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A Microcomputer Spreadsheet Technique for Analyzing Multimodal Particle Size Distributions

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Particle size distributions (PSD) of industrial aerosols are usually described in the literature by a single geometric mean (GM) and geometric standard deviation (GSD) determined using log-probability (probit) analysis, thus implying the existence of a single-mode, lognormal distribution. If a multimodal distribution is suspected, as indicated in log-probability analysis by data points that do not lie upon a straight line, a different method of analysis is necessary in order to estimate the parameters of each of the underlying distributions.

A simple method to analyze multimodal PSDs was developed. A microcomputer spreadsheet is used to fit a smooth curve to grouped (histogram) particle size data, e.g., cascade impactor data, scanning electron microscope (SEM) particle counts, and optical device particle counts. This method is based upon the assumption that multimodal PSDs can be adequately described by a linear combination of weighted, lognormal distributions. This method yields a fitted smooth curve and estimates of the parameters of each underlying distribution: GM, GSD, and the percentage each underlying distribution contributes to the overall distribution. Four steps are required. First, the distribution histogram is calculated and graphically displayed. Second, the number of underlying distributions, the GM and GSD of each distribution, and the percentage contribution of each underlying distribution to the overall distribution are estimated by visual inspection. Third, the smooth, multimodal distribution frequency curve is calculated, using these estimates, and superimposed upon the histogram. Last, the estimates of the underlying distribution parameters are changed until a satisfactory fit is obtained. The parameters estimated using this technique will not represent a unique solution. Therefore, the existence of modes should be verified by obtaining multiple samples, through source assessment, or by corroboration using different measuring devices. **Hewett, P.; McCawley, M.A.: A Microcomputer Spreadsheet Technique for Analyzing Multimodal Particle Size Distributions. Appl. Occup. Environ. Hyg. 6:865–873; 1991.**

Introduction

Industrial aerosols are usually described in the literature using the parameters of a single-mode, lognormal distribution. The typical method of analysis, particularly for cascade impactor data, is to plot the cumulative mass data or count data on log-probit (probability) paper and estimate the geometric mean and geometric standard deviation from a straight line fit through the data points. However, aerosols in industrial and environmental settings often consist of mixtures of different size distributions. These distributions may have multiple peaks, indicating the presence of two or more underlying distributions. Traditional log-probit analysis of particle size distribution data from such distributions requires that a straight line be forced through the data points. The original distributions may be poorly described by the resulting single geometric mean and geometric standard deviation. A reasonable description of a multimodal distribution requires estimates of the geometric means and geometric standard deviations for each mode and the percentage contribution of each mode to the overall distribution. This information better describes the general shape of the observed multimodal distribution. However, estimation of the parameters of the underlying distributions is not easily accomplished, even with computer programming techniques.

This article describes a simple, intuitive technique for fitting a smooth curve to a size distribution with presumed underlying multiple modes using a microcomputer spreadsheet. This technique yields, in addition to the fitted smooth curve, estimates of the parameters of the underlying distributions: geometric means, geometric standard deviations, and the percentage contribution of each underlying distribution to the overall distribution. These estimates will, in most cases, be far more descriptive than the single geometric mean and geometric standard deviation derived using log-probit analysis. The technique described is generic and can be applied to grouped size distribution data from any source: inertial impactors, single-particle light-scattering instruments, or scanning electron microscope (SEM) image analysis. Since inertial impactors are the most common means of estimating particle size distributions and, in fact, provide the most utilitarian

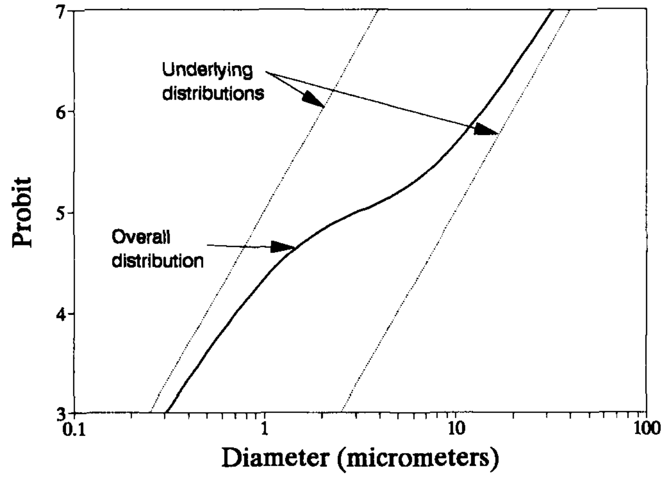

FIGURE 1. Log-probit plot of a bimodal distribution: 0.5LN(1,2) and 0.5LN(10,2).

data in terms of estimating potential health risk, the technique is illustrated using a microcomputer spreadsheet designed for an eight-stage cascade impactor.

Background

The basic assumption when analyzing cascade impactor data, or other particle size data, with a log-probit (probability) plot is that the true particle size distribution is lognormal. Such distributions are well described by the geometric mean (GM) and the geometric standard deviation (GSD) calculated from the log-probit plot.[1] If a multimodal distribution, composed of several lognormal distributions, is present, a straight-line fit may not be a reasonable representation of the true nature of such data. For example, Figure 1 is a log-probit plot of a bimodal distribution where 50 percent of the mass of the distribution comes from each of two lognormal distributions: LN(1,2) and LN(10,2). [It is standard convention to define a normal distribution using N(μ,σ) notation. This style will be followed when designating lognormal distributions: LN(GM, GSD). In addition, Table I provides definitions to abbreviations used throughout.] The underlying distributions are represented by dotted lines. The GM and GSD estimated from a straight line forced through the points

TABLE I. List of Abbreviations

AED	Aerodynamic equivalent diameter
a_n	Distribution coefficient, percentage of the overall distribution
d	Particle diameter, micrometers
d_{50}	Impactor stage particle size cutoff
$f_c(\log_{10}d)$	Frequency distribution of a combined or multimodal particle size distribution ($\log_{10}$ scale)
$f_n(\log_{10}d)$	Frequency distribution of the *n*th underlying distribution of a combined or multimodal particle size distribution ($\log_{10}$ scale)
e^2	Error squared, a measure of goodness-of-fit
GM	Geometric mean
GM_n	Geometric mean of the *n*th underlying distribution
GSD	Geometric standard deviation
GSD_n	Geometric standard deviation of the *n*th underlying distribution
LN(GM,GSD)	Log normal distribution

on this curve is not accurately descriptive of the original multimodal distribution. In the case of trimodal and quadrimodal distributions, estimation of the underlying distribution parameters from a log-probit plot is nearly impossible. If a multimodel distribution is suspected, a log-probit plot should not be used to estimate the parameters of the underlying distribution. In these cases, a frequency distribution plot is the recommended means for determining if a mixed distribution is present.[2,3]

The standard procedure for plotting a mass or count frequency distribution can be found in any standard aerosol science reference.[1,4,5] The mass (count) per stage is normalized against the total mass (count) collected and standardized for the interval width (in $\log_{10}$ units). This results in "percent mass (total counts) per $\log_{10}$ unit," which can be plotted as a mass (count) frequency histogram. A curve can be fit by hand to the histogram by connecting a line through the midpoints of each histogram bar. This procedure, while widely used, provides no information regarding the parameters of the underlying distributions; however, it does provide an estimation of the general shape of the overall distribution curve.

A spline fit procedure has been used to represent distributions observed in different areas of coal mines.[2] The spline fit is considered more accurate than the above procedure. The resulting curve may closely resemble the true distribution, however, it does not provide estimates of the underlying distribution parameters.

A microcomputer spreadsheet program was used to simplify development of a technique to fit a smooth curve to a multimodal histogram and generate estimates of the multimodal parameters. It is recommended for this application because it allows easy and rapid development of what is essentially a computer "program" without the need to become proficient in the use of a programming language. The choice of spreadsheet program is not critical as nearly all perform the required functions.

Theory

A multimodal particle size distribution can be described by a linear combination of weighted lognormal distributions:

$$f_c(\log_{10}d) = a_1f_1(\log_{10}d) + a_2f_2(\log_{10}d) + \ldots + a_nf_n(\log_{10}d) \quad (1)$$

where: $f_c(\log_{10}d)$ = combined frequency distribution
$f_n(\log_{10}d)$ = frequency distribution for the *n*th underlying distribution
a_n = the distribution coefficient, or fraction of the overall distribution, for the *n*th underlying distribution

The frequency at any diameter will be the sum of the fractional contributions of each underlying distribution. The distribution coefficients sum to unity by definition. For example, Figure 2 is the frequency distribution plot resulting from the combination of two underlying distributions. The sum of the frequencies of each mode yields the overall distribution curve. The area under each dotted line corresponds to the distribution coefficient and the two distribution coefficients sum to unity.

The frequency for each distribution, at any particle diameter (d), can be calculated:

$$f_n(\log d) = \frac{1}{\log_{10}GSD\sqrt{2\pi}}\exp\left(\frac{-(\ln d - \ln GM_n)^2}{2(\ln GSD_n)^2}\right) \quad \textbf{(2)}$$

where: GM_n = geometric mean of the nth underlying distribution
GSD_n = geometric standard deviation of the nth underlying distribution

Note that within the exponential term any log base can be used, i.e., $\log_{10}$ or ln. The ratio will remain the same. It is convention to use the natural log (ln). However, $\log_{10}GSD$ must be used outside the exponential term in order to plot curves using a $\log_{10}$ scale as the abscissa. See reference 4 for an introduction to particle size distribution theory.

The output of any particle size measuring device is essentially grouped data such as "mass per stage" or "total counts per channel." Such data are usually presented as a histogram. A histogram can be used to approximate any distribution. This is illustrated in Figure 3. A theoretical, multimodal distribution was generated using Equation 1: 0.5LN(1,2) and 0.5LN(10,2). The corresponding histogram was calculated after first defining the interval widths for the histogram. Since cascade impactors are commonly used to estimate particle size distributions, intervals corresponding to those of an eight-stage personal cascade impactor were used.[6] For purposes of illustration, a lower boundary of 0.1 μm was defined for the final filter and 100 μm as an upper boundary for the uppermost stage. The area under the theoretical curve was calculated for each interval using the trapezoidal rule and standardized to the interval width. This ideal histogram is what results if an ideal (lognormal) aerosol, distributed according to the above distribution, was sampled by an ideal cascade impactor.

This histogram has three properties worth noting. First, nearly 100 percent of the area under the curve is contained within the upper and lower boundaries of the histogram. Second, the area in each histogram bar, or interval, is equal to the corresponding area under the theoretical curve. Third, a log-probit plot of the cumulative frequency points (determined by summing the areas for each stage of this ideal impactor) will fall along the log-probit plot of the theoretical distribution. The first two properties, derived from considering the output of an ideal sampling device when sampling a theoretical lognormal distribution, are the bases of the procedure described in this article. The third property indicates that a histogram can exactly describe a lognormal distribution or a combination of lognormal distributions. Therefore, a curve fit to a histogram derived from a lognormal distribution should closely describe the sampled distribution.

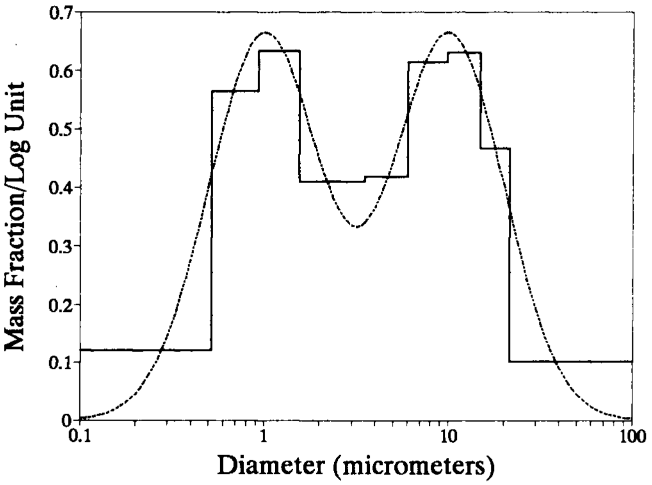

FIGURE 3. Mass frequency plot of a bimodal distribution and corresponding histogram as determined by an ideal eight-stage cascade impactor.

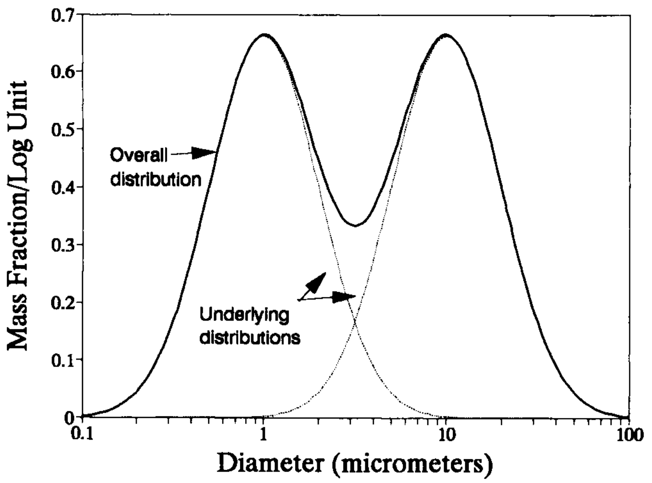

FIGURE 2. Mass frequency plot of a bimodal distribution: 0.5LN(1,2) and 0.5LN(10,2).

Since no device measures an aerosol distribution without error or bias, it cannot be known if any workplace aerosol is truly lognormal. Consequently, real particle size distribution data may not be perfectly fit by a theoretical curve. A measure of any lack of fit can be calculated by summing the squared difference between the area under the curve, per interval, and the corresponding area under the histogram. This is analogous to calculating errors in regression analysis. This measure will be called e^2. A perfect fit, such as in Figure 3, is indicated by a e^2 of zero. Since e^2 is zero, the distribution parameters of the curve represent a unique solution, relative to the first two properties described above. That is, no other set of distribution parameters will also yield a e^2 of zero.

Since a perfect fit is possible only with ideal distributions and ideal measuring devices, e^2 values for curves fit to observed (real) particle size distribution data will be greater than zero. A "best fit" curve for a particular "real" histogram will be obtained if e^2 is minimized. However, this requires an iterative computer program. The procedure described here allows the user to approach a "best fit" curve easily and rapidly. Given the level of uncertainty inherent in all particle size measuring devices, this procedure will yield curve fits that are sufficiently accurate for most purposes.

Estimating the Underlying Distributions

The procedure for fitting a smooth curve, calculated using Equation 1, to a histogram derived from cascade impactor data (or any other PSD measuring device) can be summarized as follows:

1. The distribution histogram is calculated and graphically displayed.
2. The number of underlying distributions, the distribution geometric means, geometric standard deviations, and the distribution coefficients are estimated by visual inspection.
3. The combined distribution frequency curve is calculated, using these estimates, and superimposed upon the histogram.
4. Changes are made in the estimates of the distribution parameters until the smooth curve fits the histogram satisfactorily, as indicated by a low e^2 value.

Appendix A is a sample spreadsheet designed for an eight-stage personal impactor[7] and the corresponding cell formulae are listed in Appendix B. The general procedure is the same for both mass (impactor) and count data. Figure 4 depicts a simulated example where a curve is fit to a bimodal distribution. Each revision in the parameter estimates results in a better fit and a smaller e^2 value. The final e^2 value of 0.0 indicates a perfect fit, an unlikely result with real data.

The first step in designing the spreadsheet is defining the range of values over which to calculate the frequency distribution, $f_c(\log_{10}d)$. The upper stage cutoff for the cascade impactor in the present example was given as 21.3 μm AED. A reasonable upper size limit of 100 μm was selected for demonstration purposes. The lower impactor stage cutoff was 0.97 μm AED. A lower size limit of 0.1 μm was selected for the mass collected on the final filter. Thus, the range for this spreadsheet should be from $\log_{10}(0.1)$ to $\log_{10}(100)$, or from −1 to +2 $\log_{10}$ units. An interval should be set for incrementally stepping from −1 to +2 $\log_{10}$ units. An increment of 0.05 or 0.025 results in 60 or 120 diameters for which $f_c(\log_{10}d)$ values will be calculated.

The spreadsheet in Appendix A was designed to fit distributions having one to three underlying distributions. There is no limit to the number of distributions that can

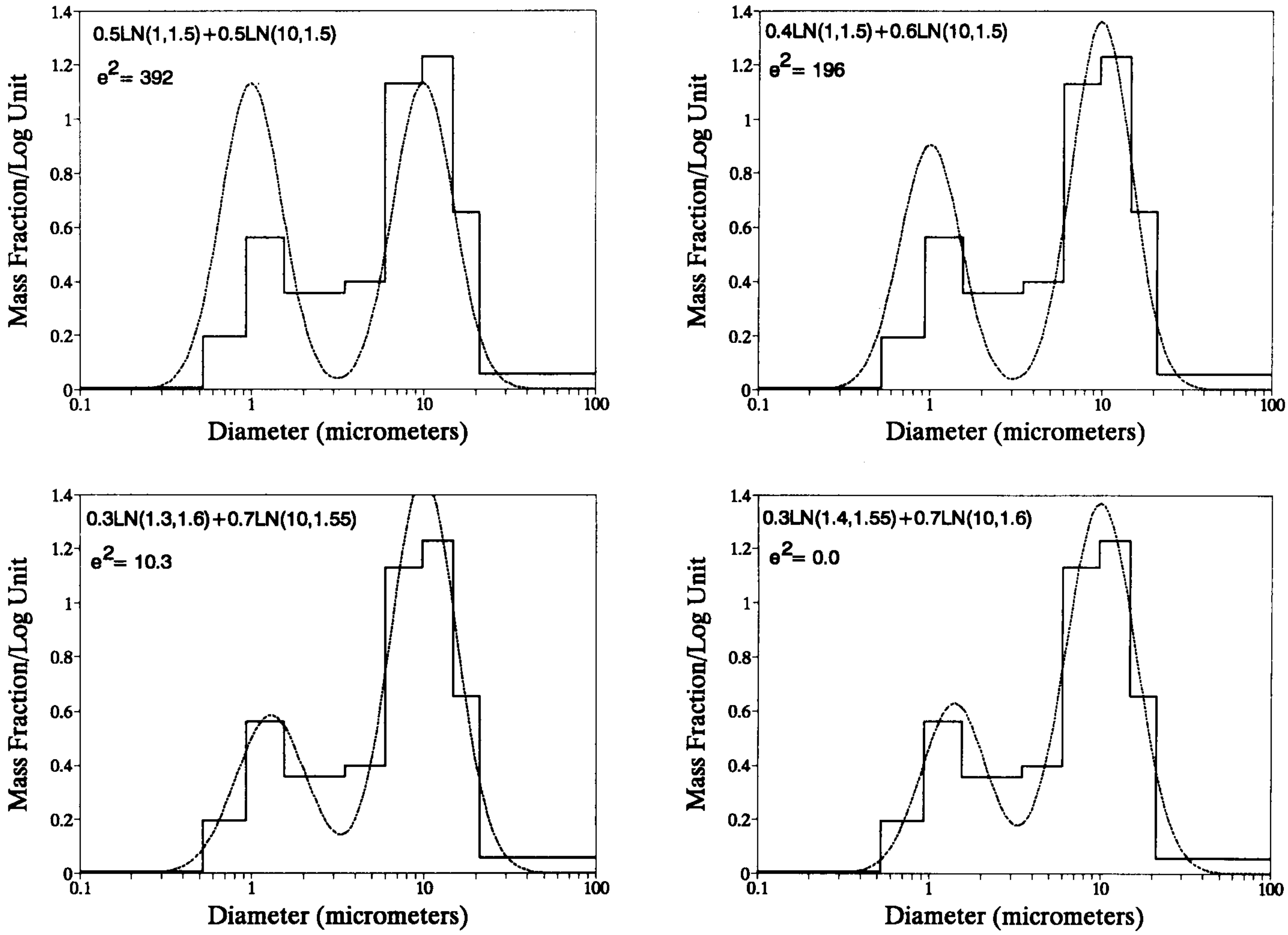

FIGURE 4. Example illustrating procedure for fitting a smooth curve to a bimodal histogram. Each change in the distribution parameter estimates results in a reduction in the goodness-of-fit measure, e^2. The perfect fit in the last plot is highly unlikely with real data.

be fitted to a histogram. However, four modes and a 0.025 stepping interval results in spreadsheets that will tax the memory limit of 640K MS-DOS® microcomputers. As a practical matter, three underlying distributions are sufficient for an eight-stage impactor. Other instruments, however, may be capable of resolving more distributions.

With experience, a satisfactory curve can be fit to a histogram in less than ten iterations. The spreadsheet can also be designed so that the underlying distributions are plotted along with the overall distribution curve to aid in estimating geometric means and standard deviations. This simply requires each underlying distribution in Equation 1 be calculated in a separate column, which can be graphed along with the histogram and the overall curve. These frequencies are summed to give the overall distribution curve.

Two rules are recommended for fitting a multimodal distribution curve to a histogram. First, nearly 100 percent of the area under the curve should be within the bounds of the histogram; that is, between the arbitrary lower cutoff for the backup filter and the aribtrary upper cutoff for the uppermost stage. In the example used here, this is between 0.1 and 100 μm. Second, the area represented by each histogram bar and the corresponding area under the curve should be nearly equal.

An objective measure of the "goodness of fit," can be obtained by comparing the area of each histogram bar with the corresponding area under the fitted curve. This area is calculated using a numerical integration technique known as the trapezoidal rule (cells E28 through E164). In the spreadsheet in Appendix A, the "percent [of total] area under [the] curve" (cells E13 through E21) was calculated, which can be directly compared to "percent total mass per stage" (cells B13 through B21). A good fit is obtained if these values are nearly equal. A better objective measure is the squared difference of these values (cells G13 through G21). Inspection of these e^2 values will indicate which areas of the curve require adjustment to obtain a better fit. If the e^2 value for each stage is minimized, the above rules are satisfied. The overall e^2 value (cell G23) is a measure of the overall "goodness of fit." Low overall e^2 values can be tedious to obtain, but values less than 10 generally result in a satisfactorily fitted curve.

Close inspection of the spreadsheet and cell formulae in the appendices should reveal how the spreadsheet works. The diameters (d) in cells A28 through A164 are used as the x-axis unless the spreadsheet is not capable of plotting a $\log_{10}$ scale, in which case, the $\log_{10}$ values in cells B28 through B164 should be used. In the example, the x-axis spans three orders of magnitude, from 0.1 to 100, with a stepping interval of 0.025 $\log_{10}$ units. This results in 120 lines. Additional lines were added that correspond to the impactor stage cutpoints. These cutpoints are entered twice, corresponding to the upper and lower boundaries of adjacent stages. This results in a more accurate appearing histogram. For example, cells A57 and A58 contain the cutpoint of the lowest stage. Cells B57 and B58 convert these values to $\log_{10}$. Since the diameters in cells A57 and A58 are identical, the area under the curve for the interval is zero, as indicated by cell E58.

Cells D28 through D164 calculate the frequency for the corresponding diameter in cells A28 through A164. The cell formula is Equation 1 (which uses Equation 2) for the case of three underlying distributions. The frequencies are often extremely small at the tails of the distribution and may be outside the range of the spreadsheet. If this occurs, a rounding function (typically called @ROUND) should be used to round Equation 1 to, say, 10^{-6}.

Cells E28 through E164 use the trapezoidal rule to calculate the area under the frequency curve. Cells F28 through F164 are used to plot the observed histogram. Each cell in this range references the appropriate cell in the block C13–C21. These cells convert the "percent mass per impactor stage" entries (cells B13–B21) to "% mass per $\log_{10}$ unit." Cells E13 through E21 calculate the area under the smooth curve for each interval of the histogram. Each cell sums only those cells in the range E28–E164 that correspond to the particle sizes collected by the corresponding impactor stage.

Discussion

The method described for fitting a smooth curve to a multimodal histogram is simple and intuitive. It allows the user to manipulate the underlying distribution parameters until a satisfactory fit is obtained.

The smooth curve calculated using this method may not accurately estimate the *true* distribution curve because it is limited by both the accuracy of the measurement technique and the willingness of the user in obtaining low e^2 values. For example, inertial impaction data can be biased due to poor inlet efficiencies, particle bounce, particle reentrainment, and weighing errors. Single-particle measuring instruments can be biased due to poor inlet efficiencies, line loses, different refractive indices, and instrument response limitations. It is the responsibility of the user to ensure that the input data is collected in as unbiased a manner as possible.[3] There are many papers in the literature that discuss these instrument deficiencies and offer techniques for minimizing the resulting bias. These should be consulted prior to collecting particle size distribution data.

The combined distribution curve fit to any particular histogram will not represent a unique solution due to the subjective nature of the procedure. However, similar "smooth curves" will be obtained by different individuals if the estimated distribution parameters are adjusted to yield low e^2 values, say less than 2 for each stage and less than 10 overall. If the e^2 values are low enough, the differences will be trivial compared to the errors commonly associated with all particle size measuring devices. If the goal is to estimate the "distribution parameters," then similar bimodal and trimodal solutions to bimodal and trimodal histograms should be obtained by different individuals provided care is taken to obtain low e^2 values.

It is possible to fit a histogram with more underlying distributions than necessary and yet obtain similar curves and equally low overall e^2 values. This is of little consequence if the goal is to merely describe the histogram with a smooth curve for representational purposes. If, however, the intent is to estimate the parameters of the "true" underlying distributions, caution must be exercised when concluding that an underlying distribution is real. In general, use as few underlying distributions as possible when fitting a curve to any histogram. As a rule of thumb, distributions containing less than 10 percent of the area under the curve may not be real, but merely an artifact arising from the collection characteristics of the sampling device used. The ability of the particle size measuring device to fractionate the aerosol accurately should be taken into account when making this decision. Knowledge of the potential particulate sources will, no doubt, be of help in deciding if a mode is real or not. For example, the presence of a source of condensation aerosols assists in verifying the presence of a submicrometer mode. Curves should be fit to the average of several measurements, rather than basing a set of estimated parameters on the analysis of a single set of particle size distribution data. The results of multiple impactor measurements can be combined by averaging the "% mass" for each stage so that an average distribution for an area can be calculated. Corroboration of the presence, location, and spread of modes by source sampling or sampling with different instrumentation is highly recommended.(8)

This method has been used to analyze data from various cascade impactors, the TSI Incorporated Aerodynamic Particle Sizer (APS); the Particle Measuring Systems, Inc., Laser Aerosol Spectrometer (LAS-X); the Micro Orifice Uniform Deposit Impactor (MSP Corporation); and the particle number distribution data from a scanning electron microscope automated image analyzer. This technique can be adapted to the output of nearly any particle size measuring system.

Obviously, the number of data points, or intervals, available will limit the number of modes possible to resolve. As a rule of thumb, single-mode distributions require at least three data points, with the central data point being the largest value. Bimodal distributions will require at least five data points, with the second and fourth data points being the largest values. Similarly, trimodal distributions will require at least seven data points, with the second, fourth, and sixth data points being the largest values. Since real multimodal distributions may not match the intervals of the measuring device, modes may be hidden or obscured, thus limiting the number of modes capable of being resolved. This is a problem that resides with the device used to measure the particle size distribution. Generally, better results will be obtained from instruments having a large number of stages or intervals.

Figure 5 represents a bimodal distribution from a dieselized coal mine measured using a MOUDI (Micro Orifice Uniform Deposit Impactor) device.(9) The submicrometer mode is believed to be comprised almost entirely of diesel particulate, while the other mode apparently results from the mechanical generation of coal dust during the mining process. Estimates of the diesel mode parameters were necessary for designing an impactor to collect primarily diesel particulate. The procedure described here was used to estimate these parameters.

Additional, underlying distributions can be used to obtain a better fit to the observed histogram. For example, Figure 5 is clearly bimodal but required three underlying distributions before a satisfactory fit was obtained. As previously stated, whether or not each underlying distribution is real is up to the user. In this case, the upper mode may be skewed because the true mass distribution is skewed, or because of measurement errors and biases. Further sampling and source assessment is necessary before concluding that the actual mass distribution is well described by the combination of these three underlying distributions.

It is a relatively simple procedure to adapt this spreadsheet to calculate and graph log-probit plots, cumulative distributions, and the various weighted distributions, such as mass, volume, and surface area. Log-probit plots can be calculated using the inverse normal function (available in texts on statistics) to convert cumulative data to the corresponding z-value.

This technique may have potential applications in other areas of occupational health. This technique could be adapted to the analysis of lognormally distributed exposure data, particularly data covering different exposure zones. In addition, the data analyzed does not have to be lognormally distributed. Equation 2 can be changed to apply to normally distributed data:

$$f_n(x) = \frac{1}{\sigma_n \sqrt{2\pi}} \exp\left(\frac{-(x - \mu_n)^2}{2\sigma_n^2}\right)$$

where: μ = the arithmetic mean of the *n*th mode
σ = standard deviation of the *n*th mode

If it is desired to plot the histogram and curve on a linear scale, rather than on the $\log_{10}$ scale, Equation 2 must be

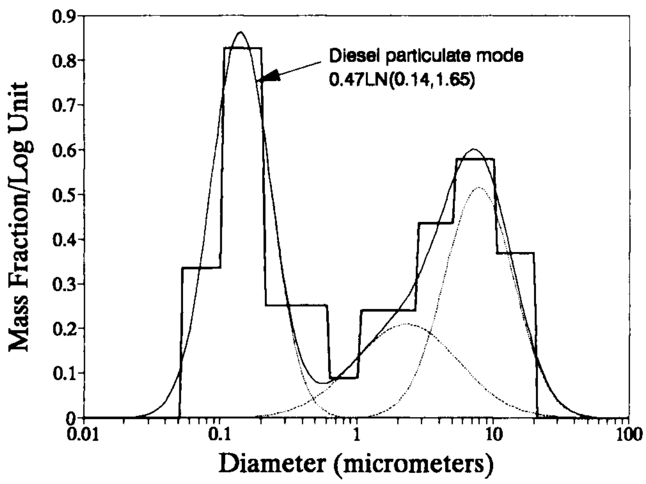

FIGURE 5. Smooth curve fit to cascade impactor data from a dieselized coal mine. The underlying distributions are indicated by the dashed lines. The parameters of the mode felt to represent diesel particulate were estimated to be 0.47LN(0.14,1.65).

changed:[4]

$$f_n(d) = \frac{1}{d\ \ln GSD \sqrt{2\pi}} \exp\left(\frac{-(\ln d - \ln GM_n)^2}{2(\ln GSD_n)^2}\right)$$

The $\log_{10}$ scale should be used when fitting a curve to a histogram, although a graph with a linear scale may be more appealing for presentation purposes.

The literature contains numerous papers describing the "inversion" problem; that is, the problem of starting with grouped data and estimating the distribution from which it was derived.[7,10,11] The method described here is conceptually similar to the procedures discussed in these papers, but it has the advantage of being intuitive and simple to implement.

Summary

The procedure described here allows the user 1) to fit a smooth curve to the histogram output of any particle size distribution measuring device and 2) to estimate the parameters of the distribution. This procedure is particularly useful if the histogram indicates a multimodal distribution. Estimates of the underlying distributions can be used to generate hypotheses regarding possible sources, modes of formation, and contribution to the overall distribution. This procedure is easily implemented using a microcomputer spreadsheet and provides an attractive alternative to other more complicated published curve-fitting procedures. Before drawing conclusions regarding the parameter estimates for the underlying distributions, users should verify the existence of modes and underlying distributions by obtaining multiple samples, through source assessment, or corroboration using different measuring devices.

Copies of a sample program are available by addressing correspondence to Paul Hewett, Division of Respiratory Disease Studies, National Institute for Occupational Safety and Health, 944 Chestnut Ridge Road, Morgantown, WV 26505. A postage-prepaid, 3.5-inch computer diskette MUST accompany request.

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Editors Note: Appendices A and B appear on the two subsequent pages.

APPENDIX A

(boxed cells indicate data entered by the user)

	A	B	C	D	E	F	G
1							
2		Worksheet for fitting a multi-modal frequency					
3		curve to a cascade impactor histogram					
4							
5		Mode 1	Mode 2	Mode 3		Sum of Coefficients	
6	Coeff =	0.3	0.7	0		1	
7	GM =	1.4	10	1			
8	GSD =	1.55	1.6	2			
9							
10		% mass/	% mass per		% area		e2
11	Stage	stage	log_unit		under curve		
12							
13	1	3.78	5.63		3.78		0.00
14	2	10.38	65.67		10.38		0.00
15	3	22.03	123.06		22.03		0.00
16	4	24.10	113.09		24.10		0.00
17	5	9.36	39.98		9.36		0.00
18	6	12.59	35.60		12.59		0.00
19	7	12.48	56.27		12.48		0.00
20	8	4.91	19.44		4.91		0.00
21	Filter	0.36	0.50		0.36		0.00
22							
23	TOTAL	100.00		TOTAL	100.00	TOTAL	0.00
24							
25				(Equation 1)	area under	Observed	
26	d	log(d)		fc(logd)	curve	Frequency	
27							
28	0.1	-1.0000		0.0000	0.0000	0.0050	
29	0.10593	-0.9750		0.0000	0.0000	0.0050	
30	0.11220	-0.9500		0.0000	0.0000	0.0050	
	.	.		.	.	.	
.	.	.		.	.	.	
56	0.50119	-0.3000		0.0403	0.0009	0.0050	
57	0.52	-0.2840		0.0489	0.0007	0.0050	
58	0.52	-0.2840		0.0489	0.0000	0.1944	
59	0.53088	-0.2750		0.0544	0.0005	0.1944	
.	.	.		.	.	.	
.	.	.		.	.	.	
135	21.13489	1.3250		0.3852	0.0106	0.6567	
136	21.3	1.3284		0.3751	0.0013	0.6567	
137	21.3	1.3284		0.3751	0.0000	0.0563	
138	22.38721	1.3500		0.3145	0.0075	0.0563	
.	.	.		.	.	.	
.	.	.		.	.	.	
164	100	2.0000		0.0000	0.0000	0.0563	

APPENDIX B

Cell	Cell Formula
F6:	+ B6 + C6 + D6
C13:	+ B13/(@LOG(100) − @LOG(21.3))
E13:	@SUM(E137..E164)*100
G13:	(B13 − E13)^2
C14:	+ B14/(@LOG(21.3) − @LOG(14.8))
E14:	@SUM(E128..E136)*100
G14:	(B14 − E14)^2
C15:	+ B15(@LOG(14.8) − @LOG(9.8))
E15:	@SUM(E119..E127)*100
G15:	(B15 − E15)^2
C16:	+ B16(@LOG(9.8) − @LOG(6))
E16:	@SUM(E109..E118)*100
G16:	(B16 − E16)^2
C17:	+ B17/(@LOG(6) − @LOG(3.5))
E17:	@SUM(E97..E108)*100
G17:	(B17 − E17)^2
C18:	+ B18/(@LOG(3.5) − @LOG(1.55))
E18:	@SUM(E81..E96)*100
G18:	(B18 − E18)^2
C19:	+ B19/(@LOG(1.55) − @LOG(0.93))
E19:	@SUM(E70..E80)*100
G19:	(B19 − E19)^2
C20:	+ B20/(@LOG(0.93) − @LOG(0.52))
E20:	@SUM(E58..E69)*100
G20:	(B20 − E20)^2
C21:	+ B21/(@LOG(0.52) − @LOG(0.1))
E21:	@SUM(E28..E57)*100
G21:	(B21 − E21)^2
B23:	@SUM(B13..B21)
E23:	@SUM(E13..E21)
G23:	@SUM(G13..G21)
A28:	10^B28
B28:	− 1
D28:	1/@SQRT(@PI*2)*($B$6/@LOG($B$8)*@EXP((− (@LN(A28) − @LN($B$7))^2)/(2*@LN($B$8)^2)) + $C$6/@LOG($C$8)*@EXP((− (@LN(A28) − @LN($C$7))^2)/(2*@LN($C$8)^2)) + $D$6/@LOG($D$8)*@EXP((− (@LN(A28) − @LN($D$7))^2)/(2*@LN($D$8)^2)))
E28:	0.025/2*(D27 + D28)
F28:	0.01*$C$21
A29:	10^B29
B29:	+ B28 + 0.025
D29:	1/@SQRT(@PI*2)*($B$6/@LOG($B$8)*@EXP((− (@LN(A29) − @LN($B$7))^2)/(2*@LN($B$8)^2)) + $C$6/@LOG($C$8)*@EXP((− (@LN(A29) − @LN($C$7))^2)/(2*@LN($C$8)^2)) + $D$6/@LOG($D$8)*@EXP((− (@LN(A29) − @LN($D$7))^2)/(2*@LN($D$8)^2)))
E29:	(B29 − B28)/2*(D28 + D29)
F29:	0.01*$C$21
A57:	0.52
B57:	@LOG(A57)
E57:	(B57 − B56)/2*(D56 + D57)
F57:	0.01*$C$21
A58:	0.52
B58:	@LOG(A58)
E58:	(B58 − B57)/2*(D57 + D58)
F58:	0.01*$C$20
A59:	10^B59
B59:	+ B56 + 0.025
E59:	(B59 − B58)/2*(D58 + D59)
F59:	0.01*$C$20
A164:	10^B164
B164:	+ B163 + 0.025
D164:	1/@SQRT(@PI*2)*($B$6/@LOG($B$8)*@EXP((− (@LN(A164) − @LN($B$7))^2)/(2*@LN($B$8)^2)) + $C$6/@LOG($C$8)*@EXP((− (@LN(A164) − @LN($C$7))^2)/(2*@LN($C$8)^2)) + $D$6/@LOG($D$8)*@EXP((− (@LN(A164) − @LN($D$7))^2)/(2*@LN($D$8)^2)))
E164:	(B164 − B163)/2*(D163 + D164)
F164:	0.01*$C$13