

BIBLIOGRAPHIC DATA
SHEET

1. Report No.

BuMines IC 8571

2.

PB220495



4. Title and Subtitle

Subsidence Due to Underground Mining (In Two Parts). 1.
Theory and Practices in Predicting Surface Deformation

March 1973

6. Performing Organization Code

7. Author(s)

Gerhard Braeumer

8. Performing Organization Rept.
No.

BuMines IC

9. Performing Organization Name and Address

Department of Mining Engineering
Michigan Technological University
Houghton, Michigan

10. Project/Task/Work Unit No.

11. Contract/Grant No.

12. Sponsoring Agency Name and Address

Denver Mining Research Center
Bureau of Mines, U.S. Department of the Interior
Building 20, Denver Federal Center
Denver, CO 8022513. Type of Report & Period
Covered Information
Circular

14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstracts

The report gives a review of the methods of predicting mine subsidence that are presently used, particularly in European coalfields. The methods are subdivided into two groups, based on mathematical expressions either for the trough profile or for the influence of infinitesimal extraction elements. The fundamental relationships common to all methods described herein are the principles of superposition and of equivalent extraction areas. The involved parameters are derived from field observations. For calculating horizontal displacements, their relation to the slope of the subsidence profile is applied. The time-dependent development of the subsidence process is characterized by the empirical "time factor." Modifications for the case of inclined strata are considered. A brief review of experimental and theoretical model investigations is given.

17. Key Words and Document Analysis. 17a. Descriptors

Subsidence

Underground Mining

Surface Movement

Subsidence

Prediction

Surface Deformation, Predicting

Fundamental Methods

17b. Identifiers/Open-Ended Terms

17c. COSATI Field/Group 08I

18. Distribution Statement Sold by Sup. Docs., GPO for \$.85,
Stock No. 2404-01329: sold by NTIS in microfiche
only for \$.85. ~~1.4~~REPRODUCED BY:
U.S. Department of Commerce
National Technical Information Service
Springfield, Virginia 22161

NTIS

19. Security Class (This
Report)

UNCLASSIFIED

20. Security Class (This
Page)

UNCLASSIFIED

21. No. of Pages

56 62

22. Price

Information Circular 8571

Subsidence Due to Underground Mining

(In Two Parts)

1. Theory and Practices in Predicting Surface Deformation

By Gerhard Brauner

Denver Mining Research Center, Denver, Colo.



UNITED STATES DEPARTMENT OF THE INTERIOR
Rogers C. B. Morton, Secretary

BUREAU OF MINES
Elburt F. Osborn, Director

This publication has been cataloged as follows:

Bräuner, Gerhard

Subsidence due to underground mining (in two parts).

1. Theory and practices in predicting surface deformation.
[Washington] U.S. Dept. of the Interior, Bureau of Mines [1973]

56 p. illus. (U.S. Bureau of Mines. Information circular 8571)

Includes bibliography.

1. Subsidence (Earth movements). 2. Mine safety. I. Title.
II. Title: Theory and practices in predicting surface deformation.
(Series)

TN23.U71 no. 8571 622.06173

U.S. Dept. of the Int. Library

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SUBSIDENCE DUE TO UNDERGROUND MINING

(In Two Parts)

1. Theory and Practices in Predicting Surface Deformation

by

Gerhard Bräuner¹

ABSTRACT

The report gives a review of the methods of predicting mine subsidence; these methods are used particularly in European coalfields. The methods are subdivided into two groups, based on mathematical expressions either for the trough profile or for the influence of infinitesimal extraction elements. The fundamental relationships common to all methods described herein are the principles of superposition and of equivalent extraction areas. The involved parameters are derived from field observations. For calculating horizontal displacements, their relation to the slope of the subsidence profile is applied. The time-dependent development of the subsidence process is characterized by the empirical "time factor." Modifications for the case of inclined strata are considered. A brief review of experimental and theoretical model investigations is given.

INTRODUCTION

Surface movements due to mining are a major problem, especially when stratified deposits are completely extracted. Even then, they vary widely, depending on the mining and strata conditions. Mining thick seams or groups of seams under soft rock sometimes induces intense cracking or caving, whereas mining in virgin areas under hard rock or in steep measures at larger depths may cause no appreciable movements at all. The normal case, however, is a relatively continuous surface deformation which is definitely measurable as vertical and horizontal displacements of surface points. This case has especially been investigated in European coalfields, so that the surface deformation can be precalculated to some extent. The precalculation methods which are used in present practice are based on these investigations, and they refer to (1) stratified deposits, mainly coal seams, (2) complete extraction, and (3) extraction areas the sizes and depths of which are large compared with the seam thickness. This report will deal with the principles of these methods, which are generally based on certain empirical statements on the relations between surface deformation and underground extraction. Some

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fundamental quantities, like the maximum possible subsidence, can then be determined directly. If these empirical relationships are generalized, the complete deformation profiles can be calculated, subject to the following restrictions:

1. Geometrically simple extraction areas, particularly rectangles;
 2. The final state of the movement process;
- and 3. Horizontal stratification.

General aspects of the problem involving (1) extraction areas of any shape, (2) temporary states of the movement process, and (3) inclined stratification are treated by means of more extensive generalizations and simplifying assumptions. The empirical findings are rather uniform in most mining districts, but there is a considerable variety of mathematical approaches, which are used as complementary relationships and reflect the different local conditions. The part of the problem that has been treated most extensively is the terminal state of surface movement in the case of flat stratification. The next paragraphs will deal with this case.

ACKNOWLEDGMENTS

This report was prepared for the Denver Mining Research Center, Denver, Colo., of the Bureau of Mines, U.S. Department of the Interior, under Grant Agreement G0110427 with the Michigan Technological University, Houghton, Mich. The Project Director was Walfrid Been, Professor and Department Head, Department of Mining Engineering.

FUNDAMENTAL RELATIONSHIPS

The Subsidence Trough

In case of a single extraction area in a flat seam, the final state of surface deformation is characterized by the following features:

1. The vertical displacements or "subsidence" of surface points form a "subsidence trough" which normally extends, in plan view, beyond the edges of the mined area in all directions.
2. The subsidence is accompanied by horizontal displacements which are appreciably larger than could be expected from only the curvature due to the subsidence. Their magnitudes and directions are approximately proportional to the slope of the subsidence profiles.
3. If the extraction area has a geometrically regular shape, the maximum subsidence occurs over the center. Horizontal displacement of that point is zero and the whole distribution of vertical and horizontal displacements is symmetrical. Some observations, however, have indicated that the slopes and horizontal displacements in the direction of the working advance are different from those in the opposite direction, but this difference is normally disregarded.

Concerning surface damage, the derivatives of the displacement curves are of special interest. They are (1) the slopes of subsidence profiles, (2) the curvatures of subsidence profiles, and (3) the strains--extensions and compressions--which result from the horizontal displacements.

The derivatives will be dealt with here only briefly, because they are described by the magnitudes and directions of the displacements. It is convenient to consider the vertical and the horizontal displacement components separately, because the vertical component, the subsidence, has a definite upper limit which depends on the seam thickness and is an important basis of calculation. This limit value is referred to as "maximum possible subsidence" or "full subsidence." It only occurs if the extraction area has a certain minimum size, the so-called "critical area." The critical area produces full subsidence at but one surface point. Areas that produce full subsidence at more points are super-critical; areas that produce no full subsidence at all are subcritical. Figure 1 illustrates these three extraction cases. If the length of the extraction area perpendicular to the plane of the figure is large enough, a state of approximately plane deformation exists in the central region of the trough, the main features are as follows:

In case of the critical area (fig. 1A), the maximum subsidence at the center is associated with zero curvature and zero strain. The horizontal displacements reach their maximum possible values in the region over the ribsides.

Basic Empirical Relationships and Their Generalizations

The fundamentals of the precalculation methods can be reduced to two empirical statements, both of them being rather well proven by measurement, and their generalizations.

The first statement is that the subsidence is proportional to the seam thickness extracted; that is, the full subsidence S_{max} is determined by the linear expression

$$S_{max} = a m, \quad (1)$$

where "a" is the "subsidence factor" and "m" is the seam thickness.² This principle does not apply in the above linear form if subsidence is due to the escape of water from the undermined strata, and it may be modified if older workings exist at a higher level in the affected rock mass.

The second statement is that the maximum subsidence over a subcritical area depends on the size of that area in relation to its depth. Considering the two-dimensional case, other quantities being equal, this means that all extraction areas having the same width-to-depth ratio produce the same maximum subsidence. This equivalence is illustrated in figure 2: Any extraction area determined by an isosceles triangle would cause the same subsidence at the top of the triangle, that is, at the surface point P over the extraction center.

²A list of the symbols used in this report is to be found in the appendix.

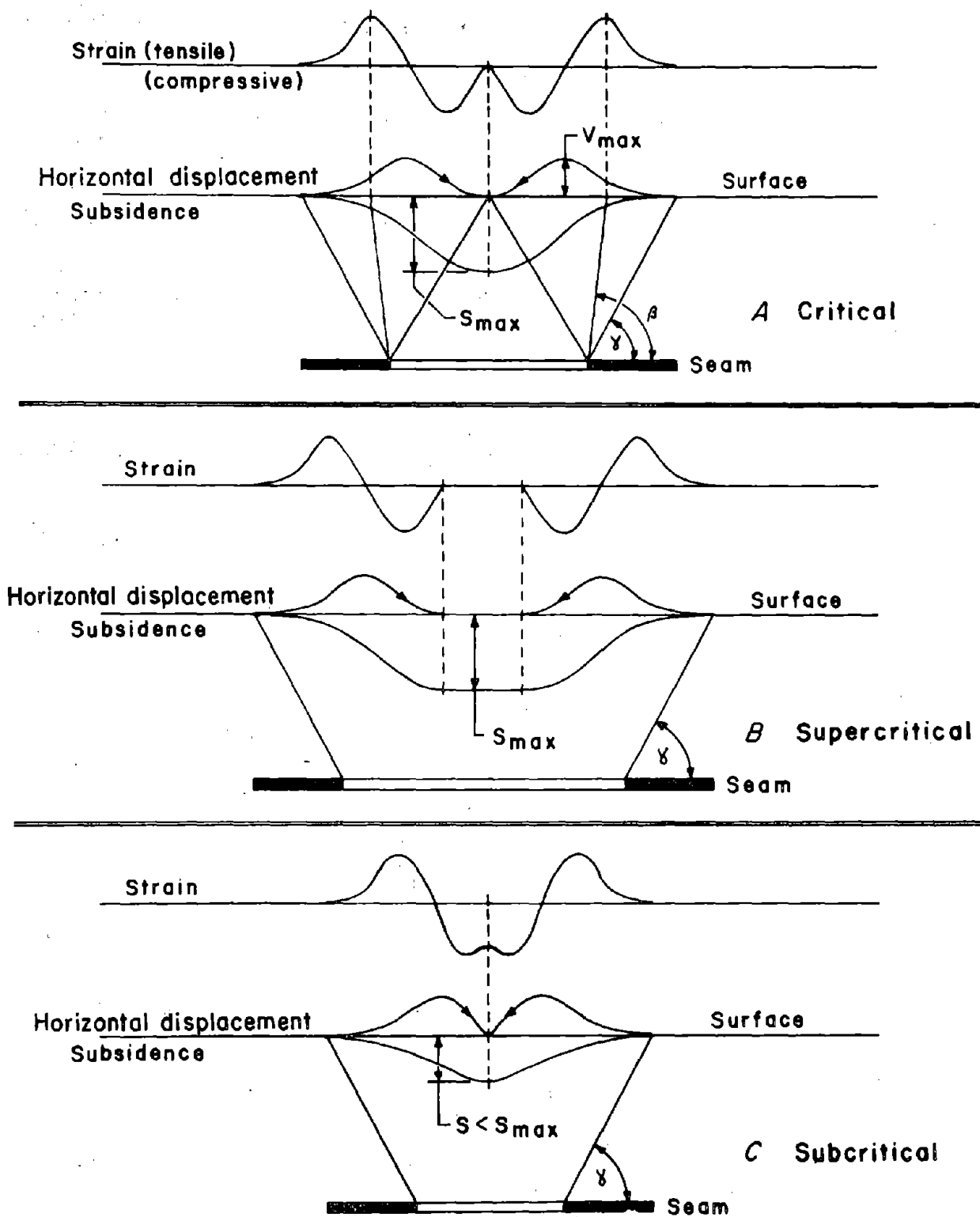


FIGURE 1. - Extraction Areas and Surface Movements.

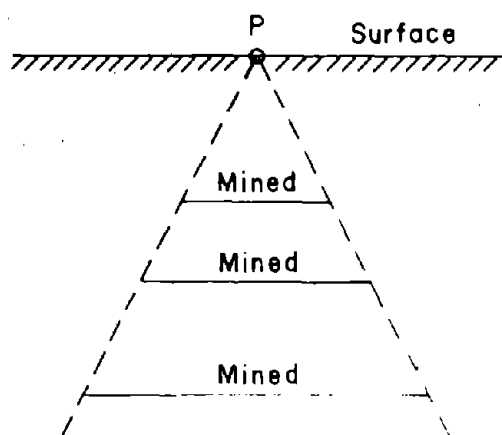


FIGURE 2. - Equivalent Subcritical Areas.

a subsidence s_1 . If only the area A_2 was mined instead of A_1 , the subsidence at P would be s_2 . The law of superposition means that mining both areas produces at P the subsidence $s = s_1 + s_2$.

This principle is true only approximately, since every subsidence process changes the consistency of the affected rock mass, and the response of that mass to a following extraction may become different. In many observation examples, however, the deviations have proven tolerable.

The linear case of equation 1 is only a special case of superposition: Let S_{max_1} be the full subsidence due to a single extraction of a seam having thickness m_1 and S_{max_2} the full subsidence due to a seam of thickness m_2 .

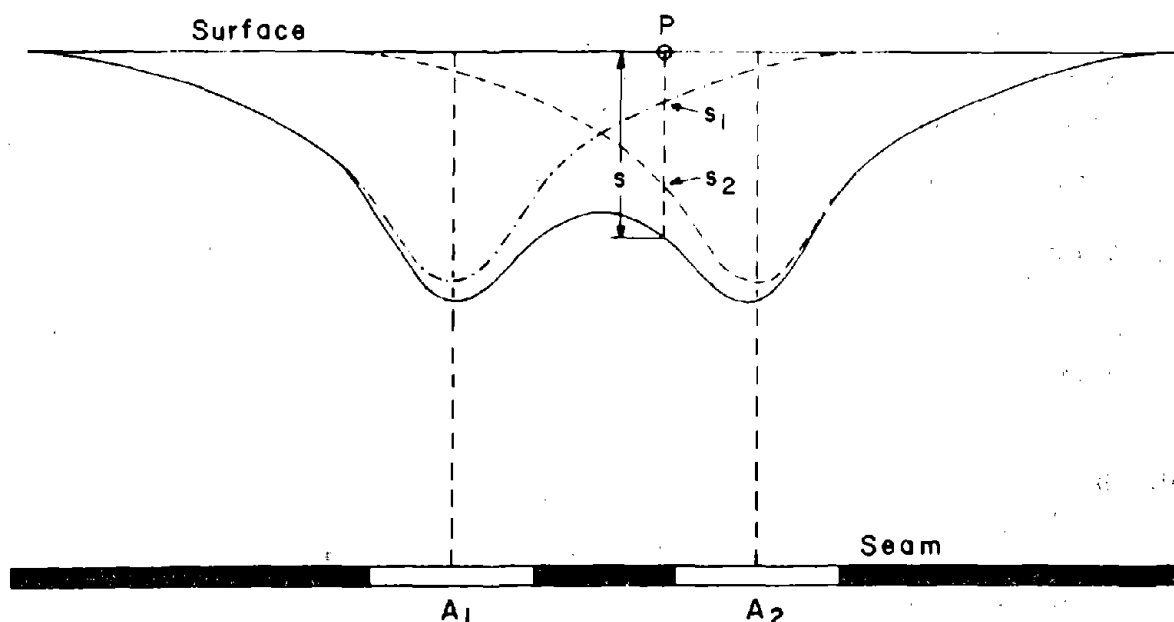


FIGURE 3. - Superposition of Surface Subsidences.

This relation becomes, of course, questionable if the upper strata are substantially different from the lower ones.

The first important generalization is the principle of undisturbed super-position of surface displacements caused by different extraction areas. This principle assumes that the displacements due to any extraction are not affected by the existence of displacements due to other extractions. Consequently, the displacements which result from more than one extraction area are simply the sum of the displacements that would occur if each area was mined alone. Figure 3 illustrates this principle: If the area A_1 was mined alone, a surface point P would undergo

Superposition of both subsidences would give the maximum subsidence

$$S_{\max 1} + S_{\max 2} = a m_1 + a m_2.$$

If the seams are so close together that they can be considered as one seam of thickness $m_1 + m_2$, the law of linearity gives for the maximum subsidence $a(m_1 + m_2)$, that is, the same result as given by the law of superposition.

The second important generalization is the principle of the equivalence of extraction areas in relation to a surface point. That is to say, any extraction area which is delimited by straight lines outgoing from the surface point produces the same displacements at that point (fig. 4). Considering the two-dimensional case, an extraction area A_1 may produce a certain percentage of full subsidence, say 10 percent at the surface point P. The same percentage would be caused by extracting a smaller area A_2 at a smaller depth h_2 , or a larger area A_3 at a correspondingly larger depth h_3 . In the three-dimensional case, equivalent extraction areas are determined by similar cones instead of similar triangles. This principle assumes that the rock properties which are responsible for the movement process do not substantially vary with the depth.

Calculation Principles and Parameters

Most precalculation methods refer to the final subsidences in case of approximately horizontal formations; some of them have been extended to horizontal displacements, temporary states, and inclined formations.

In the case of final subsidence with flat strata, the fundamental quantity is always the full subsidence as determined by equation 1 according to the principle of linearity. The subsidences of complete trough profiles are expressed as fractions of the full subsidence. The corresponding calculation methods apply the principle of equivalence and, additionally, either the law of superposition or certain more or less empirical alternatives. They are furthermore based on mathematical expressions of either (1) the subsidence profile caused by two-dimensional critical extraction areas, or (2) the influence of an infinitesimal element of extraction. Functions of the first kind

will be referred to as profile functions, those of the second kind as influence functions. Accordingly, the precalculation methods will be classified into two main groups.

For calculating displacements caused by more than one extraction area in one or more seams, the law

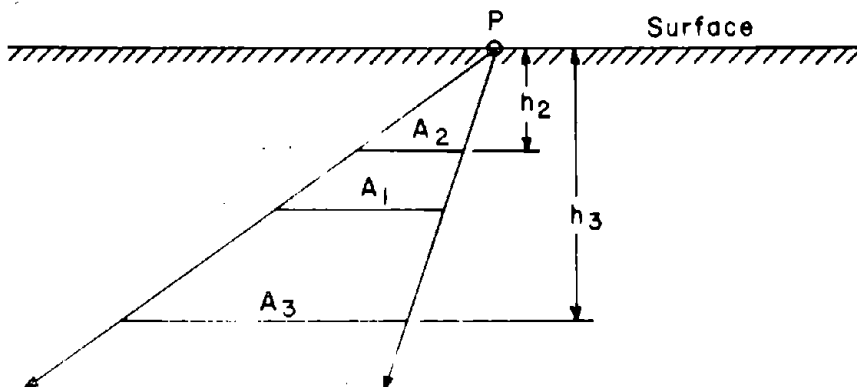


FIGURE 4. - Principle of Equivalence.

of superposition is used, that is, the effects of each working are calculated separately and then simply added up.

The quantitative parameters of calculation are the depth and the extracted thickness of the seam, the dimensions of the extraction area, and the parameters which are affected by or characteristic for the undermined strata. The latter parameters include the subsidence factor and the quantitative parameters of the profile functions or influence functions.

The subsidence factor is the ratio of full subsidence to seam thickness extracted. It is dependent on the kind of packing as well as on the mining method and should be determined by measuring the full subsidence over supercritical areas. Determinations from underground measurements of goaf convergence are more difficult and more liable to misinterpretation. Subsidence factors reported from different mining districts vary considerably especially the values for caving. This variation indicates a considerable influence of the particular strata conditions. Some values of the subsidence factor "a" are as follows:³

<u>Coalfield and method of packing</u>	<u>a</u>
British coalfields (39):	
Solid stowing.....	0.45
Caving or strip-packing.....	.90
Ruhr coalfield, Germany (14, 51):	
Pneumatic stowing.....	.45
Other solid stowing.....	.50
Caving.....	.90
North and Pas de Calais coalfield, France (17):	
Hydraulic stowing.....	0.25-0.35
Pneumatic stowing.....	.45- .55
Caving.....	.85- .90
Upper Silesia (25):	
Hydraulic stowing.....	.12
Caving.....	.70
U.S.S.R. (15): ¹	
Donets and Kizelov districts.....	.60
Kuznetsk and Karaganda districts.....	.70
Chelybinsk district.....	.90

¹Mining without stowing, which implies caving.

Observations in American coalfields indicate rather small subsidence factors for caving. From measurements in Pennsylvania (35), values between 0.50 and 0.60 can be derived. Earlier investigations have been described in reference 9.

³Underlined numbers in parentheses refer to items in the list of references at the end of this report.

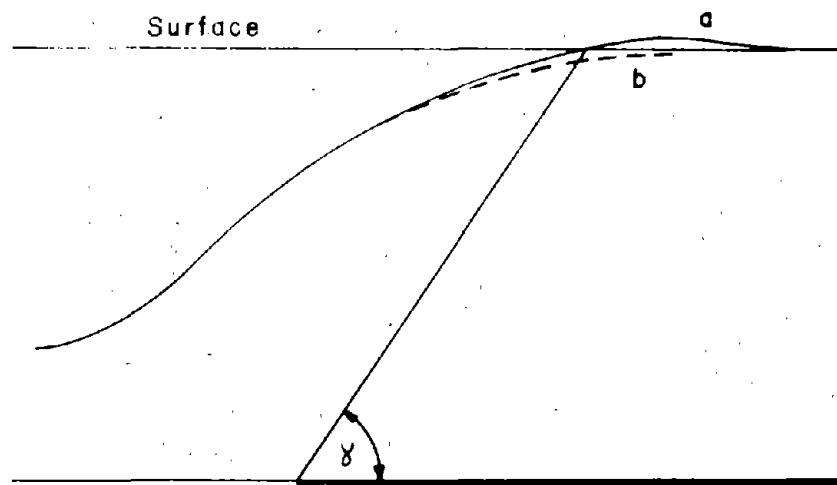
The effect of the extraction depth is uncertain. A Russian suggestion (15) is the empirical formula

$$S_{max} = \frac{25m}{25 + \sqrt{h}} \cos \alpha,$$

where α is the angle of dip and h is the dimensionless depth of the seam. The form of the equation indicates a decreasing subsidence factor with increasing depth h . Similar formulas have also been suggested for the sub-critical maximum. (See for example reference 9.) Other suggestions imply increasing subsidence factors with increasing depth (17, 40, 59). In most mining districts, the subsidence factor is treated as independent of depth.

The influence of strata conditions on the movement process is mainly expressed by the form of the profile function or influence function and by the quantitative parameters of these functions. The form of the function is normally chosen to fit the general features of the subsidence trough in a mining district. This procedure is comparable to the choice of a stress-strain equation for characterizing the rheological behavior of a material--in either case the experimental findings are expressed by a generalizing and simplifying equation. Local strata conditions are reflected quantitatively by the values of the functional parameters, the most important of which is the limit angle. As a material parameter, however, the limit angle is rather inadequate, at least in its usual meaning. It is normally understood as indicating either (1) the limit of movements causing damages to structures, (2) the limit of measurable movements, or (3) the beginning of zero movement.

The first definition implies that the limit angle may refer to the individual sensitivity of a structure and the second that it reflects the accuracy of measurement. In either case, it is not a pure rock characteristic. The third definition raises the problem of measurability, since zero quantities cannot be directly measured on principle. Indirect determination is possible only if there is a transition from subsidence to elevation at the edge of the



trough. Slight elevations of one centimeter or so have often been observed in that region; then the point of zero movement can be found by interpolation as illustrated by a in figure 5.

Measurable elevations, however, do not always occur. When taking a small finite subsidence (53), say 1 cm, as an arbitrary limit (b in figure 5), the limit angle varies

FIGURE 5. - Definitions of the Limit Angle.

with the thickness of the extracted seam: A point at which extraction of a 100-cm seam causes 1 cm subsidence will undergo 2 cm subsidence when the extracted thickness is 200 cm (principle of linearity) and the limit angle becomes smaller with increasing seam thickness. This may be the reason for variations in the order of $\pm 5^\circ$ which sometimes have been observed under apparently equal local conditions. Hence, the limit angle does not quite meet the elementary requirements of a material parameter, which are measurability and sole material-dependence.

This applies, of course, to most parameters that are used in structural rock mechanics, and may not be too serious in the case of the limit angle. An improvement, however, can be obtained when replacing the limit angle by a distance parameter which refers to certain percentages of full subsidence. Most convenient seems to be a distance approximately equal to the radius of the critical area. This parameter will be referred to as the critical radius and can be used more justifiably since it may be measured independently of the limit angle. It may be interpreted as the product of the extraction depth and the cotangent of the limit angle, but then the limit angle does not necessarily refer to the point of zero subsidence, and it normally varies with the thickness of the extracted seam. The value of the critical radius as a functional parameter can be found from any measured subsidence since any mathematical operation for calculating subsidence can be inverted for calculating functional parameters from measured subsidences. This will be illustrated in the following paragraph.

Other parameters that occur in calculation formulas must also normally be determined from measured subsidences by inverting the calculation formula. Only the angle of break is directly indicated by the maximum extension, but it is rarely used as a calculation parameter. Summarizing, the magnitudes of all precalculation parameters must be determined from measured displacements. If no data from observations are available, values found under similar geological conditions in other mining districts may provide a basis.

Average values of the limit angle have been reported from various coalfields, but they are not always comparable because of varying interpretation. Relatively small values of 35° (42) and 40° (56) have been reported in single cases. Most values range between 45° and 65° ; the corresponding values of the critical radius B and $0.5h$ and $1.0h$, respectively. Limit angles of 45° to 55° have been found in the Limburg coalfield, the Netherlands (12); of 45° to 60° in the Ruhr coalfield, Germany (14, 42); of 55° in Northern France (17), of 60° in most coalfields in the Soviet Union (15); and of 55° to 65° in Great Britain (40). In earlier reports (9), considerably larger angles are mentioned, even values of more than 90° ("negative draw"). Such unusual observations may be due to less accurate observation techniques. Relatively large values of about 70° are also found from measurements in Pennsylvania (35). The influence of noncarboniferous younger strata which overlie the coal measure rocks is sometimes expressed by different limit angles for the overburden and the carboniferous rock. Major faults are often predetermined planes of limit or of break; examples are described in references 9 and 42. The effects of old workings on the subsidence due to extraction of a deeper seam are sometimes allowed for by modifying limit angles and subsidence factors.

SUBSIDENCE CALCULATION THROUGH PROFILE FUNCTIONS

Principles

The distribution of displacements or their derivatives over two-dimensional critical extraction areas are mathematically expressed by profile functions. A subsidence profile function is an equation of one-half the subsidence profile, ranging from full subsidence to zero subsidence. The subsidence is expressed in terms of full subsidence and the location of the profile points

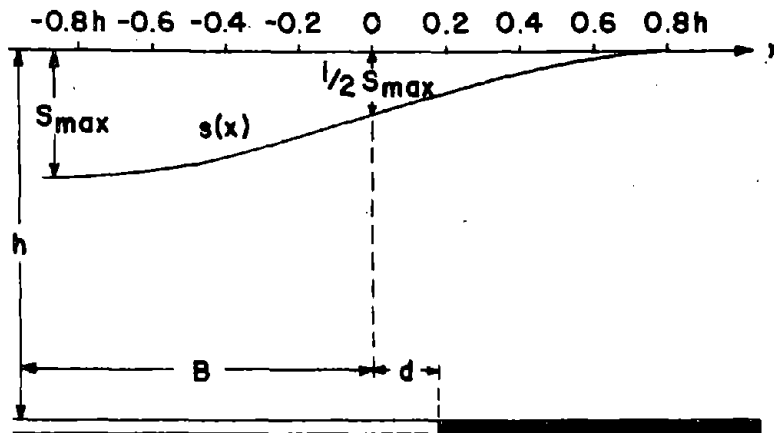


FIGURE 6. - Subsidence Profile Function.

in terms of extraction depth or a proportional quantity. In case of supercritical extraction, the central region of the trough has the constant subsidence S_{max} . In case of subcritical extraction, the particular subcritical profile is determined from the critical profile by using certain empirical and/or mathematical relations. The method is normally restricted to rectangular extraction areas in which the profile coincides with the centerline.

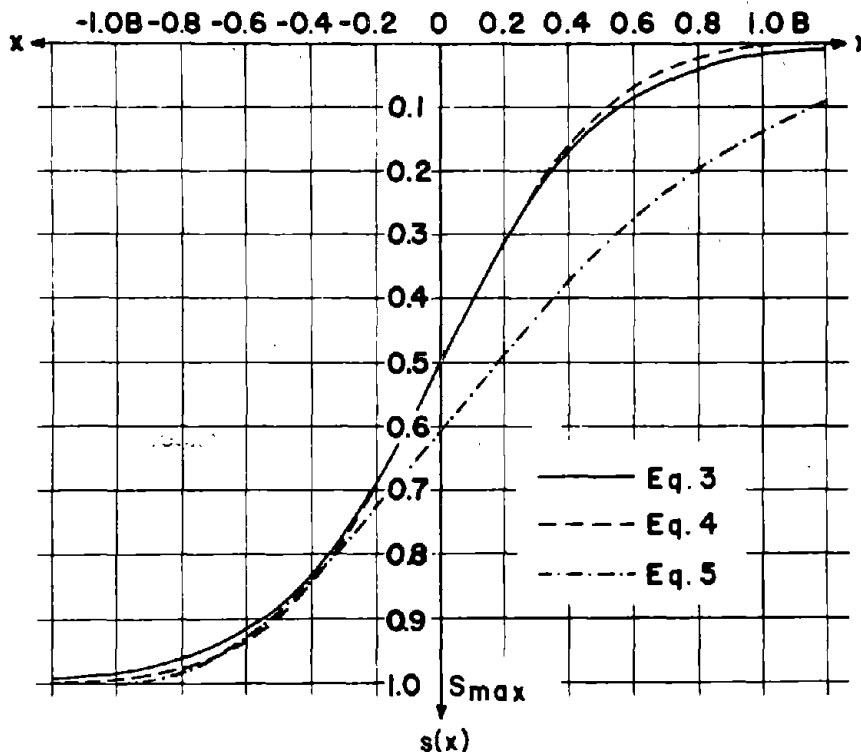


FIGURE 7. - Asymptotic Profile Functions.

The general relations are shown in figure 6. The profile function $S = f(x)$ is normally derived from the results of field observations or model investigations. In most cases, it is antisymmetrical about its point of inflection, and half-full subsidence occurs at that point. The horizontal distance d between the inflection point and the ribside must be known empirically; it can be directly measured, and it seems to be proportional to the extraction depth. The

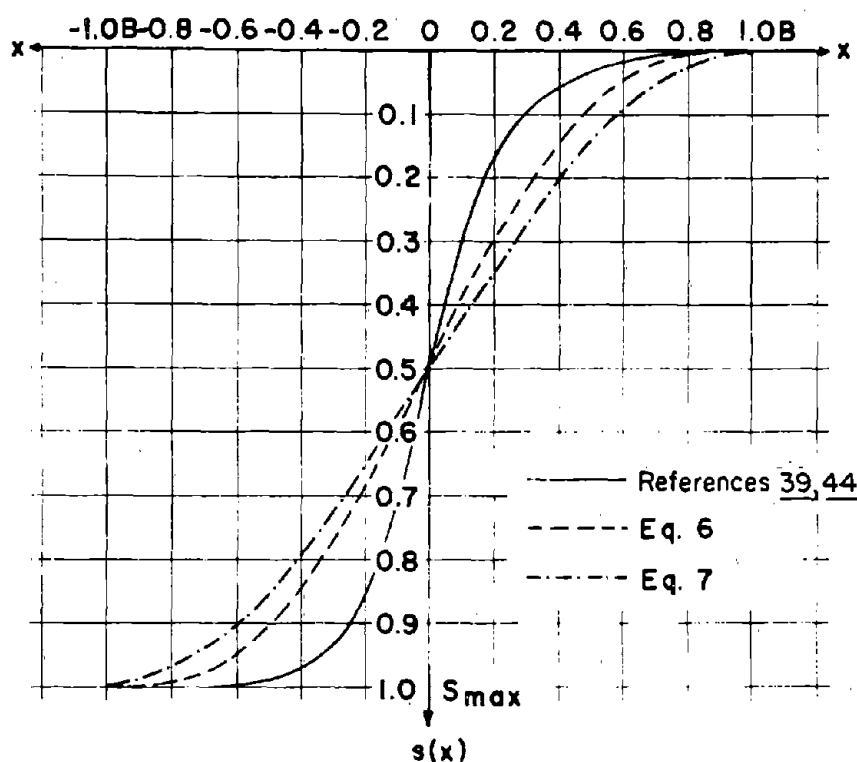


FIGURE 8. - Nonasymptotic Profile Functions.

tion, the measurement of the distance between the inflection point and the nearest point of full subsidence, is inaccurate because of the vanishing slope at the trough center.

Particular Profile Functions

For comparing different profile functions, it is convenient to express the variable x in terms of B and to take the inflection point as $x = 0$, with positive values of x indicating the direction over the unmined seam. (See fig. 6.) The functions can be subdivided into those which are asymptotic to $s = 0$ and those which reach $s = 0$ at finite values of x . The former group is shown in figure 7, the latter in figure 8.

Hyperbolic functions have been suggested based on model and theoretical investigations (11, 24, 58). They can be expressed by

$$s = \frac{1}{2} S_{\max} \left[1 - \tanh\left(\frac{cx}{B}\right) \right], \quad (2)$$

where c and B are the functional parameters. The parameter c seems to be about 2, hence the equation is

$$s = \frac{1}{2} S_{\max} \left[1 - \tanh\left(\frac{2x}{B}\right) \right]. \quad (3)$$

particular shape of the curve is also a function of that depth and will be characterized in the following by the critical radius B . Then the subsidence s of a point is a function of the position x of that point, the full subsidence $S_{\max}$, the distance d , and the critical radius B , as follows:

$$s = f(B, x, S_{\max}, d).$$

If the extraction area is subcritical, this expression also contains the width-depth ratio. The parameter B can be calculated from measured slopes. Its direct determina-

To calculate the parameter B from measured subsidences s, the equation 3 is solved for B, as follows:

$$B = \frac{2x}{\tanh^{-1} \left[1 - \frac{2s}{s_{max}} \right]},$$

where the value of x is introduced in terms of the extraction depth. To calculate B from the maximum slope s'_{max} , which can easily be observed, the function is differentiated with respect to x, and for $x = 0$ it is found that

$$B = \frac{s_{max}}{s'_{max}}.$$

A profile function used in the Upper Silesian coalfield (25) is

$$s = \frac{s_{max}}{B} \int_x^{\infty} \exp \left[-\pi \left(\frac{\xi}{B} \right)^2 \right] d\xi, \quad (4a)$$

where ξ is the integration variable. Since the integral is usually tabulated for the interval from 0 to a finite value--the "error function"--the equation may be transformed into

$$a = \frac{1}{2} s_{max} \left[1 - \frac{2}{\sqrt{\pi}} \int_0^{\frac{\pi x}{B}} e^{-u^2} du \right] \quad (4b)$$

(u = integration variable). The parameter B can again be expressed by $B = s_{max}/s'_{max}$ where s'_{max} is the maximum slope of the function 4 (at $x = 0$). Values between $B = \frac{1}{3} h$ and $B = \frac{2}{3} h$ for Upper Silesia are reported (25).

Equations 3 and 4a represent functions which are asymptotic to $s = 0$ and $s = s_{max}$. Thus the critical area and the zero subsidence are theoretically never obtained here, and the critical radius B indicates the distance between the inflection point and the point of a certain percentage of full subsidence. This percentage is 98 percent in case of equation 3 and 99 percent in case of equation 4. This difference is of no practical concern.

A function suggested for the Hungarian coalfields (37) is

$$s = s_{max} \exp \left[-\frac{1}{2} \left(\frac{x+B}{B} \right)^2 \right]. \quad (5)$$

This function does not give half of the full subsidence at the inflection point ($x = 0 : s = 0.61 s_{max}$) and is asymptotic only to $s = 0$. A similar expression has also been used for British coalfields (36). Solving the equation for B gives the quadratic equation

$$\left(1 - 2 \ln \frac{S_{\max}}{s}\right) B^2 + 2 \times B + x^2 = 0,$$

from which B can be calculated when introducing measured subsidences. Values obtained are $B = 0.27h$ and $B = 0.42h$ and for the position d of the inflection point over extraction area $d = 0.09h$ and $d = 0.27h$.

The profile functions of the second group reach full and zero subsidence at finite abscissas. The first graph in figure 8 shows a curve which has been derived from observations in British coalfields (39, 44). The function has not been expressed mathematically. The inflection point lies at $d = 0.14h$ over the extraction area.

For the Donets coalfield, the Soviet Union, the profile function

$$s = \frac{1}{2} S_{\max} \left[1 - \frac{x}{B} - \frac{1}{\pi} \sin\left(\pi \frac{x}{B}\right) \right] \quad (6)$$

is used (15). Another trigonometric equation, which has been derived from model investigations, is (21)

$$s = S_{\max} \sin^2 \left[\frac{\pi}{4} \left(\frac{x}{B} - 1 \right) \right]. \quad (7)$$

In both expressions, x ranges from $x = -B$ to $x = +B$. B is defined by $B = h \cot \gamma$, but it can, of course, again be calculated from any measured subsidence.

Subsidence Profiles Over Subcritical Extraction

Most methods for determining subcritical profiles comprise two steps: First, the finding of the particular maximum subsidence S, and second, the determination of the half profile the amplitude of which is S. Three rather different methods will be considered here.

References 39 and 44 describe an empirical method which is entirely based on field observations. The maximum subsidence S, expressed as a fraction of seam thickness, is represented graphically as a function of the width-depth ratio in case of caving, strip packing, and solid stowing. The profile for a given width-depth ratio is determined from another empirically derived graph which provides the subsidences in terms of S and the horizontal distances from the point of half maximum subsidence in terms of extraction depth. The position of that point with respect to the ribside is also given in dependence on w/h. A similar but less particularized method has been used in the Saar coalfield, Germany (29, 34).

A more complex method has been developed for the Ukrainian Donets coalfield (15). As for the maximum subsidence S, the empirical and quite plausible relationship is

$$\frac{S}{S_{\max}} = \sqrt{\frac{A}{A_{\text{crit}}}}, \quad (8)$$

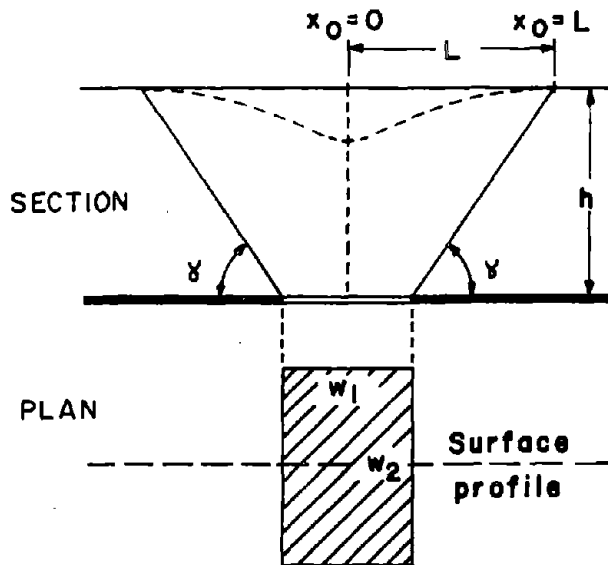


FIGURE 9. - Rectangular Subcritical Area.

where A is the size of the subcritical and A_{crit} is the size of the critical area. Assuming a rectangular subcritical area, as shown in figure 9, the area A can be expressed by the product of its widths w_1 and w_2 . The side length of a square critical area under the conditions of the Donets coalfield is empirically $\frac{h}{0.9}$, thus from equation 8

$$S = S_{max} \sqrt{0.9 \frac{w_1}{h} \cdot 0.9 \frac{w_2}{h}}.$$

When introducing the coefficients

$$n_1 = 0.9 \frac{w_1}{h} \text{ and } n_2 = 0.9 \frac{w_2}{h} \quad (9)$$

the result is

$$S = S_{max} \sqrt{n_1 n_2}. \quad (10)$$

If one of the coefficients n becomes greater than unity, which means that the width in the corresponding direction is supercritical, the value $n = 1$ is introduced. Equation 10 gives the maximum subsidence over the center of the extraction area.

For a profile through the center and parallel to one pair of the ribsides (fig. 9) a modification of equation 6 is used. The origin of the abscissa is here more conveniently chosen at the point of maximum subsidence. Then the distance of the point of zero subsidence from the origin is $L = \frac{1}{2} w + h \cot \gamma$, where the limit angle γ is given. The abscissa x_0 is expressed as a fraction of L and varies from $x_0 = 0$ to $x_0 = 1$. The equation of the profile is

$$s = S \left[n^2 \left(1 - x_0 + \frac{\sin 2\pi x_0}{2\pi} \right) + \frac{1 - n^2}{4} \left(1 + \cos \pi x_0 \right)^2 \right]. \quad (11)$$

If the profile is parallel to the side the length of which is w_1 , the coefficient n is n_1 according to equation 9. If it is parallel to w_2 , $n = n_2$ is introduced. This equation is used for $n > \frac{1}{3}$. In case of $n > \frac{1}{3}$, it is replaced by the uniform expression

$$s = \frac{1}{4} S (1 + \cos \pi x_0)^2. \quad (12)$$

If, for example, the side lengths are 600 and 1,000 ft, the depth $h = 800$ ft, and the limit angle $\gamma = 55^\circ$, the profile parallel to the shorter side is determined by taking $w_1 = 600$ ft. The calculation is as follows: From equation 9, $n_1 = 0.675$ and $n_2 > 1$ are found. Then equation 10 gives $S = 0.82 S_{max}$. Since $w_1 = 0.75h$, the length L of the half profile is $L = (0.75 + \cot 55^\circ)h = 1.075h$. Since $n = 0.675 > \frac{1}{3}$, equation 11 applies.

Thus

$$s = 0.82 S_{\max} \left[0.456 \left(1 - x_0 + \frac{\sin 2\pi x_0}{2\pi} \right) + 0.135 (1 + \cos \pi x_0)^2 \right],$$

where x_0 must be expressed as a fraction of L .

Figure 10 shows this profile in comparison with the corresponding profiles calculated by the empirical methods (British coalfields and Saar coalfield). The differences are considerable, but are explained by the differences in local conditions.

The third important method is derived from the principle of superposition (26, 59). It is illustrated in figure 11. A complete critical profile s_1 would occur if no ribside were at A_2 , and if A_1 were the edge of a semi-infinite extraction area. Reestablishing the seam with its other edge at A_2 would create an inverse critical profile s_2 . According to the principle of superposition, the difference $s_1 - s_2$ would remain as the profile resulting from the subcritical area between A_1 and A_2 . In case of the hyperbolic function according to equation 3 and assuming the inflection point vertical over the ribside A_1 (fig. 11), the function s_1 is given by equation 3. The function s_2 , with its inflection point above A_2 , is then

$$s_2 = \frac{1}{2} S_{\max} \left[1 - \tanh \frac{2(x+w)}{B} \right].$$

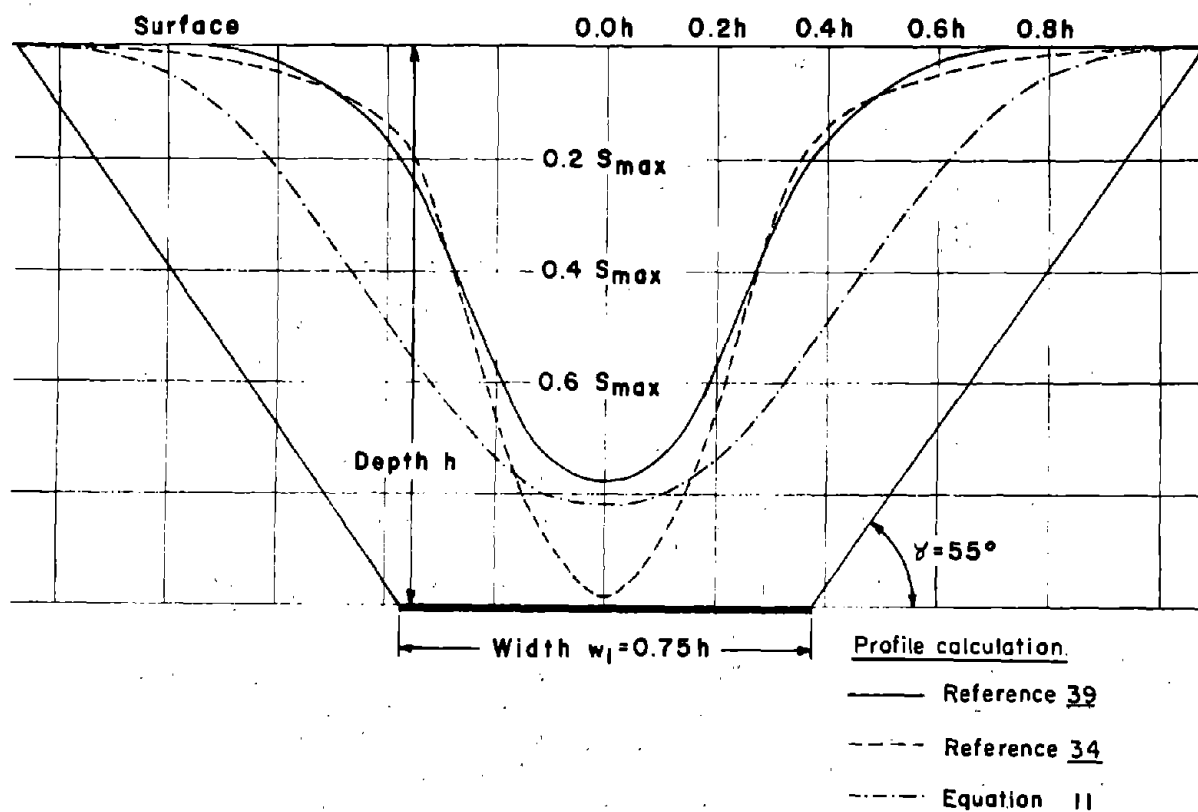


FIGURE 10. - Subsidence Profiles Determined by Different Methods.

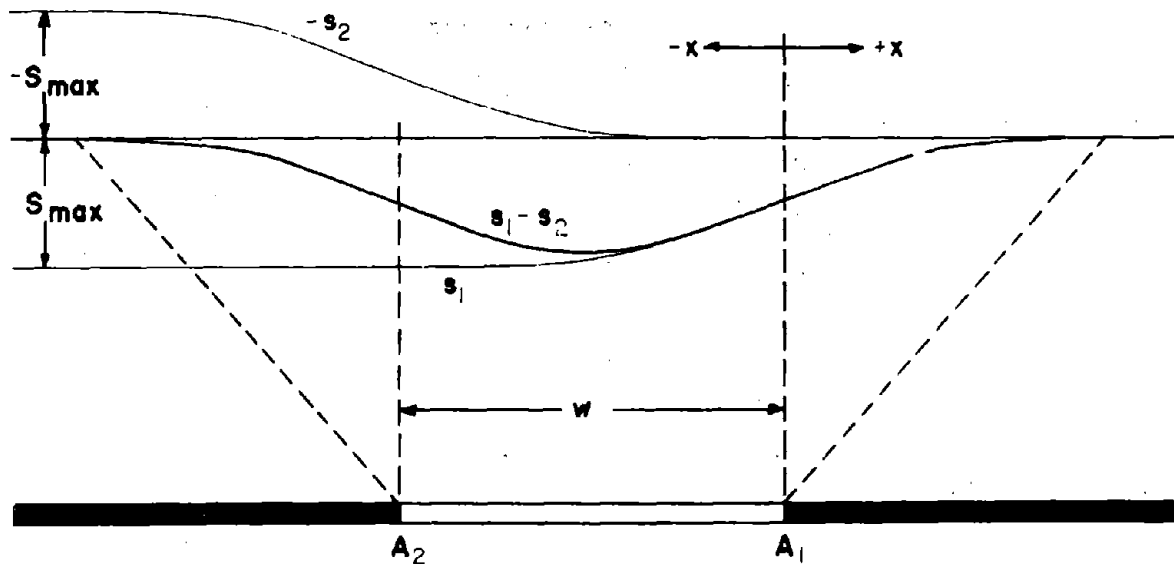


FIGURE 11. - Superposition of Two Critical Profiles.

The resulting subcritical profile is

$$s = s_1 - s_2 = \frac{1}{2} S_{\max} \left[\tanh \frac{2(x+w)}{B} - \tanh \frac{2x}{B} \right]. \quad (13)$$

The maximum subsidence occurs at $x = -\frac{1}{2} w$ and is

$$S = S_{\max} \tanh \frac{w}{B}.$$

Thus no separate determination of S is necessary here. This method is much simpler, but of course, more theoretical than the previous ones.

SUBSIDENCE CALCULATION THROUGH INFLUENCE FUNCTIONS

Principles

In the influence function approach, the principle of superposition is applied to the influence of infinitesimal parts of an extraction area. Since any area can be considered as consisting of an infinite number of infinitesimal elements, there are no geometric restrictions on the shape of the extraction area. Figure 12 illustrates the idea: The trough is imagined to be composed of infinitesimal troughs which are produced by the extraction elements. Accordingly, the subsidence of a point P is the sum of the individual subsidences due to each element. This contribution of an extraction element to the subsidence of a surface point can be expressed as the product of its area, dA , and a value p which indicates the magnitude of the influence of dA on P (fig. 13). This influence p is, of course, a function of the horizontal distance r between the point P and the element dA . This function,

$$p = f(r), \quad (14)$$

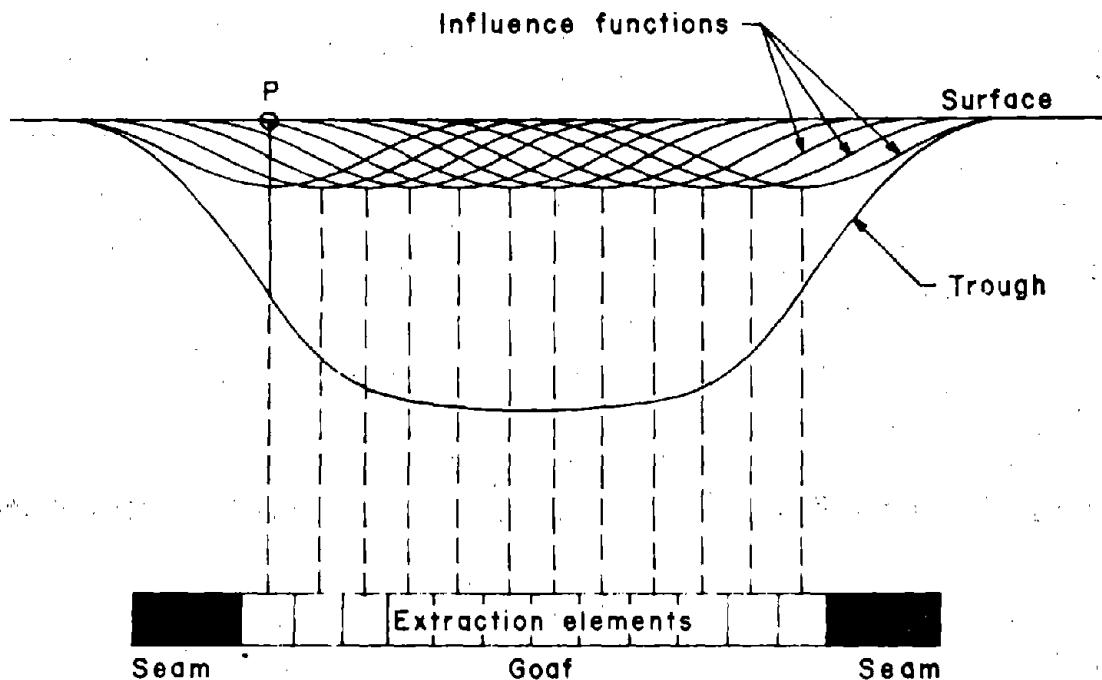


FIGURE 12. - Superposition of Infinitesimal Influences.

will be referred to as the influence function. Since the influence value p refers to the surface point P , that point is chosen as the origin of the radial variable r (cylindrical coordinates). The extraction element vertically below P has the greatest influence, hence the function $p(r)$ reaches its maximum value at $r = 0$. If P lies over the center of a critical area (fig. 14), it is subject to the influence of all extraction elements and undergoes full subsidence S_{max} . S_{max} is therefore represented by the complete area under the curve $p(r)$. If P lies over the edge, it is subject to only half the sum of all possible influences and therefore undergoes only half the

sum of all possible influences and therefore undergoes only half the full subsidence. It is also the point of inflection of the profile. Consequently, the horizontal distance B between the point of full subsidence and the inflection point is equal to half of the width of the critical area.⁴

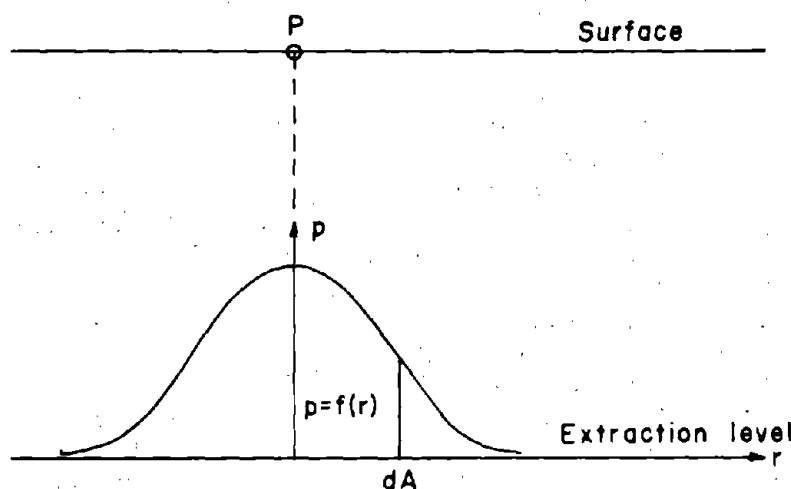


FIGURE 13. - Influence Function.

⁴In practical calculation, this can be corrected by introducing a compensation zone, as explained on page 28.

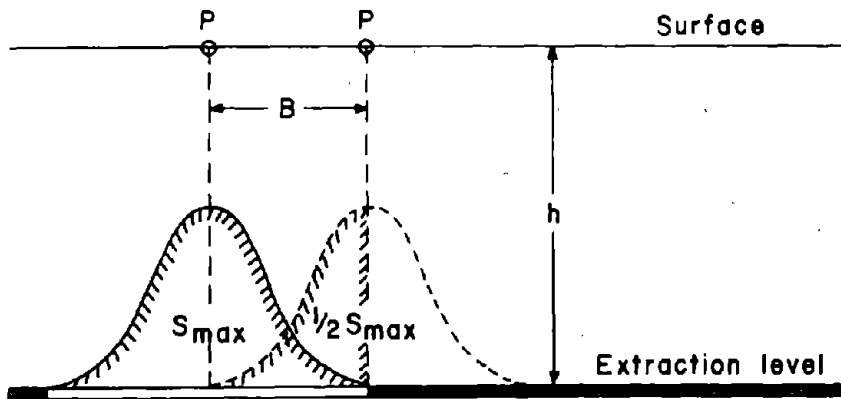


FIGURE 14. - Influence Function and Subsidence.

In three dimensions, the full subsidence is accordingly represented by the volume of the solid of revolution of the curve $p(r)$ around the z axis. The radius of that solid is the critical radius B , if p becomes zero at $r = B$, and is infinite, if p tends asymptotically to zero. Thus the full subsidence is expressed by either

$$S_{\max} = 2\pi \int_0^B r p(r) dr \quad (15a)$$

or

$$S_{\max} = 2\pi \int_0^{\infty} r p(r) dr. \quad (15b)$$

Particular Influence Functions

There are a considerable number of precalculation methods that consist of superposing the effects of equivalent extraction elements. They are mainly used in the Central European coalfields. For each of these methods, an influence function $p(r)$ can be formulated. The variable r is expressed in terms of the critical radius B , and the influence value p in terms of $S_{\max}/B^2$. B is again the parameter which characterizes the local strata conditions and can be interpreted as $B = h \cot \gamma$. In some functions, another independent rock parameter is used.

The qualitative equations of the influence functions result from experiences in particular regions and/or from mathematical considerations. Six functions will be considered in the following paragraphs.

One of the first methods (23) is based on the angle of break β (fig. 1A) and the limit angle γ as independent strata parameters. The critical area is subdivided as shown in figure 15, and it is assumed that the extraction of the inner zone produces $\frac{2}{3} S_{\max}$ and the extraction of the outer zone $\frac{1}{3} S_{\max}$ at the surface point P . As an influence function, this can be expressed by

$$p = \frac{2 \tan^2 \beta}{3\pi \tan^2 \gamma} \frac{S_{\max}}{B^2} \text{ for } 0 \leq r \leq \frac{\tan \gamma}{\tan \beta} B \quad (16a)$$

and

$$p = \frac{\tan^2 \beta}{3\pi(\tan^2 \beta - \tan^2 \gamma)} \frac{S_{max}}{B^2} \quad \text{for } \frac{\tan \beta}{\tan \gamma} B \leq r \leq B.$$

If, for example, $\gamma = 55^\circ$ and $\beta = 70^\circ$, the result is

$$p = 0.785 S_{max} / B^2 \quad \text{for } 0 \leq r \leq 0.52 B$$

and

$$p = 0.145 S_{max} / B^2 \quad \text{for } 0.52 B \leq r \leq B. \quad (16b)$$

B may be replaced by $B = h \cot \gamma$; thus there are but two independent parameters.

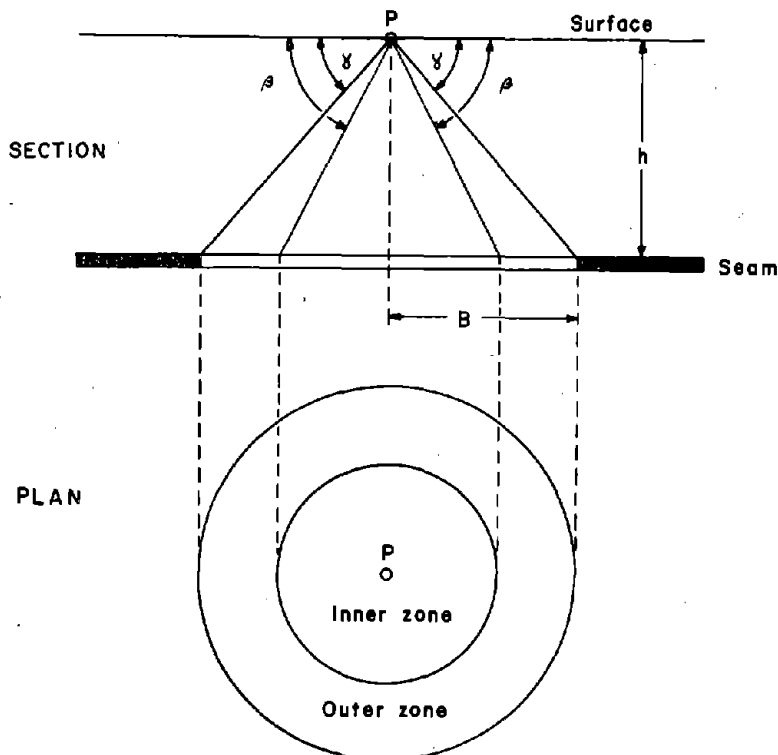
Another method which has widely been used in practice (2) can be derived from the influence function

$$p = \frac{S_{max}}{\pi(\sin \gamma \cos \gamma + \frac{\pi}{2} - \gamma)} \frac{B^3 \tan^3 \gamma}{r(r^2 + B^2 \tan^2 \gamma)^2} \quad (17a)$$

where $0 \leq r \leq B$. Since $B = h \cot \gamma$, only one parameter is used here.

For $\gamma = 55^\circ$, the equation takes the form

$$p = \frac{0.86 B^2 S_{max}}{r(r^2 + 2.04 B^2)^2}. \quad (17b)$$



Another function using only one parameter is (5)

$$p = \frac{3}{\pi} \frac{S_{max}}{B^2} \left[1 - \left(\frac{r}{B} \right)^2 \right]^2, \quad (18)$$

where $0 \leq r \leq B$.

These three influence functions are interpreted between $r = 0$ and $r = B$ only, and for $r > B$ the value of p is considered zero. Consequently, introduction into equation 15a and integrating between $r = 0$ and $r = B$ gives the full subsidence S_{max} . In case of equation 17a, p becomes infinite at $r = 0$, but the solid of revolution has a finite volume. Graphs of the three functions are shown in figure 16.

FIGURE 15. - Zones of Influence Given by the Angle of Break and the Limit Angle.

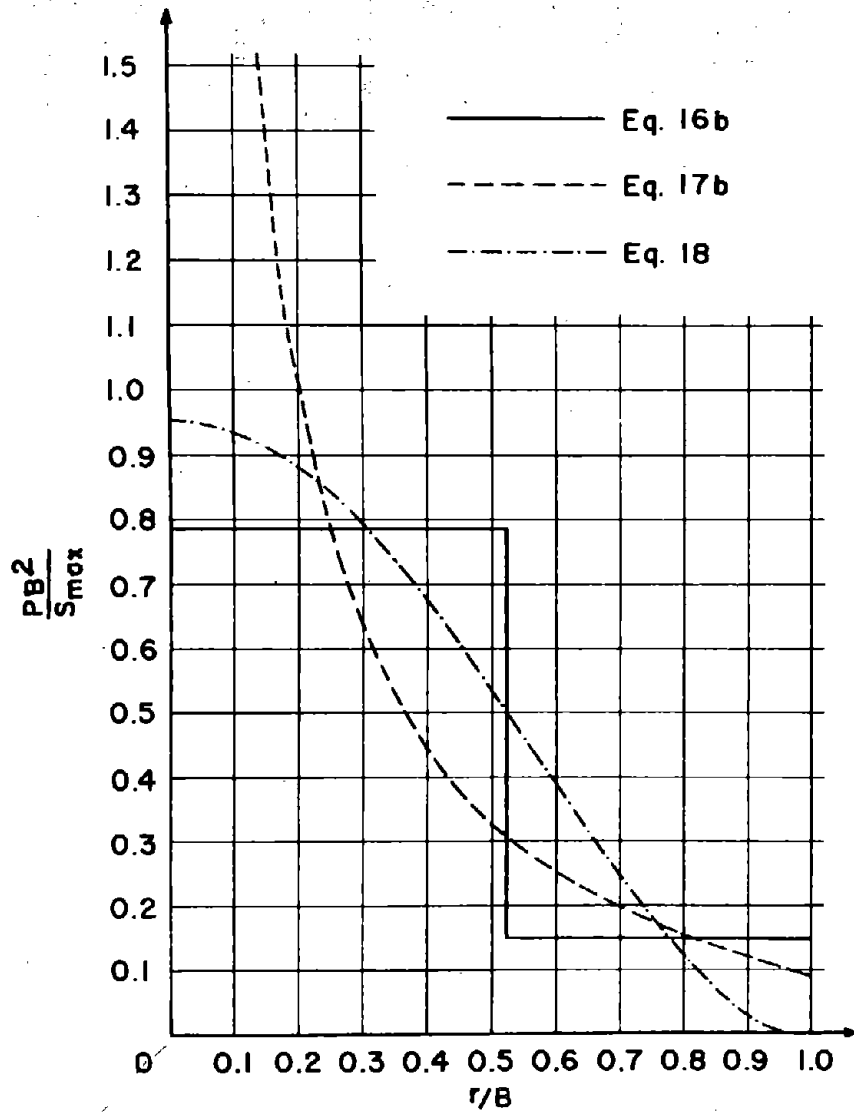


FIGURE 16. - Nonasymptotic Influence Functions.

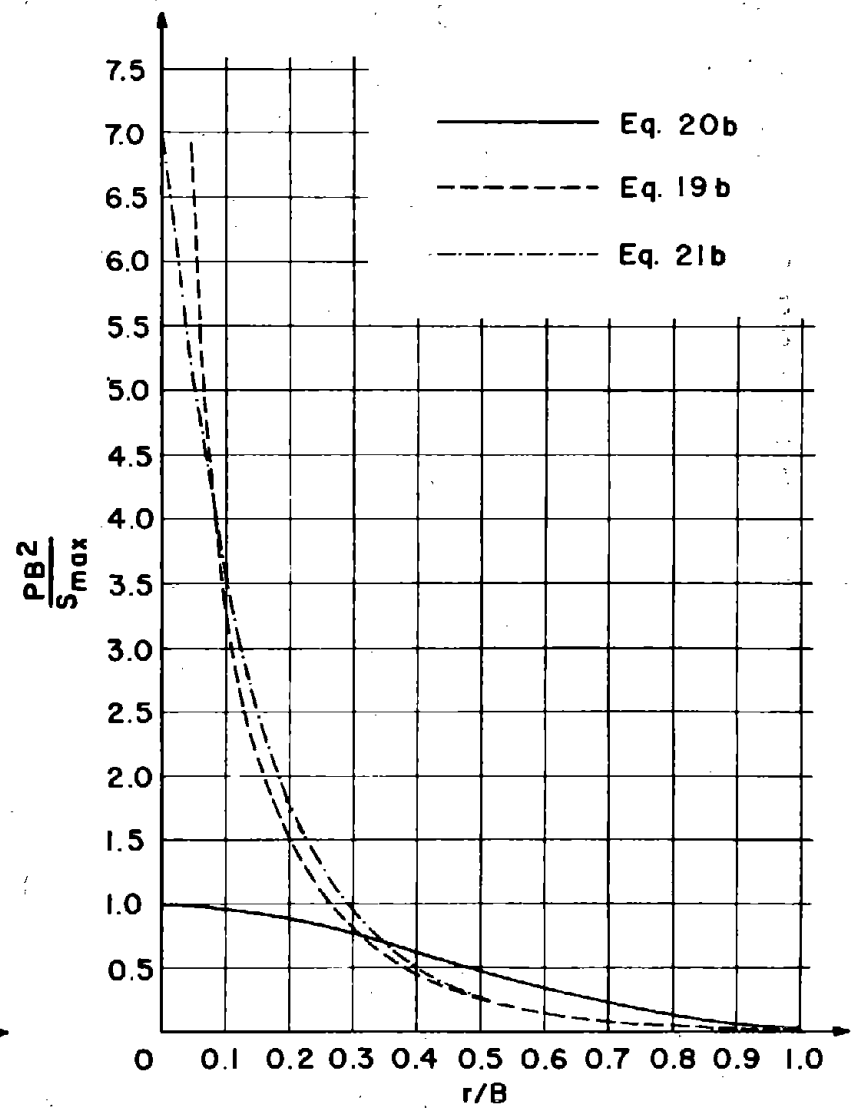


FIGURE 17. - Asymptotic Influence Functions.

The next three functions are asymptotic to $p = 0$ and are interpreted between zero and positive infinite values of r , thus they must satisfy the condition of full subsidence expressed by equation 15b.

From probability considerations, functions of the form

$$p = \text{Const} \frac{1}{r} \exp \left[-4 \left(\frac{r}{B} \right)^{2n} \right]$$

have been derived (49). The exponent n is the second parameter for characterizing strata conditions. The constant can be calculated from equation 15b. Then the equation takes the form

$$p = \frac{n^n \sqrt{2} S_{\max}}{\pi B \Gamma \left(\frac{1}{2n} \right)} \frac{\exp \left[-4 \left(\frac{r}{B} \right)^{2n} \right]}{r}, \quad (19a)$$

where Γ is the gamma function.

In case of $n = 1$, which seems to be the normal value, it is found that

$$p = \frac{2 S_{\max}}{\pi \sqrt{\pi} B} \frac{1}{r} \exp \left[-4 \left(\frac{r}{B} \right)^2 \right]. \quad (19b)$$

Introducing this into equation 15b and integrating between zero and infinity gives full subsidence, whereas integrating between $r = 0$ and $r = B$ gives about $0.99 S_{\max}$. Hence, B is practically the radius of the critical area. If, for example, $n = 3$, the function becomes

$$p = 0.216 \frac{S_{\max}}{B} \frac{1}{r} \exp \left[-4 \left(\frac{r}{B} \right)^6 \right]. \quad (19c)$$

A second group of influence functions, derived from observational results and also from probability considerations, can be expressed by

$$p = \frac{n S_{\max}}{B^2} \exp \left[-n \pi \left(\frac{r}{B} \right)^2 \right], \quad (20a)$$

where n is again the second independent parameter. If $n = 1$, the function is

$$p = \frac{S_{\max}}{B^2} \exp \left[-\pi \left(\frac{r}{B} \right)^2 \right] \quad (20b)$$

This equation can also be derived from the theory of stochastic rock movements (31). Extraction of a circular area having the radius B produces here $0.96 S_{\max}$ at the central surface point. When introducing the condition that B is the radius of an extraction area which produces $0.99 S_{\max}$ (13),

Equation 20 takes the form

$$p = \frac{4.6 S_{\max}}{\pi B^2} \exp\left[-4.6 \left(\frac{r}{B}\right)^2\right] \quad (20c)$$

Finally, relatively steep subsidence troughs are found when taking $n = 2$, that is,

$$p = \frac{2 S_{\max}}{B^2} \exp\left[-2 \pi \left(\frac{r}{B}\right)^2\right]. \quad (20d)$$

Extraction of a circular area of radius B produces $0.998 S_{\max}$ in this case.

A third kind of exponential functions has been suggested for the Silesian coalfield (27). The general equation may be written

$$p = \frac{n S_{\max}}{2\pi r_0^2 \Gamma\left(\frac{2}{n}\right)} \exp\left(-\frac{r}{r_0}\right)^n \quad (21a)$$

with r_0 and n as independent parameters. For comparing this function with the former ones, the special case $n = 1$ will be taken and it will be assumed that $r = B$ is again the radius of the area that produces $0.99 S_{\max}$. This condition is satisfied if $B = 6.65 r_0$, and equation 21a becomes

$$p = \frac{7 S_{\max}}{B^2} \exp\left(-6.65 \frac{r}{B}\right). \quad (21b)$$

Figure 17 illustrates the functions 19b, 20b, and 21b. Generally, the functions which have comparatively small maximum values at $r = 0$, such as equations 16b, 18, and 20b, apply to flat trough profiles, whereas especially equation 19b refers to relatively steep profiles. The different functions reflect not only different general conditions in mining districts but also different theoretical conceptions. Exponential expressions as represented by equation 20a are frequently used in practice.

Determination of Subsidence and Parameters

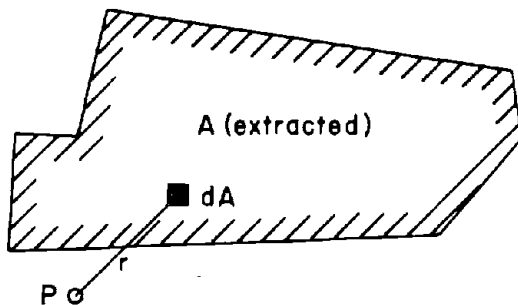


FIGURE 18. - Irregular Extraction Area.

Superposition of infinitesimal influences can be applied to any shape of extraction area. If an element dA (fig. 18) produces the infinitesimal subsidence $p dA$ of the surface point P at the horizontal distance r , then the complete subsidence s of P due to the whole extraction area A is the integral of the influence function $p(r)$ over the area A , as follows:

$$s = \iint_A p(r) dA. \quad (22)$$

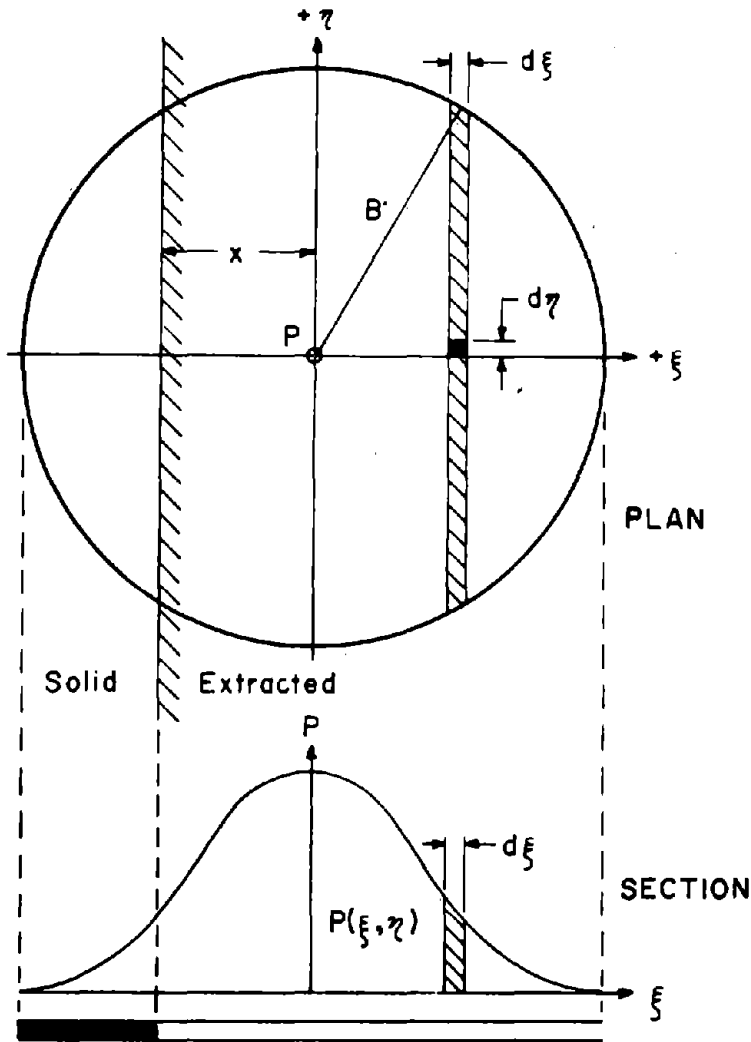


FIGURE 19. - Integration of Influence Functions.

The inverse relation can be used for finding the form of the influence function or the values of its parameters from measured subsidence s . For this purpose, analytical solutions of the area integral are useful, but they can be obtained only in the simplest cases. Most convenient is the case of two-dimensional critical extraction, the solution of which gives the relation between the influence function $p(r)$ and the profile function $s(x)$. This relation is found as follows:

The surface point P in figure 19 is located at a distance x from the ribside, and an influence function with $p = 0$ at $r = B$ is assumed. Introducing rectangular coordinates ξ, η with P as the origin, the influence function is expressed by $p(\xi, \eta)$. Then the infinitesimal element $d\xi d\eta$ produces the infinitesimal subsidence $p(\xi, \eta) d\xi d\eta$ at P . A line parallel to the η -axis as indicated has the length $2\sqrt{B^2 - \xi^2}$ and the width $d\xi$ and thus produces the subsidence

$$ds = d\xi \int_{-\sqrt{B^2 - \xi^2}}^{+\sqrt{B^2 - \xi^2}} p(\xi, \eta) d\eta.$$

Summing up the infinitesimal subsidences between $\xi = x$ and $\xi = B$ gives the complete subsidence at P , as follows:

$$s(x) = \int_x^B \int_{-\sqrt{B^2 - \xi^2}}^{+\sqrt{B^2 - \xi^2}} p(\xi, \eta) d\eta d\xi, \quad (23)$$

which is the general equation of the profile function $s(x)$ resulting from the influence function $p(r)$ resp. $p(\xi, \eta)$. If the influence function is

asymptotic for $p = 0$, the intervals of integration become infinite, and allowing $B \rightarrow \infty$ gives

$$s(x) = \int_x^\infty \int_{-\infty}^\infty p(\xi, \eta) d\eta d\xi. \quad (24)$$

The solution can also be expressed in polar coordinates, which will be illustrated in the third example given below.

Example 1--Influence function equation 19. The radial variable r is replaced by ξ and η which are expressed as fractions of B , hence $B = 1$ and

$$\left(\frac{r}{B}\right)^2 = \xi^2 + \eta^2,$$

$$p(r) = p(\xi, \eta) = \frac{3 S_{max}}{\pi} (1 - \xi^2 - \eta^2)^2.$$

The first integration according to equation 23 gives

$$\int_{-\sqrt{B^2 - \xi^2}}^{+\sqrt{B^2 - \xi^2}} (1 - \xi^2 - \eta^2)^2 d\eta = \frac{16}{15} (1 - \xi^2)^{\frac{5}{2}}.$$

Thus equation 23 takes the form

$$s(x) = \frac{16 S_{max}}{5\pi} \int_x^1 (1 - \xi^2)^{\frac{5}{2}} d\xi.$$

The indefinite integral is

$$\int (1 - \xi^2)^{\frac{5}{2}} d\xi = \frac{1}{6} \xi \sqrt{1 - \xi^2} \left[(1 - \xi^2)^2 + \frac{5}{4} (1 - \xi^2) + \frac{15}{8} \right] + \frac{5}{16} \arcsin \xi + \text{Const.}$$

Working out the definite integral and introducing the result gives the profile function

$$s(x) = \frac{S_{max}}{\pi} \left\{ \frac{\pi}{2} - \frac{x}{15} \sqrt{1 - x^2} \left[8x^4 - 26x^2 + 33 \right] - \arcsin x \right\}, \quad (25)$$

where x must be expressed as a fraction of B . This subsidence function resembles the functions shown in figure 8: At $x = 0$ it gives $s = \frac{1}{2} S_{max}$; at $x = 1$, $s = 0$; and at $x = -1$, $s = S_{max}$.

Example 2--Influence function equation 20b. With $r^2 = \xi^2 + \eta^2$, the function can be written

$$p(\xi, \eta) = \frac{S_{max}}{B^2} \exp\left(-\frac{\pi}{B^2} \xi^2\right) \exp\left(-\frac{\pi}{B^2} \eta^2\right).$$

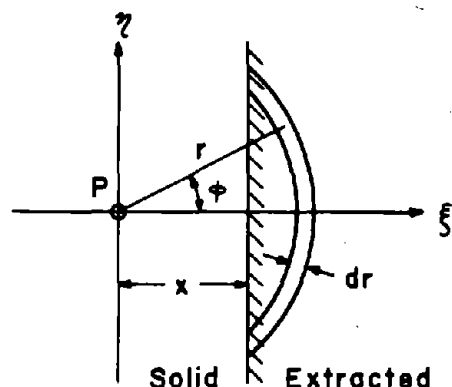
Introducing into equation 24 gives

$$s(x) = \frac{S_{max}}{B^2} \int_x^\infty \int_{-\infty}^\infty \exp\left(-\frac{\pi}{B^2} \eta^2\right) d\eta \exp\left(-\frac{\pi}{B^2} \xi^2\right) d\xi$$

or

$$s(x) = \frac{S_{max}}{B} \int_x^\infty \exp\left(-\frac{\pi}{B^2} \xi^2\right) d\xi$$

that is, the profile function equation 4.



Example 3--Influence function equation 19. This problem is better treated in polar coordinates as indicated in figure 20. The extracted region is subdivided into infinitesimal arcs the argument ϕ of which ranges between $-\arccos \frac{x}{r}$. Then the area integral equation 24 can be written as

$$s(x) = \int_x^\infty \int_{-\arccos \frac{x}{r}}^{+\arccos \frac{x}{r}} p(r) r d\phi dr$$

or, since $p(r)$ is independent of ϕ ,

$$s(x) = 2 \int_x^\infty \arccos\left(\frac{x}{r}\right) p(r) r dr. \quad (26)$$

FIGURE 20. - Integration When Using Polar Coordinates.

Introducing equation 19a gives

$$s(x) = \frac{2n\sqrt{2} S_{max}}{\pi \Gamma\left(\frac{1}{2n}\right) B} \int_x^\infty \arccos\left(\frac{x}{r}\right) \exp\left[-4\left(\frac{r}{B}\right)^{2n}\right] dr \quad (27a)$$

In case of $n = 1$ (equation 19b) the result is

$$s(x) = \frac{4 S_{max}}{\pi \sqrt{\pi} B} \int_x^\infty e^{-4\left(\frac{r}{B}\right)^2} \arccos\left(\frac{x}{r}\right) dr. \quad (27b)$$

The equations of two-dimensional profiles over subcritical areas can be found by the superposition procedure indicated in figure 11. The full profile functions derived from the above six influence functions are geometrically not very different, but the differences increase for subcritical areas with decreasing width. Figure 21 shows an example for a width equal to 0.4 of the critical radius B , which is equivalent to a width-depth ratio of 0.4 in case of $\gamma = 45^\circ$. The curves have been calculated from such different influence

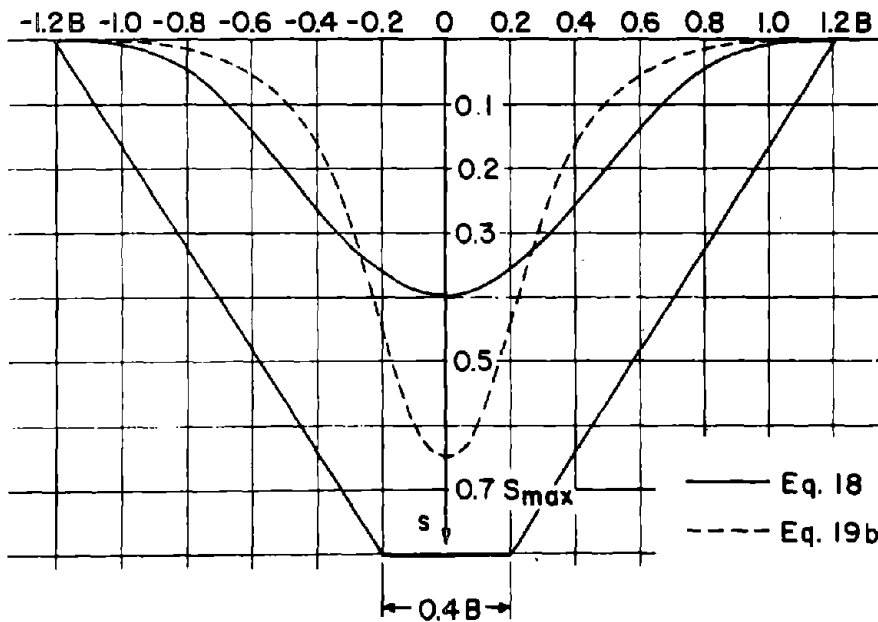


FIGURE 21. - Subcritical Profiles Derived From Influence Functions.

The center of the plan is placed on the point for which the subsidence is to be found, and the ratio of either zone covered by extraction areas is measured or estimated. Since an extraction ratio of $b_o = 1$ of the outer zone produces the subsidence $\frac{1}{3} S_{max}$, a smaller ratio b_o produces $\frac{b_o}{3} S_{max}$. For the inner zone, the subsidence contribution is $\frac{2b_i}{3} S_{max}$, so the whole subsidence is found from

$$s = \frac{S_{max}}{3} (b_o - 2b_i).$$

When using a continuous influence function, the solid of revolution which represents S_{max} is usually subdivided into concentric rings and radial sectors of equal volumes. In plan view, this gives an integration grid the parts of which are of equal weight. The radii R of the zones are found from the equation of the volume of the solid of revolution, as follows:

$$s = 2\pi \int_0^R r p(r) dr, \quad (28)$$

assuming, for example, $s = 0.1 S_{max}$, $0.2 S_{max}$, etc.

In case of the exponential equation 20a, the relation is

$$s = \frac{2\pi n}{B^2} S_{max} \int_0^R r \exp \left[-n\pi \left(\frac{r}{B} \right)^2 \right] dr.$$

functions as equations 18 and 19b, and large differences in subsidence, slope, and curvature are evident.

The general case, the subsidence due to extraction areas of arbitrary shapes is treated by numerical or graphical solution of the fundamental equation 22. Digital computers are often used for numerical solutions (13, 20), but graphic methods are also convenient. The plan in figure 15, for example, can directly be used for graphic integration.

By integrating and solving for R it is found

$$R = B \sqrt{\frac{1}{n \pi} \ln \frac{S_{max}}{S_{max} - s}}$$

The further procedure will be illustrated for the special case $n = 1$ (equation 20b), which means

$$R = B \sqrt{\frac{1}{\pi} \ln \frac{S_{max}}{S_{max} - s}}$$

When introducing $s = 0.1 S_{max}$, $0.2 S_{max}$ etc., the radii R of the 10 concentric zones are found as listed in the following tabulation:

Zone	s/S_{max}	R/B
1	0.1	0.18
2	.2	.27
3	.3	.34
4	.4	.40
5	.5	.47
6	.6	.54
7	.7	.62
8	.8	.72
9	.9	.86
10	.99	1.21

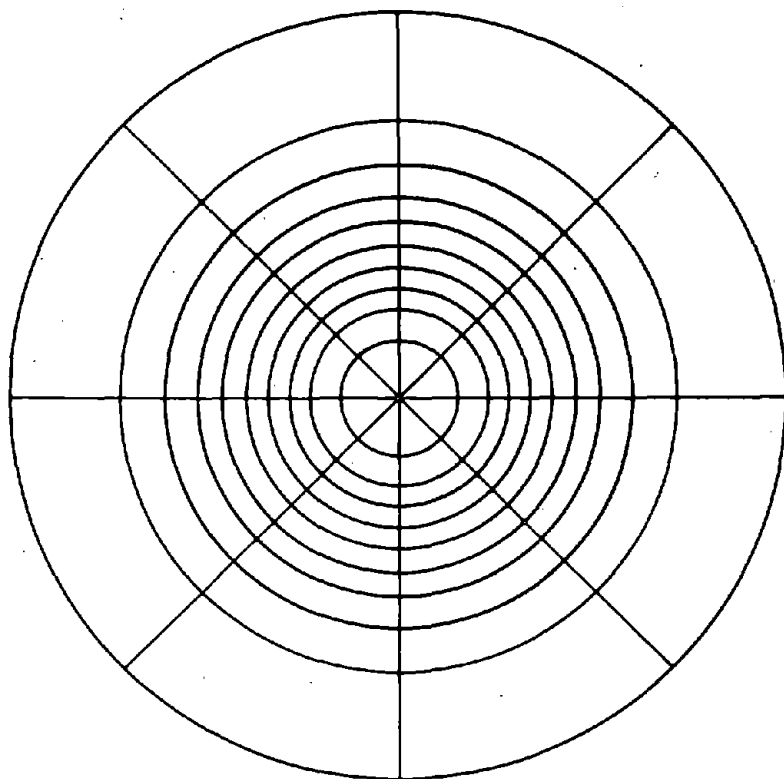


FIGURE 22. - Integration Grid for Subsidence.

The integration grid is shown in figure 22. It is furthermore subdivided into eight radial sectors, thus being composed of 80 parts. Extraction of each part produces $1/80 S_{max}$. For practical calculation, the grid is drawn on transparent paper on the same scale as the plan of the extraction areas, and its center is placed on the point under consideration on that plan. The extraction ratio b is then estimated for each part, $b = 1.00$ meaning $1/80 S_{max}$. The whole subsidence s of the surface point is found from the sum of all ratios b according to

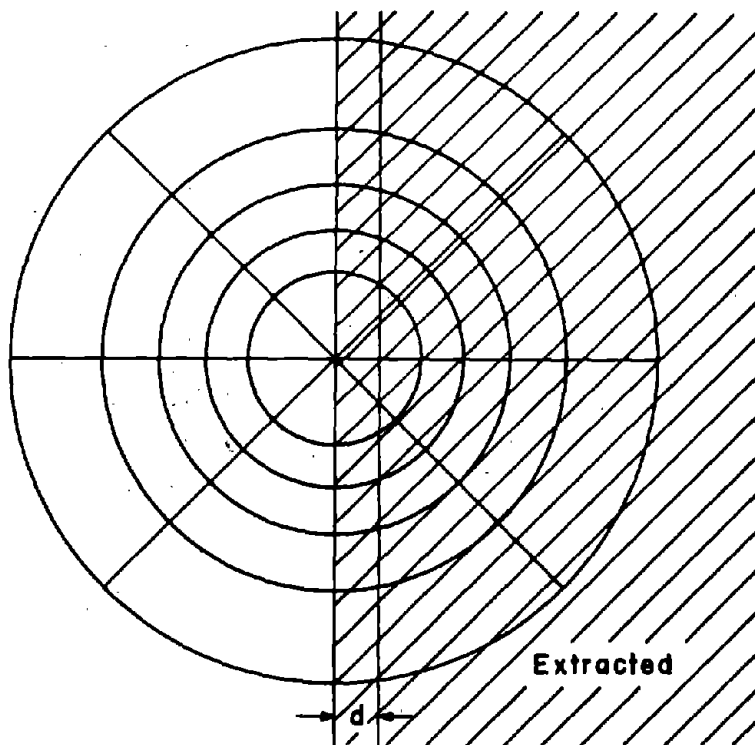
$$s = \frac{\sum b}{80} S_{max}.$$

When using equation 20b, the inner zones become

relatively large and the calculated profiles become relatively flat. If surface observations indicate steeper profiles a factor $n > 1$ in equation 20a may be chosen. Particularly steep profiles are found through influence functions as represented by equation 19a. For $n = 1$ (equation 19b), the radii of an integration grid are given as follows:

Zone	s/S_{max}	R/B
1	0.1	0.04
2	.2	.09
3	.3	.13
4	.4	.19
5	.5	.24
6	.6	.30
7	.7	.37
8	.8	.46
9	.9	.58
10	.996	1.00

It is desirable to introduce a modification of the extraction area over which an influence function is integrated. For a surface point vertically over the edge of a critical or supercritical area, one half of the grid will be covered by extracted areas (fig. 23), thus indicating 50 percent of full subsidence and inflection of the profile. The predicted subsidence will consequently deviate from what actually occurs in many coalfields where the



inflection point of critical profiles lies over the mined area, as indicated by the distance d in figure 6. In such cases the simplest way of correction is to deduct a so-called "compensation zone" from the extraction area. The width of that zone is equal to the distance d (fig. 23). For calculation purposes, the compensation zone is then treated as if it were unmined. Some depth-dependent values of the distance d have already been mentioned when dealing with profile functions. Values of 10 to 30 meters are recommended for parts of the Ruhr coalfield (14). In many parts of French and Dutch coalfields, d is nearly zero.

FIGURE 23. - Allowing for a Compensation Zone.

PROFILE FUNCTIONS OR INFLUENCE FUNCTIONS?

Profile functions can be directly derived from observations. They can be modified for simple subcritical profiles according to field evidence. General three-dimensional problems can only be treated to a limited extent and then only by introducing additional mathematical assumptions.

Influence functions cannot be directly found by measurement. The calculation procedures are entirely based on the law of superposition and can be applied to any extraction area and any surface point. Deriving an influence function from observed subsidence implies acceptance of the law of superposition.

The other premises are equal in either case, and so are the meanings of the parameters. In a recent example (61), comparative calculations have shown no remarkable differences. In both calculation methods, however, the law of superposition was applied. When using influence functions, the critical two-dimensional subsidence profile becomes invariably antisymmetrical about its inflection point. Then the angle between the horizontal plane and the line joining the edge of the working to the point of full subsidence--indicated in figure 1A--will be equal to the limit angle. If a compensation zone is introduced, the edge of this zone takes the place of the edge of the working. This equality has not always been confirmed by observations, although most differences were only slight. The same result is found when using antisymmetrical profile functions--and with the exceptions of the National Coal Board (Great Britain)--curve (39) and the curve expressed by equation 5 (37), the profile curves given in pp. 10-16 are in fact antisymmetrical about their inflection point. Moreover, determining a subcritical profile by using a profile function and the law of superposition (fig. 11) is quite the same thing as integrating an influence function. It has already been shown that, for example, the profile equation 4 can be found as an area integral of the influence equation 20b. Hence, there will be the same results in this case, but the influence function method can be used for any shape of extraction area.

Now the results of multiple extraction are generally calculated by superposing the effects of the individual workings, and the main question concerning influence functions is to what extent the superposition of the effects of small areas is justified. A very small area (that is, having a small width:depth ratio), when mined alone, will produce a different effect than as a part of a larger extraction area, but this can be allowed for by the compensation zone. Moreover, a small width:depth ratio, say 0.2, means small subsidences which are often of no practical interest. Generally speaking, calculation using profile functions may be expected to give more accurate results in case of geometrically simple problems. When extending the method to more complex cases--if this is possible at all--the results will be no better than those obtained using influence functions.

HORIZONTAL DISPLACEMENTS AND DEFORMATION (FLAT STRATIFICATION)

The Maximum Displacement V_{max}

Unlike the subsidences, the horizontal displacements have no evident maximum value determined by the seam thickness. Several theoretical attempts have been made to derive the maximum possible displacement V_{max} from the full subsidence, but the results are rather hypothetical. An example is illustrated in figure 24. Experience has shown that V_{max} occurs at the inflection point of the subsidence profile over critical or larger extraction areas. It is now assumed that the subsidence process has not changed the volume of the rock mass between the inflection point and the edge of the trough, hence the sum of the horizontal displacements between the surface $z = 0$ and the extraction level $z = h$ must be equal to the sum of the subsidence between $x = 0$ and $x = \infty$ (hatched areas in figure 24). This condition of volume incompressibility may be written

$$\int_0^h v(z) dz = \int_0^{\infty} s(x) dx,$$

but the function $v(z)$ is rather uncertain. When assuming a linear function and $v = 0$ at the seam level, its integral will be $\frac{1}{2} h V_{max}$. For the hyperbolic subsidence profile (equation 3), the condition of incompressibility then becomes

$$\frac{1}{2} h V_{max} = \frac{1}{2} S_{max} \int_0^{\infty} \left[1 - \tanh \left(\frac{2x}{R} \right) \right] dx.$$

But $\int \tanh \left(\frac{2x}{R} \right) = \frac{R}{2} \ln \cosh \left(\frac{2x}{R} \right)$ and $\lim_{u \rightarrow \infty} \ln \cosh u = u - \ln 2$,

therefore

$$V_{max} = \frac{R \ln 2}{2h} S_{max}.$$

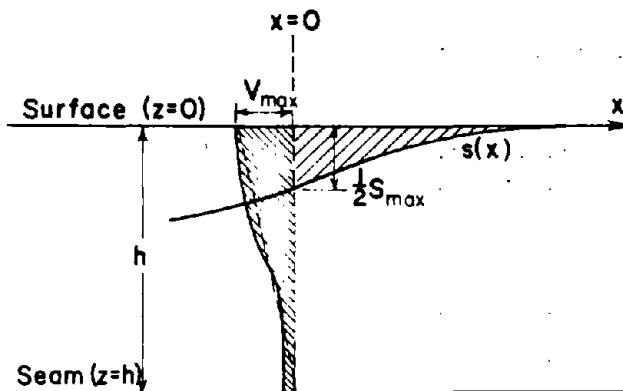


FIGURE 24. - Relations Between Vertical and Horizontal Displacements.

For $R = h$ this gives $V_{max} = 0.35 S_{max}$ and for $R = 2/3 h$ $V_{max} = 0.23 S_{max}$. These values are in the order of magnitude of observed values, but generally a definite dependence of V_{max} on the R/h ratio has not yet been confirmed by observations.

Other theoretical investigations arrive at V_{max} as a function of certain mechanical rock parameters, but measurement of these parameters is more difficult than measuring V_{max} itself. So the simplest and most reliable way to determine V_{max} is to do so from direct observation.

According to results obtained previously, V_{max} is proportional to S_{max} . Some values of V_{max}/S_{max} are as follows:

<u>Coalfield and location</u>	<u>V_{max}/S_{max}</u>
Ruhr, Germany.....	0.35-0.45
Donets, USSR (15).....	.30
Kizel, USSR (15).....	.30
Chelyabinsk, USSR (15).....	.30
Karaganda, USSR (15).....	.30
Kuznetsk, USSR (15).....	.35
British fields ¹	.16
Northern French coalfields (17).....	.40

¹Calculated from graphs in reference 39.

Calculation of Horizontal Displacement Profiles

The most important reason for calculating the horizontal displacements is their relation to the slope of the subsidence profile. In particular, in the two-dimensional case the displacements v are proportional to the slope, that is,

$$v = \text{Const} \frac{\partial s(x)}{\partial x} . \quad (29)$$

This relationship has been confirmed by observation results in various coalfields. It has also been applied to subcritical profiles (1). For the present, it provides the best way for calculating horizontal displacements. Analogously to subsidence calculation, profile functions or influence functions can be derived. Earlier methods which are based on other relationships have been described in references 42 and 50.

A profile function for the horizontal displacements is found by direct differentiation of a given subsidence profile function. This method has been used in the Donets coalfield (15); differentiation of equation 11 with respect to x_0 gives

$$\frac{\partial s}{\partial x_0} = s'(x) = -S[2n^2 \sin^2 \pi x_0 + \frac{\pi}{2} (1-n^2) \sin \pi x_0 (1 + \cos \pi x_0)] . \quad (30)$$

The negative sign only defines the direction of the slope of the semiprofile. For the critical profile, that is, $n = 1$, the equation becomes

$$s'(x_0) = 2 \sin^2(\pi x_0) S_{max} ,$$

the maximum value of which is $2 S_{max}$. Since the maximum possible horizontal displacement is V_{max} , the constant in equation 29 must be $\frac{V_{max}}{2 S_{max}}$. Thus the equation of horizontal displacements is

$$v(x_0) = \frac{V_{max}}{2 S_{max}} s'(x_0) , \quad (31)$$

where V_{max} must be known empirically. Equation 30 gives $s'(x_0)$, with $n < 1$ for subcritical and $n = 1$ for critical or supercritical extraction. The origin $x_0 = 0$ is at the point of maximum subsidence, where $v = 0$.

The profile functions give the displacements only in the direction of the profile. A general calculation method can again be developed by using influence functions. If an extraction element dA , as shown in figure 25, causes the infinitesimal horizontal displacement $q dA$ at the point P, the component in a given direction (arrow in figure 25) will be $q dA \cos \phi$, where $dA = r d\phi dr$. The displacement caused by all extraction elements at the distance r between $\phi = -\frac{\pi}{2}$ and $\phi = +\frac{\pi}{2}$ is

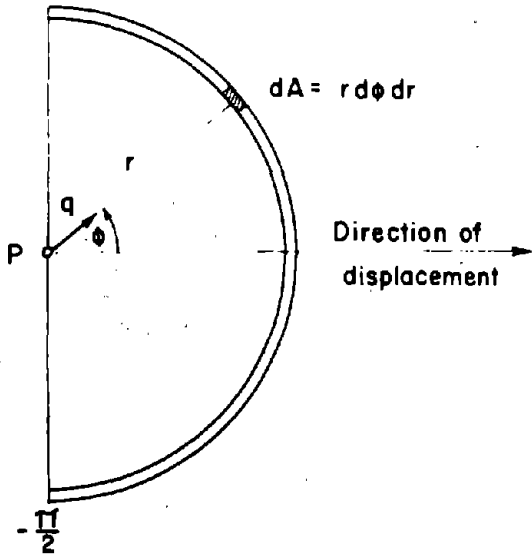
$$dv = q(r) r dr \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \cos \phi d\phi = 2 q(r) r dr. \quad (32)$$

If a whole semicircle between $r = 0$ and $r = R$ is extracted, the resulting displacement is

$$v = 2 \int_0^R q(r) r dr, \quad (33)$$

and the maximum possible displacement will occur when the radius of that semicircle becomes infinite:

$$V_{max} = 2 \int_0^R q(r) r dr. \quad (34)$$



These formulas have already been given in references 5 and 8.

For determining the influence function $q(r)$, equation 29 can be applied to the axially symmetric case, replacing x by r , v by q , and s by p :

$$q(r) = \text{Const} \frac{\partial p(r)}{\partial r}. \quad (35)$$

This result is also found from the theory of stochastic movements and the condition of volume incompressibility (8, 31). As an example, the subsidence influence function equation 20b will be introduced. Its derivative with respect to r is

FIGURE 25. - Integration of Horizontal Displacements.

$$q(r) = \text{Const } r \exp \left[-\pi \left(\frac{r}{B} \right)^2 \right].$$

From the condition that introduction into equation 34 and integration must give $V_{\max}$, the constant is found to be $\frac{2\pi}{B^3} V_{\max}$, hence

$$q(r) = \frac{2\pi}{B^3} V_{\max} r \exp \left[-\pi \left(\frac{r}{B} \right)^2 \right] \quad (36)$$

is the influence function for horizontal displacements.

The horizontal displacement of a surface point in a given direction is found by integrating the components of q in that direction over the extraction area. In the general three-dimensional case, the graphic integration is again done the simplest way. The radii of an integration grid can be calculated by solving equation 33 for the radius R and introducing v in fractions of $V_{\max}$. When taking equation 36 as the influence function, equation 33 takes the form

$$v = \frac{4\pi}{B^3} V_{\max} \int_0^R r^2 \exp \left[-\pi \left(\frac{r}{B} \right)^2 \right] dr.$$

By integrating

$$\int r r \exp \left[-\pi \left(\frac{r}{B} \right)^2 \right] dr$$

by parts and substituting

$$u = \sqrt{\pi} \frac{r}{B} \text{ and } U = \sqrt{\pi} \frac{R}{B},$$

this integral can be transformed to

$$v = \frac{2}{\sqrt{\pi}} V_{\max} \left(\int_0^U e^{-u^2} du - U e^{-U^2} \right).$$

The expression

$$\frac{1}{\sqrt{\pi}} \int_0^U e^{-u^2} du - U e^{-U^2} = \Psi(U)$$

can be calculated with the help of a table of the error function. A graphic

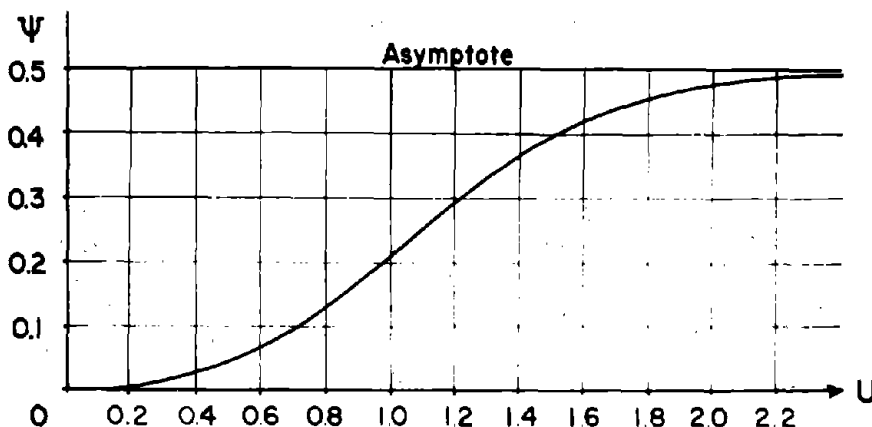


FIGURE 26. - Graph of the Function $\Psi = (U)$.

representation is shown in figure 26. Now for a given value of v , the value of Ψ is calculated from

$$\Psi = \frac{v}{2 V_{\max}},$$

the corresponding value U is found from the graph (fig. 26), and the radius R is

$$R = \frac{U}{\sqrt{\pi}} B.$$

For a 10-zone grid, that is, for $v = 0.1 V_{max}$, $0.2 V_{max}$, etc., the radii are as follows:

<u>Zone</u>	<u>v/V_{max}</u>	<u>R/B</u>
1	0.1	0.30
2	.2	.40
3	.3	.47
4	.4	.55
5	.5	.61
6	.6	.68
7	.7	.76
8	.8	.85
9	.9	.99
10	.98	1.20

The resulting integration grid is shown in figure 27. The arrow indicates the direction of the displacement. The angles of radial subdivision into equivalent sectors with respect to that direction are found from the sine function: one-quarter of displacement will result when integrating equation 32 between $\varphi = 0$ and $\varphi = \pm 30^\circ$. Thus the diagonal lines for radial subdivision make an angle of 30° with respect to the direction of displacement. For practical calculation, the center of the transparent grid is placed on the calculation point and the arrow is turned in the direction in which the displacement is to

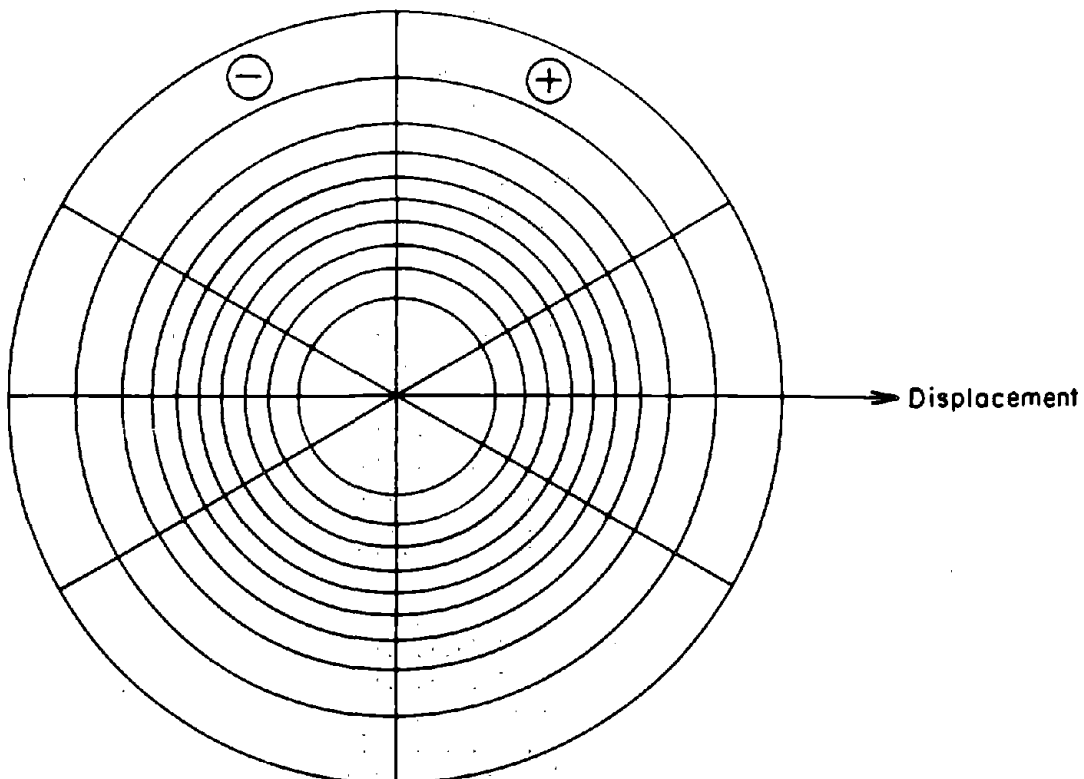


FIGURE 27. - Integration Grid for Horizontal Displacement.

be found. Extraction in the right semicircle causes displacement in that direction; extraction in the left semicircle causes displacement in the opposite direction. Extraction of one complete semicircle means the maximum possible displacement V_{max} in the corresponding direction. From this it is evident that extraction of only half the critical area produces the displacement V_{max} . Extraction of both semicircles, or critical extraction, causes displacements V_{max} in both opposite directions, thus zero displacement. Complete extraction (extraction ratio $b = 1$) of one part of the grid produces $\frac{1}{40} V_{max}$, partial extraction ($b < 1$) $\frac{b}{40} V_{max}$. If the extraction ratios of the grid parts of the right semicircle are denoted as b_r , and those of the left semicircle as b_l , the resulting displacement v is

$$v = \frac{\sum b_r - \sum b_l}{40} V_{max}.$$

A positive value means displacement in the direction of the arrow, a negative value, displacement in the opposite direction.

THE DERIVATIVES OF THE DISPLACEMENTS

If the vertical and horizontal displacements of profile points are known, the slope, the curvature, and the horizontal strain of that profile can easily be calculated. Small distances between the observation points are recommended in the region of maximum slope of the displacement curves (about 2 percent of the extraction depth). Slopes and strains are usually expressed in terms of S_{max}/h , and the curvature is expressed in terms of S_{max}/h^2 or by the radius of curvature. The curvature is practically the second derivative of the subsidence with respect to the lateral distance. If the horizontal displacements are proportional to the slope, at least in the two-dimensional case, the curvature is proportional to the strain. Slopes and strains are dimensionless (usually millimeters per meter), whereas the curvature has the dimension of an inverse length, for example, 5×10^{-7} meter⁻¹.

Subcritical extraction is of particular interest here because it may induce extreme curvatures and compressive strains. In case of an extraction area of half the critical width, that is, when $w = B$, the horizontal displacement over the unmined seam is the same as in case of critical extraction, but the decrease from V_{max} to zero over the mined area is much steeper. The result is high compressive strains at the center of the subcritical trough. The width of the subcritical area over which the maximum possible compressive strain occurs is normally 0.4 h to 0.5 h . It has also been expressed as a function of the angle of break (47). The value of the maximum strain ranges between 0.7 S_{max}/h and 1.7 S_{max}/h in different coalfields.

If the extraction area is critical, the maximum extensions and contractions are normally approximately equal. Values between 0.5 S_{max}/h and 0.9 S_{max}/h have been reported. The position of the point of maximum tensile strain is indicated by the angle of break β (fig. 1A). Values of β between 55° (soft shale) and 85° (thick-bedded sandstone) have been observed, often in good agreement with precalculations. The order of magnitude of slopes and curvatures for critical extraction is 1.5 to 3.0 $\frac{S_{max}}{h}$ resp. 3 to 15 $\frac{S_{max}}{h^2}$.

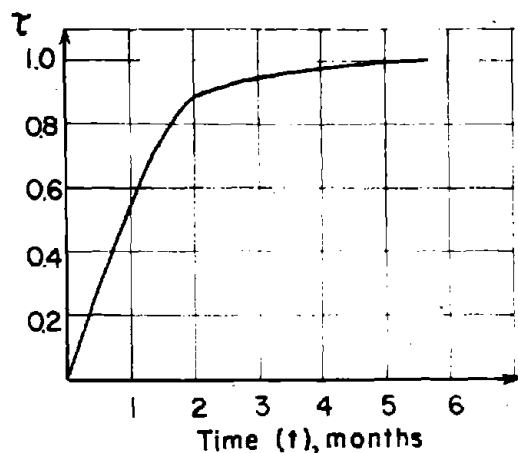


FIGURE 28. - Time Factor for Soft Strata.

final subsidence at the time of complete critical extraction. The depths varied from 150 to 700 meters. The same percentage was found in Northern France (17) for depths from 260 to 410 meters; the duration of aftereffects was about 6 months.

Apart from the size of the critical area and the rate of advance, the duration of movement depends on the following factors:

(1) Strata conditions: The process lasts longer in case of stiff and/or thick-bedded coal measures or overburdens than in case of compliant and/or thin-bedded strata. Durations of up to 7 years (aftereffect--2 years) have been observed for thick-bedded sandstone in Upper Silesia (42). For thin-bedded coal measures and soft tertiary overburden (14) in the Lower Rhine region, Germany, only a few months of movement were observed. In the Ruhr coalfield (cretaceous overburden), 4 to 5 years are normal, including about 1 year for aftereffects.

(2) Depth of extraction: The movements begin later and last longer when the depth is large. At shallow depths, they often begin after only a few days from the beginning of extraction. Exceptions are cases in which the undermined strata remain stable. (See reference 28.) In two Siberian coalfields, the duration of movements is about 2 years in case of shallow extraction (18, 54) but 4 to 5 years if the extraction is deeper than 400 meters (54).

(3) Kind of packing: The duration of movements is usually shorter in case of caving than in case of stowing.

(4) Former extractions: Strata which have already been deformed by earlier workings react more rapidly than strata affected for the first time.

In order to avoid consideration of the rate of advance, it is convenient to consider the development of subsidence due to small extraction areas which have been mined in a very short time. Figure 28 shows an example for relatively

THE EFFECTS OF TIME

The effects of time are perhaps the most uncertain factors in precalculation. The movements begin days to years after the beginning of extraction and last weeks to years after its end. With regard to a surface point which is completely undermined, the time taken for full subsidence to occur depends, of course, on the time it takes for extraction of the critical area, which in turn depends on the rate of advance and the depth of extraction, that is, the size of the critical area. Normally the greater proportion of subsidence occurs during that time, whereas the aftereffects are rather small. Extensive observations in British coalfields (57) have shown 94 percent of

mobile strata. It is assumed that extraction has been completed within the first month; then the ratio τ of instantaneous subsidence to final subsidence at any time t is as follows:

τ_1	= 0.55	at $t = 1$ month
τ_2	= .89	at $t = 2$ months
τ_3	= .94	at $t = 3$ months
τ_4	= .97	at $t = 4$ months
τ_5	= .99	at $t = 5$ months
τ_6	= 1.00	at $t = 6$ months

This ratio τ is also referred to as the time factor. An approximate analytical expression has been derived from the assumption that the rate of subsidence is proportional to the difference between final subsidence s and temporary subsidence s_t (25):

$$\frac{ds_t}{dt} = \text{Const} (s - s_t).$$

The solution is

$$s_t/s = \tau = 1 - e^{-ct} \quad (37)$$

where the factor c is a mechanical characteristic. Its dimension is time^{-1} . A similar time factor can be found for horizontal displacements, but only few observations have been reported as yet.

Figure 29 illustrates the qualitative effects of time. The working may have ceased after 11 months (March). For the point P_2 , the critical area is extracted, hence it will finally undergo full subsidence. If the area were mined instantaneously, no temporary slope, curvature, or strain would occur at P_2 . If the units indicated were not mined within a month but within, say, 5 years each, P_2 would consecutively undergo the final values of displacements

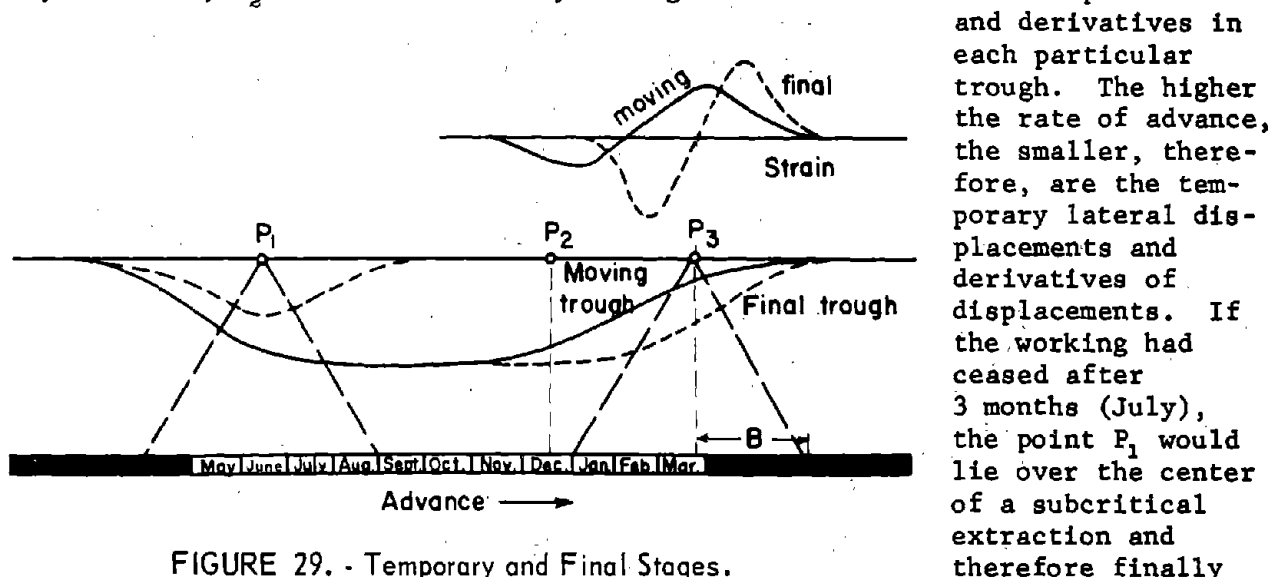


FIGURE 29. - Temporary and Final Stages.

undergo considerable curvature and compressive strain. Since the working continues, only part of that curvature occurs, and that part will be smaller, the higher is the rate of advance.

The movements accompanying the advancing face are illustrated for the surface point P_3 in figure 29. At the moment at which the working ceases (March), the "traveling" curvature and tensile strain are relatively high at P_3 . In the final trough, no curvature or strain remains at P_3 (but there is maximum slope at this point). A surface structure at P_3 could be protected by extracting the critical area for P_3 at a rate at which the temporary tensile strains are small enough to cause no damage. The necessary rate of advance can be calculated from the final values of displacements and the time factor. For idealized two-dimensional problems, analytical solutions can be developed (25, 46, 60). When using, for example, the profile function defined by equation 4 and the time factor according to equation 37, and applying the law of superposition, the maximum convex curvature $C_{t \max}$ over an advancing face is (25)

$$C_{t \max} = 1.52 \frac{S_{\max}}{B_2} \left[1 - \exp \left(- \frac{cB}{u} \right) \right],$$

where u is the rate of advance. The factor 1.52 has been found for Upper Silesia.

Example (from 25):

$$S_{\max} = 87 \text{ cm}, h = 275 \text{ meters}, B = 0.58 h = 160 \text{ meters},$$

$c = 1 \text{ year}^{-1}$. The final curvature ($u \rightarrow \infty$) is

$$C_{\max} = 1.52 \frac{0.87}{160^2} = 52 \times 10^{-8} \frac{\text{millimeter}}{\text{meter}^2}$$

from which severe damages to structures must be expected. If the permissible curvature is $C_{\text{perm}} = 20 \times 10^{-8}$, the necessary rate u is found from the condition

$$C_{\text{perm}} = C_{\max} \left[1 - \exp \left(- \frac{160}{u} \right) \right].$$

The result is $u = 326$ meters per year, which is the slowest permissible rate of advance.

In general, however, numerical solutions must be used. By applying the law of superposition, the displacement-time curves of single points are determined from the time-dependent contributions of extraction parts. These contributions in turn are found by multiplying the final displacement values by the time factor. The calculation procedures are lengthy and time-consuming, and use of electronic computers is more convenient. As an illustration, the subsidence of the point P_3 in figure 29 will be considered.

The area affecting point P_3 is subdivided into three parts, each of which is mined within one month. First, the contribution Δs_1 of each part to the final subsidence s of P_3 is calculated, using one of the methods described before. The results may be $\Delta s_1 = 45$ mm (for the part mined in January), $\Delta s_2 = 162$ mm (February), and $\Delta s_3 = 364$ mm (March). Thus the complete final subsidence of P_3 is $s = 571$ mm. The time factor represented in figure 28 may apply, the unit of time being one month.

At $t = 1$ (end of January), the subsidence due to the first extraction part is $\tau_1 \times \Delta s_1 = 0.55 \times 45$ mm. At $t = 2$, it is $\tau_2 \times \Delta s_1 = 0.89 \times 45$ mm. But at the same time, extraction of the second part is complete, the time factor being τ_1 . Thus the subsidence s_{t2} of P_3 at $t = 2$ is

$$s_{t2} = \tau_2 \Delta s_1 + \tau_1 \Delta s_2.$$

Accordingly, the complete system of equation is as follows:

$$s_{t1} = \tau_1 \Delta s_1$$

$$s_{t2} = \tau_1 \Delta s_2 + \tau_2 \Delta s_1$$

$$s_{t3} = \tau_1 \Delta s_3 + \tau_2 \Delta s_2 + \tau_3 \Delta s_1$$

$$s_{t4} = \tau_2 \Delta s_3 + \tau_3 \Delta s_2 + \tau_4 \Delta s_1$$

$$s_{t5} = \tau_3 \Delta s_3 + \tau_4 \Delta s_2 + \tau_5 \Delta s_1$$

$$s_{t6} = \tau_4 \Delta s_3 + \tau_5 \Delta s_2 + \tau_6 \Delta s_1$$

$$s_{t7} = \tau_5 \Delta s_3 + \tau_6 \Delta s_2 + \tau_6 \Delta s_1$$

$$s_{t8} = \tau_6 \Delta s_3 + \tau_6 \Delta s_2 + \tau_6 \Delta s_1$$

The final subsidence s is given by s_{t8} , since $\tau_8 = 1.00$. The calculation is shown in the following table. The last column gives the result, which is the subsidence s_t at the time t . All calculations are given in millimeters.

t, months	Δs_1	$\tau_1 \Delta s_1$	$\tau_2 \Delta s_1$	$\tau_3 \Delta s_1$	$\tau_4 \Delta s_1$	$\tau_5 \Delta s_1$	$\tau_6 \Delta s_1$	$\tau_6 \Delta s_1$	$\tau_6 \Delta s_1$	s_t
1 (January).	45	25	-	-	-	-	-	-	-	25
2 (February)	162	89	40	-	-	-	-	-	-	129
3 (March)...	364	200	144	42	-	-	-	-	-	386
4 (April)...	-	-	324	152	44	-	-	-	-	520
5 (May)....	-	-	-	342	157	45	-	-	-	544
6 (June)...	-	-	-	-	353	160	45	-	-	558
7 (July)...	-	-	-	-	-	360	162	45	-	567
8 (August).	-	-	-	-	-	-	364	162	45	571

The inverse procedure is used for determining the values of the time factor from measured subsidences s_t .

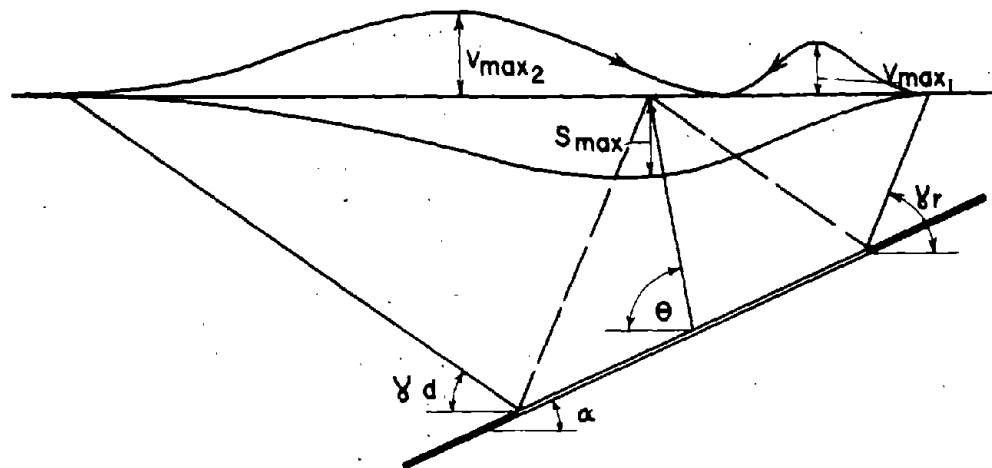


FIGURE 30. - Displacements Over an Inclined Seam.

SURFACE DISPLACEMENTS OVER INCLINED SEAMS

Extraction of inclined seams causes unsymmetrical trough profiles, as illustrated in figure 30. The main features are--

1. The maximum subsidence does not occur over the middle of the mined area and is often accompanied by horizontal displacement in the direction of the rise of the beds.
2. With increasing inclination of the beds,
 - a. The limit angle to the rise increases;
 - b. The limit angle to the dip decreases;
 - c. The horizontal displacements in the direction of the rise increase;
 - d. The horizontal displacements in the direction of the dip decrease.

Critical, supercritical, and subcritical extraction are defined as in case of flat seams. Figure 30 refers to critical extraction. For characterizing the profile, the following terms can be used:

- α = angle of dip;
- γ_r = limit angle to the rise;
- γ_d = limit angle to the dip;
- θ = angle between the horizontal and the line joining the center of the extraction area to the surface point of maximum subsidence;

V_{max_1} = maximum possible horizontal displacement in the direction of the dip;

and V_{max_2} = maximum possible horizontal displacement in the direction of the rise.

In the direction of the strike of the seam, the value of the limit angle lies between γ_d and γ_r . If the position of the point of full subsidence with respect to the ribsides is determined by lines parallel to the limit lines (dashed line in figure 30), the angle θ becomes a function of the limit angles. This determination, which results from the law of superposition, is not generally valid.

Normally the inclined carboniferous strata have an overburden of flat younger formations which affects the values of the limit angles. This may be allowed for by different directions of the limit lines in the coal measures and in the overburden, as indicated by the dotted lines in figure 31. This distinction can also be drawn in case of flat seams. The values of γ_r and γ_d observed at the surface are then a function of the depth of extraction. Even if there is no discordant overburden, the dependence of the angles γ_r , γ_d and θ on the dip of the seam can only be found from local observations and cannot be expressed uniformly.

The usual precalculation methods are generalizations of the methods developed for flat stratification. When using a profile function, the first step is to express the full subsidence in terms of the dip of the seam. Several formulas are in use, the oldest and most common of which is

$$S_{max} = a m \cos \alpha, \quad (38)$$

where a is the subsidence factor for horizontal strata. The simplest way of calculating a complete profile is to apply the same profile function as in case of flat strata, taking different lengths for both semiprofiles according to the position of the point of maximum subsidence. This will be illustrated

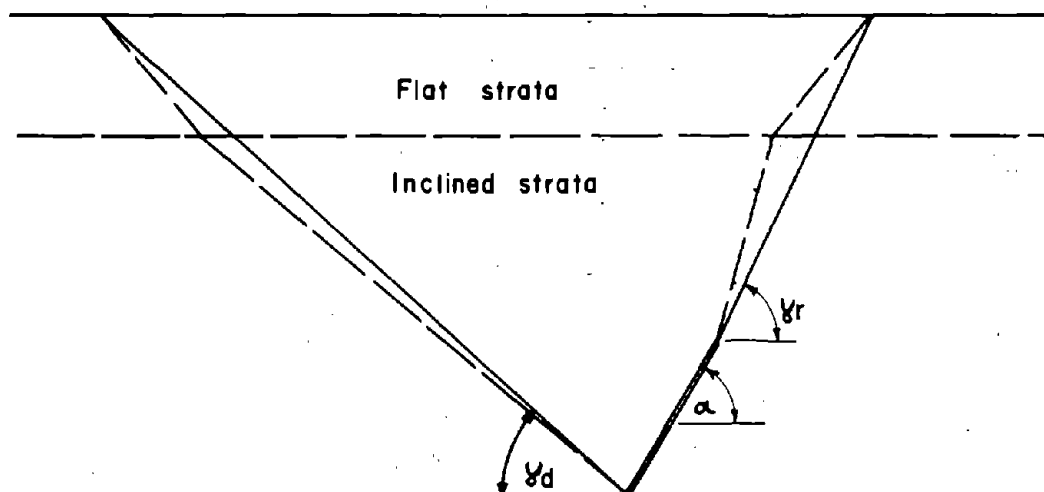


FIGURE 31. - Influence of Flat Overburden.

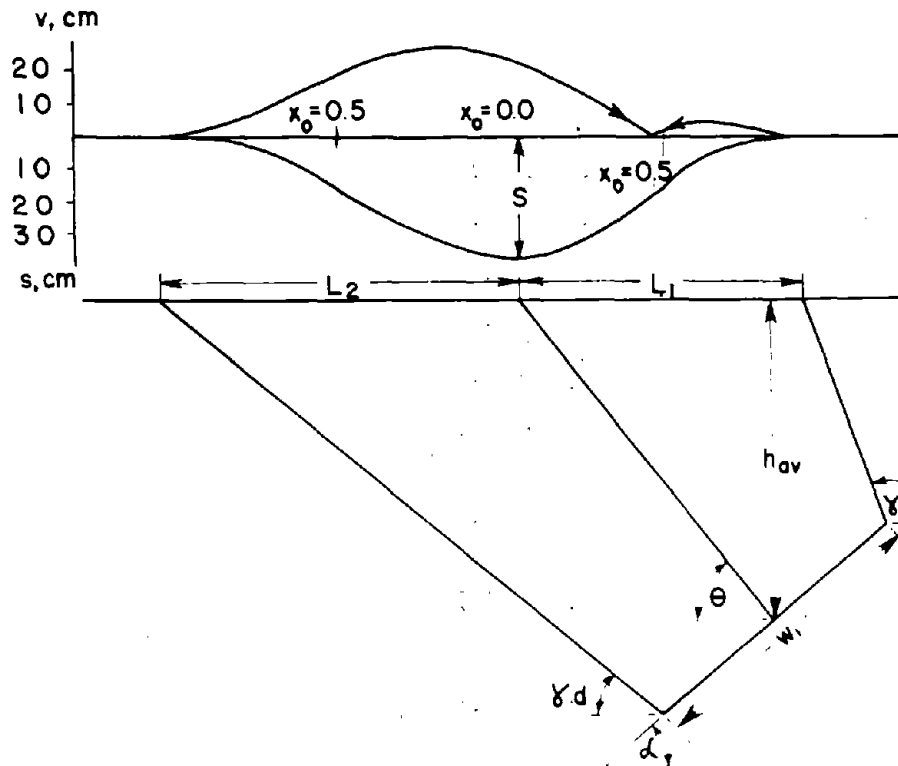


FIGURE 32. - Calculation Through Profile Functions.

must be known. From the data of the extraction area and the angles γ_r , γ_d and θ , the position of the point of maximum subsidence is determined. Its distance from the edges of the trough is L_1 in the direction of the rise and L_2 in the direction of the dip. The full subsidence is found from equation 38 and the individual maximum subsidence from equation 10. Now the profiles are calculated from equations 11 and 31, taking the different values L_1 and L_2 as the lengths of the semiprofiles. For the horizontal displacements, an additional component is superimposed which depends on the dip and on the thickness h_b of the horizontal overburden and is proportional to the subsidence. If v_f is the displacement which would occur in case of horizontal strata, from equation 31, then the entire displacement v is expressed by the empirical equation

$$v = ks \pm v_f,$$

where

$$k = \tan \alpha - 0.02 \frac{h_b}{S}.$$

The positive sign indicates the direction of the rise of the seam. The calculation procedure is shown in the following example (fig. 32):

for the profile function equation 6, from which a calculation method for inclined stratification has been derived in reference 15. (See figure 32.)

The extraction area is characterized by the angle of dip (α), the depth at both edges (h_1 , h_2), the width in both directions (w_1 , w_2), and the thickness of the extracted seam (m). As parameters, the angles γ_r , γ_d , and θ , the subsidence factor, and the ratio V_{max}/S_{max}

Example (from 15)--Given:

$$\begin{array}{lll} \gamma_r = 110^\circ & m = 90 \text{ cm} & w_1 = 90 \text{ meters} \\ \gamma_d = 41^\circ & \alpha = 40^\circ & w_2 = 450 \text{ meters} \\ \theta = 50^\circ & h_1 = 71 \text{ meters (upper edge)} & \\ & h_2 = 129 \text{ meters (lower edge)} & \end{array}$$

Values in case of horizontal strata: $a = 0.6$

$$V_{max} = 0.3 S_{max}$$

1. Full subsidence (equation 38): $S_{max} = 41 \text{ cm}$

2. Average depth: $h_{av} = 100 \text{ meters}$

3. Lengths of semiprofile: $L_1 = 88 \text{ meters}$
 $L_2 = 105 \text{ meters}$

4. Coefficients n (equation 9) $n_1 = 0.9 \frac{w_1}{h_{av}} = 0.8$
 $n_2 = 0.9 \frac{w_2}{h_{av}} > 1.0$

5. Maximum subsidence (equation 10): $S = 37 \text{ cm}$

6. Semiprofile of subsidence (equation 11):

$$s = 37 \left[0.64 \left(1 - x_0 + \frac{\sin 2\pi x_0}{2} \right) + 0.09 (1 + \cos \pi x_0)^2 \right],$$

where x_0 = ranges between 0 (maximum subsidence) and 1 (zero subsidence).

$x_0 = 1$ is the length L_1 or L_2 , respectively.

when $x_0 = 0.5$, for example, $0.5 \times L_1 = 44 \text{ meters}$ for the semiprofile in the direction of the rise of the seam, and 52.5 meters for the other half. $s(x_0 = 0.5)$ is found to be 15.2 cm.

7. Semiprofile of lateral displacement, flat strata (equation 31):

$$v_f = 55 \left[1.28 \sin^2 \pi x_0 + 0.57 \sin \pi x_0 (1 + \cos \pi x_0) \right],$$

which gives, for example, at $x_0 = 0.5$ $v_f (x_0 = 0.5) = 10.2 \text{ cm}$. Now the thickness of horizontal overlying strata has been found to be 5 meters, thus $k = 0.6$ and

$$v = 0.6 s \pm v_f$$

It is based on the principle of equivalence as shown in figure 33: According to the general equivalence (see fig. 4), the common effect of three horizontal extraction areas dA at different depths is equal to the effect of a uniform horizontal extraction area A_2 . Now when an inclined extraction area A_1 is composed of an infinite number of infinitesimal horizontal elements, its entire effect on the surface point P will also be equal to the effect of the horizontal area A_2 . Thus, if the influences of the elements of a horizontal critical area A_2 on the point P are given by an influence function, these influences can be transferred to the elements of an inclined critical area by central projection. For transferring the zonal subdivision of an integration grid, the procedure is illustrated in figure 34, as follows:

It is assumed that the critical area AB is given by the limit angles γ_r and γ_d . The line PM is drawn which bisects any horizontal line between the limit lines outgoing from P . A horizontal line CD is drawn through M , indicating a fictitious horizontal extraction area equivalent to the real extraction area AB . Then CD is subdivided into zones (here five zones) using the same subdivision as in case of horizontal stratification. By central projection, with P as the center, these zones are transferred to the inclined critical area AB . The subdivision in plan view is then found by vertical parallel projection from AB to the horizontal. The projection of the surface point P does not coincide with the center M of the subdivision, as in the case of horizontal strata. The zones in the direction of the strike of the seam are transferred by the corresponding procedure, assuming a circular horizontal reference area CD . The angles of the diagonal sectorial lines, which are 45° for vertical and 30° for horizontal displacements in case of horizontal strata (figs. 22 and 27), change their values when being transferred to AB . In plan view, the zones become elliptic and can easily be constructed since four points of each ellipse are known. If the angle α of dip is equal to or greater than the limit angle γ_r in the direction of the rise, the outer envelope of the grid becomes parabolic or hyperbolic. Figure 35 shows a complete subsidence grid, using different limit angles for the coal measures and the overburden. Extraction of each part causes $\frac{1}{40} S_{max}$. For practical calculation, the point P in the grid must be placed on the point the subsidence of which is to be determined, and the arrow must point to the rise of the seam. Since the dimensions of the grid are proportional to the vertical distance between the surface point and the inclined seam plane, one and the same grid can only be used for a profile in the direction of the strike; otherwise the grid must be made smaller or larger proportional to the depth of the seam below the point under consideration. For calculating horizontal displacements, the values of V_{max1} and V_{max2} must be known, and the weight of the grid parts which represent the upper half of the critical area becomes greater than the weight of the lower half (which, in plan view, covers a much larger area). The full subsidence is here calculated from equation 1, introducing subsidence factors between 0.3 and 0.5 (stowing), and the only strata parameters are the limit angles γ_r and γ_d . For inclinations of more than 50° , the full subsidence is practically only reached when mining at shallow depths.

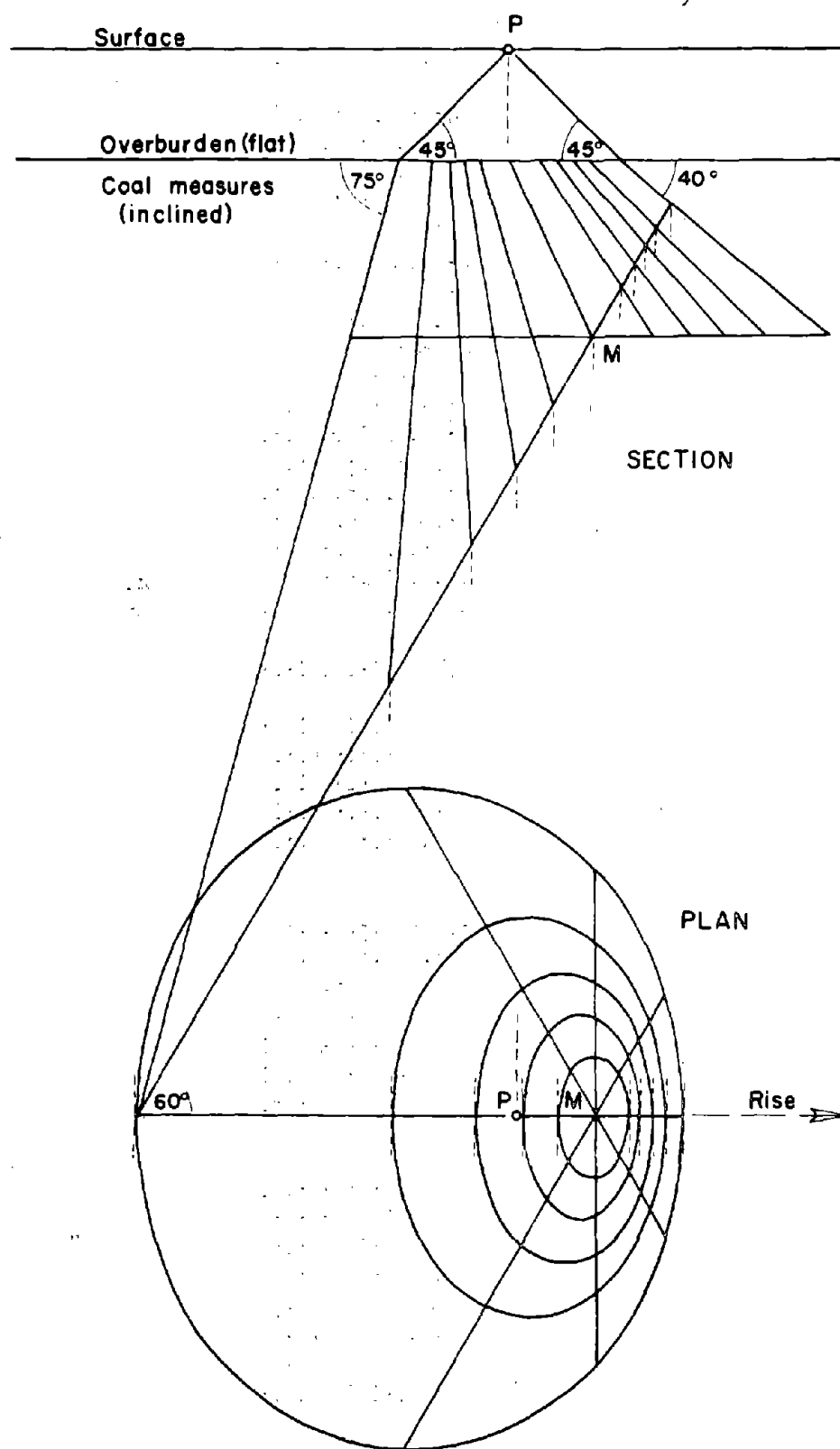


FIGURE 35. - Subsidence Integration Grid.

RANGE OF APPLICATION AND PRECISION

Compared with measured values, the results of precalculations are better for subsidence than for horizontal displacements, for final than for instantaneous states, and for horizontal than for inclined stratification. Deviations in profile shape are possible, especially in case of temporary profiles and inclined strata, whereas final profiles over flat seams mostly deviate only in magnitude. These quantitative differences seem to result from the uncertainty in determining the subsidence factor and the critical radius rather than from errors introduced by simplifying assumptions. In central trough regions, agreements within ± 20 percent for subsidences and ± 30 percent for horizontal displacements have often been obtained. In the edge regions, especially for displacements less than 10 cm, the relative deviations usually become larger. As for temporary values or displacements over inclined seams, differences of ± 50 percent are a good result; only the maximum subsidence can be predicted more accurately. Some of the calculation methods are also applied to levels below surface, especially in order to predict the effects on shafts. The general experience is then that the discrepancies increase with decreasing distance from the level of extraction, a comprehensible result in view of the simplifications inherent in the basic relationships.

PHYSICAL MODEL INVESTIGATIONS

Model investigations have been used to find or check fundamental relations concerning the development of the subsidence process from the working to the surface. They are practically restricted to the two-dimensional case because three-dimensional models are expensive and hardly accessible to observation. Any quantitative interpretation involves the problem of mechanical similitude.

A relatively successful quantitative approach was obtained in Russian experiments with "equivalent" model materials (43). In an actual problem, the natural strata were imitated at a geometric scale of 1:100 by model materials cemented by gypsum. The approximate uniaxial compressive and tensile strength of the material was chosen according to mechanical similitude, thus the model can be considered approximately equivalent with respect to strength. A good quantitative agreement between model results and field measurements of subsidence was found.

Most model experiments, however, are concerned with more qualitative questions, especially the justification of basic concepts. As continuous model materials, solidified gelatin (24) and partially elastic "Moltopren" layers (21) were used. More frequently, the material has been more or less granular, like sand (45), stratified sand (6, 33), granulated glass (38), and granulated metal (52). For evaluating problems such as the applicability of the principle of superposition, the depth-dependence of the limit angle, or the effects of previous extractions, model experiments can be qualitatively informative.

ABSTRACT MODEL INVESTIGATION

Idealized model concepts are introduced for applying analytical methods of conventional mechanics. They are either abstractions of general stress-strain relations, or they refer specifically to ground movements.

The first group is based on general physical conditions like those of equilibrium and continuity and on particular boundary conditions. For specifying the material, the concepts of elasticity, viscoelasticity, and viscoplasticity have been used. When comparing these methods with the methods described before, the main practical difference is that the abstract models are based on laboratory and field measurements of material properties, whereas the previous methods were based on field observations of actual ground movements. The question arises, however, whether the values of the parameters selected are appropriate for any given instance, because these parameters are seldom measurable on the scale of the rock mass. It is sometimes possible to back-calculate appropriate physical parameters inasmuch as the subsidences are known from field studies and useful generalizations may then be derived theoretically. The models generally represent continuous⁵ materials, and the analyses can be subdivided as follows:

1. Elastic behavior

- a. Isotropic elasticity (4, 19, 30, 55): The main differences of the particular solutions lie in the boundary conditions.
- b. Anisotropic elasticity (3, 4, 48): Different assumptions are introduced concerning the friction and cohesion between the elastic layers.

2. Viscoelastic behavior: In reference 22, an analysis is made that includes the cases of isotropy and of horizontal stratification.

3. Viscoplastic behavior: In reference 11, a solution for the final state of deformation is derived, referring to an incompressible nonlinear model (Bingham-Schwedoff).

Two concepts have been devised with special regard to the mine subsidence process and seem to be adaptable especially to discontinuous materials. They are--

4. The "Stochastic" medium (6, 31-33), which is characterized by basic relationships between the displacements instead of stress-strain relationships, such as the principle of superposition, and illustrated to some extent by granular materials. Figure 36 shows a subsidence trough for elastic, stochastic, and combined behavior (16), the latter case representing a rock mass that has broken in its lower part and remained continuous in its upper part. The concept of gap-diffusion (10), which interprets the propagation

⁵There are several theoretical models employing finite element methods which treat discontinuous materials.

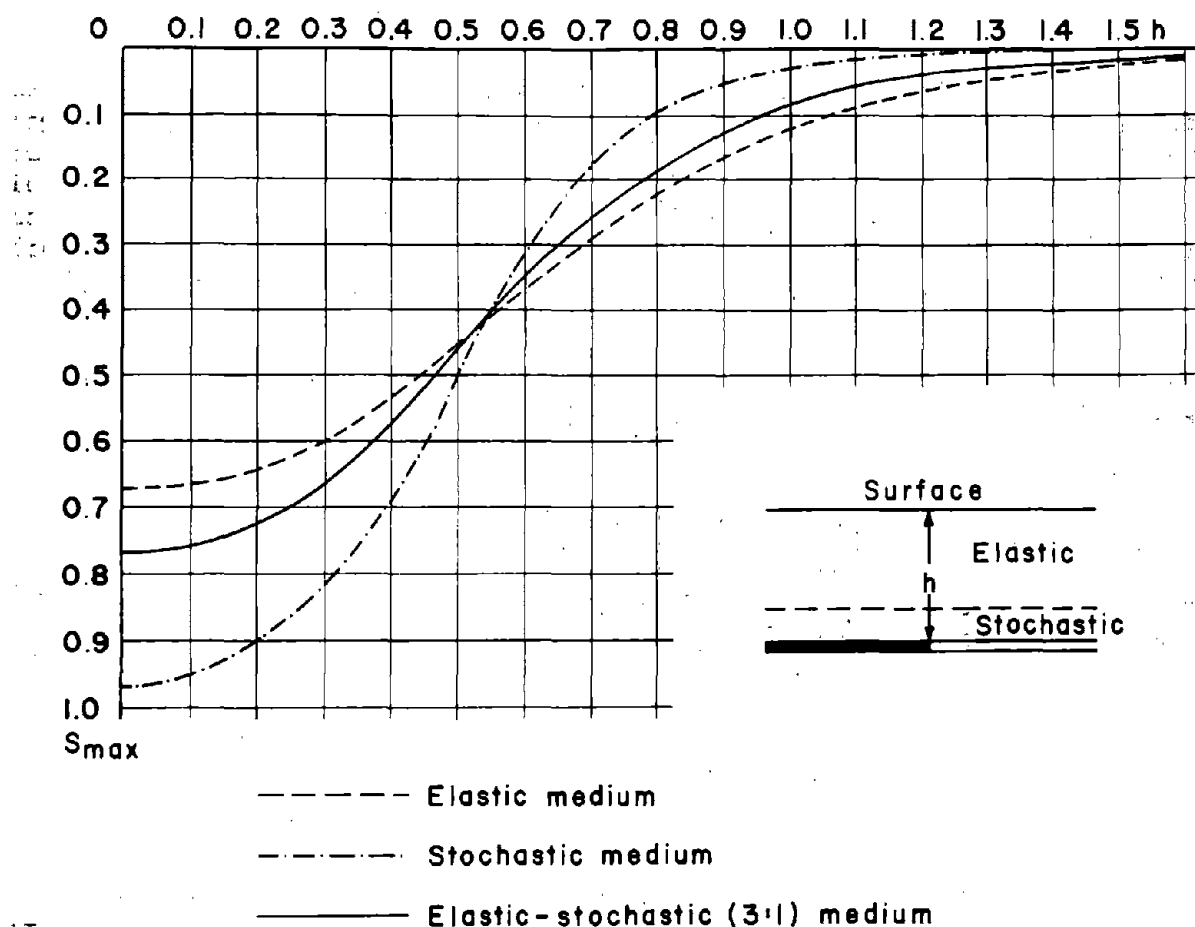


FIGURE 36. - Theoretical Semiprofiles of Subsidence Trough (16).

of displacements in the rock mass as a diffusion of mass deficiencies outgoing from the mined seam as the initial deficiency. Particular solutions have not yet been worked out.

As mechanical interpretations of the subsidence process, the abstract models go deeper than the usual methods. Application to concrete problems requires a number of simplifying assumptions and raises the question of adaptation to real rock and of determining the parameters. This may be a reason why they have been rarely used in practice.

CONCLUSION

In all precalculation methods, the parameters for characterizing the rock mass must be determined from observed displacements. This may be a disadvantage, but it is questionable if there are more appropriate ways of getting representative quantitative information. Most adaptable seems to be the influence function method, which is not necessarily an alternative to abstract models, because influence functions, for example of the exponential form (equation 20), can be derived also from such model analyses. The problems

involved concern fundamental relations rather than mathematical refinement: of particular interest are, for example, the range within which the law of superposition applies, the dependence of the subsidence factor on the distance from the ribsides, and the effects of older workings. These problems occur in any empirical or analytical treatment of the subsidence process, and can be investigated by underground observations and by model experiments.

REFERENCES⁶

1. Avershin, S. G. (Strata Displacement Caused by Underground Extraction.) Llgletekhizdat, Moscow, 1947 (in Russian).
2. Bals, R. (A Contribution to the Problem of Precalculating Mining Subsidences.) Mitteilungen aus dem Markscheidewesen, v. 42/43, 1931/1932, pp. 98-111 (in German).
3. Berry, D. S. A Theoretical Elastic Model of the Complete Region Affected by Mining a Thin Seam. Proc. 6th Symp. Rock Mech., Rolla, Mo., 1964, pp. 310-329.
4. Berry, D. S., and T. W. Sales. An Elastic Treatment of Ground Movement Due to Mining. J. Mech. Phys. Solids, pt. 1, v. 8, 1960, pp. 280-292; pt. 2, v. 9, 1961, pp. 52-62; pt. 3, v. 10, 1962, pp. 73-83.
5. Beyer, F. (On Predicting Ground Deformations Due to Mining Flat Seams.) Habilitation Thesis, Tech. Univ., Berlin, 1945 (in German, unpublished, but described in reference 42).
6. Bodziony, J., J. Litwiniszyn, and A. Smolarski. New Research Into Rock Masses Treated as Media Characterized by Stochastic Equations. Proc. Internat. Conf. Strata Control, Paris, 1960, pp. 137-150.
7. Bräuner, G. (On Precalculating Ground Movements in Case of Inclined Extraction.) Mitteilungen aus dem Markscheidewesen, v. 66, No. 2, 1959, pp. 35-47 (in German).
8. _____. (On the Application of Subsidence and Displacement Grids to the Stochastic Theory of Rock Movements in Flat Formations.) Mitteilungen aus dem Markscheidewesen, v. 68, No. 1, 1961, pp. 1-16 (in German).
9. Briggs, H. Mining Subsidence. Arnold & Co., London, 1929, 215 pp.
10. Buchheim, W. (The Mathematical Treatment of Subsidence Processes in Loose Masses as a Process of Diffusion of Rock Gaps.) Proc. Symposium Rock Mech., Cracow (Poland), 1966, pp. 67-74 (in German).
11. Cherny, G. J. (Determination of Surface Subsidence and Deformation by Rock Movements in Form of Rheologic Flow.) Izv. VUZ, Gorny Zh., v. 9, No. 7, 1966, pp. 3-9 (in Russian).
12. Drent, S. Some Considerations on the Connection Between Time-Curves and the Thickness of the Non-Carboniferous Overburden in the South Limburg Coalfield. Proc. Europ. Cong. Ground Movement, Leeds, 1957, pp. 49-57.
13. Ehrhardt, W., and A. Sauer. (Precalculation of Subsidence, Tilt, and Curvature Over Extractions in Flat Formations.) Bergbauwissenschaften, v. 8, 1961, pp. 415-428 (in German).

⁶Titles enclosed in parentheses are translations from the language in which the item was originally published.

14. Fläschentragner, H. Considerations in Ground Movement Phenomena Based on Observations Made in the Left Bank Lower Rhine Region. Proc. Europ. Cong. Ground Movement, Leeds, 1957, pp. 58-73.
15. General Institute of Mining Surveying. (The Movements of the Rock Masses and of the Surface in the Main Coalfields of the Soviet Union.) Ugletekhizdat, Moscow, 1958, 250 pp. (in Russian).
16. Gil, H. (The Distribution of Displacements in Horizontal Zones Being Composed of a Granular and an Elastic Medium.) Archiwum górnictwa, v. 11, 1966, pp. 279-310 (in Polish).
17. Grard, C. (Mining Subsidence and the Means Permitting the Limiting of Their Effects on the Surface.) Revue de l'Industrie Minière, v. 51, January 1969, pp. 35-70 (in French).
18. Grigorovich, V. T., and others. Surface Subsidence During the Working of a Sequence of Seams at the 'Kaiekan' Pit of the Noril'sk Coalfield. Soviet Min. Sci., No. 2, 1965, pp. 86-93.
19. Hackett, P. An Elastic Analysis of Rock Movements Caused by Mining. Trans. Inst. Min. Engrs., v. 118, pt. 7, April 1959, p. 421.
20. Hiramatsu, Y., and Y. Oka. Precalculation of Ground Movements Caused by Mining. Internat. J. Rock Mech. Min. Sci., v. 5, 1968, pp. 399-414.
21. Hoffman, H. The Effects of Direction of Working and Rate of Advance on the Scale-Deformation of a Self-Loaded Stratified Model of a Large Body of Ground. Proc. Internat. Conf. Strata Control, New York, 1964, pp. 397-411.
22. Imam, H. F. A Viscoelastic Analysis of Mine Subsidence in Horizontally Laminated Strata. Ph. D. Thesis, Univ. of Minnesota, 1965, 126 pp.
23. Keinhorst, H. (Considerations on the Problem of Mining Damages.) Glückauf, v. 70, 1934, pp. 149-155 (in German).
24. King, H. J., and J. T. Whetton. Mechanics of Mine Subsidence. Proc. Europ. Cong. Ground Movement, Leeds, 1957, pp. 27-38.
25. Knothe, S. (Rate of Advance and Ground Deformation.) Bergakademie, v. 5, No. 12, 1953, pp. 513-518 (in German).
26. _____. Observations of Surface Movements Under Influence of Mining and Their Theoretical Interpretation. Proc. Europ. Cong. Ground Movement, Leeds, 1957, pp. 210-218.
27. Köchmanski, T. (Integral-Theory of Ground Movements Resulting From Extraction.) Freiburger Forschungshefte, A 118, April 1959, pp. 36-56 (in German).

28. Kumar, S. R., and B. Singh. Investigations on Surface Subsidence in Mine No. 1. Central Min. Res. Station, Dhanbad, Bihar, India, Research Paper 34,, May 1967, 17 pp.
29. Lenge, A. Inspection and Correction of Damage to Railways Caused by Mining in the Saar. Proc. Europ. Cong. Ground Movement, Leeds, 1957, pp. 106-114.
30. Lisowski, A. (Rock Displacements According to the Theory of Elasticity.) Archiwum górnictwa, v. 1, 1956, pp. 76-94 (in Polish).
31. Litwiniszyn, J. The Theories and Model Research of Movements of Ground Masses. Proc. Europ. Cong. Ground Movement, Leeds, 1957, pp. 202-209.
32. _____. Time-Space Processes in Stochastic Media. Bull. Acad. Polonaise Sci., Série des sci. techn., 1958/59.
33. _____. On Certain Linear and Nonlinear Strata Theoretical Models. Proc. Internat. Conf. Strata Control, New York, 1964, pp. 384-394.
34. Loos, W. (The Occurrence of the Subsidence Trough in the Saar Coalfield.) Mitteilungen aus dem Markscheidewesen, v. 67, No. 5, 1960, pp. 264-265 (in German).
35. Maize, E. R., E. Thomas, and H. P. Greenwald. Studies of Roof Movement in Coal Mines. 4. Study of Subsidence of a Highway Caused by Mining Coal Beneath. BuMines Rept. of Inv. 3562, 1941, 11 pp.
36. Marr, J. E. A New Approach to the Estimation of Mining Subsidence. Trans. Inst. Min. Eng., v. 118, 1958/59, pp. 692-706.
37. Martos, F. Concerning an Approximate Equation of the Subsidence Trough and Its Time Factor. Proc. Internat. Strata Control Cong., Leipzig, 1958, pp. 191-205.
38. _____. (On A Statistical Method of Precalculating Ground Movements Caused by Mining.) Freiburger Forschungshefte, A 404, 1966, pp. 319-330 (in German).
39. National Coal Board--Production Department. Subsidence Engineers' Handbook, 1966, 130 pp.
40. _____. Principles of Subsidence Engineering. Bull. 63/240, 1963, 27 pp.
41. Niederhofer, G. (A New Method of Precalculating Surface Subsidences, Particularly Over Steep Workings.) Mitteilungen aus dem Markscheidewesen, v. 69, No. 2, 1962, pp. 33-75 (in German).
42. Niemczyk, O. Bergschadenkunde (Study of Mining Damages). Verlag Glückauf, Essen, W. Germany, 1949, 291 pp. (in German).

43. Ogloblin, D. M. (Basic Laws Governing the Process of Rock Movement in Working Several Shallow-Dip Seams.) Freiburger Forschungshefte, A 185, 1960, pp. 63-76 (in German).
 44. Orchard, R. J., and W. S. Allen. Ground Curvature Due to Coal Mining. The Chartered Surveyor, v. 97, May 1965.
 45. Pariseau, W. G., and H. D. Dahl. Mine Subsidence and Model Analysis. Trans. AIME, 1968, v. 241, pp. 488-494.
 46. Perz, F. (The Influence of Time on Ground Movements Above Workings.) Mitteilungen aus dem Markscheidewesen, v. 55, 1948, pp. 92-117 (in German).
 47. Rellensmann, O., and E. Wagner. The Effect on Railways of the Ground Movements Due to Mining. Proc. Europ. Cong. Ground Movement, Leeds, 1957, pp. 74-82.
 48. Salamon, M. D. G. Elastic Analysis of Displacements and Stresses Induced by the Mining of Seam or Reef Deposits. J. S. African Inst. Min. Met., v. 64, 1963/64, pp. 128-149, 197-218, 468-500.
 49. Sann, B. (Considerations on Precalculating Ground Subsidences Due to Coal Mining.) Bergbau-Rundschau, 1949, pp. 163-168 (in German).
 50. Schleier, O. (Precalculation of Ground Movements.) Der deutsche Steinkohlenbergbau. Essen, W. Germany, 1956, pp. 452-508 (in German).
 51. Schulte, H. F. The Effects of Subsidence on the Strata Immediately Above a Working, With Different Types of Packing and in Level Measures. Proc. Europ. Cong. Ground Movement, Leeds, 1957, pp. 188-198.
 52. Shalaghinov, N. F. (The Limit Angles of Ground Movement.) Ugol, No. 11, 1963, p. 40 (in Russian).
 53. Singh, T. N., and B. Singh. Angle of Draw in Mine Subsidence. J. Mines, Metals & Fuels, v. 16, 1968, pp. 253-258.
 54. Strelnikov, K. V., and V. P. Samarin. (The Duration of Surface Subsidence When Mining Seams at Larger Depths in the Kizelov District.) Gorn davlenie i metodika marksheydersk. rabot (VNIMI, 56), L, 1966, pp. 139-144 (in Russian).
 55. Szefer, G. (The Effect of Mining on the State of Stress and Strain of the Rock Mass According to the Theory of Elasticity.) Zeszyty problemowe górnictwa, No. 1, 1964, pp. 3-54 (in Polish).
- VNIMI. See reference 15.
56. Voight, B., and W. Pariseau. The Nature of Prediction in Subsidence Engineering. ASCE Meeting, 1968, Pittsburgh, Preprint 762, 42 pp.

57. Wardell, K. Some Observations on the Relationship Between Time and Mining Subsidence. Trans. Inst. Min. Eng., v. 113, 1953-54, pp. 471-483, disc. p. 799.
58. _____. Surface Ground Movements Associated With the Total and Partial Extraction of Stratified Mineral Deposits. M.S. Thesis, University of Nottingham, 1965, 167 pp.
59. Wardell, K., and N. E. Webster. Some Surface Observations and Their Relationship to Movement Underground. Proc. Europ. Cong. Ground Movement, Leeds, 1957, pp. 141-148.
60. Weir, A. M. Subsidence--the Interpretation of Traveling Strain Observations. Colliery Guardian, v. 212, 1966, pp. 576-577.
61. Zenc, M. Comparison of Bals' and Knothe's Methods of Calculating Surface Movements Due to Underground Mining. Internat. J. Rock Mech. Min. Sci., v. 6, 1969, pp. 159-90.

APPENDIX.--MAIN SYMBOLS USED IN GROUND MOVEMENT CALCULATIONS

The symbols are largely the same as suggested by the British National Coal Board (39).

- a Subsidence factor
- d Horizontal distance of the inflection point of a subsidence profile from the edge of the area of extraction
- h Depth of mining (depth below surface)
- h_c Critical depth: depth at which extraction causes ground deformations equal to values allowable for given structures.
- p Value of a subsidence influence function
- q Value of an influence function for horizontal displacements
- m_s Seam thickness extracted
- s Subsidence of a point
- s'_0 Maximum possible slope under given conditions
- t Time
- v Horizontal displacement of a point in a specified direction
- B Radius of a critical area of extraction
- E_{max} Maximum possible tensile strain under given conditions
- E_t Maximum traveling tensile strain
- S_{max} Maximum possible subsidence; also, full subsidence
- V_{max} Maximum possible horizontal displacement under given conditions
- α Angle of dip
- β Angle of break
- γ Limit angle
- ϵ Normal strain
- ρ Radius of curvature
- τ Time factor