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# THE EFFECT OF RESPIRATOR DEAD SPACE AND LUNG RETENTION ON EXPOSURE ESTIMATES

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*This paper develops, tests, and applies equations that predict the magnitude of the effect of lung retention and respirator dead space on average inhalation concentration and other related quantities. The equations were validated by numerical simulation and experimental measurement with a respirator on a mannequin connected to a breathing machine. Experimental data are presented verifying the applicability of the equations. The authors present applications of the equations and procedures to various types of respirator performance measurements and to a predictive respirator performance model. Graphs are presented giving correction factors. In all cases the correction factors are less than 2. Under typical conditions of workplace protection factor measurement with half-mask respirators, average inhalation concentration will be 105% to 125% of full-cycle average concentration.*

**R**espirator dead space is the volume of air inside a respirator external to the wearer's face. Usually, after the first breath exhaled air fills the respirator cavity at the end of each exhalation (the beginning of the next inhalation). As the next inhalation proceeds, incoming filtered air and aerosol that penetrates the filter mixes with the exhaled air remaining in the mask. The aerosol concentration in exhaled air is lower than that in the previous inhaled breath because of loss of particles due to lung retention during inhalation. This mixing process serves to reduce the concentration of aerosol that is inhaled compared with what it would have been had there been no dead space and consequently no mixing. The amount by which the average concentration inside the mask during inhalation (average inhalation concentration) is reduced by this process depends primarily on lung retention expressed as the fraction of inhaled particles that deposit in the respiratory system,  $F_{dep}$ , and the ratio of the volume of the respirator dead space to

tidal volume,  $V_{ds}/V_t$ . Lung retention and respirator dead space are closely intertwined and both must be present for this effect to manifest itself.

The respirator wearer's actual exposure and dose are directly proportional to the average concentration inside the respirator during inhalation.<sup>(1)</sup> If a respirator performance evaluation measured the average contaminant concentration in the air entering the mouth or nose, no specific correction for the effect of dead space would be necessary to estimate average inhalation concentration. Various measures of respirator performance involve quantities that can be corrected for the effect of lung retention and dead space to estimate more accurately average inhalation concentration. Standard corn oil quantitative fit tests (QNFT) use peak penetration in the calculation of fit factors.<sup>(2)</sup> Workplace protection factor (WPF) evaluations measure average mass concentration during the entire breathing cycle. A computer model described by Hinds and Bellin<sup>(3)</sup> estimates combined filter and leak penetration into a respirator during inhalation without taking into account the effect of mixing of exhaled air with inhaled air in the respirator dead space. Performance estimates based on this type of model require correction for the effect of dead space to improve the accuracy of their prediction. Information and equations derived here can be used to correct these respirator performance measurements for the effect of lung retention and dead space on average inhalation concentration.

As developed further below, these corrections—although not large (the theoretical maximum is a factor of 2)—are definable and represent a systematic error that can be corrected. There are many situations in which it is desirable to know the actual exposure of a respirator wearer, such as in an epidemiological study, an industrial hygiene follow-up to observed occurrence of symptoms, or good industrial hygiene practice (the evaluation component of recognition, evaluation, and control). The present study develops equations that can be applied to assess the effect of respirator dead space and lung retention on inhalation exposure. It also provides experimental data verifying the applicability of these equations, and presents applications of these equations

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and procedures to various types of respirator performance measurements and to a predictive respirator performance model.

## PREVIOUS WORK

In 1967, Thomas developed a model to assess the effect of the dead space of his apparatus on experimental measurements of pulmonary deposition.<sup>(4)</sup> He developed an equation that allows one to calculate the true pulmonary deposition from measured deposition using an empirical value relating tidal volume to dead-space volume. Thomas's model requires measurement of aerosol concentration entering a facemask from all sources and leaving the mask through the exhalation valve as an index of apparent deposition. The latter quantities are not readily available in most studies of respirator performance.

Myers et al. in 1986 discussed sampling bias, and the averaging of contaminant concentration during inhalation and exhalation for sampling within a facepiece.<sup>(1)</sup> Although they included the effect of pulmonary retention on full-cycle average contaminant concentration in a respirator, they did not discuss the effect of respirator dead space on this concentration.

In 1989, Campbell and Myers presented an analytical model to explain discrepancies in their sampling bias study measurement system.<sup>(5)</sup> Their model tracks dead space, leak, inlet, and pulmonary volumes through an inhalation and exhalation cycle. Although not specifically intended to estimate the effect of dead space on inhalation concentration, it can be used for that. An advantage of their model is that it does not require any assumptions about the nature of the mixing or streaming process inside the respirator. A disadvantage is that it requires the user to estimate five independent "flushing factors." These are defined as the fraction of the facepiece and leak volumes left in the facepiece during inhalation and the fraction of the facepiece, leak, and inlet (filter) volumes passing through the exhalation valve during exhalation. No values or guidelines are given for estimating these quantities. This severely limits the ability of their model to provide insight into the effect of dead space and lung retention on the performance of respirators.

Three studies have estimated the bias that occurs in measuring fit factors and protection factors by sampling the full respiratory cycle rather than inhalation only.<sup>(6-8)</sup> This bias is the result of lung retention and dead space. Taken together they estimate that full-cycle sampling gives fit factors (or protection factors) that are 5% to about 60% higher than those based on average inhalation concentration. A recent paper by Johnston et al. gave guidelines for WPF studies that included a recommendation to consider correction for lung retention.<sup>(9)</sup>

In 1976, James summarized the literature on the physiological effect of respirator dead space.<sup>(10)</sup> Rebreathing a portion of the exhaled carbon dioxide leads to an increase in depth and frequency of breathing. The increase in tidal volume ranges from 50% to 90% of the respirator dead-space

volume.<sup>(11)</sup> The effect manifests itself for respirator dead space of 100 mL or more.<sup>(10)</sup> This would include virtually all air-purifying respirators, except perhaps "mouthpiece" respirators.

The objective of the present study is to derive, test, and demonstrate the applicability of equations that allow one to calculate the magnitude of the effect of lung retention and respirator dead space on various measures of inhaled dose and respirator performance. This allows one to determine whether the presence of respirator dead space and lung retention causes a significant error in exposure estimates or respirator performance estimates in a given situation. The nature of the mixing and streaming process is assumed and the equations derived are verified experimentally. There is a scientific basis for all quantities used and for the underlying assumptions and theory. No flushing factors or adjustable constants need to be estimated.

## THEORY

As contaminated air enters a respirator through filters or facial-seal leaks it will mix to a varying degree with the air in the respirator dead space. In considering this mixing process it is useful to treat the flow entering the respirator through the filters separately from that entering through facial-seal leaks because, as explained below, it is likely they operate by different mechanisms and are independent of one another. Flow entering through the filters can be either well-mixed or plug flow (no mixing), or something in between. The well-mixed and plug flow cases represent the physical extremes in mixing. They are described mathematically below.

For the well-mixed case, for filter flow the instantaneous concentration in the respirator dead space,  $C_F(V)$ , after a volume  $V$  has entered the respirator during an inhalation is given by

$$C_F(v) = C_o P_F - (C_o P_F - C_i) e^{-V/V_{ds}} \quad (1)$$

where  $C_o$  is the outside concentration;  $C_i$  is the initial concentration in the dead space at the beginning of the inhalation;  $P_F$  is the fractional penetration of aerosol through the filters; and  $V_{ds}$  is the volume of the respirator dead space. Concentration is defined here as the mass of aerosol particles in the respirator dead space divided by the volume of the respirator dead space. The quantity  $C_o P_F$  is the concentration in the flow entering the respirator dead space through the filters and also the concentration in the respirator dead space after a large volume ( $V \gg V_{ds}$ ) has entered the respirator in the absence of facial-seal leaks. The quantity  $V/V_{ds}$  represents the number of air changes that have occurred in the respirator dead space as a result of a volume  $V$  entering that space. Equation 1 is equivalent to the concentration buildup equations used to describe dilution ventilation.<sup>(12)</sup>

At the other extreme is plug flow, where there is no mixing. This is visualized best by thinking of a respirator as a long tube with a filter at one end and the mouth at the other. As filtered air enters the tube it pushes the plug of dead-space

air along the tube, without mixing, into the mouth. A sharp boundary exists between the incoming filtered air and the dead-space air. The instantaneous concentration inside the mask, as defined above, increases linearly with the volume of air entering the mask until the respirator is filled with filtered air. Thereafter, the concentration remains constant for the remainder of the inhalation.

$$C_F(V) = C_i + (C_o P_F - C_i)(V/V_{ds}) \quad \text{for plug flow and } V \leq V_{ds} \quad (2)$$

$$C_F(V) = C_o P_F \quad \text{for plug flow and } V > V_{ds} \quad (3)$$

As will be shown below, the experimental data presented here support the view that the mixing situation for that portion of the total flow entering respirator dead space through the filters is best approximated by the well-mixed model (Equation 1).

The situation for facial-seal leaks is more complicated. If aerosol entering the respirator through facial-seal leaks mixes with the air in the dead space, then an equation equivalent to Equation 1 with  $P_F$  replaced by total penetration (filter + leak) can be used. The more likely case—based on work by Myers et al.<sup>(1)</sup> and Oestenstad et al.<sup>(13)</sup>—is where the leak flow streams directly into the mouth without appreciable mixing with the air in the dead-space volume. For this situation we can define the leak flow fraction  $F_L$  as

$$F_L = Q_L/Q_T \quad (4)$$

where  $Q_L$  is the flow rate through the facial-seal leak(s) and  $Q_T$  is the total flow entering the respirator, filter flow plus leak flow. Furthermore, one can make the reasonable assumption that the fraction of the dead-space volume occupied by the leak-stream path is equal to the ratio of leak flow to total flow. Thus, the volume of the leak-stream path  $V_L$  is  $F_L V_{ds}$ . When a volume  $V$  has entered the respirator, a volume  $F_L V$  will have entered through the leak stream and a volume  $(1 - F_L)V$  will have entered through the filters. The leak flow through a facial-seal leak will be plug flow through this leak-stream volume. The concentration in this leak stream volume  $C_L$  is characterized by equations equivalent to Equations 2 and 3 for the leak stream volume. Equation 2 becomes

$$C_L(F_L V) = C_i + (C_o P_L - C_i) \frac{F_L V}{V_L} \quad \text{for } F_L V \leq V_L \quad (5)$$

where  $P_L$  is the fractional penetration of aerosol particles through the facial-seal leak and  $V_L = F_L V_{ds}$ .

It is convenient to express Equation 5 in terms of the total volume entering the respirator,  $V$ . Equation 5 becomes

$$C_L(V) = C_i + (C_o P_L - C_i)(V/V_{ds}) \quad \text{for } V \leq V_{ds} \quad (6)$$

Similarly, for  $V > V_{ds}$

$$C_L(V) = C_o P_L \quad (7)$$

Because the expressions for the concentration in the respirator due to filter flow and leak flow are independent, an expression for the combined concentration can be obtained by weighted addition:

$$C(V) = (1 - F_L)C_F(V) + F_L C_L(V) \quad (8)$$

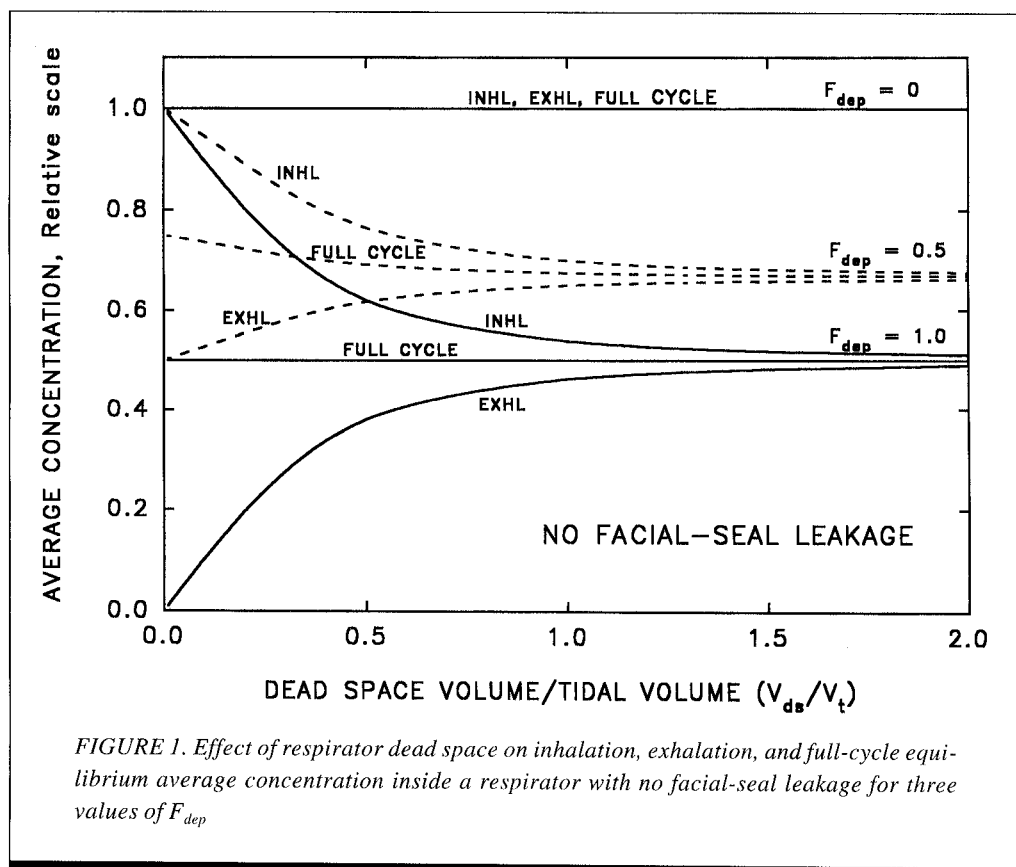
where  $C_F(V)$  is given by Equation 1 and  $C_L(V)$  is given by Equations 6 or 7.

Equation 8 with Equations 1 and 6 or 7 form the basic equations for modeling the concentration in the dead-space volume. They are based on the assumption that air and aerosol that has entered the dead-space volume through the filters is well-mixed with the dead-space air (actually  $1 - F_L$  of the dead-space air), and that air and aerosol entering through facial-seal leaks streams directly to the mouth.

In the Appendix, Equation 8 is expanded to provide working equations for instantaneous concentration during inhalation and exhalation. These are integrated over the breathing cycle to get average concentration during inhalation and exhalation. All these equations depend on lung retention and the ratio of the dead-space volume to the volume entering the respirator. As such, they allow one to investigate the effect of different values of lung retention and dead space on instantaneous and average concentrations in the respirator. They also permit the development of simplified equations, given in the Appendix, for the case in which tidal volume  $V_t$  is greater than three times the dead-space volume.

Figure 1 shows the effect of respirator dead space on average concentration (inhalation, exhalation, and full cycle) at equilibrium for a respirator with filter penetration but no facial-seal leaks as a function of  $V_{ds}/V_t$  for fractional respiratory deposition  $F_{dep}$  of 0, 0.5, and 1.0. Figure 1 was calculated stepwise for each breath using Equations A10-A12 with  $F_L = 0$  until equilibrium was reached. When dead space is zero no mixing occurs, and when  $F_{dep}$  is 1.0 the greatest difference between inhalation and exhalation concentration occurs. When dead space volume is large, there is more extensive mixing of inhalation air with exhalation air, which reduces the difference between  $\bar{C}_{IN}$  and  $\bar{C}_{EX}$ . Although figures could have been plotted with air changes per breath on the horizontal axis,  $V_{ds}/V_t$  is used here because it allows the plotting of the zero dead-space point, the scale is proportional to dead-space volume, and it shows a wide range of conditions in a compact scale.

Figure 2 is the same as Figure 1 except that it is for a respirator with facial-seal leaks but no contaminant penetration through the filters. It is based on equations A10-A12 with  $P_F = 0$ . It is assumed that aerosol entering through facial-seal leaks streams directly to the mouth during inhalation and that mixing occurs during exhalation. As with Figure 1, when the dead space is zero the maximum difference is observed. Average concentrations shown in Figure 2 (leak-only condition) are greater than in Figure 1 because of the nature of plug flow in the leak-only condition. With plug flow, a greater proportion of aerosol can enter the mask but not be inhaled and thus not be retained in the respiratory



system. In-facepiece concentrations consequently are somewhat greater for plug flow than for well-mixed flow.

The temporal profile for instantaneous concentration inside the respirator was modeled by a BASIC computer program. The program calculates concentration buildup and decay (purging) for a sequence of inhalations and exhalations using Equations A1 and A2. Figure 3 presents curves showing instantaneous aerosol concentration inside the respirator for the first three breaths and the 15th breath for the extreme values of dead space volume relative to tidal volume  $V_{ds}/V_t$  given in Table I. Conditions for Figure 3 are  $F_{dep} = 1.0$  and  $V_{ds}/V_t = 0.12$  and 1.73, the smallest and largest values given in Table I and a leak fraction of 0.01 (approximately equivalent to a QNFT fit factor of 100). Under these conditions steady state (defined here as an average value within 5% of the ultimate equilibrium value) is reached after one breath for small dead space and after three breaths for the large dead space. When dead space is small, the concentration inside the respirator quickly approaches  $C_o P_m$  during inhalation and  $C_o P_m(1 - F_{dep})$  during exhalation, where  $P_m$  is total penetration into the respirator. When dead space is large, there is a buildup with each breath as shown by the dashed line in Figure 3. This program also was run for 80

conditions over the range  $0.0 \leq F_L < 1.0$ ;  $0.0 \leq F_{dep} \leq 1.0$ ; and  $0.1 \leq V_{ds}/V_t \leq 2.0$ . For these conditions the average inhalation concentration was found to be within 5% and 1% of equilibrium values after 3 and 5 breaths, respectively, for half-face masks and after 8 and 12 breaths for full-face masks, which is consistent with the results of Campbell and Myers.<sup>(5)</sup> For a given respirator the maximum number of breaths required to reach equilibrium occurs when  $F_{dep} = 0$ , the condition associated with the greatest change in concentration in going from an initial concentration of zero to its equilibrium concentration.

One difficulty in applying the equations presented here to a given respirator situation is that the value of  $F_{dep}$  must be known to use the equations. For gases and vapors this can be determined from published values of uptake, but for aerosols

it requires knowing or estimating the particle size distribution inside the respirator, which almost always will be different than the ambient size distribution. The equations, however, can be used directly to estimate the effect of dead space on average inhalation concentration for one or more monodisperse aerosols. In this case,  $F_{dep}$  for each size can be estimated from tables<sup>(14)</sup> or equations.<sup>(15)</sup>

Alternatively, a predictive model such as the one described by Hinds and Bellin<sup>(3)</sup> can be used to estimate the inside size distribution and its average deposition. An analysis of 35 occupational aerosol size distributions using this model<sup>(16)</sup> reveals that for the average of three brands of dust, fume, and mist (DFM) dual cartridge respirators approved by the National Institute for Occupational Safety and Health, the inside size distributions show much less variability than their outside size distributions. For the usual case of (1) an outside mass median diameter less than 10  $\mu m$ ; (2) a

TABLE I. Dead-Space Volumes and Dead Space to Tidal Volume Ratios

Mask Type	Dead Space Volume, mL	Dead-Space Volume/Tidal Volume			
		$WR^A = 0^B$ $V_t = 724^C$	208 981 <sup>C</sup>	415 <sup>B</sup> 1320 <sup>C</sup>	622 kg $\times$ m/min 1620 <sup>C</sup> mL
Quarter	190	0.26	0.19	0.14	0.12
Half	270	0.37	0.28	0.21	0.17
Full	1250	1.73	1.27	0.95	0.77

<sup>A</sup>WR is the work rate of the respirator wearer. Values used are those given by Silverman et al.<sup>(17)</sup>

<sup>B</sup>Work rates used for experimental validation

<sup>C</sup>Tidal volumes for the indicated work rates, from Silverman et al.<sup>(17)</sup>

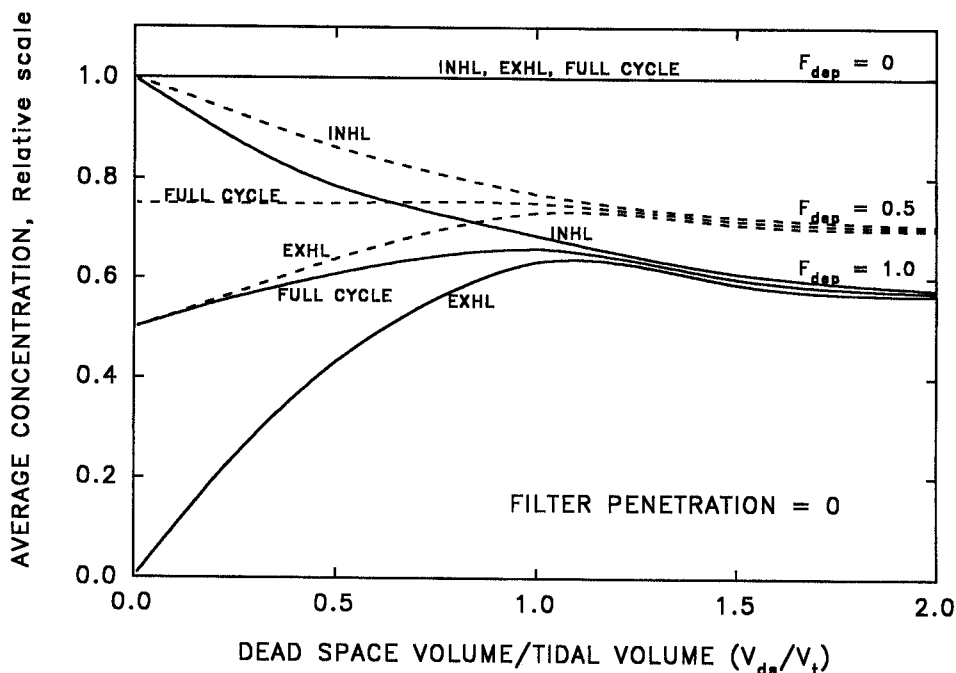


FIGURE 2. Effect of respirator dead space on inhalation, exhalation, and full-cycle equilibrium average concentration inside a respirator with no filter penetration for three values of  $F_{dep}$ .

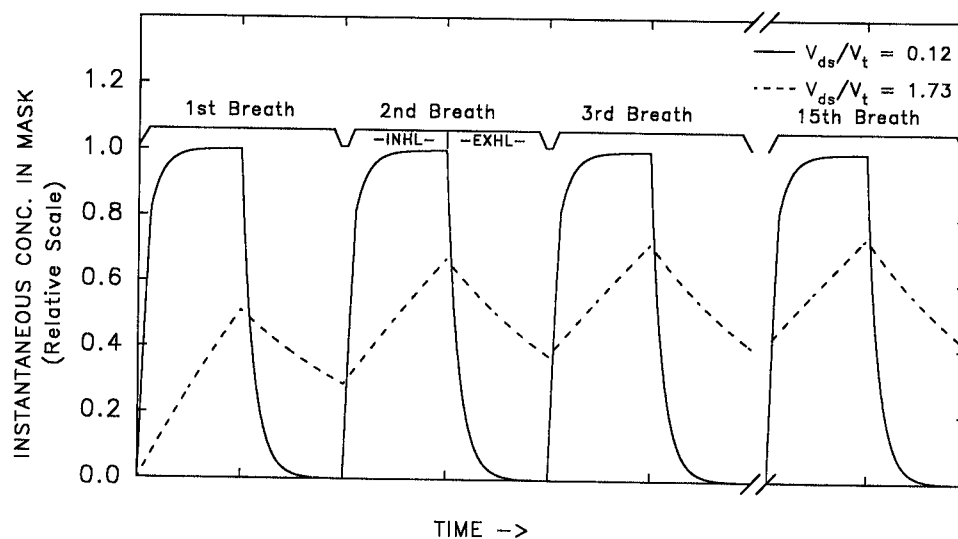


FIGURE 3. Instantaneous concentration inside a respirator with large and small dead-space volumes for the first three and 15th breaths.  $F_L = 0.01$ ,  $P_F = 0.01$ , and  $F_{dep} = 1.0$ .

geometric standard deviation greater than 2.0; and (3) a QNFT fit factor greater than 50, the inside mass median diameters range from 0.1 to 2  $\mu\text{m}$ —a range with relatively constant pulmonary deposition of 20% to 40%. In the absence of other information, therefore, a value of 30% for  $F_{dep}$  would be a reasonable estimate when the outside mass

median aerodynamic diameter (MMAD) is 1 to 10  $\mu\text{m}$  and a value of  $F_{dep} = 20\%$  is reasonable when the outside MMAD is less than 1  $\mu\text{m}$ .

## NUMERICAL VALIDATION

Equations A10-A12 were validated using full iterative numerical simulation.  $\bar{C}_{IN}$  and  $\bar{C}_{EX}$  were calculated stepwise by Equations A10-A12, one calculation for each inhalation and exhalation, until equilibrium was reached.  $\bar{C}_{IN}$  and  $\bar{C}_{EX}$  also were calculated by a numerical simulation that integrates the mass entering, leaving, and remaining in the respirator dead-space volume. The latter calculations involve 1000 steps for each inhalation or exhalation. Equilibrium results agreed within 1% over a wide range of conditions:  $0 \leq F_L \leq 1.0$ ;  $0.1 \leq V_{ds}/V_t \leq 2.0$ ; and  $0 \leq F_{dep} \leq 1.0$ .

The explicit equation (Equation A14) was used to calculate  $\bar{C}_{IN}$  and the results compared with stepwise calculations using Equations A10-A12 for 120 conditions over the range  $0 \leq F_L \leq 1.0$ ;  $0.15 \leq V_{ds}/V_t \leq 0.4$ ; and  $0 \leq F_{dep} \leq 1.0$ . Equation A14 agreed with the more rigorous Equations A10-A12 within 6% for all conditions where  $V_{ds}/V_t \leq 0.3$  and within 14% for  $V_{ds}/V_t \leq 0.4$ .

## EXPERIMENTAL METHODS

We conducted experiments to validate Equations A10-A12 by conducting a laboratory evaluation

of respirator performance using a mannequin. Respirators were sealed with hot melt adhesive to the face section of a fiber glass mannequin and thus represented a no facial-seal leak condition. The mannequin was placed in an aerosol exposure chamber, and samples were taken outside and inside the respirator. A dual piston mechanical breathing machine

was used to simulate typical human respiration at 0 and 415 kg-m/min work rates. Experiments were conducted under two conditions: (1) normal breathing conditions in which the exhaled air was filtered (Fisher G6 glass fiber filter) before being returned to the respirator through the mouth, and (2) with the exhaled air diverted from the mask. The latter condition (periodic inhalation with no exhalation through the mask) is equivalent to a no dead-space situation. It is equivalent to allowing  $V_i$  to become much greater than  $V_{ds}$ . The filtered exhalation air is the most severe case, equivalent to 100% aerosol deposition in the respiratory system. The ratio of the aerosol mass entering the mouth with normal breathing and with diverted exhalation is a measure of the effect of lung retention and dead space on average inhalation concentration. It is equivalent to the ratio of the average inhalation concentration with dead space to that without dead space.

Three mask types were evaluated: a quarter-face, a half-face, and a full-face mask (without a nose cup). A NIOSH-approved dust and mist filter was used with the quarter mask. Half- and full-face masks were used with DFM filters and organic vapor cartridges with paint prefilters. Each mask was evaluated at two work rates: 0 and 415 kg-m/min. Measured tidal volumes were 700 mL for a work rate of 0 kg-m/min and 1260 mL for 415 kg-m/min. These values are slightly less than those reported by Silverman et al.<sup>(17)</sup> because of the resistance of the sampling filter between the mask and the breathing machine. Breathing frequency was approximately 20 breaths/min at 0 kg-m/min and 23 breaths/min at the 415 kg-m/min work rate. The three masks and two work rates allowed evaluation of the effect of dead space for six values of  $V_{ds}/V_t$  that covered more than a 10-fold range from 0.14 to 1.73, as shown in Table I. This range covers nearly all situations likely to be encountered with air-purifying respirators.

Dead-space volume was evaluated by measuring the volume of water needed to completely fill the mask cavity while the mask was glued to the mannequin. Dead space observed under these conditions is expected to be equal to or slightly greater than that obtained under working conditions. The hot melt adhesive formed a bead seal around the facepiece, raising the mask slightly from the mannequin surface. Also, the strap tension on a real face slightly compresses both the mask and the skin. Respirator dead space for quarter and half masks has been reported to be between 100 and 250 mL for masks worn by human subjects,<sup>(10)</sup> similar to the measured

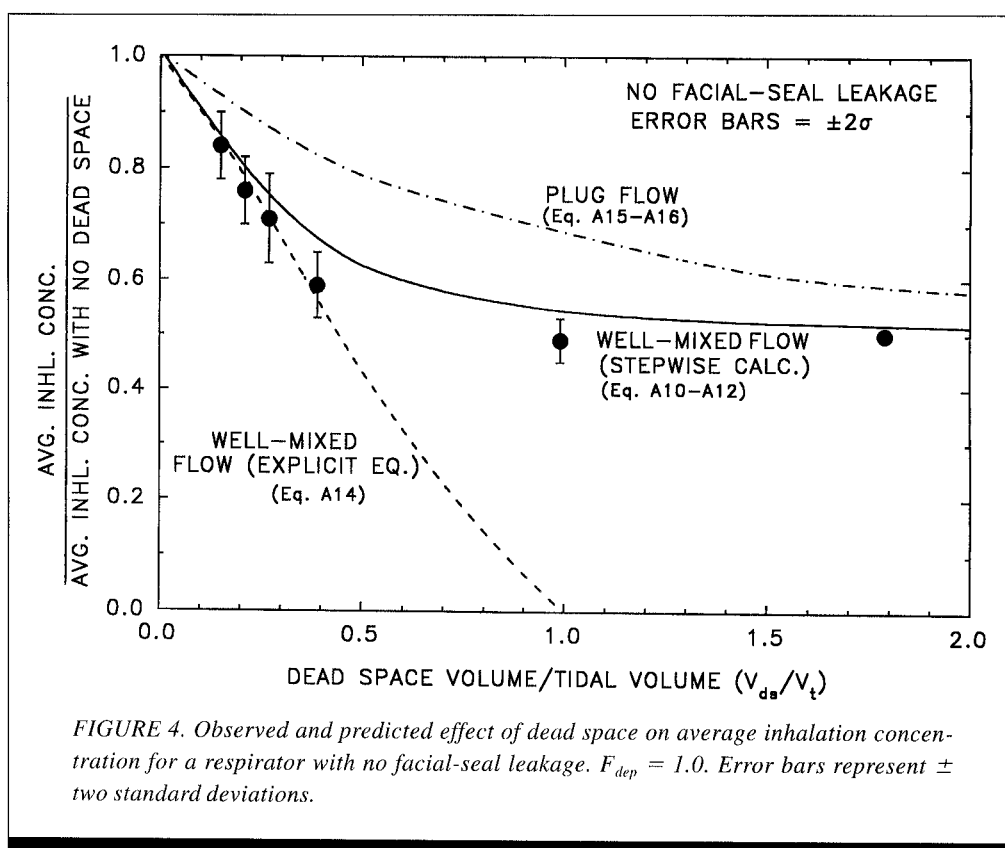


FIGURE 4. Observed and predicted effect of dead space on average inhalation concentration for a respirator with no facial-seal leakage.  $F_{dep} = 1.0$ . Error bars represent  $\pm$  two standard deviations.

values given in Table I for the quarter-face and half-face masks.

Experimental measurements were made using a bench-scale aerosol test chamber described previously.<sup>(18)</sup> The 109-L chamber has a top mixing section, a honeycomb flow laminator section, a 52-L transparent plastic cylindrical aerosol test section (40-cm diameter), and a bottom exhaust plenum. The vertical airflow velocity (about 4 cm/sec) is uniform within 20% of the mean throughout the test section, and test aerosol concentrations is uniform within 5% and stable within 5% for more than 1 hour. Measurements were made using three test aerosols: a polydisperse aerosol having an MMAD of 0.51  $\mu\text{m}$  and a geometric standard deviation (GSD) of 2.1, and monodisperse aerosols of 2.1 and 4.2  $\mu\text{m}$  aerodynamic diameter. The aerosol material was oleic acid tagged with uranine dye (sodium fluorescein). The polydisperse aerosol was generated with a TSI, Inc., constant output aerosol generator; the monodisperse aerosols were generated with a TSI, Inc., vibrating orifice aerosol generator. Samples were extracted in 10 mL of buffered distilled water and fluorescence emission measured at 515 nm for excitation at 490 nm.

Aerosol concentrations inside and outside the mask during inhalation were sampled simultaneously. All samples were taken as simultaneous inside-outside pairs and concentration expressed as a ratio of inside-to-outside fluorescence. The outside sample was taken at 4.6 L/min on a 37-mm open-face cassette mounted on the mannequin's head. This sampling arrangement meets Davies criteria for still-air sampling for particles less than 20  $\mu\text{m}$ .<sup>(19)</sup> The inside concentration was determined by sampling the entire inhalation flow through a

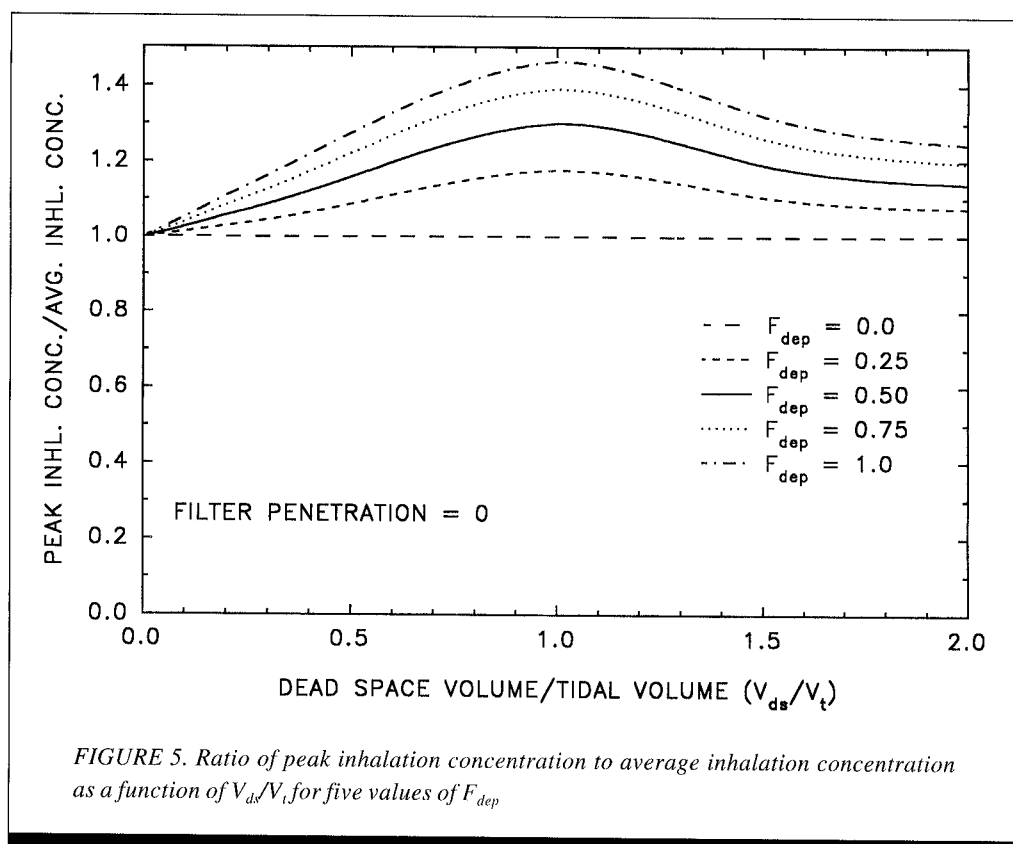


FIGURE 5. Ratio of peak inhalation concentration to average inhalation concentration as a function of  $V_{ds}/V_t$  for five values of  $F_{dep}$ .

90-mm glass fiber filter. All flow entering the mouth passed through this sampling filter. This arrangement avoids any in-facepiece sampling errors associated with leak streaming, such as those described by Myers et al.<sup>(1)</sup>

The ratio of average penetration observed with exhalation flow passing through the mask to average penetration observed with exhalation flow diverted was calculated using the ratio of averages method described by Cochran.<sup>(20)</sup> This ratio represents the factor by which average inhalation concentration is reduced due to rebreathing exhaled air contained in the respirator mask cavity.

## RESULTS

Figure 4 shows the observed and predicted effect of dead space on average inhalation concentration for the six dead space ratios tested. Predicted effects were determined by stepwise calculation using Equation A10-A12 (solid line), which assumes well-mixed flow for the filter flow. Also shown in Figure 4 are predicted concentration ratios for plug filter flow (dash-dot line), and for the explicit Equation A14 (dashed line), which assumes well-mixed filter flow.

## DISCUSSION OF RESULTS

Overall, the experimental data shown in Figure 4 agree most closely with the well-mixed curve (solid line) and did not follow the plug-flow curve indicating that the air

entering through the filter mixes fully with the dead-space air. The experimental results show the best agreement with the stepwise calculations for small and large values of  $V_{ds}/V_t$ . In the central region ( $V_{ds}/V_t$  from 0.4 to 1) experimental results are about 10% lower but closest to the well-mixed (filter flow) curve (solid line). At large ratios, typical of full-face masks without nose cups, experimental results show good agreement to the predictive equations indicating that significant mixing is taking place. Note that for  $V_{ds}/V_t < 0.4$  both stepwise calculation with Equations A10-A12 and the explicit Equation A14 are within the error bars of the data, indicating equally useful equations. The average coefficient of variation of the measurements was 5.3%. Overall, the stepwise calculations with well-mixed filter flow

agree with experimental data within 16% over the range of ratios of dead-space volumes to tidal volume used, 0.14 to 1.73. It is clear from Figure 4 that when  $V_{ds}/V_t > 0.4$  the stepwise method (Equations A10-A12) is the appropriate procedure to calculate the average inhalation concentration.

## APPLICATIONS

Stepwise calculations using Equations A10-A12 (or Equation A14 when  $V_t > 3V_{ds}$ ) can be used to analyze the change in average inhalation concentration caused by respirator dead space and lung retention for several common measurement situations.

### Fit Factor Evaluations

QNFT data provide a fit factor that indicates how well a particular respirator fits an individual wearer. It is of interest in risk assessment and epidemiologic studies to estimate inhalation exposure among a group of workers wearing respirators who have undergone fit testing. There are, however, many sources of error in attempting to use fit factors to estimate average inhalation concentration; for example, filter type, particle size distribution, and work rate can be very different for fit test and use conditions, and these differences can result in significant differences in exposure to the wearer.<sup>(16)</sup> There also is a modest systematic error in measured fit factor associated with lung retention and respirator dead space, as described here. Corn oil fit tests frequently use calculations based on average peak penetration of a

submicrometer aerosol leaking into a probed respirator. For each breath, peak penetration occurs at the end of inhalation, just before exhalation.

Figure 5 shows the effect of respirator dead space and lung retention on the ratio of peak inhalation concentration to average inhalation concentration as a function of  $V_{ds}/V_t$ . The ordinate represents the ratio of the maximum instantaneous concentration in the respirator to the average concentration in the respirator during inhalation. The latter is what the wearer is exposed to; the former often is used in QNFT measurement as a conservative measure of fit factor. The curves show a maximum at a  $V_{ds}/V_t$  value of approximately 1.0. This is a result of competing effects. When there is no dead space, there is no difference between peak concentration and average concentration. When  $V_{ds}/V_t$  is large, there is little difference between peak and average concentration because there is extensive mixing, as shown in Figures 1 and 2.

### Respirator Performance Model

A predictive respirator performance model described by Hinds and Bellin predicts respirator performance based on a measured fit factor from a standard QNFT and a given exposure particle size distribution and work rate.<sup>(3)</sup> This model predicts the concentration of contaminant entering the respirator dead space during inhalation, which will equal  $\bar{C}_{IN}$  only for a mask with no dead space. Figure 6 compares the concentration predicted by the model with the average inhalation concentration for the no facial-seal leak condition. When  $V_{ds}/V_t$  and  $F_{dep}$  are known or can be estimated, Figure 6 can be used to correct such model calculations for the effect of dead space and lung retention to obtain an estimate of average inhalation concentration.

### Workplace Protection Factor Studies

Workplace protection factors customarily are evaluated by sampling continuously inside and outside a respirator while it is being used properly. Without correction, WPF data might not provide an accurate measurement of inhalation exposure, since the exhaled breath is sampled along with the inhaled breath. WPF measurements can be corrected for the effect of dead space and lung retention to provide an estimate

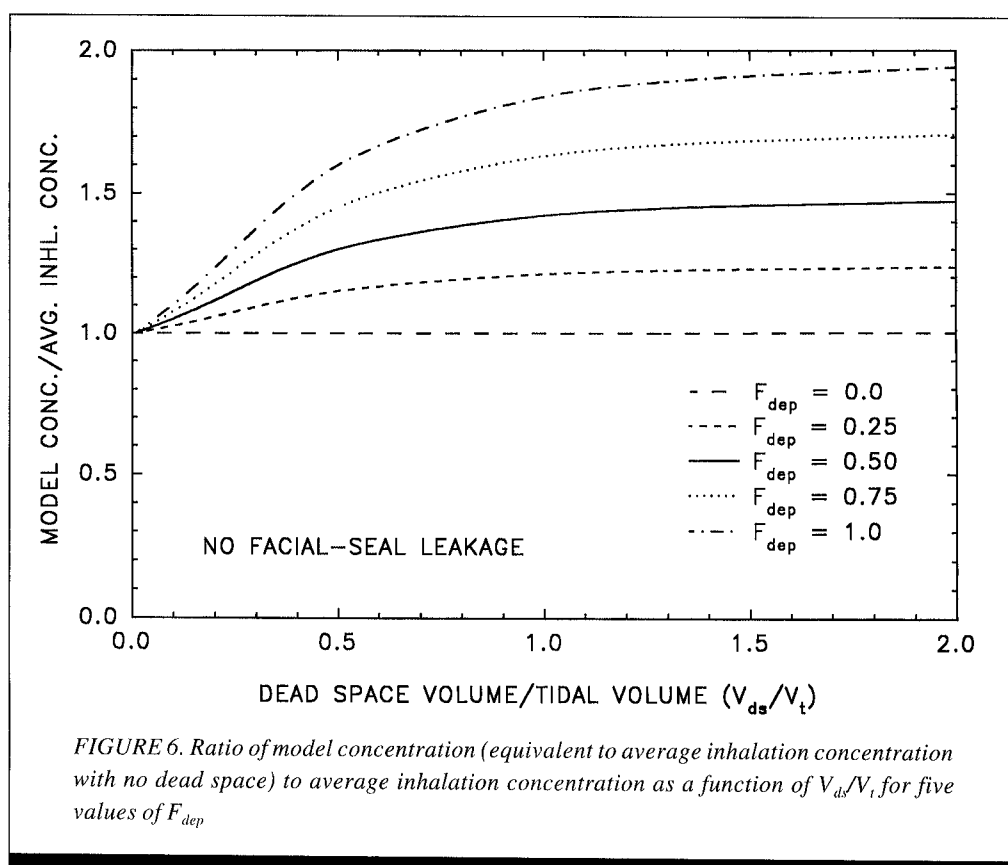


FIGURE 6. Ratio of model concentration (equivalent to average inhalation concentration with no dead space) to average inhalation concentration as a function of  $V_{ds}/V_t$  for five values of  $F_{dep}$ .

of average inhalation concentration. To do so requires knowing or estimating the respirator dead space (expressed as a fraction of tidal volume), lung retention, and inhalation/exhalation timing. In practice, these factors usually are not known. Reasonable estimates, however, can be made for these factors if particle size distribution and work rate are known or can be estimated.

WPF measures full-cycle average respirator cavity concentration  $\bar{C}_{avg}$ , given by

$$\bar{C}_{avg} = K_i \bar{C}_{IN} + K_e \bar{C}_{EX} \quad (9)$$

where  $K_i$  and  $K_e$  represent the time fraction of each breath spent in inhalation and exhalation, respectively.

The values of  $K_i$  and  $K_e$  have a relatively narrow range.  $K_i$  ranges from 0.43 to 0.51.<sup>(18)</sup> The maximum error in  $\bar{C}_{avg}$  introduced by setting  $K_i = K_e = 0.5$  is less than 16%. Figures 7 and 8 show the effect of dead space and lung retention on the relationship between WPF data and the average inhalation concentration  $\bar{C}_{IN}$  for  $K_i = K_e = 0.5$ . The correction increases with increasing lung retention because of the lowered concentration during exhalation. The correction is reduced as dead-space volume increases because the difference in average concentration between exhaled and inhaled air decreases as dead space volume increases. This decrease in concentration difference is a result of greater mixing of inhaled air with exhaled air. Figures 7 and 8 can be used to correct WPF measurements to estimate inhaled dose. Figures 7 and 8 represent extreme values of fit, and most practical respirator use situations will fall between them.

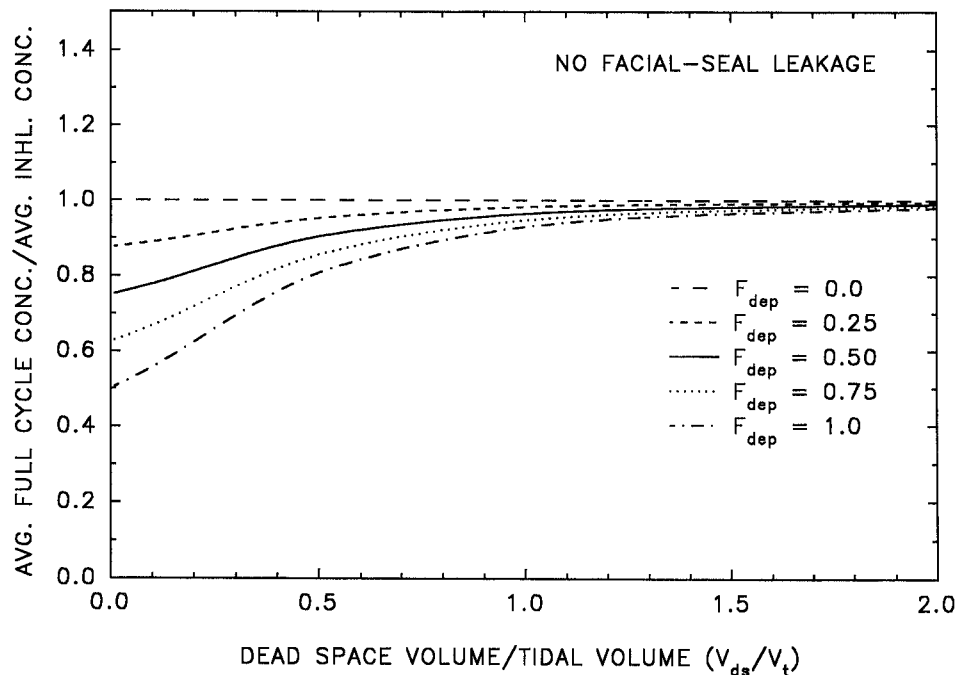


FIGURE 7. Ratio of average full-cycle concentration to average inhalation concentration as a function of  $V_{ds}/V_t$  for five values of  $F_{dep}$  for a respirator with no facial-seal leakage

#### Application Example: WPF evaluation

A welder is wearing a DFM dual cartridge half-face mask air-purifying respirator. The welder is working at a work rate of 415 kg-m/min and is exposed to welding fume with an approximate median size of 0.5  $\mu\text{m}$ . Quantitative measurement of fit give a fit factor greater than 100. Continuous measurements of welding fume concentration during a 4-hour period give an outside concentration of 5.0  $\text{mg}/\text{m}^3$  and an inside concentration of 0.12  $\text{mg}/\text{m}^3$ .

From Table I,  $V_{ds}/V_t = 0.21$  (or  $V_t = 4.9 V_{ds}$ )  $F_{dep}$  for  $d_a = 0.5 \mu\text{m}$  is 0.18 from Hinds (1982)<sup>(21)</sup>

Because fit factor is greater than 100, Figure 7 can be used to estimate  $\bar{C}_{FULL}/\bar{C}_{IN}$ . From Figure 7,  $\bar{C}_{FULL}/\bar{C}_{IN} = 0.93$ ; the uncorrected WPF =  $C_o/\bar{C}_{FULL} = 5.0/0.12 = 42$ .

Applying the above correction factor gives the WPF corrected for the effect of lung retention and dead space =  $C_o/\bar{C}_{IN} = 5.0/(0.12/0.93) = 0.93 \times 42 = 39$ .

The corrected value (39) represents the WPF on which exposure or dose estimates should be based. This correction is relatively small, primarily because  $F_{dep}$  is small, as it usually will be because of the small aerosol size distribution inside the mask. It is, however, a systematic bias that can be removed from the data.

At the other extreme, consider a respirator manufacturer testing the same respirator mounted on a mannequin exposed to the same aerosol. The respirator is operated with a breathing machine at the same breathing rate as above. The inhalation air is filtered after entering the mannequin's mouth and before being exhaled through the mask. This is equivalent to a  $F_{dep}$  of 1.0. Continuous measurement of welding fume concentration is made inside and outside as before. If the same

inside and outside concentrations are measured, one enters Figure 7 with  $V_{ds}/V_t$  of 0.21 and  $F_{dep}$  of 1.0 and gets a  $\bar{C}_{FULL}/\bar{C}_{IN}$  ratio of 0.63. The uncorrected PF is 42 as before,

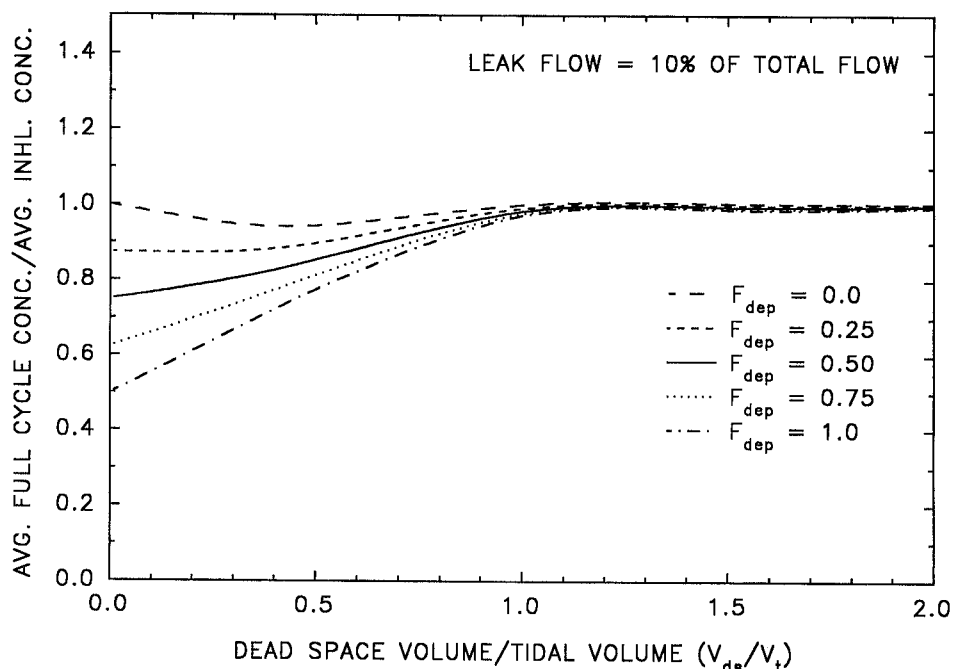


FIGURE 8. Ratio of average full-cycle concentration to average inhalation concentration as a function of  $V_{ds}/V_t$  for five values of  $F_{dep}$  for a respirator with 10% facial-seal leakage

but the corrected PF is  $42 \times 0.63 = 26$ . The latter, a PF of 26, represents the protection actually provided by the respirator under these conditions.

## DISCUSSION OF APPLICATIONS

For the usual corn oil QNFT of half-mask respirators,  $F_{\text{dep}}$  would be approximately 0.2 and  $V_{\text{ds}}/V_t$  would range from 0.2 to 0.4. From Figure 5, one can conclude that the use of peak penetration instead of average inhalation concentration for QNFT would contain a conservative error of less than 7%.

WPF measurements reflect average contaminant concentration in the respirator for the full breathing cycle, inhalation and exhalation. As dead space increases, the average concentration in the mask during exhalation increases and the average inhalation concentration decreases, as shown in Figure 1. When dead space is large relative to tidal volume (such as for a full-face respirator without a nose cup) or when pulmonary retention is low, WPF measurements differ little from average inhalation concentration. Correction factors required to estimate the average inhalation concentration from WPF measurements depend primarily on  $F_{\text{dep}}$  and the ratio  $V_{\text{ds}}/V_t$ . The largest adjustment to  $\bar{C}_{\text{avg}}$  is for the case with no respirator dead space and pulmonary deposition equal to 1.0, an unlikely situation. The dip in the curves of Figure 8 for  $V_{\text{ds}} < V_t$  is caused by the plug-flow effects mentioned in connection with Figure 2.

Smith et al. estimated in their evaluation of respirator performance (an effective protection factor study) among workers exposed to cadmium that the error introduced by sampling during both inhalation and exhalation was about 10%.<sup>(22)</sup> They assumed that  $K_i = 0.4$  and  $F_{\text{dep}} = 0.2$ . This compares favorably with the correction value of 7–10% obtained from Figures 7 and 8 for  $V_{\text{ds}}/V_t = 0.3$  and  $F_{\text{dep}} = 0.2$ .

Many full-face mask respirators are equipped with a nose cup to reduce fogging of the face shield. A well-fitting nose cup separates inhalation and exhalation flow, creating a condition equivalent to a small dead space. Full-face mask respirators without nose cups have a large physical respirator dead space volume, and the experimental data presented here indicate that over several breaths the entire volume is involved in mixing. Powered air-purifying respirators provide a steady flow of filtered air to the face. This design reduces the amount of exhaled air that is rebreathed and is equivalent to a very small dead space.

In addition to the respirator dead space there is a physiological dead space of approximately 150 mL.<sup>(10)</sup> This physiological dead space represents air that is inhaled and exhaled without participating in respiratory gas exchange. It is present whether or not a respirator is worn. Wearing a respirator adds its dead space to the physiological dead space to give a larger effective physiological dead space. Consequently, respirators with dead space of 100 mL or more cause a compensatory increase in the depth and frequency of breathing.<sup>(10)</sup>

The effect of respirator dead space considered here addresses how it modifies the inhaled or measured concentration in the mask. Because flow in the respiratory system is independent of mixing in the respirator, the physiological

dead space does not add to the volume in which mixing takes place, so its effect cannot be included in any simple way. The quantity  $F_{\text{dep}}$  represents fractional aerosol deposition under normal breathing conditions, which includes the effect of physiological dead space. At the end of an inhalation, the air in the physiological dead space is the last to have been inhaled and the first to be exhaled. As such, it will have a higher aerosol concentration than the air in the other parts of the respiratory system. It is higher during inhalation because it is the least affected by respirator dead space and higher during exhalation because that portion has lower deposition. Physiological dead space thus should serve to reduce slightly the exposure predictions given here.

To evaluate the effect of physiological dead space, the numerical simulation equations described above were modified so that the first 150 mL of exhaled air had a concentration equal to the average for the last 150 mL of air inhaled. This is equivalent to no deposition for this portion of exhaled air. The remaining exhaled air is treated the same as before, except deposition is increased to compensate for the 150 mL with no deposition. The latter is necessary for mass balance. Results indicate that although  $\bar{C}_{\text{EX}}$  is increased, there is little change in  $\bar{C}_{\text{IN}}$ , except at high values of  $V_{\text{ds}}/V_t$ , low tidal volumes, and  $F_{\text{dep}} = 1.0$  where  $\bar{C}_{\text{IN}}$  can be increased by up to 10%.

## CONCLUSION

Whenever one attempts to estimate inhaled dose for respirator wearers based on average inhalation concentration and minute volume, one should consider the effect of lung retention and respirator dead space on average inhalation concentration. The extent of the effect depends on the ratio of dead-space volume to tidal volume and fractional aerosol retention in the respiratory system. Based on the work presented here, the authors conclude that lung retention and respirator dead space can reduce the average concentration in the respirator during inhalation by as much as a factor of 2—although generally much less—compared with the no dead-space situation. The larger the respirator dead space relative to tidal volume, and the greater the respiratory retention of contaminant, the greater the effect.

Equations are derived that allow one to calculate the influence of lung retention and respirator dead space on average inhalation concentration and other related quantities under usual conditions of respirator use and testing. Experimental validation of these equations confirms that contaminant entering the facepiece through the filters mixes with the air in the respirator dead space. The equations show that equilibrium concentration profiles for half-mask respirators are reached after three breaths for the usual range of  $V_{\text{ds}}$  and  $V_t$  values.

Graphs are presented giving correction factors that allow one to calculate average inhalation concentration based on measurements such as peak inhalation concentration, full-cycle average concentration, and computer models based on penetration through filters and leaks. In all cases the correction factors are less than 2. Under typical conditions of WPF

measurement with dual cartridge half-mask respirators, average inhalation concentration will be 105% to 125% of full-cycle average concentration measurement.

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## APPENDIX

The equations developed in the Theory section above are combined and expanded here to provide working equations for the instantaneous concentration in the respirator during inhalation. Equations for concentration in the respirator during exhalation also are developed and expanded to provide equations for instantaneous concentration in the respirator during exhalation. These equations are integrated over the breathing cycle to get average concentration in the respirator during inhalation and during exhalation. Combining Equation 8 with Equations 1, 6, and 7 gives

$$C(V) = (1 - F_L)C_F + F_L[C_i + (C_oP_L - C_i)(V/V_{ds})] \quad \text{for } V \leq V_{ds} \quad (A1)$$

and

$$C(V) = (1 - F_L)C_F + F_L C_o P_L \quad \text{for } V > V_{ds} \quad (A2)$$

where

$$CF = C_o P_F - (C_o P_F - C_i)e^{-V/V_{ds}} \quad (A3)$$

During exhalation, streaming would not occur and instantaneous concentration  $\hat{C}(V)$  is given by an equation similar to Equation 1 with  $C_o P_F$  replaced by  $C_R$ , the average concentration exhaled into the respirator.

$$C_R = (1 - F_{dep}) \frac{M_{INHL}}{V_t} \quad (A4)$$

where  $F_{dep}$  is the fractional deposition of aerosol in the respiratory system,  $M_{INHL}$  is the mass of aerosol inhaled during the previous inhalation, and  $V_t$  the volume inhaled (tidal volume).

$M_{INHL}$  is obtained by integrating an expression for the instantaneous mass entering the mouth, from filter and leak streams, over the previous inhalation.

Expanding Equation A4 with  $M_{INHL}$  yields the aerosol concentration in exhaled air entering the mask  $C_R$

$$C_R = \left( \frac{1 - F_{dep}}{V_t} \right) [(1 - F_L)\bar{C}_F + F_L C_i V_t] \quad \text{for } V_t \leq V_{ds} \quad (A5)$$

$$C_R = \left( \frac{1 - F_{\text{dep}}}{V_t} \right) \left[ (1 - F_L) \bar{C}_F + F_L C_i V_{\text{ds}} + F_L C_o P_L (V_t - V_{\text{ds}}) \right] \quad \text{for } V_t > V_{\text{ds}} \quad (\text{A6})$$

where

$$\bar{C}_F = C_o P_F V_t - (C_o P_F - C_i) V_{\text{ds}} (1 - e^{-V_t/V_{\text{ds}}}) \quad (\text{A7})$$

and  $C_i$  is the initial concentration for the previous inhalation. The instantaneous concentration during exhalation  $\dot{C}(V)$  becomes

$$\dot{C}(V) = C_R - (C_R - \dot{C}_i) e^{-V/V_{\text{ds}}} \quad (\text{A8})$$

The initial concentration  $\dot{C}_i$  is the concentration in the mask at the beginning of exhalation, equal to that at the end of the previous inhalation as given by Equation A1 or A2 for  $V = V_t$ .

The average concentration during inhalation  $\bar{C}_{\text{IN}}$  is obtained by integrating Equations A1 and A2 over the inhalation volume  $V_t$ .

$$\bar{C}_{\text{IN}} = \frac{1}{V_t} \int_0^{V_t} C(V) dV \quad (\text{A9})$$

where  $C(V)$  is given by Equation A1 as  $V$  goes from 0 to  $V_{\text{ds}}$  and by Equation A2 as  $V$  goes from  $V_{\text{ds}}$  to  $V_t$ .

$$\bar{C}_{\text{IN}} = \left( \frac{1 - F_L}{V_t} \right) \bar{C}_F + F_L \left( C_i + (C_o P_L - C_i) \left( \frac{V_t}{2V_{\text{ds}}} \right) \right) \quad \text{for } V_t \leq V_{\text{ds}} \quad (\text{A10})$$

and

$$\bar{C}_{\text{IN}} = \left( \frac{1 - F_L}{V_t} \right) \bar{C}_F + \left( \frac{F_L V_{\text{ds}}}{V_t} \right) (C_i + (C_o P_L - C_i)/2) + \left( \frac{F_L C_o P_L}{V_t} \right) (V_t - V_{\text{ds}}) \quad \text{for } V_t > V_{\text{ds}} \quad (\text{A11})$$

Following the same procedure for exhalation, starting with Equation A8, gives average concentration in the respirator during exhalation  $\bar{C}_{\text{EX}}$

$$\bar{C}_{\text{EX}} = C_R - (C_R - \dot{C}_i) \left( \frac{V_{\text{ds}}}{V_t} \right) (1 - e^{-V_t/V_{\text{ds}}}) \quad (\text{A12})$$

Equations A10-A12 are general forms that apply to all situations; however, they require knowing the initial concentration for each inhalation and exhalation. Thus, their application requires stepwise calculation for each inhalation and exhalation in sequence, using the results of each previous step, until equilibrium is reached, usually after a few breaths. For each segment the initial concentration must be calculated by Equations A1, A2, or A3. This general form thus lacks an explicit solution.

When  $V_t > 3V_{\text{ds}}$ , such as for most half-mask and quarter-mask respirators, there are more than three air changes of the respiratory dead space volume during each inhalation and each exhalation. For this situation, equilibrium or steady state concentrations are nearly reached at the end of each inhalation and exhalation, and an approximate explicit solution can be obtained.

Under equilibrium conditions, when  $V_t \gg V_{\text{ds}}$ , the initial concentration for inhalation  $C_i$  is the final concentration for the previous exhalation, which has a steady-state value of

$$C_i = (1 - F_{\text{dep}})[(1 - F_L)C_o P_F + F_L C_o P_L] \quad (\text{A13})$$

Combining Equations A7 and A13 with A11 gives an approximate explicit equation for  $\bar{C}_{\text{IN}}$  for the case in which  $V_t > 3V_{\text{ds}}$ .

$$\begin{aligned} \bar{C}_{\text{IN}} \cong & \frac{C_o(1 - F_L)}{V_t} \\ & \cdot [P_F V_t - (P_F - (1 - F_{\text{dep}}))[(1 - F_L)P_F + F_L P_L]] V_{\text{ds}} \\ & \cdot (1 - e^{-V_t/V_{\text{ds}}}) + \frac{F_L C_o V_{\text{ds}}}{V_t} \\ & \cdot [(1 - F_{\text{dep}})][(1 - F_L)P_F + F_L P_L] \\ & + \frac{1}{2} (P_L - (1 - F_{\text{dep}}))[(1 - F_L)P_F + F_L P_L] \\ & + \frac{F_L P_L C_o}{V_t} (V_t - V_{\text{ds}}) \quad \text{for } V_t > 3V_{\text{ds}} \quad (\text{A14}) \end{aligned}$$

A similar expression can be obtained for  $\bar{C}_{\text{EX}}$  when  $V_t > 3V_{\text{ds}}$  by substituting Equation A6, A7, and A13 into Equation A12 and noting that for  $V_t \gg V_{\text{ds}}$ ,  $\dot{C}_i = (1 - F_L)C_o P_F + F_L C_o P_L$ .

Equations, equivalent to A10 and A11, for plug flow through a respirator with no facial-seal leaks were derived for use in Figure 4. The derivation follows the procedure used for Equations A10 and A11 but starting with Equations 2 and 3.

$$\bar{C}_{\text{IN}} = C_i \left( 1 - \frac{V_t}{2V_{\text{ds}}} \right) + \frac{C_o P_F V_t}{2V_{\text{ds}}} \quad \text{for plug flow and } V_t \leq V_{\text{ds}} \quad (\text{A15})$$

and

$$\bar{C}_{\text{IN}} = C_i \left( \frac{V_{\text{ds}}}{2V_t} \right) + C_o P_F \left( 1 - \frac{V_{\text{ds}}}{2V_t} \right) \quad \text{for plug flow and } V_t > V_{\text{ds}} \quad (\text{A16})$$

In the application of Equations A15 and A16 for Figure 4 a well-mixed model (Equation A8) was used for exhalation.