 \sum X_{1j} U_j - b_2 \sum X_{2j} U_j - b_3 \sum X_{3j} U_j. \quad (1.24)$$

The square of the multiple correlation coefficient R (50) is given by

$$R^2 = 1 - \frac{\sum e_j^2}{\sum U_j^2}. \quad (1.25)$$

The standard errors S_E in the stress components b_1 , b_2 , and b_3 (50) are given by

$$S_{b1} = S_E (c_{11})^{1/2}, \quad (1.26)$$

$$S_{b2} = S_E (c_{22})^{1/2}, \quad (1.27)$$

and
$$S_{b3} = S_E (c_{33})^{1/2}. \quad (1.28)$$

From the values of b_1 , b_2 , and b_3 , the secondary principal stresses P and Q and the stress directions can be calculated from the following equations (47):

$$P = 1/2(b_1 + b_2) + 1/2 \left[(b_1 - b_2)^2 + 4 b_3^2 \right]^{1/2}, \quad (1.29)$$

$$Q = 1/2(b_1 + b_2) - 1/2 \left[(b_1 - b_2)^2 + 4 b_3^2 \right]^{1/2}, \quad (1.30)$$

and
$$\theta_1 = 1/2 \tan^{-1} \left[\frac{2 b_3}{b_1 - b_2} \right]. \quad (1.31)$$

In a rotated coordinate system aligned with the secondary principal stresses P and Q , the computed errors S_E , S_{b1} , S_{b2} , and S_{b3} are the same as in the original coordinate system when the measure displacements are taken at 60° intervals (47). Since the angle in the P - Q coordinate system is zero, the minimum probable error (S_{θ_1}), in the original angle θ_1 is given by

$$S_{\theta_1} = 1/2 \tan^{-1} \left[\frac{2 S_{b3}}{P - Q} \right]. \quad (1.32)$$

The confidence intervals for the magnitude and direction of the secondary principal stresses P and Q (as given by equation 1.32) are $P \pm t_\alpha S_{b1}$, in the direction $\theta_1 \pm t_\alpha S_{\theta_1}$, and $Q \pm t_\alpha S_{b2}$, in the direction $(\theta_1 + 90^\circ) \pm t_\alpha S_{\theta_1}$, where t_α is the table value of Student's t -statistic for the desired probability (α) with $n - 3$ degrees of freedom, and θ_1 is measured from the x direction.

It may be of interest to determine, statistically, if the secondary principal stresses P and Q are significantly different at a particular site (30, 50). It can be said that P differs significantly from Q if the following relationship is true (50):

$$\frac{|P - Q|}{S_E (c_{11} - 2c_{12} + c_{22})^{1/2}} > t_\alpha. \quad (1.33)$$

This relationship confirms whether the difference between P and Q represents a biaxial stress condition.

A comparison can also be made between any stress component determined from the least squares analysis of data set at one site to the corresponding stress component determined at another site. By analogy to the statistics of straight lines, if the following is true:

$$\frac{|P - P'|}{(c_{11} + c_{11'})^{1/2} \left[\frac{(n-3) S_E^2 + (n'-3) S_{E'}^2}{n + n' - 6} \right]^{1/2}} > t_{\alpha(n+n'-6)} \quad (1.34)$$

and
$$\frac{|Q - Q'|}{(c_{11} + c_{11'})^{1/2} \left[\frac{(n-3) S_E^2 + (n'-3) S_{E'}^2}{n + n' - 6} \right]^{1/2}} > t_{\alpha(n+n'-6)}, \quad (1.35)$$

where the primed and unprimed values correspond to the different data sets, then P and P' or Q and Q' are significantly different (9, 47).

DEFORMATION AND STRESSES IN THREE DIMENSIONS

The stress ellipsoid (the three-dimensional state of stress) can be calculated from deformation measurements obtained from three nonparallel boreholes. Using the overcoring method previously described, the deformation measurements from three boreholes can be combined into a single set of data to provide the average three-dimensional stress components. The solution for this situation is based on the least squares principle; more specifically, the minimum sum of squared deviations from the fitted regression line (9).

The three-dimensional state of stress is defined by specifying the magnitudes of the three components of normal stress (σ_1 , σ_2 , and σ_3), and the three components of shearing stress (τ_{12} , τ_{23} , and τ_{31}) (fig. 1.4). A total of six independent stress components are required to define the stress tensor. The τ_{12} and τ_{32} , which act parallel to the axis of the borehole, have negligible influence on the diametral deformation of the borehole because a linear

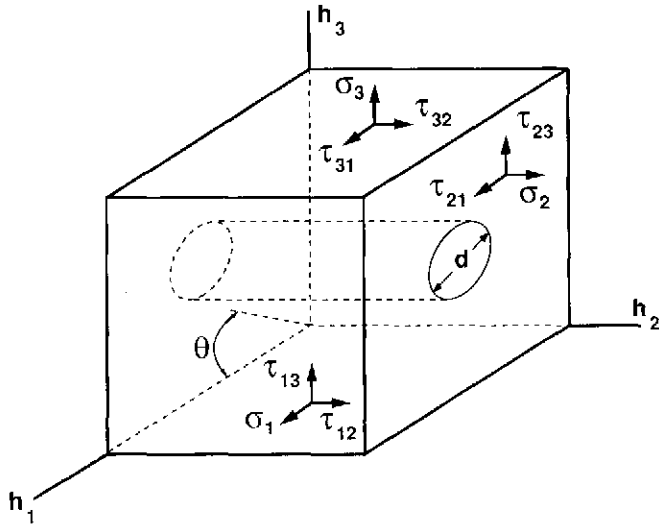


Figure 1.4.—Coordinate system for any drill hole.

element that lies parallel or at right angles to the borehole experiences no change in length owing to a shearing stress (51). Therefore, the diameter change of the borehole is a function only of σ_1 , σ_3 , and τ_{13} ($\tau_{13} = \tau_{31}$), which act normal to the borehole axis, and σ_2 , which acts parallel to the borehole axis. For these stated conditions, the plane strain condition applies.

The local rectangular coordinate system presented in figure 1.4 is used for each borehole. The axes h_1 , h_2 , and h_3 form a right-handed coordinate system with h_2 parallel to the borehole axis. The angle θ is with respect to h_1 and measured in the h_1 - h_3 plane, the positive direction of θ being as shown in figure 1.4.

When the diametral deformations are measured in more than one borehole, it is necessary to relate the deformation measurements from each local coordinate system to a common coordinate system. The common coordinate system consists of a set of right-handed rectangular coordinates (x , y , and z) that is assigned to the medium, chosen in any convenient direction, such as parallel to the east-west, north-south, and vertical directions or parallel to the dip of the ore body.

The three normal stresses (σ_x , σ_y , and σ_z) and the three shearing stresses (τ_{xy} , τ_{yz} , and τ_{zx}) in the selected coordinate system are the six independent components for the stress tensor in the medium and are related to the deformation measurements by the following equation:

$$U = b_1 J_1 + b_2 J_2 + b_3 J_3 + b_4 J_4 + b_5 J_5 + b_6 J_6, \quad (1.36)$$

$$b_1 = \sigma_x,$$

$$b_2 = \sigma_y,$$

$$b_3 = \sigma_z,$$

$$b_4 = \tau_{xy},$$

$$b_5 = \tau_{yz},$$

$$b_6 = \tau_{zx},$$

$$J_1 = f_1 l_1^2 + f_2 l_2^2 + f_3 l_3^2 + f_4 l_1 l_3,$$

$$J_2 = f_1 m_1^2 + f_2 m_2^2 + f_3 m_3^2 + f_4 m_1 m_3,$$

$$J_3 = f_1 n_1^2 + f_2 n_2^2 + f_3 n_3^2 + f_4 n_1 n_3,$$

$$J_4 = 2f_1 l_1 m_1 + 2f_2 l_2 m_2 + 2f_3 l_3 m_3 + f_4 (l_1 m_3 + l_3 m_1),$$

$$J_5 = 2f_1 m_1 n_1 + 2f_2 m_2 n_2 + 2f_3 m_3 n_3 + f_4 (m_1 n_3 + m_3 n_1),$$

$$\text{and } J_6 = 2f_1 n_1 l_1 + 2f_2 n_2 l_2 + 2f_3 n_3 l_3 + f_4 (n_1 l_3 + n_3 l_1).$$

$$\text{where } f_1 = d(1 + 2 \cos 2\theta)(1 - \nu^2)/E + d\nu^2/E,$$

$$f_2 = -d\nu/E,$$

$$f_3 = d(1 - 2 \cos 2\theta)(1 - \nu^2)/E + d\nu^2/E,$$

$$f_4 = d(4 \sin 2\theta)(1 - \nu^2)/E,$$

and l_i , m_i , n_i = direction cosines for positive h_i axis with respect to positive X , Y , and Z axes, respectively, and similarly for h_2 and h_3 axes.

Equation 1.36 expresses the diametral deformation as a linear function of the stress components b_i . To each distinct orientation of U , there is a corresponding distinct set of values (J_i) that depend only on the direction of measurement and the elastic constants E and ν . Equation 1.36 applies for any number of deformations from any number of boreholes. By measuring U for six independent orientations and substituting the six sets of values in equation 1.36, six independent simultaneous equations were obtained that can be used to solve for the b_i unknowns. The solution requires six different deformation measurements, a minimum of two from each of three boreholes.

When the orientation of the boreholes is chosen, the contribution of each of the six stress components relative to the diametral deformation should be considered. From

this standpoint, it would be desirable for each borehole to be influenced similarly by each of the six stress components; thus, the magnitude of the six J_i in equation 1.36 should be approximately equal, numerically (9). To accomplish a balance between the determination of the six stress components, a combination of measured directions of angle θ within the boreholes and the hole orientation with respect to the X, Y, and Z axes is required. The directions chosen should coincide with the assumed directions of highest stress or could be the directions of primary importance. For example, the two directions chosen for a 2-C BHG could be (1) $\theta = 0$ and 90 , or (2) $\theta = 45$ and 135 ; for a 3-C BHG, the three directions chosen could be (1) $\theta = 0, 60$ and 120 , or (2) $\theta = 30, 90$, and 150 ; and for a 4-C BHG, the four directions chosen could be $\theta = 0, 45, 90$, and 135 .

Measuring four or more equally spaced directions in each borehole provides significantly better balance than only two or three directions.

Least Squares Solution in Three Dimensions

The least squares analysis for calculating the six stress components provides information to the precision of the stress values as well as an average value for each of the six stress components (9). From equation 1.36 for measured U , the average sample values for b_i can be calculated so that the value of U predicted by equation 1.36 will have minimum deviation from the actual measured values as stated below. Assuming a sample set of U values is randomly selected from a population, each measured value of U can be given by

$$U = \beta_1 J_1 + \beta_2 J_2 + \dots + \beta_6 J_6 + \epsilon \quad (1.37)$$

$$= b_1 J_1 + b_2 J_2 + \dots + b_6 J_6 + e, \quad (1.38)$$

where β = population parameter,

ϵ = true error of each U measurement,

b = respective estimate of β ,

and e = residual about regression line.

A set of estimates b_i determined from the least squares solution minimizes the sum of squares of the residuals e . The b_i are obtained from the following equations:

$$\begin{aligned} b_1 \sum J_1^2 + b_2 \sum J_1 J_2 + b_3 \sum J_1 J_3 + \dots + b_6 \sum J_1 J_6 \\ = \sum U J_1, \end{aligned}$$

$$\begin{aligned} b_1 \sum J_2 J_1 + b_2 \sum J_2^2 + b_3 \sum J_2 J_3 + \dots + b_6 \sum J_2 J_6 \\ = \sum U J_2, \end{aligned}$$

$$\begin{aligned} b_1 \sum J_6 J_1 + b_2 \sum J_6 J_2 + b_3 \sum J_6 J_3 + \dots \\ + b_6 \sum J_6^2 = \sum U J_6. \end{aligned} \quad (1.39)$$

Developing the matrix A , let $a_{ij} = \sum J_i J_j$ ($i, j = 1, 2, 3, \dots, 6$) be substituted in equation 1.39, where $a_{21} = a_{12}$, $a_{31} = a_{13}$, and so forth, and $g_j = \sum U J_j$ be substituted in equation 1.39, where $c_{12} = c_{21}$, $c_{13} = c_{31}$, and so forth, so that the a_{ij} of equation 1.39 are defined in the following matrix:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{16} \\ a_{21} & a_{22} & a_{23} & \dots & a_{26} \\ \dots & \dots & \dots & \dots & \dots \\ a_{61} & a_{62} & a_{63} & \dots & a_{66} \end{bmatrix} \quad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \end{bmatrix}.$$

Then equation 1.39 may be represented by

$$Ab = g. \quad (1.40)$$

By analogy to the usual algebraic operation, b is obtained by

$$b = A^{-1}g, \quad (1.41)$$

where A^{-1} = inverse of A .

Let $C = A^{-1}$, then equation 1.41 becomes

$$b = Cg. \quad (1.42)$$

The solution can now be obtained from equation 1.39 (9). The computer code for solving equation 1.39 is presented in appendix G.

Statistical Interpretations

The standard error S_E about the fitted regression line (50) for equation 1.36 is

$$S_E = \left[\frac{\sum e^2}{n - p} \right]^{1/2}, \quad (1.43)$$

where n = total number of U measurements

and p = number of b_i calculated from data.

The error sum of squares for a set of U measurements can be calculated from (52)

$$\sum e^2 = \sum U^2 - (b_1 g_1 + b_2 g_2 + \dots + b_6 g_6), \quad (1.44)$$

where $g_j = \sum U_j$

and $j = 1, 2, \dots, 6$.

The square of the multiple correlation R (50) is given by

$$R^2 = 1 - \frac{\sum e^2}{\sum U^2}. \quad (1.45)$$

The standard errors S_E in the stress components b_i (50) are given by

$$\begin{aligned} S_{b_1} &= S(c_{11})^{1/2}, \\ S_{b_2} &= S(c_{22})^{1/2}, \\ &\dots\dots\dots, \\ S_{b_6} &= S(c_{66})^{1/2}, \end{aligned} \quad (1.46)$$

where S is given by equation 1.43, and c_{11} to c_{66} are coefficients of A^{-1} .

If desired, it may be determined, statistically, whether the lateral stress is the same in all directions. If b_1 and b_2 are the two horizontal stress (σ_h) components, then it can be said that b_1 differs significantly from b_2 if the following relationship is true (50):

$$\frac{|b_1 - b_2|}{S_{b_1} - b_2} > t_\alpha, \quad (1.47)$$

$$\text{where } S_{b_1} - b_2 = S(c_{11} - 2c_{12} + c_{22})^{1/2}, \quad (1.48)$$

S is given by equation 1.43, and t_α is the table value of Student's t -statistic for the desired probability α with $n - 6$ degrees of freedom.

It may be of interest to determine, statistically, if a given stress component b_1 is significantly different from

some theoretically expected or assumed value B (e.g., for overburden, B may be zero). It can be said that b_1 differs significantly from B if the following relationship is true (50):

$$\frac{|b_1 - B|}{S_{b_1}} \leq t_\alpha. \quad (1.49)$$

It also can be determined, statistically, if a given stress component b_2 is significantly different from a corresponding stress component b_2' determined at a different time or a different site in the mine. It can be said that b_2 differs significantly from b_2' if the following relationship is true (50):

$$\frac{|b_2 - b_2'|}{S_{b_2} - b_2'} \leq t_\alpha, \quad (1.50)$$

where $S_{b_2} - b_2' =$

$$\left(\frac{1}{n} + \frac{1}{n'} \right) \left[\frac{n(S_{b_2})^2 c_{22} + n'(S_{b_2'})^2 c_{22'}}{(n + n') - (p + p')} \right]^{1/2}. \quad (1.51)$$

EFFECTS OF ANISOTROPY ON IN SITU ROCK STRESS

Several reports have been published (48, 53-54) on a plane elasticity solution for a circular hole in an infinite orthotropic elastic medium. During stress relief by overcoring, the radial displacement u at the boundary of a hole is of particular interest. The exact plane stress or plane strain displacement in an orthotropic medium subjected to a uniaxial stress field is given (54) for the hole axis oriented along one of the elastic axes. The result may be extended to the biaxial case. Figure 1.5 illustrates the geometry. The u caused by the secondary principal stress Q at right angles to the secondary principal stress P (54) yields u at $r = a$. In terms of the diametral deformation, $U = 2u$.

For an anisotropic medium having elastic axes (X_1 and X_2), the relationship between the diametral deformation of a cylindrical hole to the secondary principal stresses P and Q is given by

$$U = A \cos^2 \theta + B \sin \theta \cos \theta + C \sin^2 \theta, \quad (1.52)$$

where the coefficients A , B , and C for plane strain condition (axial strain $\epsilon_3 = 0$) are given by

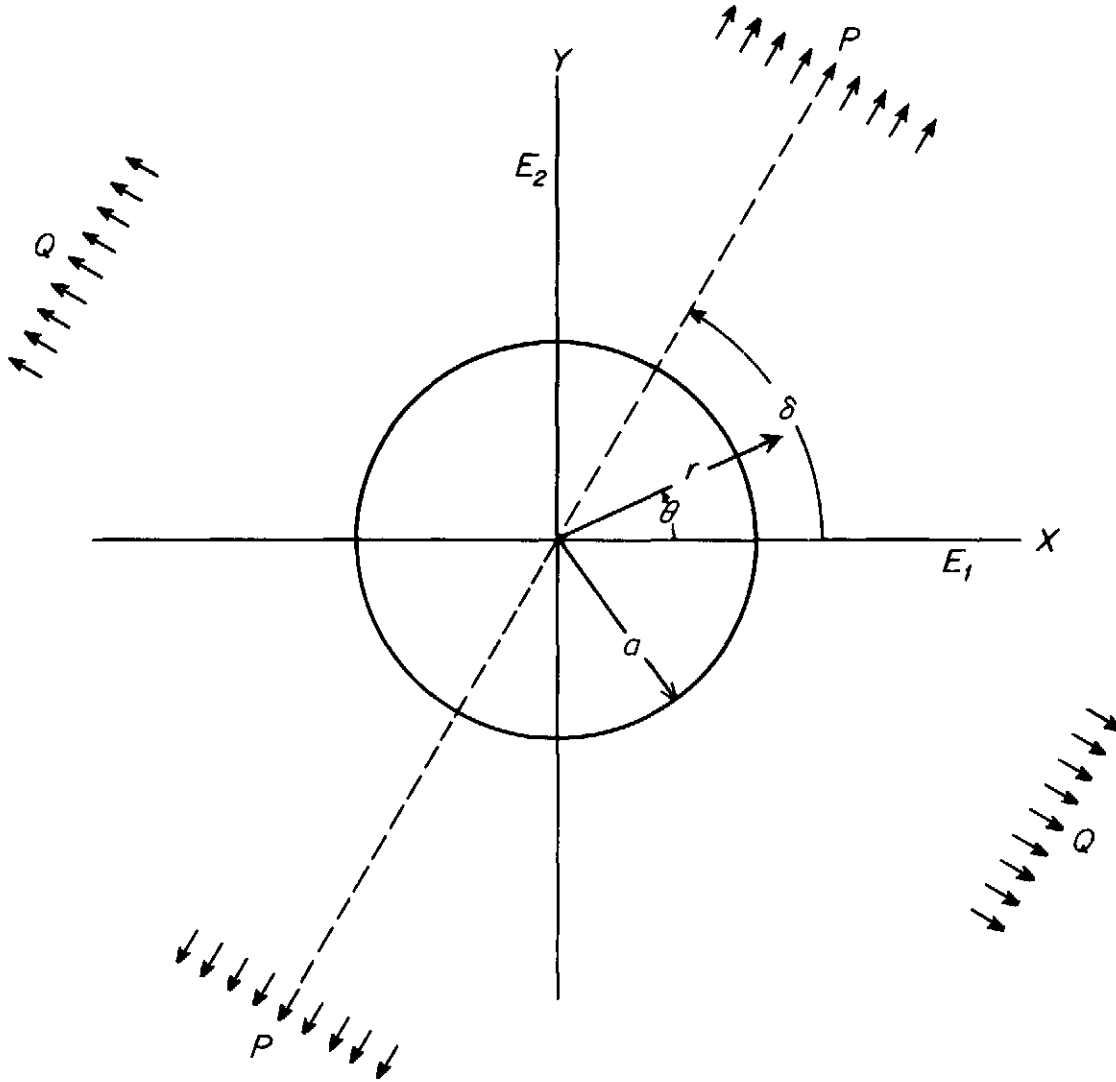


Figure 1.5.—Circular hole in orthotropic solid subject to biaxial field.

$$A = d \left[\left(2\alpha_{11} + \sqrt{\alpha_{11}\alpha_{22}} \right) (P \cos^2 \delta + Q \sin^2 \delta) - \sqrt{\alpha_{11}\alpha_{22}} (P \sin^2 \delta + Q \cos^2 \delta) \right],$$

$$\alpha_{11} = \frac{1 - \nu^2}{E_1},$$

and

$$\alpha_{22} = \frac{1 - \nu^2}{E_2}.$$

$$B = d \left[(P - Q) \left(\alpha_{11} + 2\sqrt{\alpha_{11}\alpha_{22}} + \alpha_{22} \right) \sin 2\delta \right],$$

$$\text{and } C = d \left[\left(2\alpha_{22} + \sqrt{\alpha_{11}\alpha_{22}} \right) (P \sin^2 \delta + Q \cos^2 \delta) - \sqrt{\alpha_{11}\alpha_{22}} (P \cos^2 \delta + Q \sin^2 \delta) \right],$$

where θ = angle measured positive in counter-clockwise direction from E_1 to U_1 ,
 δ = angle measured positive in counter-clockwise direction from E_1 to P ,

The coefficients A , B , and C can be determined from U_1 , U_2 , and U_3 determined at different values of angle θ . The value of θ is measured from an elastic axis, which is generally determined in the laboratory from the overcore; the actual value of θ is unknown in the field. Only the increments between the angles can be set in the field. The increment most commonly used is 60° . From field and laboratory tests, the three sets of angles and deformations are known and substituted into A , B , and C . By solving for these constants simultaneously, the values of A , B , and C are calculated. The initial estimates of angle δ and

secondary principal stresses P and Q are obtained from the following equations:

$\delta =$

$$\tan^{-1} \left[\frac{C\alpha_{11} - A\alpha_{22}}{B\sqrt{\alpha_{11}\alpha_{22}}} \pm \sqrt{\left(\frac{C\alpha_{11} - A\alpha_{22}}{B\sqrt{\alpha_{11}\alpha_{22}}} \right)^2 + 1} \right], \quad (1.53)$$

$$P = \frac{L - K \cos^2 \delta}{\sin^2 \delta - \cos^2 \delta}, \quad (1.54)$$

$$\text{and} \quad Q = K - P, \quad (1.55)$$

where L and K = constants,

$$K = \frac{A \sqrt{\alpha_{11}\alpha_{22}} + C\alpha_{11}}{\alpha_{11}d \left(\alpha_{22} + \sqrt{\alpha_{11}\alpha_{22}} \right)}, \quad (1.56)$$

$$\text{and} \quad L = \frac{A\alpha_{22} + C \left(2 \sqrt{\alpha_{11}\alpha_{22}} + \alpha_{22} \right)}{2d \left(\alpha_{22} + \sqrt{\alpha_{11}\alpha_{22}} \right)^2}. \quad (1.57)$$

Corrections for the effects of axial strain ϵ_3 on the field measurements can be made. If the axial stress σ_3 is known or can be estimated and Poisson's ratios ν_{31} and

ν_{32} acting along X_1 and X_2 , respectively, and the compliance coefficients S_{31} and S_{32} (ν divided by the axial modulus of elasticity E_3) are determined from laboratory testing of the overcore (chapter 4), then A_k and C_k from the relationships

$$A_k \sim A - S_{31}d \left[\sigma_3 - \nu_{31} (P \cos^2 \delta + Q \sin^2 \delta) - \nu_{32} (P \sin^2 \delta + Q \cos^2 \delta) \right],$$

$$\text{and } C_k \sim C - S_{32}d \left[\sigma_3 - \nu_{31} (P \cos^2 \delta + Q \sin^2 \delta) - \nu_{32} (P \sin^2 \delta + Q \cos^2 \delta) \right]$$

are substituted for A and C in equations 1.53, 1.56, and 1.57. The two relationships, A_k and C_k , are approximated because the axial strain ϵ_3 is not measured during the overcoring process. In the isotropic solution for plane strain biaxial stress (55),

$$\alpha_{11} = \alpha_{22} = \frac{1 - \nu^2}{E}.$$

The computer code for solving isotropic and anisotropic secondary principal stresses P and Q is presented in appendix E.

CHAPTER 2.—INSTRUMENTATION

INTRODUCTION

A fundamental part of rock mechanics is to measure and interpret the in situ state of stress. One of the most useful instruments to accomplish this objective is the BHG developed by the U.S. Bureau of Mines. The BHG (10, 56) is designed for absolute (in situ) measurement, and when it is used with any procedure for producing strain relief, it can measure the resulting diametral borehole deformation. From the borehole deformation, the magnitude and direction of the principal stresses S_1 , S_2 , and S_3 can be calculated.

To take an absolute measurement, the in situ load acting on the rock mass must be removed. Overcoring (1-2, 5, 7-8) is the technique used to produce this stress or strain relief. The in situ stress is calculated from the difference between the first and last BHG measurements obtained while overcoring.

Gages used to obtain absolute measurements are divided into two classes—those measuring surface strain (stress cannot be measured on the surface) and those measuring strain-stress within the rock mass through boreholes or slots. The only practical method to take substantially deep measurements is to use a borehole. However, drilling a hole in the rock mass will alter the absolute or virgin stress field. It is necessary, therefore, when the U are interpreted, to either incorporate a theoretical relationship between the medium strain or stress or to calibrate the instrument in the laboratory. The theoretical relationship is obtained from either the strain on the borehole wall or the deformation of the borehole. During the calibration, a known stress applied to a block of rock containing the BHG (8, 57) results in an empirical two-dimensional solution for the deformation of a hole in a particular medium (21). The BHG does not exert sufficient force to disturb the stress distribution in the rock; thus, the gage does not interact with the hole.

Although the originally designed BHG provides satisfactory information for engineering mine design, new gage designs and modifications, described in this chapter, have substantially increased the flexibility and reliability of stress measurement.

GAGE DESIGN

To achieve the necessary range, sensitivity, and stability for the BHG, the following specifications were established (56):

1. BHG diameter: 1.375 in.
2. Borehole (gage hole) diameter: 1.460 in.

3. Range of measurement for a piston: 0.012 in (0.024 in for the diametral deformation).
4. Piston length: 0.325 in (adjustable to 0.395 in).
5. Gage sensitivity: $1 \mu\text{in/in}$.
6. Piston contact force: 2 lb, minimum deflection; 10 lb, maximum deflection.
7. Temperature stability: $2 \mu\text{in/in}/^\circ\text{F}$.

The gage diameter selected fits inside an EX-size (1.460-in bit diameter) borehole, which satisfies the minimum hole diameter d . Generally, a diamond reamer is used to produce a more uniform hole and to increase the hole diameter to approximately 1.5 in. The adjustable-length piston accommodates gage hole variations up to 1.590 in. Usually, the d increases when softer rock is drilled. Drilling speeds and feed rates can also affect the d . Although the temperature in a borehole is relatively stable, the temperature of the drilling water (or air) may vary considerably. Hence, the gage should be relatively insensitive to temperature changes.

Because of typically poor operating environments, the gage must be rugged, moisture proof, dustproof, and should be corrosion resistant.

Mechanical Gage Construction

A BHG was constructed to satisfy the above specifications. (Details for the construction of the gage appears in appendix B.) The gage can be constructed from either of the AISI Type 303 stainless steels (selected for its machining characteristics) or grade 2, commercially pure, centerless ground and annealed titanium. Titanium reduces the gage weight by 43 pct, which increases the depth of overcoring in vertical upholes. A piston contacts the borehole wall and the cantilever on which two resistance strain gages are bonded. These strain gages are connected to two strain gages on the opposite cantilever, forming a four-arm bridge. Gage resistance of the four strain gages in the bridge is matched to within 0.01 ohm to maintain temperature stability of $2 \mu\text{in/in}/^\circ\text{F}$. The bending strain of the cantilever is converted by the resistance strain gages into an electrical analog signal that is measured on a standard strain indicator. Hence, the cantilevers are termed "transducers." Opposite transducers in the gage form a component that measures a diametral deformation of the borehole. Since three diametral deformations are necessary to calculate the magnitude and direction of the stress normal to the axis of the borehole, the gage is constructed with six transducers and six pistons. This gage design, with only four moving parts per component, provides a durable mechanical device (fig. 2.1).

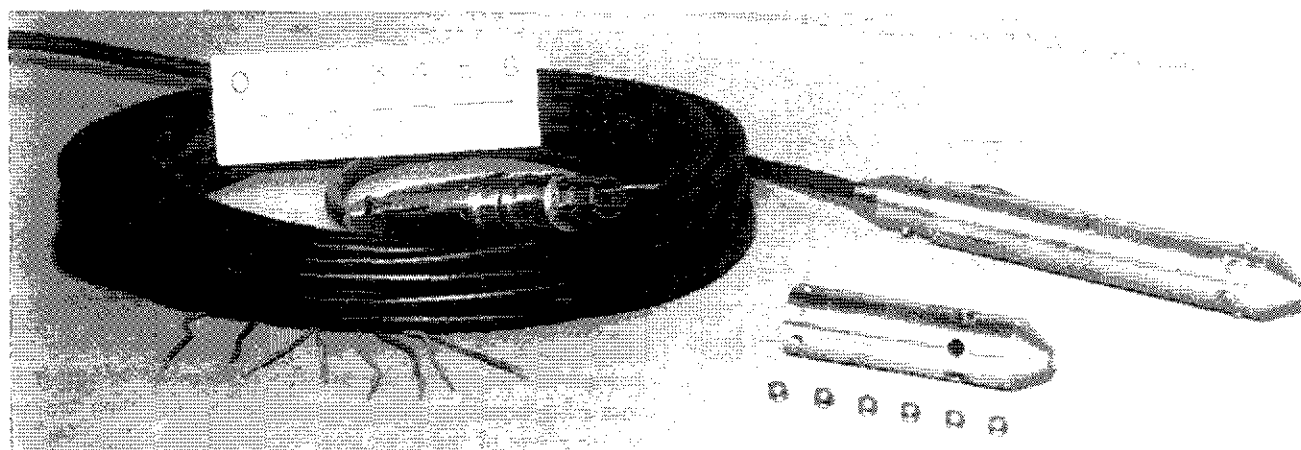


Figure 2.1.—Three-component BHG.

The piston and gage body O rings, together with the cable grommet, provide a waterproof and dustproof seal for the interior of the gage. Gage springs, together with the force exerted by the pistons on the borehole wall, prevent gage movement during the overcoring process. (The contact force of the pistons must be greater than the contact force of the springs against the wall of the borehole. This prevents axial and tangential deformations from introducing error in the diametral deformation.) A shielded eight-conductor neoprene cable carries the gage output signal to the strain indicators. This cable is secured in the BHG by tightening the cap (clamping nut) against the rubber grommet, forcing it into the tapered end of the gage body (fig. I.3).

Any movement between the gage case and the gage body affects both the sensitivity and accuracy of the gage. To maintain rigidity of the gage, the gage case is fastened to the gage body either by stainless steel screws or by threading the case onto the gage body.

External Four-Arm Strain Gage Bridge

Time and temperature stability of the BHG depends mostly on the strain gage characteristics, the type of adhesive and method of bonding the strain gage to the cantilever, and the type of circuit (bridge) selected.

Each four-arm bridge circuit (Wheatstone bridge), consisting of four strain gages, one gage on each side of two cantilevers (fig. 2.2), is temperature compensated from a change in temperature of the wires in the eight-conductor cable.

Constantan alloy grid strain gages with two high-endurance lead wires, fully encapsulated in epoxy and temperature compensated for the transducer material,

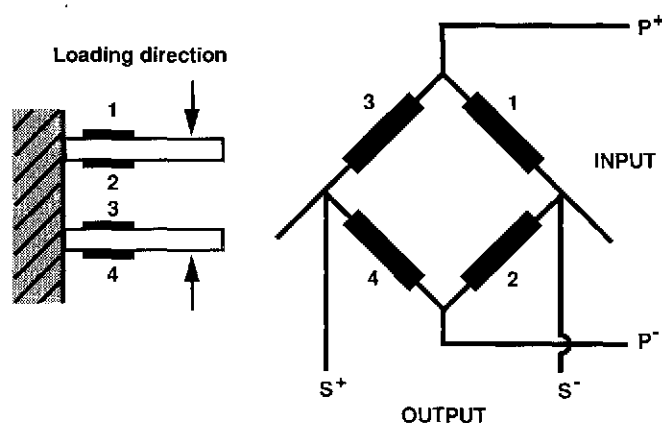


Figure 2.2.—Schematic of strain gage bridge wiring for cantilever pair. (1-4 = strain gages; P⁺, P⁻, S⁺, and S⁻ = strain indicator terminals.)

were selected. This type of gage is moistureproof and useful over a wide temperature range and in extreme environments. The cyclic endurance of the gage is sufficient to permit a maximum strain level of 2,000 $\mu\epsilon$. After considerable experimenting, an elevated-temperature-cure epoxy-resin cement or an elevated-temperature-cure, two-component, 100-pct-solid epoxy adhesive was found to produce the best results, yielding little or no creep.

CANTILEVER DESIGN

The strain produced in the transducer (cantilever) is directly proportional to the piston displacement (borehole deformation). To achieve a desirable range without exceeding the mechanical stability of the cantilever, the following design relationships between the deflection of

the cantilever, the dimensions of the cantilever, and the applied stress, strain, or force were used (56):

$$y = \frac{2L^2 \epsilon}{3t}, \quad (2.1)$$

$$y = \frac{2L^2 \sigma}{3Et}, \quad (2.2)$$

$$y = \frac{4WL^3}{bEt^3}, \quad (2.3)$$

and
$$\sigma = \frac{6WL}{bt^2}, \quad (2.4)$$

where y = cantilever deflection, in,

L = cantilever length, in,

ϵ = maximum strain, $\mu\text{in/in}$,

t = cantilever thickness, in,

σ = maximum stress, psi,

E = modulus of elasticity, psi,

W = applied force to end of cantilever, lb,

and b = cantilever width, in.

A width-to-thickness ratio of greater than 3 gives good lateral stability to a cantilever. The cantilever material selected is the most important consideration in the design and should contain the following properties: (1) a low modulus of elasticity (E), (2) a high proportional limit, (3) a linear stress-strain curve, and (4) no hysteresis. The material found to most closely satisfy these conditions is a half-hard, cold-worked beryllium-copper alloy. The E is 19×10^6 psi, the proportional limit is 90,000 psi, the stress-strain curve is virtually linear to a stress level of 60,000 psi, and the material does not take a set.

By staying within the internal limits of the BHG diameter, the following cantilever dimensions were established: $L = 1.3125$, $t = 0.1$, and $b = 0.315$.

By using equation 2.3, for this cantilever and for a minimum contact force of 2 lb, $y_2 = 0.003$ in, and for a maximum contact force of 10 lb, $y_{10} = 0.015$ in.⁸ Thus, the measuring range for a single cantilever is $y = y_{10} - y_2 = 0.012$ in. Since two cantilevers (transducers) are necessary to measure the U , the total measuring range is 0.024 in.

⁸When setting the 3-C BHG, the minimum force for the six pistons is 12 lb and the maximum force is 60 lb.

The bending stress in the cantilever at maximum deflection (0.015 in) is 23,000 psi.

GAGE DESIGN MODIFICATIONS

Modifications in the BHG have greatly improved the versatility of the gage (11).

Extended Placement End

When incompetent rock is overcored in a vertical down-hole, pieces of rock may fall into the EX-size borehole behind the gage and prevent the placement and retrieval tool from hooking onto the orientation pins, thus preventing retrieval of the gage from the borehole. By lengthening the placement end of the gage by 2 in (fig. 2.3), the orientation pins are allowed to extend out of the EX-size borehole into the 6-in-diam hole to facilitate engagement of the retrieval tool.

Reversed Borehole Deformation Gage

The springs on the standard BHG are 6 in from the plane of measurement of the gage. Thus, the gage must be positioned at least 6 in into the gage hole (EX-size borehole) for the springs to contact the borehole (fig. 2.3). When unfractured rock is overcored, the BHG plane of measurement is set between 6 and 9 in inside the pilot hole and a 12- to 18-in core is drilled. In fractured rock and under conditions that cause premature breaking of the core, a complete overcore may not be possible. Finite-element modeling of overcoring at depth in a medium provides the necessary compensation for the U when the plane of measurement is set between 6 in and the collar of the pilot hole (11). By making the necessary gage modification to take measurements less than 6 in from the collar of the hole, the "reversed" gage was developed.

By lengthening the standard BHG case, springs are placed in the case ahead of the pistons (fig. 2.3). This reversed case enables the plane of measurement to be set at any distance inside the gage hole. The reversed gage allows for a complete stress relief from a much shorter length of overcore than is required by the standard gage.

Two-Component Borehole Deformation Gage

Since the calculated E_1 and E_2 axes of the core are at right angles and the 3-C BHG pistons are oriented at 60° , two triaxial tests are necessary. In some cases, the core condition will not allow for a second test. This disadvantage led to the construction of a laboratory 2-C BHG. The 2-C BHG (fig. 2.4) measures deformation along the E_1 and E_2 axes of the stress-relieved core during the triaxial test. The reduced cable size and the modified placement end of the gage enhance gage placement in the triaxial chamber. Transducer size is the same as for the

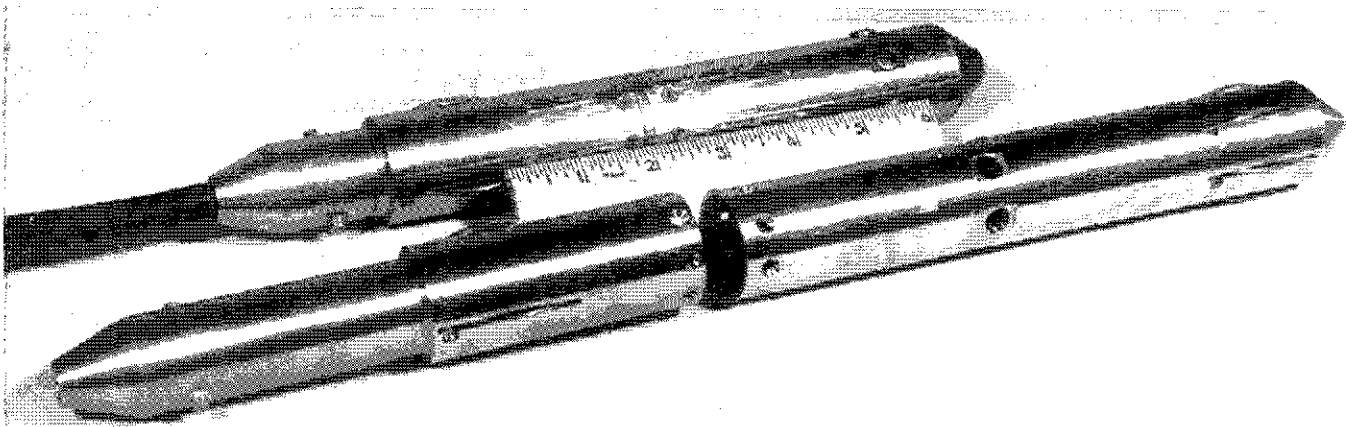


Figure 2.3.—Three-component BHG (top), extended-placement end (lower left), and reversed case (lower right).

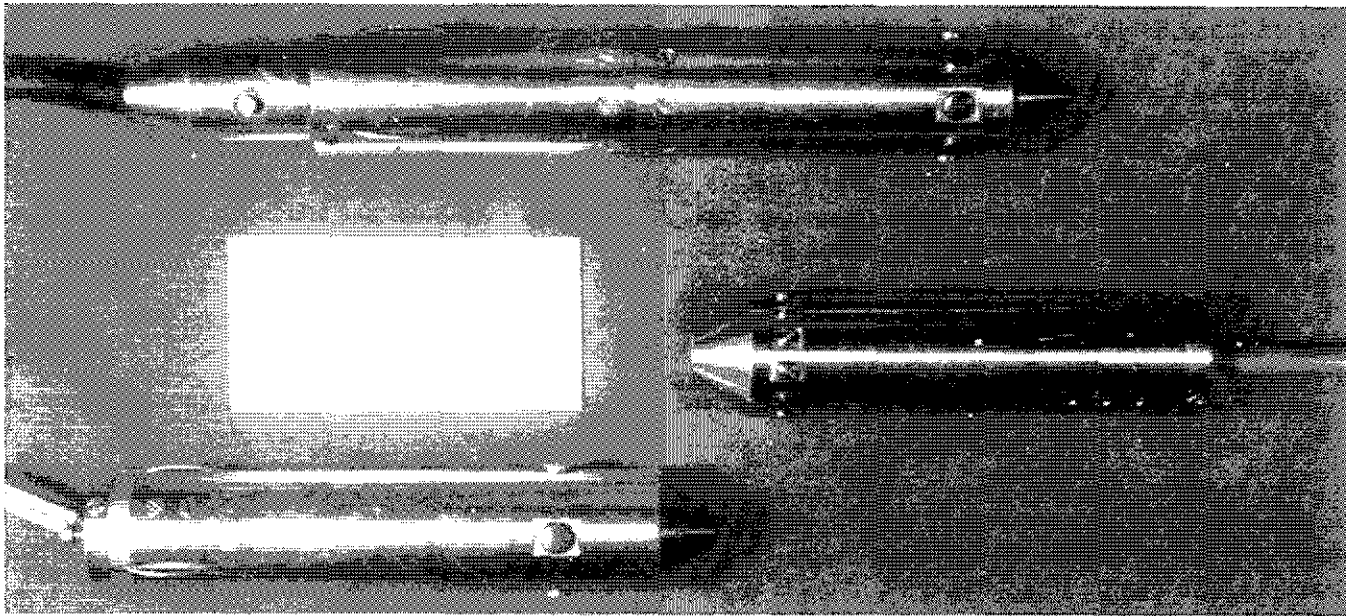


Figure 2.4.—Four-component BHG (top), six-component BHG (center right), and two-component BHG (bottom).

3-C BHG. The 2-C BHG body and cantilevers were machined from a solid piece of 17-4 precipitation hardening (PH) stainless steel, providing a creep-free gage. The gage could, however, be constructed using taper-mounted beryllium-copper cantilevers.

High-Temperature Three-Component Borehole Deformation Gage

If the BHG is used for measuring short-term stress changes at temperatures between 200° F (95° C) and 400° F (205° C), several of the gage components must be

replaced with components that can withstand the elevated temperature. Substitutes are needed for the adhesive, solder, connecting wire, neoprene jacket cable, and connecting plug. The cantilever material should be a heat-treated (1,150° F) 17-4 PH stainless steel. The modulus of elasticity E is 28.5×10^6 psi, the proportional limit is 170,000 psi, and the stress-strain curve is practically linear within the designed range. Dimensions for the 17-4 PH stainless steel cantilever are $L = 1.3125$ in, $t = 0.065$ in, and $b = 0.250$ in. The measuring range is 0.050 in, the bending stress is 52,000 psi, and the maximum total contact force is 42 lb.

Four-Component Borehole Deformation Gage

A 4-C BHG to measure four diametral deformations U , 45° apart, of a $1\frac{1}{2}$ -in.-diam borehole was constructed and calibrated (fig. 2.4). Cantilever dimensions, measuring range, and bending stress for this gage are calculated from equations 2.1 to 2.4. This gage provides an additional U to permit calculation of stresses from any three measurements. Also, U_1 , U_2 , U_3 , and U_4 can be used to calculate the stresses from equations 1.23 to 1.25.

Six-Component Borehole Deformation Gage

A 6-C BHG designed for laboratory triaxial testing was constructed and calibrated (fig. 2.4). This gage allows for more efficient biaxial and triaxial testing of the stress-relief core. The six cantilevers measure six diametral deformations U at 60° intervals, reducing from four to two biaxial tests and from two to one triaxial tests. Cantilever dimensions, measuring range, and bending stress for this gage are calculated from equations 2.1 to 2.4.

The 2-C, 4-C, and 6-C BHG's can be constructed from the 3-C BHG drawings included in appendix B with only minor changes necessary. For the 2-C BHG, four tapered holes 90° apart are drilled into the gage body instead of six tapered holes 60° apart. For the 4-C and 6-C BHG's, smaller tapered holes are used to accommodate the smaller cantilevers. The piston holes located in the gage case are relocated so that the pistons will make contact with the shorter length cantilevers. Smaller diameter pistons (0.311-in.-diam) are used for the 4-C BHG.

TRANSDUCER PREPARATION AND GAGE ASSEMBLY

This section provides the instructions and descriptions for installing the strain gages onto the cantilever and for assembling the 3-C BHG (58). Basic instructions for installing the strain gages are also available from the strain gage manufacturers.

The materials listed below are necessary for the fabrication of the BHG transducers and would be needed before starting the installation of the strain gage:

1. Beryllium-copper rod(s) in the d of the transducer width.
2. Strain gages and terminal strips.
3. Adhesive.
4. Degreasing agent (e.g., trichloroethylene).
5. Spring clamps and pads.
6. Self-adhering Teflon⁹ fluorocarbon polymer or Mylar JG tape.
7. Gauze sponges.

⁹Reference to specific products does not imply endorsement by the U.S. Bureau of Mines.

8. Strain gage application kit.
9. Temperature-controlled soldering iron, solder, rosin flux, and rosin solvent.
10. Fume hood for cleaning the cantilever and tools, and an oven for heat treating the adhesive.

Because beryllium-copper will oxidize, the strain gages should be epoxied to the cantilever and placed into the oven within 1 h after the cantilever is etched and air dried.

The following list of materials will be needed to assemble the 3-C BHG. Obtain all the components before starting the assembly.

1. BHG body, case, and placement end.¹⁰
2. Six transducers and locking nuts.
3. Shielded eight-conductor neoprene cable with No. 18 wire about 100 ft long (figs. 2.5-2.6).
4. Male and female (5/8-in.-diam) nine-pin miniature hexagon connector (manufactured by the Amphenol

¹⁰Drawings of this equipment appear in appendix B.

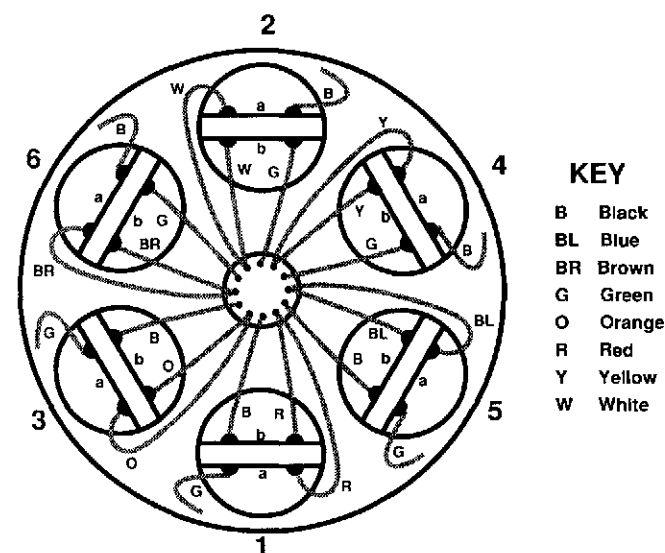


Figure 2.5.—Transducer location and color code. (1-6 = cantilevers.)

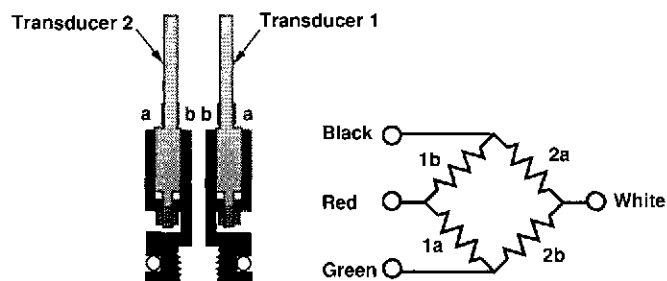


Figure 2.6.—Wiring diagram for one component in 3-C BHG. (1a and 1b = strain gages on transducer 1; 2a and 2b = strain gages on transducer 2.)

Products Div. of Allied Signal, Inc.) with gold-plated pins and sockets (fig. 2.7).

5. No. 30 stranded hookup wire.
6. Clamping nut, rubber grommet, and concave washer.
7. Gage body and piston O rings.
8. Stainless steel screws.
9. No. 23 music wire for gage springs.
10. Custom connector placement tool and custom piston nut wrench.

The following items are needed for the gage to function but are not required for the gage assembly.

1. Eight pistons.
2. Piston washers, about 48 0.005 in and 12 0.015 in thick.

Figure 2.5 shows the location of the transducers in the BHG body and figure 2.6 shows the color code for wiring the transducers, which can be helpful in making electrical repairs or replacing a transducer.

BOREHOLE GAGE CALIBRATION

The following list describes the procedure for calibrating the 3-C BHG. The gage should be calibrated before and after use to verify calibration and to ensure that the gage is functioning correctly.

1. Grease all pistons.
2. Put the pistons into the gage.
3. Place the gage into the calibration jig (fig. 2.8).
4. Line up the pistons for component 1 (U_1) with the micrometers by positioning a piston from component 2 at the "viewing hole" in the jig.
5. Tighten the three wing nuts.

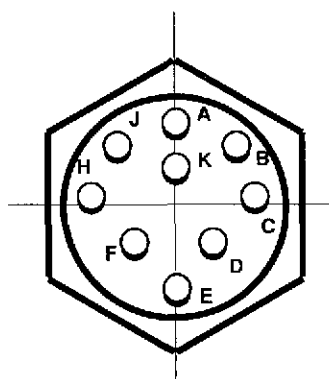


Figure 2.7.—Wire hookup to connector.

KEY

- A Black
- B Orange
- C Brown
- D Blue
- E Yellow
- F Green
- H Red
- J White
- K Not used

6. Check the piston alignment through the "viewing hole."

7. Set the strain indicator on "full bridge," center the balance knob if there is one, and set the gage factor (G.F.) at 0.40 or the G.F. desired. (The G.F. setting for calibration must be the same as that used in the field for over-coring and biaxial testing.)

8. Hook the wires for component 1 (U_1) (black, green, white, and red) to the indicator as shown in figure 2.9 and balance it.

9. Turn one micrometer in until the needle of the indicator begins to move. The micrometer is now in contact with the pistons.

10. Do the same with the opposite micrometer.

11. Rebalance the indicator if necessary.

12. Record this no-load reading (zero displacement).

13. Turn each micrometer in 0.0160 in (a total of 0.0320 in displacement for component 1). (The micrometer reads to ten-thousandths of an inch.)

14. Balance the indicator.

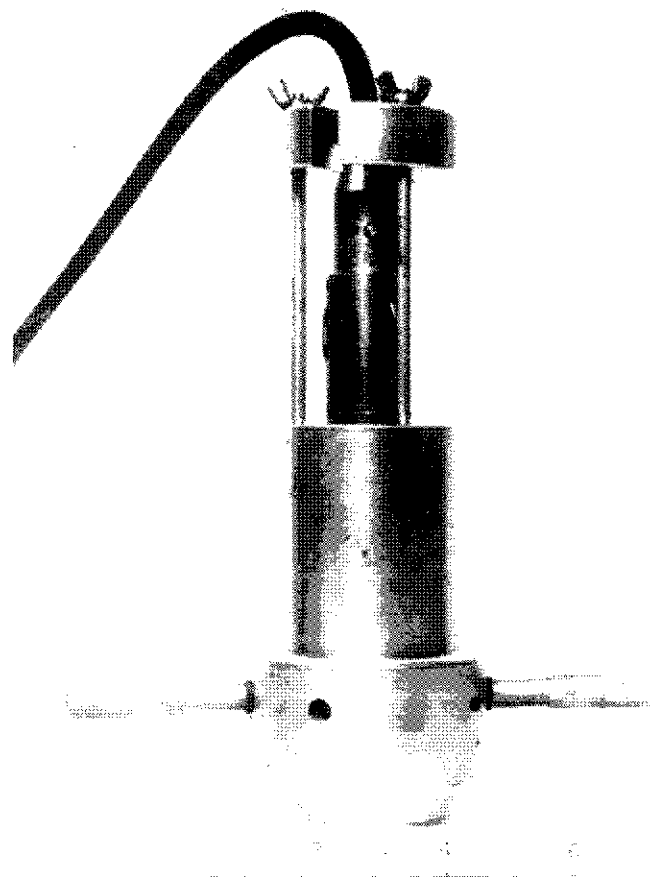


Figure 2.8.—BHG calibration jig.

Sensitivity knob: turn full clockwise.
Balance knob: put in midrange (5 turns of the 10-turn potentiometer).
Bridge switch: to full.

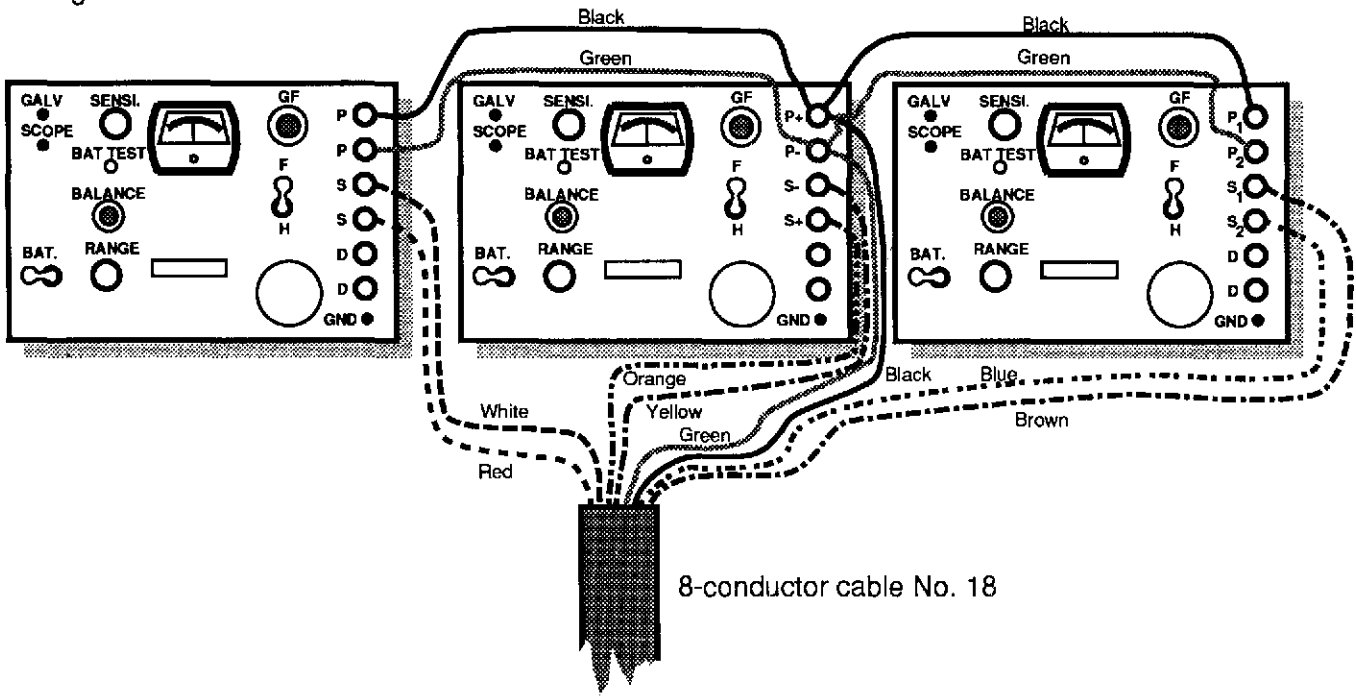


Figure 2.9.—Wire hookup to strain indicators.

- 15. Record the reading.
- 16. Wait 2 min and check the combined creep for the two transducers. The creep for 2 min should not exceed 20 μ in/in.
- 17. Record the new reading.
- 18. Back off each micrometer 0.0040 in (a total of 0.0080 in for the component).
- 19. Balance and record.
- 20. Continue this procedure until the micrometers are back to the zero position. This reading is also the zero displacement reading for the second cycle.
- 21. Repeat steps 13 through 20 for the second cycle.
- 22. Loosen the wing nuts and rotate the gage clockwise to line up component 2 (U_2) with the micrometers.
- 23. Retighten the wing nuts.
- 24. Hook up the wires for component 2 (U_2) (black, green, yellow, and orange) to the strain indicator (fig. 2.9) and balance it.
- 25. Repeat steps 9 through 21 for component 2.
- 26. Calibrate component 3 (U_3) in a similar manner.

Table 2.1 is an example of a calibration for one component, at a G.F. of 0.40. The calibration should consist of at least two cycles.

Table 2.1.—Example of calibration for one component at G.F. of 0.40

Displacement, in	Indicator reading, μ in/in	Difference, μ in/in
CYCLE 1 ¹		
0.0000	17,036	NAp
0.0320	48,480	(²)
0.0320	48,473	31,333
0.0240	40,558	23,418
0.0160	32,437	15,297
0.0080	24,398	7,258
0.0000	17,140	NAp
CYCLE 2 ¹		
0.0320	48,487	(²)
0.0320	48,476	31,296
0.0240	40,584	23,404
0.0160	32,422	15,242
0.0080	24,340	7,160
0.0000	17,180	NAp

NAp Not applicable.

¹Calibration factor: $\frac{\text{Known displacement } (\mu\text{in})}{\text{Indicator units}} = \mu\text{in/indicator unit.}$

²Wait 2 min to check for excessive creep.

Subtract the zero displacement reading (last reading of each cycle) from all the other indicator readings to determine differences. These values are in microinches per inch. To eliminate any effects of friction between the piston O ring and the wall of the hole in the gage case, the least difference value (in cycle 1, 7,258 $\mu\text{in/in}$ at 0.0080-in displacement) is subtracted from the largest difference value (31,333 $\mu\text{in/in}$ at 0.0320 in). This difference (24,075 $\mu\text{in/in}$) is divided into the deformation differences for the given range; in this case, 0.0240 in or 24,000 μin . The result is the calibration factor. Repeat these calculations for the second cycle and average them to obtain the calibration factor for component 1. Thus, for cycle 1, 24,000 divided by 24,075 equals 0.997, and for cycle 2, 24,000 divided by 24,136 equals 0.994. Use a calibration factor of 1.00 for the component. Repeat the above procedure to determine the calibration factors for components 2 and 3.

The above method is a faster means of calculation and yields results accurate to within 1 pct of the least squares method. However, if desired, a least squares determination can be made using a plot of micrometer displacement versus indicator units.

TEMPERATURE CALIBRATION

By matching the resistance of the four strain gages in the Wheatstone bridge to within 0.01 ohm, temperature stability for the BHG should be achieved. To determine whether temperature stability has been achieved for a particular BHG, a temperature calibration should be conducted using the following procedure:

1. Hook the wires from the BHG to the strain indicators.
2. Remove all pistons from the gage.
3. Record the strain reading for each component and the temperature.
4. Place the gage in a refrigerator.
5. After the gage comes to equilibrium, record the strain readings and temperature.
6. Subtract these strain readings, taken while the gage was in the refrigerator, from the room-temperature strain readings.
7. Subtract the refrigerator temperature from the room temperature.
8. Divide this temperature difference into the strain reading differences (step 6) to obtain the temperature calibration factor for each component.

An indicated strain of 2 $\mu\text{in/in}/^\circ\text{F}$ or less is acceptable. The same procedure is performed at elevated temperature by placing the gage in a laboratory oven. Oven temperature should not exceed 120° F (49° C) unless calibrating the "high temperature" BHG.

Temperature differences generally are not prevalent underground and corrections for temperature are not necessary. Surface work may require either making a temperature correction by using the temperature calibration or adjusting the water temperature to that of the rock. This is accomplished by using ice to lower the temperature or a torch to increase the temperature (32).

CHAPTER 3.—OVERCORING EQUIPMENT AND TECHNIQUES USED IN ROCK STRESS DETERMINATIONS

INTRODUCTION

Stress-relief techniques and instrumentation have been developed by the U.S. Bureau of Mines through many years of research and are successfully used to determine the in situ state of stress in rock. The drilling equipment and accessories used in the overcoring operation, along with a description for operating the BHG and for performing the overcoring procedures, are discussed in detail by Hooker and Bickel (1) and Bickel (2). The information presented in this chapter will facilitate understanding the techniques used for performing overcoring and for operating the BHG.

The Bureau-developed 3-C BHG (10), which measures three diameters 60° apart and has all sensing elements in the same plane, is designed to measure diametral deformations U of an EX-size (1½-in.-diam) borehole during stress relief by overcoring (59). The process essentially consists of (1) drilling an EX-size pilot borehole with a diamond bit and reamer, (2) positioning the BHG in the pilot hole, and (3) drilling a larger concentric borehole (e.g., 6-in.-diam)¹¹ over the BHG and recording U_1 , U_2 , and U_3 . The core is cut in one continuous drilling operation, and U_1 , U_2 , and U_3 are taken at the start, during, and at the end of the drilling. Generally, U_1 , U_2 , and U_3 are taken every 1/2 in of drill penetration. This will provide some data if the core breaks prematurely. Also, plotting U_1 , U_2 , and U_3 ¹² will reveal any abnormality that could effect the accuracy of the measurements. Drilling is concluded when the borehole deformation no longer occurs, indicating that the stress is relieved from the core (fig. 3.1). The BHG and core are removed from the borehole and the procedure is repeated. The core orientation and the position of the gage are marked on the core. Figure 3.2 shows a general configuration, after a completed overcore, of the pilot borehole, BHG, and diamond overcoring bit (barrel). Field results show that overcoring bits ranging from 4- to 12-in.-diam could be used, but the 6-in.-diam bit is considered the optimum size from both an economical and practical standpoint.

The marked core is then tested in a biaxial chamber (13) to determine the modulus of elasticity E and anisotropy of the rock. The elastic properties and U_1 , U_2 , and U_3 from each overcore are used to calculate the two-dimensional state of stress in the plane normal to the axis of the borehole. The core is optionally tested in a triaxial

chamber (14) at the same stress level it experienced in situ (which is not always possible during biaxial testing) to determine Poisson's ratio ν and a more accurate E (60). A least squares method has been developed to calculate the average rock stress components from multiple overcoring tests (47). If deformation measurements U are obtained from at least three nonparallel holes, the three-dimensional state of stress can be determined (9).

DRILLING EQUIPMENT

The following drilling equipment is recommended for in situ stress measurements. The number and lengths of drill rods and core barrels needed will depend on the objectives and purpose for making the measurements.

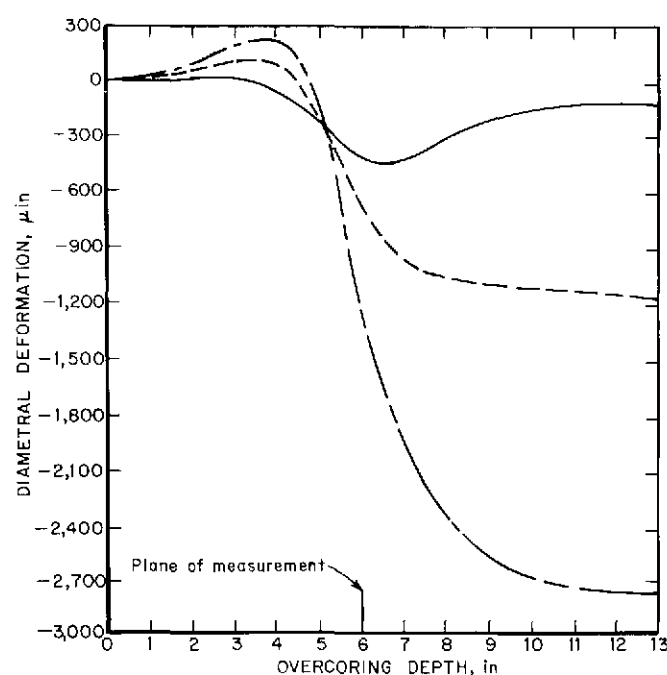


Figure 3.1.—Deformation measurements for three diameters from typical overcoring process.

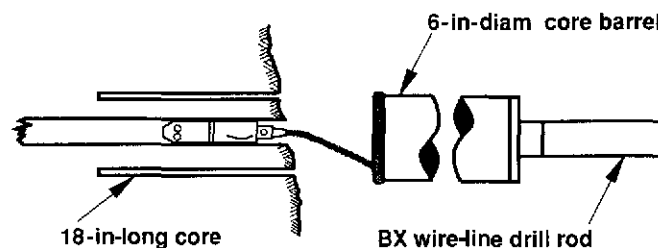


Figure 3.2.—Position of BHG in pilot borehole.

¹¹A 6-in.-diam thin-wall diamond masonry bit produces a 5½-in.-diam core.

¹²A computer code for plotting U_1 , U_2 , and U_3 is presented in appendix C.

1. Drill that can provide chuck speed of 120 rpm and penetration rate of 1/2 to 1 in/min using 6-in-diam overcoring bit. The chuck and quill must be large enough to accommodate an EW¹³ drill rod (fig. 3.3). Since drills do not normally have chuck speeds as low as 120 rpm, a change in gear ratio may be necessary. For a typical 20-hp air drill, the recommended range of rotation speed is 0 to 1,000 rpm, with feed ratios of 200, 300, 500, and 800 rpi.

2. EWG¹⁴ (EWX) double-tube, swivel-type core barrels, diamond bit, and reamer (fig. 3.4).

3. BX wire-line and EW drill rods and either fixed (fig. 3.5) or clamp-on-type drill rod stabilizers. Drill rods used for overcoring must be large enough in diameter to allow for the BHG and placement tool to pass through. The pin in the EW drill rod that passes through the drill and chuck must be drilled out to 9/16-in diam to accommodate the gage cable and drilling water.

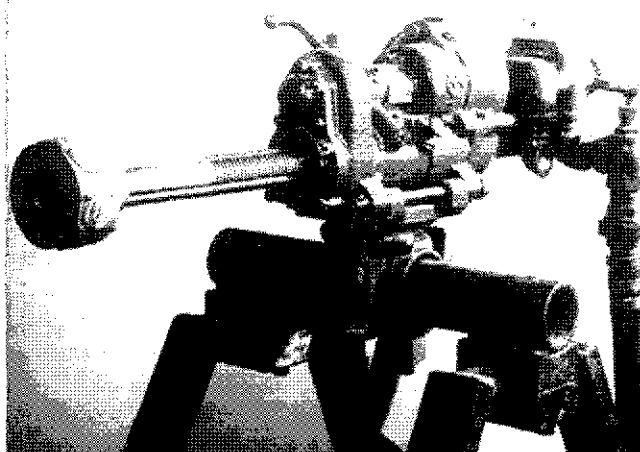


Figure 3.3.—Type CP-65 air drill.

¹³E, EW, B, and N denote diameter or size.

¹⁴G and X denote manufacturer's series of equipment.

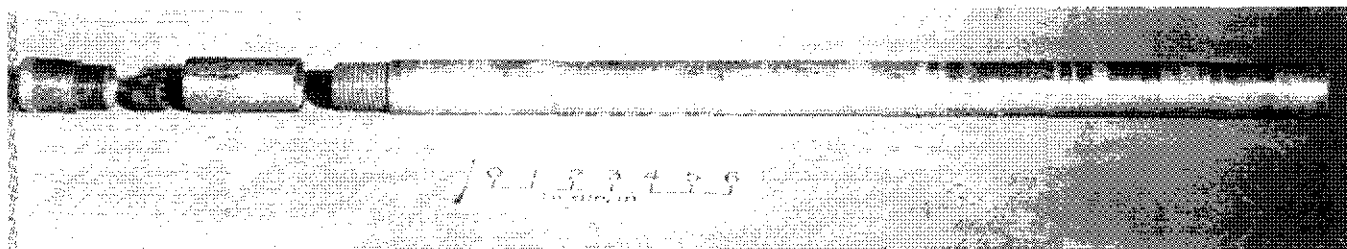


Figure 3.4.—EX-size bit with core spring, reamer, and 2-ft EWX core barrel.

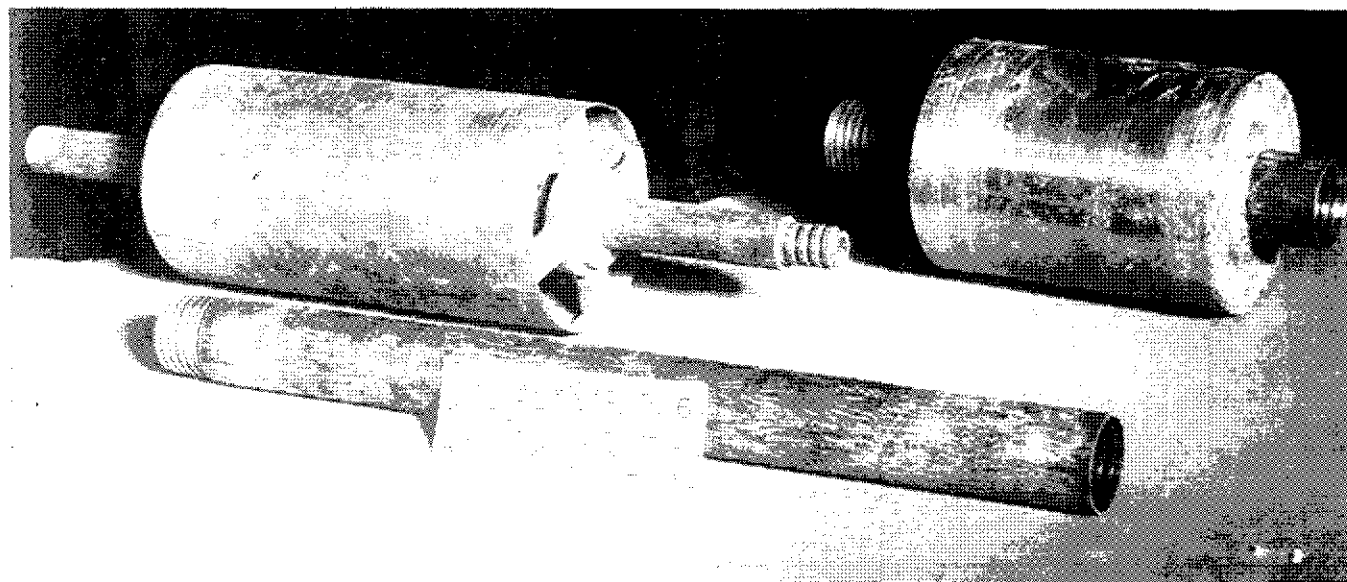


Figure 3.5.—Stabilizer on 2-ft section of EW drill rod (upper left), stabilizer on 1-ft section of BX wire-line drill rod (upper right), and 2-ft piece of BX wire-line drill rod (bottom).

4. Modified water swivel in which the waterway passing through the swivel is drilled out to 9/16-in diam and the water intake valve and pipe fitting are relocated to enter the swivel at 90° (fig. 3.6). Also required are a solid plug to seal the water swivel during normal drilling, and a plug with a 1/2-in hole to allow exit of the gage cable during the overcoring operation. Rubber washers [5/8-in outside diameter (OD), 7/16-in inside diameter (ID)] are placed inside the water swivel in front of the plug to minimize the amount of water squirting out around the cable.

5. Open-end, 6-in-diam thin-wall masonry overcoring bit and expander head (fig. 3.7). Open-end overcoring bits allow for easy removal of expanded or broken core.

6. Starter bit, 6-in diam (fig. 3.8).

7. Centering stabilizer (fig. 3.9).

8. Rod hook to fit BX wire-line drill rod for overcoring in an uphole.

9. Core breaker and retrieving tools (figs. 3.10-3.11).

INSTRUMENTATION

The following instruments are required for the overcoring operation, for testing drilled core, and for calibrating the BHG.

1. Standard or modified BHG (also, if available, the 2-C BHG), special piston pliers (fig. 3.12), 0.005- and 0.015-in-thick brass washers, and silicone grease.

2. Three Vishay model P-350 or equivalent strain indicators.

3. BHG placement and retrieval tools (fig. 3.13).

4. BHG calibration jig (fig. 2.8).

5. Biaxial pressure chamber (fig. 1.8).

6. Triaxial pressure chamber (optional) (fig. 1.9).

SITE SELECTION

To calculate the complete three-dimensional state of stress, deformation measurements must be obtained in at least three nonparallel holes. Typical layouts of boreholes and directions of measurement are shown in figure 3.14. Three orthogonal boreholes provide the best configuration of three boreholes for determining the six stress components with uniform precision (fig. 3.14A). It is possible to obtain good results from configurations 3.14B, C, and D, where the angle between the horizontal holes has been decreased from 90° to not less than 60°. Borehole configurations 3.14A, B, and C require only one drill setup. Deformation measurements should be made outside the zone of influence of the mined opening. This distance is usually taken to be one d of the mined opening. Geologic or structural conditions may require that the zone of stress measurement be confined to a small area. In these cases, configuration 3.14D will provide good engineering estimates of the stress components, but a high standard deviation (SD) will exist for any calculated principal stress component with an orientation parallel or subparallel to the axis of any of the boreholes. This configuration also requires three drill setups.

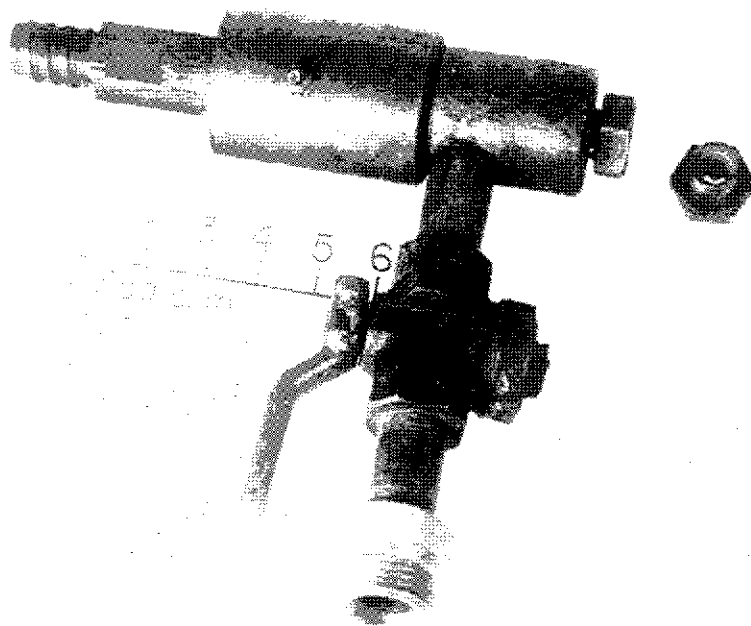


Figure 3.6.—Water swivel with solid plug and plug (right) used during overcoring.

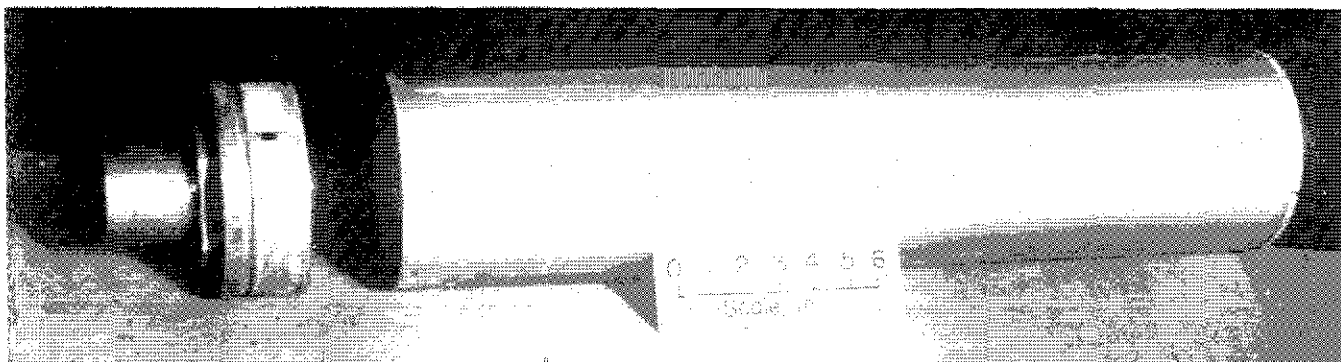


Figure 3.7.—Expander head assembled (adapted for BX wire-line drill rod) and 6-in-diam core bit.

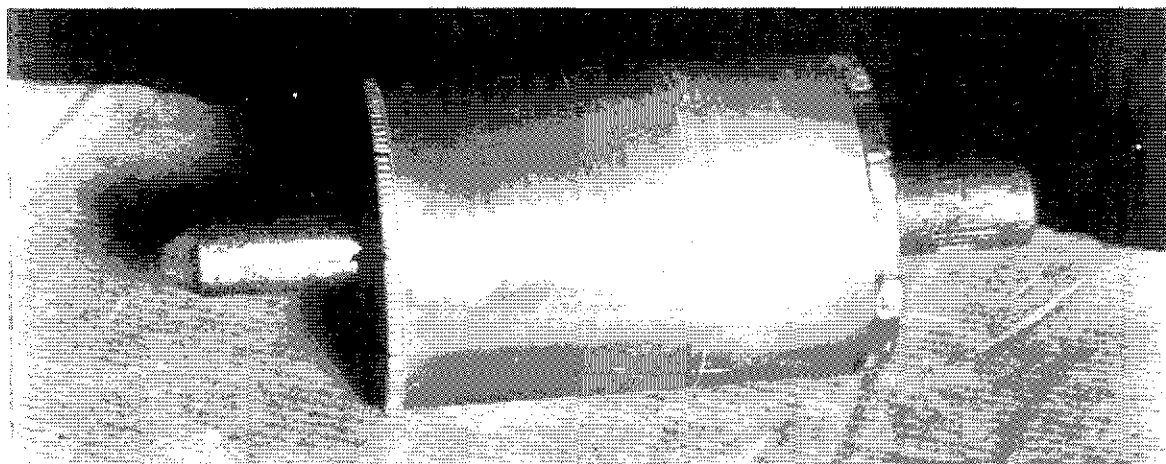


Figure 3.8.—Starter bit (6 in. in diam by 1 ft long) with pilot and expander head adopted for EW drill rod.

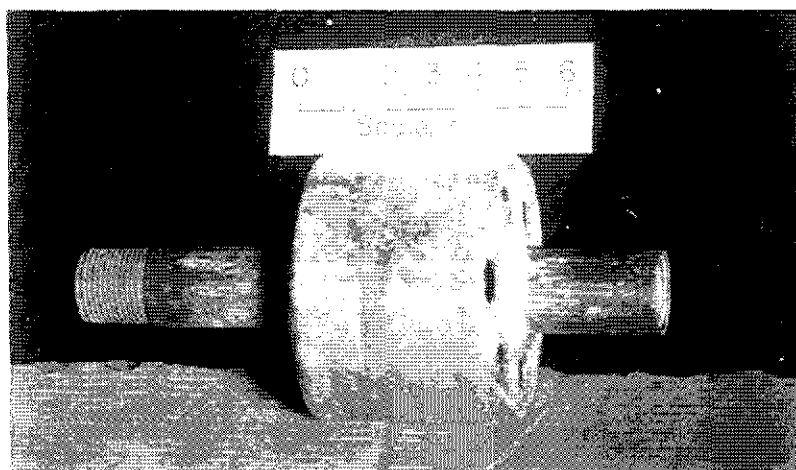


Figure 3.9.—Typical centering stabilizer.

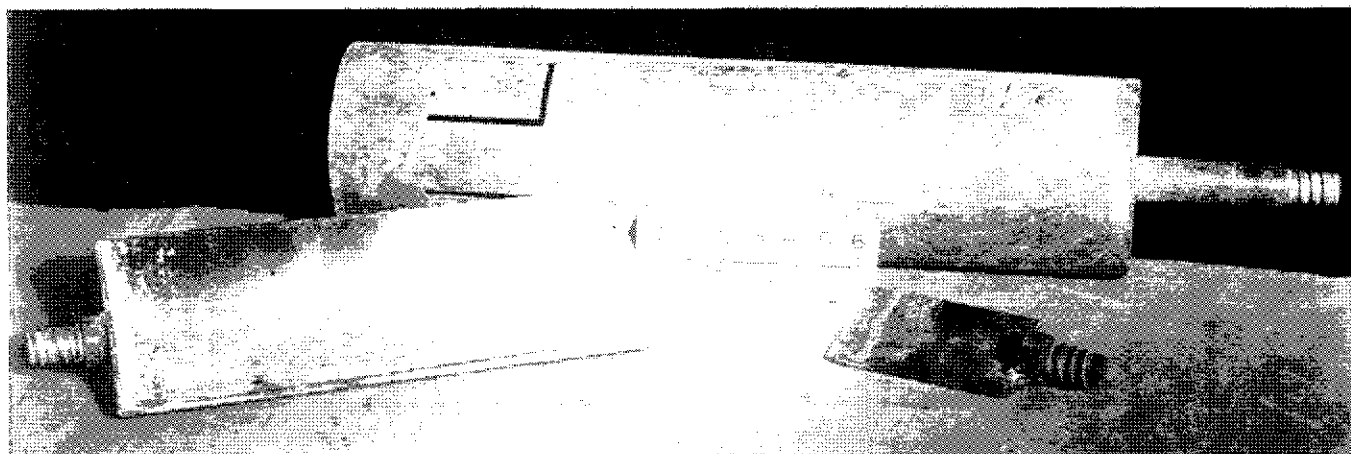


Figure 3.10.—Core puller (top), core shovel (lower left), and core breaker (lower right).

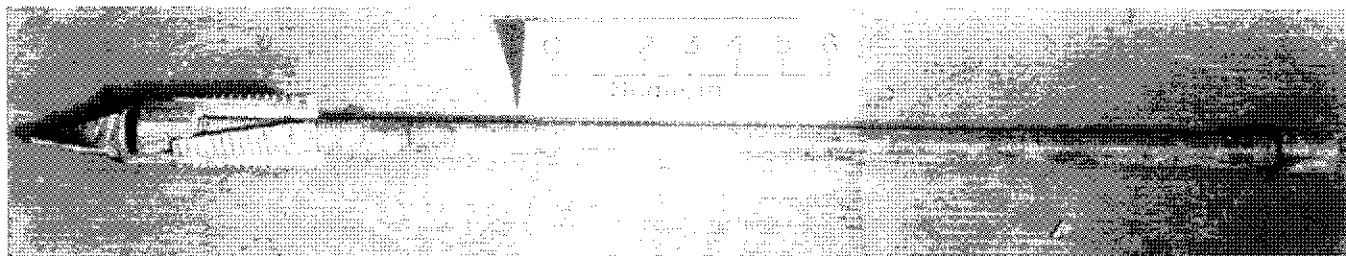


Figure 3.11.—Retrieval tool used in place of core puller to retrieve 6-in-diam core from vertical hole that has EX-size hole through core.

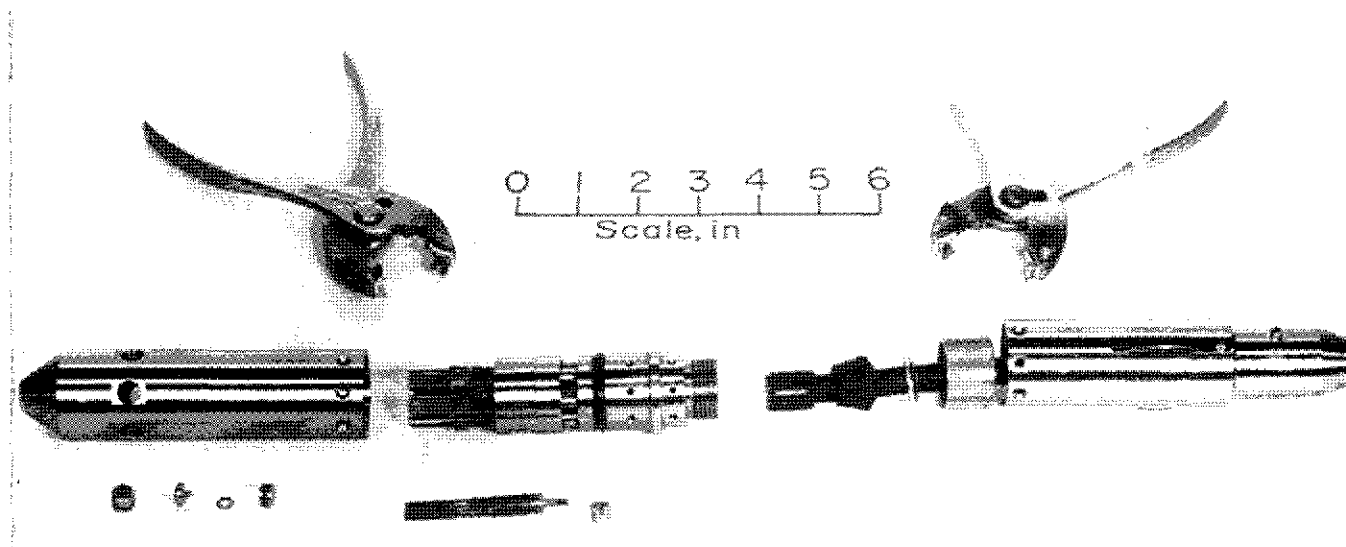


Figure 3.12.—Special pliers (top), Bureau-developed disassembled 3-C BHG (center), piston, disassembled piston and washer, and transducer with nut (bottom).

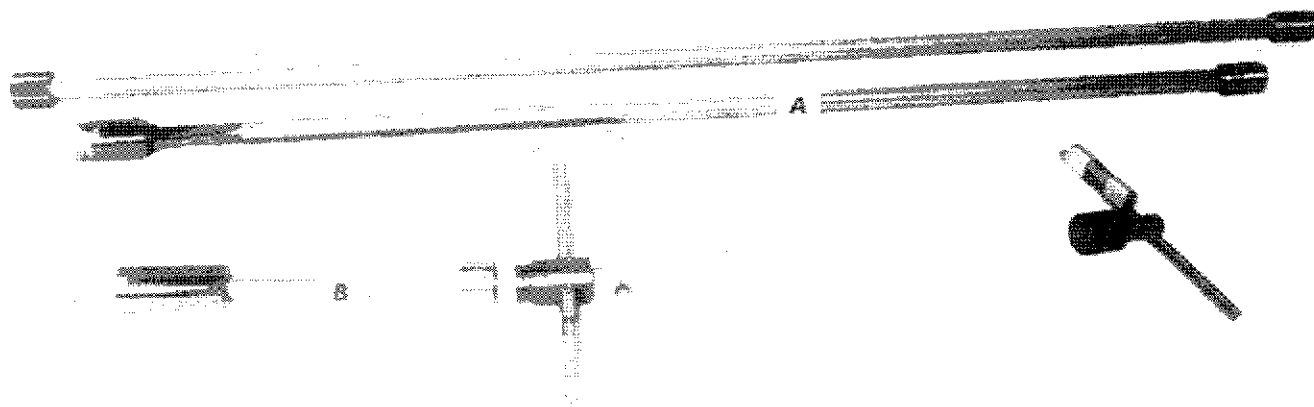


Figure 3.13.—Field placement and retrieval tool (A); biaxial test orientation tool (B).

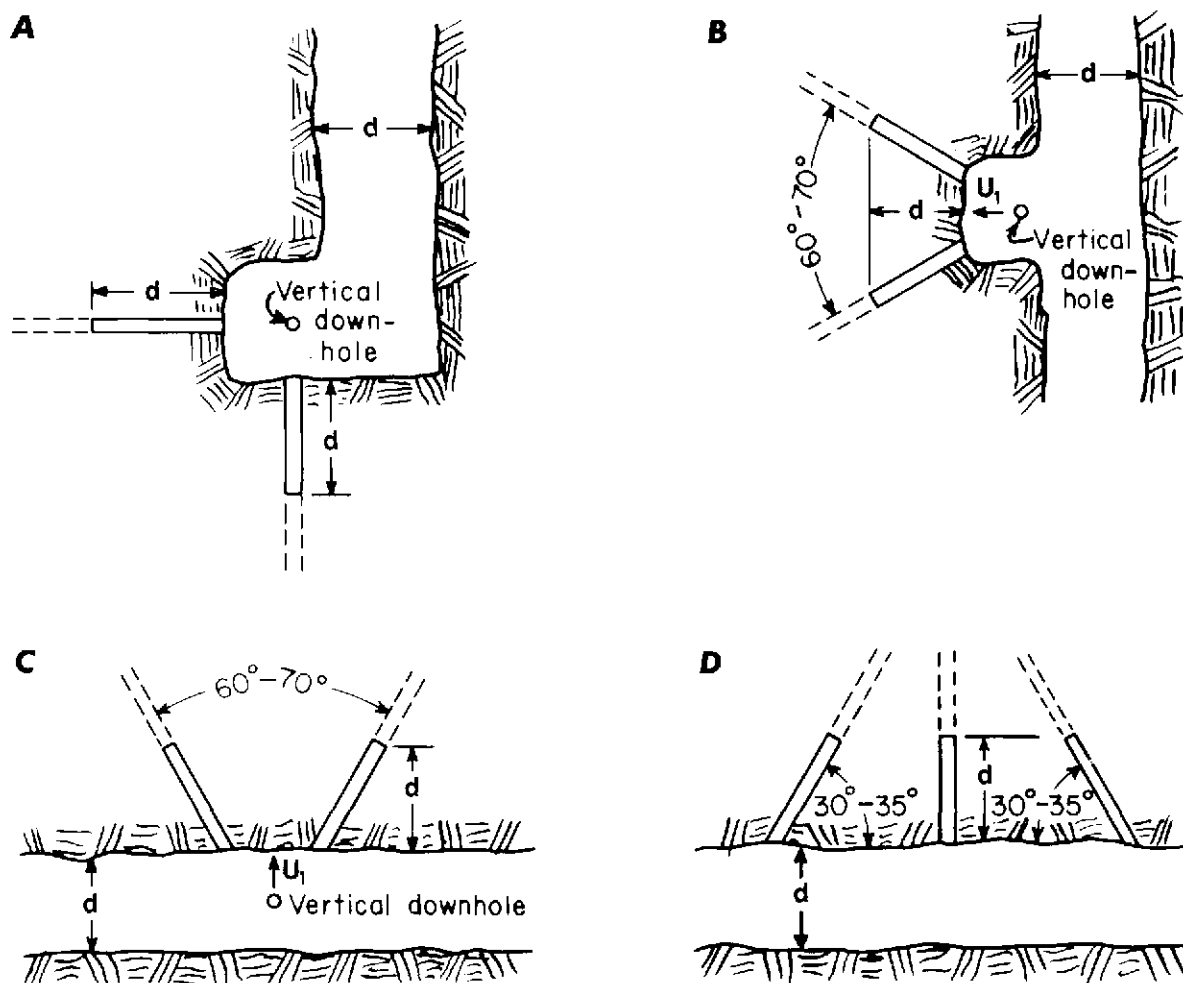


Figure 3.14.—Drill hole configurations for optimum setup (A), alternative setups (B-C), and setup when measurements in vertical boreholes are not obtainable (D).

puller (fig. 3.10). Mark the orientation, depth, and plane of measurement on the core. Cores should be tested in a biaxial chamber (chapter 4) as soon as conveniently possible after recovery to determine the modulus of elasticity E .

14. Place the overcoring bit back into the hole and repeat steps 5 through 13 until sufficient overcore stress reliefs are obtained to meet the project objectives. (Caution: Always check the inside of the overcoring bit for pieces of rock or rock disk before inserting the bit back into the hole.)

When the overcoring operation is performed in fractured rock, zones of mineralized veins, or in rock that is diskings, a minimum core length of 12 in or gage placement of at least 6 in may not be possible. By using the reversed case, the plane of measurement can be set at any distance inside the gage hole. However, a correction must be made to the deformation measurements since the gage depth relative to the collar of the borehole determines the shape of the borehole deformation versus drill depth curve (stress-relief curve). Figure 3.20 shows stress-relief curves

as constructed from finite-element calculations (12-13), which represent a typical overcore curve. The top curve represents a "standard" overcoring stress relief. The bottom curve represents an overcoring stress relief with the plane of measurement of the gage positioned 1 in inside the EX-size borehole. Generally, a complete overcoring stress relief is achieved with the plane of measurement of the gage positioned at least 6 in inside the EX-size borehole. Referring to curve A in figure 3.20, the section of curve from 3 to 8.5 in represents the effects of the stress concentration that occur around the EX-size borehole ahead of the overcoring bit. Figure 3.21 is a finite-element representation of overcoring at depth in a medium showing the area of the next 6-in-diam core affected by stress concentration (11). This stress concentration will affect the magnitude of the measured U during the next overcoring operation if the plane of measurement is within this section of the borehole (e.g., figure 3.20, curve B). Hence, the resultant stress-relief deformations will include the applied stress and stress concentration. Thus, it is necessary to subtract the stress concentration from the

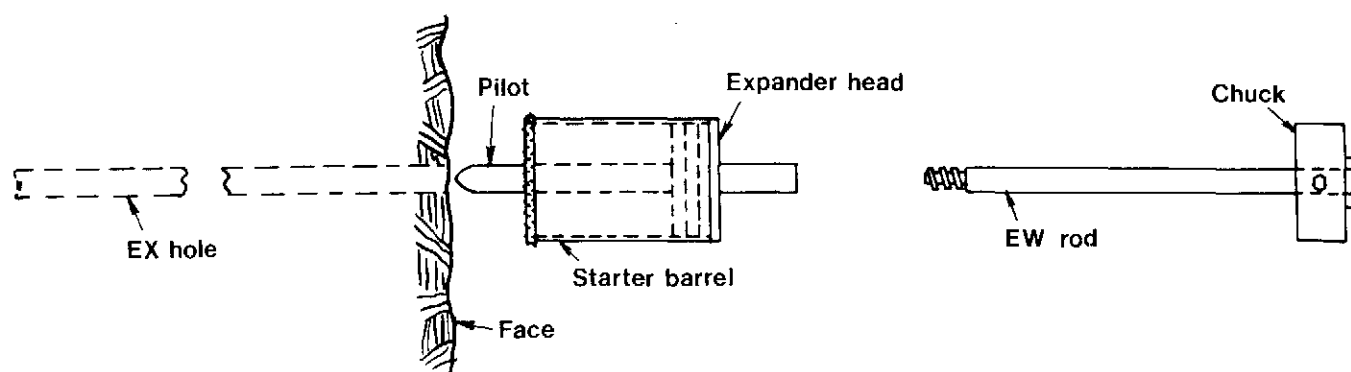


Figure 3.16.—Starting 6-in-diam overcoring hole.

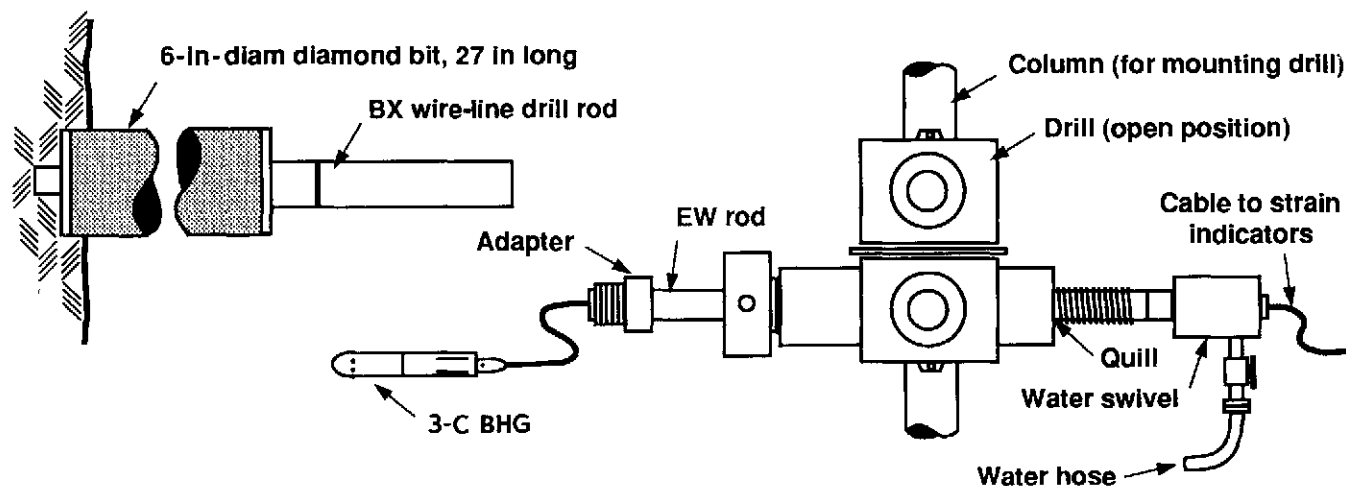


Figure 3.17.—Setup for 3-C BHG emplacement.

Hole No. _____ Date _____ Orientation: U_1 _____

Gage No. _____ Calibration factor U_1 _____

Gage factor _____ U_2 _____

True bearing of hole _____ U_3 _____

Temperature: Rock _____ Water _____

DEPTH			DEFORMATION			REMARKS
Gage	Hole (+)		INDICATOR READING			
			U ₁	U ₂	U ₃	
		Zero				TIME: Gage Set Overcore Start
9"	Face	Bias				
	1/2"					
	1"					
	1 1/2"					
	2 "					
	2 1/2"					

	7"					
	7 1/2"					
	8"					
	8 1/2"					
Pistons	9"					
	9 1/2"					
	10"					
	10 1/2"					
	11"					

	13"					
	13 1/2"					
	14"					
	14 1/2"					
	15"					
	15 1/2"					
	16"					
	16 1/2"					
	17"					
	17 1/2"					
	18"					

Figure 3.18.—Example of field data sheet.

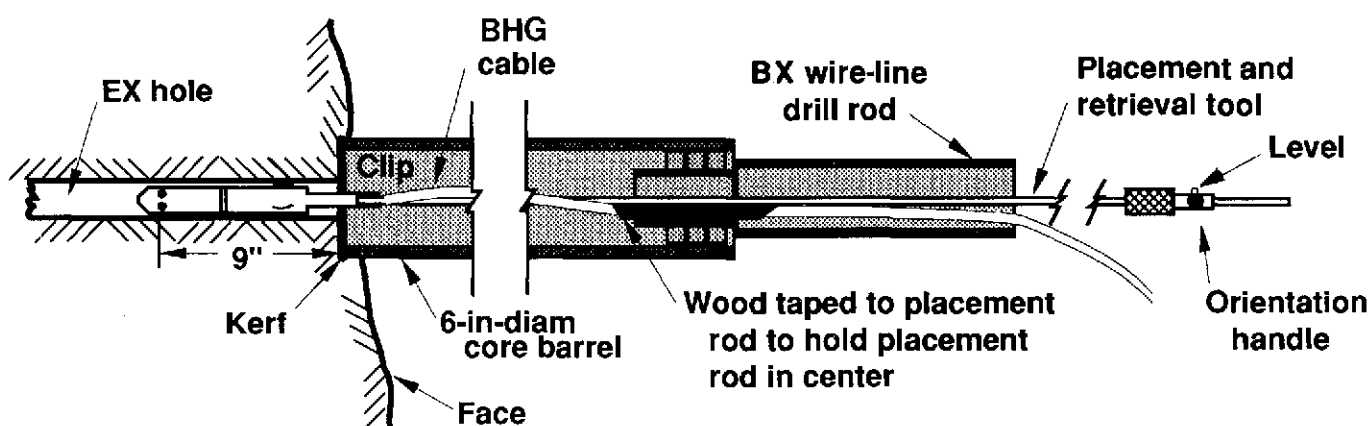


Figure 3.19.—Positioning 3-C BHG with placement rod through 6-in-diam core bit and drill rod.

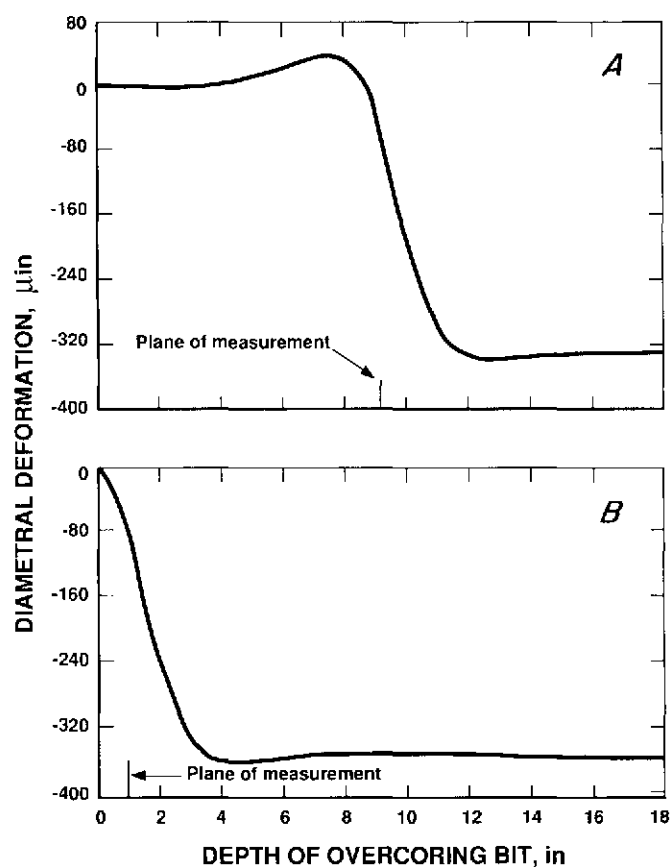


Figure 3.20.—Plot of diametral deformation versus depth of overcoring bit taken from finite-element calculations for typical overcore (A) and overcore with plane of measurement at 1 in (B).

measured deformation by utilizing the curves in figure 3.22 (11). This figure represents the results of finite-element calculations of U for a range of Poisson's ratio ν . The percentage increase in deformation is, for all practical purposes, independent of the E and the stress level (11); therefore, only the ν of the rock must be specified to

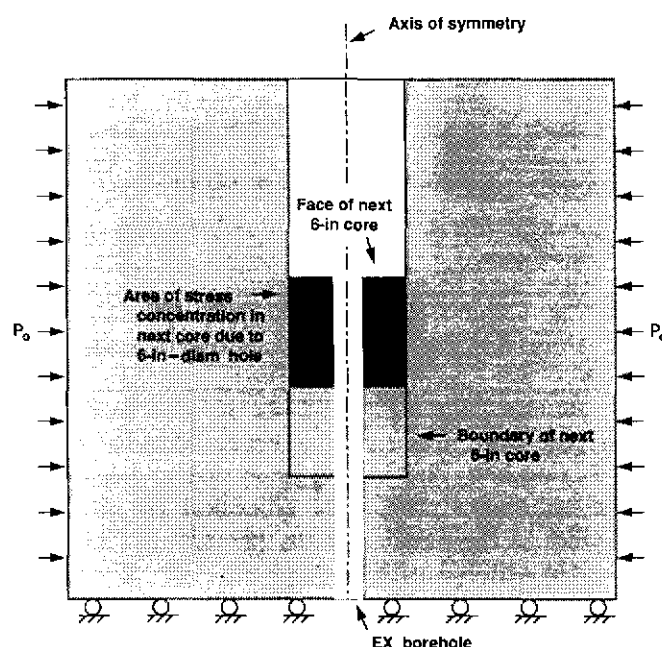


Figure 3.21.—Finite-element model of overcoring at depth.

determine the correction factor for any gage placement inside the EX-size borehole (11). For an overcoring operation performed at the surface of a medium, this concentration of stress does not exist and a correction in the measured deformation is not required.

PROCEDURE FOR OPERATING THREE-COMPONENT BOREHOLE GAGE

The procedures described in this section have been developed through use of the BHG over a long period of time. It is important that the details are followed closely to minimize any chance of damage to the gage and to provide the most reliable data acquisition.

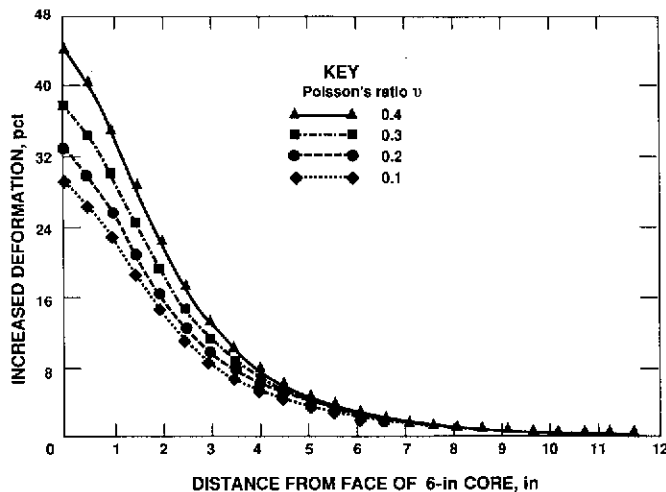
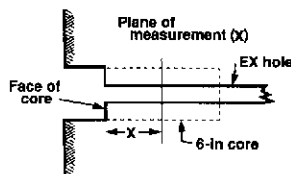


Figure 3.22.—Increased deformation versus distance from face of 6-in-diam core.

1. Hook the gage to the indicators, as shown in figure 2.9.

2. Remove all the pistons from the gage with special pliers (fig. 3.12). The special pliers are made by brazing a 1/2-in-diam by 3/8-in-long brass rod to the jaws of a standard pair of pliers. The cylindrical axis of the rod is parallel to the handles of the pliers.¹⁵ An 11/32-in-diam hole is drilled axially through the center of the brass. The pliers are opened by sawing through the brass along the diameter perpendicular to the handles of the pliers, leaving a half-round piece of brass attached to each jaw. The outside edges are then filed to fit inside the 1/2-in-wide milled flats of the gage case.

3. Record the zero reading for each of the three diametral components. Label them U_1 , U_2 , and U_3 zero readings.

4. Grease the O ring on each piston and insert the pistons into their proper place in the gage.

5. Place the gage into the EX-size hole or into a test specimen if the work is being done in the laboratory. (Caution: Do not force the gage into the EX-size hole.) If it fits very tightly (maximum allowable force is 42 lb), remove washers from the pistons as follows:

a. Remove one piston of a diametral pair with the special pliers.

b. Unscrew the piston sections with both pliers, being careful not to grasp the O ring with the pliers.

c. Remove a washer and screw the piston firmly back together.

d. Grease the O ring and insert the piston into the gage.

e. Repeat steps a through d for the remaining two diametral pairs. Reset the gage in the EX-size hole. If the gage is still too tight, remove a washer from the remaining three pistons as stated in steps a through d. If the gage is very loose inside the EX-size hole when it is initially inserted, remove the pistons one at a time and add a washer using steps a through d.

6. When the gage is properly sized, it should offer minimal to moderate resistance as it is placed into the EX-size hole. After the gage is inserted to the proper depth, position it in the desired orientation.

7. Read all three components (U_1 , U_2 , and U_3).

When stress is relieved, as in field overcoring, the bias set on each component should be between 13,000 and 20,000 indicator units (microinches per inch) with a G.F. of 0.40. When stress is applied to a specimen, as in the laboratory or in the biaxial chamber, the bias set on each of the components should be between 8,000 and 15,000 indicator units for the same G.F.

The following tabulation shows typical zero readings and calibration factors for a G.F. of 0.40:

Zero readings (pistons out), $\mu\text{in/in}$	Calibration factors, $\mu\text{in/in}$
$U_1 \approx -4,000$	$U_1 = 1.00$
$U_2 \approx +3,100$	$U_2 = 1.00$
$U_3 \approx +4,100$	$U_3 = 1.04$

Care must be exercised not to overload the transducers, since not all BHG's have the element safety plug installed in the gage case. Maximum load on any component must not exceed 50,000 indicator units with a G.F. of 0.40. The purpose of the element safety plug is to restrict the end displacement of the transducer to less than 0.025 in.

Lowering the G.F. increases sensitivity. A G.F. of 0.40 provides a reading of one indicator unit for 1 μin of deformation. When a G.F. other than 0.40 is used, the working range and calibration factor must be determined in advance of operating.

¹⁵Diagrams of the 3-C BHG showing pliers appear in appendix B.

TROUBLESHOOTING BOREHOLE GAGE MALFUNCTIONS

Lack of Balance on One or More Indicators

If balance on one or more of the indicators is not achieved, recheck the wiring hookup to see that it is in accordance with the wiring diagrams in figure 2.9 and that all connections are tight. If balancing is still not achieved, the problem may be in the plug-in cable connection to the BHG. This is checked by removing all of the screws from the placement end of the gage, removing the end, turning off the knurled clamping nut, and pushing in the cable to ensure good connection. With the indicators set to their respective zero readings, each component will provide the indicator balance when a good connection is made. Screw the knurled clamping nut very tightly into place and replace the end. (Imbalance may occur when too much pull has been exerted on the cable accidentally or intentionally during gage retrieval after overcoring. In a vertical hole, sometimes cuttings or broken rock drop into the EX-size hole, impeding the retrieval tool from hooking onto the orientation pins of the gage. Do not attempt to retrieve the gage by pulling the cable. Instead, remove the drilling rod and the 6-in-diam core barrel from the hole and break off the core with the gage still in place. Remove the gage cable from the drill, the 6-in-diam core barrel, and the drill rod. String the cable through one of the holes in the end of the core puller and then retrieve both core and gage with the core puller.)

Insensitivity of One or More Elements on Indicator

If elements become insensitive to deflection of the pistons or nonresponsive to the turning of the indicator dial, the probability exists that moisture has entered the

gage either through the piston(s) or through the cable connecting plug. In this case, perform the following:

1. Remove the pistons and the BHG case to check for water. If water is present, check for a piston that may have been placed into the gage backwards; then check the piston O rings for a possible cut that may have occurred from gripping the O ring with pliers. Replace the O ring if necessary, grease all O rings, and replace the case.
2. Remove the placement end as described previously. If water is found inside the gage or in the cable connecting plug, check the grommet seal and the cable for cut(s); repair or replace as necessary. Dry out the gage and/or connecting plug completely, regrease the cable where it passes through the grommet, and tighten the knurled nut firmly. Replace the placement end.

Lack of Component Balance

If one component does not balance anywhere on the indicator dials or balances intermittently, then this is an indication that there is a disconnected wire or possibly a poor solder joint. Remove the BHG case and check all wires going to this component, including the plug-in cable connector. Solder where needed and replace the gage case.

Sensitivity of Indicator When Touched

If an indicator(s) is sensitive when it is touched, then this sensitivity generally occurs after the instrument is used for a period of time in very humid conditions. Use plastic or other insulating material underneath the indicators. Also, keep the indicators covered with plastic when they are not in use during the working day. Each night the indicators should be brought out of the mine and allowed to dry.

CHAPTER 4.—DETERMINING ELASTIC CONSTANTS FROM STRESS-RELIEVED CORE

INTRODUCTION

To compute the in situ rock stress from borehole deformation measurements in plane strain, it is necessary to determine the modulus of elasticity E and Poisson's ratio ν of the stress-relieved core obtained during overcoring. The E and elastic constants of the stress-relieved core (a thick-wall cylinder of rock) can be obtained by applying hydraulic pressure to the external surface of the core and measuring the borehole deformation U_1 , U_2 , and U_3 (2, 13, 15-17, 59). A biaxial device (13) and/or a triaxial device (14) are used to apply the external hydraulic radial pressure σ_p on the thick-wall cylinder of rock. The biaxial test procedure using the biaxial device (biaxial pressure chamber) provides a convenient method for determining the E and anisotropy (2, 15) of the rock and the effects caused by the gage hole not remaining concentric during the overcoring process. This apparatus is portable, and, therefore, measurements can be taken in the field, which is advantageous when rock prone to deterioration is tested. Furthermore, no core preparation is required. Because the ends of the core are unconstrained, a radial load (P_o) applied at the level of the in situ stress can cause the core to fail owing to the tensile strain that develops in the axial direction. Therefore, the maximum P_o must be low enough that the induced axial strain ϵ_3 does not fracture the rock in a plane normal to the axis of the core. Values for the stress level should range from 1,500 psi for granites down to 250 psi for tuff and layered rock. Values for the stress level applied to fractured rock that is still intact should not exceed 150 psi. The triaxial test eliminates this limitation by using an axial load to constrain the ends of the core before P_o is applied, thus preventing failure. The triaxial test, however, requires core preparation. The ν is determined from either the triaxial test or the laboratory uniaxial test. The triaxial test can also account for the non-linearity of the stress-relieved core by testing the core at the same stress level as the rock experienced in situ.

It is recommended that all stress-relieved core be biaxially tested at the minesite. A representative sampling of the core should be returned to the laboratory for preparation and triaxial testing.

From the biaxial test data, the anisotropic parameters α_{11} and α_{22} are calculated. These values are used with the field deformation measurements U_1 , U_2 , and U_3 to determine the secondary principal stresses P and Q normal to the axis of the borehole in plane stress. Triaxial testing provides the elastic constants necessary to calculate the secondary principal stresses P and Q in plane strain. If

deformation measurements and the elastic constants of the rock are obtained from three nonparallel holes, the three-dimensional state of stress can be determined. A least squares solution of the data provides a more accurate representation of the results (9).

BIAXIAL TESTING OF STRESS-RELIEVED CORES

During the biaxial test, a section of the stress-relieved core is radially loaded (fig. 4.1) and the corresponding center hole deformation is measured using the 3-C BHG. The relationship between the modulus of elasticity E and the center hole deformation U (fig. 4.2) is given by (13, 51, 61):

$$E = - \frac{4ab^2P_o}{(b^2 - a^2)U}, \quad (4.1)$$

where b = outer radius.

Briefly, the biaxial method to determine the modulus of elasticity E from stress-relieved core (2) consists of:

1. Positioning the core into the biaxial pressure chamber with the in situ plane of measurement in the center third of the chamber; preferably, in the center of the chamber.
2. Connecting the BHG cable wires to the three strain indicators and zeroing the indicators.

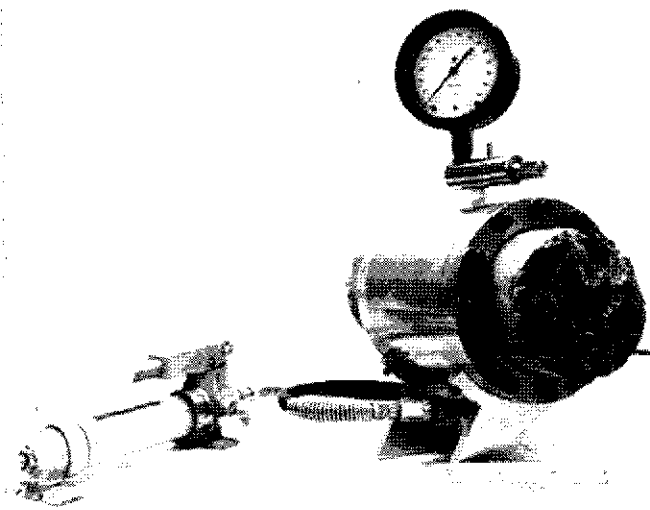


Figure 4.1.—Biaxial chamber with core and pump.

3. Inserting the 3-C BHG from the collar end to the plane of measurement with the gage pistons positioned at the same depth and orientation as in situ.

4. Checking for sufficient bias and recording the three no-load readings.

5. Applying a radial load P_o to the core and recording U_1 , U_2 , and U_3 .

6. Releasing P_o and recording the three no-load readings.

7. Repeating this loading and measuring process with the gage rotated 15° , 30° , 45° , and 60° counterclockwise from the original orientation.

The last cycle (60°) is redundant to evaluate core quality and is averaged with the first cycle (0°) to provide 12 data points for calculating (equation 4.1) the maximum, E_1 , and minimum, E_2 , modulus of elasticity.¹⁶

Procedure and Apparatus

The cores recovered from the overcoring stress relief should be biaxially tested (2) as soon as possible to determine the modulus of elasticity E . Generally, the cores recovered from an overcoring hole are tested while the setup is made on the second hole and preliminary drilling performed. At least 6 in of intact core are required for the test. Cores cut with a used bit will fit the testing apparatus properly. Those cut with a new bit may require one wrap of 0.015-in-thick plastic to fit properly. The

core, either wrapped or unwrapped, is inserted into the biaxial chamber¹⁷ with the plane of measurement centered in the chamber and with the orientation mark (position U_1) at the 12-o'clock position (fig. 4.3). If it is not possible to center the plane of measurement, then it should be positioned in the center third of the chamber, keeping U_1 in the 12-o'clock position. If there is a clearance of more than approximately $1/16$ in, then the core must be centered in the chamber with wooden wedges (fig. 4.3) to prevent the liner from "popping out" and releasing oil when pressure is applied. This would require repairing the liner. The core is centered by using four pieces of wood approximately $1/32$ in thick, $1/8$ in wide, and 4 to 6 in long, with the last inch beveled to an edge. Place two wedges at each end between the core and the chamber at the 4:30 and 7:30 clock positions, inserting the wood about $3/4$ in. Then apply about 100 psi to the core, while opening the valve slowly, to remove any air entrapped in the system. Repressure the core (about 50 psi) to hold it rigid; then emplace the BHG and orient it with the biaxial chamber orientation tool (fig. 3.13B). The stress-relieved core is now ready for the test cycle.

Test Cycle

The following steps describe the procedure for biaxially testing the stress-relieved core.

1. Preload the core three times to a selected stress level.
2. Record the no-load (zero) reading for each component (U_1 , U_2 , and U_3) on the data sheet shown in figure 4.4.
3. Raise the radial load P_o to the selected level and record the deformation measurements U_1 , U_2 , and U_3 .

¹⁷Diagrams of the biaxial chamber and the placement tool appear in appendix B.

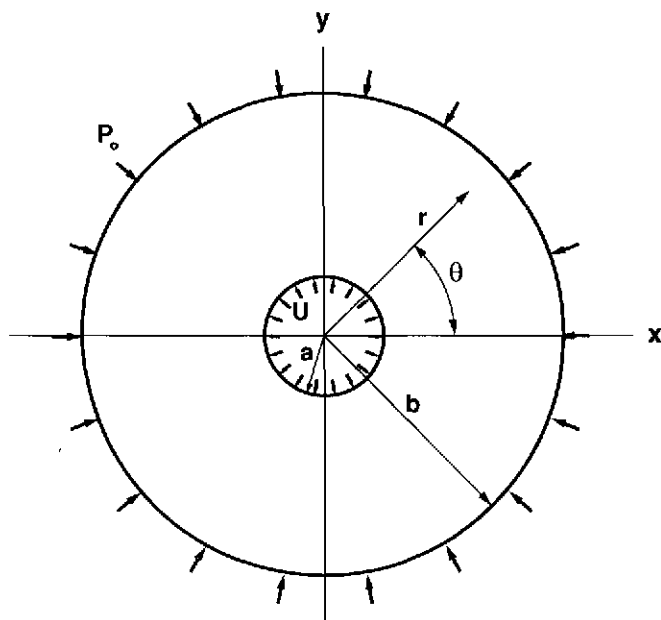


Figure 4.2.—Thick-wall cylinder section.

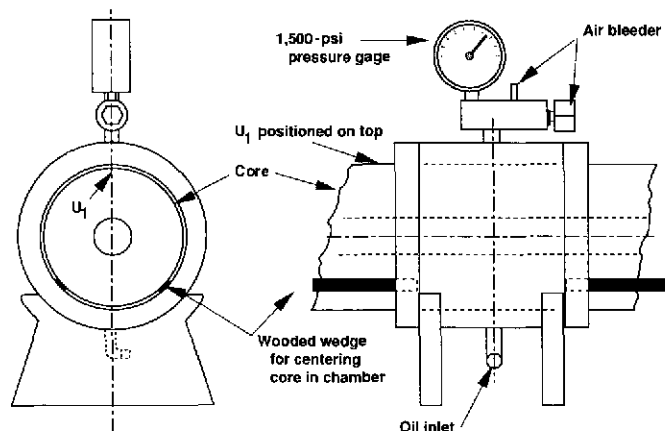


Figure 4.3.—Centering core in biaxial chamber.

4. Release the pressure and record the no-load readings.

5. Using the orientation tool, rotate the gage in the core 15° counterclockwise.

6. Preload the core twice.

7. Record the no-load reading for each component ($U_1 + 15^\circ$, $U_2 + 15^\circ$, and $U_3 + 15^\circ$). (See figure 4.4.)

8. Repeat steps 3 through 6.

9. Record the no-load readings ($U_1 + 30^\circ$, $U_2 + 30^\circ$, and $U_3 + 30^\circ$).

10. Repeat steps 3 through 6.

11. Record the no-load readings ($U_1 + 45^\circ$, $U_2 + 45^\circ$, and $U_3 + 45^\circ$).

12. Repeat steps 3 through 6.

13. Record the no-load readings ($U_1 + 60^\circ$, $U_2 + 60^\circ$, and $U_3 + 60^\circ$).

This fifth cycle is a repetition of the first cycle, except that component U_3 ($U_3 + 60^\circ$ reading) is now positioned at U_1 , component U_1 ($U_1 + 60^\circ$ reading) is at U_2 , and component U_2 ($U_2 + 60^\circ$ reading) is at U_3 . This last cycle, when compared with the first cycle, can give an indication of the condition of the core. Also, if the applied stress level (P_0) selected was not too high, the no-load (zero) readings should return to within 10 μ in from the starting zero readings. Any sudden loss of pressure during a cycle indicates core failure. After a core is removed from the biaxial chamber, always check the inside ends of the chamber to ensure that the liner has not "popped" out of position.

Failure to push back the liner into position will prevent removal of the next core tested and possible tearing of the liner. Dismantling of the chamber will be required to remove the core from the chamber and to replace the liner if necessary.

The no-load readings obtained from step 4 are subtracted from the load readings of step 3. The value from the unload cycle is used in the calculation for the modulus of elasticity E , because during the overcoring process, the core is relieved, unloading the transducer in the BHG. These differences in the readings are then multiplied by the calibration factors of the gage. The maximum and minimum E_1 and E_2 values or the anisotropic parameters α_{11} and α_{22} of the core are calculated from either a least squares fit to an ellipse using the computer code in appendix D or by plotting the deformation readings on polar coordinate paper.

After biaxial testing, NX-size core should be drilled axially from the stress-relieved core and tested uniaxially to determine the axial modulus of elasticity E_3 and Poisson's ratio ν .

When overcoring is performed in poor quality rock where at least 6-in-long stress-relief core is not obtainable for field biaxial testing, solid 5 $\frac{5}{8}$ -in-diam core is required from the overcoring hole or from an adjacent parallel drill hole not more than several feet away. In the laboratory, NX-size core is drilled radially in two directions, 90° apart, from the solid core, prepared (18), and tested in uniaxial compression to determine modulus of elasticity E . Tests

MINE NAME _____ DATE _____

GAGE USED _____; C.F. $U_1 =$ _____, $U_2 =$ _____, $U_3 =$ _____

σ_p	U_1	U_1+15	U_1+30	U_1+45	U_1+60	U_2	U_2+15	U_2+30	U_2+45	U_2+60	U_3	U_3+15	U_3+30	U_3+45	U_3+60	REMARKS
	HOLE NO. _____ DEPTH _____ RELIEF NO. _____															
0																
0																
Diff																
X C.F.																
	HOLE NO. _____ DEPTH _____ RELIEF NO. _____															
0																
0																
Diff																
X C.F.																
	HOLE NO. _____ DEPTH _____ RELIEF NO. _____															
0																
0																

Figure 4.4.—Example of typical biaxial data sheet. (Diff = difference; C.F. = calibration factor.)

on core from both directions will provide two moduli, but not necessarily the maximum E_1 and minimum E_2 moduli. These two moduli are averaged for an isotropic solution.

If diskings occur along the entire length of the 6-in-diam borehole, then other means for determining the rock's in situ physical properties must be considered. Core diskings refers to the formation of disks or wafers, generally of uniform thickness, that rupture or fracture on surfaces approximately normal to the axis of the core (fig. 4.5). Disking is a function of axial stress σ_3 , radial strain e_r , and the physical properties of the rock (62-63). In some cases, however, diskings that occurs in stress-relieved core may not occur in solid NX-size core; therefore, an NX-size hole can be drilled adjacent to the overcoring hole to obtain the rock properties. If the stress field is known, core diskings can be used to estimate material properties.

CALCULATING MODULUS OF ELASTICITY

Data from the biaxial test are tabulated onto a data sheet (fig. 4.6) for input into a computer program or for plotting on polar coordinate graph paper to obtain the

anisotropic parameters (fig. 4.7) α_{11} and α_{22} $\left(\frac{1}{E_{11}} \text{ and } \frac{1}{E_{22}} \right)$ (15, 17).

A sample set of biaxial data is presented in table 4.1 and tabulated in table 4.2. Results from table 4.1 are transferred to table 4.2, starting with data point $U_3 + 60^\circ$ and ending with data point $U_1 + 15^\circ$. (Present computer code requires this order of data input; however, the opposite direction could be used by altering the computer code.) Since the deformation readings for $U_3 + 60^\circ$ (328) and U_1 (320) are redundant readings, they are averaged

(324) and recorded in table 4.2, data column 3, row 1. The deformation reading for $U_3 + 45^\circ$ is recorded in row 2, $U_3 + 30^\circ$ is recorded in row 3, etc. The deformation reading for U_3 (264) and $U_2 + 60^\circ$ (288), the redundant readings, are averaged (276) and recorded in row 5. Also, U_2 and $U_1 + 60^\circ$ are averaged and recorded in row 9. Figure 4.7 is a computer printout¹⁸ of the anisotropic parameters α_{11} and α_{22} for this sample set of biaxial data. These results are used as input for the stress analysis. The value for the points plotted (fig. 4.7) are determined from equation 4.1. A least squares fit is applied to the 12 points to obtain the E_1 and E_2 anisotropic axes.

If biaxial data are obtained from a vertical uphole, the data from table 4.2 must be entered in the computer code in reverse order; that is, data point 1 (0°), then data point 12 (165°), data point 11 (150°), etc. The reason for this is that when the BHG is placed in a vertical uphole, components 2 and 3 switch positions as viewed from the mine opening. The stress calculations, however, are relative to the ground surface and not from the mine opening where the measurements are taken.

TRIAXIAL TESTING OF STRESS-RELIEVED CORES

Stress-relieved cores are triaxially tested (61) to determine the elastic constants, ν , E_3 , and the compliance coefficients S_{31} and S_{32} . Because an axial stress σ_3 can be applied to the core during triaxial testing, the radial load (P_o) applied can be made equal to the average stress level the rock experiences in situ. This is not always possible during the biaxial test because of the tensile loading of the core caused by the applied P_o .

¹⁸See footnote 16.

Table 4.1.—Sample biaxial test data for deformation measurements U_1 , U_2 , and U_3 at 15° intervals

(Hole 3; relief 3; depth, 13 ft, 2 in; gage 3-A; Dec. 12, 1984)

σ_p , psi	U_1	$U_1 + 15$	$U_1 + 30$	$U_1 + 45$	$U_1 + 60$	U_2	$U_2 + 15$	$U_2 + 30$
Initial readings (0 psi)	36,952	34,971	34,574	33,249	33,270	38,389	37,872	39,554
Load readings (300 psi)	37,277	35,301	34,913	33,573	33,575	38,699	38,167	39,850
No-load readings (0 psi)	36,951	34,964	34,573	33,245	33,264	38,387	37,863	39,551
Diff	326	337	340	328	311	312	304	299
Diff $\times$ C.F.	320	330	333	321	305	300	292	287
	$U_2 + 45$	$U_2 + 60$	U_3	$U_3 + 15$	$U_3 + 30$	$U_3 + 45$	$U_3 + 60$	
Initial readings (0 psi)	41,398	42,982	35,496	35,683	35,950	35,137	33,697	
Load readings (300 psi)	41,691	43,278	35,766	35,970	36,264	35,449	34,023	
No-load readings (0 psi)	41,392	42,978	35,494	35,680	35,950	35,135	33,685	
Diff	299	300	272	290	314	314	338	
Diff $\times$ C.F.	287	288	264	281	305	305	328	

C.F. (calibration factor): $U_1 = 0.98$; $U_2 = 0.96$; $U_3 = 0.97$.

Diff (difference) = load reading minus no-load (zero) reading.

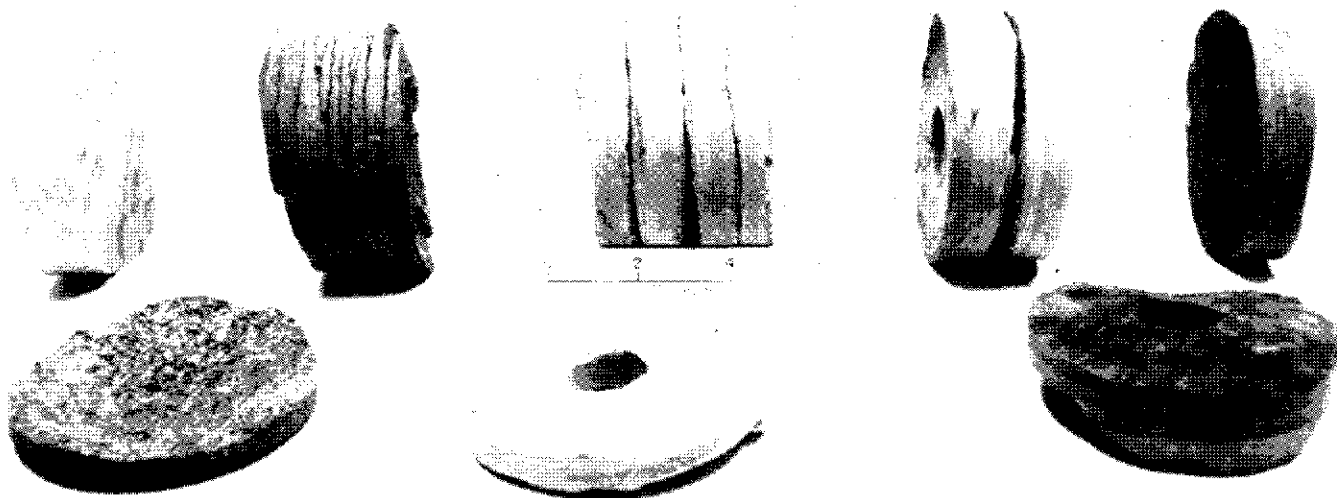


Figure 4.5.—Core disks.

Table 4.2.—Tabulation of biaxial data¹

(Hole 3; relief 3; depth, 13 ft, 2 in;
P_o = 300 psi)

Measurement	θ, deg	U _i , μin/in
1	0	324
2	15	305
3	30	305
4	45	281
5	60	276
6	75	287
7	90	287
8	105	292
9	120	303
10	135	321
11	150	333
12	165	330

θ = angle measured counterclockwise from U_i.

¹See table 4.1.

For a thick-walled cylinder (fig. 4.2) composed of an orthotropic elastic material loaded axially (P₃) and laterally with an external radial load P_o, the deformation U of the inner hole for plane strain (axial strain ε₃ equals zero) is given by (15)

$$U = - \frac{4ab^2P_o}{b^2 - a^2} (\alpha_{11} \cos^2\theta + \alpha_{22} \sin^2\theta), \quad (4.2)$$

where α₁₁ and α₂₂ are the elastic parameters related to E₁ and E₂, respectively. The elastic moduli E₁ and E₂ are determined by

$$E_i = \frac{4ab^2P_o}{(b^2 - a^2)U_i}, \quad (4.3)$$

where U_i = diametral deformation in the direction parallel to E_i.

Briefly, the triaxial method of determining the elastic constants from stress-relieved core consists of (1) cutting the core to a length-to-diameter ratio of approximately 2, (2) cementing four 90° rosette resistance strain gages on the exterior surface of the core along the E₁ and E₂ axes (90° intervals) at the center plane, (3) centering the core inside the triaxial pressure chamber, (4) inserting the 2C-BHG¹⁹ in the center hole of the core oriented to measure deformation along the E₁ and E₂, (5) stringing the resistance strain gage leads and BHG cable through the top bearing plate and connecting these gage wires to strain indicators, and (6) applying first an axial load, then a radial load P_o to obtain the axial stress σ₃, the radial stress σ_r, the axial strain ε₃, and the measurement U.

Procedure and Apparatus

After the stress-relieved core is biaxially tested at the minesite and the E₁ and E₂ axes of anisotropy are determined, representative core(s) from each overcoring hole is

¹⁹If a 2-C BHG is not available, then the 3-C BHG can be used. However, when using the 3-C BHG, one component is oriented on one axis of anisotropy (the other two components are not used) and the test performed. The BHG must then be rotated 90° to the other axis of anisotropy and another test performed.

Hole Depth			Relief $P_o =$		
Data:	θ	U			
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					

Hole Depth			Relief $P_o =$		
Data:	θ	U			
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					

Hole Depth			Relief $P_o =$		
Data:	θ	U			
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					

Hole Depth			Relief $P_o =$		
Data:	θ	U			
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					

Figure 4.6.—Example of data sheet for biaxial test results.

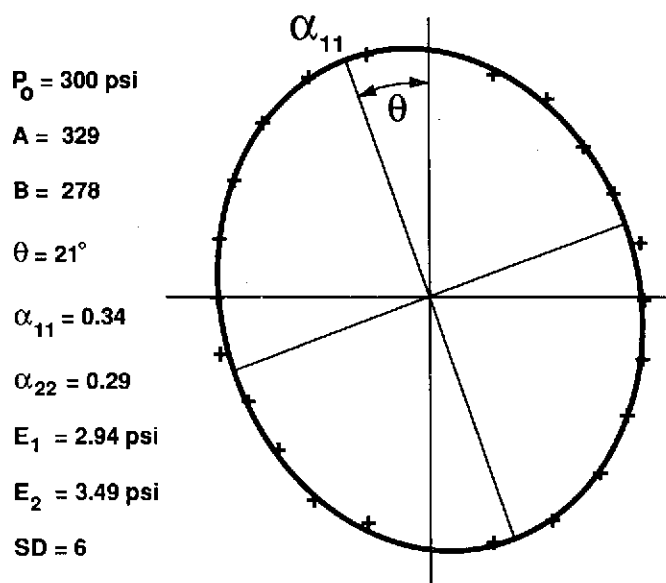


Figure 4.7.—Computer printout of anisotropic parameters.

selected and prepared for triaxial testing. The triaxial pressure chamber²⁰ will accommodate a core length to 15½ in. Generally, a core length-to-diameter ratio of 2 is used. For a core of less than 2:1 length to diameter, consult the ASTM standards for allowable tolerance (18).

Figure 4.8 gives the coordinate system and nomenclature used for triaxial testing. The location of E_1 and E_2 axes, which are 90° apart, are marked on the collar end and side of the core. Four (AX-5-1 type) 90° rosette strain gages are epoxied on the side along the axes on the center plane (fig. 4.9).

Use an insulated No. 22 stranded copper wire to connect the four-rosette strain gages and as lead wires from the strain gages to a strain indicator (or switching unit). Teflon fluorocarbon polymer tape is placed under and over all solder connections to prevent shorting of the strain gage to the core and to provide a moisture barrier. The two longitudinal gages positioned on the E_1 axis will provide axial strain 31 (fig. 4.10) and the two longitudinal gages aligned along the E_2 axis will provide axial strain 32. Similarly, the two transverse gages aligned along the E_1 axis will provide lateral strain 31 and the two transverse gages aligned along the E_2 axis will provide lateral strain 32. The wiring for the four 90° rosette strain gages is shown in figure 4.10. All wires should be taped to the core (test specimen) to allow the specimen to be freely placed into the triaxial pressure chamber without tearing any of the wires or strain gage leads.

Now the specimen with four 90° rosette strain gages installed is positioned in the center of the triaxial pressure chamber with the collar end up. Because the test specimens may be of different lengths, steel plates (spacers)

varying in thicknesses from 1/2 to 2 in are used to ensure centering of the core in the chamber. A 10-in-long threaded rod attached to a mechanical roof bolt anchor can be used to place and remove the core and steel plates from the triaxial chamber. A 2-C BHG is positioned in the 1½-in center hole to measure deformation at the center plane normal to the axis of the hole. The position of the gage is oriented along the E_1 and E_2 axes of the specimen (fig. 4.9). Wires from the strain gages exit the chamber through the slots cut in the outer edge of the top 2-in-thick steel bearing plate, which rests on the top of the specimen, and extend out of the chamber. The BHG cable passing through the center of this top bearing plate is bent 90° and positioned in the slot of the 1/2-in-thick slotted bearing plate, which rests on the top bearing plate, and will contact the swivel head of the press when the axial load is applied (fig. 4.11). Wires from the strain gages are connected to a multichannel strain indicator, and the 2-C BHG is connected to two single-channel strain indicators. A second specimen with four 90° rosette strain gages epoxied to it is also connected to the multichannel strain indicator. These gages are used as "dummy gages" and, when connected with the test specimen gages, will complete the two-arm bridge circuit. The test specimen is now ready for the test cycle.

Test Cycle

The following steps describe the procedure for triaxially testing the stress-relieved core.

1. An axial load is applied to the specimen to seat the bearing plates and to ensure satisfactory strain gage installation. Satisfactory strain gage installation is confirmed when the strains return to the zero strain reading after the axial load is released. Zero strain readings are recorded on the triaxial data sheet (fig. 4.12).

2. With the radial load $P_0 = 0$, the axial stress σ_3 is applied. The value selected for σ_3 is the measure or estimated stress parallel to the borehole. σ_3 is decreased in four or five increments and strain readings are recorded at each increment.

3. The value of the radial load P_0 is determined by applying P_0 after the axial load is reapplied,²¹ until the average deformation, $1/2(U_\theta + U_{\theta+90})$, measured by the 2-C BHG is equal to the average in situ deformation, $1/3(U_\theta + U_{\theta+60} + U_{\theta+120})$, measured during overcoring by the 3-C BHG. Thus, the applied radial stress (σ_r) equals the average in situ lateral stress.

4. With the radial load P_0 still applied, the axial load is slowly released until the axial strain ϵ_3 is zero, establishing plane strain condition and simplifying the determination of the elastic parameters. Then, P_0 is released.

²¹The axial load P_3 is always applied first to the specimen to prevent the bearing plates from separating from the specimen, causing possible tensile failure of the specimen.

²⁰Diagram of the triaxial chamber appears in appendix B.

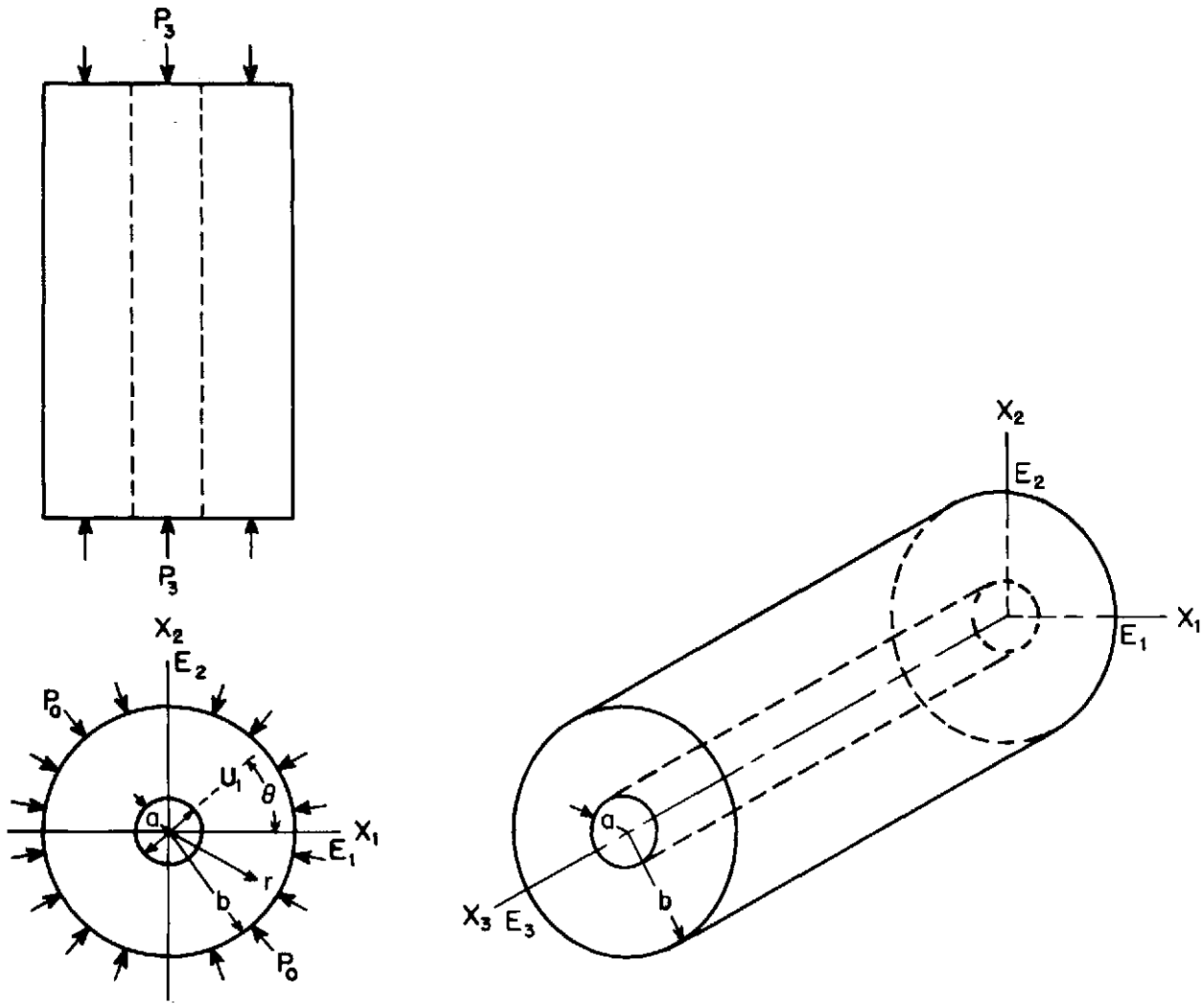


Figure 4.8.—Coordinate system and nomenclature for orthotropic thick-wall cylinder under load.

5. With the axial load still set at the established value, the two zero (no-load) readings from the BHG are recorded. The radial load P_0 is applied to the previously determined value and the two deformation readings are recorded. Then, P_0 is incrementally decreased and the deformations are recorded at each increment.

6. If considered necessary, the cycle is repeated.

From step 2 of the test cycle, the constants E_3 , ν_{31} , and ν_{32} are determined using the measured average axial strain ϵ_3 and the measured tangential strains 31 and 32 (ϵ_{θ_1} and ϵ_{θ_2}) obtained from the four 90° rosette strain gages. The equations for calculating the elastic constants are (17)

$$E_3 = \frac{\sigma_3}{\epsilon_3}, \quad \nu_{31} = \frac{\epsilon_{\theta_1}}{\epsilon_3}$$

and
$$\nu_{32} = \frac{\epsilon_{\theta_2}}{\epsilon_3}. \quad (4.4)$$

The compliance coefficients S_{31} and S_{32} are determined from the following equations:

$$S_{32} = -\frac{\nu_{32}}{E_3}$$

and
$$S_{31} = -\frac{\nu_{31}}{E_3}. \quad (4.5)$$

For the elastic axes, the following relationships exist:

$$\alpha_{11} = \frac{1 - \nu_{31}^2}{E_1}$$

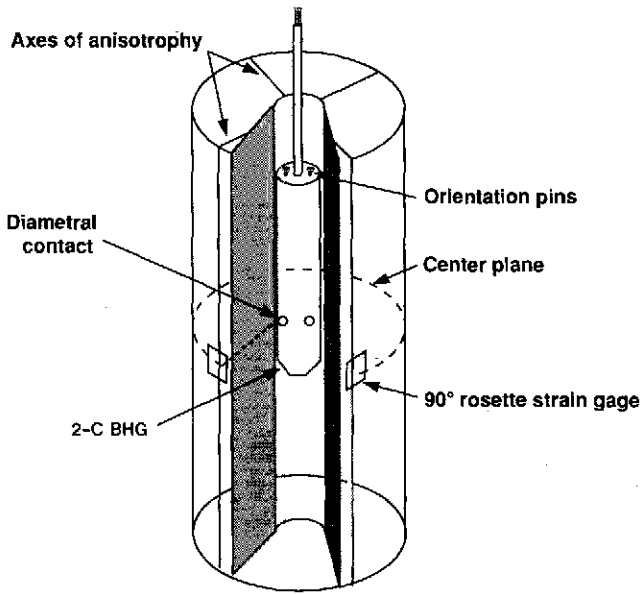


Figure 4.9.—Cutaway of stress-relieved core showing BHG and strain gage locations.

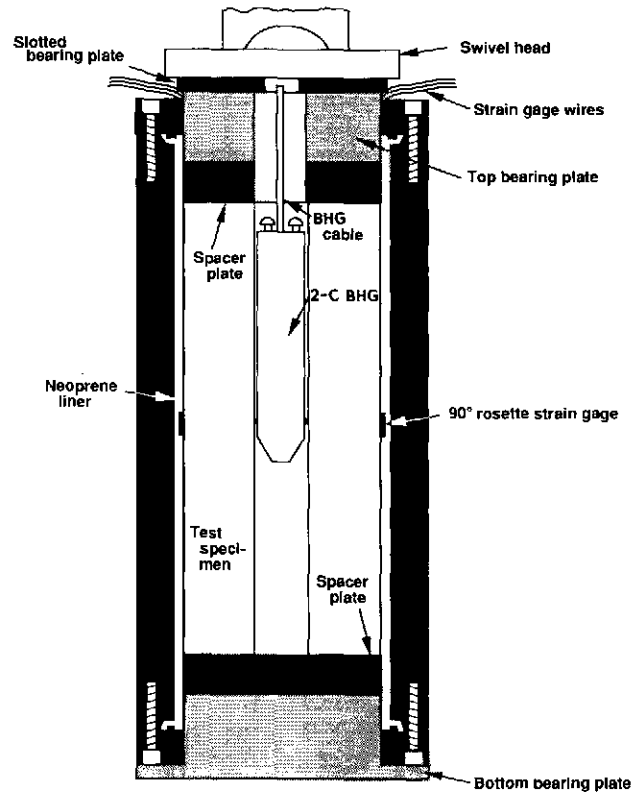


Figure 4.11.—Cross section of stress-relieved core and triaxial chamber assembly.

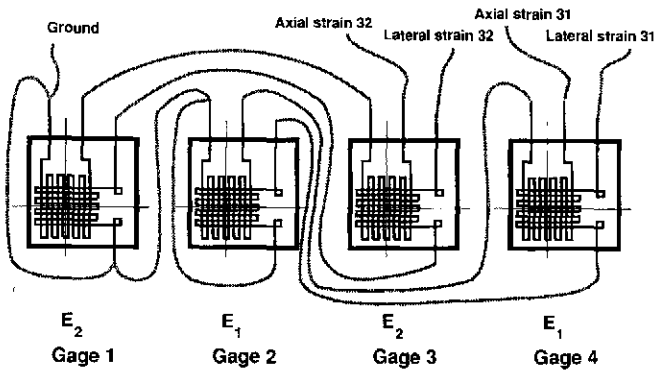
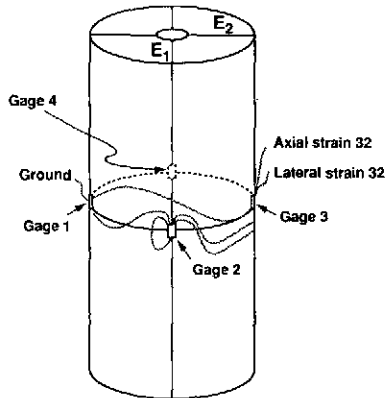


Figure 4.10.—Wiring diagram for 90° rosette strain gages.

and
$$\alpha_{22} = \frac{1 - \nu_{32}^2}{E_2}. \quad (4.6)$$

For the isotropic case, $\alpha_{11} = \alpha_{22} = \frac{1 - \nu^2}{E}.$

Determined stresses using both anisotropic and isotropic solutions can be compared. From the above equations and relationships,

$$\alpha_{11} = \frac{1 - \nu_{13}\nu_{31}}{E_1},$$

$$\alpha_{22} = \frac{1 - \nu_{23}\nu_{32}}{E_2},$$

$$\frac{\nu_{31}}{E_3} = \frac{\nu_{12}}{E_1},$$

and
$$\frac{\nu_{23}}{E_2} = \frac{\nu_{32}}{E_3}, \quad (4.7)$$

Location depth	σ_z	Load	Axial strain 31			Axial strain 32			E unloading
			Read	ϵ	Corr ϵ	Read	ϵ	Corr ϵ	

Lateral strain 31			Lateral strain 32			P_o	Axial load	Deformation		ν_{31}	ν_{32}
Read	ϵ	Corr ϵ	Read	ϵ	Corr ϵ			U_1	U_2		

Figure 4.12.—Example of typical triaxial data sheet. (Read = reading; Corr = corrected.)

the following equations are obtained:

$$E_1 = \frac{1}{\alpha_{11} - S_{31}\nu_{31}},$$

$$E_2 = \frac{1}{\alpha_{22} - S_{32}\nu_{32}},$$

$$E = \frac{1}{3}(E_1 + E_2 + E_3),$$

$$\nu_{13} = \frac{1 - E_1\alpha_{11}}{\nu_{31}},$$

$$\nu_{23} = \frac{1 - E_2\alpha_{22}}{\nu_{32}},$$

$$\bar{\nu} = \frac{1}{4}(\nu_{31} + \nu_{13} + \nu_{32} + \nu_{23}),$$

$$\bar{S} = \frac{\bar{\nu}}{E},$$

$$\text{and} \quad \bar{\alpha} = \frac{1 - \bar{\nu}^2}{E}, \quad (4.8)$$

where $\alpha_{11}, \alpha_{22}, \nu_{31}, \nu_{32}, S_{31}, \text{ and } S_{32}$ = quantities used for the anisotropic evaluation

and $\bar{\alpha}, \bar{\nu}, \text{ and } \bar{S}$ = quantities used for the isotropic evaluation of stresses.

CALCULATING ANISOTROPIC PARAMETERS

A sample set of data obtained from a laboratory triaxial test for a Minnesota granite core and the calculated results are presented in table 4.3. This table also presents data from the two-element 90° rosette strain gages and from the 2-C BHG. The apparent strain readings provided by a rosette gage differ from the actual strain components present by a small amount that depends on the transverse sensitivity of each gage element. Conversion from apparent to real strain may be accomplished by means of the following formulas:

$$e_1 = R_1 - \frac{1}{b}R_2 \quad (4.9)$$

and
$$e_2 = R_2 - \frac{1}{b}R_1, \quad (4.10)$$

where e_1, e_2 = actual axial and lateral strain components along respective gage axes, respectively,

R_1, R_2 = apparent axial and lateral strains indicated by each gage, respectively,

and b = value furnished with rosette strain gage.

Sample calculation for the axial strain E_3 along axis 31 for $\sigma_z = 2,000$ psi using equations 4.9 and 4.10 is as follows:

$$R_1 = -234 \mu\text{in/in},$$

$$R_2 = 48 \mu\text{in/in},$$

and
$$e_1 = -234 - \frac{1}{25} 48 = -235.9 \mu\text{in}.$$

A minus value for e_1 and e_2 denotes compression. This value is recorded in table 4.3 in the column "Corrected ϵ "

under "Axial strain 31." All the remaining axial and lateral strains measured are corrected and the results recorded in table 4.3.

From the corrected axial strains, the modulus of elasticity E is calculated. Sample calculation for $\sigma_z = 2,000$ psi is

$$E = \frac{\sigma_z}{\epsilon_{31} + \epsilon_{32}} = \frac{-2,000}{\frac{(-235.9) + (-245.4)}{2}}$$

$$= 8.311 \times 10^6 \text{ psi}.$$

where $\epsilon_{31}, \epsilon_{32}$ = strains along axes of anisotropy 31 and 32, respectively.

From the axial and lateral strains along axis 31, the Poisson's ratio ν in direction 32 is calculated. For $\sigma_z = 2,000$ psi,

$$\nu_{32} = \frac{\text{lateral strain 31}}{\text{axial strain 31}} = \frac{57.4}{235.9} = 0.243.$$

All values for ν_{31} and ν_{32} are calculated and the results recorded in table 4.3.

Table 4.3.—Data and results from laboratory triaxial tests for Minnesota granite

(Hole 3; depth, 74 in; $b = +25$; $K = \frac{4ab^2}{b^2 - a^2} = 3.2$; calibration factor: $U_1 = 1.15, U_2 = 1.13$)

σ_z , psi	Load, lb	Axial strain 31, $\mu\text{in/in}$			Axial strain 32, $\mu\text{in/in}$			Unloading, $E \times 10^6$ psi	Lateral strain 31, $\mu\text{in/in}$		
		Reading	ϵ	Corrected ϵ	Reading	ϵ	Corrected ϵ		Reading	ϵ	Corrected ϵ
0	0	7,000	NAP	NAP	7,000	NAP	NAP	NAP	7,000	NAP	NAP
2,000	46,650	7,234	-234	-235.9	7,244	-244	-245.4	8.311	6,952	48	57.4
1,500	35,000	7,183	-183	-184.4	7,192	-192	-193.0	7.949	6,965	35	42.3
1,000	23,300	7,129	-129	-130.0	7,136	-136	-136.7	7.499	6,976	24	29.2
500	11,650	7,070	-070	-70.5	7,074	-74	-74.5	6.897	6,988	12	14.8
0	0	7,004	NAP	NAP	7,004	NAP	NAP	NAP	6,999	NAP	NAP
		Lateral strain 32, $\mu\text{in/in}$					Deformation, μin				
		Reading	ϵ	Corrected ϵ	P_0 , psi	P_3 , lb	U_1	U_2	ν_{31}	ν_{32}	
0		7,000	NAP	NAP	0	16,200	-39,920	-35,113	NAP	NAP	
2,000		6,966	34	43.8	1,750	16,200	-39,299	-34,492	0.178	0.243	
1,500		6,975	25	32.7	0	16,200	-39,928	-35,116	.169	.229	
1,000		6,982	18	23.4	NAP	NAP	¹ 629	¹ 624	.171	.225	
500		6,991	9	12.0	NAP	NAP	² 723.4	² 705.1	.161	.210	
0		6,999	NAP	NAP	NAP	NAP	NAP	NAP	NAP	NAP	

NAP Not applicable.

¹Difference between no-load and load readings.

²No-load readings times calibration factor.

The no-load deformation reading obtained from the 2-C BHG ($U_1 = -39,928$) is subtracted from the load deformation reading ($U_1 = -39,299$). This change in deformation ($629 \mu\text{in}$) is multiplied by the calibration factor for component 1 (1.15), resulting in a deformation of $723.4 \mu\text{in}$ along axis 31. The same procedure is performed for component 2.

Figure 4.13 is a data sheet for the elastic constants (17). Results from the biaxial and triaxial tests are recorded on the data sheet. Values from the data sheet, along with the overcoring deformation measurements, are used to calculate the in situ stresses.

NONLINEARITY IN STRESS-RELIEVED OVERCORES

Testing of stress-relieved cores obtained from the overcoring techniques is based on the assumption that the

rock is a linearly elastic material, which is not generally true. Thus, varying degrees of elastic nonlinearity must be accepted or a procedure used that corrects for elastic nonlinearity. Physical property tests can be conducted in such a manner as to eliminate the effects of elastic nonlinearities (60).

For most rock, the stress-strain curve is nonlinear as is the diametral deformation versus lateral load curve for a thick-wall cylinder. Figure 4.14 represents a typical load-deformation curve for a rock cylinder. The nonlinear portion of the curve between points O and A is generally attributed to microfracture closure. From point A to C, the curve becomes more linear and, for some rocks, may be assumed to be linear without appreciable error.

For a thick-wall cylinder of rock or overcore, equation 4.1 is applicable only for the straight-line segment between points B and C. The slopes of the lines $\overline{OA}$, $\overline{OB}$, and $\overline{OC}$ are represented by KE_a , KE_b , and KE_c . The constant K is expressed by

$$K = \frac{b^2 - a^2}{4ab^2} \quad (4.11)$$

For a linear elastic material, the load-deformation curve is a straight line and $E_a = E_b = E_c$.

Because the overcoring procedure measures deformation during unloading from a given stress level to zero stress, the elastic properties should be measured in the same manner. This unload curve represents the secant modulus of the rock. To account for the rock nonlinearity during the testing, the load applied to the overcore must be the same as that which produced the deformation during overcoring stress relief. The secant modulus calculated at this load level should be used in equations 4.4 to 4.8.

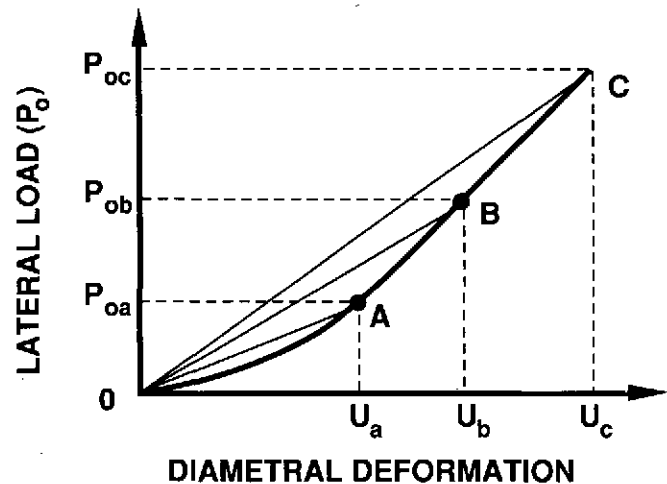


Figure 4.14.—Typical load-deformation curve for thick-wall cylinder of rock.

SIGNATURE

TITLE

SUBJECT Static Elastic Constants

DATE

$\sigma_z =$

$P_0 =$

$\nu_{31} =$

$E_3 =$

$\nu_{32} =$

$\alpha_{11} =$

$\alpha_{22} =$

$$S_{31} = -\frac{\nu_{31}}{E_3} =$$

$$S_{32} = -\frac{\nu_{32}}{E_3} =$$

$$E_1 = \frac{1}{\alpha_{11} - S_{31} \nu_{31}} =$$

$$E_2 = \frac{1}{\alpha_{22} - S_{32} \nu_{32}} =$$

$$\nu_{13} = \frac{1 - E_1 \alpha_{11}}{\nu_{31}} =$$

$$\nu_{23} = \frac{1 - E_2 \alpha_{22}}{\nu_{32}} =$$

$$\bar{E} = \frac{1}{3} (E_1 + E_2 + E_3) =$$

$$\bar{\nu} = \frac{1}{4} (\nu_{31} + \nu_{13} + \nu_{32} + \nu_{23}) =$$

$$\bar{S} = -\frac{\bar{\nu}}{\bar{E}} =$$

$$\bar{Q} = \frac{1 - \bar{\nu}^2}{\bar{E}} =$$

Figure 4.13.—Example of data sheet for elastic constants.

CHAPTER 5.—DETERMINING STRESSES

INTRODUCTION

This chapter presents instructions to determine the secondary principal stresses P and Q and the three-dimensional state of stress from field deformation measurements.

Two methods are also presented for estimating secondary principal stresses P and Q from core diskings. Since a direct relationship exists, within certain loading limitations, between axial stress σ_3 and radial stress σ_r at failure, the lower limits of the applied stress field can be determined.

SECONDARY PRINCIPAL STRESS CALCULATIONS

The biaxial state of stress has been commonly evaluated from borehole deformation measurements by using isotropic relations. If the rock is anisotropic in the plane normal to the borehole, an average Young's modulus,

$$E = \frac{E_1 + E_2}{2}, \quad (5.1)$$

is used in the calculations. The isotropic plane strain solutions using diametral deformation measurements are obtained from the following (chapter 1) basic equations (27):

$$P = \frac{E}{6d(1 - \nu^2)} \left\{ (U_1 + U_2 + U_3) + \frac{\sqrt{2}}{2} \left[(U_1 - U_2)^2 + (U_2 - U_3)^2 + (U_1 - U_3)^2 \right]^{1/2} \right\}, \quad (5.2)$$

$$Q = \frac{E}{6d(1 - \nu^2)} \left\{ (U_1 + U_2 + U_3) - \frac{\sqrt{2}}{2} \left[(U_1 - U_2)^2 + (U_2 - U_3)^2 + (U_1 - U_3)^2 \right]^{1/2} \right\}, \quad (5.3)$$

$$\text{and} \quad \tan 2\theta = \frac{-\sqrt{3} (U_2 - U_3)}{2U_1 - U_2 - U_3}. \quad (5.4)$$

Any effect from axial stress σ_3 is neglected.

The biaxial state of stress can be evaluated from borehole deformation measurements by using anisotropic relations. Depending on the degree of anisotropy of the

rock, large errors may occur (17) from the isotropic evaluation of the state of stress. Anisotropic plane stress and plane strain equations are presented in chapter 1 for calculating the state of stress in a plane normal to the axis of the borehole; effects of axial stress σ_3 may be included. Laboratory tests on the overcore (chapter 4) provide the elastic constants ν_{31} , ν_{32} , and E_3 and the compliance coefficients S_{31} and S_{32} required for the plane strain solution.

A computer code for calculating the secondary principal stresses P and Q for either isotropic or anisotropic conditions appears in appendix E.

Secondary principal stresses P and Q are calculated from each set of deformation measurements U_1 , U_2 , and U_3 . These resulting stress magnitudes and directions can be plotted using the computer code in appendix E. If more than three diametral deformation measurements of any combination are available, a least squares method of calculating the average rock stress components can be performed (47). This least squares analysis provides a means of determining the multiple correlation coefficient R , the variance, the scatter for a set of deformation measurements, and the confidence limits for P and Q magnitudes and directions. Statistical information obtained from this method of analysis is very useful in the interpretation of results. The computer code for the least squares solution of P and Q , including a sample calculation, appears in appendix F.

THREE-DIMENSIONAL STATE OF STRESS CALCULATIONS

A computer code for calculating the maximum, intermediate, and minimum principal stresses S_1 , S_2 , and S_3 , including a sample calculation and plot, appears in appendix G. This computer code also provides the average ground stress components with respect to the X-Y-Z coordinate system, the standard deviation SD of the ground stress components, and precision of data, which includes deformation sum of squares, residual sum of squares, multiple correlation of coefficients R squares, and SD of the fitted data. For the X-Y-Z coordinate system, the plus X axis is east, the plus Y axis is north, and the plus Z axis is vertically up; for S_1 , S_2 , and S_3 , the bearing is taken from north, with the inclination of 0° vertically down, 90° horizontal, and 180° vertically up.

ESTIMATING SECONDARY PRINCIPAL STRESSES FROM CORE DISKING

Core diskings occur when drilling is performed in a substantial stress level relative to the strength of the rock.

From laboratory tests (63-68), modeling studies (69), and field observations (8, 26, 35, 59, 66, 70-72), relationships between stress levels and strength properties have been developed to calculate the lower limits of the applied stress field (62-63, 73). One such relationship is that a core will disk when the average compressive stress in a plane normal to the axis of the borehole is approximately equal to one-half the unconfined compressive strength of the rock. The thickness of the disks that are formed under this criterion will be roughly one-fourth of the core diameter.

A second relationship between core diking and shear strength S_o (62, p. 426; 63), with the applied stress acting

normal to the axis of the borehole, is given by the following equation:

$$S \geq -3,400 - 2.0 S_o - 0.7 \gamma h, \quad (5.5)$$

where S = stress component, psi,

S_o = shear strength, psi,

γ = density of overburden, lb/ft³,

and h = height or depth of overburden, ft.

CHAPTER 6.—ALTERNATIVE METHODS FOR ABSOLUTE ROCK STRESS DETERMINATIONS

INTRODUCTION

With increasing concerns in recent years for improving mine design and for studying ground control problems in existing mines, endeavors to achieve reliable means of measuring in situ stresses have led to the development of many devices and techniques (74-78). The techniques most commonly used and accepted by the scientific community, besides the overcoring techniques using the 3-C BHG developed by the U.S. Bureau of Mines, are presented in this chapter.

Each technique or method has advantages and disadvantages with respect to a particular application; however, the different methods can complement each other. To determine which stress-determining method to use, several factors should be addressed: (1) the theoretical soundness of each available method, (2) the limitations, practicability, or ease of use of each available method, and (3) the cost of equipment and time involved for collecting the data. A

technique using epoxy or grout slows the rate at which data can be gathered.

The most versatile methods are those using a borehole as compared with methods limited to surface measurements. Borehole techniques permit measurements beyond the influence of a mined opening. Also, a stress profile for studying stress distribution in a pillar, rib, floor, or roof can be obtained.

All methods will give good estimates of the stresses, providing the prescribed techniques are followed, ground conditions are suitable, and a degree of skill is obtained.

HYDRAULIC FRACTURING TECHNIQUE

The hydraulic fracturing technique determines underground in situ stress magnitudes and stress directions in a plane perpendicular to the drill hole (79-80). This technique uses packers to isolate a test section of a drill hole while fluid pressure generates, propagates, sustains, and reopens fractures in the rock (fig. 6.1). The measured

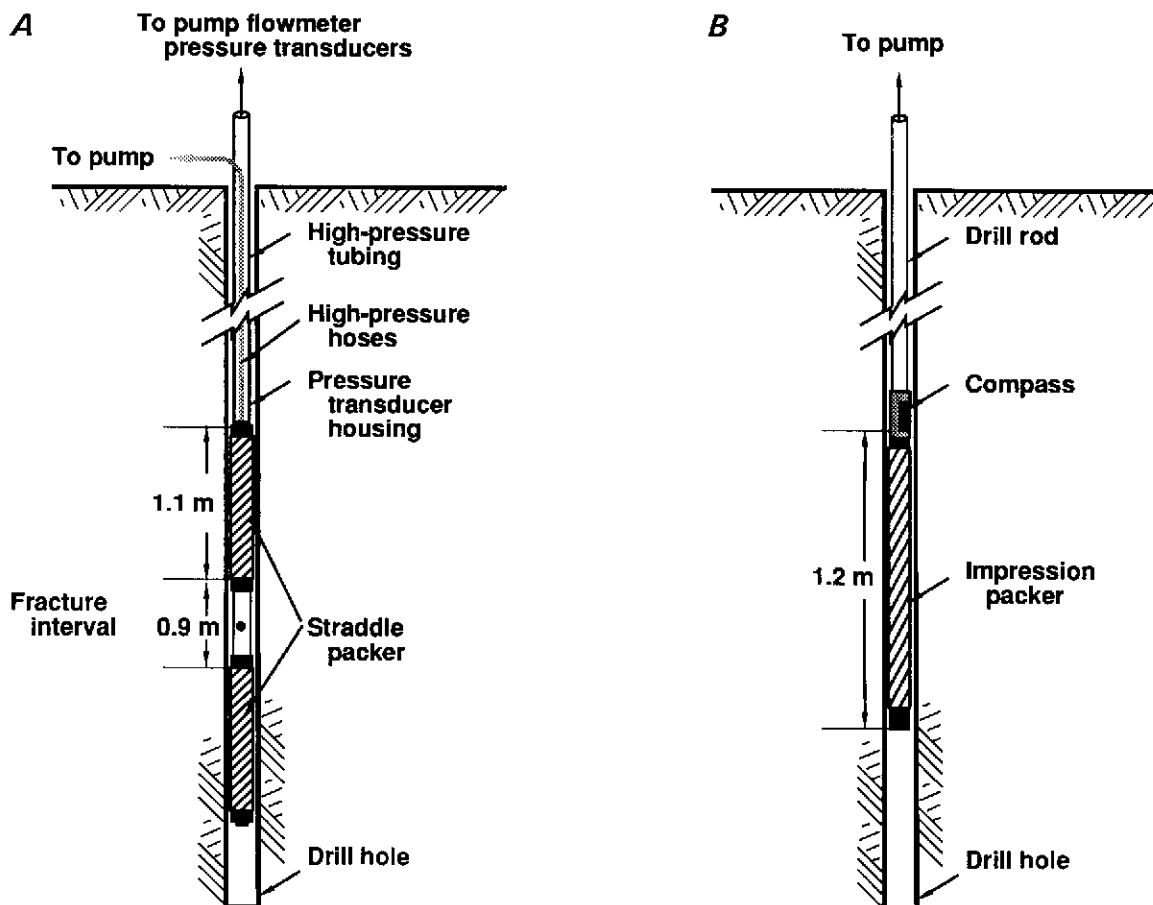


Figure 6.1.—Schematic of hydraulic fracturing tool (A) and impression packer (B).

pressures are related to the existing field stresses (81-82). Elastic properties of the rock are not required to calculate the stress magnitudes. However, a knowledge of the tensile strength of the rock in the fractured test section is required. Direction can be obtained by observing and measuring the orientation of the induced fracture plane. This application is suitable for measurements in vertical drill holes at depths (more than 150 ft) greater than that obtainable by most other techniques that use some type of strain or stress relief by overcoring. This method is considered to measure stresses over a relatively large area (more than 1 to 3 ft) rather than a point. The more closely the rock tested approaches brittle, homogenous, isotropic, elastic, and nonporous media in behavior, the more accurate the method is. This method assumes that the axis of the drill hole is aligned with a principal stress direction. If the drill-hole direction is more than $\pm 15^\circ$ from a principal stress direction, the results become questionable (77).

FLAT JACK TECHNIQUE

The flat jack method determines stress parallel to and near the exposed rock surface in an excavation (83). Since only one direction is measured, at least six measurements in distinct directions are required to calculate the stress tensor. This technique consists of either measuring the change of length between pairs of measuring pins when a slot is cut into the rock between the pins or using two hydraulic borehole pressure cells in place of the pins. A thin hydraulic cell called a flat jack is grouted into the slot and pressurized until the initial measured distance is obtained (fig. 6.2). The pressure in the flat jack is then considered equal to the preexisting stress. This condition must be reached to obtain valid results.

The flat jack method does not require any knowledge of the elastic properties of the rock; hence, it is considered a true stress-measuring method (62). Very often relief of the state of stress seriously disturbs the mechanical properties of the rock, and consequently the rock samples used to determine these properties may not be representative of the rock mass (84). The flat jack design has an averaging effect over a comparatively large area of the flat jack; therefore, local variations in the rock stress are minimized. This method is also better adapted to measurements in inelastic rock.

Because of the difficulty in cutting deep flat jack slots, the flat jack method is restricted to near-surface measurements. Thus, the determined stresses are representative only of the rock mass immediately surrounding the excavation and not the virgin stress, although this method can provide an estimate of the in situ stress and reveal general trends. For precise in situ absolute stress, a different method should be considered. This method can be used in highly stressed areas and rock in which insufficient core

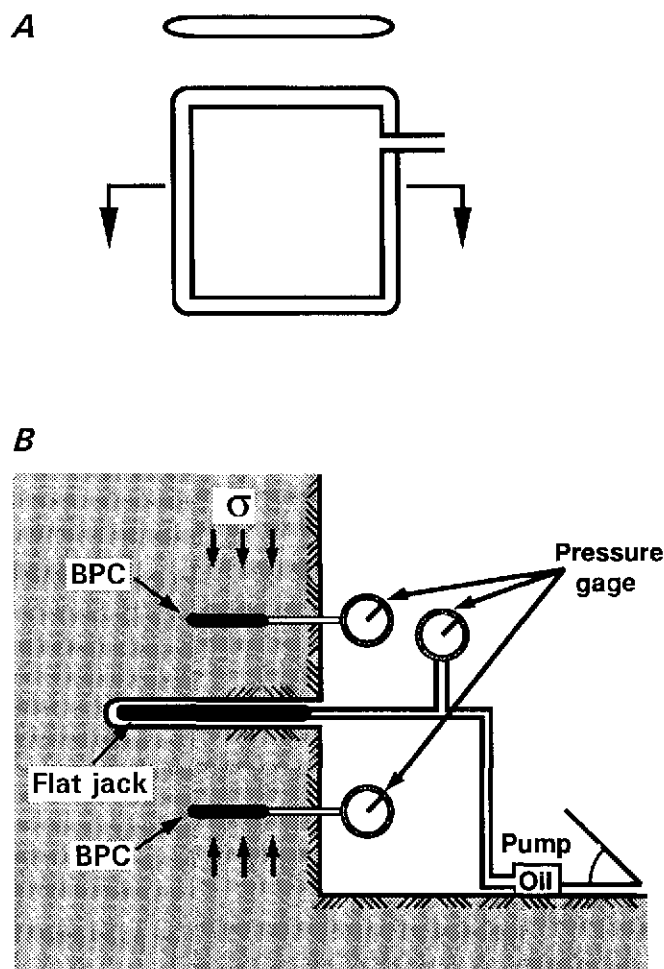


Figure 6.2.—Typical flat jack (A) and flat jack test configuration (B).

recovery would eliminate those methods that require drill core recovery.

The main causes of error inherent in the flat jack method are nonrecoverability of strain under load and the influence of shear stresses in the test area (85). A special application, which uses several flat jacks simultaneously, is able to measure tensile stresses, monitor stress changes, and measure stresses deeper in the rock (85).

DOORSTOPPER METHOD USING ROSETTE STRAIN GAGE OR CSIR- OR CSIRO-TYPE CELL

The doorstopper method determines rock stress by measuring the strain occurring at the end or in the walls of a borehole when the stresses are relieved by overcoring (86-87). Rock behavior should approximate that of a homogeneous, elastic medium. Although actual rock conditions are seldom ideal, a decision must be made whether the actual condition is sufficiently close to an elastic medium for stress determinations. The cell or rosette

strain gage should be located in a fracture free section of rock where core length of two to three times the diameter of the overcoring hole can be obtained. When testing is conducted under anisotropic conditions, the cell with 12 strain gages will provide a more representative analysis than the cell with 9 strain gages or the 4-element rosette strain gage.

Rosette Strain Gage

The doorstopper technique consists of epoxying a four-element, 45° rosette strain gage to the end of the borehole that has been ground flat and smooth with a flat-face diamond-impregnated bit (fig. 6.3). Overcoring a distance

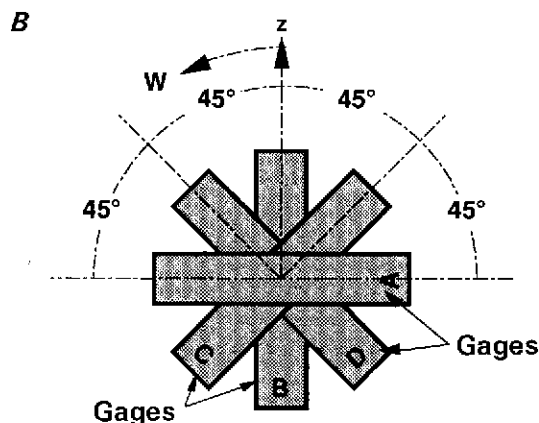
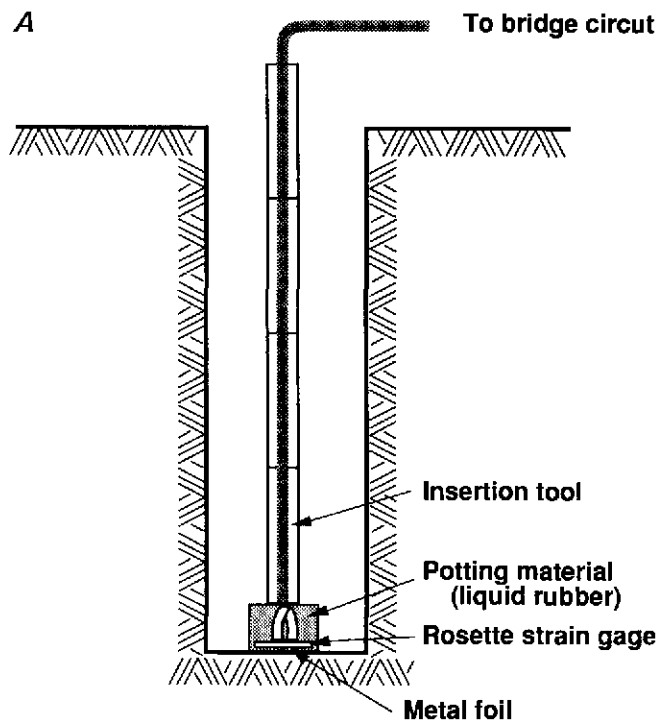


Figure 6.3.—Strain gage installation (A) and strain gage configuration viewed from axis of borehole (B).

of two or three times the borehole diameter is required to relieve the stress at the end of the borehole containing the strain gage. Because of stress concentrations, a correction factor must be applied to the strain reading obtained during the overcoring. These strain readings and the elastic properties of the rock are used to calculate the stresses in a plane normal to the axis of the borehole. Three non-parallel boreholes are necessary to determine the complete three-dimensional stress field. Questions regarding the value to use the stress concentration correction factor and the soundness of the theoretical basis have been raised (88). The doorstopper method can be used in deep mines, but concern exists as to the adequacy of the epoxy bond of the strain gage or the cell under conditions of high humidity and elevated temperature. In a downhole, other methods should be considered because of the difficulty of removing water and maintaining a dry borehole. The doorstopper method using the rosette strain gage and a very fast setting epoxy is a relatively inexpensive method for measuring the field stress (89).

CSIR-Type Triaxial Strain Cell

The CSIR-type triaxial strain cell (fig. 6.4) consists of three groups of three strain gages mounted on the circumference of a tubular body. After the strain gages are epoxied to the wall of the EX-size pilot hole, the rock containing the cell is overcored similar to the procedure used with the BHG. The elastic properties and the strain changes from overcoring are used to calculate the in situ state of stress.

CSIRO-Type Hollow Inclusion Cell

The CSIRO-type hollow inclusion cell (fig. 6.5) is similar in construction and concept to the CSIR-type triaxial strain cell. This cell consists of three three-element rosette strain gages encapsulated in a thin-walled epoxy pipe. The nine strain measurements allow for some redundancy of measurements. Both the CSIR- and CSIRO-type cells require only one borehole to determine the complete three-dimensional state of stress. Therefore, this method is advantageous for stress determination in a relatively high stress field when overcoring is possible in only one direction.

HYDRAULIC BOREHOLE PRESSURE CELL METHOD

Hydraulic borehole pressure cells determine in situ stress and changes in stress (76, 90) in a plane perpendicular to a drill hole from pressure convergence-divergence tests using a combination of one cylindrical and two encapsulated flat hydraulic borehole pressure cells oriented 90° apart (91). This combination of pressure cells is

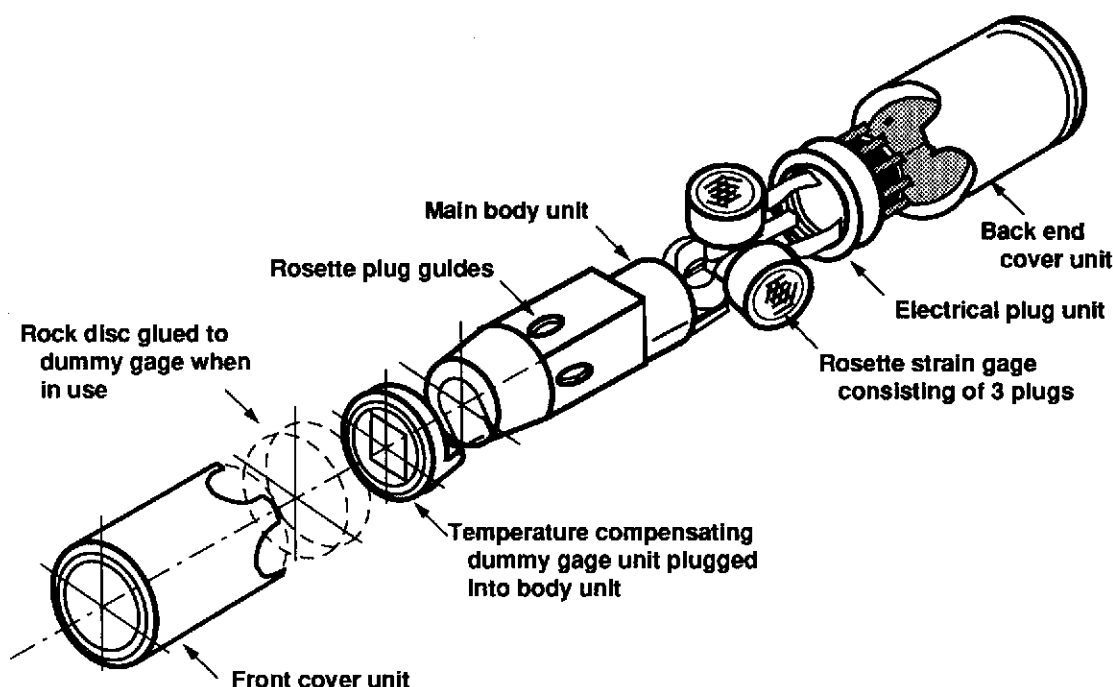


Figure 6.4.—Exploded view of CSIR-type triaxial strain cell.

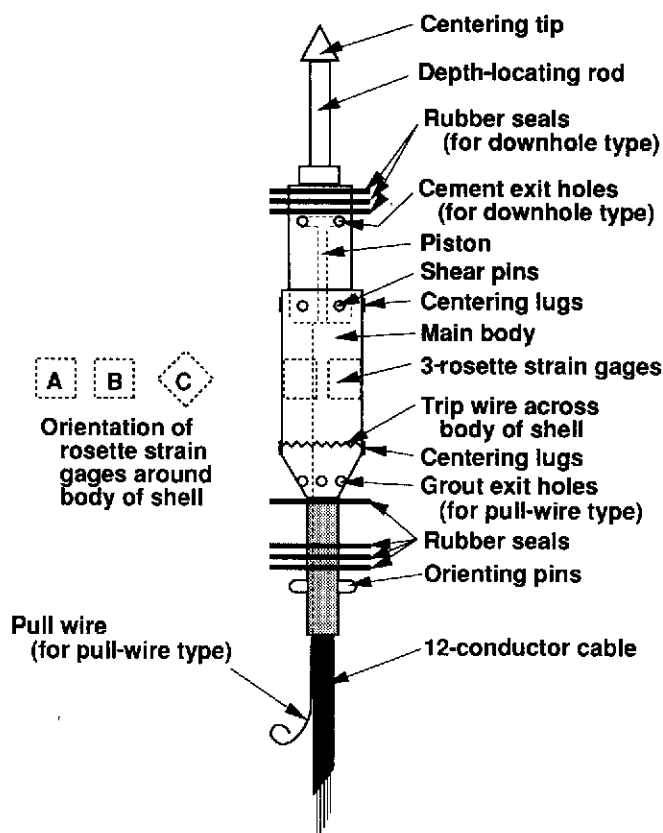


Figure 6.5.—Schematic of CSIRO-type hollow inclusion cell.

referred to as a cell package (fig. 6.6). Calculations are based on the elastic infinite plate and thick-walled cylinder theories. Providing the rock mass is homogenous and isotropic and the stresses are uniform in the test section of the rock, the cylindrical cell gives the sum of the stresses and the two flat cells give the ratio of the biaxial stresses. If pressure readings are obtained from the instrumentation in three nonparallel boreholes and the principal stress directions are known, the three-dimensional magnitudes can be calculated. Providing the stress direction is known or is assumed, the triaxial rock stress magnitudes can be estimated from readings in a pair of orthogonal boreholes (92-93).

The hydraulic borehole pressure cell method consists of pressurizing and expanding a pressure cell in a borehole until the outer shell of the cell is in full contact with the wall of the borehole. In layered strata, it is recommended to install the cells in separate holes and at the same depth to ensure that the cell placement is in the same rock type. Over time, the cell pressure stabilizes to a cell equilibrium pressure. The cell equilibrium pressures and Poisson's ratio ν are used to calculate the magnitudes of the secondary principal stresses P and Q . Changes in stress are determined from the change in cell pressure. Calibration of the cell package before and after installation allows for determination of the in situ ν . Some concern exists in selecting the setting pressure for the cell since the setting pressure value affects the response of the cell.

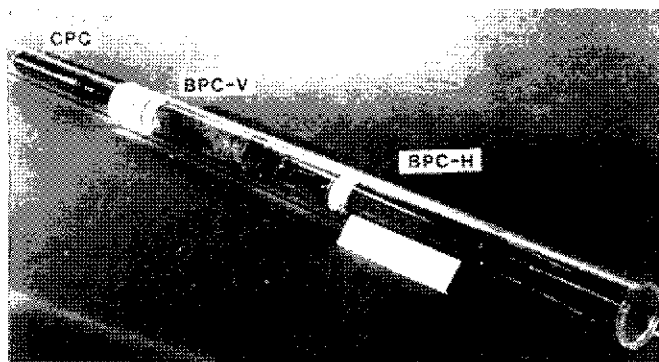


Figure 6.6.—Pressure cell package. (CPC = cylindrical pressure cell; BPC-V = borehole pressure cell in vertical direction; BPC-H = borehole pressure cell in horizontal direction.)

SURFACE ROSETTE METHOD

The surface rosette method (fig. 6.7), which is also called the displacement rosette method or undercoring, determines absolute stress and stress changes on the surface around an opening (11). This technique uses six pins installed in pairs across three diameters spaced 60° apart (fig. 6.8). After initial measurements are taken across the three diameters using a Whittemore-type strain gage,²² the surface rock stress is relieved by drilling a hole concentric to the six pins (referred to as undercoring). Final measurements are then taken across the three diameters. Based on the elastic infinite plate theory, the change in length of the diametral distances and the elastic modulus of the rock are used to calculate the surface stresses. Generally, rosette pins are placed on 10-in centers and undercored with a 6-in-diam bit. The rosette diameter can be varied to meet the measurement precision required.

The surface rosette method is a simple and economical technique for determining the absolute stresses and the change of stress on the surface around an opening. If stress measurements outside the influence of the mined opening are required, then other methods should be considered. The surface rosette method is not suitable for heavily jointed rock or rock that has been severely damaged by blasting.

Table 6.1 is a summary of the various in situ stress-measuring methods.

²²The Whittemore-type strain gage is a dial extensometer. A gage design to measure 10 in can be read accurately to $100\ \mu\text{in}$ and interpolated to $10\ \mu\text{in}$.

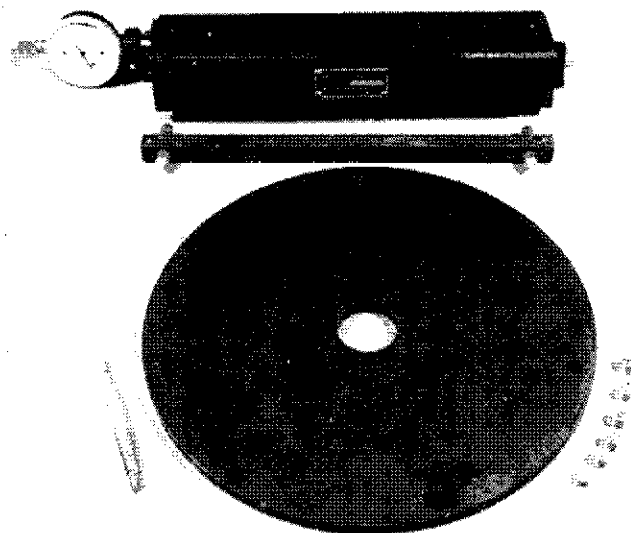


Figure 6.7.—Equipment for surface rosette displacement measurements.

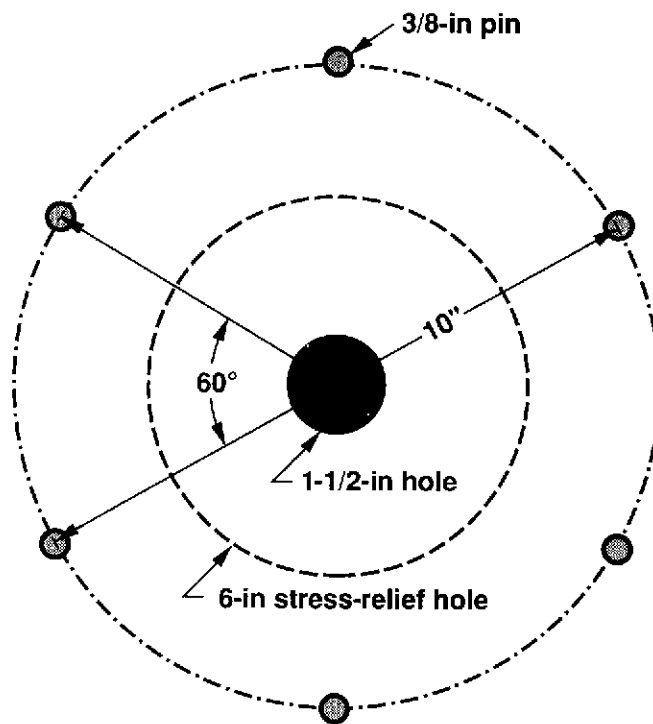


Figure 6.8.—Surface rosette configuration.

Table 6.1.—Summary of stress-measuring methods

Method	Requirements	Application	Difficulties	Comments
3-C BHG	Borehole overcoring. Need rock to core. Elastic properties of rock.	Measurements can be made up to several hundred feet deep. Stress profile or 2-dimensional stresses from single hole. 3-dimensional stress field from 3 nonparallel holes.	In fractured rock or rock that disks.	Does not need a dry hole. Good estimate of stress from core length-to-diameter ratio of 1:2.
Hydraulic fracturing	Borehole. Know principal stress direction or be within 15°. Must have orientation of fractures. Know tensile strength of rock.	Measures in very deep holes from the surface.	Less accurate if material is not brittle, homogenous, elastic, isotropic, or porous. Straight hole and proper hole diameter for seating packers.	Not generally used underground. Elastic properties of rock not required.
Flat jack	Slot. Minimum of 6 tests (flat jacks) measuring in different directions. Epoxy pins on surface.	Measurement restricted to near surface. Does not disturb stress field. Measures over larger area.	Trapped air in grout could invalidate results. Nonrepeatable stress-strain behavior could produce considerable error at low in-site stresses.	Elastic properties of rock not required.
Rosette strain gage	Dry borehole overcoring. Must use a "not too common" fast-setting epoxy to be time efficient. Elastic properties of rock.	Measures in a shorter section of rock than other methods requiring a borehole. 3-dimensional stress field from 3 nonparallel holes. Suitable in high-stress zones.	Clean and dry hole. Value to use for the "stress concentration factor" not certain.	Temperature effects measurement and must be monitored.
CSIR-type cell ¹	Dry borehole overcoring. Need rock to core. Cell is needed for each overcore. Material should be homogeneous and elastic. Elastic properties of rock.	3-dimensional stress field from a single borehole.	Trapped air in adhesive over a strain gage invalidates that particular strain measurement. Material that is not fractured for a length of 2-3 times the diameter of the overcoring hole.	Fast setting adhesive is recommended to become time efficient. Suitable when coring is only possible in 1 direction.
Hydraulic pressure cell . .	Borehole. Poisson's ratio ν .	Measures stress magnitude in softer material (i.e., coal) than other methods. Measures in plastic material. If principal stress direction is known, can calculate 3-dimensional stress magnitude from 3 nonparallel holes.	May take up to a month to reach "equilibrium pressure." Not as accurate as most methods. "Setting pressure" not certain.	Young's modulus E not required. In situ Poisson's ratio ν can be measured if cell is calibrated in laboratory and in situ. Should use 3 separate holes in layered strata.
Surface rosette	Epoxy pins on surface. Undercoring. Elastic properties of rock.	Simple and economical method for measuring stresses on surface.	Holding pins in place on a wet roof or back until epoxy sets. Not as accurate as most methods. Not suitable in heavily jointed or blast-damaged rock.	Restricted to surface.

¹CSIRO-type cell has same characteristics.

CHAPTER 7.—STRESS DETERMINATIONS IN THE UNITED STATES

INTRODUCTION

For more than three decades, the U.S. Bureau of Mines has determined field stresses throughout the United States, both on the surface and in underground mines. At these minesites, either the complete three-dimensional state of stress and/or the horizontal secondary principal stresses P and Q were determined. If stress measurements were performed on more than one level at a mine, both stress values are given in the tables. Stress measurements performed at more than one site on or near the same level in a mine are averaged. Variations in the stresses may occur within a mine when local anomalies are present.

EXCESS HORIZONTAL STRESSES

Figure 7.1 is a representation of excess horizontal stress σ_h distributions in the United States compiled by the Bureau (26, 94). Excess σ_h is the σ_h in excess of those induced by gravity loading (Poisson's effect) or temperature changes. Excess σ_h has been determined to exist at the surface of the earth as well as at depth. The major and minor axes of each ellipse in figure 7.1 represent the excess P and Q secondary principal stresses at each site. The site numbers correspond to those in tables 7.1 and 7.2.

The gravity-induced portion of the in situ stress field can be calculated from the following equations:

$$\sigma_v = \gamma h \quad (7.1)$$

$$\text{and} \quad \sigma_h = \left[\frac{\nu}{1 - \nu} \right] \sigma_v, \quad (7.2)$$

where σ_v = vertical stress, psi,

and σ_h = horizontal stress, psi.

The derivation of equations 7.1 and 7.2 is based on lateral confinement of the rock mass.

By subtracting the gravity-induced stress from the measured stress, the excess stress is given. Excess horizontal stresses σ_h are presented for the purpose of comparing σ_h measured at various depths. However, the measured stress would be used for mine design.

From the maps in figure 7.1, several generalities are evident. Horizontal stresses σ_h are prevalent at the surface, ranging from 415 psi at Rock Springs, WY, to 3,248 psi at Babbitt, MN (table 7.1), and are directional. Near the eastern coast of the United States, the maximum compressive σ_h tends to parallel the Appalachian Mountains. Near the Great Lakes, the maximum σ_h tends to be tangential to the Michigan Basin. In the Rocky Mountain

States (figure 7.1, bottom) the maximum σ_h tend to parallel the Rocky Mountains but vary greatly in magnitude, ranging from 262 psi at Craig, CO, to 3,718 psi at Sunnyside, UT (table 7.2). The highest compressive horizontal stress value (6,258 psi at site 20) is located in the Coeur d'Alene District of Idaho. The combination of this extremely high σ_h , the geology, and the mining system used in this region makes the mines prone to rock burst (94). Coal mines in West Virginia and the Rocky Mountain region, to name a few, have experienced directional ground control problems that have correlated with the direction of the σ_h (23, 26-28, 94). The effects of depth, temperature, and topography on the in situ stress field have also been investigated. At the Homestake Mine in South Dakota (site 19), the three-dimensional stress field was determined at two different depths (table 7.3). The

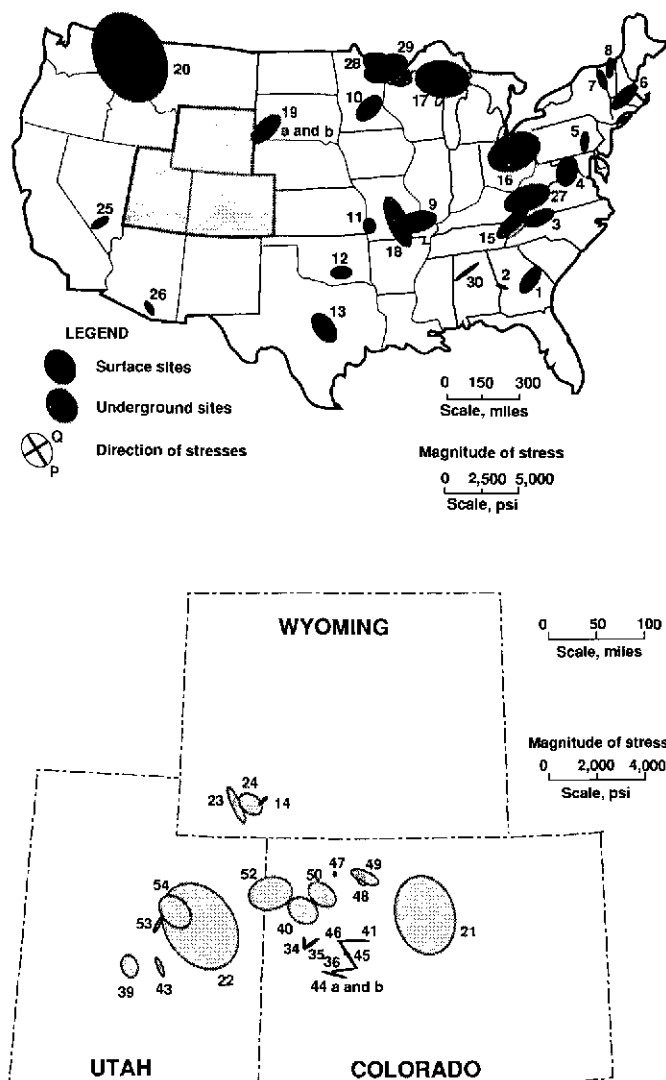


Figure 7.1.—Excess horizontal stress map.

effects of yearly temperature variations on the σ_h (32) were measured in a granite quarry at Mount Airy, NC (site 3 in table 7.1). The influence of mountainous topography on the stress field (35) was studied at the Henderson Mine in Colorado (site 21).

THREE-DIMENSIONAL STATE OF STRESS

In several of the underground sites shown in figure 7.1, the complete three-dimensional state of stress was determined from overcoring measurements obtained in three nonparallel boreholes. Results for the principal stresses (S_1 , S_2 , and S_3), along with the bearing and the inclination for each stress component, are presented in table 7.3. The standard deviation (SD) of the principal stress components is presented to give a level of confidence to the principal stress values. As shown in table 7.3, the SD of S_3 for both the Galena Mine and the Urad Mine is greater than the calculated stress value. Thus, the resulting stresses, bearings, and inclinations may or may not be representative of the in situ state of stress. The three principal stress components can be represented vectorially as shown for the Henderson Mine in figure 7.2.

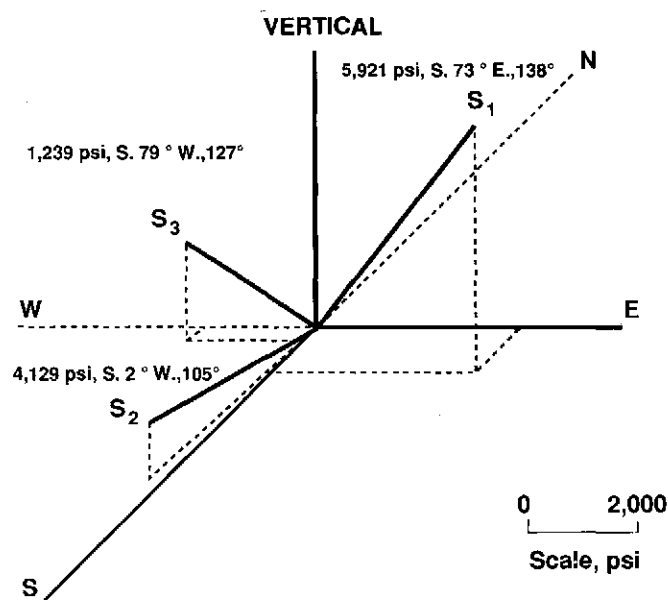


Figure 7.2.—Principal stresses in Henderson Mine in Colorado.

Table 7.1.—Excess horizontal compressive stress at surface sites

Site	Rock mass and location	Direction of P	Magnitude of P, psi	Magnitude of Q, psi	Average depth of measurement, ft	References
1	Lithonia gneiss, Lithonia, GA.	N. 49° E.	1,639	941	18.1	17
2	Douglasville gneiss, Douglasville, GA.	N. 64° W.	512	285	1.8	17
3	Mount Airy granite, Mount Airy, NC.	N. 87° E.	2,464	1,191	13	17
4	Virginia diabase, Rapidan, VA.	N. 6° E.	1,678	1,385	8.6	17
5	French Creek norite, St. Peters, PA.	N. 14° E.	820	335	4.8	17
6	Chelmsford granite, West Chelmsford, MA.	N. 56° E.	2,133	1,113	61.9	17
7	Proctor dolomite, Proctor, VT.	N. 4° W.	1,328	516	1.2	17
8	Barre granite, Barre, VT.	N. 14° E.	1,734	791	151.2	17
9	St. Francois granite, Graniteville, MO.	N. 77° E.	3,190	1,397	4.7	17
10	Diamond-pink granite, St. Cloud, MN.	N. 50° E.	2,190	1,534	4.9	11
11	Burlington limestone, Carthage, MO.	N. 2° E.	1,066	777	4	17
12	Troy granite, Troy, OK.	N. 84° W.	1,075	519	4.5	17
13	Granite Mountain granite, Marble Falls, TX.	N. 33° W.	2,219	1,491	4.7	17
14	Green River oil shale, Rock Springs, WY.	N. 42° E.	415	171	10	26, 94
29	Biwabik iron formation, Babbitt, MN.	N. 77° E.	3,248	1,639	14	94
31	Devitrified breccia, Amchitka, AK.	N. 67° W.	65	-59	8	42

NOTE.—A tensile (–) minimum stress results when the effects of overburden are subtracted out and does not necessarily indicate a tensile in situ stress.

Table 7.2.—Excess horizontal compressive stress at underground sites

Site	Location	Direction of P	Magnitude of P, psi	Magnitude of Q, psi	Depth of overburden, ft	References
15	Immel Mine, Knoxville, TN.	N. 58° E.	3,007	551	925	(¹)
16	Limestone Mine, Barberton, OH.	N. 77° E.	4,000	2,500	2,300	59
17	Mather "B" Mine, Ishpeming, MI.	N. 82° W.	3,822	2,937	3,200	(¹)
18	Fletcher Mine, Bunker, MO.	N. 17° W.	3,682	1,595	1,000	37
19a	Homestake Mine, Lead, SD.	N. 30° E.	2,784	1,058	6,200	(¹)
19b	Do.	N. 43° E.	2,668	826	2,943	(¹)
20	Crescent Mine, ² Wallace, ID.	N. 27° W.	6,258	4,966	5,300	45
21	Henderson Mine, Empire, CO.	N. 15° W.	3,398	2,283	3,127	35
22	Sunnyside No. 1 Mine, Sunnyside, UT.	N. 31° W.	3,718	2,898	1,060	(¹)
23	Alchem Trona Mine, Green River, WY.	N. 21° W.	1,768	631	1,600	(¹)
24	Big Island Mine, ^{2,3} Green River, WY.	N. 38° W.	1,054	705	850	(¹)
25	Nevada Test Site, Rainier Mesa, NV.	N. 47° E.	972	345	1,250	(¹)
26	Lakeshore Mine, Casa Grande, AZ.	N. 17° W.	502	160	1,570	(¹)
27	Beckley No. 1 Mine, Bolt, WV.	N. 69° E.	2,973	1,466	700	26, 94
28	Minnamax Mine, Babbitt, MN.	N. 59° W.	3,958	1,824	1,700	94
30	North River Mine, Berry, AL.	N. 60° E.	1,830	9	500	94
32	Lee's Summit Quarry, Kansas City, MO.	N. 49° E.	2,079	-113	80	(¹)
33	Kerford Quarry, Weeping Water, NE.	N. 83° E.	596	474	90	(¹)
34	Logan Wash Mine, DeBeque, CO.	N. 6° E.	308	25	650	(¹)
35	Anvil Points Mine, ² Rifle, CO.	N. 58° E.	757	7	500	(¹)
36	Bear Mine, Somerset, CO.	N. 69° E.	961	-125	1,250	(¹)
37	Beckley No. 2 Mine, Sabine, WV.	N. 46° W.	1,652	1,199	350	23
38	Kitt No. 1 Mine, Philippi, WV.	N. 68° E.	1,777	1,377	700	(¹)
39	Sufco No. 1 Mine, Salina, UT.	N. 16° W.	915	462	1,000	(¹)
40	Bureau of Mines Bored Shaft, Meeker, CO.	N. 62° W.	1,247	861	2,035	(¹)
41	Thompson Creek Mine, Carbondale, CO.	N. 87° E.	1,007	105	1,625	(¹)
42	Elkhart Mine, Elkhart, IL.	N. 37° E.	996	540	275	(¹)
43	Emery No. 50 Mine, Emery, UT.	N. 22° W.	1,012	124	450	(¹)
44a	Orchard Valley Mine, Paonia, CO.	N. 69° E.	1,037	145	350	(¹)

See explanatory notes at end of table.

Table 7.2.—Excess horizontal compressive stress at underground sites—Continued

Site	Location	Direction of P	Magnitude of P, psi	Magnitude of Q, psi	Depth of overburden, ft	References
44b	Orchard Valley Mine, Paonia, CO.	N. 83° W.	1,172	16	1,625	(¹)
45	Coal Basin No. 5 Mine, Redstone, CO.	N. 52° W.	412	198	1,000	(¹)
46	Dutch Creek No. 1 Mine, Redstone, CO.	N. 29° W.	1,564	-170	2,700	(¹)
47	Eagle No. 5 Mine, Craig, CO.	N. 4° W.	262	43	463	29
48	Apex No. 2 Mine, Oak Creek, CO.	N. 53° W.	291	232	410	29
49	Foidel Creek Mine, Oak Creek, CO.	N. 57° W.	1,020	405	405	29
50	Rienau No. 2 Mine, Meeker, CO.	N. 34° W.	1,098	745	765	29
51	Peabody No. 10, Mine, Pawnee, IL.	N. 73° E.	1,070	549	360	(¹)
52	Deserado Mine, Rangely, CO.	N. 77° E.	1,822	1,268	605	(¹)
53	King No. 4 Mine, Hiawatha, UT.	N. 20° E.	698	114	1,220	(¹)
54	Castle Gate No. 3 Mine, Helper, UT.	N. 29° W.	1,522	1,144	2,200	(¹)
55	Southfield Mine, Canon City, CO.	N. 7° W.	1,386	781	1,410	(¹)

¹Previously unpublished.²Horizontal stresses determined from the three-dimensional stress ellipsoid calculations.³Measurements taken in trona.

NOTE.—A tensile (-) minimum stress results when the effects of overburden are subtracted out and does not necessarily indicate a tensile in situ stress.

Table 7.3.—Three-dimensional state of stress at underground sites

Site	Location	Depth of overburden, ft	Principal stresses, psi	SD, ¹ psi	Bearing, ² deg	Inclination, ³ deg
15	Immel Mine, Knoxville, TN.	925	$S_1 = 3,580$ $S_2 = 1,333$ $S_3 = 1,015$	202 152 159	238 197 146	102 16 100
17	Mather "B" Mine, Ishpeming, MI.	3,200	$S_1 = 3,895$ $S_2 = 2,470$ $S_3 = 2,207$	451 414 348	290 257 194	114 28 104
19a	Homestake Mine, Lead, SD.	2,942	$S_1 = 3,816$ $S_2 = 2,925$ $S_3 = 1,841$	487 332 316	222 232 133	112 23 86
19b	Do.	6,454	$S_1 = 7,960$ $S_2 = 5,307$ $S_3 = 3,427$	289 414 544	152 33 122	166 97 78
21	Henderson Mine, Empire, CO. ⁴	3,579	$S_1 = 5,921$ $S_2 = 4,129$ $S_3 = 2,740$	778 639 831	107 182 259	138 105 127
23	Alchem Trona Mine, Green River, WY.	1,600	$S_1 = 2,758$ $S_2 = 2,328$ $S_3 = 1,187$	115 67 56	317 308 248	169 80 86
24	Big Island Mine, Green River, WY.	850	$S_1 = 1,634$ $S_2 = 1,366$ $S_3 = 1,202$	146 87 128	120 34 209	92 24 67
25	Nevada Test Site, Rainier Mesa, NV.	1,250	$S_1 = 1,239$ $S_2 = 819$ $S_3 = 345$	73 64 133	227 222 136	110 20 92
26	Lakeshore Mine, Casa Grande, AZ.	1,570	$S_1 = 1,708$ $S_2 = 1,091$ $S_3 = 847$	109 118 156	339 346 255	156 66 87
28	Minnamox Mine, Babbitt, MN.	1,700	$S_1 = 4,739$ $S_2 = 2,657$ $S_3 = 1,837$	428 352 158	121 30 44	86 74 163
34	Logan Wash Mine, DeBeque, CO.	650	$S_1 = 462$ $S_2 = 309$ $S_3 = 125$	48 55 66	43 11 103	165 77 82
35	Anvil Points Mine, Rifle, CO.	500	$S_1 = 903$ $S_2 = 773$ $S_3 = 148$	163 52 134	238 241 149	101 11 89
56	Urad Mine, Empire, CO. ⁴	2,046	$S_1 = 2,643$ $S_2 = 1,769$ $S_3 = 1,109$	201 239 1,373	70 308 218	173 94 96
57	Galena Mine, Wallace, ID.	3,700	$S_1 = 5,155$ $S_2 = 4,501$ $S_3 = -305$	397 416 1,325	271 259 176	132 42 96

¹Standard deviation of the principal stresses.²Bearing is referenced using true north as 0° and increases clockwise to 360°.³Inclination is referenced using vertical down as 0°, horizontal as 90°, and vertical up as 180°.⁴Reference 35.

SUMMARY

The 3-C BHG developed by the U.S. Bureau of Mines is an accurate device for measuring the diametral deformation of a borehole during stress relief by overcoring. If the procedures are followed as described in this bulletin, the BHG can be temperature insensitive and time stable for at least a month. The BHG measures three diametral deformations U_1 , U_2 , and U_3 (60° apart) of a $1\frac{1}{2}$ -in-diam EX-size borehole as the gage is overcored with a 6-in-diam diamond drill bit. Since a relationship exists between the applied stress and the diametral deformation of a borehole in rock, the stresses normal to the axis of the borehole can be calculated. Stress-relief deformation measurements obtained from three nonparallel boreholes provide the complete three-dimensional state of stress. Biaxial testing of the stress-relieved core provides a convenient method for obtaining the modulus of elasticity E and the rock anisotropy in the field. Laboratory triaxial testing of the stress-relieved core compensates for

nonlinearity of the rock and provides the elastic constants necessary to determine the in situ state of stress. Temperature variations can be accounted for in the stress calculations. A least squares method of calculating the average rock stress components provides statistical information that is very useful in the interpretation of results.

Several design modifications in the BHG have enhanced the versatility of the gage.

For certain applications, alternative methods for determining the field stress are available. All methods presented give good estimates of the stresses, providing the prescribed techniques are followed, ground conditions are suitable, and a degree of skill is applied.

The Bureau's 3-C BHG, using the overcoring technique of stress relieving the rock, is recognized worldwide by the mining industry as the standard for determining the absolute state of stress and has been included in the ASTM industry standards annual book (18, pp. 1084-1102).

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APPENDIX A.—DERIVATIONS FOR THE DEFORMATION OF A HOLE IN A PLATE

By Wilbur I. Duvall¹

The equations for the deformation of holes in an infinite plate that is perfectly elastic, homogeneous, and isotropic are presented in part A of this appendix. The cases for uniaxial and biaxial stress fields in plane stress and plane strain are considered.

The equations relating the principal stresses in a medium to the deformation of a hole as measured along three diameters 45° or 60° apart are derived in part B of this appendix. These equations are called the 45° and 60° deformation rosette equations.

Part A

Case 1: Deformation of a hole in a plate subjected to a unidirectional stress field under the condition of plane stress

To obtain the deformation of a hole in a plate under plane stress, the Kirsch equations (3)² for the stress distributions around a hole are substituted in Hooke's law equations for plane stress and integrated.

The relationships for the strain components and displacements in polar coordinates are

$$e_r = \frac{\partial u}{\partial r}, \quad (A-1)$$

$$e_\theta = \frac{u}{r} + \frac{\partial v}{r \partial \theta}, \quad (A-2)$$

$$\text{and} \quad \lambda_{r\theta} = \frac{\partial u}{r \partial \theta} + \frac{\partial v}{\partial r} - \frac{v}{r}. \quad (A-3)$$

Hooke's law equations for plane stress are

$$e_r = \frac{1}{E} (\sigma_r - \nu \sigma_\theta), \quad (A-4)$$

where σ_θ = tangential stress,

$$e_\theta = \frac{1}{E} (\sigma_\theta - \nu \sigma_r), \quad (A-5)$$

$$\text{and} \quad \lambda_{r\theta} = \frac{2(1 + \nu)}{E} \tau_{r\theta}, \quad (A-6)$$

where $\lambda_{r\theta}$ = shearing strain.

The Kirsch equations for the stress distributions around a circular hole (unidirectional stress field, $T = 0$) are

$$\sigma_r = \frac{S}{2} \left[1 - \frac{a^2}{r^2} \right] + \frac{S}{2} \left[1 - \frac{4a^2}{r^2} + \frac{3a^4}{r^4} \right] \cos 2\theta, \quad (A-7)$$

$$\sigma_\theta = \frac{S}{2} \left[1 + \frac{a^2}{r^2} \right] - \frac{S}{2} \left[1 + \frac{3a^4}{r^4} \right] \cos 2\theta, \quad (A-8)$$

$$\text{and} \quad \tau_{r\theta} = -\frac{S}{2} \left[1 + \frac{2a^2}{r^2} - \frac{3a^4}{r^4} \right] \sin 2\theta. \quad (A-9)$$

Combining equations A-1 and A-4, equations A-2 and A-5, and equations A-3 and A-6 yields

$$\frac{\partial u}{\partial r} = \frac{1}{E} (\sigma_r - \nu \sigma_\theta), \quad (A-10)$$

$$\frac{u}{r} + \frac{\partial v}{r \partial \theta} = \frac{1}{E} (\sigma_\theta - \nu \sigma_r), \quad (A-11)$$

$$\text{and} \quad \frac{\partial u}{r \partial \theta} + \frac{\partial v}{\partial r} - \frac{v}{r} = \frac{2(1 + \nu)}{E} \tau_{r\theta}. \quad (A-12)$$

Substituting equations A-7 and A-8 into equation A-10 gives

$$\begin{aligned} \frac{\partial u}{\partial r} = \frac{1}{E} & \left[\frac{S}{2} \left[1 - \frac{a^2}{r^2} \right] + \frac{S}{2} \left[1 - \frac{4a^2}{r^2} + \frac{3a^4}{r^4} \right] \cos 2\theta \right] \\ & - \frac{\nu}{E} \left[\frac{S}{2} \left[1 + \frac{a^2}{r^2} \right] - \frac{S}{2} \left[1 + \frac{3a^4}{r^4} \right] \cos 2\theta \right]. \end{aligned} \quad (A-13)$$

Integrating equation A-13 gives

$$\begin{aligned} u = \frac{1}{E} & \left[\frac{S}{2} \left[r + \frac{a^2}{r} \right] + \frac{S}{2} \left[r + \frac{4a^2}{r} - \frac{a^4}{r^3} \right] \cos 2\theta \right] \\ & - \frac{\nu}{E} \left[\frac{S}{2} \left[r - \frac{a^2}{r} \right] - \frac{S}{2} \left[r - \frac{a^4}{r^3} \right] \cos 2\theta \right] \\ & + g_1(\theta), \end{aligned} \quad (A-14)$$

where $g_1(\theta)$ = arbitrary function of θ only.

¹Physicist, Denver Research Center, U.S. Bureau of Mines, Denver, CO, (retired); derived part of the equations presented in this appendix.

²Italic numbers in parentheses refer to items in the list of references preceding this appendix.

Substituting equations A-7, A-8, and A-14 into equation A-11 and combining terms gives

$$\begin{aligned} \frac{\partial v}{\partial \theta} = & \frac{1}{E} \left[-\frac{S}{2} \left(2r + \frac{4a^2}{r} + \frac{2a^4}{r^3} \right) \cos 2\theta \right] \\ & - \frac{\nu}{E} \left[\frac{S}{2} \left(2r - \frac{4a^2}{r} + \frac{2a^4}{r^3} \right) \cos 2\theta \right] \\ & - g_1(\theta). \end{aligned} \quad (\text{A-15})$$

Integrating equation A-15 gives

$$\begin{aligned} v = & \frac{1}{E} \left[-\frac{S}{2} \left(r + \frac{2a^2}{r} + \frac{a^4}{r^3} \right) \sin 2\theta \right] \\ & - \frac{\nu}{E} \left[\frac{S}{2} \left(r - \frac{2a^2}{r} + \frac{a^4}{r^3} \right) \sin 2\theta \right] \\ & - \int g_1(\theta) d\theta + g_2(r), \end{aligned} \quad (\text{A-16})$$

where $g_2(r)$ = function of r only.

Differentiating equation A-14 with respect to θ gives

$$\begin{aligned} \frac{\partial u}{\partial \theta} = & \frac{1}{E} \left[-S \left(r + \frac{4a^2}{r} - \frac{a^4}{r^3} \right) \sin 2\theta \right] \\ & - \frac{\nu S}{E} \left(r - \frac{a^4}{r^3} \right) \sin 2\theta + \frac{\partial g_1(\theta)}{\partial \theta}. \end{aligned} \quad (\text{A-17})$$

Differentiating equation A-16 with respect to r gives

$$\begin{aligned} \frac{\partial v}{\partial r} = & \frac{1}{E} \left[-\frac{S}{2} \left(1 - \frac{2a^2}{r^2} - \frac{3a^4}{r^4} \right) \sin 2\theta \right] \\ & - \frac{\nu}{E} \left[\frac{S}{2} \left(1 + \frac{2a^2}{r^2} - \frac{3a^4}{r^4} \right) \sin 2\theta \right] \\ & + \frac{\partial g_2(r)}{\partial r}. \end{aligned} \quad (\text{A-18})$$

Substituting equations A-9, A-16, A-17, and A-18 into equation A-12 and collecting terms gives

$$\frac{\partial g_1(\theta)}{\partial \theta} + \frac{r \partial g_2(r)}{\partial r} + \int g_1(\theta) d\theta - g_2(r) = 0. \quad (\text{A-19})$$

Since $g_1(\theta)$ is a function of θ only and $g_2(r)$ is a function of r only, the terms containing θ and the terms containing r must both be equal to zero; thus,

$$\frac{r \, dg_2(r)}{dr} - g_2(r) = 0 \quad (\text{A-20})$$

$$\text{and} \quad \frac{d^2 g_1(\theta)}{d\theta^2} + g_1(\theta) = 0, \quad (\text{A-21})$$

where total derivatives are used because g_2 is a function of r only, and g_1 is a function of θ only. The solution of equations A-20 and A-21 are

$$g_2(r) = Cr \quad (\text{A-22})$$

$$\text{and} \quad g_1(\theta) = A \sin \theta + B \cos \theta. \quad (\text{A-23})$$

As there can be no tangential displacements v on either the x or y axis, the boundary conditions are $v = 0$ for all values of r when $\theta = 0, \frac{\pi}{2},$ and π . Substituting equations A-22 and A-23 and the boundary condition into equation A-16 gives $A = 0, B = 0,$ and $C = 0$. Because the constants in equations A-22 and A-23 are zero, the arbitrary functions $g_1(\theta)$ and $g_2(r)$ are zero; thus, the displacements u and v , from equations A-14 and A-16, are

$$\begin{aligned} u = & \frac{1}{E} \left[\frac{S}{2} \left(r + \frac{a^2}{r} \right) + \frac{S}{2} \left(r + \frac{4a^2}{r} - \frac{a^4}{r^3} \right) \cos 2\theta \right] \\ & - \frac{\nu}{E} \left[\frac{S}{2} \left(r - \frac{a^2}{r} \right) - \frac{S}{2} \left(r - \frac{a^4}{r^3} \right) \cos 2\theta \right] \end{aligned} \quad (\text{A-24})$$

$$\begin{aligned} \text{and} \quad v = & -\frac{1}{E} \left[\frac{S}{2} \left(r + \frac{2a^2}{r} + \frac{a^4}{r^3} \right) \sin 2\theta \right] \\ & - \frac{\nu}{E} \left[\frac{S}{2} \left(r - \frac{2a^2}{r} + \frac{a^4}{r^3} \right) \sin 2\theta \right]. \end{aligned} \quad (\text{A-25})$$

Equations A-24 and A-25 describe radial u and tangential displacements v anywhere in the medium. To obtain the change in any diameter U of the hole (deformation), substitute $r = a = \frac{d}{2}$ into equation A-24; thus,

$$U = \frac{Sd}{E} (1 + 2 \cos 2\theta). \quad (\text{A-26})$$

Case 2: Deformation of a hole in a plate subjected to a unidirectional stress under the condition of plane strain

To obtain the deformation of a hole in a plate in plane strain, the Kirsch equations A-7, A-8, and A-9 are substituted in the Hooke's law equations for plane strain, which are

$$e_r = \frac{1}{E} \left[(1 - \nu^2)\sigma_r - \nu(1 + \nu)\sigma_\theta \right], \quad (\text{A-27})$$

$$e_\theta = \frac{1}{E} \left[(1 - \nu^2)\sigma_\theta - (1 + \nu)\sigma_r \right], \quad (\text{A-28})$$

and
$$\lambda_{r\theta} = \frac{2(1 + \nu)}{E} \tau_{r\theta}. \quad (\text{A-29})$$

Equations A-1, A-2, A-3, A-7, A-8, and A-9 are valid under conditions of both plane stress or plane strain. Hooke's law equations for plane strain differ from the plane stress equations only by the terms $(1 - \nu^2)$ and $(1 + \nu)$. Therefore, the method of solution and the boundary conditions are valid for both cases and the radial u and tangential displacements v anywhere in the medium are

$$u = \frac{1 - \nu^2}{E} \left[\frac{S}{2} \left(r + \frac{a^2}{r} \right) + \frac{S}{2} \left(r + \frac{4a^2}{r} - \frac{a^4}{r^3} \right) \cos 2\theta \right] - \frac{\nu(1 + \nu)}{E} \left[\frac{S}{2} \left(r - \frac{a^2}{r} \right) - \frac{S}{2} \left(r - \frac{a^4}{r^3} \right) \cos 2\theta \right] \quad (\text{A-30})$$

and

$$v = -\frac{1 - \nu^2}{E} \left[\frac{S}{2} \left(r + \frac{2a^2}{r} + \frac{a^4}{r^3} \right) \sin 2\theta \right] - \frac{\nu(1 + \nu)}{E} \left[\frac{S}{2} \left(r - \frac{2a^2}{r} + \frac{a^4}{r^3} \right) \sin 2\theta \right]. \quad (\text{A-31})$$

To obtain diameter U , $r = a = \frac{d}{2}$ is substituted into equation A-30; thus,

$$U = \frac{Sd}{E} (1 - \nu^2) (1 + 2 \cos 2\theta). \quad (\text{A-32})$$

Case 3: Deformation of a hole in a plate subjected to a biaxial stress field and for the condition of plane stress

The equations for the stress distribution around a hole are obtained by superposition; therefore, equations A-7, A-8, and A-9 become

$$\sigma_r = \frac{S + T}{2} \left[1 - \frac{a^2}{r^2} \right] + \frac{S - T}{2} \left[1 - \frac{4a^2}{r^2} + \frac{3a^4}{r^4} \right] \cos 2\theta, \quad (\text{A-33})$$

$$\sigma_\theta = \frac{S + T}{2} \left[1 + \frac{a^2}{r^2} \right] - \frac{S - T}{2} \left[1 + \frac{3a^4}{r^4} \right] \cos 2\theta, \quad (\text{A-34})$$

$$\text{and } \tau_{r\theta} = -\frac{S - T}{2} \left[1 + \frac{2a^2}{r^2} - \frac{3a^4}{r^4} \right] \sin 2\theta. \quad (\text{A-35})$$

The solution for the displacements is obtained by the same method as used for case 1; the displacements are

$$u = \frac{1}{E} \left[\frac{S + T}{2} \left(r + \frac{a^2}{r} \right) + \frac{S - T}{2} \left(r + \frac{4a^2}{r} - \frac{a^4}{r^3} \right) \cos 2\theta \right] - \frac{\nu}{E} \left[\frac{S + T}{2} \left(r - \frac{a^2}{r} \right) - \frac{S - T}{2} \left(r - \frac{a^4}{r^3} \right) \cos 2\theta \right] \quad (\text{A-36})$$

$$\text{and } v = -\frac{1}{E} \left[\frac{S - T}{2} \left(r + \frac{2a^2}{r} + \frac{a^4}{r^3} \right) \sin 2\theta \right] - \frac{\nu}{E} \left[\frac{S - T}{2} \left(r - \frac{2a^2}{r} + \frac{a^4}{r^3} \right) \sin 2\theta \right]. \quad (\text{A-37})$$

To obtain the deformation of the hole, $r = a = \frac{d}{2}$ is

$$U = \frac{d}{E} [(S + T) + 2(S - T) \cos 2\theta]. \quad (\text{A-38})$$

Case 4: Deformation of a hole in a plate subjected to biaxial stress and for the conditions for plane strain

The equations for the stress distributions around a hole in a plate subjected to a biaxial stress field, equations A-33, A-34, and A-35, are substituted into equations A-27, A-28, and A-29. Then the procedures outlined under case 1 are followed to obtain

$$U = \frac{(1 - \nu^2)d}{E} [(S + T) + 2(S - T) \cos 2\theta]. \quad (\text{A-39})$$

PART B

Case 1: The 60° deformation-rosette equations for a biaxial stress field and plane stress

The equations for the deformation of a hole in a plate in biaxial stress fields are used to relate the stresses in the plate to the deformations measured across three diameters of the hole.

Equation A-38 determines the deformation of a hole in a plate in a biaxial stress field. By letting $K = \frac{d}{E}$, $A = S + T$, and $B = S - T$, equation A-38 becomes

$$U = KA + 2KB \cos 2\theta. \quad (\text{A-40})$$

To obtain U_1 , U_2 , and U_3 ,

$$U_1 = KA + 2KB \cos 2\theta_1, \quad (\text{A-41})$$

$$U_2 = KA + 2KB \cos 2\theta_2, \quad (\text{A-42})$$

$$\text{and } U_3 = KA + 2KB \cos 2\theta_3. \quad (\text{A-43})$$

For a 60° rosette, $\theta_2 = \theta_1 + 60^\circ$ and $\theta_3 = \theta_1 + 120^\circ$; equations A-41, A-42, and A-43 become

$$U_1 = KA + 2KB \cos 2\theta_1, \quad (\text{A-44})$$

$$U_2 = KA + 2KB \cos 2(\theta_1 + 60^\circ), \quad (\text{A-45})$$

$$\text{and } U_3 = KA + 2KB \cos 2(\theta_1 + 120^\circ). \quad (\text{A-46})$$

The cosine of the sum of the angles in equations A-45 and A-46 is

$$\cos(2\theta_1 + 120^\circ) = -\frac{1}{2} \cos 2\theta_1 - \frac{\sqrt{3}}{2} \sin 2\theta_1 \quad (\text{A-47})$$

$$\text{and } \cos(2\theta_1 + 240^\circ) = -\frac{1}{2} \cos 2\theta_1 + \frac{\sqrt{3}}{2} \sin 2\theta_1. \quad (\text{A-48})$$

Substituting equations A-47 and A-48 in equations A-45 and A-46 and solving for A and B gives

$$A = \frac{U_1 + U_2 + U_3}{3K} \quad (\text{A-49})$$

$$\begin{aligned} \text{and } B = \frac{\sqrt{2}}{6K} & \left[(U_1 - U_2)^2 + (U_2 - U_3)^2 \right. \\ & \left. + (U_1 - U_3)^2 \right]^{1/2}. \end{aligned} \quad (\text{A-50})$$

Substituting equations A-47 and A-48 into equations A-44, A-45, and A-46 and solving for θ_1 gives

$$\tan 2\theta_1 = -\frac{\sqrt{3}(U_2 - U_3)}{2U_1 - U_2 - U_3}. \quad (\text{A-51})$$

Because $A = S + T$ and $B = S - T$, the sum and difference of the stresses are

$$S + T = \frac{E}{3d} (U_1 + U_2 + U_3) \quad (\text{A-52})$$

$$\begin{aligned} \text{and } S - T = \frac{\sqrt{2}E}{6d} & \left[(U_1 - U_2)^2 + (U_2 - U_3)^2 \right. \\ & \left. + (U_1 - U_3)^2 \right]^{1/2}. \end{aligned} \quad (\text{A-53})$$

Hence, the stresses in the medium can be obtained from a measurement of U_1 , U_2 , and U_3 . The angle between the stresses and the angle at which U_1 is measured are obtained from equation A-51.

The conditions for plane strain can be obtained by dividing equations A-52 and A-53 by the term $(1 - \nu^2)$.

Case 2: The 45° deformation-rosette equations

The solution for the 45° deformation rosette is obtained by the same steps as used for the solution of the 60°

deformation rosette. The solutions for the sum and difference of the stresses are

$$S + T = \frac{E}{2d} (U_1 + U_3) \quad (\text{A-54})$$

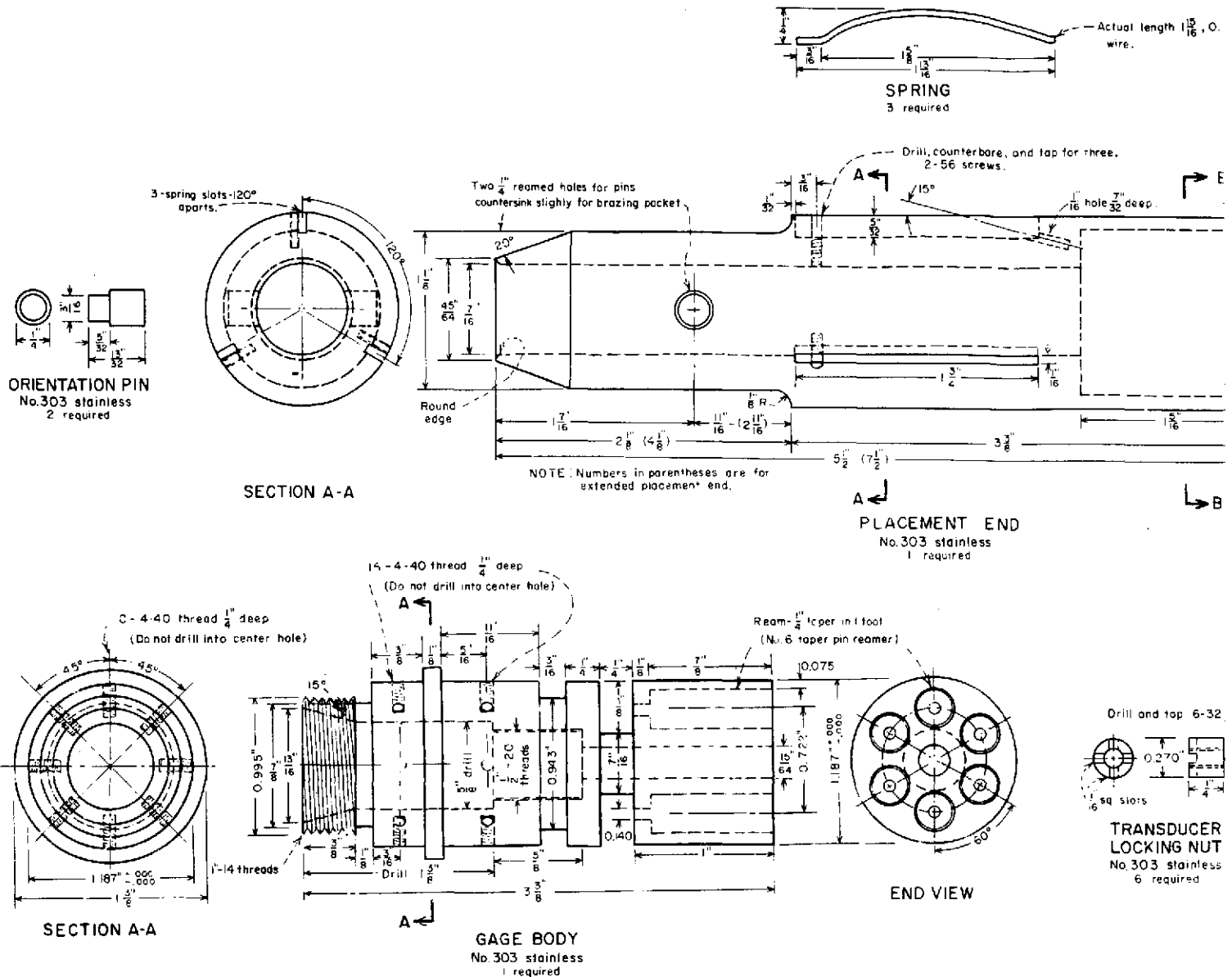
$$\text{and } S - T = \frac{E}{2\sqrt{2}d} \left[(U_1 - U_2)^2 + (U_2 - U_3)^2 \right]^{1/2}. \quad (\text{A-55})$$

The tangent of twice the angle between the stress S and the deformation U_1 is

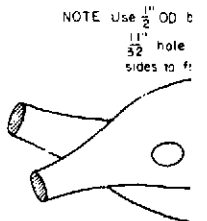
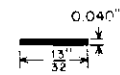
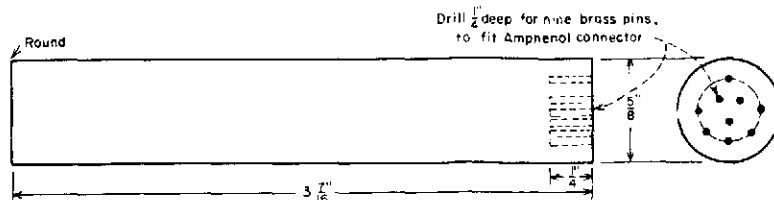
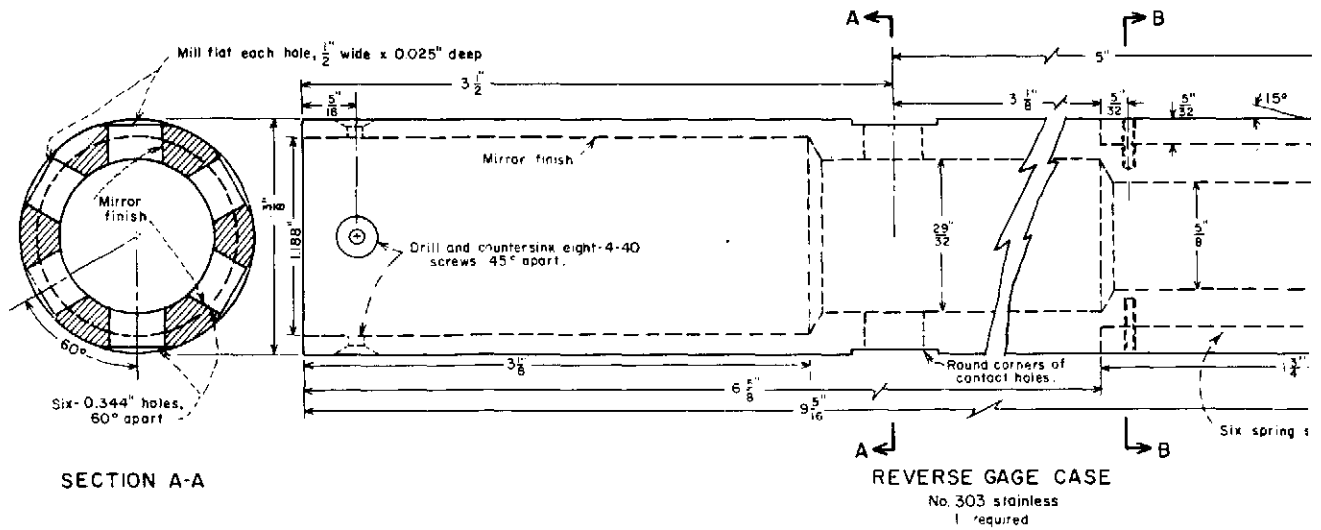
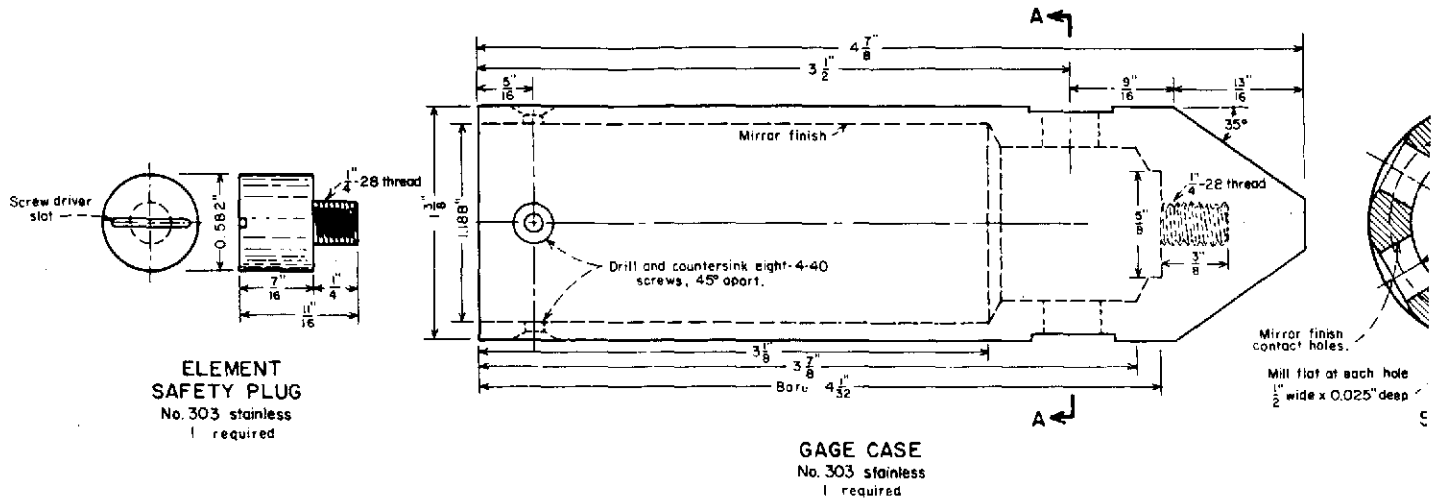
$$\tan 2\theta_1 = - \frac{2U_2 - U_1 - U_3}{U_1 - U_3}. \quad (\text{A-56})$$

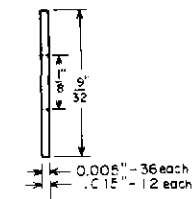
Again, for plane strain, equations A-54 and A-55 can be used by dividing by the term $(1 - \nu^2)$.

APPENDIX B.—EQUIPMENT DIA



- LIST OF MATERIALS**
- 1-3/8" 303 stainless steel rod
 - 5/8" brass rod
 - 0.040" brass pins
 - 3/8" cold-worked beryllium copper (Beryco 33-25) 1/2 H rod
 - Tungsten-carbide (Kennametal) wear button
 - Cable - Neoprene jacket, shielded, 8-conductor, No. 18 wire, 0.42" OD
 - Connector - "Amphenol" No. 126-013 female socket fits into gage body
 - No. 126-220 male socket fit into gage body
 - Rubber Grommet - "Crouse-Hinds" No. 93
 - O-Ring "Buna" No. 2-213 for gage body. No. 2-009 for piston
 - Machine screws - No. 4-40 stainless steel slotted flat head 1/4"
 - 2-56 stainless steel slotted pan head 3/16"
 - Music wire - No. 23 (0.051")
 - Piston washers - 9/32" x 0.005"
 - 9/32" x 0.015"
 - Strain gage - WA-09-125AD-120 options B-87 and RR Micro-M
 - Terminals - No. T-18 or No. CEG-50C

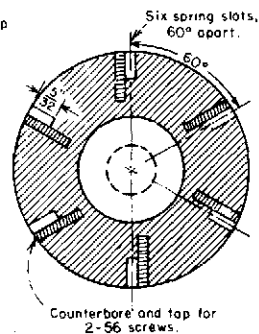




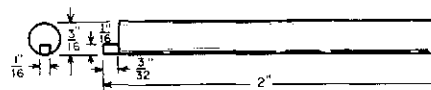
WEAR
BUTTON
Kennametal 6
6 required
Scale 5:1

PISTON ASSEMBLY
No 303 stainless
6 required
Scale: 5:1

PISTON
WASHER
Brass
Scale: 5:1

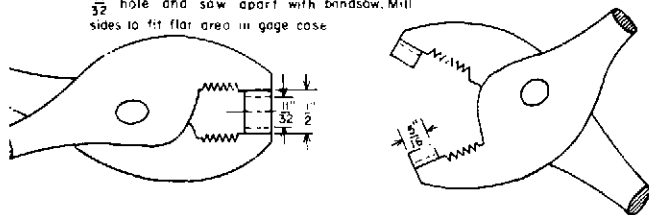


SECTION B-B



LOCKING NUT WRENCH
Top | steel |

NOTE Use $\frac{1}{2}$ " OD brass rod and braise to pliers. Drill $\frac{11}{32}$ " hole and saw apart with bandsaw. Mill sides to fit flat area in gage case.



MODIFIED PLIERS

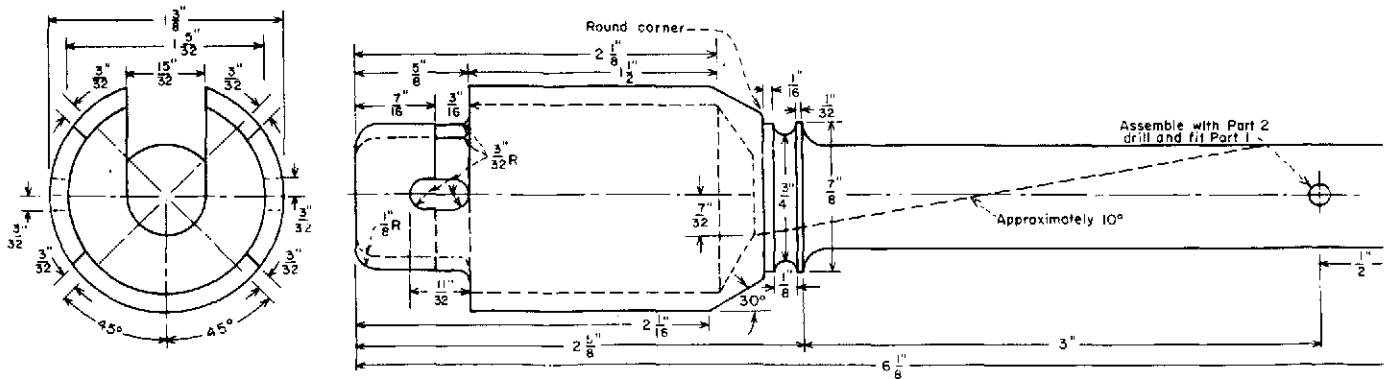
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DENVER MINING RESEARCH CENTER
DENVER, COLORADO

THREE-COMPONENT BOREHOLE GAGE

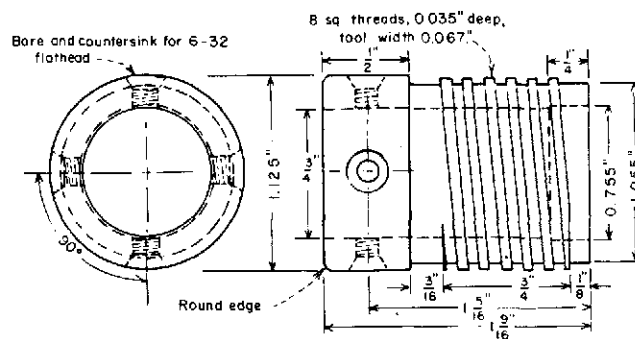
SCALE 2:1

DRAWING 2 of 2

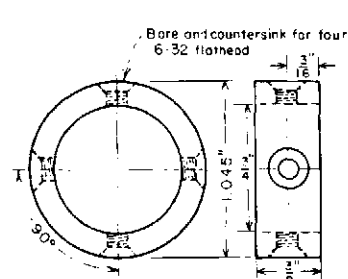
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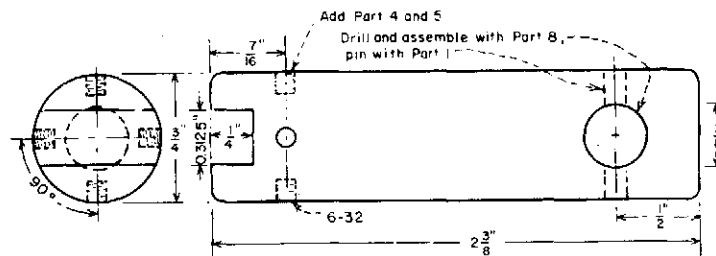
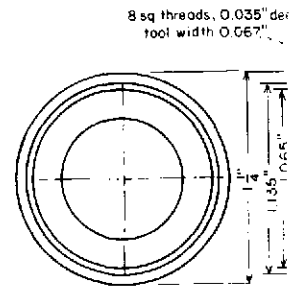
Denver-Type Head
No. 303 stainless
1 required



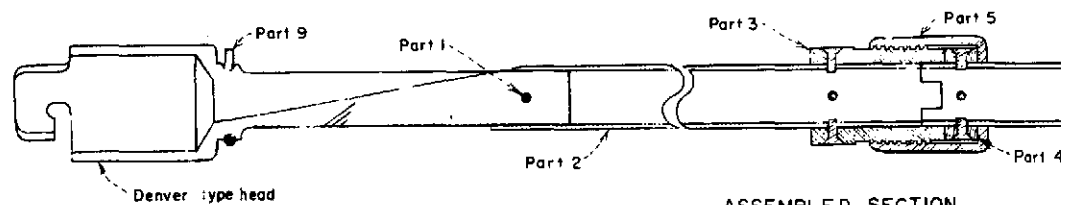
Part 3
Brass
13 required



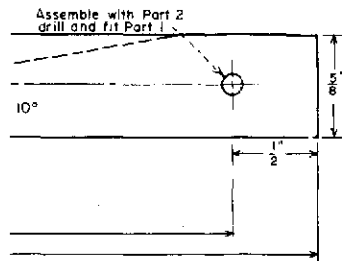
Part 4
Brass
13 required



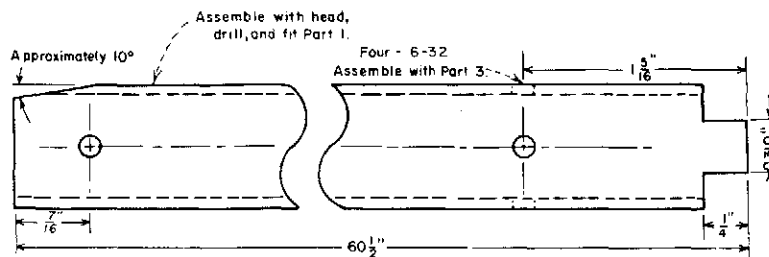
Part 7
No. 303 stainless
1 required



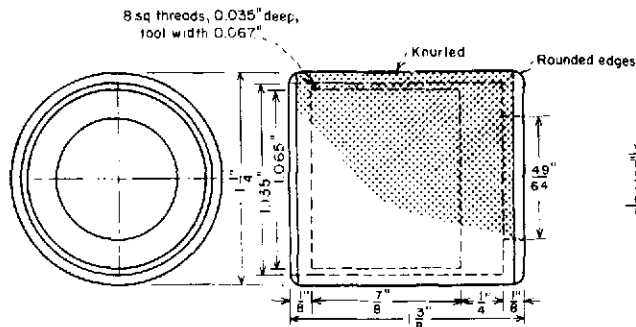
ASSEMBLED SECTION



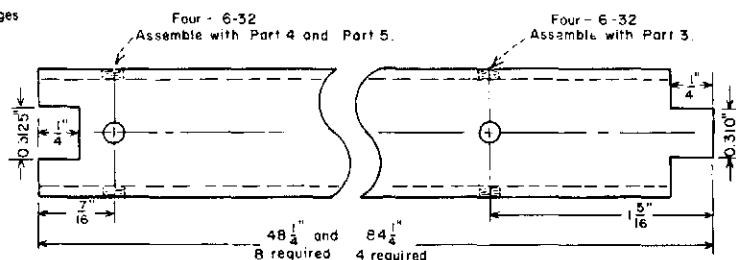
Part 1
Drill rod
2 required



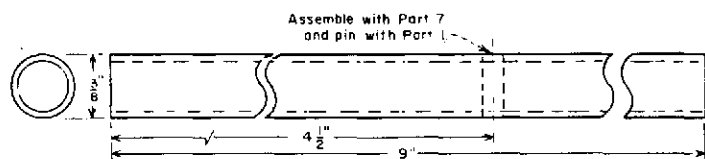
Part 2
3/4" Aluminum tubing
1 required



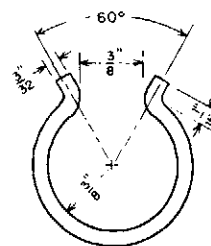
Part 5
Brass
13 required



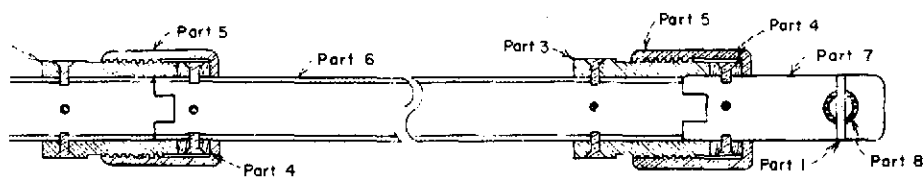
Part 6
3/4" Aluminum tubing
4 required



Part 8
Stainless pipe
1 required



Part 9
Drill rod
1 required



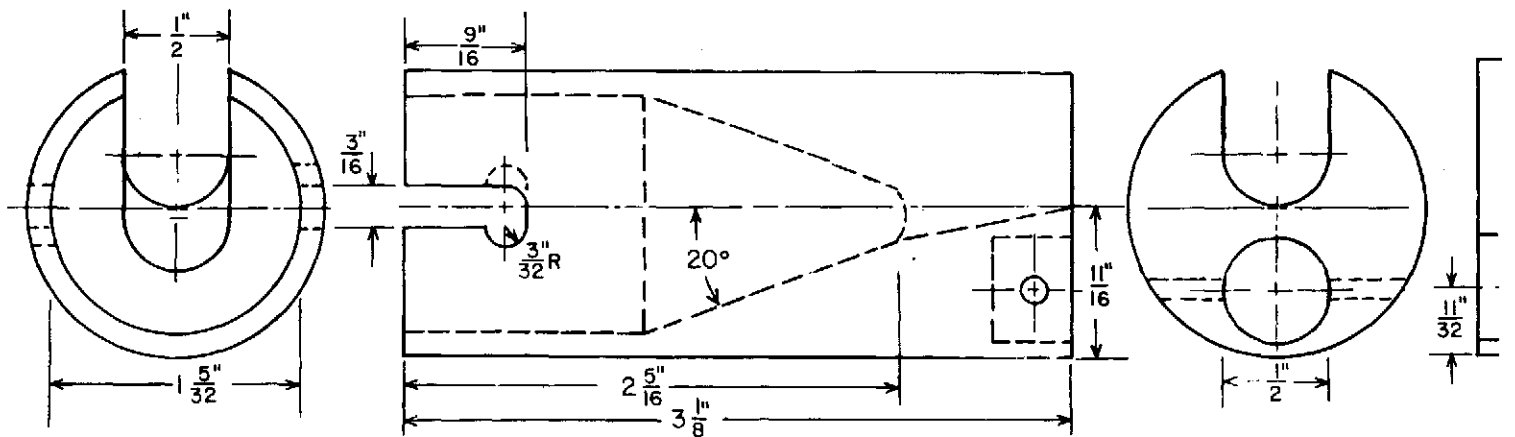
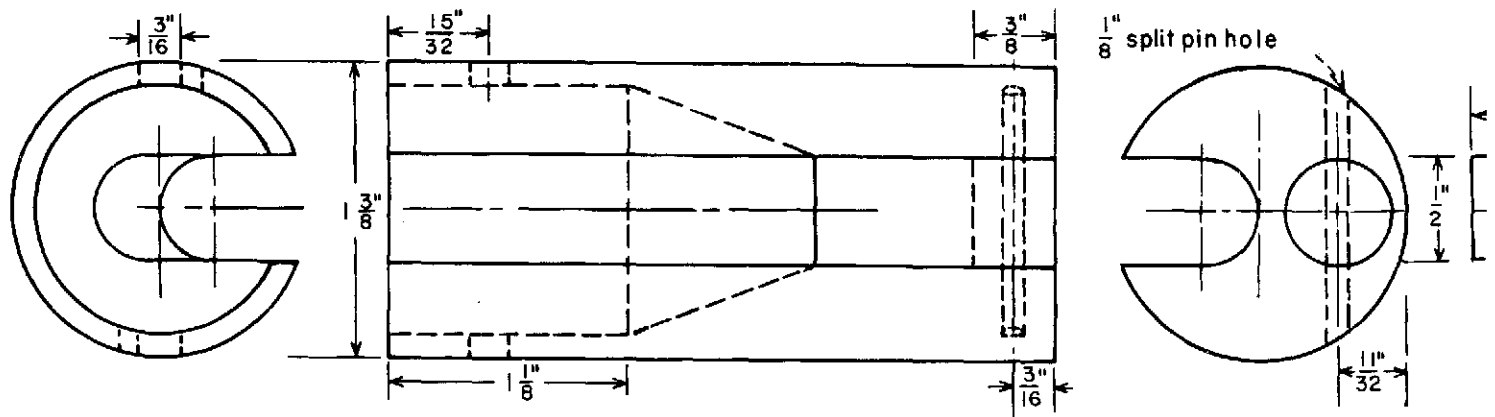
MBLED SECTION

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BOREHOLE GAGE PLACING AND RETRIEVING TOOL

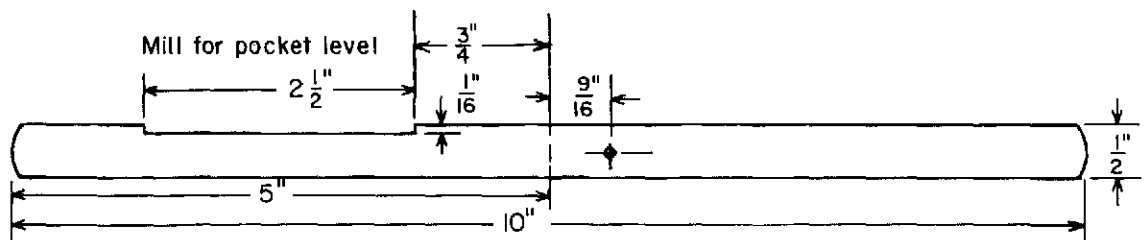
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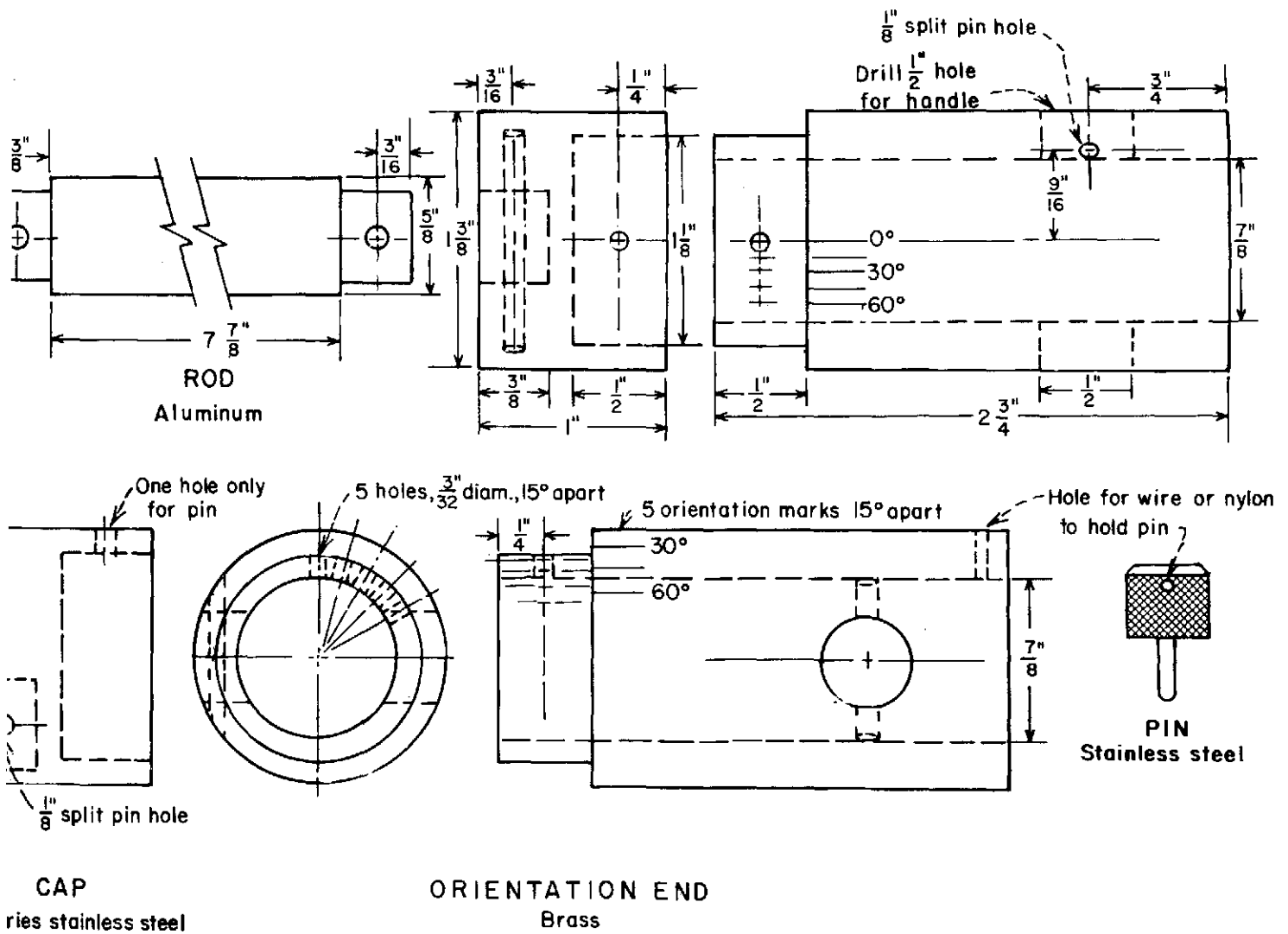


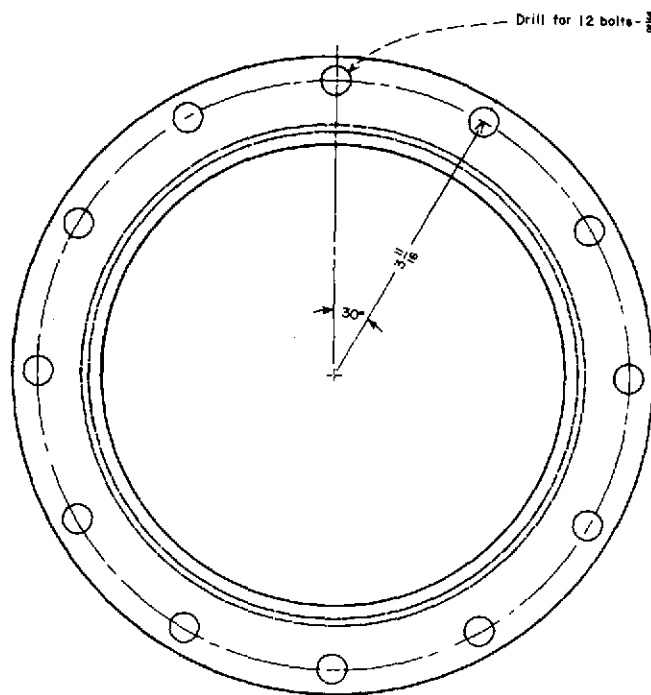
HEAD
300 series stainless steel

300 :

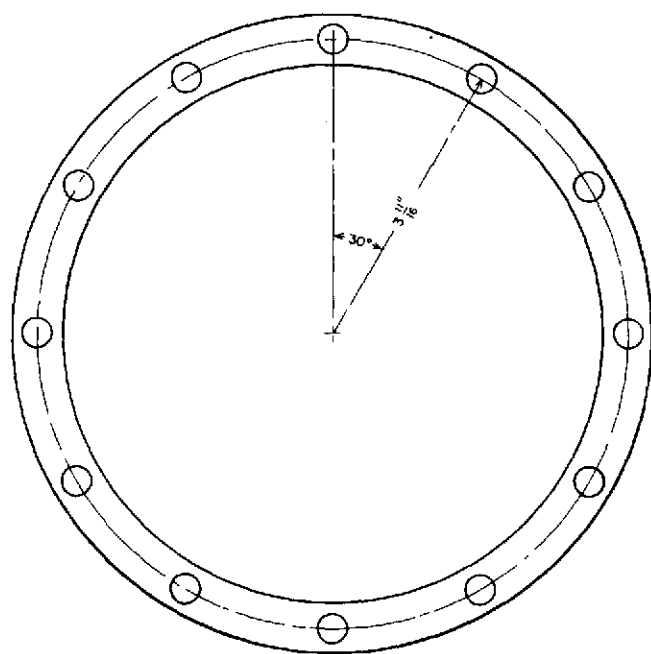
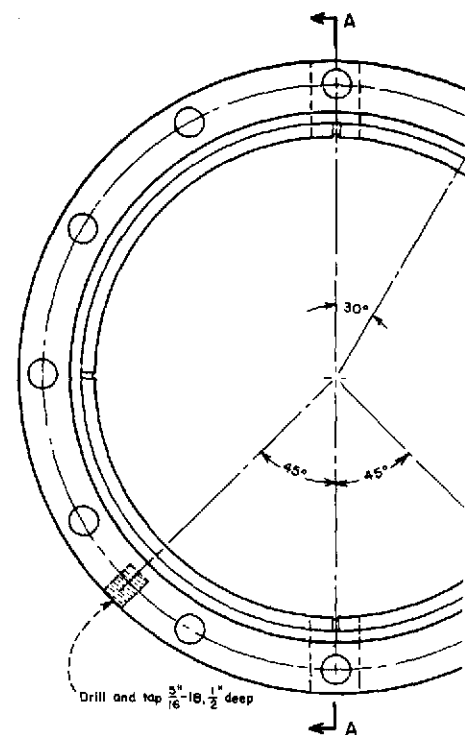
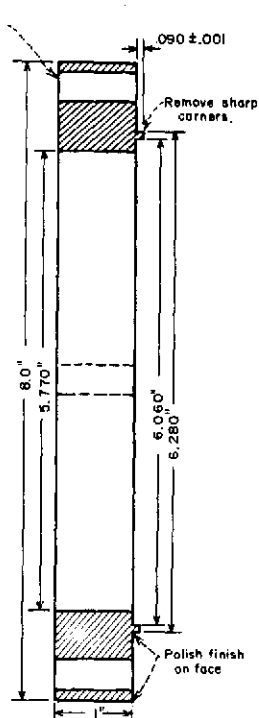


HANDLE
Aluminum

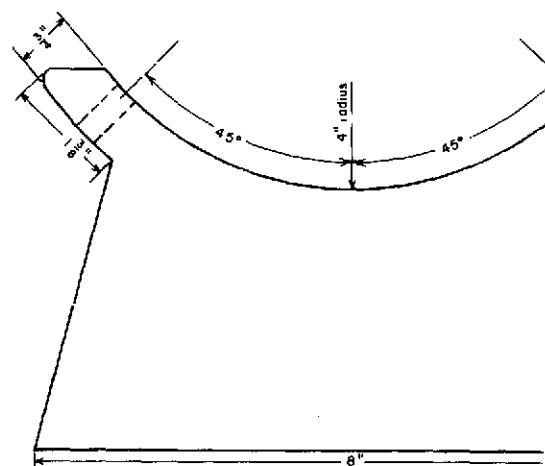
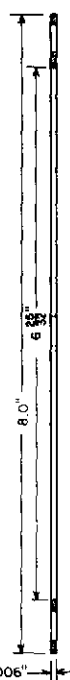




Part 1
Stainless steel
2 required



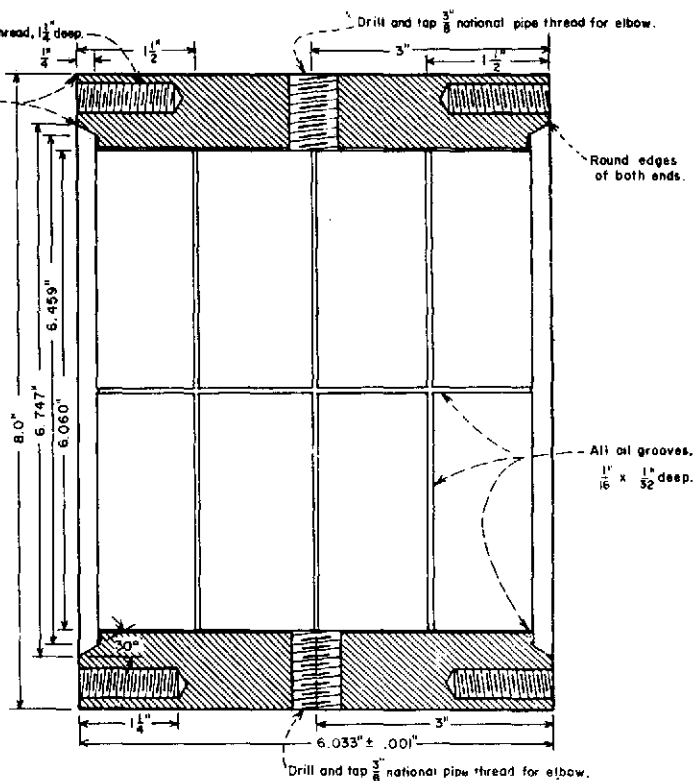
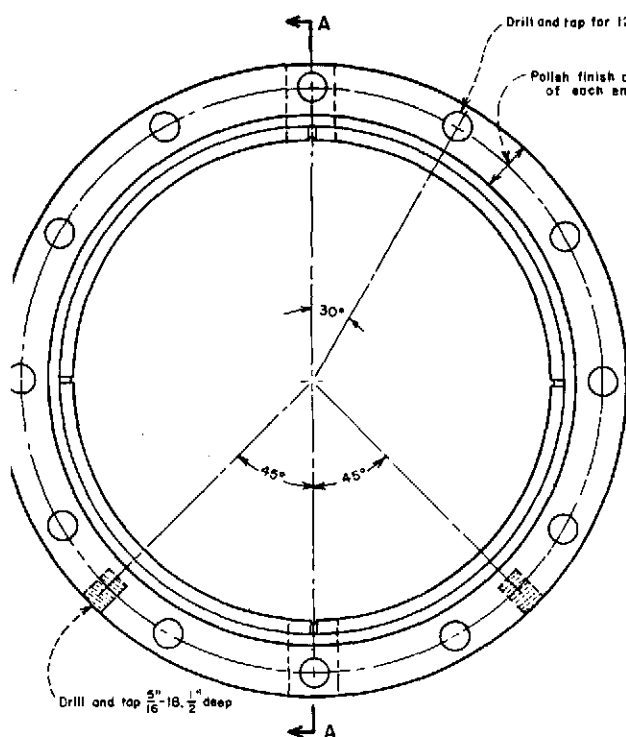
Part 3
Brass
2 required



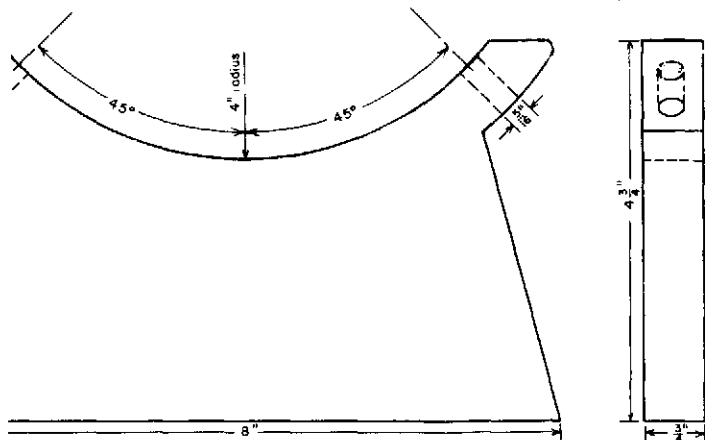
Part 4
Aluminum
2 required



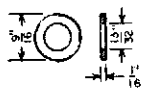
Part 5
Copper or brass
24 required



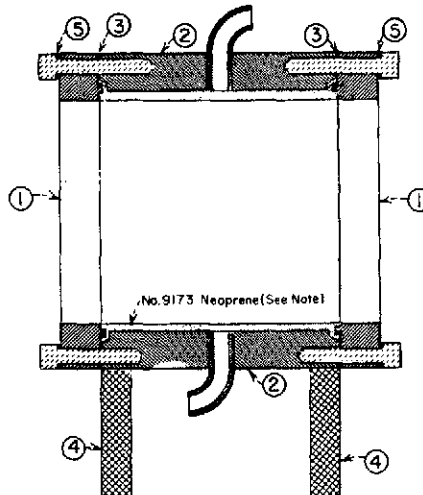
Part 2
Stainless steel
1 required



Part 4
Aluminum
2 required



Part 5
Copper or brass
24 required



ASSEMBLED CROSS SECTION
Half Scale

NOTE: No. 9173 Neoprene liner needed to fit this biaxial chamber is made by Colorado Molded Products Company, Denver from their mold No. CM 997. The chamber is made from 303, 304, or 316 stainless steel tubing, however, 7075-T6 aluminum seamless tubing will reduce the weight by 2/3 and is recommended if applied pressure does not exceed 3,000 psi. Allen head bolts are torqued to 45ft-lbf in the steel and 35ft-lbf in the aluminum.

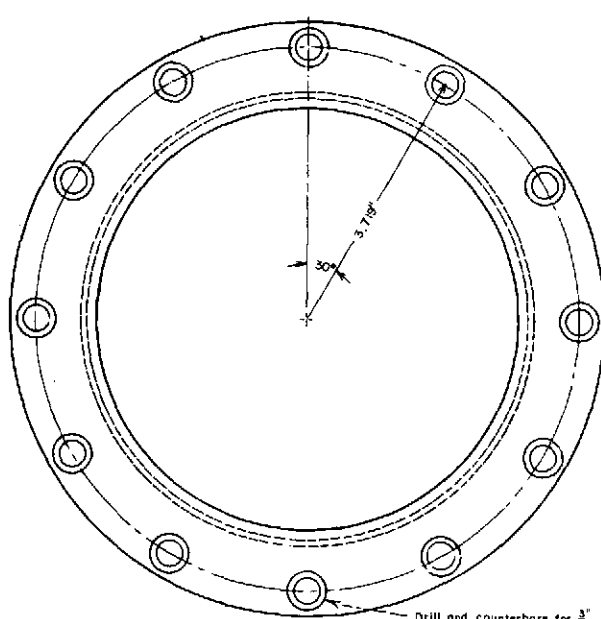
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BIAXIAL PRESSURE CHAMBER

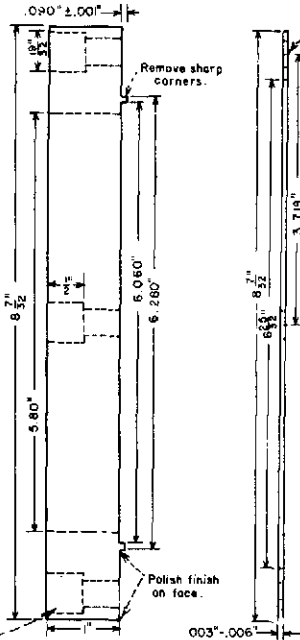
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No 1009

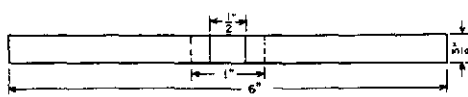
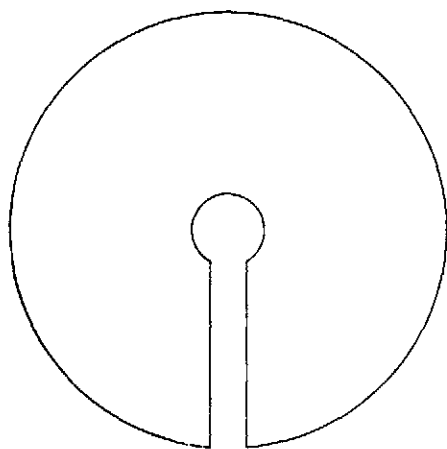
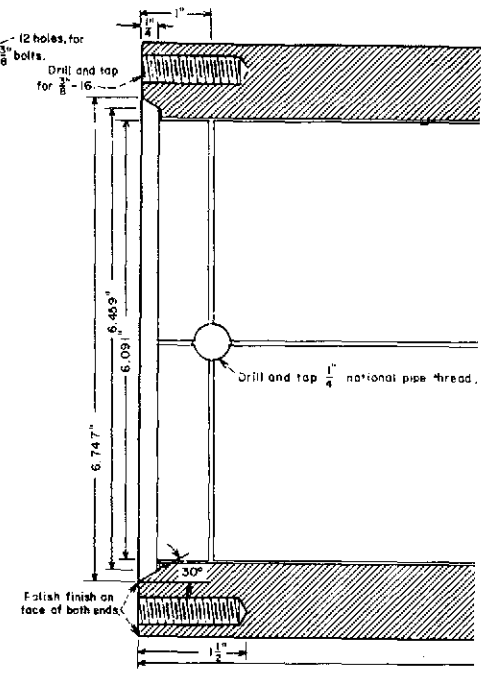
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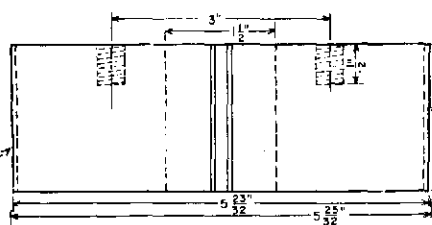
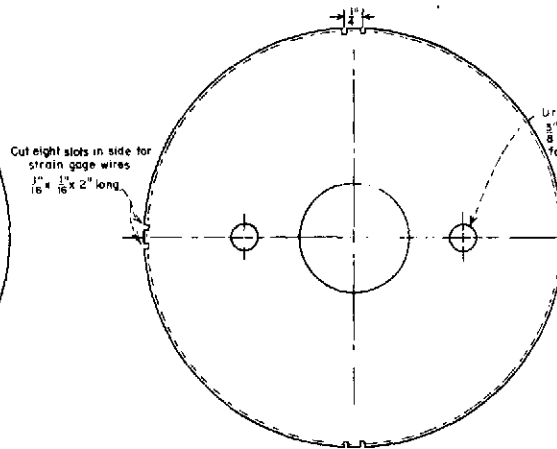
Part 1
Stainless steel
1 required



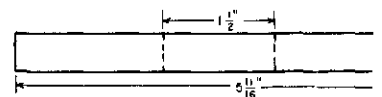
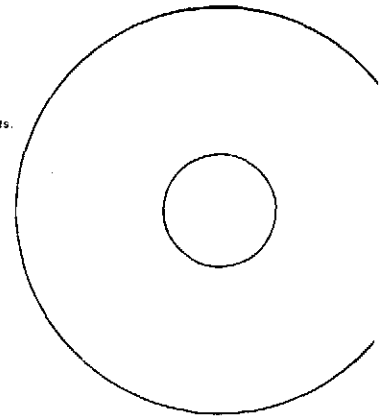
Part 4
Brass
2 required



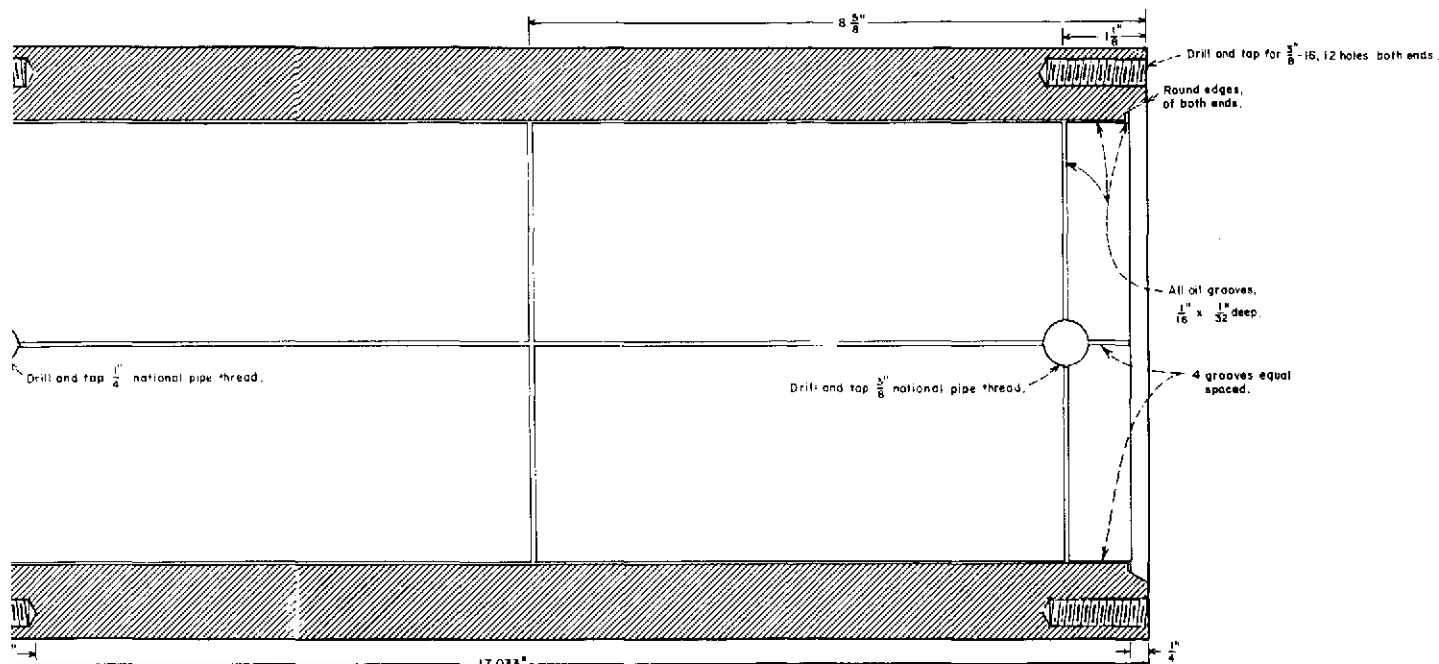
Part 6
Stainless steel
1 required



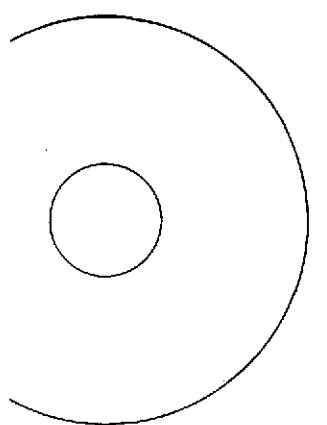
Part 7
Stainless steel
1 required



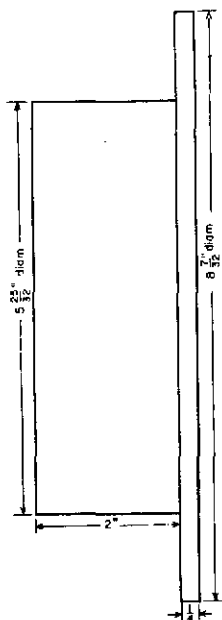
Part 8
Stainless steel
X = 1/2 1 required
X = 1 2 required
X = 2 2 required



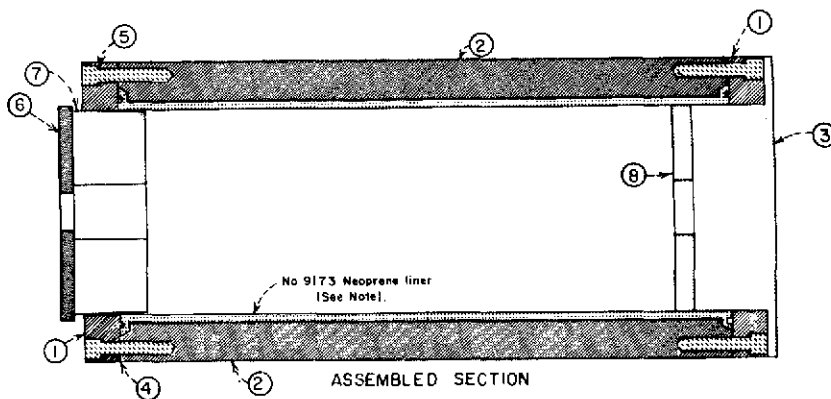
SECTION- Part 2
Stainless steel
1 required



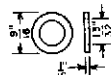
Part 8
Stainless steel
X = 1 1/2" 1 required
X = 1" 2 required
X = 2" 2 required



Part 3
Stainless steel
1 required



NOTE: No 9173 Neoprene liner needed to fit this triaxial chamber is made by Colorado Molded Products Company from their mold. Mold No. CM 1427. Torque the Allen socket head bolts to 60 ft-lb. 304 stainless steel seamless tubing is recommended. 303 or 316 can be used also.



Part 5
Copper or Brass
24 required

U. S. BUREAU OF MINES
DENVER MINING RESEARCH CENTER
DENVER, COLORADO

TRIAxIAL PRESSURE CHAMBER

SCALE: 1/1

No. 1010

DATE: 7/23/73

APPENDIX C.—COMPUTER CODE FOR PLOTTING DEFORMATION MEASUREMENTS

By Dennis R. Dolinar¹ and Meng-Cherng Sun¹

```

$INCLUDE: 'FGRAPH.FI'
      PROGRAM PLOTDEF
C *****
C * Program for deformation plot -overcoring          *
C * Dennis R. Dolinar, USBM - DRC                      *
C * PC FORTRAN version by Meng-Cherng Sun, 11/12/1991  *
C *****
C      integer u
      common /parm/ u(3,40),b(40),desc,isets,numpts,xidr,xid
      common /fnam/ fname
      character desc*80,ans,fname*12
      fname= '
C
      call init
1 write(*,1001)
1001 format (' Input from keyboard or file ?(k or f): '\)
      read (*,'(a)') ans
      if(ans.eq.'k' .or. ans.eq.'K') then
      call kinput
      else if(ans.eq.'f' .or. ans.eq.'F') then
      call finput
      else
      write(*,*) ' Please answer "k" or "f"'
      go to 1
      end if
C      check if you need to edit data
      call edit
      call calc
C      write(*,*)
C      write(*,*) ' Here are the values after calculation'
C      call display
C      -----
      call plot
C      -----
      stop ' End of Plotdef Program - No Errors.'
      end

      subroutine init
C *****
C * initializes array b and u          *
C *****
      integer u
      dimension u(3,40),b(40)

```

¹Mining engineer, Denver Research Center, U.S. Bureau of Mines, Denver, CO.


```

        do 2 i=1,40
2 b(i)=0.0
        do 3 i=1,3
        do 3 j=1,40
3 u(i,j)=0.0

        return
        end

        subroutine kinput
c *****
c * Input data from keyboard *
c *****
        integer u
        common /parm/ u(3,40),b(40),desc,isets,numpts,xidr,xid
        character desc*80,ans
        logical correct
c
        correct=.false.
1111 write(*,*) 'Enter title: '
        read(*,1003) desc
        if(correct) goto 10
c
        11 write(*,1001)
1001 format(' Enter number of sets of data (1 to 3)? '\)
        read(*,'(i1)') isets
        if ((isets.lt.1) .or. (isets.gt.3)) then
            write(*,*) ' Sorry...'
            write(*,*) ' This program only handles up to 3',
+                ' sets of data'
            goto 11
        end if
        if(correct) goto 10
12 write(*,1002)
1002 format(' Inches drilled per reading? '\)
        read(*,*) xidr
        if(correct) goto 10
13 write(*,1003)
1003 format(' Total inches drilled? '\)
        read (*,*) xid
        if(xid.lt.xidr) then
            print*, ' Sorry...'
            print*, ' You entered a larger value for inches drilled/reading'
            print*, ' than for inches drilled. Please try again. '
            go to 12
        end if
        if(correct) goto 10
14 continue
10 print*
        write(*,2000) desc
2000 format(' Title: ',a70)
        write(*,2001) isets

```

```

2001 format(' number of sets of data      = ',i5)
    write(*,2002) xidr
2002 format(' inches drilled per reading = ',f5.1)
    write(*,2003) xid
2003 format(' Total inches drilled      = ',f5.1)
    write(*,2005)
2005 format(' Is this correct? (y or n) '\)
    read(*,'(a)') ans
    if((ans.eq.'n').or.(ans.eq.'N')) then
        print*,' Which number to change: '
        print*,'          1 = Number of sets of data'
        print*,'          2 = Inches drilled per reading'
        print*,'          3 = Total inches drilled'
        print*,'          4 = Title'
        read(*,100) n
        correct=.true.
        go to (11,12,13,1111) n
    else if ((ans.eq.'y').or.(ans.eq.'Y')) then
        numpts=xid/xidr+1
        if(numpts.gt.40) then
            print*,' Sorry.....'
            print*,' This program only handles up to 40 data points'
            go to 11
        end if
        do 1 i=1,issets
            write(*,3001) i
3001 format(' Enter initial reading for data set #', i1,':')
            read(*,*) u(i,1)
            write(*,3002) i
3002 format(' Enter readings for data set # ',i1,'---')
            write(*,3003) numpts-1
3003 format(' Input ',i2,' more readings: ')
            do 2 j=2,numpts
                2 read(*,*) u(i,j)
                1 continue

                b(1)=0.0
                do 3 j=1,numpts
                    3 if(j.gt.1) b(j)=b(j-1)+xid
                    else
                        print*,' Please answer "y" or "n"'
                        go to 10
                    end if
                end if
            end do
        end do
    end if
C
C SAVING INPUT DATA?
WRITE(*,4001)
4001 FORMAT(' Do you want to save the input data? (y or n) '\)
    read(*,'(a)') ans
    if(ans.eq.'y' .or. ans.eq.'Y') then
        call output
    endif

```

```

100  format(i1)
101  format(f6.2)
102  format(a10)
103  format(a80)
104  format(a1)
     return
     end

     subroutine finput
C *****
C *   input data from a file   *
C *****
C
     integer u
     common /parm/ u(3,40),b(40),desc,isets,numpts,xidr,xid
     common /fnam/ fname
     character ans,fname*12,desc*80
     logical ex,op

11  write(*,*) ' Enter file name..'
     read(*,'(a12)') fname
1   write(*,*) ' The file name is : ',fname
     write(*,1001)
1001 format(' Is this correct? (y* or n) '\)
     read(*,104) ans
     if((ans.eq.'n').or.(ans.eq.'N')) go to 11
C
     inquire(file=fname,exist=ex,opened=op)
C
C ** start read loop
C =====
C     if(ex) then
C         open(5,file=fname,status='old')
C         read(5,103) desc
C         read(5,100) isets
C -----
C         if ((isets.lt.1) .or. (isets.gt.3)) then
C             print*, ' Sorry...'
C             print*, ' This program only handles up to 3 sets of data'
C             print*, ' You indicated : ',isets
C             goto 99
C         endif
C -----
C         read(5,*) xidr
C         read(5,*) xid
C -----
C         if(xid.lt.xidr) then
C             print*, ' Sorry...'
C             print*, ' You entered a larger value for inches drilled/reading'
C             print*, ' than for inches drilled. Please try again. '
C             go to 99
C         endif

```

```

c -----
10 print*, ' Title -- '
   print*,      desc
   print*, ' Number of sets of data = ', isets
   print*, ' Inches drilled per reading = ', xidr
   print*, ' Total inches drilled = ', xid
   print*
   write(*,1002)
1002 format(' Is This Correct? (y* or n)'\)
   read(*,104) ans
   if((ans.eq.'n').or.(ans.eq.'N')) go to 99
   numpts=xid/xidr+1
c -----
   if(numpts.gt.40) then
     print*, ' Sorry.....'
     print*, ' This program only handles up to 40 data points'
     go to 99
   endif
c -----
   do j=1,numpts
     read(5,105,end=97) (u(i,j),i=1,isets)
   enddo
     b(1)=0.0
     do j=1,numpts
       if(j.gt.1) b(j)=b(j-1)+xidr
     enddo
     go to 98
c =====
   else
     print*, ' Sorry... The file you indicated doesn''t exist.'
     print*, ' Please check the file name.'
     print*
     go to 11
   end if
c =====

C ** Not enough data points in file
97   print*, ' Your file indicates :', numpts, ' data points'
   print*, ' But there appears to only be ', j-1, ' data points.'

C ** Bailout
99   print*
   print*, ' Please make the corrections required to the file:'
   print*, fname, ' with a text editor restart the program.'
   stop ' Termination of program Plotdef with error '

C ** No problems
98   close(5)
   return

100   format(i1)
102   format(a32)

```

```

103  format(a80)
104  format(a1)
105  format(i6,:i6,:i6)
    end

    subroutine edit
c *****
c *   Edit the data   *
c *****
    integer u
    common /parm/ u(3,40),b(40),desc,isetc,numpts,xidr,xid
    character desc*80,ans,ansl

c
    call display
    print*
    write(*,1001)
1001 format(' Do you need to edit these data? (y or n*): '\)
    read(*,104) ansl
    if(ansl.eq.'y' .or. ansl.eq.'Y') goto 11
    return

c   Start to edit data
    11 write(*,2001)
    2001 format(' To change the data----')
    write(*,2002)
    2002 format(' Enter data set # (1 ,2 or 3) = '\)
    read(*,100) isetc
    write(*,2003)
    2003 format(' Enter data point # = '\)
    read(*,101) iptc
    write(*,*) ' The current value is: ', u(isetc,iptc)
    write(*,2004)
    2004 format(' Is this the point you wish to change? (y* or n) '\)
    read(*,104) ans
    if(ans.eq.'n' .or. ans.eq.'N') go to 12
    write(*,2005)
    2005 format(' Enter the new U value = '\)
    read(*,*) u(isetc,iptc)
    12 call display
    write(*,2006)
    2006 format(' Do you want to edit data any more? (y or n)'\)
    read(*,104) ansl
    if(ansl.eq.'y' .or. ansl.eq.'Y') goto 11

c
C   SAVING INPUT DATA?
    WRITE(*,4001)
    4001 FORMAT(' Do you want to save the edited data? (y or n*) '\)
    read(*,'(a)') ans
    if(ans.eq.'y' .or. ans.eq.'Y') then
        call output
    endif

```

```

      return
100  format(i1)
101  format(i2)
104  format(a1)
      end

      subroutine calc
c *****
c *   Initializes data for calculation   *
c *****
c
      integer u
      common /parm/ u(3,40),b(40),desc,isets,numpts,xidr,xid
      character desc*80
c
      do 5 i=1,isets
      do 4 j=2,numpts
4 u(i,j)=u(i,j)-u(i,1)
5 continue
      do 6 i=1,isets
6 u(i,1)=0.0
      return
      end

      subroutine display
c *****
c *   Display data on screen   *
c *****
c
      integer u
      common /parm/ u(3,40),b(40),desc,isets,numpts,xidr,xid
      character desc*80

      write(*,100) 'Inches','U1','U2','U3'
      if(numpts.gt.20) then
      do 1 j=1,20
1 write(*,101) j,b(j),(u(i,j), i=1,isets)
      print*,' Press return to continue.....'
      read(*,*)
      do 2 j=21,numpts
2 write(*,101) j,b(j),(u(i,j), i=1,isets)
      else
      do 4 j=1,numpts
4 write(*,101) j,b(j),(u(i,j), i=1,isets)
      end if
c
100 format(6x,a6,8x,a2,6x,a2,6x,a2)
101 format(i4,f6.2,4x,i8,:i8,:i8)
      return
      end

```

```

      subroutine output
c *****
c *   write data into a file   *
c *****
c
      integer u
      common /parm/ u(3,40),b(40),desc,isets,numpts,xidr,xid
      common /fnam/ fname
      character ans,fname*12,desc*80
      logical ex

      if(fname .eq. ' ')goto 11
      write(*,1001) fname
1001 format(' The file name = ',a12)
      write(*,1002)
1002 format(' Is it o.k. to over-write on this file? (y or n*)'\)
      read(*,'(a)') ans
      if(ans.eq.'y' .or. ans.eq.'Y') goto 22
c
11 WRITE(*,*)
   WRITE(*,*) ' Enter output file name ..'
   WRITE(*,*) ' (For saving input data)'
   read(*,'(a12)') fname
1 write(*,*) ' The file name is : ',fname
   write(*,2002)
2002 format(' Is this correct? (y* or n)'\)
   read(*,'(a)') ans
   if((ans.eq.'n').or.(ans.eq.'N')) go to 11
   inquire(file=fname,exist=ex)
   if(ex) then
4 write(*,2003)
2003 format(' That file already exists.'
+ ' Do you want to replace it? (y* or n)'\)
      read(*,'(a)') ans
      if(ans.eq.'n' .or. ans.eq.'N') go to 11
      endif
c
c   save input data into a file for later use.
22 open(5,file=fname)
   write(5,103) desc
   write(5,100) isets
   write(5,*) xidr
   write(5,*) xid

      do j=1,numpts
         write(5,105) (u(i,j),i=1,isets)
      enddo
c
c ** No problems
98   return

100  format(i1)
102  format(a32)
103  format(a80)

```

```

104  format(al)
105  format(i6,:i6,:i6)
    end

    subroutine plot
c *****
c *   Use USBM fortran utility and graphics library *
c *   and Microsoft Fortran Graphical Lib.         *
c *****
    INCLUDE 'FGRAPH.FD'
    RECORD /videoconfig/ screen
    COMMON screen

c
    integer u
    common /parm/ u(3,40),b(40),desc,issets,numpts,xidr,xid
    character desc*80

c
c   Initialization
c
    call initplot

c
c   Plot data
c
    CALL plotdata

c
c   set video back and stop program
c
    call closeplot
    return
    end

    subroutine plotdata
c *****
c *   Set window and plots drill hole data. *
c *****
    INCLUDE 'FGRAPH.FD'

c
    integer u
    integer*2 vxmin,vxmax,vymin,vymax
    INTEGER*2 dummy
    INTEGER*2 xwidth,yheight,cols,rows
    common /parm/ u(3,40),b(40),desc,issets,numpts,xidr,xid
    common/viewtext/ xwidth, yheight, cols, rows
    common /vpor/ vxmin,vxmax,vymin,vymax
    common /wind/ wxmin,wxmax,wymin,wymax
    character desc*80
    character xlabel*23, ylabel*24

    RECORD /videoconfig/ screen
    RECORD /rccoord/      curpos
    RECORD /wxycoord/     wxy
    COMMON screen

```

```

c
c-- Find min. max. and interval for u and b data --
    IUMIN = u(1,1)
    IUMAX = u(1,1)
    do 10 i=1, isets
    do 10 j=1, numpts
        IUMIN = MIN0(IUMIN, u(i,j) )
        IUMAX = MAX0(IUMAX, u(i,j) )
    10 continue
c*****DEBUG*****
C    write(*,1000) IUMIN,IUMAX
C 1000 format(2I10)
c*****
    umin=iumin
    umax=iumax
    nint=10      !approx. number of intervals
    call niceint(umin,umax,nint,dymin,dymax,dyint)
c    define min., max. and interval for xdata
    dxmin = b(1)
    dxmax = b(numpts)
    dxint = xidr*2.
c    number of intervals along axes
    nxint = (dxmax - dxmin)/dxint
    nyint = (dymax - dymin)/dyint
c
c*****DEBUG*****
C    write(*,1001) dxmin,dxmax,dxint,nxint
C    write(*,1001) dymin,dymax,dyint,nyint
C 1001 format(3f10.1,i5)
c*****
    write(*,*) ' Hit Return to Continue'
    read(*,*)
c
c    Set viewport (screen in pixels) for data
    sxmin = 100
    sxmax = 551
    symin = 55
    symax = 380
c
    vxmin = sxmin - 100
    vxmax = sxmax + 25
    vymin = symin - 25
    vmax = symax + 100

    xvtod = (dxmax - dxmin)/(sxmax - sxmin)
    yvtod = (dymax - dymin)/(symax - symin)
c
c    Set window (x- and y-data)
    wxmin = dxmin - xvtod*100
    wxmax = dxmax + xvtod*25
    wymin = dymin - yvtod*100
    wymax = dymax + yvtod*25

```

```

c
C*****DEBUG*****
c      print viewport to see
c      write(*,*) vxmin,vxmax
c      write(*,*) vymin,vymax
c      read(*,*)
C*****
c      call Data window
c      call clearscreen($GCLEARSCREEN)
c      call window

c
c      Print Title in color on the screen.
c      CALL settextposition(2,17,curpos)
c      dummy = settextcolor( 6 ) !brown color
c      CALL outtext (desc)

c
c      Draw a bordered rectangle around the graph
c      dummy = setcolor(1)      ! blue
c      dummy = rectangle_w( $GBORDER, dxmin, dymin, dxmax, dymax )

c
c      Draw Data curves with different color
c      CALL curve

c
c      Draw a dashed line at data value=0
c      dummy = setcolor(5)      !magenta
c      CALL setlinestyle (#F0F0)
c      CALL moveto_w(dxmin,0.,wxy)
c      dummy = lineto_w(dxmax,0.)
c      Call setlinestyle (#FFFF)

c
c      plot x-axis and tic marks, and labels
c
c      xstr = dxmin
c      xend = dxmax
c      xint = dxint
c      yloc = dymin
c      xlabel = 'DEPTH OF OVERCORING, in'
c      ndec=0
c      call xaxis(xstr,xend,xint,yloc,xlabel,ndec)

c
c      Plot y-axis and tic marks, and labels
c
c      ystr = dymin
c      yend = dymax
c      yint = dyint
c      xloc = dxmin
c      ylabel = 'DEFORMATION, microinches'
c      ndec=0
c      call yaxis(ystr,yend,yint,xloc,ylabel,ndec)
c      return
c      END

```

```

c      subroutine curve
c *****
c * draws curve based on x- and y-data *
c *****
c * programmed: Meng-Cherng Sun      *
c * Date: 11/27/91                  *
c *****
      INCLUDE 'FGRAPH.FD'
c
      integer u
      INTEGER*2 dummy
      common /parm/ u(3,40),b(40),desc,isets,numpts,xidr,xid
      dimension bb(40),uu(40)
      character desc*80
      character cno,lleg*2
      RECORD /videoconfig/ screen
      COMMON screen

      do 20 i=1,isets
c-- get data -- bb=x-data, uu=y-data
      do 19 j=1,numpts
        bb(j) = b(j)
      19 uu(j) = u(i,j)
c
c      set color for different curve
      dummy = setcolor(i+1)
c
c      Plot curves.
      call lines(bb,uu,numpts)
c      legend
      xleg=bb(numpts-2)
      yleg=uu(numpts-2)-3.
      write(cno,'(il)') i
      lleg='U'//cno
      call label_gtx(xleg,yleg,6,15,7, i+1, lleg)
      20 continue
      return
      end

```

Sample data for plotting the deformation measurements

Minnamax Mine - Hole 1 Depth 12'2"

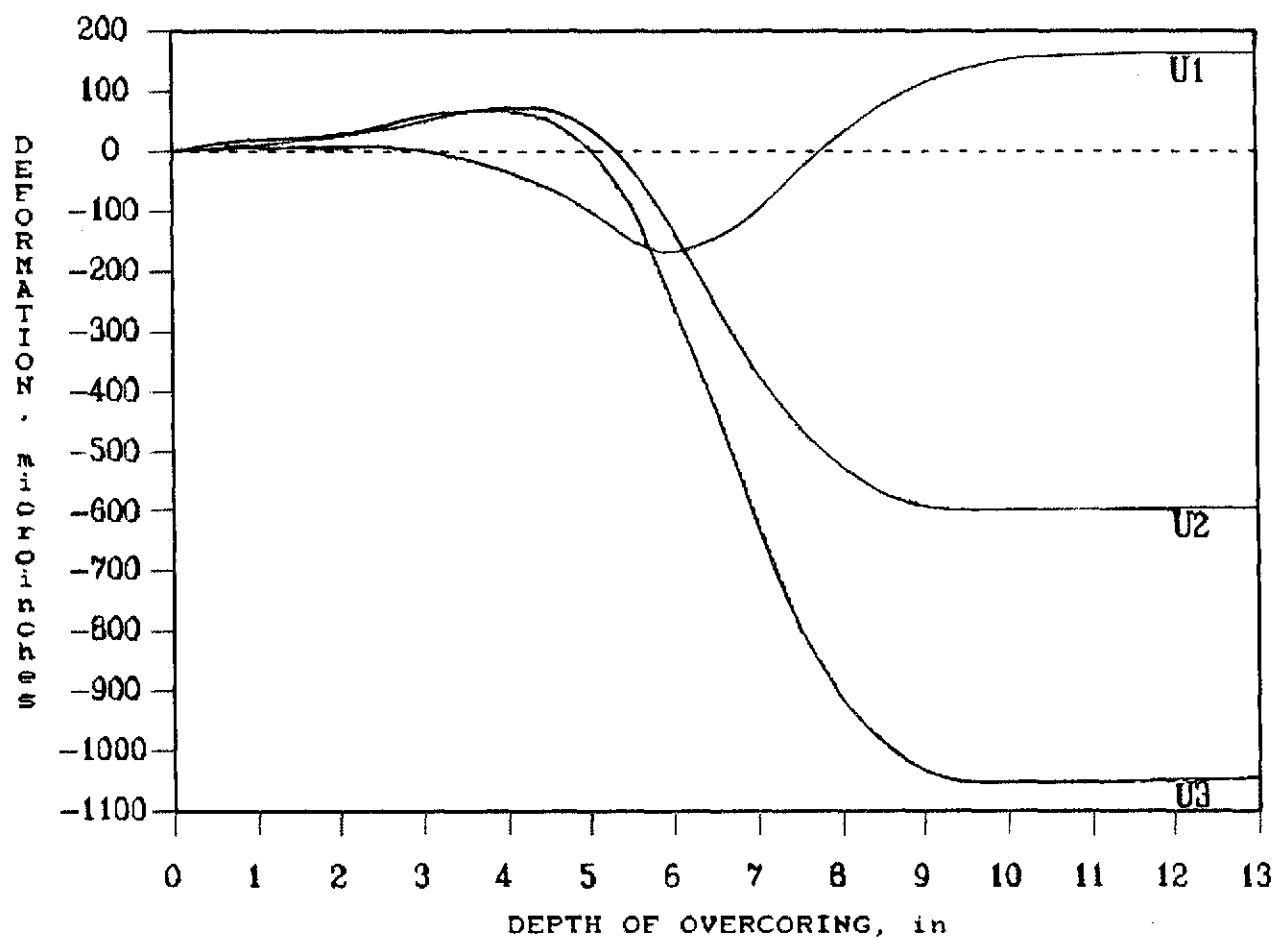
3

5.000000E-01

13.000000

0	0	0
5	10	2
8	18	6
6	21	13
5	24	21
9	36	40
2	48	56
-11	64	66
-33	72	66
-59	71	54
-97	43	7
-148	-29	-93
-174	-134	-256
-150	-253	-425
-100	-374	-624
-32	-460	-796
27	-526	-915
75	-569	-990
110	-589	-1031
134	-599	-1054
151	-592	-1053
157	-598	-1050
159	-603	-1052
162	-598	-1052
164	-596	-1045
165	-591	-1045
164	-592	-1044

Minnamax Mine - Hole 1 Depth 12' 2"



APPENDIX D.—COMPUTER CODE FOR CALCULATING LEAST SQUARES ELLIPSE FROM BIAxIAL TEST DATA

By Dennis R. Dolinar¹ and Meng-Cherng Sun¹

```

$INCLUDE: 'FGRAPH.FI'
      program plotbiax
c *****
c * program calculates and plots the      *
c * Bi-axial Data - an ellipse.          *
c * Dennis R. Dolinar, USBM - DRC        *
c *****
c * Programmed for PC by Meng-Cherng Sun *
c * Date: 01/06/1992                    *
c *****
c
      integer*2 vxmin, vymin, vxmax, vmax
      common /idata/ title,p0,ndef,l(12),u(12)
      common /odata/ a2,b2,z1,o2,o3,o4,o5,y
      common /vpwr/ vxmin,vxmax,vymin,vymax
      common /wind/ wxmin,wxmax,wymin,wymax
      common /dwin/ dxmin,dxmax,dymin,dymax
      character title*56,ans

      11 write(*,99)
      99 format(' Program plots an ellipse for bi-axial data')
      write(*,100)
      100 format(' Input from keyboard or file? (k or f*): '\)
      read (*,'(a)') ans
      if (ans .eq. 'k' .or. ans .eq. 'K') then
        call kinput
      else
        call finput
      write(*,200)
      200 format(' Are these data O.K.? (y* or n): '\)
      read (*,'(a)') ans
      if (ans .eq. 'n' .or. ans .eq. 'N') then
        write(*,*) ' Enter input data from keyboard again!'
        call kinput
      endif
c      END OF INPUT DATA
      endif
c
c      calculate
c
c      call calc

```

¹Mining engineer, Denver Research Center, U.S. Bureau of Mines, Denver, CO.

```

c
c   find max. value of data to plot
c   call maxval(u,ndef,umax)
c   call roundb(umax,rmax)
c*****debug*****
c   write(*,*) umax,rmax
c   read(*,*)
c*****
c
c   set data range by maximum data value
c
c   dxmin = -rmax
c   dxmax =  rmax
c   dymin = -rmax
c   dymax =  rmax
c
c   set window for plot based on rmax
c
c   wxmin = -rmax - rmax*0.15      ! min.x.w
c   wxmax =  rmax + rmax*0.15      ! max.x.w
c   wymin = -rmax - rmax*0.15      ! min.y.w
c   wymax =  rmax + rmax*0.15      ! max.y.w
c
c   define viewport on screen
c
c   vxmin = 100                    ! min.x.v
c   vxmax = 640 - 100              ! max.x.v
c   vymin = 20                     ! min.y.v
c   vymax = 480 - 20              ! max.y.v
c
c   call initplot
c
c   call window
c
c   call plot
c
c   call closeplot
c
c   write(*,2200)
c 2200 format(' Do you want to plot more? (y or n*)'\)
c   read (*,'(a)') ans
c   if (ans .eq. 'y' .or. ans .eq. 'Y') goto 11
c
c   write(*,*) ' Program stops!'
c   end
c   subroutine kinput
c *****
c * input data from keyboard      *
c *****
c   common /idata/ title,p0,ndef,l(12),u(12)
c   character title*56,ans
c
c   write(*,1001)

```

```

1001 format(' Enter Title (up to 56 characters)-')
    read(*,'(a56)') title

    write(*,1003)
1003 format(' Enter pressure (p0) in psi. = '\)
    read (*,*) p0

    write(*,1004)
1004 format(' Enter 12 readings for every 15 deg.')
    ndef=12

    ml=15
    do i=1, ndef
        write(*,1005) i
1005 format(' Enter U',i2,' = '\)
        read(*,*) u(i)
        l(i) = (i-1)*(-ml)
    end do

C    SAVING INPUT DATA?
    WRITE(*,2003)
2003 FORMAT(' Do you want to save the input data? (y or n*) '\)
    read(*,'(a)') ans
    if(ans.eq.'y' .or. ans.eq.'Y') then
        call fsave
    endif
    return
end
subroutine finput
C *****
C * Input data from a file *
C *****
C
    common /idata/ title,p0,ndef,l(12),u(12)
    character ans,fname*12,title*56
    logical ex

    11 write(*,12)
    12 format(' Enter file name : '\)
        read(*,'(a12)') fname
        write(*,101) fname
101 format(' The file name is : ', al2,
+ ' Is this correct? (y* or n) '\)
        read(*,'(a1)') ans
        if((ans.eq.'n').or.(ans.eq.'N')) go to 11
C
        inquire(file=fname,exist=ex)
        if(ex) then
            open(5,file=fname,status='old')
            read(5,'(a56)') title
            write(*,*) title
C
            read (5,'(f10.0)') p0
            write(*,*) p0

```

```

      read (5,'(i5)') ndef
      write(*,*) ndef
c
      do i=1, ndef
      read (5,'(i5,f10.0)') l(i),u(i)
      write(*,'(i5,f10.2)') l(i),u(i)
      end do
c
      else
      write(*,*) ' File does not exist, Please check file name!'
      goto 11
      endif
c
      return
      end

      subroutine fsave
c *****
c * save input data into a file      *
c *****
      common /idata/ title,p0,ndef,l(12),u(12)
      character title*56,fname*12,ans
      logical ex

      11 write(*,*)
      write(*,*) ' Enter output file name ..'
      write(*,*) ' (For saving input data)'
      read(*,'(a12)') fname
      write(*,101) fname
101 format(' The file name is : ', al2,
+ ' Is this correct? (y or n) '\)
      read(*,'(a)') ans
      if((ans.eq.'n').or.(ans.eq.'N')) go to 11
c
      inquire(file=fname,exist=ex)
      if(ex) then
      write(*,*) ' That file already exists. Do you want to replace it?'
      write(*,103)
103 format(' Enter y or n '\)
      read(*,'(a)') ans
      if((ans.eq.'n').or.(ans.eq.'N')) go to 11
      endif
c
      save input data into a file for later use.
      open(5,file=fname)
      write(5,'(a56)') title

      write(5,'(f10.0)') p0

      write(5,'(i5)') ndef

      do i=1, ndef
      write(5,'(i5,f10.2)') l(i),u(i)
      end do

```

```

        return
    end
    subroutine calc
c *****
c *   calculation   *
c *****
        common /idata/ title,p0,ndef,l(12),u(12)
        common /odata/ a2,b2,z1,o2,o3,o4,o5,y
        character title*56
c
        dtr=3.14159265/180.
        rtd=180./3.1419265
        s1=0
        s2=0
        s3=0
        s4=0
        s5=0
        s6=0
        s7=0
        s8=0
        s9=0
        N=0
        do 10 i=1, ndef
            if(u(i) .eq. 0.0) goto 10
            N=N+1
            si = sin(l(i)*dtr)
            co = cos(l(i)*dtr)
            s1 = s1 + co**4
            s2 = s2 + si**4
            s3 = s3 + si**2 * co**2
            s4 = s4 + si**3 * co
            s5 = s5 + co**3 * si
            s6 = s6 + u(i) * co**2
            s7 = s7 + u(i) * si * co
            s8 = s8 + u(i) * si**2
            s9 = s9 + u(i)**2
        10 continue

        d =s1*s3*s2+s5*s4*s3+s3*s5*s4-s3*s3*s3-s1*s4*s4-s2*s5*s5
        g1=s6*s3*s2+s7*s4*s3+s5*s4*s8-s8*s3*s3-s6*s4*s4-s7*s5*s2
        g2=s1*s7*s2+s5*s8*s3+s6*s4*s3-s3*s7*s3-s1*s8*s4-s5*s6*s2
        g3=s1*s3*s8+s4*s5*s6+s3*s5*s7-s3*s3*s6-s5*s5*s8-s1*s4*s7

        a=g1/d
        b=g2/d
        c=g3/d

```

```

z1=atan(b/(a-c))/2.
z2=z1
z1=z1*rtd
d1=cos(z2)**4 -sin(z2)**4
a1=a*cos(z2)**2 - c*sin(z2)**2
b1=c*cos(z2)**2 - a*sin(z2)**2
a2=a1/d1
b2=b1/d1

o1=4*p0*0.75*(2.8125**2)/(2.8125**2-0.75**2)
o2=a2/o1
o3=b2/o1
o4=1/o2
o5=1/o3
ss=s9 -a*s6 -b*s7 -c*s8
y = sqrt(ss/(N-3))
c
c*****debug*****
c      write(*,*) ' A2 =', a2
c      write(*,*) ' B2 =', b2
c      write(*,*) ' Z1 =', z1
c      write(*,*) ' P0 =', p0
c      write(*,*) ' a1=', o2
c      write(*,*) ' a22=', o3
c      write(*,*) ' e1=', o4
c      write(*,*) ' e22=', o5
c      write(*,*) ss
c      write(*,*) ' SD =', Y
c      read(*,*)
c*****
      return
      end
      subroutine plot
c *****
c * plots points and ellipse *
c *****
c * Programmer: Meng-Cherng Sun *
c * Date: 01/06/1992 *
c *****
c
      INCLUDE 'FGRAPH.FD'
      RECORD /rccoord/ s
c
      integer*2 vxmin,vxmax,vymin,vymax,dummy
      integer*2 irow,icol
      common /idata/ title,p0,ndef,l(12),u(12)
      common /odata/ a2,b2,z1,o2,o3,o4,o5,y
      common /vpor/ vxmin,vxmax,vymin,vymax
      common /wind/ wxmin,wxmax,wymin,wymax
      common /dwin/ dxmin,dxmax,dymin,dymax
      character title*56,cht*1
      character theta*1,alpha*1,angthe*11,alpha1*7,alpha22*7
      dimension xp(2),yp(2)

```

```

c      alpha = char(224)
      theta = char(233)
c
c      Draw a box around the graph
c
c      dummy = setcolor(1)
      dummy = rectangle_w($GBORDER, wxmin,wymin,wxmax,wymax)
c      write figure and title
c
c      fonts/character size in screen and window coordinates
      icw=8
      ich=10
      xcw= icw*((wxmax-wxmin)/(vxmax-vxmin))
      ycw= ich*((wymax-wymin)/(vymax-vymin))
c      write title onto plot
      xcenter=wxmin+(wxmax-wxmin)/2.
      xtitle = xcenter-xcw*(len_trim(title)/2)
      ytitle = wymax-ycw
c**   call label_gtx(xtitle,ytitle,6,ich,icw,15,title)
      call label_tx(1,18,5,title)
c      plot ellipse
c
      dtr=3.14159265/180.
c      draw axes passing through (0,0)
      call axes
c
      do 10 i = 1, ndef
        IF(u(i) .eq. 0.0) GOTO 10
        do 11 j = 1, 2
          l(i)=l(i)+(j-1)*180.
          x1 = u(i)*cos((l(i)+90)*dtr)
          y1 = u(i)*sin((l(i)+90)*dtr)
c
          dummy = setcolor(4)      ! red
          call cross(x1,y1,5)
11      continue
10      continue
c      Draw ellipse
      tl=0.
      r = a2*cos((-zl+tl)*dtr)**2 + b2*sin((-zl+tl)*dtr)**2
      x2 = r*cos((+tl+90)*dtr)
      y2 = r*sin((+tl+90)*dtr)
c      get first point
      xp(1)=x2
      yp(1)=y2
      dummy = setcolor(2)      ! green
      do tl=1,360,2
        r = a2*cos((-zl+tl)*dtr)**2 + b2*sin((-zl+tl)*dtr)**2
        x2 = r*cos((+tl+90)*dtr)
        y2 = r*sin((+tl+90)*dtr)
        xp(2)=x2
        yp(2)=y2
        call lines(xp,yp,2)

```

```

c    move to next point
      xp(1)=x2
      yp(1)=y2
      end do
c    Draw alphall axis
      dummy = setcolor(3)
      xp(1)=a2*cos((z1+90)*dtr)
      yp(1)=a2*sin((z1+90)*dtr)
      xp(2)=-xp(1)
      yp(2)=-yp(1)
      call lines(xp,yp,2)
c    Print alphall symbol
      if(xp(1).lt.0.) xalpha = xp(1)-2*xcw
      if(xp(1).ge.0.) xalpha = xp(1)+3*xcw
      yalpha = yp(1)+0.25*ycw
      alphall=alpha//'ll'
      call getvfw(xalpha,yalpha,vx,vy)
      call getcfv(vx,vy,icol,irow)
      call setttextposition(irow,icol,s)
      write(*,'(lx,a3\')') alphall

      xthe= xp(1)/2+0.15*xcw
      ythe= yp(1)-1.5*ycw

      cht = theta
      call getvfw(xthe,ythe,vx,vy)
      call getcfv(vx,vy,icol,irow)
      call setttextposition(irow,icol,s)
      WRITE(*,'(lx,a1\')') cht
      dummy = setcolor(1)      !blue
c    draw axes passing through (0,0) again
      call axes

      dummy = setcolor(3)
c    Draw alpha22 axis
      xp(1)=b2*cos(z1*dtr)
      yp(1)=b2*sin(z1*dtr)
      xp(2)=-xp(1)
      yp(2)=-yp(1)
      call lines(xp,yp,2)

c    legend
      irow=2
      icol=1
c    call moveto (irow,icol,vxy)
      call setttextposition(irow,icol,s)
      write(*,*)
      write(*,*)
      write(*,*)
      ip0=p0
      write(*,2999) ip0
2999 format(' Po = ',i5,' psi')
      ia2=a2
      write(*,3000) ia2

```

```

3000 format(' A = ',i5)
    ib2=b2
    write(*,3001) ib2
3001 format(' B = ',i5)
    angthe = ' '//theta//' ='
    izl=z1
    write(*,3002) angthe,izl
3002 format(a6,i5,' deg')
    alphall=' '//alpha//'11='
    write(*,3003) alphall,o2
3003 format(a6,f5.2)
    alpha22=' '//alpha//'22='
    write(*,3004) alpha22,o3
3004 format(a6,f5.2)
    write(*,3005) o4*1000000.
3005 format(' E1 = ',lp,e9.2)
    write(*,3006) o5*1000000.
3006 format(' E2 = ',lp,e9.2)
    write(*,3007) ifix(y)
3007 format(' SD = ',i5,' deg')
c
    return
    end
c
    subroutine getvfw(wx,wy,vx,vy)
c *****
c * Gets point on screen/viewport(vx,vy) *
c * from window coordinates (wx,wy)      *
c *****
c
    integer*2 vxmin, vymin, vxmax, vmax
    common /vpor/ vxmin,vxmax,vymin,vymax
    common /wind/ wxmin,wxmax,wymin,wymax
c
    vx = vxmin+((vxmax - vxmin)/(wxmax - wxmin))*(wx-wxmin)
    vy = vymin+((vymin - vmax)/(wymax - wymin))*(wy-wxmin)
    return
    end

    subroutine getcfv(vx,vy,icol,irow)
c *****
c * Gets point in text window (icol,irow) *
c * from viewport window (vx,vy)          *
c *****
c
    integer*2 vxmin, vymin, vxmax, vmax
    common /vpor/ vxmin,vxmax,vymin,vymax
    common /wind/ wxmin,wxmax,wymin,wymax
    integer*2 icol,irow
c
    icol = vx/(640/80)
    irow = vy/(480/35)
    return
    end

```

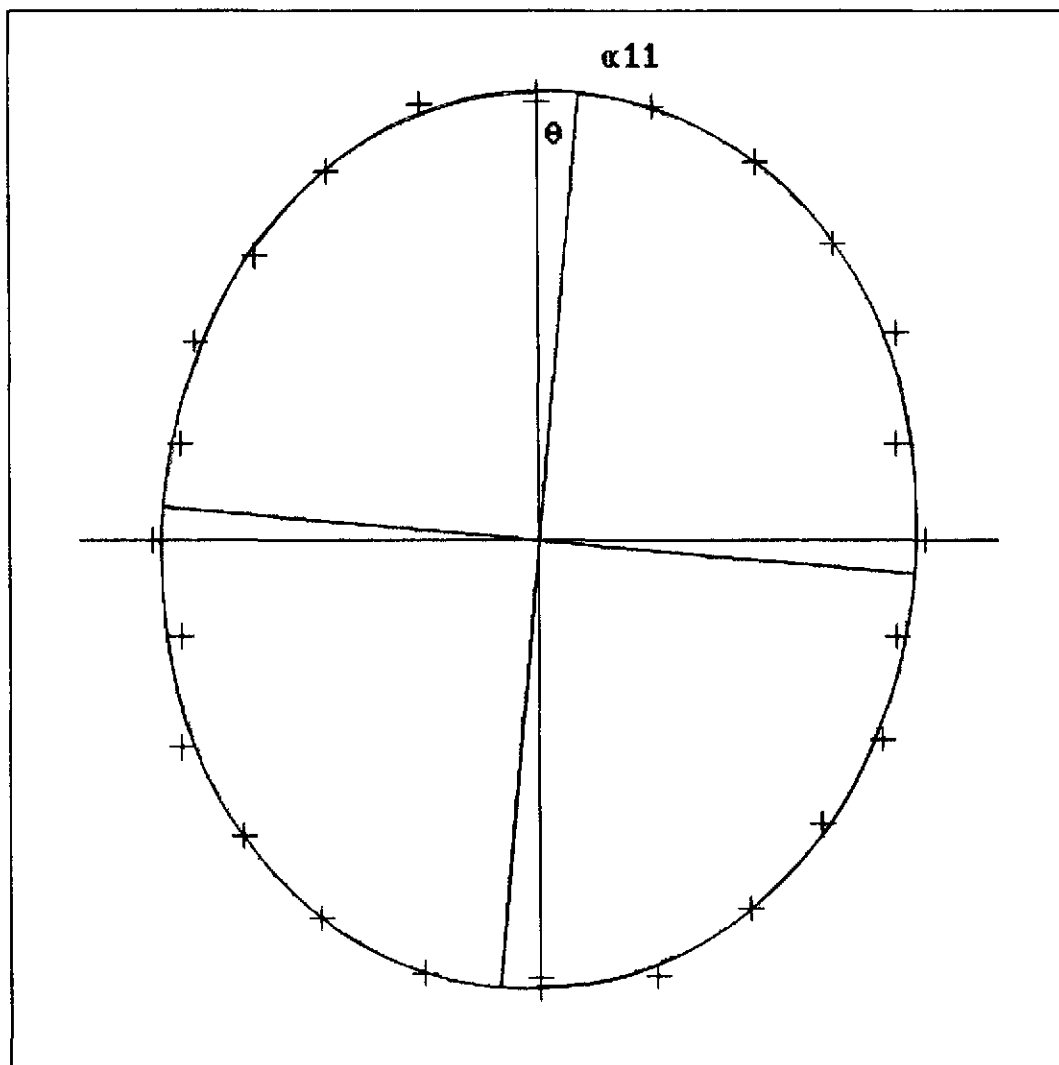
Sample data for plotting the biaxial ellipse

Minnamax Mine - Hole 1 Depth 18'2"
1000.

12	
0	380.00
-15	389.00
-30	379.00
-45	364.00
-60	359.00
-75	323.00
-90	337.00
-105	322.00
-120	345.00
-135	349.00
-150	370.00
-165	392.00

Minnamax Mine - Hole 1 Depth 18'2"

Po = 1000 psi
A = 390
B = 328
 θ = -5 deg
 α_{11} = .12
 α_{22} = .10
E1 = 8.28E+06
E2 = 9.85E+06
SD = 8 deg



APPENDIX E.—COMPUTER CODE FOR DETERMINING ISOTROPIC AND ANISOTROPIC SECONDARY PRINCIPAL STRESSES

By Dennis R. Dolinar¹ and Meng-Cherng Sun¹

```

$INCLUDE: 'FGRAPH.FI'
      program plotpqd
c *****
c * program calculates Anisotropic or Isotropic      *
c * stress and plots the P, Q and their directions. *
c * By Dennis R. Dolinar, USEM - DRC                 *
c *****
c * Programmed for PC Fortran by Meng-Cherng Sun     *
c * Date: 01/22/1992                                *
c *****
c
      integer*2 vxmin, vymin, vxmax, vmax
      character ans
      common /vpor/ vxmin,vxmax,vymin,vymax
      common /wind/ wxmin,wxmax,wymin,wymax

11 write(*,22)
22 format(' Program calculates the Anisotropic or Isotropic'
      # ' Stress/' and plots the P, Q and their Directions')
      write(*,33)
33 format(' Just plot p, q and delta data? (y or n*)'\)
      read(*,'(a)') ans
      if (ans.eq.'y' .or. ans .eq.'Y') goto 333

      write(*,100)
100 format(' Input from keyboard or file? (k or f*): '\)
      read (*,'(a)') ans
      if (ans .eq. 'k' .or. ans .eq. 'K') then
        call kinput
      else
        call finput
      write(*,200)
200 format(' Are these data O.K.? (y* or n): '\)
      read (*,'(a)') ans
      if (ans .eq. 'n' .or. ans .eq. 'N') then
        write(*,*) ' Enter input data from keyboard again!'
        call kinput
      endif
c      END OF INPUT DATA
      endif
c
c      calculation
      call calc

```

¹Mining engineer, Denver Research Center, U.S. Bureau of Mines, Denver, CO.

```

c
c      Save p, q and delta
      write(*,301)
301 format(' Do you want to save p, q and delta? (y* or n) '\)
      read(*,'(a)') ans
      if (ans.eq.'n' .or. ans.eq.'N') goto 444
      call savepqd
      goto 444

c
c      read in p, q and delta from a file
333 call readpqd

c
444 call plot

c
      write(*,2200)
2200 format(' Do you want to plot more? (y or n*)'\)
      read (*,'(a)') ans
      if (ans .eq. 'y' .or. ans .eq.'Y') goto 11

c
      write(*,*) ' Program stops!'
      end
      subroutine kinput
c *****
c * input data from keyboard      *
c *****
      character title*56,ans
      common /idata/ title,angul,npts,depth(20),theta(20)
      common /oparm/ up1(20),up2(20),up3(20),all(20),a22(20),
#                      sigmaz(20),v31(20),v32(20),s31(20),s32(20),
#                      nitera(20)

      write(*,1001)
1001 format(' Enter Title (up to 56 characters)-')
      read (*,'(a56)') title

      write(*,1011)
1011 format(' Enter Bearing/angle of U1 in degree = '\)
      read(*,*) angul

      write(*,1002)
1002 format(' Enter number of readings (points) ='\)
      read (*,*) npts
      do n = 1, npts
      write(*,1003) n
1003 format(' For R-',i2)
      write(*,1013)
1013 format(' Enter depth in Ft. = ',\)
      read(*,*) depth(n)

      write(*,1004)
1004 format(' Enter u1,u2,u3 = '\)
      read (*,*) up1(n),up2(n),up3(n)

```

```

        write(*,1005)
1005 format(' Enter theta angle in Degrees = '\)
        read(*,*) theta(n)

        write(*,1006)
1006 format(' Enter alphall,alpha22 = '\)
        read(*,*) all(n),a22(n)

        write(*,1007)
1007 format(' Enter sigmaz, v31,v32 = '\)
        read(*,*) sigmaz(n),v31(n),v32(n)

        write(*,1008)
1008 format(' Enter s31,s32 = '\)
        read(*,*) s31(n),s32(n)

        write(*,1009)
1009 format(' Enter number of iterations = '\)
        read(*,*) nitera(n)
        enddo

c
C   SAVING INPUT DATA?
        WRITE(*,2003)
2003 FORMAT(' Do you want to save the input data? (y or n*) '\)
        read(*,'(a)') ans
        if(ans.eq.'y' .or. ans.eq.'Y') then
            call fsave
        endif
        return
        end
        subroutine finput
c *****
c *   Input data from a file           *
c *****
c
        logical ex
        character title*56,fname*12,ans
        common /idata/ title,angul,npts,depth(20),theta(20)
        common /oparm/ up1(20),up2(20),up3(20),all(20),a22(20),
        #               sigmaz(20),v31(20),v32(20),s31(20),s32(20),
        #               nitera(20)
        common /filnm/ fname

11 write(*,12)
12 format(' Enter file name : '\)
        read(*,'(a12)') fname
        write(*,101) fname
101 format(' The file name is : ', al2,
        + ' Is this correct? (y* or n) '\)
        read(*,'(a1)') ans
        if((ans.eq.'n').or.(ans.eq.'N')) go to 11
c

```

```

    inquire(file=fname,exist=ex)
    if(ex) then
        open(5,file=fname,status='old')

        read (5,'(a56)') title
        write(*,'(a56)') title

        read (5,'(f10.2)') angul
        write(*,'(f10.2)') angul

        read (5,'(i2)')  npts
        write(*,'(i2)')  npts
c
        do n = 1, npts
c
            read (5,'(f10.2)')  depth(n)
            write(*,'(f10.2)')  depth(n)

            read (5,'(3f10.2)') up1(n),up2(n),up3(n)
            write(*,'(3f10.2)') up1(n),up2(n),up3(n)

            read (5,'( f10.2)') theta(n)
            write(*,'( f10.2)') theta(n)

            read (5,'(2f10.3)') all(n),a22(n)
            write(*,'(2f10.3)') all(n),a22(n)

            read (5,'(f10.2,2f10.3)') sigmaz(n),v31(n),v32(n)
            write(*,'(f10.2,2f10.3)') sigmaz(n),v31(n),v32(n)

            read (5,'(2f10.3)') s31(n),s32(n)
            write(*,'(2f10.3)') s31(n),s32(n)

            read (5,'( i2)'    )  nitera(n)
            write(*,'( i2)'    )  nitera(n)
            enddo
        else
            write(*,*) ' File does not exist, Please check file name!'
            goto 11
        endif
        return
    end

    subroutine fsave
c *****
c * save input data into a file      *
c *****
    character title*56,fname*12,ans
    logical ex
    common /idata/ title,angul,npts,depth(20),theta(20)
    common /oparm/ up1(20),up2(20),up3(20),all(20),a22(20),
#               sigmaz(20),v31(20),v32(20),s31(20),s32(20),
#               nitera(20)
    common /filnm/ fname

```

```

11 write(*,*)
   write(*,*) ' Enter output file name ..'
   write(*,*) ' (For saving input data)'
   read(*,'(a12)') fname
   write(*,101) fname
101 format(' The file name is : ', al2,
+ ' Is this correct? (y* or n) '\)
   read(*,'(a)') ans
   if((ans.eq.'n').or.(ans.eq.'N')) go to 11
c
   inquire(file=fname,exist=ex)
   if(ex) then
   write(*,102)
102 format(' That file already exists. Do you want to replace it?'
#           ' (y* or n) '\)
   read(*,'(a)') ans
   if((ans.eq.'n').or.(ans.eq.'N')) go to 11
   endif
c   save input data into a file for later use.

   open(6,file=fname)

   write(6,'(a56)') title
   write(6,'(i2)') npts

   do n = 1, npts
   write(6,'( f10.2)') depth(n)
   write(6,'( f10.2)') angul
   write(6,'(3f10.2)') up1(n),up2(n),up3(n)
   write(6,'( f10.2)') theta(n)
   write(6,'(2f10.3)') all(n),a22(n)
   write(6,'(f10.2,2f10.3)')      sigmaz(n),v31(n),v32(n)
   write(6,'(2f10.3)') s31(n),s32(n)
   write(6,'( i2)')      nitera(n)
   enddo
c
   close(6)
   return
   end
   subroutine calc
c *****
c *   calculation   *
c *****
   character title*56
   common /idata/ title,angul,npts,depth(20),theta(20)
   common /oparm/ up1(20),up2(20),up3(20),all(20),a22(20),
#               sigmaz(20),v31(20),v32(20),s31(20),s32(20),
#               nitera(20)
   common /pqd/   p(20),q(20),delta(20)
   common /filnm/ fname
   character fname*12,itrfname*12,filename*8

```

```

c      open a file to save output for checking iteration results
      filename = fname(1:8)
      call packname(filename, '.itr', itrfname)
      open(15,file=itrfname)
      write(15,'(a56)') title

      dtr=3.14159265/180.
      rtd=180./3.1419265

c
      do n = 1, npts

      u1 = up1(n)
      u2 = up2(n)
      u3 = up3(n)

      d1 = all(n)
      d2 = a22(n)
      d3 = -theta(n)
      d4 = sigma2(n)
      d5 = v31(n)
      d6 = v32(n)
      d7 = s31(n)
      d8 = s32(n)

      cos1 = cos( d3      *dtr)
      cos2 = cos((d3+60) *dtr)
      cos3 = cos((d3+120)*dtr)

      sin1 = sin( d3      *dtr)
      sin2 = sin((d3+60) *dtr)
      sin3 = sin((d3+120)*dtr)

      f1 = cos1**2
      f2 = cos2**2
      f3 = cos3**2
      f4 = cos1*sin1
      f5 = cos2*sin2
      f6 = cos3*sin3
      f7 = sin1**2
      f8 = sin2**2
      f9 = sin3**2

      g1 = f1*f5*f9 +f2*f6*f7 +f4*f8*f3 -f3*f5*f7 -f1*f6*f8 -f2*f4*f9
      g2 = u1*f5*f9 +u2*f6*f7 +f4*f8*u3 -u3*f5*f7 -u1*f6*f8 -u2*f4*f9
      g3 = f1*u2*f9 +f2*u3*f7 +u1*f8*f3 -f3*u2*f7 -f1*u3*f8 -f2*u1*f9
      g4 = f1*f5*u3 +f2*f6*u1 +f4*u2*f3 -f3*f5*u1 -f1*f6*u2 -f2*f4*u3

      a = g2/g1
      b = g3/g1
      c = g4/g1

```

```

c      Start to iterations
      it = 0
      cl = c
      al = a
      sqdd =sqrt(d1*d2)
c      iterate
1000 ck = (a*sqdd+c*d1)/(d1*1.5*(d2+sqdd))
      cl = (a*d2+c*(2*sqdd+d2))/(2*1.5*(d2+sqdd)**2)
      hl = (c*d1-a*d2)/(b*sqdd) + sqrt(((c*d1-a*d2)/(b*sqdd))**2+1)
      h2 = atan(hl)

      p(n) = (cl-ck*cos(h2)**2)/(sin(h2)**2 - cos(h2)**2)
      q(n) = ck-p(n)

      write(15,1001) it
1001 format(i3, ' iteration')
      write(15,1002) p(n), q(n), h2*rtd
1002 format(' P=',f10.2,' psi, Q=',f10.2,' psi, Delta=',f5.1,' DEG')

      v1 = d5*(p(n)*cos(h2)**2 + q(n)*sin(h2)**2)
      v2 = d6*(p(n)*sin(h2)**2 + q(n)*cos(h2)**2)

      a = al-1.5*d7*(d4-v1-v2)
      c = cl-1.5*d8*(d4-v1-v2)

      if(it.eq.nitera(n)) goto 2000
      it = it+1
      goto 1000

2000 write(15,*) '          Final Stress Values'
      delta(n) = h2*rtd
      write(15,1002) p(n), q(n), delta(n)
      enddo
      close(15)
      return
      end
      subroutine savepqd
c *****
c *      save p, q and delta for plots *
c *****
      character title*56, fname*12, pqdfname*12, filename*8
      common /idata/ title, angl, npts, depth(20), theta(20)
      common /pqd/   p(20), q(20), delta(20)
      common /filnm/ fname
      logical ex
c
      filename = fname(1:8)
      call packname(filename, '.pqd', pqdfname)

      write(*,101) pqdfname
101 format(' The file name is : ', a12,
+ ' Is this correct? (y* or n) '\)
      read(*, '(a1)') ans
      if(ans.eq.'y' .or. ans.eq.'Y') goto 2000

```

```

c   read in the file name instead
11 write(*,*)
   write(*,*) ' Enter output file name ..'
   write(*,*) ' (For saving input data)'
   read(*,'(a12)') fname
   write(*,102) fname
102 format(' The file name is : ', al2,
+ ' Is this correct? (y* or n) '\)
   read(*,'(a)') ans
   if((ans.eq.'n').or.(ans.eq.'N')) go to 11
c
   inquire(file=pqdfname,exist=ex)
   if(ex) then
   write(*,103)
103 format(' That file already exists. Do you want to replace it?'
#           ' (y* or n) '\)
   read(*,'(a)') ans
   if((ans.eq.'n').or.(ans.eq.'N')) go to 11
   endif

c   save input data into a file for later use.
200 open(5,file=pqdfname)

   write(5,'(a56)') title
   write(5,'(f10.2)') angul
   write(5,'(i2)') npts

   do n = 1, npts
   write(5,'(5f10.2)') depth(n),p(n),q(n),delta(n),theta(n)
   enddo
   return
   end

   subroutine readpqd
c *****
c * Inputs p, q and delta from .pqd file *
c *****
   character title*56,fname*12,pqdfname*12,filename*8
   common /idata/ title,angul,npts,depth(20),theta(20)
   common /pqd/ p(20),q(20),delta(20)
   common /filnm/ fname
   logical ex

c
11 write(*,12)
12 format(' Enter pqd data file name : '\)
   read(*,'(a12)') fname
   filename = fname(1:8)
   call packname(filename, '.pqd', pqdfname)

   write(*,101) pqdfname
101 format(' The file name is : ', al2,
+ ' Is this correct? (y* or n) '\)
   read(*,'(a1)') ans
   if(ans.eq.'n' .or. ans.eq.'N') go to 11

```

```

c
  inquire(file=pqdfname,exist=ex)
  if(ex) then
    open(5,file=pqdfname,status='old')

    read (5,'(a56)')  title
    write(*,'(a56)')  title
    read (5,'(f10.2)') angul
    write(*,'(f10.2)') angul
    read (5,'(i2)')   npts
    write(*,'(i2)')   npts
    do n = 1, npts
      read (5,'(5f10.2)') depth(n),p(n),q(n),delta(n),theta(n)
      write(*,'(5f10.2)') depth(n),p(n),q(n),delta(n),theta(n)
    enddo

c
    else
      write(*,*) ' File does not exist, Please check file name!'
      goto 11
    endif

c
    return
  end

  subroutine plot
c *****
c *   Initialize, plot and close....*
c *****
    integer*2 vxmin, vymin, vxmax, vymax
    character title*56
    common /vpor/  vxmin,vxmax,vymin,vymax
    common /wind/  wxmin,wxmax,wymin,wymax
    common /idata/ title,angul,npts,depth(20),theta(20)
    common /pqd/   p(20),q(20),delta(20)

c
    initialize graphics video
    call initplot

c
c
c    set first window for p and q data
    call setwinpq
    call plotpq

c
c
c    set second window for delta data
    call setwindd
    call plotdd

c
    close graphics video
    call closeplot

c
    return
  end

```

```

      subroutine setwinpq
c *****
c *   Find max. and min. values of x and y data *
c *   and set viewport and window               *
c *****
      INCLUDE 'FGRAPH.FD'

      integer*2 vxmin, vymin, vxmax, vmax
      character title*56
      common /vpor/ vxmin,vxmax,vymin,vymax
      common /wind/ wxmin,wxmax,wymin,wymax
      common /dminx/ dxmin,dxmax,dxint,dymin,dymax,dyint
      common /idata/ title,angul,npts,depth(20),theta(20)
      common /pqd/   p(20),q(20),delta(20)

c
c   plot compression(-) into (+) value
      do n = 1, npts
      p(n) = -p(n)
      q(n) = -q(n)
      enddo

c
c   reverse p and q value, if p<q
      do 10 n = 1, npts
      if (p(n) .gt. q(n)) goto 10
      temp = p(n)
      p(n) = q(n)
      q(n) = temp
10 continue

c
c   find min. and max. value of data points (p and q)
      dmax = p(1)
      dmin = p(1)
      do n = 1, npts
      dmax = max (p(n),dmax)
      dmax = max (q(n),dmax)
      dmin = min (p(n),dmin)
      dmin = min (q(n),dmin)
      enddo

c   round off to nice number for y-data(p or q)
      nint=8      ! approx. number of intervals
      call niceint(dmin,dmax,nint,dymin,dymax,dyint)
c*****debug*****
c   write(*,*) dmin,dymin, dmax,dymax
c   read(*,*)
c*****
c   round off to nice number for x-data
      nint = 15
      call niceint(depth(1),depth(npts),nint,dxmin,dxmax,dxint)
      dxmin=0
      dxmax=20
c*****debug*****
c   write(*,*) depth(1),dxmin,depth(npts),dxmax
c   read(*,*)
c*****

```

```

write(*,*) ' Hit Return to Continue'
read(*,*)

c
c   Set viewport (screen in pixels) for data
sxmin = 100
sxmax = 551
symin = 155
symax = 380

c
vxmin = sxmin - 100
vxmax = sxmax + 25
vymin = symin - 25
vymax = symax + 100

xvtd = (dxmax - dxmin)/(sxmax - sxmin)
yvtod = (dymax - dymin)/(symax - symin)

c   Set window (x- and y-data)
wxmin = dxmin - xvtd*100
wxmax = dxmax + xvtd*25
wymn = dymin - yvtod*100
wymax = dymax + yvtod*25

c
C*****DEBUG*****
c   print viewport to see
c   write(*,*) vxmin,vxmax
c   write(*,*) vymin,vymax
c   read(*,*)
C*****
c   call Data window
c   call clearscreen($GCLEARSCREEN)
c   call window
c   return
c   end
c   subroutine plotpq
c *****
c *   Plot q and p curves   *
c *****
c   INCLUDE 'FGRAPH.FD'

integer*2 vxmin, vymin, vxmax, vmax
character title*56,xlabel*10,ylabel*12
integer*2 dummy
common /vpor/ vxmin,vxmax,vymin,vymax
common /wind/ wxmin,wxmax,wymn,wymax
common /dmimx/ dxmin,dxmax,dxint,dymin,dymax,dyint
common /idata/ title,angul,npts,depth(20),theta(20)
common /pqd/ p(20),q(20),delta(20)

record /rccoord/ curpos
record /wxycoord/ wxy

c
c   Print Title in color on the screen.
c   CALL settxtposition(2,14,curpos)

```

```

        dummy = setttextcolor( 6 ) !brown color
        CALL outtext (title)
c
c      Draw a bordered rectangle around the graph
        dummy = setcolor(1)      ! blue
        dummy = rectangle_w( $GBORDER, dxmin, dymin, dxmax, dymax )
c
c      Draw Data curves with different color
        CALL curve
c
c      Draw a dashed line at data value=0
        dummy = setcolor(5)      !magenta
        CALL setlinestyle (#F0F0)
        CALL moveto_w(dxmin,0.,wxy)
        dummy = lineto_w(dxmax,0.)
        Call setlinestyle (#FFFF)
c
c      plot x-axis and tic marks, and labels
c
        xstr = dxmin
        xend = dxmax
        xint = dxint
        yloc = dymin
        xlabel = 'DEPTH, ft'
        ndec=0
c      dummy = setcolor (8) ! dark gray
        call xaxis(xstr,xend,xint,yloc,xlabel,ndec)
c
c      Plot y-axis and tic marks, and labels
c
        ystr = dymin
        yend = dymax
        yint = dyint
        xloc = dxmin
        ylabel = 'STRESS, psi'
        ndec=0
c      dummy = setcolor(8)      ! dark gray
        call yaxis(ystr,yend,yint,xloc,ylabel,ndec)
        return
        END
c
        SUBROUTINE curve
c *****
c * draws curve based on x- and y-data *
c *****
c * programmed: Meng-Cherng Sun      *
c * Date: 11/27/91                  *
c *****
        INCLUDE 'FGRAPH.FD'
c
        character title*56
        common /idata/ title,angul,npts,depth(20),theta(20)
        common /pgd/   p(20),q(20),delta(20)
        INTEGER*2 dummy

```

```

dimension y(20)
character lleg*2
RECORD /videoconfig/ screen
COMMON screen

do i =1, 2
c  plot p curve with x--depth
c  set color for different curve
  dummy = setcolor(i+1)
  do n =1,npts
    if (i .eq. 1) y(n)=p(n)
    if (i .eq. 2) y(n)=q(n)
  enddo
c
c  draw a (+) symbol at point
  call lines(depth,y,npts)
  do n = 1, npts
    call cross(depth(n),y(n),5)
  enddo
c
c  legend
  xleg=depth(npts-1)
  yleg=  y(npts-1)-20
  lleg=' P'
  if(i.eq.2) lleg=' Q'
  call label_gtx(xleg,yleg,6,15,7, i+1, lleg)
c
  enddo
  return
end

subroutine setwindd
c *****
c * Find max. and min. values of x and y data *
c * and set viewport and window *
c *****
  INCLUDE 'FGRAPH.FD'

  integer*2 vxmin, vymin, vxmax, vymax
  character title*56
  common /vpor/ vxmin,vxmax,vymin,vymax
  common /wind/ wxmin,wxmax,wymin,wymax
  common /dmimx/ dxmin,dxmax,dxint,dymin,dymax,dyint
  common /idata/ title,angul,npts,depth(20),theta(20)
  common /pqd/ p(20),q(20),delta(20)
c
c  Set viewport (screen in pixels) for data
  vxmin = 100
  vxmax = 551
  vymin = 50
  vymax = 150
c
c  dymin=0.
c

```

```

c      Set window (x- and y-data)
      wxmin = dxmin
      wxmax = dxmax
      wymin = dymin
      wymax = dymax

c      call Data window
      call window

c
      return
      end
      subroutine plotdd
c *****
c *   Plot p and q stress and direction *
c *****
      INCLUDE 'FGRAPH.FD'

      integer*2 vxmin, vymin, vxmax, vymax
      character title*56
      integer*2 dummy
      common /vpor/ vxmin,vxmax,vymin,vymax
      common /wind/ wxmin,wxmax,wymin,wymax
      common /dminx/ dxmin,dxmax,dxint,dymin,dymax,dyint
      common /idata/ title,angul,npts,depth(20),theta(20)
      common /pqd/   p(20),q(20),delta(20)
      dimension angp(20),xp(2),yp(2)

c
      dtr=3.14159265/180.
      rtd=180./3.1419265
c      ratio/scale for x-coordinate
      y2x=((dxmax-dxmin)/(dymax-dymin))
      #      *(float(vymax-vymin)/float(vxmax-vxmin))

c
c      Draw a bordered rectangle around the graph
      dummy = setcolor(1)      ! blue
      dummy = rectangle_w( $GBORDER,dxmin,dymin,dxmax,dymax )

c
c      compute angle
c
      do n = 1, npts
      ang = delta(n) + theta(n)
      if(ang . ge. 90.) ang = ang-90.
      angp(n) = ang + angul
      enddo
c ***** debug *****
c      write(*,*) angul
c      do n =1, npts
c      write(*,*) p(n),q(n),delta(n),theta(n),angp(n)
c      enddo
c      read(*,*)
c *****
c
c      Draw p and q magnitude with direction (angp)
c

```

```

do n = 1, npts
xc = depth(n)
yc = dymin + (dymax-dymin)/2.

c  draw p stress
dummy = setcolor(2)
dx = 0.5*p(n)*sin(angp(n)*dtr)*y2x
dy = 0.5*p(n)*cos(angp(n)*dtr)
xp(1) = xc - dx
yp(1) = yc + dy
xp(2) = xc + dx
yp(2) = yc - dy
call lines(xp,yp,2)

c  draw q stress
c  set color
dummy = setcolor(3)
if(q(n) .lt. 0.) q(n)=-q(n)
dx = 0.5*q(n)*cos(angp(n)*dtr)*y2x
dy = 0.5*q(n)*sin(angp(n)*dtr)
c  write(*,*) dx,dy
xp(1) = xc - dx
yp(1) = yc - dy
xp(2) = xc + dx
yp(2) = yc + dy
call lines(xp,yp,2)
enddo

return
END

```

Sample data for calculating the secondary principal stresses

Southfield Mine - Roof (hole 1) May 1990 Plane Strain

-12.00

5

1.40		
-524.00	-195.00	277.00
29.00		
.251	.258	
-19.00	.207	.207
-.093	-.093	

3

11.00		
-2133.00	-988.00	-485.00
43.00		
.414	.459	
-945.00	.245	.245
-.087	-.087	

3

14.50		
-4258.00	-2057.00	-2148.00
-5.00		
1.766	.934	
-1255.00	.170	.170
-.121	-.121	

3

15.80		
-3404.00	-2006.00	-1699.00
-22.00		
.436	3.371	
-1307.00	.150	.150
-.106	-.106	

3

17.80		
-1399.00	-531.00	62.00
39.00		
.280	.269	
-1337.00	.140	.140
-.039	-.039	

3

Sample data for plotting the secondary principal stresses

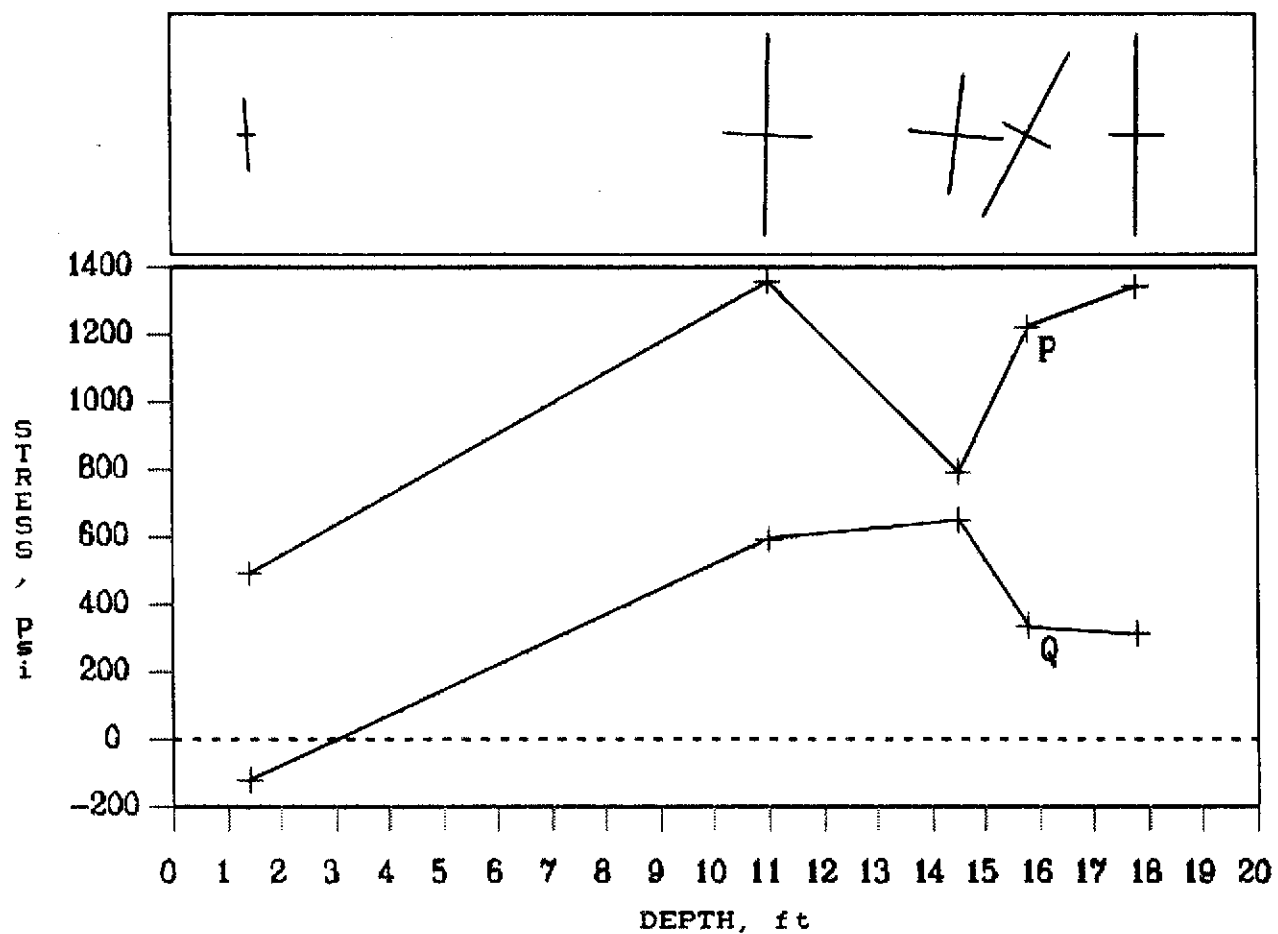
Southfield Mine - Roof (hole 1) May 1990 Plane Strain

-12.00

5

1.40	119.41	-491.91	78.98	29.00
11.00	-590.40	-1355.32	57.41	43.00
14.50	-792.24	-650.49	10.34	-5.00
15.80	-1221.49	-336.26	5.90	-22.00
17.80	-318.34	-1339.15	62.51	39.00

Southfield Mine - Roof (hole 1) May 1998 Plane Strain



APPENDIX F.—COMPUTER CODE FOR DETERMINING LEAST SQUARES METHOD OF SECONDARY PRINCIPAL STRESSES

By Dennis R. Dolinar¹ and Meng-Cherng Sun¹

```

      program lsquare
c *****
c *   Program for Least Squares Calculation   *
c *   By Dennis R. Dolinar                   *
c *****
c *   Programmed for PC by Meng-Cherng Sun   *
c *   Date: 02/10/1992                      *
c *****
c
      character title*56,ans
      integer u
      common/idata/ title,angle(3),depth,hdiam,numrf,u(20,3),e(20),v(20)
c
      write(*,99)
      99 format(' Program for least squares Analysis ')
      write(*,100)
      100 format(' Input from keyboard or file? (k or f*): '\)
      read (*,'(a)') ans
      if (ans .eq. 'k' .or. ans .eq. 'K') then
        call kinput
      else
        call finput
      write(*,200)
      200 format(' Are these data O.K.? (y* or n): '\)
      read (*,'(a)') ans
      if (ans .eq. 'n' .or. ans .eq. 'Y') then
        write(*,*) ' Enter input data from keyboard again!'
        call kinput
      endif
c      END OF INPUT DATA
      endif
c
      call calc
c
      call tablein
c
      call table
c

```

¹Mining engineer, Denver Research Center, U.S. Bureau of Mines, Denver, CO.

```

c      write(*,*) ' Program stops!'
      end
      subroutine kinput
c *****
c * input data from keyboard      *
c *****
      character title*56,ans
      integer u
      common/idata/ title,angle(3),depth,hdiam,numrf,u(20,3),e(20),v(20)

      write(*,*) ' Enter Title (up to 56 characters)--'
      read(*,'(a56)') title

      write(*,1001)
1001 format(' Enter Bearing/Angle of U1 in degree = '\)
      read (*,*) angle(1)
      write(*,1011)
1011 format(' Enter Bearing/Angle of U2 in degree = '\)
      read (*,*) angle(2)
      write(*,1021)
1021 format(' Enter Bearing/Angle of U3 in degree = '\)
      read (*,*) angle(3)
c
      write(*,1002)
1002 format(' Overburden Depth (ft) = '\)
      read(*,*) depth
      write(*,1003)
1003 format(' Borehole diameter (in)= '\)
      read(*,*) hdiam

      write(*,2001)
2001 format(' Enter Number of Reliefs = '\)
      read(*,*) numrf
c
      do i = 1, numrf
      write(*,2002) i
2002 format(' For Relief ',i2,' --')

      do j = 1, 3
      write(*,2003) j
2003 format(' Deformation - U',i1,' = ' '\)
      read(*,*) u(i,j)
      enddo ! i loop

      write(*,2004)
2004 format(' Elastic Modulus = '\)
      read(*,*) e(i)
      write(*,2005)
2005 format(' Poisson's Ratio = '\)
      read(*,*) v(i)
      enddo !i loop

```

```

c
c   Saving input data.
c   write(*,3003)
3003 format(' Do you want to save the input data? (y or n) '\)
c   read(*,'(a)') ans
c   if(ans.eq.'y' .or. ans.eq.'Y') then
c       call fsave
c   endif
c
c   return
c   end
c   subroutine finput
c *****
c * Input data from a file *
c *****
c
c   integer u
c   character fname*12,title*56
c   common/idata/ title,angle(3),depth,hdiam,numrf,u(20,3),e(20),v(20)
c   common/infile / fname
c   character ans
c   logical ex
c
c   11 write(*,12)
c   12 format(' Enter file name : '\)
c   read(*,'(a12)') fname
c   write(*,101) fname
c   101 format(' The file name is : ', al2,
c   + ' Is this correct? (y* or n) '\)
c   read(*,'(a1)') ans
c   if((ans.eq.'n').or.(ans.eq.'N')) go to 11
c
c   inquire(file=fname,exist=ex)
c   if(ex) then
c       open(5,file=fname,status='old')
c       read(5,'(a56)') title
c       write(*,*) ' Title -- '
c       write(*,*) title
c   Angle of U1,U2and U3
c       read (5,'( 3f10.2)') (angle(i),i=1,3)
c       write(*,'( 3f10.2)') (angle(i),i=1,3)
c   Overburden Depth
c       read (5,'( f10.2)') depth
c       write(*,'( f10.2)') depth
c   Borehole diameter (in)
c       read (5,'( f10.2)') hdiam
c       write(*,'( f10.2)') hdiam
c
c   Number of Reliefs
c       read (5,'(i2)') numrf
c       write(*,'(i2)') numrf
c
c   do i = 1, numrf
c   For Relief--

```

```

c      Deformation--
      read (5,'(3i10)') (u(i,j), j=1,3)
      write(*,'(3i10)') (u(i,j), j=1,3)

c      Elastic Modulus
      read (5,'( e10.3)') e(i)
      write(*,'( e10.3)') e(i)
c      Poisson's Ratio
      read (5,'( f5.2)') v(i)
      write(*,'( f5.2)') v(i)
      enddo !i loop

c
      else
      write(*,*) ' File does not exist, Please check file name!'
      goto 11
      endif

c
      close(5)
      return
      end

      subroutine fsave
c *****
c * save input data into a file      *
c *****
      character title*56,fname*12
      integer u
      common/idata/ title,angle(3),depth,hdiam,numrf,u(20,3),e(20),v(20)
      common/infile / fname
      character ans
      logical ex

      11 write(*,*)
      write(*,*) ' Enter output file name ..'
      write(*,*) ' (For saving input data)'
      read(*,'(a12)') fname
      write(*,101) fname
      101 format(' The file name is : ', a12,
      + ' Is this correct? (y or n) '\)
      read(*,'(a)') ans
      if((ans.eq.'n').or.(ans.eq.'N')) go to 11

c
      inquire(file=fname,exist=ex)
      if(ex) then
      write(*,*) ' That file already exists. Do you want to replace it?'
      write(*,103)
      103 format(' Enter y or n '\)
      read(*,'(a)') ans
      if((ans.eq.'n').or.(ans.eq.'N')) go to 11
      endif

c      save input data into a file for later use.
      open(6,file=fname)
      write(6,'(a56)') title

```

```

c      Angles of U1, U2 and U3
      write(6,'( 3f10.2)') (angle(i),i=1,3)
c      Overburden Depth (ft)
      write(6,'( f10.2)') depth
c      Borehole diameter (in)
      write(6,'( f10.2)') hdiam
c      Number of Reliefs
      write(6,'(i2)') numrf
      do i = 1, numrf
c      For each relief--
c      Deformation--
      write(6,'(3i10)') (u(i,j), j=1,3)

c      Elastic Modulus
      write(6,'( el0.3)') e(i)

c      Poisson's Ratio
      write(6,'( f5.2)') v(i)
      enddo      !i loop
      close(6)
      return
      end

      subroutine calc
c *****
c *      calculations      *
c *****
      character title*56
      integer u
      common/idata/ title,angle(3),depth,hdiam,numrf,u(20,3),e(20),v(20)
      common/odata/ p,q,theta,setheta,s1,s2,s3,se1,se2,se3,nset,
#                  sum,err,rr,std,c11,c22,c12

c
      dtr = 3.1416/180.0
      rtd = 180.0/3.1416
c**      angle(1) = 90.
c**      angle(2) = 150.
c**      angle(3) = 30.
c
c      compute
c
      nset = numrf*3
c
      a1 = 0.
      a2 = 0.
      a3 = 0.
      a4 = 0.
      a5 = 0.
      a6 = 0.
      a7 = 0.
      a8 = 0.
      a9 = 0.
      a10= 0.
      a11= 0.

```

```

al2= 0.
sum = 0.0
n = 0
do i = 1, numrf
do j = 1, 3
n = n+1
uu = 0.000001*float(u(i,j)) + v(i)*hdiam*depth/e(i)
vv = uu*uu
tt = 2.*angle(j)* dtr

xx = hdiam/e(i)
yy = 2.*(1.-v(i)*v(i))

x1 = xx*(1. + yy*cos(tt))
x2 = xx*(1. - yy*cos(tt))
x3 = xx*(2. * yy*sin(tt))

a1 = a1 + x1 * x1
a2 = a2 + x1 * x2
a3 = a3 + x1 * x3
a4 = a4 + x1 * uu
a5 = a5 + x2 * x1
a6 = a6 + x2 * x2
a7 = a7 + x2 * x3
a8 = a8 + x2 * uu
a9 = a9 + x3 * x1
a10=a10 + x3 * x2
a11=a11 + x3 * x3
a12=a12 + x3 * uu

sum=sum + vv
enddo ! j-loop
enddo ! i-loop
c
ss = a1 *a6 *a11 + a5 *a10*a3 + a9 *a2 *a7
# -a9 *a6 *a3 - a5 *a2 *a11 - a1 *a10*a7

s1 =(a4 *a6 *a11 + a8 *a10*a3 + a12*a2 *a7
# -a12*a6 *a3 - a8 *a2 *a11 - a4 *a10*a7)/ss

s2 =(a1 *a8 *a11 + a5 *a12*a3 + a9 *a4 *a7
# -a9 *a8 *a3 - a5 *a4 *a11 - a1 *a12*a7)/ss

s3 =(a1 *a6 *a12 + a5 *a10*a4 + a9 *a2 *a8
# -a9 *a6 *a4 - a5 *a2 *a12 - a1 *a10*a8)/ss
c
c
c
compute P, Q AND THETA

s12 = 0.5*(s1+s2)
s22 = 0.5*(sqrt((s1-s2)**2 + 4*s3*s3))
p = s12 + s22
q = s12 - s22
theta = 0.5* atan(2.*s3 /(s1-s2)) * rtd

```

```

c
err = sum - (s1*a4 + s2*a8 + s3*a12)
std = sqrt(err/(nset-3))
rr = 1-err/sum
c11 = (a6 * a11 - a7 * a7)/ss
c22 = (a1 * a11 - a3 * a3)/ss
c12 = (a1 * a6 - a2 * a2)/ss
sel = std * sqrt(c11)
se2 = std * sqrt(c22)
se3 = std * sqrt(c12)
setheta = 0.5* atan(2.*se3/(p-q))*rtd

c
if(q .lt. p) then
temp = q
q = p
p = temp
theta=90. + theta
endif
return
end

subroutine tablein
c *****
c * output input-data in table form *
c *****
character title*56,fname*12
integer u
common/idata/ title,angle(3),depth,hdiam,numrf,u(20,3),e(20),v(20)
common/infile / fname
common/outfile/ pfile
dimension iang(3)
character ofile*8,pfile*12

c
open a file for output data/print-out.
ofile=fname(1:8)
nc=len_trim(ofile)
pfile=ofile(1:nc)//'.out'

open(7,file=pfile,status='unknown')
write(7,*) 'Input data for least-squares analysis'
write(7,'(1x,a56)') title
do i = 1, 3
iang(i) = ifix(angle(i))
enddo
write(7,*)
write(7,10) (iang(i),i=1,3)
10 format(' ( U1 is',i3,', U2 is',i4,', U3 is',i3 ' deg.)')
write(7,20)
20 format(' -----')
# '-----')
write(7,21)
21 format(' Relief ; Deformation, uin ; Ex10(6), psi |'
# ' Poisson's' )
write(7,22)

```

```

22 format('          |-----|          |'
# ' ratio'      )
write(7,23)
23 format('          |  U1  |  U2  |  U3  |          |'
#          )
write(7,20)

n = 0
do i = 1, numrf
write(7,55) i, (u(i,j), j=1,3), e(i)/10**6, v(i)
55 format(i2, '.....', 3(2x, i5, 1x), 2x, f10.3, 5x, f5.2)
enddo
write(7,20)
write(7,65) depth
65 format(' Overburden depth, ', f10.0, ' ft.')
write(7,*)
return
end

subroutine table
c *****
c * output in table form *
c *****
character title*56, pfile*12
integer u
common/idata/ title, angle(3), depth, hdiam, numrf, u(20,3), e(20), v(20)
common/outfile/ pfile
common/odata/ p, q, theta, setheta, s1, s2, s3, sel, se2, se3, nset,
# sum, err, rr, std, c11, c22, c12
c
write(7,*) 'Results from least-squares plane-strain solution'
write(7, '(1x,a56)') title
write(7,1000)
1000 format(' -----')
write(7,1001) ifix(p), ifix(q), theta, setheta
1001 format(' P.....psi.. ', i10, /
# ' Q.....psi.. ', i10, /
# ' Theta.....deg.. ', f10.0, /
# ' SE Theta...deg.. ', f10.0 )

c Stress components with respect to X,Y coordinate system
write(7,1002) ifix(s1), ifix(s2), ifix(s3)
1002 format(' Sigma X....psi.. ', i10, /
# ' Sigma Y....psi.. ', i10, /
# ' Tau XY.....psi.. ', i10 )

write(7,1003) sel, se2, se3
c Standard Deviations of Stress Components-
1003 format(' SE (X)....psi.. ', f10.0 /
# ' SE (Y)....psi.. ', f10.0 /
# ' SE (XY)....psi.. ', f10.0 )

write(7,1004) nset
1004 format(' n..... ', i10)

```

```

        write(7,1005) sum,err,rr,std
c    Presicion of Data-
1005 format(' Sum u2.....in.. ',e10.4,/
#         ' Sum e2.....in.. ',e10.4,/
#         ' R2.....in.. ',f10.4,/
#         ' Se.....in.. ',e10.4 )
        write(7,1006) c11,c22,c12
1006 format(' C11..... ',e11.6,/
#         ' C22..... ',e11.6,/
#         ' C12..... ',e11.6 )
        write(7,1000)
        write(7,*)
c
c    print out explanations
c
        write(7,2001)
2001 format(' P      Maximum secondary principal compressive stress.'/
#         ' Q      Minimum secondary principal compressive stress.'/
#         ' Theta  Positive angle measured counterclockwise from '
#         ' U1 to Q.'/
#         ' SE      Standard error.'/
#         ' SigmaX  Stress component, east, with respect to x-y, '
#         ' coordinates system.'/
#         ' SigmaY  Stress component, north, with respect to x-y, '
#         ' coordinates system.'/
#         ' Tau XY  Shear stress component.'/
#         ' n      Number of deformation measurements.'/
#         ' Sum u2  Deformation sum of squares.'/
#         ' Sum e2  Residual sum of squares.'/
#         ' R2      Multiple correlation coefficient squared.'/
#         ' Se      Standard error at 95% confidence level.'/
#         ' C11, C22, C12  Constants from least-squares '
#         ' calculation.' )
c
        write(*,5000) pfile
5000 format(// ' The results of least-squares calculation has been'
#         ' saved '/' into " ' ,a12, '" file.')
c
        close(7)
        return
        end

```

Sample data for lease-squares calculation

Apex No.2	Mine - Site 1	Hole 1	in floor
90.00	150.00	30.00	
-420.00			
1.50			
4			
-217	-161	-268	
.292E+07			
.26			
-132	-186	-213	
.281E+07			
.26			
-148	-161	-184	
.275E+07			
.26			
-361	-99	-209	
.265E+07			
.26			

Sample output data from least-squares calculation

Input data for least-squares analysis

Apex No.2 Mine - Site 1 Hole 1 in floor

(U1 is 90, U2 is 150, U3 is 30 deg.)

Relief	Deformation, uin			Exl0(6), psi	Poisson's ratio
	U1	U2	U3		
1.....	-217	-161	-268	2.920	.26
2.....	-132	-186	-213	2.810	.26
3.....	-148	-161	-184	2.750	.26
4.....	-361	-99	-209	2.650	.26

Overburden depth, -420. ft.

Results from least-squares plane-strain solution

Apex No.2 Mine - Site 1 Hole 1 in floor

```

P.....psi..      -257
Q.....psi..      -213
Theta.....deg..    59.
SE Theta...deg..    16.
Sigma X....psi..    -224
Sigma Y....psi..    -246
Tau XY....psi..     -19
SE (X)....psi..     22.
SE (Y)....psi..     22.
SE (XY)....psi..    14.
n.....           12
Sum u2.....in..    .8253E-06
Sum e2.....in..    .4042E-07
R2.....           .9510
Se.....in..        .6702E-04
C11.....          .112500E+12
C22.....          .112501E+12
C12.....          .410777E+11

```

```

P      Maximum secondary principal compressive stress.
Q      Minimum secondary principal compressive stress.
Theta  Positive angle measured counterclockwise from U1 to Q.
SE     Standard error.
SigmaX Stress component, east, with respect to x-y, coordinates system.
SigmaY Stress component, north, with respect to x-y, coordinates system.
Tau XY Shear stress component.
n      Number of deformation measurements.
Sum u2 Deformation sum of squares.
Sum e2 Residual sum of squares.
R2     Multiple correlation coefficient squared.
Se     Standard error at 95% confidence level.
C11, C22, C12 Constants from least-squares calculation.

```

APPENDIX G.—COMPUTER CODE FOR DETERMINING THREE-DIMENSIONAL STATE OF STRESS

By Dennis R. Dolinar¹ and Meng-Cherng Sun¹

```

$INCLUDE: 'FGRAPH.FI'
      program plot3d
c *****
c * Program plots 3-D principal stresses      *
c * By Dennis R. Dolinar                      *
c *****
c * Programmed for PC by Meng-Cherng Sun      *
c * Date: 10/12/1991                          *
c *****
c
      integer*2 v1,v2,v3,v4
      common/idata/ title,s(3),b(3),c(3)
      common/pdata/ x(3),y(3),z(3)
      common/rdata/ r1(3),r2(3),r3(3)
      common/vpor/ v1,v2,v3,v4
      common/wind/ w1,w2,w3,w4
      character title*56,ans
      common/mdata/ q3,q4
c
      write(*,99)
      99 format(' Program plots 3-D Principal Stresses '
+ 'and their directions --' )
      write(*,100)
      100 format(' Input from keyboard or file? (k or f*): '\)
      read (*,'(a)') ans
      if (ans .eq. 'k' .or. ans .eq. 'K') then
        call kinput
      else
        call finput
      write(*,200)
      200 format(' Are these data O.K.? (y* or n): '\)
      read (*,'(a)') ans
      if (ans .eq. 'n' .or. ans .eq. 'Y') then
        write(*,*) ' Enter input data from keyboard again!'
        call kinput
      endif
c      END OF INPUT DATA
      endif
c
      call calc
c
      find max. r's

```

¹Mining engineer, Denver Research Center, U.S. Bureau of Mines, Denver, CO.

```

c
    smax = abs(r1(1))
    do j = 1, 3
        smax = max(smax,abs(r1(j)))
        smax = max(smax,abs(r2(j)))
        smax = max(smax,abs(r3(j)))
    end do
c*****DEBUG write *****
c    write(*,*) 'smax=',smax
c*****
c
c    set window for plot based on smax
c
c    q1 = smax
    call round(smax,anice)
c    q2 = float(ifix(q1/anice))
    q3 = anice
    q4 = q3
c*****DEBUG WRITE *****
c    write(*,*) smax,anice,q3,q4
c*****
c
    w1 = -q3 - q3*0.15      ! min.x.w
    w2 =  q3 + q3*0.15      ! max.x.w
    w3 = -q3 - q3*0.15      ! min.y.w
    w4 =  q3 + q3*0.15      ! max.y.w
c*****debug*****
c    write(*,2001) w1,w2,w3,w4
c 2001 format(' wxmin = ',f10.2, ' wxmax = ',f10.2/
c    +          ' wumin = ',f10.2, ' wymax = ',f10.2)
c    write(*,*) ' Hit return key to continue'
c    read(*,*)
c*****
c
c    define viewport on screen
c
    v1 = 100                ! min.x.v
    v2 = 640 - 100          ! max.x.v
    v3 = 20                 ! min.y.v
    v4 = 480 - 20           ! max.y.v
c
c    call initplot
c
c    call clearscreen( $GCLEARSCREEN )
c    call window
c
c    call plot
c
c    call closeplot
c
c    write(*,*) ' Program stops!'
end

```

```

      subroutine kinput
C *****
C * input data from keyboard      *
C *****
      character title*56,ans
      common/idata/ title,s(3),b(3),c(3)

      write(*,*) ' Enter Title (up to 56 characters)--'
      read(*,'(a56)') title
      write(*,*) ' Enter stresses and their directions--'
C
      do j=1,3
      write(*,1001) j
1001 format(' Principal Stress - S',il,' = ' \)
      read(*,*) s(j)
      write(*,1002)
1002 format(' Bearing = ' \)
      read(*,*) b(j)
      write(*,1003)
1003 format(' Inclination = ' \)
      read(*,*) c(j)
      write(*,*)
      end do
C      SAVING INPUT DATA
      WRITE(*,2003)
2003 FORMAT(' Do you want to save the input data? (y or n) '\)
      read(*,'(a)') ans
      if(ans.eq.'y' .or. ans.eq.'Y') then
      call fsave
      endif
      return
      end
      subroutine finput
C *****
C * Input data from a file      *
C *****
C
      common /idata/ title,s(3),b(3),c(3)
      character ans,fname*12,title*56
      logical ex

      11 write(*,12)
      12 format(' Enter file name : '\)
      read(*,'(a12)') fname
      write(*,101) fname
101 format(' The file name is : ', al2,
+ ' Is this correct? (y* or n) '\)
      read(*,'(a1)') ans
      if((ans.eq.'n').or.(ans.eq.'N')) go to 11
      inquire(file=fname,exist=ex)
      if(ex) then
      open(5,file=fname,status='old')
      read(5,'(a56)') title
      write(*,*) ' Title -- '

```

```

        write(*,*) title
        do j = 1, 3
            read (5,102) s(j),b(j),c(j)
            write(*,102) s(j),b(j),c(j)
        end do
102 format(3f10.2)
c
        else
            write(*,*) ' File does not exist, Please check file name!'
            goto 11
        endif
c
        return
    end

    subroutine fsave
c *****
c * save input data into a file      *
c *****
        common /idata/ title,s(3),b(3),c(3)
        character ans,fname*12,title*56
        logical ex

11 write(*,*)
        write(*,*) ' Enter output file name ..'
        write(*,*) ' (For saving input data)'
        read(*,'(a12)') fname
        write(*,101) fname
101 format(' The file name is : ', al2,
+ ' Is this correct? (y or n) '\)
        read(*,'(a)') ans
        if((ans.eq.'n').or.(ans.eq.'N')) go to 11
c
        inquire(file=fname,exist=ex)
        if(ex) then
            write(*,*) ' That file already exists. Do you want to replace it?'
            write(*,103)
103 format(' Enter y or n '\)
            read(*,'(a)') ans
            if((ans.eq.'n').or.(ans.eq.'N')) go to 11
        endif
c
        save input data into a file for later use.
        open(5,file=fname)
        write(5,'(a56)') title
        do j=1, 3
            write(5,104) s(j),b(j),c(j)
        end do
104 format(3f10.2)
        return
    end

    subroutine calc
c *****
c * calculations *
c *****

```

```

common/idata/ title,s(3),b(3),c(3)
common/pdata/ x(3),y(3),z(3)
common/rdata/ r1(3),r2(3),r3(3)
character title*56
c
c   compute directions
c
  do j = 1, 3
    s(j)=-s(j)
    p = (90.0 - b(j))*3.1416/180.
    t = (180.0 - c(j))*3.1416/180.
    x(j) = s(j)*sin(t)*cos(p)
    y(j) = s(j)*sin(t)*sin(p)
    z(j) = s(j)*cos(t)
  end do
c
c   compute stresses and their directions
c
  do j = 1, 3
    r1(j) = x(j) + y(j)*sin(45.*3.1416/180.)
    r2(j) = y(j)*sin(45.*3.1416/180.)
    r3(j) = z(j)
  end do
c
  return
end

subroutine plot
c *****
c * plots stress and their directions *
c *****
c * Programmer: Meng-Cherng Sun *
c * Date: 12/12/1991 *
c *****
c
  INCLUDE 'FGRAPH.FD'
c
  integer*2 v1,v2,v3,v4,dummy
  common/vpor/ v1,v2,v3,v4
  common/wind/ wxmin,wxmax,wymmin,wymax
  common/idata/ title,s(3),b(3),c(3)
  common/pdata/ x(3),y(3),z(3)
  common/rdata/ s1(3),s2(3),s3(3)
  common/mdata/ q3,q4
  dimension xp(2),yp(2)
  character str*10,cno,units*12
  character title*56
c
c   fonts/character size in window coordinates
c
  ich=12
  icw=9
  call getcsz_w(ich,icw,ycw,xcw)
c

```

```

c   write Title
c
c   call label_tx(1,14,5,title)
c
c   Draw a box around the graph
c
c   dummy = setcolor(1)
c   dummy = rectangle_w($GBORDER, wxmin,wymn,wxmax,wymax)
c
c   draw axes
c
c   vertical axis
c   xp(1)=0.
c   yp(1)=0.
c   xp(2)=0.
c   yp(2)=q4
c   dummy = setcolor(7)
c   call lines(xp, yp, 2)
c   str = 'Vertical'
c   call label_gtx(xp(2)-4*xcw,yp(2)+2*ycw,6,ich,icw,4,str)
c
c   E - axis
c   xp(2)=q3
c   yp(2)=0.
c   dummy = setcolor(7)
c   call lines(xp, yp, 2)
c   str = 'E'
c   call label_gtx(xp(2)+0.5*xcw,yp(2)+0.5*ycw,6,ich,icw,4,str)
c
c   S - axis
c   xp(2)=-q4
c   yp(2)=-q4
c   dummy = setcolor(7)
c   call lines(xp,yp,2)
c   str = 'S'
c   call label_gtx(xp(2)-xcw,yp(2),6,ich,icw,4,str)
c
c   N - axis - dashed line
c   xp(2)=q4
c   yp(2)=q4
c   dummy = setcolor(7)
c   call setlinestyle (#F0F0)
c   call lines(xp,yp,2)
c   str = ' N'
c   call label_gtx(xp(2),yp(2)+0.5*ycw,6,ich,icw,4,str)
c
c   W - axis - dashed line
c   xp(2)=-q3
c   yp(2)=0.
c   dummy = setcolor(7)
c   call lines(xp,yp,2)
c   str = 'W'
c   call label_gtx(xp(2)-1.5*xcw,yp(2)+0.5*ycw,6,ich,icw,4,str)
c   call setlinestyle (#FFFF)

```

```

c
c   Now plot stresses and directions
c
    ich=10
    icw=5
    call getcsz_w(ich,icw,ycw,xcw)
c
    do j = 1,3
        r1 = s1(j)
        r2 = s2(j)
        r3 = s3(j)
c
c   Draw dashed line
        xp(1)=x(j)
        yp(1)=0.
        xp(2)=r1
        yp(2)=r2
        dummy = setcolor(2)
        call setlinestyle(#F0F0)
        call lines(xp,yp,2)
c   Draw dashed line for
        xp(1)=r1
        yp(1)=r2
        xp(2)=r2
        yp(2)=r2
        call lines(xp,yp,2)
c   Draw dashed line for
        xp(1)=r1
        yp(1)=r2
        xp(2)=r1
        yp(2)=r2+r3
        call lines(xp,yp,2)
        call setlinestyle(#FFFF)
c   Draw solid line for s
        xp(1)=0.
        yp(1)=0.
        xp(2)=r1
        yp(2)=r2+z(j)
        call lines(xp,yp,2)
        write(cno,'(il)') j
        str = 'S'//cno
        call label_gtx(xp(2)-xcw,yp(2)+ycw,4,ich,icw,4,str)
    end do
c
c   Draw scale and units
c
    scale=1000
    units=' Scale, psi'
    xscale=wxmax-1000.-3.*xcw
    yscale=wymn+4* ycw
c   Draw a line for scale
        xp(1)=xscale
        yp(1)=yscale
        xp(2)=xp(1)+scale

```

```

        yp(2)=yp(1)
        dummy = setcolor(7)
        call lines(xp,yp,2)
c      tic mark - 0
        xp(1)=xscale
        yp(1)=yscale
        xp(2)=xp(1)
        yp(2)=yp(1)+ycw/2.
        dummy = setcolor(7)
        call lines(xp,yp,2)
        str='0'
        call label_gtx(xp(2)-xcw,yp(2)+ycw,6,ich,icw,2,str)
c      tic mark - scale=1000
        xp(1)=xp(1)+scale
        yp(1)=yscale
        xp(2)=xp(1)
        yp(2)=yp(1)+ycw/2.
        dummy = setcolor(7)
        call lines(xp,yp,2)
        str='1000'
        call label_gtx(xp(2)-2*xcw,yp(2)+ycw,6,ich,icw,2,str)
c      print units
        call label_gtx(xscale,yscale-0.5*ycw,4,ich,icw,4,units)
c
        return
        end

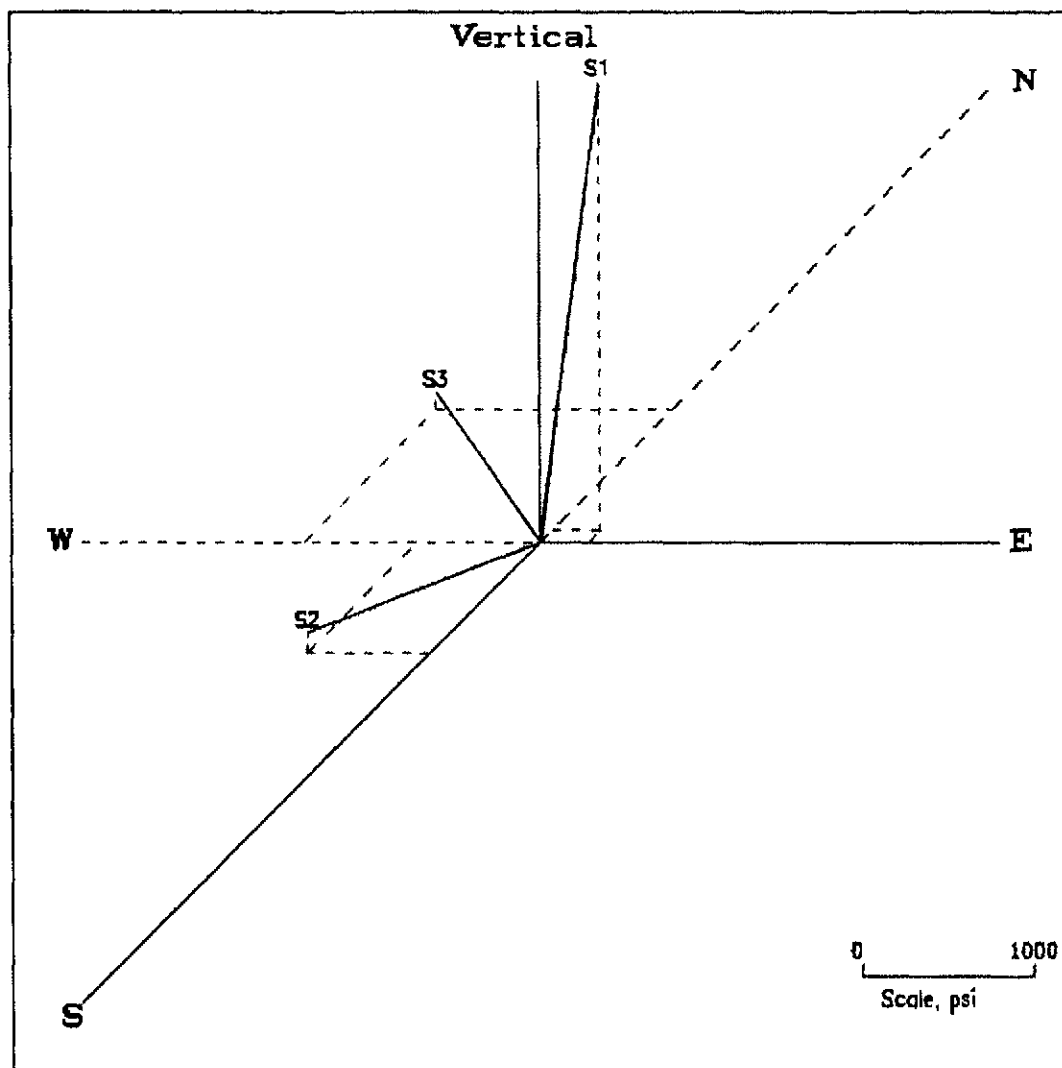
```

Sample data for plotting 3-D principal stresses

Urad Mine Principal Stresses for Site 1.

-2643.00	70.40	173.40
-1169.00	218.00	95.50
-1769.00	308.30	93.50

Urad Mine Principal Stresses for Site 1



APPENDIX H.—SYMBOLS AND ABBREVIATIONS USED IN THIS BULLETIN

NOTE.—This list does not include the unit of measure abbreviations listed at the front of this bulletin.

A	matrix	E_3	axial modulus of elasticity
a	inner radius of hole	G.F.	gage factor
A, B	constants (appendix A, part B)	g_j	sum of deformation $\times J_j$, where $j = 1, 2, \dots, 6$
A, B, C	coefficients	$g_1(\theta)$	arbitrary function of θ only
a_{ij}	elements in matrix	h	height or depth of overburden
ASTM	American Society for Testing and Materials	h_1, h_2, h_3	axes of rectangular coordinate system
B	theoretical or assumed value for overburden load (chapter 1)	ID	inside diameter
b	outer radius of core; cantilever width (chapter 2); value furnished with rosette strain gage (chapter 4)	J_i	distinct set of values corresponding to each distinct orientation of U
BHG	borehole deformation gage	K, L	constants
b_i	stress components	L	cantilever length (chapter 2)
D	determinant	l, m, n	direction cosines
d	diameter of hole	n	number of measurements
E	modulus of elasticity (Young's modulus)	OD	outside diameter
e	residual about regression line; error sum of squares of residual	P	maximum secondary principal stress
e_j	error in each equation	p	number of b_i
e_r	radial strain	PH	precipitation hardening
e_θ	tangential strain (appendix A)	P_o	radial load
E_1	maximum modulus along axes of anisotropy	P_3	axial load
e_1	actual axial strain component	Q	minimum secondary principal stress
E_2	minimum modulus along axes of anisotropy	R	multiple correlation coefficient
e_2	actual lateral strain component	r	radial distance
		R_1	apparent axial strain
		R_2	apparent lateral strain

S	stress component (chapter 5)	δ	angle measured counterclockwise from E_1 to P
SD	standard deviation	ϵ	strain; true error of U measurement (chapter 1); maximum strain (chapter 2)
S_E	standard error of data about regression	ϵ_{θ_1}	tangential strain along 31 axis of anisotropy
S_o	shear strength	ϵ_{θ_2}	tangential strain along 32 axis of anisotropy
S, T	perpendicular applied stresses (chapter 1 and appendix A)	ϵ_3	axial strain
S_x	normal stress in X direction	ϵ_{31}	strain along 31 axis of anisotropy
S_y	normal stress in Y direction	ϵ_{32}	strain along 32 axis of anisotropy
S_{θ_1}	minimum probable error in original angle θ_1	θ	angle measured counterclockwise from S to r and E_1 to U_1
S_1, S_2, S_3	principal stresses	θ_1	original angle measured counterclockwise from S to U_1 or P to U_1
S_{31}, S_{32}	compliance coefficients	$\lambda_{r\theta}$	shearing strain
t	cantilever thickness	ν_{31}	Poisson's ratio along 31 axis of anisotropy
t_α	table value of Student's t -statistic for desired probability α	ν_{32}	Poisson's ratio along 32 axis of anisotropy
U	diametral deformation of hole	σ	stress
u	radial displacement	σ_h	horizontal stress
U_1, U_2, U_3, U_4	diametral deformation measurements	σ_p	radial pressure
v	tangential displacement	σ_r	radial stress
W	force at end of cantilever	σ_v	vertical stress
X, Y, Z	global coordinate system	$\sigma_x, \sigma_y, \sigma_z$	three normal stresses
x, y, z	local coordinate system	σ_θ	tangential stress
X_1, X_2	elastic axes	σ_3	axial stress
y	cantilever deflection	$\tau_{r\theta}$	shearing stress
α_{11}, α_{22}	anisotropic parameters		
β	population parameter		
γ	density of overburden		

τ_{xy}	shear stress in X-Y plane	3-C BHG	three-component borehole deformation gage
τ_{yz}	shear stress in Y-Z plane	4-C BHG	four-component borehole deformation gage
τ_{zx}	shear stress in Z-X plane	6-C BHG	six-component borehole deformation gage
$\tau_{12}, \tau_{13}, \tau_{23}, \tau_{31}$	shearing stress components		
1j, 2j, 3j	different diametral deformations (chapter 1)		
2-C BHG	two-component borehole deformation gage		