

DEPOSITION FROM CHARGED AEROSOL FLOWS THROUGH A TWO-DIMENSIONAL BEND

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Abstract—An analysis is presented on the deposition from a charged aerosol passing through a two-dimensional bend due to the simultaneous action of inertial and electrostatic forces. Collection efficiency is found for an idealized rotational flow. The results are compared qualitatively with available experimental data.

INTRODUCTION

Knowledge of inertial deposition of aerosol particles in a bend is important in a number of practical applications. Several experimental and theoretical studies (Johnson and Muir, 1973; Yeh, 1974; Cheng and Wang, 1975; Johnston *et al.*, 1977; Crane and Evans, 1977) have recently been made on this subject in order to obtain the collection efficiency of the bend. The mechanism for the particle collection in this case is the inertial force acting on the particle. This force drives the particle away from the streamline towards the wall. As a result some of the particles will collide with the wall before they reach the exit of the bend.

If the particle carries an electric charge, an additional electrostatic force will be exerted on the particle and its trajectory will be altered. This leads to a change in the collection efficiency of the bend. Melandri *et al.* (1977) and Chan *et al.* (1977) have made such measurements in the context of lung deposition. Significant increase of the collection efficiency was observed.

In this paper, an analysis is presented on the deposition of a monodisperse aerosol which has a uniform electric charge per particle as the aerosol flows through a bend. To simplify the analysis, the case of a two-dimensional bend will be considered. The flow field in a two-dimensional bend in the fully developed region was given by Goldstein (1938). The entry flow problem was studied by So (1976). In a two-dimensional channel, secondary flows do not occur because the paths the fluid particles would have to follow in order to reach the inside wall would be infinitely long. However, due to the presence of the centrifugal force, So has found that the entrance length for a curved two-dimensional channel is increased by the factor $1 + 0.4125 \alpha$ compared to that of a straight channel where α is the ratio of the channel width to the diameter of curvature of the inner wall.

The velocity field to be used in this paper is an ideal one. We neglect the entrance effect and assume that the velocity varies linearly with the distance from the center of curvature of the channel. It has been shown (Crane and Evans, 1977) that such an ideal velocity profile gives a very good result for the collection efficiency for an aerosol flow through a pipe bend.

BASIC EQUATIONS

Assuming that the particle has a much larger mass density than the air and neglecting the Basset term, the equation of the motion for a particle has the form (Hidy and Brock, 1970)

$$m \frac{d\mathbf{u}}{dt} = f(\mathbf{U} - \mathbf{u}) + \mathbf{F}_e \quad (1)$$

where m and \mathbf{u} are, respectively, the mass and velocity of the particle, t is the time, \mathbf{U} is the air velocity, $f = 6\pi\mu a$ in which μ is the air viscosity and a is the particle radius, and \mathbf{F}_e is the electrostatic force on the particle.

We consider a charged aerosol of uniform concentration entering the bend as shown in Fig. 1. In the polar coordinates (r, θ) , equation (1) takes the form

$$m(\ddot{r} - r\dot{\theta}^2) = -f\dot{r} + F_e \quad (2a)$$

$$m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) = f(U - r\dot{\theta}) \quad (2b)$$

where the dot denotes the differentiation with respect to t , and F_e and U have only a component in the r and θ direction, respectively.

We let $2h$, R and θ_0 be, respectively, the width, the radius of curvature and the angle of the bend. We also let $T = R\theta_0/u$ be the residence time of a particle in the bend, and $\tau = m/f$ be the relaxation time of the particle. Then, typically, we have $\tau/T \ll 1$. If we consider only a narrow bend such that $h/R \ll 1$, equations (2a) and (2b) may be simplified by keeping the lowest order of terms to give

$$-mr\dot{\theta}^2 = -f\dot{r} + F_e \quad (3a)$$

$$U - r\dot{\theta} = 0. \quad (3b)$$

From equations (3a) and (3b) we may eliminate $\dot{\theta}$ to obtain a single equation in r . The result is

$$\dot{r} - \frac{\tau U^2}{r} - \frac{F_e}{f} = 0. \quad (4)$$

For $h/R \ll 1$, the expression for F_e may be approximated by the formula of the image force between two conducting parallel plates. It has the form (Yu and Chandra, 1978)

$$F_e = \frac{Q^2 h \rho}{4\pi\epsilon_0} \left[\frac{1}{(h^2 - \rho^2)^2} + A \right] \quad (5)$$

where Q is the particle charge, ϵ_0 is the permittivity of the air, and

$$A = \sum_{n=1}^{\infty} (2n+1) / [(2n+1)^2 h^2 - \rho^2]^2$$

is a weak function of ρ in which $\rho = r - R$. Equation (5) neglects the interparticle interaction, which is not important for a dilute aerosol (Yu, 1977).

COLLECTION EFFICIENCY

Equation (4) can be integrated directly to give

$$t = \int_{r_c}^{r_w} \frac{dr}{\frac{\tau U^2}{r} + \frac{F_e}{f}} \quad (6)$$

where r_c is the critical radius at the entrance of the bend such that any particle entering the bend at $r = r_c$ will be deposited at the wall $r = r_w$ at the exit ($\theta = \theta_0$). Since $r_w = R + h$ and $R - h$, there will be two solutions for r_c from equation (6). The two limiting particle trajectories are shown in Fig. 1. Any particle which has a trajectory in the shaded areas will be deposited in the bend. The existence of two limiting particle trajectories is due to the fact that the centrifugal force and the electrostatic force acting on a particle are additive for deposition on the outside wall. For deposition on the inside wall, these two forces act in opposite directions.

In terms of r_{c1} and r_{c2} , the collection efficiency of the bend has the form

$$\eta = \frac{\int_{R-h}^{r_{c2}} U(r)dr + \int_{r_{c2}}^{R+h} U(r)dr}{\int_{R-h}^{R+h} U(r)dr} \quad (7)$$

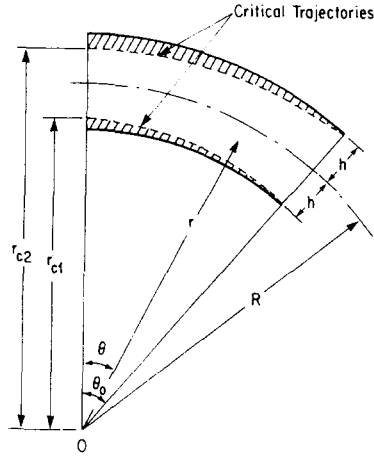


Fig. 1. Geometry of the two-dimensional bend.

Assuming an idealized rotational flow in the form $U(r) = \alpha r$ where α is a constant, equation (7) is simplified to

$$\eta = 1 - \frac{r_{c2}^2 - r_{c1}^2}{(R + h)^2 - (R - h)^2} \tag{8}$$

where r_{c1} and r_{c2} are determined from equation (6) by taking $t = \theta_0/\alpha$.

When $F_e = 0$ (i.e., the particle is neutral), equation (6) can be integrated exactly. In this case, η becomes

$$\eta = \frac{1}{4} \left(2 + S + \frac{1}{S} \right) [1 - \exp(-2I)] \tag{9}$$

where $S = R/h$ and $I = Stk \theta_0/S$ in which Stk is the Stokes number given by $\tau \alpha R/h$. When $I \ll 1$ and $S \gg 1$, we have, from equation (9),

$$\eta = Stk \theta_0/2. \tag{10}$$

This result which shows that η is independent of S has been previously found by Yeh (1974) and Crane and Evans (1977). Figure 2 shows the collection efficiency for a 90° bend calculated from equation (9) vs the Stokes number for various values of S . When $S > 20$, it is seen that equation (10) becomes a good approximation of equation (9).

When $F_e \neq 0$, r_{c1} and r_{c2} have to be determined numerically from equation (6). η then depends upon I , S and an electrostatic force parameter E , defined by

$$E = \frac{Q^2}{4\pi\epsilon_0 R^3 m \alpha^2}. \tag{11}$$

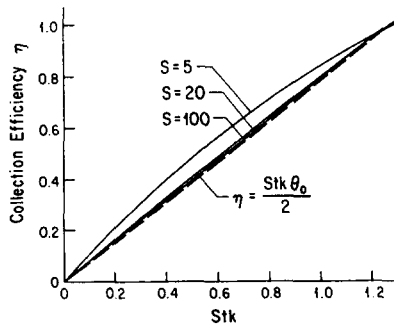


Fig. 2. Collection efficiency vs Stokes number for various values of S .

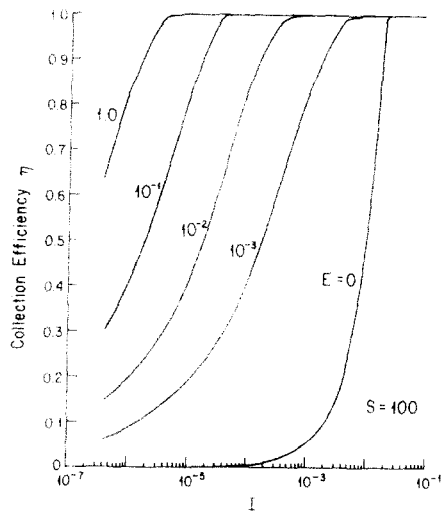


Fig. 3. Collection efficiency of a 90° bend vs I for $S = 100$ and several values of E .

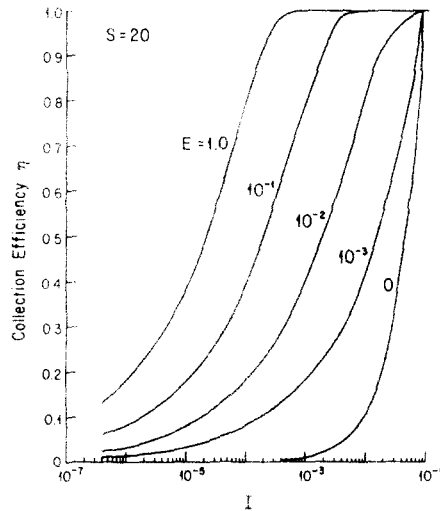


Fig. 4. Collection efficiency of a 90° bend vs I for $S = 20$ and several values of E .

In Figs 3 and 4, we show the collection efficiency η vs I for various values of E at $S = 100$ and 20. It is seen that η increases with both increasing I and E as expected. For the same I and E , η has a higher value at $S = 100$ than that at $S = 20$. This is because $S = 100$ corresponds to a narrower bend which gives both higher inertial and electrostatic depositions.

In order to compare the present result qualitatively with the experimental data obtained from a lung cast by Chan *et al.* (1978), we consider an aerosol with $1 \mu\text{m}$ aerodynamic diameter and 300 electronic charges per particle flowing through a 90° bend of 1 cm width. The collection efficiencies are calculated for various values of Stk and S and are shown in Fig. 5. The values of η for $Q = 0$ are also calculated for comparison. It is seen that the particle charge effect is insignificant for $Stk > 0.02$. This agrees with the experimental finding. At a given Stk , the calculated efficiency is considerably lower than the measured value because the two-dimensional bend was used in the analysis. However, the dependence of η on Q has a similar pattern as that observed by the experiment.

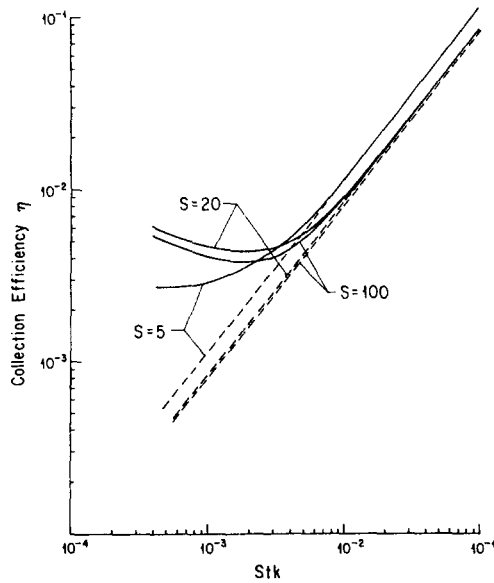


Fig. 5. Collection efficiency of a 90° bend vs Stokes number for several values of S when an aerosol of $1\ \mu\text{m}$ aerodynamic diameter is flowing through a bend of 1 cm width. The dotted lines are for $Q = 0$ and the solid lines are for $Q = 300$ electronic charges.

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