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Error Analysis in Assessing Respirator Protection Factors

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Two methods of investigation are presented which can be useful in quantitative industrial hygiene. Propagation of error theory is used to predict the precision of air concentration measurements and to demonstrate the major source of imprecision when measuring aerosols using gravimetric analysis. Predictable precision is shown to vary from ±5 to ±25 percent with the largest proportion of this imprecision changing from the gravimetric measurement of mass at low aerosol concentrations to the control of air sample flow rate at high concentrations. When evaluating workplace protection factors (WPFs), both of these conditions can exist simultaneously as in-mask and ambient concentrations, respectively, and their combined effect is investigated. The use of factor analysis is then demonstrated by investigating the impact of noncompliance with respirator use on WPFs. This impact varies with the level of effective protection (EPF), the fraction of time the wearer is not in compliance, and whether the compliance and noncompliance contaminant concentrations are equal or unequal and differ by a ratio D. For example, noncompliance in a uniform environment for an interval of only 0.02 percent of the total work time will cause a 25 percent bias error between a measured EPF of 1000 and the corresponding WPF had compliance been complete. On the other hand, if the environment of compliance has higher concentration conditions than the environment of noncompliance by a ratio of 5:1, then a 10 percent interval of noncompliance is required to generate this same 25 percent error for a measured EPF of 10. Application of these quantitative tools can greatly aid the efficient allocation of a hygienist's limited time and attention. POPENDORF, W.: ERROR ANALYSIS IN ASSESSING RESPIRATOR PROTECTION FACTORS. APPL. OCCUP. ENVIRON. HYG. 10(7):606–615; 1995.

Industrial hygienists are often concerned with the effects of experimental or methodological errors on the precision and accuracy of their evaluation results when selecting acceptable analytic procedures,[1] when making compliance or other decisions,[2] or when trying to separate the effects of environmental variability from those caused by analytical variability.[3,4] Hygienists often want to minimize variability within their methods to improve the precision of their estimates, but they are often unaware that their efforts to improve overall precision can be inefficient or squandered if resources are apportioned on steps or variables within a given procedure which are not major contributors to the imprecision of the calculated result.

The influence of the relative contribution of variability became an issue during the conduct and interpretation of the results of a field-based respirator effectiveness project.[5] How precise were individual results? Could imprecision cause a high frequency of low protection factors? How significant was intermittent momentary noncompliance on the measured protection factors?

Two techniques presented here were used to investigate the potential impact of methodological variability upon the precision and accuracy of measured air contaminant concentrations (C) and workplace protection factors (WPFs). The first technique is most suitable to analyze the effect of relatively small, random errors (imprecision) within multiple, independent variables upon the precision of a result calculated from those variables. The second technique is more suitable to analyze the effects of larger variations more characteristic of bias and inaccuracy in the result. Of particular interest was the influence of partial noncompliance with respirator wear upon the resulting measured protection factor.[5] Finally, both techniques are combined to demonstrate how propagation of error theory can be used to test the precision of a bias estimation. Both techniques are generalizable to other calculated results of interest to quantitative industrial hygienists.

The Theory of Propagating Small Random Errors

Propagation of error theory provides researchers and other collectors of environmental data with a powerful tool with which to investigate or predict the effect that random error, uncertainty, or variability in individually measured or controlled variables has on the precision or variability of a calculated result.[4,6,7] For instance, the theory can be applied:

1. to anticipate the expected magnitude of experimental or methodological uncertainty in a result (R) to be measured using a new procedure;
2. to compare the predicted methodological variability with either the natural variability expected in an environment or the magnitude of variability observed within a set of results (e.g., a set of measured contaminant concentrations);
3. to apportion measured variability within a set of results to or among multiple independent variables used within the method or technique; or/and
4. to identify the major source(s) of experimental variability within a given experimental procedure or analytic method for the purpose of improving the precision of that method.

In each case a propagation of error analysis would start with an estimate of the uncertainty or imprecision of each measured component, evaluate its propagation through the equation, and yield the individual and combined effects of these imprecisions upon the calculated result (referred to herein generically as R). The effect of any correlation between components (i.e.,

where the magnitude of errors in one variable will depend upon the value of another interdependent variable within the equation) can also be assessed, although such correlations are often not easily predicted.

This method is parametric in that it assumes a distribution of errors around a central value which can be described by some parameter like a standard or geometric deviation.[7,8] The accuracy with which the propagation of error theory can predict the precision of R depends critically upon the ability of the user to define the precision (or imprecision) of each component in a consistent manner. For example, one could use either the standard deviation ($\pm S$), the geometric standard deviation (S_g or GSD), 95 percent confidence limits of either of the above ($\pm 1.96S$ or $S_g^{1.96}$), or the range. It is important not to interchange among these optional parameters within a given error analysis. Given consistently defined estimates of the precision of each variable, the calculated precision in R will be a reliable predictor of that chosen definition.

When R is calculated using an equation of k variables X_i with each variable having an estimated uncertainty or imprecision (expressed herein generically as S_i), then error propagation theory holds that the uncertainty of the result S_R (or more specifically its variance S_R^2) may be estimated by Equation 1.[6,7]

$$S_R^2 = \sum_{i=1}^{k} \left(\frac{\delta R}{\delta X_i}\right)^2 S_i^2 + \sum_{j=1}^{k,k\neq i} \left(2\rho_{X_jX_i} \frac{\delta R}{\delta X_j} \frac{\delta R}{\delta X_i}\right) S_i S_j \qquad (1)$$

where:

$\delta R/\delta X_i$ = the partial differential of the equation used to calculate R with respect to each independent X_i

$\rho_{X_iX_i}$ = the correlation coefficient between any two variables that are intercorrelated

Of course, for independent errors (i.e., $\rho = 0$), Equation 1 simplifies greatly by the deletion of the right-hand summation.

It is also sometimes useful but often more cumbersome to apply this basic error propagation theory to determine the geometric variance of lognormally distributed data. It is possible to make a logarithmic transformation of some calculation formulas to provide a direct estimate of the relative variance of R throughout a potentially wide range of X_i. Unfortunately, the mathematics of the logarithmic approach can easily become unwieldy even for certain otherwise simple equations such as $R = X_1 + X_2$. But one useful example is worth pointing out. The generic relative variance for a rational equation in which the i variables appear only with powers (such as X^{y_i}) is shown in Equation 2.

$$\left(\frac{S_R}{R}\right)^2 = \sum_{i=1}^{k} \left(y_i\right)^2 \left(\frac{S_i}{X_i}\right)^2 \qquad (2)$$

It is useful to note in Equation 2 that the predicted result's relative variance is simply the sum of the squares of the relative errors of each component (S_i/X_i) times the square of the power of that respective variable within the original calculation formula (y_i). This relationship yields two useful rules of thumb to improve experimental error most efficiently:

1. If the precisions of all component variables are either equal or unknown and assumed equal, allocate resources to those variables which appear with the largest powers (e.g., squares, cubes, etc.) and disregard those with the smallest powers (e.g., square roots, etc).
2. If the formula is linear (where all powers are unity), allocate resources to that component with the highest relative error, since the predicted relative variance is simply the sum of the component variances.

It can also be shown that for small variations (GSD $\leq$ 1.35), the normal and lognormal distributions are practically indistinguishable.[8] Thus, in practice it is usually adequate to determine the linear deviation S_R using Equation 1 within a relatively narrow region of each X_i as shown in the examples below, and then divide S_R by the corresponding R to yield the relative precision S_R/R. An important limitation of this latter approach is that the conclusion may be valid only within the vicinity of the assumed set of operating conditions. Hewson and Martin[4] apparently used this method without explaining it to calculate total error values for their low respirator protection factors, but they did not explore the wider range of experimental conditions encountered by others, as reviewed by Popendorf *et al.*[5] The examples which follow will show that even this limitation can be overcome by calculating an array of S_R/R values to cover any desired range of X_i and R of interest to a given investigator. As stated above, the only real trick to applying this method is to define the errors of each variable accurately and consistently.

Small Random Errors Underlying Concentration Measurements

An initial example of interest to most industrial hygienists is the assessment of air contaminant concentration as calculated by Equation 3. Which factor should one emphasize in quality control?

$$R = C = \frac{\text{mass of contaminant collected}}{\text{air flow rate} \times \text{time}} = \frac{m}{Q \times t} \qquad (3)$$

As a first step, the investigator must define typical or mean conditions of interest for each variable. Table 1 gives two examples of interest. One is a moderately dusty environment (4.2 mg/m^3) in which working with a respirator might be advisable[9] or required.[10] The other condition is a low dust environment (0.21 mg/m^3); although it could be any low dust environment, it was chosen to represent the inside of a respirator with a WPF of 20 worn in the above environment.

In the next step, values for the precision or uncertainty of each variable are assigned in either absolute terms (S_i) or relative terms (S_i/X_i). Values assigned herein are the absolute precisions of each component's method, from which relative precisions were derived applicable to each condition. Independent error contributions from measured m, Q, and t are assumed in this example (i.e., all values of ρ in Equation 1 = 0), although it is recognized that some investigators or study protocols may sample highly contaminated air for a shorter time, thus creating an intercorrelation between m and t.

The third step is to apply the basic error propagation Equation 1 to the equation defining the result. Applying Equation 1 to Equation 3 yields Equation 4 (three equivalent forms of Equation 4 are shown for illustration only).

TABLE 1. Instrument Component Precisions (S_i) and Example Values (X_i) Used to Calculate the Predicted Precision of Measuring Aerosol Concentrations Via Gravimetric Sampling in Two Conditions

Variable	m	Q	t	C
	Mass	Flow rate	Time	Concentration
Instrument	Balance	Rotameter	Watch	Calculation
S_i	±0.02 mg	±0.1 L/min ±0.0001 m^3/min	±1 minute	?
Example A: High dust environment (e.g., outside a mask)				
X_i	1.5 mg	1.5 L/min 0.0015 m^3/min	240 minutes	4.2 mg/m^3
S_i/X_i	±0.013	±0.067	±0.004	?
Example B: Low dust environment (e.g., inside a mask)				
X_i	0.075 mg	1.5 L/min 0.0015 m^3/min	240 minutes	0.21 mg/m^3
S_i/X_i	±0.27	±0.067	±0.004	?

? represents values to be calculated via Equations 3 or 4. The two examples are at a ratio of concentrations corresponding to those outside versus inside a mask with a protection factor of 20.

$$S_C^2 = \left(\frac{\delta C}{\delta m}\right)^2 S_m^2 + \left(\frac{\delta C}{\delta Q}\right)^2 S_Q^2 + \left(\frac{\delta C}{\delta t}\right)^2 S_t^2 \quad (4a)$$

$$S_C^2 = \left(\frac{1}{Qt}\right)^2 S_m^2 + \left(\frac{-m}{Q^2 t}\right)^2 S_Q^2 + \left(\frac{-m}{Qt^2}\right)^2 S_t^2 \quad (4b)$$

A useful variation on Equation 4b is to multiply each ratio top and bottom as needed to yield R in each numerator. Since in this case R equals the concentration C = m/Qt, substitution yields Equation 4c.

$$S_C^2 = \left(\frac{C}{m}\right)^2 S_m^2 + \left(\frac{-C}{Q}\right)^2 S_Q^2 + \left(\frac{-C}{t}\right)^2 S_t^2 \quad (4c)$$

If the relative variance is desired, then one may divide both sides of the equation by C^2 and bring the component variances $(S_i)^2$ inside the parentheses to yield Equation 5.

$$\left(\frac{S_C}{C}\right)^2 = \left(\frac{S_m}{m}\right)^2 + \left(\frac{S_Q}{Q}\right)^2 + \left(\frac{S_t}{t}\right)^2 \quad (5)$$

The fourth and last step is to solve for S_R or S_R/R. Example A from Table 1 is first solved using Equation 4c to predict the precision when measuring the high dust concentration (outside the mask).

$$S_C^2 = \left(\frac{4.2}{1.5}\right)^2 0.02^2 + \left(\frac{4.2}{0.002}\right)^2 0.0001^2 + \left(\frac{4.2}{240}\right)^2 1^2$$

$$= \underset{\text{for mass}}{0.0031} + \underset{\text{for flow}}{0.0784} + \underset{\text{for time}}{0.0003}$$

$$S_C^2 = 0.0818 = (\pm 0.29 \text{ mg/m}^3)^2$$

The equivalent relative precision $S_C/C = \pm 7$ percent can be derived above by dividing ±0.29 mg/m^3 by the 4.2 mg/m^3 mean concentration. It could also have been calculated directly using Equation 5:

$$(S_C/C)^2 = (0.013)^2 + (0.067)^2 + (0.004)^2$$

$$= \underset{\text{for mass}}{0.00017} + \underset{\text{for flow}}{0.00449} + \underset{\text{for time}}{0.000016}$$

$$(S_C/C)^2 = 0.00467 = (\pm 0.068)^2$$

Solving for the low dust (or inside the mask) case cited as example B in Table 1 would proceed in exactly the same manner. Only the step using Equation 5 is shown for illustration.

$$(S_C/C)^2 = (0.27)^2 + (0.067)^2 + (0.004)^2$$

$$= \underset{\text{for mass}}{0.0729} + \underset{\text{for flow}}{0.00449} + \underset{\text{for time}}{0.000016}$$

$$(S_C/C)^2 = 0.07740 = (\pm 0.278)^2 \text{ or } S_C/C = \pm 28\%$$

In the above two cases, the relative precision of the method varies from ±7 percent when sampling moderately high dust environments to ±28 percent when sampling low dust environments. Over this range of conditions, the major contributor to these errors changes from the measurement of flow rate in the dirtier environment (contributing 0.00449/0.00467 = 96% of the error to the result) to the measurement of dust mass in the cleaner environment (0.0729/0.0774 = 94% of the error).

The results of Equation 5 are also plotted in Figure 1 for long-term (240 minutes) and short-term (24 minutes) air sampling over a range of concentration conditions. For fixed sampling parameters (Q and t), each concentration corresponds to a fixed mass of contaminant collected from the air. The example results are also indicated in Figure 1 as circled A and B, respectively. Somewhat different but analogous results could be predicted for an aerosol mass determined by other methods such as chemically, by count, etc.

Small Random Errors Underlying WPF Measurements

The basic operational definition of a measured WPF resulting from a mask being worn throughout a period of time at work has been given by Equation 6a.[11]

$$\text{WPF} = \frac{\text{average concentration outside mask}}{\text{average concentration inside mask}} = \frac{C_{\text{outside}}}{C_{\text{inside}}} \quad (6a)$$

When sampling at nominally equal Q and t both inside and outside the mask (parallel sampling), the WPF can also be

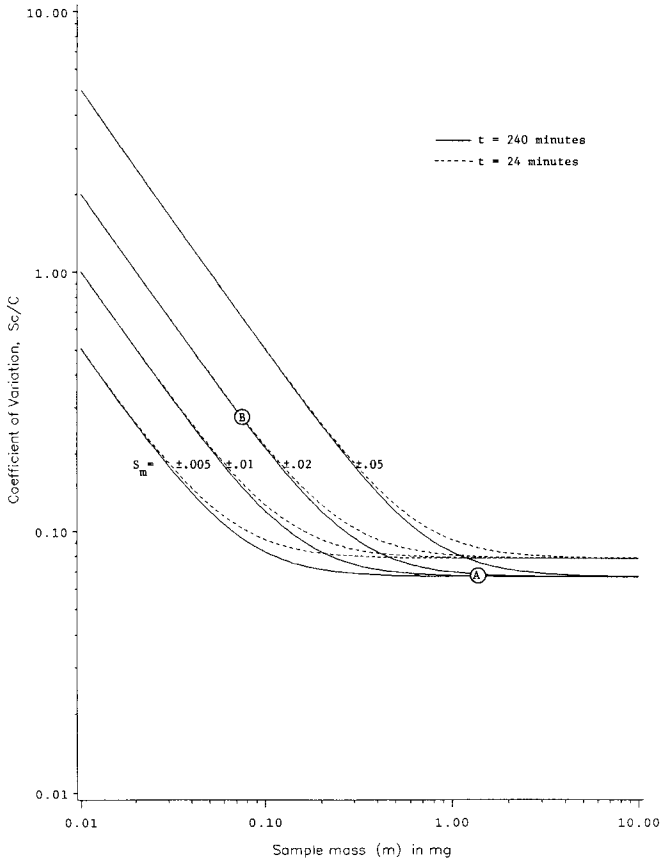

FIGURE 1. Relative precision of measured aerosol concentration (S_C/C from Equation 5) for samples collected using the methods and component precisions listed herein and in Table 1. Examples from Table 1 are marked as A and B, respectively.

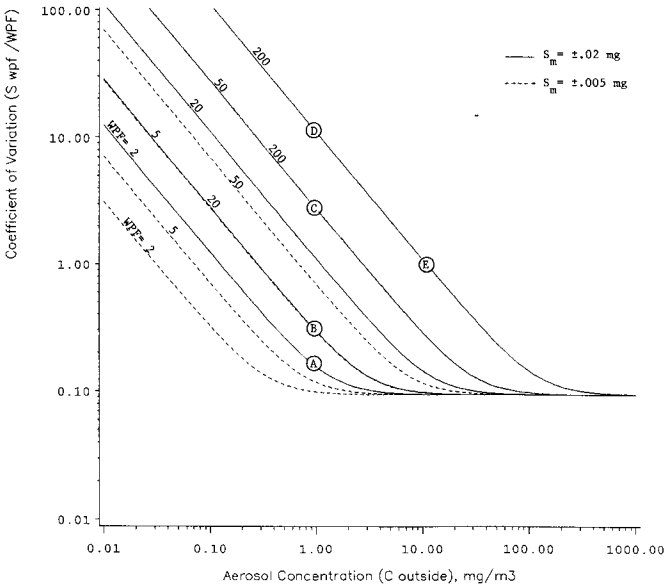

FIGURE 2. Relative precision of measured WPF (S_{WPF}/WPF from Equation 7). Solid lines depict sampling protocols similar to Table 1; dashed lines depict an $S_m = \pm 0.005$ mg.

calculated by Equation 6b as the ratio of the outside sample mass over the inside sample mass collected over the duration of the measurement. However, in reality, because the masses collected both inside and outside of a respirator are affected by independent random errors in sample flow rates Q and sample times t, Equation 6a is the appropriate place to start an error analysis of WPF.

$$WPF = \frac{m_{outside}}{Q_{outside}\, t_{outside}} \times \frac{Q_{inside}\, t_{inside}}{m_{inside}} \approx \frac{m_{outside}}{m_{inside}} \qquad (6b)$$

The steps used to derive the relative variance of a measured WPF $(S_{WPF}/WPF)^2$, shown in Equation 7, are detailed in Appendix A. Important points along the way are that the measured contaminant concentrations inside and outside the mask are considered independent, and that nominally parallel sampling methods are used inside and outside the mask. The result is expressed on the basis of an ambient contaminant concentration which is normally measured *a priori* as part of a good respirator program.[9,10]

$$\left(\frac{S_{WPF}}{WPF}\right)^2 = \left[(1 + WPF^2)\left(\frac{S_m}{m_{outside}}\right)^2\right] + 2\left[\left(\frac{S_Q}{Q}\right)^2 + \left(\frac{S_t}{t}\right)^2\right] \qquad (7)$$

The results of Equation 7 are plotted in Figure 2 for the analytic conditions in Table 1 characteristic of a WPF study against dust.[4,5] One can see that for an outside concentration of 1 mg/m^3 and an S_m of ± 0.02 mg, a measured WPF of 2 will have a relative error of only about $\pm 0.15\times$ (point marked A in Figure 2). That is, the true WPF could be 15 percent above or below the calculated WPF. This may also be expressed as a multiplying factor of $1.15\times$ yielding 95% confidence limits of $1.52 \leq WPF \leq 2.63$. This precision is within the expected range generally considered acceptable for industrial hygiene measurements. However, under these same sampling and analytic conditions, the precision of a measured WPF value of 5 would already be outside the generally accepted ± 25 percent limits (point B in Figure 2). For higher measured WPF values, the relative precision of the estimated WPF rapidly deteriorates, viz. $S_{WPF}/WPF = \pm 210$ percent for a WPF = 50 at point C, and $S_{WPF}/WPF > \pm 1100$ percent for a WPF $\geq$ 200 at point D. Or when viewed from another perspective, for a measured WPF of 200 to be precise within a relative error of even ± 100 percent using these sampling protocols (point E in Figure 2), the ambient dust concentration would have to be at least 10 mg/m^3.

This relationship indicates that the precision of high WPF measurements collected using the analytic methods described in Table 1 in conditions where respirators are not mandatory (i.e., where nuisance dust or PNOC concentrations are $\leq$5 mg/m^3) would be outside those normally acceptable to industrial hygienists. Somewhat more precise WPFs could result by

improving the absolute precision of the mass measurement to ±0.005 mg, as shown by the dashed lines of Figure 2, but only in some regions of WPF and C. Using an analytic method characterized by constant relative precision inside and outside the mask is also beneficial, as discussed briefly in Appendix A (Equation A6 and Figure 5).

Bias Errors Underlying WPF Measurements

In comparison to the random component errors described above, biasing factors are also often present in the real world. Bias errors may or may not be large, but they occur in only one direction. Their impact can be assessed more appropriately by introducing an explicit error term into the equation yielding a result and analyzing its effect.[7] A relevant example is the lack of respirator user compliance in wearing a mask 100 percent of the time.

The effect on WPF (Equation 6) of less than perfect user compliance will be estimated from a previous definition and two assumptions. The term effective protection factor (EPF) has been previously defined to differentiate the WPF from the level of protection if the mask is not worn continuously for various reasons such as comfort, communication, interference with work, etc.[11] EPF can be expressed mathematically in various ways, as explained in Appendix B. The fraction of time in noncompliance will be defined by f in Equation 8.

$$f = \frac{t_{off}}{t_{on} + t_{off}} \tag{8}$$

If compliance is less than 100 percent, one can only measure EPF and not WPF. To relate EPF quantitatively to an expected WPF had compliance been 100 percent requires two assumptions. The first is a model that relates WPF over the time that the mask is worn. The assumed model used herein is that the WPF is constant over time with no deterioration in the fit of the mask during the day. This assumption is reasonable, at least for nonfabric facepiece respirators. The second assumption is that the ratio of the contaminant concentration outside the mask when it is worn to when it is not worn is known or knowable and defined by the ratio D in Equation 9. This ratio could vary from approximately one if the user is noncompliant at random or if there are no concentration gradients within the workplace, to some rather large number if the user was noncompliant only in locally low ambient concentrations of the airborne contaminant and was compliant while in high concentrations.

$$D = \frac{C_{\text{while wearing}}}{C_{\text{while not wearing}}} = \frac{C_{high}}{C_{low}} \text{ if } D > 1 \tag{9}$$

The development of Equation 10 relating WPF to an EPF measured during a field investigation is presented in Appendix B.

$$\frac{WPF}{EPF} = \frac{D(1-f)}{D(1-f) - f(EPF-1)} \tag{10}$$

The above dependence of WPF upon EPF, D, and f is depicted in Figure 3. The horizontal portions of these plot lines are regions over which WPF is practically independent of compliance. However, at some increased value of f, the expected WPF eventually begins to rise noticeably above the measured EPF. Equation B16 defines an acceptable f value for each combination EPF and D, at which EPF would be within a given percentage of the WPF expected if compliance had been complete. Table 2 lists some acceptable f values for bias error limits of 25 percent. This f value could be used to quantify the appropriate level of urgency in enforcing rigid compliance in a field study (or its analogue in Equation B8 if WPF in day-to-day use is expected to equal a quantitative fit factor). For example, in an environment with no concentration differences (D = 1), the expected fully compliant WPF would have been within 25 percent of a measured EPF of 1000 (viz. WPF = 1.25 × 1000 = 1250) if noncompliance were for no more than 0.02 percent of the work shift, which corresponds to 3 seconds in a 4-hour work shift. As another example, if an EPF of 10 were measured in a concentration ratio D = 5:1, the same 25 percent bias error would not occur until noncompliance comprised 10 percent of the work shift or 24 minutes in 4 hours. A table similar to Table 2 could be generated from Equation B16 for any chosen EPF, D, and acceptable WPF/EPF bias criterion.

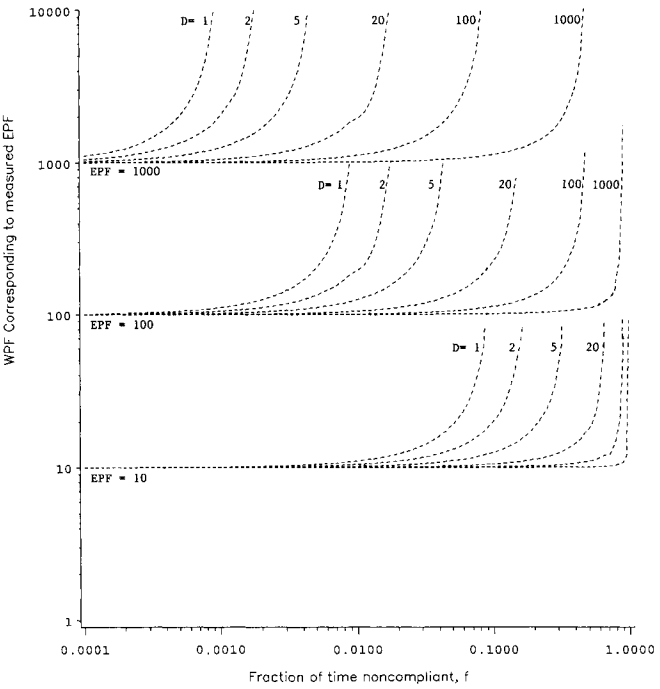

FIGURE 3. WPF values corresponding to a measured EPF as a function of the fraction of time the respirator is not worn (f) and the ratio of concentrations (D) calculated from Equation 10.

At some critical value of f, the slope of each plot line in Figure 3 becomes almost vertical, at which point the equivalent value of WPF is unknown. EPF values are interpretable over a much wider range of f values as D increases, that is, at higher ratios between concentrations where the wearer is compliant to where he or she is not compliant. However, to use a D value, a difference in concentrations must be known or defined. In the real world, data may not be available upon which to assign a value of D unless both C_{high} and C_{low} are measured separately (Equation 9). C_{high} may have been pre-

TABLE 2. Acceptable (Maximum) Noncompliance f Values for a 25 Percent Bias Error (WPF/EPF = 1.25 Derived from Equation B16)

	EPF = 5	EPF = 10	EPF = 100	EPF = 1000
D = 1	0.0476	0.0275	0.0020	0.0002
D = 2	0.0909	0.0426	0.0040	0.0004
D = 5	0.2000	0.1000	0.0100	0.0010
D = 20	0.5000	0.3077	0.0388	0.0040
D = 100	0.8333	0.6897	0.1681	0.0196
D = 1000	0.9804	0.9569	0.6689	0.1668

viously measured if the respirator is used in compliance with the National Institute for Occupational Safety and Health (NIOSH) respirator selection decision logic[9] and the Occupational Safety and Health Administration (OSHA) respiratory protection standard.[10] However, if measurements are lacking from either or both environments, one is left to speculate that a reasonable value for D will be somewhere between a theoretical lower limit and a practical upper limit.

The minimum D value shown in Equation 11 is based on the theoretical necessity for the denominator of Equation 10 to be equal to or greater than zero (similar to the limit for f in Equation B11 when D = 1). In fact, if D were to equal this theoretical lower limit, WPF would have to have been infinitely high while the mask was worn in order to have measured that EPF at that level of noncompliance f.

$$D_{minimum} > \frac{f(EPF - 1)}{(1 - f)} \tag{11}$$

A practical maximum D ratio can also be estimated by first using the fact implied by Equation 9 that the concentration measured outside the mask during a respirator protection effectiveness assessment ($C_{outside}$) is a time-weighted average of C_{high} over the fraction of time (1 − f) that the mask is worn, and C_{low} over the fraction f that the mask is not worn, as shown in Equation 12. A practical maximum D would then occur (Equation 13) if C_{low} is assumed to be the local clean air background concentration, C_{clean} for the contaminant being measured.

$$C_{outside} = C_{high}(1 - f) + C_{low}\, f \tag{12}$$

$$D_{maximum} \leq \frac{C_{outside} - C_{clean}\, f}{(1 - f)\, C_{clean}} \tag{13}$$

An interesting example of a relatively high lack of compliance reported by Harris *et al.* (f value stated to be typically 0.5) is interpreted in Table 3.[12] Equation 11 indicates that even if the WPF had been infinitely high, the D ratio between the coal dust concentrations in areas where the masks were known to be worn and the areas where they often were not worn must have been at least 2.2× in order to have a measured EPF of 3.2. Based on an average $C_{outside}$ concentration of ≈1 mg/m^3 reported in these or similar settings by Reist *et al.*[13] and an assumed mine clean air concentration of 0.1 mg/m^3, the practical maximum D ratio predicted by Equation 13 might be 95×, yielding a WPF of 3.3. A more likely D would take into account the limited range of clean air conditions available within a mine, the limited ability of mine workers to perceive high versus low respirable dust concentration differences, and the likelihood that wearers would conscientiously wear their respirators in high and not in low exposure conditions. If the range of likely D ratios were between 5× and 20×, the corresponding mean WPF would have been somewhere between 3.6 and 5.7, as listed in Table 3.

Combined Analyses of Small Errors on Bias Estimates

The two error analysis techniques presented above can be combined to assess the effects that small errors in assigning f and D values can have on estimating WPF from EPF. Because S_f and S_D are independent of each other, the first part of Equation 1 was applied to Equation 10 to yield Equation 14.

$$S^2_{WPF/EPF} = \left\{\frac{D(EPF - 1)}{[D(1 - f) - f(EPF - 1)]^2}\right\}^2 S_f^2 + \left\{\frac{f(1 - f)(EPF - 1)}{[D(1 - f) - f(EPF - 1)]^2}\right\}^2 S_D^2 \tag{14}$$

Equation 14 could be solved for any assigned values depending upon the experimental circumstances. However, for purposes of discussion it is convenient to combine terms by assuming either equal absolute precisions S_f and S_D or equal relative precisions. Since f is normally a small fraction of the observed assessment period and D is a ratio ≥1, it is more reasonable to assume circumstances where one can estimate D and f with equal relative precision, that is, to assume equal values for S_f/f and S_D/D (shown in Equation 15 as the generic S_i/X_i).

TABLE 3. Possible WPFs Using Equation 10 for the EPF Statistics Reported by Harris *et al.*[12] for the Performance of 187 Half-Mask Respirators Against Coal Dust with an Estimated Use Compliance Factor f of 0.5

D	WPF/EPF	WPF	Comments
2.2	∞	∞	Minimum D (Equation 11)
2.5	8.3	27	
3	3.75	12	
5	1.8	5.7	Lowest practical D
10	1.28	4.1	
20	1.12	3.6	Highest practical D
100	1.02	3.3	Maximum D (Equation 13)

Geometric mean EPF = 3.2 and GSD of EPFs = 1.55.

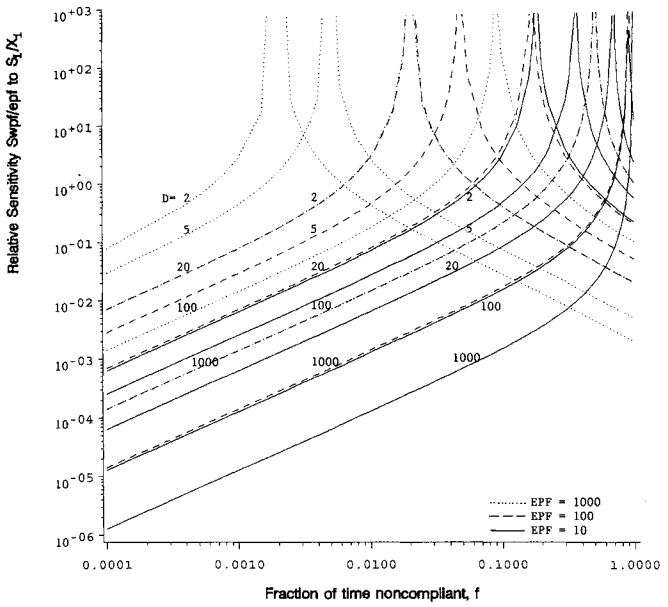

FIGURE 4. Precision of the expected WPF estimated from measured EPF as a ratio of assumed equal relative precisions of S_D/D and S_f/f calculated from Equation 15.

$$S^2_{WPF/EPF} = [1 + (1 - f)^2] \cdot \left\{\frac{D\,f(EPF - 1)}{[D(1 - f) - f(EPF - 1)]^2}\right\}^2 \left(\frac{S_i}{X_i}\right)^2 \qquad (15)$$

As implied by the derivation of Equation 15, under the circumstances of equal relative precisions, the contribution to the result's precision can be proportioned as $1/[1 + (1 - f)^2]$ for the relative precision in estimating the D ratio and $(1 - f)^2/[1 + (1 - f)^2]$ for the relative precision in estimating the f factor. Thus, for values of $f < 0.1$, the relative errors in estimating each factor have roughly equal effects on the relative precision of estimating WPF from EPF.

One can also anticipate a sensitive region in the estimated precision of WPF when the denominator of Equation 15 approaches zero, which turns out to be the same as those in Figure 3 and those discussed in Table 4. One can see the effects of this phenomenon in Figure 4, where values of $S_{WPF/EPF}/S_i/X_i$ are plotted as a function of f for some typical combinations of EPF and D. This ratio can be viewed as the relative sensitivity of expected WPF estimated from EPF in comparison to the combined effect of equal relative errors in estimating f and D. Ratios greatly exceeding 10 occur near the sensitive region of f. Ratios less than 0.1 (shown as 10^{-01}) indicate regions where the estimate is insensitive to errors in D and f. One can see that for low EPF, the result is insensitive to errors for almost any D and for f values ≤ 0.1. These were the conditions (with $D \approx 1$ and $f \leq 0.01$) encountered by Popendorf *et al.*[5]

Unfortunately, the high f situation encountered by Harris *et al.*[12] is not so favorable. At EPF = 3.2 and f = 0.5, the sensitivity of the estimate of WPF is almost equal to the generic precision S_i/X_i, and relatively more sensitive to errors in estimating D than to errors in estimating f by a factor of 4:1. Given that the likely D values in Table 3 span a range of $\pm 2\times$ about a midpoint of 10, the expected mean WPF values should span a range of no more than $\pm 2\times$ about 4.1. This analysis is not dependent upon the analytic methods used in any particular respirator effectiveness study.

Conclusions

Propagation of error analysis was used to demonstrate that large increases in relative imprecision can result from measuring low concentrations using methods with a fixed absolute precision. These small random errors place some practical limits on the precision of measuring high WPFs, but have relatively little effect when measuring low WPFs. Some practical rules of thumb were recommended on the basis of Equation 2.

Factor analysis was used to investigate the influence of bias on WPF. A lack of compliance will reduce an EPF below the WPF expected had the respirator been worn continuously, but not appreciably in all circumstances. Using an acceptance criterion of a 25 percent error, acceptable values of f were identified for a wide range of D and EPF values. Bias errors will be severe when D is very close to its lower limit as given by Equation 11 (or f is close to its critical value as given by Equation B11).

These two techniques were combined to show, for situations where f is less than 1/WPF, that the effect of errors in estimating D ratios will have almost no influence on the WPF/EPF ratio. Under unfavorable conditions of moderately low EPF values measured in low compliance (high f) conditions, estimates of expected WPF were still possible within a factor of about 2×. And perhaps most important when interpreting a field study designed to assess WPF, a short or intermittent lack of compliance (even up to 10% of the time) is unable to produce or explain a high frequency of WPF values less than 10, as has been encountered by numerous researchers.[4,5]

Acknowledgment

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Appendix A: Error Propagation in WPF Measurements

Recognizing that Equation 6a is rational permits the application of Equations 1 and 2 to begin to derive the relative variance of a measured WPF $(S_{WPF}/WPF)^2$ as shown in Equation A1.

$$\left(\frac{S_{WPF}}{WPF}\right)^2 = \left(\frac{S_{C,\,inside}}{C_{inside}}\right)^2 + \left(\frac{S_{C,\,outside}}{C_{outside}}\right)^2 - 2\rho\, WPF^2 \frac{S_{C,\,inside}}{C_{inside}} \frac{S_{C,\,outside}}{C_{outside}} \quad \text{(A1)}$$

It could be argued that because WPF is assumed to be a constant, the concentration inside a mask C_{in} should be correlated with that outside C_{out}, and that the correlation term in Equation A1 should be retained. However, in those studies where correlation was tested, they turned out to be only weakly correlated at best.[4,5,14–16] In the largest study of about 70 WPFs, the correlation ρ^2 between pairs of measured concentrations was only 0.005 for disposable masks, 0.14 for cartridge half-masks, and 0.15 for powered air purifying helmets.[5] Given the weak magnitude of this correlation, ρ will be disregarded for the purposes of the following discussion. Thus, in Equation A2 the relative precision of each concentration is broken down into its measured components:

$$\left(\frac{S_{WPF}}{WPF}\right)^2 = \left(\frac{S_{m,\,inside}}{m_{inside}}\right)^2 + \left(\frac{S_{Q,\,inside}}{Q_{inside}}\right)^2 + \left(\frac{S_{t,\,inside}}{t_{inside}}\right)^2 + \left(\frac{S_{m,\,outside}}{m_{outside}}\right)^2 + \left(\frac{S_{Q,\,outside}}{Q_{outside}}\right)^2 + \left(\frac{S_{t,\,outside}}{t_{outside}}\right)^2 \quad \text{(A2)}$$

Equation A2 reduces to Equation A3 when the inside and outside subscripts are removed for those variables which are nominally equal because virtually all WPF studies utilize parallel sampling protocols. For parallel sampling, the flow rate Q, sample collection time t, and their respective absolute variances are the same for samples collected either inside or outside the mask. Of course, if sampling conditions are maintained equal inside and outside the mask and the respirator is providing protection, the sample mass m_{inside} the mask should not equal the $m_{outside}$ (see also Equation 6b in text).

$$\left(\frac{S_{WPF}}{WPF}\right)^2 = \left(\frac{S_{m,\,inside}}{m_{inside}}\right)^2 + \left(\frac{S_{m,\,outside}}{m_{outside}}\right)^2 + 2\left[\left(\frac{S_Q}{Q}\right)^2 + \left(\frac{S_t}{t}\right)^2\right] \quad \text{(A3)}$$

It would be helpful if Equation A3 were simplified by combining terms. Two options are available depending upon the precision characteristics of the analytic method employed to measure the contaminant. For methods characterized by fixed absolute precision such as a gravimetric balance, S_m is a constant but the ratio S_m/m will be different inside and outside the mask. Good industrial hygiene practices (and OSHA regulations) dictate that the aerosol concentration (and corresponding sample mass) outside the mask should be known before a respirator is used.[9,10] Since the sample mass inside the mask is a function of that outside and the WPF (as defined by Equation 6b), m_{inside} will be removed from Equation A3 by substitution, yielding Equations A4 and A5, which should be convenient in both planning and interpreting WPF measurements using analytic methods with a fixed absolute precision.

$$\left(\frac{S_{WPF}}{WPF}\right)^2 = \left(\frac{S_m\, WPF}{m_{outside}}\right)^2 + \left(\frac{S_m}{m_{outside}}\right)^2 + 2\left[\left(\frac{S_Q}{Q}\right)^2 + \left(\frac{S_t}{t}\right)^2\right] \quad \text{(A4)}$$

$$\left(\frac{S_{WPF}}{WPF}\right)^2 = \left[(1 + WPF^2)\left(\frac{S_m}{m_{outside}}\right)^2\right] + 2\left[\left(\frac{S_Q}{Q}\right)^2 + \left(\frac{S_t}{t}\right)^2\right] \quad \text{(A5)}$$

For methods characterized by fixed relative precision such as atomic absorption or gas chromatography analyses (due to the ability to dilute liquid extracts into a linear calibration range), the two S_m/m ratios in Equation A3 can be combined directly, yielding Equation A6.

$$\left(\frac{S_{WPF}}{WPF}\right)^2 = 2\left(\frac{S_m}{m}\right)^2 + 2\left[\left(\frac{S_Q}{Q}\right)^2 + \left(\frac{S_t}{t}\right)^2\right] \quad \text{(A6)}$$

Comparing the first terms on the right side of Equations A5 and A6, it becomes apparent that a more relatively precise measurement of WPF is likely to result from a mass measurement with equal relative precision inside and out than from a method with equal absolute precision. As can be seen in Figure 5, the limiting precision of S_{WPF}/WPF is dependent upon the

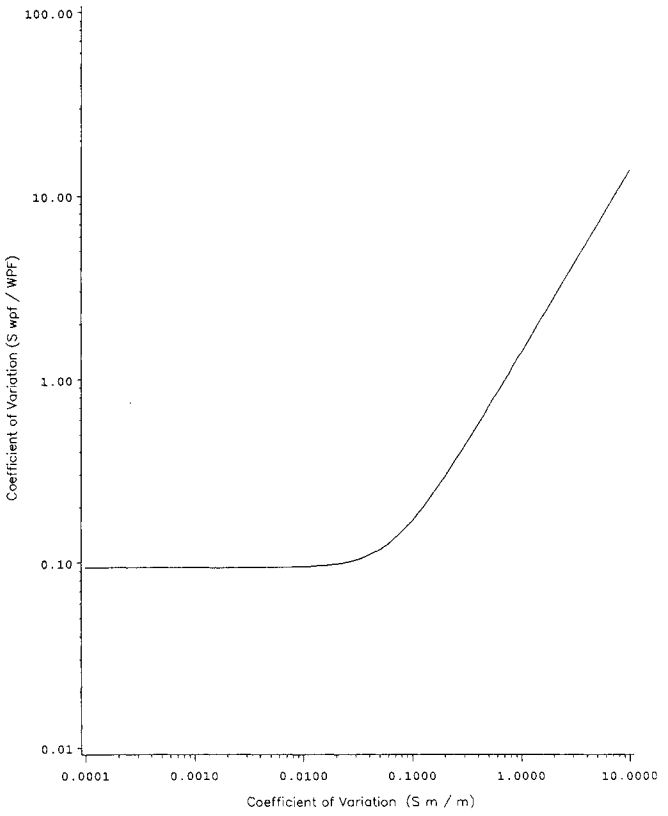

FIGURE 5. Relative precision of measured WPF (S_{WPF}/WPF from Equation A6) using protocols similar to Table 1 except for using an analytic method that yields equal S_m/m for samples inside and outside the mask.

flow rate and sample timing as long as S_m/m is less than about ±0.10 or ±10 percent.

Appendix B: Factor Analysis in WPF Measurements

EPF is the ratio of the concentrations measured outside a mask to that measured inside when it is not worn all the time. For purposes of modeling, the concentration outside $C_{outside}$ is measured and initially assumed to be a constant whether the mask is worn or not. On the other hand, the concentration measured inside the mask (the denominator in Equation B1) comprises a time-weighted average of the concentration outside the mask ($C_{outside}$) while it is not being worn (t_{off}) and a reduced concentration existing inside the mask (C_{inside}) only during the time it is being worn (t_{on}), as shown in Equation B1.

$$EPF = \frac{C_{outside}\,(t_{on} + t_{off})}{C_{outside}\,t_{off} + C_{inside}\,t_{on}} \tag{B1}$$

Note in Equations B1 through B3 that if the mask were worn continuously (i.e., $t_{off} = 0$), EPF would equal the WPF as shown in Equation 6 of the text. On the other hand, if the mask is not worn continuously, C_{inside} as defined cannot be measured. However, the assumption of a constant concentration outside the mask permits the application of Equation 6a to substitute the nominal $C_{outside}$/WPF for C_{inside} while the mask is being worn.

$$EPF = \frac{C_{outside}(t_{on} + t_{off})}{C_{outside}\,t_{on}/WPF + C_{outside}\,t_{off}} \tag{B2}$$

Thus, $C_{outside}$ also ceases to be a factor needed to calculate EPF as shown in Equation B3 (consistent with the concept of a constant outside concentration).

$$EPF = \frac{(t_{on} + t_{off})}{t_{on}/WPF + t_{off}} \tag{B3}$$

Rather than deal with the absolute values of t_{on} and t_{off}, a relative noncompliance factor f is defined as the fraction of time that the respirator is not worn (Equation B4). Thus, t_{off} in Equation B5 can be substituted into Equation B3 above to derive Equations B6 and B7.

$$f = \frac{t_{off}}{t_{on} + t_{off}} \tag{B4}$$

$$t_{off} = \frac{t_{on}\,f}{(1 - f)} \tag{B5}$$

$$EPF = \frac{[1 + (f/1 - f)]}{[1/WPF + f/(1 - f)]} \tag{B6}$$

$$\frac{EPF}{WPF} = \frac{1}{1 + f(WPF - 1)} \tag{B7}$$

Note that if f = 0 (i.e., if the mask is worn all the time), EPF in Equation B6 would again equal the WPF. Some results of this equation for $0 \leq f \leq 0.07$ were apparently plotted by Brown.[17] The ratio of EPF/WPF given in Equation B7 can be interpreted as the proportional reduction from an expected WPF to a measured EPF caused by the mask not being worn continuously in a uniform ambient concentration. For any level of noncompliance f, the proportionate reduction in EPF/WPF increases as the expected WPF increases. Alternatively, the f values needed to reduce any expected WPF to a given EPF/WPF can be found using Equation B8.

$$f = \frac{(EPF/WPF)^{-1} - 1}{WPF - 1} \tag{B8}$$

For example, an f as small as 0.0003 would reduce an expected WPF of 1000 by 25 percent (e.g., EPF/WPF = 0.75 when off for only 5 seconds in 4 hours). But for a mask with a WPF of 10, reductions don't exceed 25 percent until f exceeds 0.037 (e.g., the mask could be off for a total of nearly 9 minutes over 4 hours). This equation could be used to establish training and supervisory guidelines for acceptable EPF/WPF values due to intermittent noncompliance which should be at least equal to the ratio of a quantitative fit factor over the required protection level.

Rather than projecting the effect of f on an expected WPF, in some situations one may not know what WPF to expect. For instance, one may intend to assess WPF but actually measure EPF in the field due to an unexpected lack of participant cooperation and compliance with instructions. Alternatively, one may intend to assess EPF in the experimental design because it is simpler logistically, but may desire to know what WPF to expect had compliance been 100 percent. In either

case, unless one knows the expected WPF had the respirator been worn continuously, one cannot use Equations B6 through B8. To estimate the expected WPF in the presence of noncompliance, these equations have to be restated in terms of a measured EPF and the WPF/EPF ratio as shown in Equations B9 through B11 (analogous to Equations B6 through B8).

$$\mathrm{WPF} = \frac{\mathrm{EPF}(1 - f)}{1 - f\,\mathrm{EPF}} \tag{B9}$$

$$\frac{\mathrm{WPF}}{\mathrm{EPF}} = \frac{(1 - f)}{1 - f\,\mathrm{EPF}} \tag{B10}$$

$$f = \frac{\mathrm{WPF/EPF} - 1}{[(\mathrm{WPF/EPF})\mathrm{EPF}] - 1} \tag{B11}$$

Analogous to the examples above, one can calculate an f that will not cause the WPF to exceed the measured EPF by more than 25 percent. For example, wearing a mask with an EPF of 100 measured while breaking the mask-to-face seal for up to nearly 30 seconds over 4 hours (f = 0.002) will cause the WPF to exceed the measured EPF by no more than 25 percent. Moreover, limits can be seen in these predictive models; for instance, when the measured EPF equals 1/f the denominator of Equation B10 becomes zero. Thus, if a respirator is not worn long enough for the measured EPF to exceed (or for practical purposes even approach) 1/f, a WPF cannot be estimated via these assumptions and equations.

A logical condition necessary to avoid the above limitation and sufficient to explain experimental results where the measured EPF exceeds 1/f [e.g., Harris *et al.*[12]], is that the respirator be worn in high concentrations of airborne contaminant and taken off in less contaminated settings. This condition violates the initial concept of a constant $C_{outside}$ as stated above, but is probably realistic for many real-world settings.[4] A convenient way to define the two concentrations is by their ratio D in Equation B12 between C_{high}, in which the mask is worn during t_{on}, and C_{low}, in which it is not worn during t_{off} (the same as Equation 9 in the text).

$$D = \frac{C_{\text{while wearing}}}{C_{\text{while not wearing}}} = \frac{C_{high}}{C_{low}} \text{ if } D > 1 \tag{B12}$$

This definition of D usefully approximates potentially gradual concentration gradients in some real-world settings by a more abrupt transition to a uniformly higher concentration, and grants that the user has either the incentive at their own discretion or sufficient supervisory direction to wear the mask during those higher exposure conditions. However this ratio occurs, a difference can happen and the net time-weighted average difference can be expressed as a ratio of concentrations. This ratio can then be used to redefine EPF in Equation B13 as a time-weighted average similar to Equation B1, and to solve again for WPF, WPF/EPF, and f as shown in Equations B14 through B16, respectively. As expected, if D = 1, Equations B14 through B16 reduce to Equations B9 through B11, respectively.

$$\mathrm{EPF} = \frac{C_{high}\,t_{on} + C_{low}\,t_{off}}{C_{high}\,t_{on}/\mathrm{WPF} + C_{low}\,t_{off}} = \frac{t_{on} + t_{off}/D}{t_{on}/\mathrm{WPF} + t_{off}/D} \tag{B13}$$

$$\mathrm{WPF} = \frac{\mathrm{EPF}\,D\,(1 - f)}{(D - fD + f) - \mathrm{EPF}\,f} \tag{B14}$$

$$\frac{\mathrm{WPF}}{\mathrm{EPF}} = \frac{D(1 - f)}{D(1 - f) - f(\mathrm{EPF} - 1} \approx \frac{D}{D - f\,\mathrm{EPF}} \text{ for } f \ll 1 \ll \mathrm{EPF} \tag{B15}$$

$$f = \frac{D(\mathrm{WPF/EPF} - 1)}{[(\mathrm{WPF/EPF})(\mathrm{EPF} + D - 1)] - D} \tag{B16}$$

Similar to the situation before D was introduced, a critical value of f still exists if the denominator of Equations B14 or B15 is zero. The expected WPF becomes unpredictable under such circumstances near the critical f value determined in Equation B17. Calculated values of $f_{critical}$ are listed in Table 4 for a range of experimental conditions including those depicted in Figures 3 and 4.

$$f_{critical} = \frac{D}{(\mathrm{EPF} + D - 1)} \tag{B17}$$

TABLE 4. Examples of Critical Values of f for EPF and D Derived from Equation B17

	EPF = 2	**EPF = 5**	**EPF = 10**	**EPF = 20**	**EPF = 100**	**EPF = 1000**
D = 1	0.5000	0.2000	0.1000	0.0500	0.0100	0.0010
D = 2	0.6667	0.3333	0.1818	0.0952	0.0198	0.0020
D = 5	0.8333	0.5556	0.3571	0.2083	0.0481	0.0050
D = 20	0.9524	0.8333	0.6897	0.5128	0.1681	0.0196
D = 100	0.9901	0.9615	0.9174	0.8403	0.5025	0.0910
D = 1000	0.9990	0.9960	0.9911	0.9814	0.9099	0.5003

These values correspond to the vertical portions of the curves in Figures 3 and 4.