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Standards setting committees in the United States and Great Britain have adopted exposure limits for magnetic fields with a single frequency in the sub-radio frequency (sub-RF) range (1 Hz to 30 kHz). Since sub-RF magnetic fields in most occupational and residential settings have multiple frequencies, a more practical exposure assessment approach is derived in this article. To avoid adverse health effects, the present exposure limits control the currents induced in the body by the magnetic field. In this article, limits on induced current are shown to be equivalent to placing limits on the magnetic field's derivative dB/dt, both of which are applicable to the multifrequency fields found in the environment. The existing exposure limits are therefore used to derive bounds on dB/dt. Commercially available gaussmeters can be used to measure sub-RF magnetic fields for comparison with these limits on dB/dt exposures. To measure dB/dt, the gaussmeter needs a linear frequency response, and a correction factor must be applied to the readout. The errors in assessing induced currents with multiple frequencies are evaluated for other common methods of measuring magnetic fields. These other methods can have acceptable accuracy, but only if the field's harmonic distortion is sufficiently low. Practical methods are recommended for measuring multifrequency magnetic fields and assessing their induced body currents. The relationship between sub-RF magnetic fields and adverse health effects is not addressed in this article. BOWMAN, J.D.: MEASURING MULTIFREQUENCY MAGNETIC FIELDS FOR EXPOSURE ASSESSMENT BASED ON INDUCED BODY CURRENTS. APPL. OCCUP. ENVIRON. HYG. 10(7):628–634; 1995.

The threshold limit value (TLV) for magnetic fields in the sub-radio frequency (sub-RF) range is a formula:[1]

$$B_{TLV} = \frac{60\text{ mT-Hz}}{f} \tag{1}$$

where f is the field's frequency (in hertz) and the magnetic field is expressed as the root mean squared (rms) magnetic flux density B in the SI units of milliTesla [1 milliTesla (mT) = 10 gauss (G)]. This formula is recommended for single frequencies from 1 to 300 Hz. In the United Kingdom, the National Radiological Protection Board (NRPB) recommends the same functional form for its exposure limit: $B_{EL} = C_{EL}/f$. With the NRPB limit, the constant C_{EL} equals 94 mT-Hz.[2]

Although these exposure limits assume that the field has a single frequency, occupational and residential magnetic fields in the sub-RF range generally have multiple frequencies. Figure 1 shows the frequency spectra of occupational magnetic fields that have components above and below the electric power frequency (60 Hz in North America and Brazil; 50 Hz in the rest of the world). The amount of magnetic field at these other frequencies can be expressed as the percent harmonic distortion ($\%H_d$):

$$\%H_d = 100\% \frac{B_{\text{other frequencies}}}{B_{\text{power}}} \tag{2}$$

Measurements of the harmonic distortion in a sample of six workplaces found a mean value for $\%H_d$ of 51.3 percent (range from the 10th to the 90th percentile = 8.66 to 124%); in 18 homes the mean was 20.3 percent (range from the 10th to the 90th percentiles = 6.19 to 41.0%).[3] This widespread harmonic distortion in sub-RF magnetic fields creates difficulties in using single-frequency limits to assess occupational and environmental exposures.

To deal with multiple frequencies, this article suggests a modification of the existing exposure limits for sub-RF magnetic fields. In addition, a method is described for measuring exposures with the modified formula using existing monitors. This analysis is concerned only with measuring exposures and does not address the question of adverse health effects from exposures to sub-RF magnetic fields.[4]

Basis for Multifrequency Exposure Limits

The TLV and similar exposure limits are designed to keep electric currents induced in the body by external magnetic fields below levels that are known to affect humans. The safe limit for induced currents was originally proposed by a committee of the International Radiation Protection Association (IRPA), which has now become the International Commission for Non-Ionizing Radiation Protection (ICNIRP).[5] In the IRPA/ICNIRP criteria, the induced current density (J = current per unit area) is limited to 10 mA/m^2 rms and below.

To extend these exposure limits to multifrequency fields, the mathematical derivation of Equation 1 needs to be examined.[6] To estimate induced body currents, Faraday's law for magnetic induction is applied to a simple model of the human body in a single-frequency magnetic field. The full mathematical derivation of this exposure limit is given in Appendix A.

The physical ideas behind induced current limits can be easily understood by considering an electrical circuit placed in a magnetic field, as mentioned in the rationale for the IRPA

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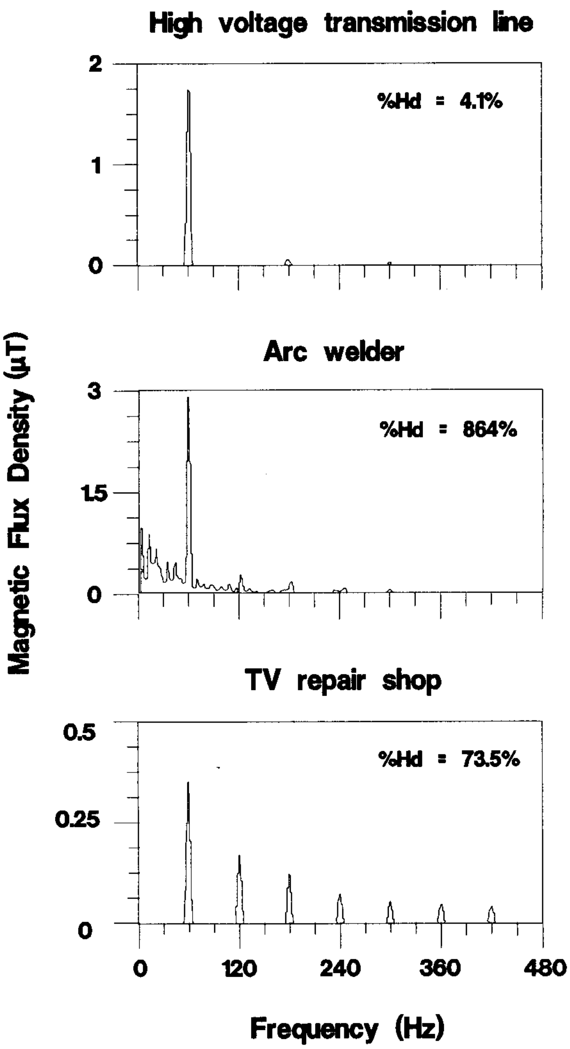

FIGURE 1. Magnetic field frequency spectra and percent harmonic distortion (%H_d) measured at workstations. The measurements were taken with an induction coil and a digital signal analyzer (Hewlett-Packard model 3561A).

guidelines.[5] According to Faraday's law,[7] the voltage V induced in this circuit is:

$$V = -A\frac{dB}{dt} \tag{3}$$

where the circuit covers an area A. Now, the current I in the circuit is given by Ohm's law: V = IR, where R is the resistance. Therefore, the induced current I is proportional to the magnetic field's derivative: $I = -(A/R)\ dB/dt$. The same result can be derived for currents induced in conductive tissues within the body (e.g., the blood stream, the lymphatic system, intercellular fluids, and the cell cytoplasm). As shown in Appendix A, the maximum current density J induced in a bodily circuit is proportional to dB/dt averaged over the circuit.

Since some adverse biological effects are known to result from high induced current densities J,[5] this result provides a rationale for the evaluation of magnetic field exposures and the control of induced body currents. Currents within the body can clearly be kept in control by limiting the magnitude of dB/dt to which people are exposed. To assess exposures, dB/dt needs to be measured over the person's body and averaged.

However, exposure limits such as the TLV and the IRPA/ICNIRP guidelines have been designed to apply directly to the magnetic field B. To accomplish this, these criteria now assume that the field is a sinusoidal function with a single frequency f:

$$B(t) = \sqrt{2}B\cos(2\pi ft) \tag{4}$$

Following electrical engineering conventions, the field's magnitude B is expressed as the root mean square (rms):

$$B = \mathrm{rms}(B) \equiv \sqrt{\frac{1}{T}\int_0^T dt|B(t)|^2} \tag{5}$$

Now, the time derivative of Equation 4 is: $dB/dt = -2\pi f \times \sqrt{2}B\sin(2\pi ft)$. Since the rms of a sine function is $1/\sqrt{2}$, the rms derivative for the single-frequency sinusoidal field is:

$$\mathrm{rms}\left(\frac{dB}{dt}\right) = 2\pi f\, B \tag{6}$$

Since the induced current density J is proportional to dB/dt, the IRPA limit on the body currents becomes:

$$2\pi fB \times \text{const.} = \mathrm{rms}(J) \leq 10\frac{\text{mA}}{\text{m}^2} \tag{7}$$

By solving Equation 7 for B, exposure limits are obtained in the form of Equation 1 with its inverse dependence on frequency. To derive the constant C_{EL} in Equation 1, standards committees use simple models of the human body to estimate the maximum constant of proportionality in Equation 7 (Appendix A). Since some judgment is required, different standard bodies have arrived at different values for C_{EL} (e.g., 60 mT-Hz for the TLV and 94 mT-Hz for the NRPB in the United Kingdom).

These limits on the magnetic field magnitude B (Equation 1) can be turned into limits on dB/dt by simple substitution into Equation 6:

$$\mathrm{rms}\left(\frac{dB}{dt}\right)_{EL} = 2\pi(f\,B_{EL}) = 2\pi\, C_{EL} \tag{8}$$

For example, the limit on dB/dt that is equivalent to the present TLV would be: $2\pi C_{EL}$ = 377 mT-Hz = 377 mT/s (3.77 kG/s).

Remarkably, Equation 8 is a valid exposure limit for magnetic fields that are not a sine wave with a single frequency. The standards committee selected the proportionality constant to get the maximum induced current which could result in their model of the human body. Therefore, other magnetic fields with the same dB/dt will induce body currents less than the estimate from Equation 7, whatever the field's frequency spectrum or wave shape (Appendix A). Since Equation 8 keeps currents induced by a sinusoidal field within the IRPA/ICNIRP safety criteria, it also provides a safety limit on dB/dt of any sub-RF magnetic field. Therefore, any exposure limit based on induced currents can be converted to a limit on dB/dt by Equation 8. Moreover, this limit applies to multifrequency fields.

Measuring Multifrequency Magnetic Fields

To compare worker exposures with Equation 8, the magnetic field derivative must be measured in the multifrequency environment encountered in most workplaces. Two methods are apparent for measuring the derivative exactly with commercially available equipment: (1) calculate the derivative from a measurement of the frequency spectrum and (2) apply a correction factor to the readout of a gaussmeter with a linear frequency response.

The first method is based on instruments that measure the components of the time-dependent magnetic field B(t) expanded as a Fourier series:[8]

$$B(t) = \sqrt{2}\sum_f B_f \cos(2\pi ft + \phi_f) \tag{9}$$

where:

B_r = the rms magnitude
ϕ_f = the phase for frequency F

The magnitudes B_f plotted as a function of frequency are the frequency spectra shown in Figure 1. Frequency spectra for sub-RF magnetic fields can be measured with digital signal analyzers (as in Figure 1), digital waveform capture instruments,[3] and a personal monitor that uses a series of bandpass filters.[9]

With the Fourier series, the time derivative of the multifrequency field and its rms can now be calculated. As shown in Appendix B, the rms derivative is:

$$\mathrm{rms}\left(\frac{dB}{dt}\right) = 2\pi\sqrt{\sum_f (fB_f)^2} \tag{10}$$

This straightforward calculation (which can be programmed into some spectrum analyzers) gives the magnetic field derivative from a measurement of the frequency spectrum. The main drawback of this method is the instrument expense and operator sophistication which accurate spectrum measurements require.

Fortunately, the derivative can be measured more easily with commercial gaussmeters that have a linear frequency response (Figure 2) and a true rms signal detector.[10] With a linear response, such a gaussmeter responds to every component in the frequency spectrum as (fB_f); with a true rms detector, the frequency components are combined just as in the formula for rms(dB/dt) (Equation 10). Linear response gaussmeters are therefore measuring the derivative, but they are calibrated to read out in units of magnetic flux density (in either Tesla or gauss). To do this, the linear meter has a calibration constant k:

$$R_{linear} = k\sqrt{\sum_f (fB_f)^2} \tag{11}$$

which is chosen so that its readout R_{linear} equals the rms magnitude rms(B) for magnetic fields with the electric power frequency f_{power}.

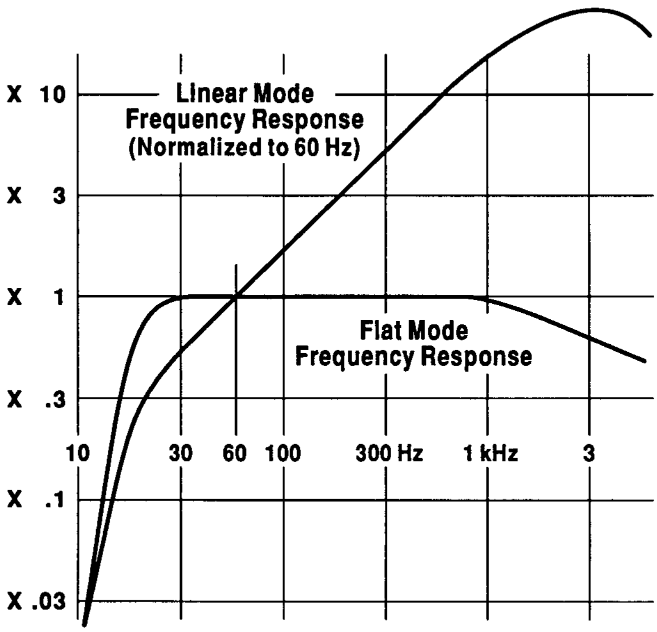

FIGURE 2. Linear- and flat-frequency response modes of a gaussmeter. (Figure used by permission of Monitor Industries, Boulder, Colorado. No endorsement by NIOSH is implied.)

The similarity between Equations 10 and 11 suggests that dB/dt can be determined exactly from the readout of a linear response gaussmeter multiplied by a correction factor. The correction factor can be determined exactly if the linear response meter is calibrated in a pure sinusoidal field with only the frequency f_{power}. The gaussmeter is then calibrated so that $R_{linear} = \mathrm{rms}(B) = B_{power}$, the rms magnitude at the power frequency. Using Equation 11, the calibration constant k must be $1/f_{power}$. Substituting this result into Equations 10 and 11 gives the desired correction factor:

$$\mathrm{rms}\left(\frac{dB}{dt}\right) = 2\pi f_{power} R_{linear} \tag{12}$$

In other words, the magnetic field derivative can be measured exactly with a linear response gaussmeter by using the correction factor of $2\pi f_{power}$. The correction factors are 377 seconds^{-1} for 60-Hz fields and 314 seconds^{-1} for 50-Hz fields.

Discussion

The dB/dt exposure limit derived above is proposed as a general approach to limiting body currents induced by sub-RF magnetic fields. The relationship between rms(J) and rms(dB/dt) is based on the combination of Faraday's and Ohm's laws, which is generally valid in the sub-RF range (Appendix A).

Magnetic induction has been studied experimentally with measurements of the currents induced in saline solution and conductive gel manikins by 60-Hz magnetic fields.[11,12] Differences between measured currents and calculations based on Faraday's and Ohm's laws are insignificant, validating their use for induction calculations in the sub-RF range.

Even with a highly nonuniform magnetic field from a wire carrying 100 A located 1 cm from the saline solution, the measured induced currents correlate with dB/dt averaged over the container.[12] This finding suggests a method for assessing exposures close to sources (e.g., an electric line worker fixing an energized line, or an arc welder whose power cable is running over a shoulder). The readings from several magnetic field monitors worn over the worker's body can be used to calculate the spatial average of dB/dt. If the source is far enough away so that the magnetic field is uniform over the worker's body, a single monitor should be sufficient.

Although the physical basis for limiting the induced body current by restricting dB/dt is sound, the numerical values for the TLV and other exposure limits based on the IRPA/ICNIRP criteria are based on several simplistic assumptions (Appendix A). More accurate limits on dB/dt can be derived from realistic calculations of the currents induced in parts of the body where biological effects originate.[13] Whether such body current limits are desirable is part of the broader debate over possible health effects of low frequency electric and magnetic fields.[4]

The proposed method for measuring dB/dt with a linear response gaussmeter is a modification of published methods for assessing residential power-frequency magnetic fields.[14–16] To adapt these methods to measuring occupational exposures to dB/dt in the sub-RF range, the frequency spectra in the workplace should first be investigated. Typically, this simply involves an understanding of the magnetic field sources. If the only electric currents in the workplace come directly from AC power and no source of other frequencies is identified, then the frequency span of the magnetic field probably lies within the bandwidth of most power-frequency gaussmeters.[10] If measurement accuracy is essential, this assumption should be validated by measuring the frequency spectrum.

For example, the magnetic field's frequency span is 60 to 300 Hz for the high voltage transmission line in Figure 1, and is 60 to 420 Hz in the TV repair shop. In these spectra, the harmonics of 60 Hz come from nonlinear circuit components, such as transformers and rectifiers. The gaussmeter in Figure 2 is designed to maintain its linear response over a 40 to 1000-Hz bandwidth that encompasses these harmonics. On the other hand, the welder's arc generates a broad spectrum (Figure 1) which lies partially below the gaussmeter's bandwidth. In such cases, the frequency spectrum should be measured and the dB/dt calculated with Equation 10. Similar problems would occur with magnetic fields with components above 1000 Hz (e.g., fields from induction welders and induction furnaces).

Note that some workplaces that have their own supply of AC electricity can have power frequencies different from the 50 or 60 Hz supplied by electric utilities. Commercial airliners, for instance, have 440 Hz electric power. In these environments, dB/dt can be measured with Equation 12. Since these magnetic fields are not well characterized, frequency spectra measurements assure that the linear response gaussmeter has adequate bandwidth, but are not needed to calculate dB/dt with Equation 12. In this equation, f_{power} is the calibration frequency for the gaussmeter, not the power frequency in the environment.

Choosing instrumentation whose frequency response and bandwidth accommodates the frequency spectra in the magnetic field environment is clearly necessary for accurate dB/dt measurements. In addition, the meter needs a true rms time response, rather than a rectified average or peak response. Using these principles, the methods described above can accurately and efficiently assess magnetic field exposures to control induced body currents.

However, approximate methods of assessing magnetic field exposures will be attractive to some. Gaussmeters without a linear frequency response have become widespread, and may be substituted for the recommended linear response gaussmeters. In addition, calibrations are sometimes done with magnetic fields with harmonic distortion instead of the recommended single-frequency field. Finally, measurements of multifrequency fields can be compared to the existing exposure limits for single frequencies. Therefore, the errors in these approximate methods need to be assessed.

Gaussmeters with flat, narrowband, or broadband frequency filters can be reasonably accurate if the workplace magnetic field environment has sufficiently low harmonic distortion. To demonstrate this, consider a rms gaussmeter with a frequency filter F(f), so its readout is

$$R_F = \sqrt{\sum_f |F(f)\, B_f|^2} \tag{13}$$

As always, this gaussmeter is calibrated to have a unit response at the power frequency [i.e., $F(f_{power}) = 1$]. By the arguments given above, R_F multiplied by the correction factor $2\pi\, f_{power}$ equals dB/dt exactly when the field's only frequency is f_{power}, and is approximately correct if the field does not have too much harmonic distortion.

The error from a gaussmeter with nonlinear frequency response is derived as a function of harmonic distortion in Appendix C. To illustrate these errors, consider a gaussmeter with a flat response [i.e., $F(f) = 1$ over its frequency bandwidth]. With magnetic fields from the high voltage transmission line in Figure 1, the 4.1 percent harmonic distortion comes primarily from the third harmonic. Using the formulas from Appendix C, the percent bias in dB/dt measured with the flat-response gaussmeter is −0.66 percent. The bias will be negative (underestimation of the exposure) if the harmonic distortion comes predominantly from frequencies greater than the power frequency.

In general, the bias from a flat-responding gaussmeter is within ±10 percent if the harmonic distortion primarily from the third harmonic is less than 17.4 percent. Since only 20 percent of the workplace measurements of harmonic distortion have $\%H_d$ less than 10 percent,[3] the flat-responding gaussmeters will have errors greater than ±10 percent in most occupational environments.

The same error calculation applies to assessing multifrequency magnetic fields with an exposure limit for a single frequency (Equation 1). Consider an industrial hygienist who uses a gaussmeter with a frequency response F(f) to assess exposures with one of the existing limits. Suppose the hygienist logically chooses the power frequency to use in the formula for the limit (Equation 1). The gaussmeter readout R_F compared to the exposure limit can then be rewritten as

$$2\pi f_{power} R_F \leq 2\,\pi\, f_{power} B_{EL} = 2\pi C_{EL} \tag{14}$$

This result parallels the dB/dt exposure limit (Equation 8) compared with the linear response gaussmeter (Equation 12). Therefore, errors will arise from the existing exposure limits used with the power frequency only if the gaussmeter has a nonlinear frequency response. With the flat-response meters, as shown above, such errors will generally be greater than ± 10 percent in occupational magnetic fields.

Finally, consider the errors resulting from a calibration device that uses AC line current, rather than the single-frequency power supply recommended for calibrating gaussmeters.[15] With sufficiently low harmonic distortion, line current can be acceptable for calibrations. The error resulting from calibration fields with harmonic distortion is calculated in Appendix D. If the calibration power supply has $\%H_d = 4.1$ percent concentrated in the third harmonic (like power from the transmission line in Figure 1), the measurement bias is -0.66 percent. For most industrial hygiene applications, the errors from using line current for calibrating the gaussmeter can therefore be acceptable.

Conclusions and Recommendations

The TLV for sub-RF magnetic fields and other exposure limits derived from the IRPA/ICNIRP criteria for induced currents are difficult to use in most occupational environments since multiple frequencies are usually present. With the flat-response gaussmeters most commonly used for exposure assessments, limits designed for single frequencies are estimated to produce errors greater than ± 10 percent in nearly 80 percent of occupational environments.

Exposure limits based on induced body currents can be modified to accommodate coincident multiple frequencies because they are equivalent to limits on the derivative dB/dt. Equivalent exposure limits for dB/dt are derived from the numerical constants in the formulas of the single-frequency limits.

The magnetic field derivative can be measured with two methods that apply simple calculations to the output of commercially available instruments. One method requires a measurement of the magnetic field's frequency spectrum, which is presently expensive and demanding. The more flexible method for measuring dB/dt uses a gaussmeter with a true rms time response and a linear frequency response over the frequency span of the magnetic field environment. A gaussmeter whose frequency response is not linear can be used with acceptable accuracy if harmonic distortion in the environmental magnetic field is sufficiently low.

The gaussmeter must be calibrated against purely sinusoidal magnetic fields whose frequency and rms magnitude cover the ranges expected in the work environment. The calibrations can be done with AC line current if its harmonic distortion is sufficiently low.

Gaussmeter measurements should be taken at several body locations, bracketing the organs relevant to the health concern. If the source is far enough away that the magnetic field is uniform over the worker's body, a single monitor should be sufficient. To obtain dB/dt, the gaussmeter's readout must be multiplied by the correction factor $2\pi f_{power}$, where the power frequency is for the magnetic field used in the calibration. The dB/dt measurements are averaged over all body locations.

The body-averaged measurement of dB/dt is then compared with $2\pi C_{EL}$, where the constant C_{EL} can be derived from existing limits on sub-RF magnetic fields. Whether these magnetic field limits based on induced currents are related to adverse chronic health effects is still a matter of research.

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Appendix A: Derivation of Exposure Limits Based on Induced Body Currents

This appendix derives more rigorously the body current density **J** induced by a sub-RF magnetic field. (**J** is a vector, as indicated by the bold type, which points in the direction of the current flow.) A simple model of the human body will be used to solve Faraday's law in its integral form for the induced electric field **E:**

$$\int_C \mathbf{E} \cdot \mathrm{dl} = -\int_A \frac{\partial \mathbf{B}}{\partial t} \cdot \mathrm{ds} \tag{A1}$$

where the integrals are over an area A with circumference C.[7] The current density vector **J** is then obtained from Ohm's law in the form: $\mathbf{J} = \sigma \mathbf{E}$, where σ is the conductivity (in units of Siemens per meter) for the various tissues through which the current flows.

If the spatial orientation of the vector $\mathrm{d}\mathbf{B}/\mathrm{dt}$ is uniform over the area A, then substituting Ohm's law into Faraday's law (Equation A1) gives:

$$\int_C \frac{1}{\sigma}\mathbf{J} \cdot \mathrm{dl} = -\left|\frac{\mathrm{d}\mathbf{B}}{\mathrm{dt}}\right| A \cos\theta \tag{A2}$$

where

$|\mathrm{d}\mathbf{B}/\mathrm{dt}|$ = the vector magnitude averaged over the area A
θ = the angle between the vector $\mathrm{d}\mathbf{B}/\mathrm{dt}$ and the plane of the current path C

As was argued above (Equation 3), this result shows rigorously that the body current **J** can be controlled by limiting $|\mathrm{d}\mathbf{B}/\mathrm{dt}|$. To assess exposures, the magnitude of the magnetic field derivative $\mathrm{d}\mathbf{B}/\mathrm{dt}$ should be measured over the person's body and averaged.

The induced body currents can be determined uniquely from Faraday's and Ohm's laws if the second-order magnetic fields generated in the body by the eddy currents **J** can be neglected. These second-order magnetic fields are negligible if the dimensions of the object are much less than the skin depth $\delta = \sqrt{(\pi\, f \mu \sigma)^{-1}}$ (i.e., the layer in which eddy currents are induced).[17] For average values of the permeability μ and conductivity σ in the human body, the skin depth for 60-Hz fields is estimated to be greater than 50 m,[18] which extrapolates to 2 to 400 m over the range from 30 kHz to 1 Hz. Since the skin depth is greater than the greatest dimension of the human body, Equation A2 is an accurate equation for the induced body currents from oscillating magnetic fields below the RF range (<30 kHz).

Equation A2 gives instantaneous limits on the magnetic field. More useful are limits on the rms field (Equation 5), which is how they are usually measured. Taking the rms of Equation A2 also makes the limit valid for any magnetic field, whatever its frequency spectrum or waveshape. The rms function even applies to pulsed fields if the averaging time covers the pulse's duration.

To solve Equation A2 for the magnitude of the rms induced current, the TLV and IRPA/ICNIRP criteria now assume that the magnetic field is sinusoidal (Equation 4), so that rms(dB/dt) is $2\pi f B$ (Equation 6). To evaluate the integral in Equation A2, simple electrical models of the human body are used: a round circuit the size of an average adult's waist in the IRPA's rationale[5] or an ellipsoid the size of an average man in the TLV rationale.[6] Both IRPA and the TLV rationale use a constant conductivity $\sigma = 0.2$ S/m, the average value for the human body at 50 Hz.[19] The result is an upper bound on the rms induced body current:

$$\mathrm{rms}(\mathbf{J}) \leq 2\pi f B \sigma \times \text{geometric factor} \tag{A3}$$

where the geometric factor depends on the model for the body.

To illustrate the calculation of the geometric factor, consider the round circuit with a radius r = 5 cm assumed by IRPA.[5] Substituting this geometry into Equation A2 gives a geometric factor of $A \cos\theta/C = (\pi r^2) \cos\theta/(2\pi r) = 0.025$ m for a magnetic field that is perpendicular to the circuit ($\theta = 0°$). This exposure condition maximizes the induced current, and is therefore the situation that must be kept under control to assure safe levels of **J**.

The more complex and more realistic ellipsoidal model in the TLV's rationale gives a maximum induced current of $J = 1.2\, f B \sigma$, so their geometric factor is $1.2/2\pi = 0.19$ m.[6] The tenfold difference between these two geometric factors is one reason that the TLV is lower than the equivalent exposure guideline from IRPA/ICNIRP.

With both the TLV and IRPA/ICNIRP rationales the rms current density is proportional to the frequency times B, as shown in Equation A3. The TLV rationale states this explicitly.[6] However, the IRPA paper only lists a series of values for the induced current density, which obey: rms(J) = 2B for "50/60 Hz magnetic fields."[5] In contrast, Equation A3 with the geometric factors from IRPA's circular model gives current densities of 1.57 B for 50 Hz and 1.89 B for 60 Hz. The IRPA committee has thus rounded off the modeled relationship to maximize the induced current, effectively adding a safety factor to its exposure guidelines. Since the IRPA/ICNIRP guidelines are not frequency dependent, the safety margin is greater for 50 Hz.

The formula for frequency-dependent exposure limits (Equation 1) is obtained by solving Equation A3 for the magnetic field magnitude B:

$$B = \left[\frac{\text{rms}\,(J)}{2\pi\sigma \times \text{geometric factor}}\right]\frac{1}{f} \tag{A4}$$

By setting rms(J) to the 10 mA/m^2 safety criterion, the quantity in brackets approximates the constant C_{EL} in these exposure limits. Substituting the parameters from the TLV rationale gives C_{EL} = 41.7 mT-Hz, while the official TLV formula (including a safety factor) uses 60 mT-Hz.[1,6]

Appendix B: Derivation of the rms Derivative for Multiple Frequencies

Taking the time derivative of the magnetic field expressed as a Fourier series (Equation 9) gives:

$$\frac{dB(t)}{dt} = -\sqrt{2}\sum_f 2\pi f\, B_f \sin(2\pi\, f t + \phi_f) \tag{B1}$$

To take the rms (Equation 5) of dB/dt, Parceval's theorem[20] is used to give:

$$\text{rms}\left(\frac{dB}{dt}\right) = 2\pi\sqrt{\sum_f (f B_f)^2} \tag{B2}$$

This result is the basis for the methods to measure the magnetic field derivative (Equations 10 and 11).

Appendix C: Errors in dB/dt Measurements with a Nonlinear Frequency Response

A gaussmeter with a nonlinear frequency response F(f) can also be used to measure dB/dt approximately by applying the correction factor $2\pi f_{power}$. Relative to the exact procedure in Equation 12, the bias with the nonlinear gaussmeter is:

$$\text{bias}\left(\frac{dB}{dt}\right) = \frac{R_F - R_{linear}}{R_{linear}} = \sqrt{\frac{\sum_f |F(f)B_f|^2}{\sum_f |f\,B_f|^2}} - 1 \tag{C1}$$

To evaluate this bias, assume that the harmonic distortion of the workplace magnetic field is dominated by the nth harmonic of the power frequency in the environment (i.e., $f = nf_{power}$). The field's harmonic distortion (defined in Equation 2) is approximately

$$H_d \equiv \frac{\%H_d}{100\%} = \frac{1}{B_{power}}\sqrt{\sum_{f\neq f_{power}} B_f^2} \cong \frac{B_n}{B_{power}} \tag{C2}$$

So the bias from using nonlinear gaussmeters is

$$\text{bias}\left(\frac{dB}{dt}\right) \cong \sqrt{\frac{1 + F(n)^2 H_d^2}{1+n^2 H_d^2}} - 1 \tag{C3}$$

For a linear response gaussmeter [i.e., F(n) = 1], the bias is negative if n > 1 (a harmonic greater than f_{power}) and positive if n < 1 (an under-harmonic). This result is also used to estimate the errors in assessing exposures to multifrequency fields with a single-frequency exposure limit.

Appendix D: Calibration with Fields with Harmonic Distortion

If the calibration field has harmonic distortion (Equation C2) in its power supply, its magnitude can be written as: rms(B) = $B_{power}\sqrt{(1 + H_d^2)}$. A linear response gaussmeter calibrated against a multifrequency field will have response R'_{linear} with a calibration constant k′ which equals:

$$k' \equiv \frac{\text{rms}(B)}{\sum_f |fB_f|^2} = \frac{1}{f_{power}}\sqrt{\frac{1 + H_d^2}{1+\sum_{n\neq 1}|nB_n/B_{power}|^2}} \tag{D1}$$

If the frequency spectrum of the calibration field (or current) is measured, then k′ can be determined explicitly, and rms(dB/dt) = $2\pi R'_{linear}/k'$. If the harmonic distortion is not measured, the approximate expression rms(dB/dt) ≈ $2\pi f_{power} R'_{linear}$ can still be used to compare exposures with the existing limits, but bias(dB/dt) = (k/k′) − 1.

The bias from this approximate calibration is estimated (as in Appendix C) by assuming that H_d is dominated by the nth harmonic:

$$\text{bias}\left(\frac{dB}{dt}\right) \approx \sqrt{\frac{1 + H_d^2}{1+n^2 H_d^2}} - 1 \tag{D2}$$

This result is used to estimate the errors from calibrating gaussmeters with fields that have harmonic distortion.