American Industrial Hygiene Association Journal

ISSN: 0002-8894 (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/aiha20

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To cite this article: Martha A. Waters , Steve Selvin & Stephen M. Rappaport (1991) A MEASURE OF GOODNESS-OF-FIT FOR THE LOGNORMAL MODEL APPLIED TO OCCUPATIONAL EXPOSURES, American Industrial Hygiene Association Journal, 52:11, 493-502, DOI: 10.1080/15298669191365108

To link to this article: https://doi.org/10.1080/15298669191365108

Published online: 04 Jun 2010.

A Measure of Goodness-of-Fit for the Lognormal Model Applied to Occupational Exposures*

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The lognormal distribution is often applied to occupational exposures, yet the assumption of lognormality is rarely verified. This lack of rigor in evaluating the appropriateness of the lognormal model has resulted, in part, from the difficulty of applying formal goodness-of-fit tests. When evaluation of model fit has been attempted, occupational hygienists have relied upon probability plotting of exposures rather than upon formal statistical methods. The goal of this work was to develop for the occupational hygienist a simple quantitative evaluation to supplement the probability plot. A measure of goodness-of-fit to the lognormal model based on the ratio of two estimators of the mean of the distribution, the simple or direct estimate of the mean and the maximum likelihood estimate of the mean of a lognormal distribution, is described. This new measure, the ratio metric, is a simple extension of calculations made routinely by many occupational hygienists. Results from using the ratio metric were compared to probability plotting and to two traditional measures of goodness-of-fit, the Lilliefors test and the W test, for two occupational exposure data sets. The results of the ratio and W tests are comparable for a variety of occupational exposure data, but the Lilliefors test is overly conservative and does not detect several cases of gross deviations from lognormality. The ratio metric is an effective alternative to the Lilliefors test and is easier to perform than the W test for the range of data usually encountered by occupational hygienists. Occupational hygienists are encouraged to use the ratio metric in conjunction with the probability plot in evaluating the lognormal assumption.

*This research was supported in part by the Health Effects Component of the University of California Toxic Substances Research and Teaching Program and by the Atlantic Richfield Corporation.

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The "lognormal" model is often applied to occupational exposures based on both empirical fits and theoretical arguments. In theory, lognormal distributions arise from multiplicative effects of random influences on exposure levels. In the case of chemical exposures, these random influences include the mobility of the worker, the generation rate of the contaminant, and the rate of contaminant concentration dilution.

Although the lognormal distribution is widely used to describe occupational exposures, the assumption of lognormality is rarely verified.[1] One reason for this is that data are seldom collected in sufficient quantity to allow formal statistical tests of goodness-of-fit to be applied. Even when sample sizes are large, however, occupational hygienists have usually relied upon probability plotting of exposures rather than formal methods. Advantages of probability plotting are simplicity of preparation and the quantity of information that can be displayed in a compact form. The major disadvantage of probability plotting is the subjectivity of the decision about how well the model fits the data. The lack of rigor in evaluating the lognormal assumption probably has resulted, in part, from the difficulty of applying formal goodness-of-fit tests, which require computer programs or special tables. Thus, the goal of this work is to provide the occupational hygienist with a simple quantitative evaluation to supplement the probability plot.

The authors suggest a measure of goodness-of-fit to the lognormal model based on the ratio of two estimators of the mean of the distribution, i.e., $\bar{y}/\bar{x}_c$, where $\bar{y}$ is the simple or direct estimate of the mean and $\bar{x}_c$ is the maximum likelihood estimate of the mean of a lognormal distribution. This measure, called the "ratio metric," derives directly from location and scale estimates of the distribution and is simple to compute. The results of probability plotting and two traditional measures of goodness-of-fit, the Lilliefors test[2] and the W test,[3] were compared with the proposed ratio metric for two occupational exposure data sets. The ratio metric is an effective alternative to the Lilliefors test and is easier to perform than the W test for the range of data usually encountered by occupational hygienists.

TESTING DISTRIBUTIONAL ASSUMPTIONS

Once a statistical model has been selected to represent physical phenomena, the appropriateness of any distributional assump-

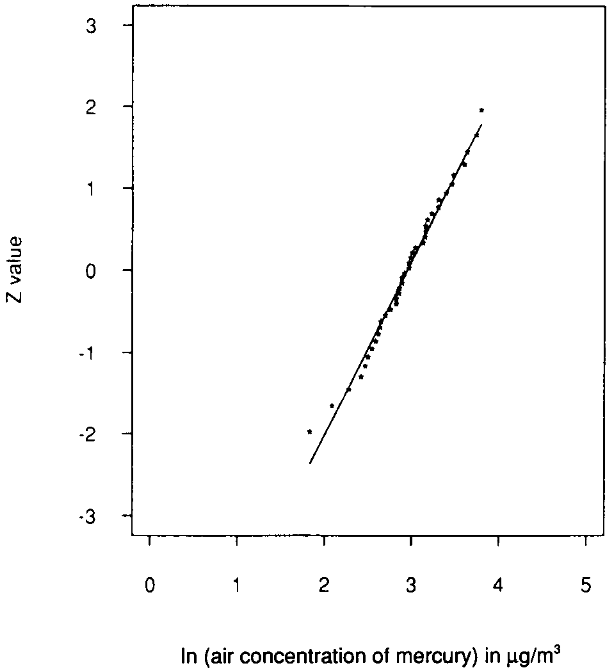

FIGURE 1. Mercury Worker 1

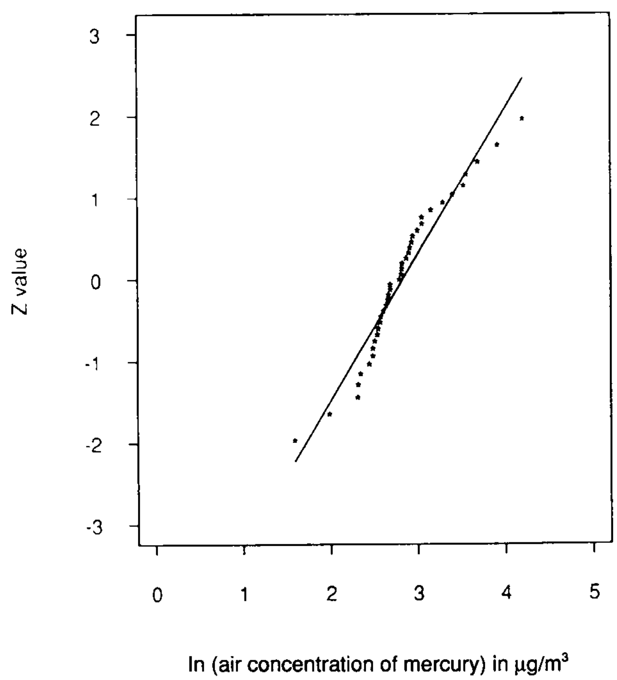

FIGURE 2. Mercury Worker 4

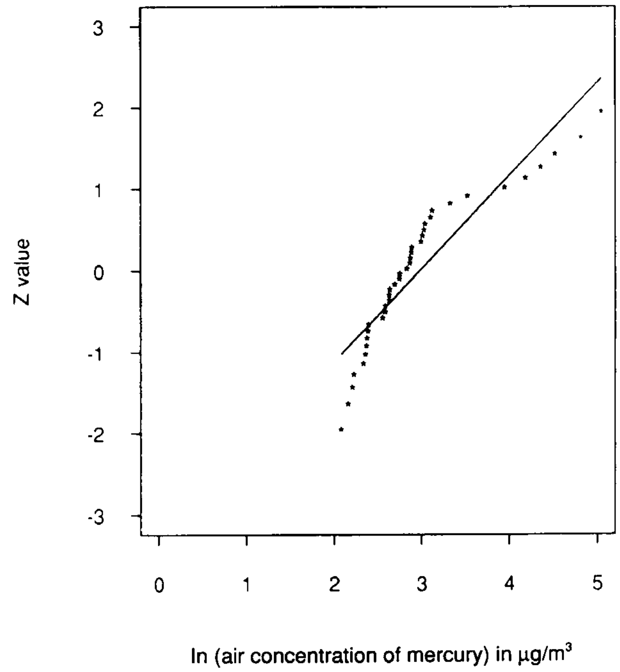

FIGURE 3. Mercury Worker 11

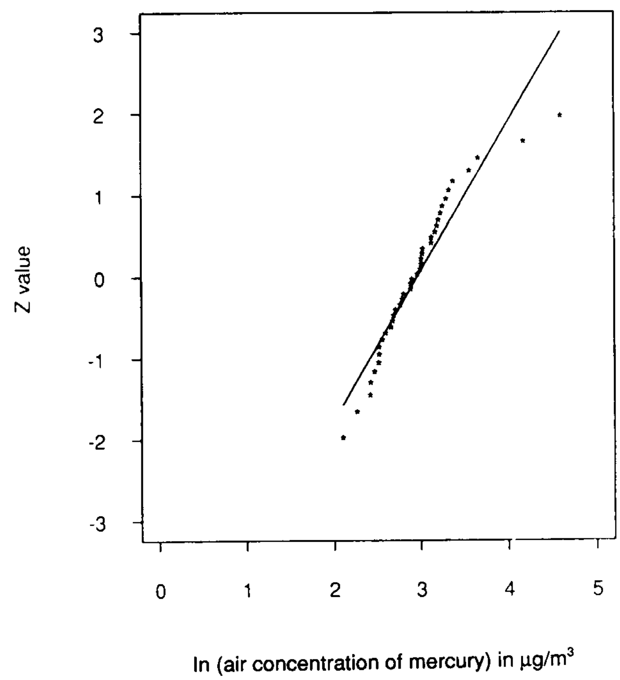

FIGURE 4. Mercury Worker 12

tion can be examined in various ways. A common method involves probability plotting where the sample order statistics are plotted versus the expected values of the order statistics from a standardized distribution (zero location and unit scale parameter).[4,5] This method provides a visual representation of the "fit" and allows qualitative evaluation of the adequacy of the assumed model. When the data fit the postulated distribution well, this plot yields a straight line. Departures from linearity indicate lack of fit. The subjectivity of the probability plotting method for evaluating goodness-of-fit lies in determining how well the data fit a straight line.

Figures 1–5 show examples of log-probability plots of workers' airborne exposures to inorganic mercury over a period of 40 workdays.[6] Each point on the plot represents a single 8-hr

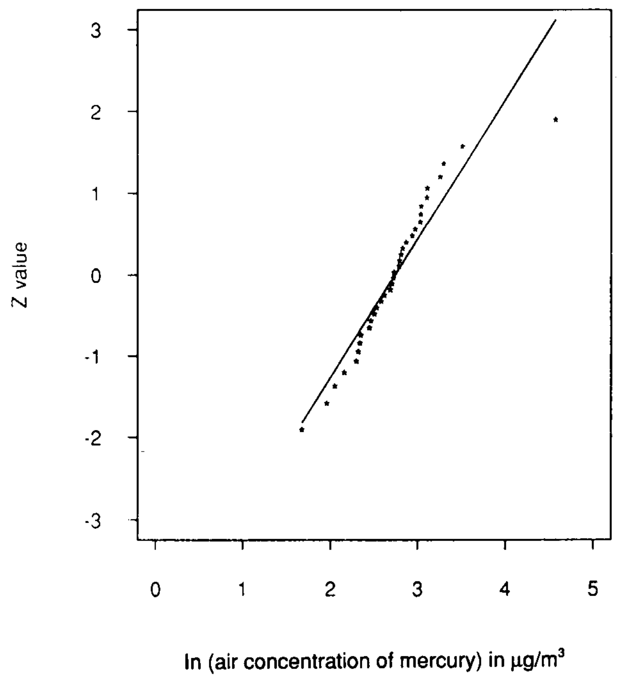

FIGURE 5. Mercury Worker 14

time-weighted average exposure. Logarithms of the 40 exposure levels were ranked in each case and plotted on a linear scale against the standard normal z variate corresponding to the rank. The straight lines in the plots represent the best-fit lines from linear regression of the logarithms of exposure levels on the standard normal z variate. The relative linearity of the plots shown in Figures 1 and 2 indicates the assumption of lognormality of exposures to be reasonable; conversely, the relative lack of linearity shown in Figures 3–5 casts doubt on the lognormality of the data.

A variety of probability plotting papers are available for different distributions to facilitate the process of evaluating various distributional models[7] where one axis is incremented in cumulative percent according to the standardized distribution and the other axis may be linear, logarithmic, or any other mathematical transformation. A good description of probability plotting techniques can be found in Hahn and Shapiro.[8] Computer programs such as STATGRAPHICS® (DOS)[9] and System S (UNIX)[10] provide the ability to quickly prepare and view probability plots for a number of distributions.

Formal statistical tests provide a rigorous basis for assessing a model's fit to the data. A variety of goodness-of-fit tests have evolved for the normal distribution[11] that can be applied to the lognormal distribution by working with the logarithms of observed data values. Power comparisons of specific tests have shown the Shapiro-Wilk W test[3] and its extensions[12,13] to be a sensitive omnibus test and for many skewed populations "clearly the most powerful."[11] (Omnibus tests evaluate the null hypothesis against all alternative hypotheses, as opposed to directional tests, which are intended to detect specified alternative hypotheses.) The W test statistic is the squared ratio of the best linear unbiased estimator for scale to the standard deviation. A simple description of its use can be found in Gilbert.[14] A disadvantage of the W test is the requirement of tables for obtaining weights and significance levels. Although Royston[13] provided an approximate normalizing transformation suitable for computer programming, this test is not easily used nor accessible to most occupational hygienists.

The Lilliefors test for normality is an adaptation of the Kolmogorov-Smirnov test for cases where the mean and variance of the postulated normal distribution are estimated from the sample.[4,15] This test employs the single largest discrepancy between the empirical distribution function estimated from the sample and the postulated distribution function as a measure of goodness-of-fit. The Kolmogorov-Smirnov tests are sensitive to outliers and are generally less powerful than tests that employ all data values (e.g., the W test).[11] However, use of the Lilliefors test is simple; it has been recommended for applications to occupational exposures[16]; and it has been used by Hines and Spear[17] and by Rappaport[1] to evaluate the lognormality of occupational exposures.

THE RATIO METRIC

As mentioned above, the proposed ratio metric compares the ratio of two estimators of the mean; i.e., the direct or simple estimate of the mean ($\bar{y}$) and the maximum likelihood estimate of the mean of a lognormal distribution ($\bar{x}_c$). If (y_1, y_2, ..., y_n) represent a set of random, independent exposures derived from a lognormal distribution with mean μ_c and variance σ_c^2, the estimates of the mean and the variance of the distribution of y are given by

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \qquad (1)$$

and

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2 \qquad (2)$$

The estimate of the mean $\bar{y}$ is unbiased regardless of the underlying distribution. If $x_i = \ln(y_i)$, then (x_1, x_2, ..., x_n) represents a sample of log-transformed exposures with a normal distribution (mean μ_L and variance σ_L^2). The estimated mean and variance from this distribution are designated $\bar{x}_L$ and s_L^2 where

$$\bar{x}_L = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad (3)$$

and

$$s_L^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x}_L)^2 \qquad (4)$$

The maximum likelihood estimate of the mean of a lognormal distribution, $\bar{x}_c$, is computed as

$$\bar{x}_c = \exp[\bar{x}_L + \frac{1}{2}s_L^2] \qquad (5)$$

This estimate depends strictly on the validity of the lognormal assumption. That is, $\bar{x}_c$ is an appropriate estimate of the mean exposure level only when the data are sampled from a lognormal distribution.[18] (A maximum likelihood estimate requires that all component estimates be maximum likelihood. The sample variance s_L^2 does not meet this requirement completely as the denominator contains n – 1 rather than n.)

The ratio, $\bar{y}/\bar{x}_c$, is easily computed directly from simple descriptive statistics available in all statistical software and on many calculators. The ratio is close to 1 when the distribution is lognormal and deviates from 1 when the distribution deviates from lognormality. The expected value of the ratio is slightly biased from 1.[18] The amount of bias depends on both the sample size and the parameters of the distribution.

The behavior of the ratio $\bar{y}/\bar{x}_c$ was examined by computer simulation. Simulations were performed by using System S software[10] on a Sun 3/50 work station (Sun Microsystems, Fremont, Calif.). One thousand samples of size n = 5 to n = 50 were drawn randomly from lognormal distributions with specified means and variances and for each sample the ratio $\bar{y}/\bar{x}_c$ was computed. Thus, the sample estimates of the mean and variance of 1000 values of the ratio were obtained for each sample size.

Table I shows values of the mean of the ratio $\bar{y}/\bar{x}_c$ for the simulated lognormal distributions. The columns reflect different values of the coefficient of variation σ_c/μ_c where μ_c and σ_c are the true mean and standard deviation of the lognormal distribution. The geometric standard deviations [$\sigma_g = \exp(\sigma_L)$] corresponding to the coefficients of variation are also shown for σ_g ranging from 1.7 to 5.4. The relation between σ_g and σ_c/μ_c is

TABLE I. Mean of the Ratio $\bar{y}/\bar{x}_c$

Sample Size	$cv_c^A=0.5$ $\sigma_g^B=1.7$	$cv_c=1$ $\sigma_g=2.3$	$cv_c=2$ $\sigma_g=3.6$	$cv_c=3$ $\sigma_g=4.6$	$cv_c=4$ $\sigma_g=5.4$
n = 5	0.9753	0.9218	0.8073	0.7279	0.6692
n = 10	0.9874	0.9546	0.8912	0.8158	0.7670
n = 20	0.9938	0.9751	0.9434	0.9010	0.8649
n = 30	0.9958	0.9844	0.9543	0.9275	0.8921
n = 50	0.9976	0.9914	0.9843	0.9679	0.9308

$^A cv_c$ = coefficient of variation = σ_c/μ_c.
$^B\sigma_g$ = geometric standard deviation.

TABLE II. Variance of the Ratio $\bar{y}/\bar{x}_c$

Sample Size	$cv_c^A=0.5$ $\sigma_g^B=1.7$	$cv_c=1$ $\sigma_g=2.3$	$cv_c=2$ $\sigma_g=3.6$	$cv_c=3$ $\sigma_g=4.6$	$cv_c=4$ $\sigma_g=5.4$
n = 5	0.00050	0.00540	0.02799	0.04457	0.06029
n = 10	0.00017	0.00307	0.02502	0.04920	0.06664
n = 20	0.00008	0.00199	0.02794	0.06530	0.07138
n = 30	0.00005	0.00147	0.02278	0.06406	0.07988
n = 50	0.00004	0.00105	0.01793	0.05463	0.05671

$^A cv_c$ = coefficient of variation = σ_c/μ_c.
$^B\sigma_g$ = geometric standard deviation.

TABLE III. Selected Percentiles of the Ratio $\bar{y}/\bar{x}_c$

		p =							
n	σ_c/μ_c	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
5	0.5	0.8898	0.9179	0.9300	0.9499	0.9943	0.9957	0.9968	0.9980
5	1	0.6618	0.7130	0.7665	0.8287	0.9877	0.9933	0.9964	1.0023
5	2	0.2483	0.3701	0.4658	0.5727	0.9745	0.9876	0.9967	1.0230
5	3	0.1609	0.2258	0.3082	0.4163	0.9597	0.9776	0.9899	1.0043
5	4	0.0796	0.1323	0.1981	0.3000	0.9571	0.9786	0.9905	0.9991
10	0.5	0.9419	0.9547	0.9635	0.9703	0.9992	1.0034	1.0069	1.0142
10	1	0.7974	0.8274	0.8544	0.8778	1.0048	1.0279	1.0496	1.0983
10	2	0.5358	0.5941	0.6511	0.7097	1.0317	1.1359	1.2187	1.3658
10	3	0.3002	0.3870	0.4650	0.5476	1.0294	1.1406	1.3286	1.4851
10	4	0.2517	0.2790	0.3569	0.4441	1.0353	1.1382	1.3260	1.5769
20	0.5	0.9697	0.9750	0.9793	0.9831	1.0029	1.0073	1.0110	1.0196
20	1	0.8672	0.8860	0.9049	0.9220	1.0209	1.0415	1.0667	1.0959
20	2	0.6322	0.6877	0.7499	0.7881	1.0880	1.1897	1.2711	1.5791
20	3	0.4752	0.5542	0.6091	0.6634	1.1217	1.2537	1.4604	1.8856
20	4	0.3946	0.4664	0.5298	0.5969	1.1635	1.3282	1.4844	1.8812
30	0.5	0.9773	0.9809	0.9839	0.9872	1.0039	1.0072	1.0111	1.0161
30	1	0.9058	0.9136	0.9269	0.9408	1.0314	1.0505	1.0691	1.1010
30	2	0.7217	0.7499	0.7771	0.8184	1.0980	1.1742	1.3001	1.5116
30	3	0.6013	0.6283	0.6735	0.7155	1.1403	1.2844	1.4897	1.9127
30	4	0.4612	0.5326	0.5792	0.6378	1.1547	1.3270	1.5585	2.1608
50	0.5	0.9839	0.9859	0.9877	0.9907	1.0047	1.0074	1.0102	1.0141
50	1	0.9309	0.9378	0.9448	0.9545	1.0302	1.0479	1.0656	1.0940
50	2	0.7960	0.8162	0.8367	0.8662	1.1214	1.2097	1.3251	1.5159
50	3	0.6598	0.7026	0.7503	0.7811	1.1656	1.3059	1.5113	1.7526
50	4	0.5899	0.6233	0.6633	0.7021	1.1876	1.3749	1.5887	1.8692

$$\frac{\sigma_c}{\mu_c} = [\exp(\sigma_L^2) - 1]^{1/2} \quad (6)$$

The rows of Table I reflect different sample sizes used to compute the two estimates of the mean and thus the ratio $\bar{y}/\bar{x}_c$. It is evident that the ratio is least biased from 1 when the sample size is large and the variability is small. As variability of the distribution increases and as sample size decreases, the ratio decreases.

In Table I the range of variances covers most exposure scenarios encountered in practice. For situations where groups of individual workers' mean exposures have been summarized by lognormal distributions, σ_g has usually been less than 3.[1]

The variance of the 1000 values of the ratio is shown in Table II. As in Table I, the columns reflect different values of the coefficient of variation ranging from 0.5 to 4 with corresponding geometric standard deviations σ_g of 1.7 to 5.4. The rows reflect the impact of the sample size on the estimated ratio $\bar{y}/\bar{x}_c$. It is evident that as sample size decreases and the coefficient of variation increases, the variance increases.

Percentiles for the ratio metric were also computed for various sample sizes and ranges of variances. Table III gives selected percentiles ranging from 1% to 99%. For example, if n = 20 and $\sigma_c/\mu_c = 1$, the ratio $\bar{y}/\bar{x}_c$ computed from a sample of lognormally distributed data would be expected to fall between 0.89 and 1.07 95% of the time. A measure of fit to the lognormal distribution, therefore, translates to determining whether the ratio computed for the group of exposures is extreme as compared with the percentiles in Table III.

Figure 6 displays the upper 97.5 and lower 2.5 percentiles of the ratio metric for different values of the coefficient of variation σ_c/μ_c. In the legend, σ_c/μ_c is abbreviated as cv_c. The upper and lower bounds represent a 95% confidence band about the ratio. It is evident from this plot that in cases where the geometric standard deviation is less than 3.6 (corresponding to $\sigma_c/\mu_c < 2$), a sample size of 20 is adequate to determine distributional fit with marginal improvements obtained by larger sample sizes.

It is important to note that the ratio metric does not constitute a rigorous statistical goodness-of-fit test because the theoretical distribution function of the test statistic is unknown. The ratio metric is best thought of as an ad hoc measure of goodness-of-fit to the lognormal distribution, which has the advantage of being an objective measure, unlike probability plotting.

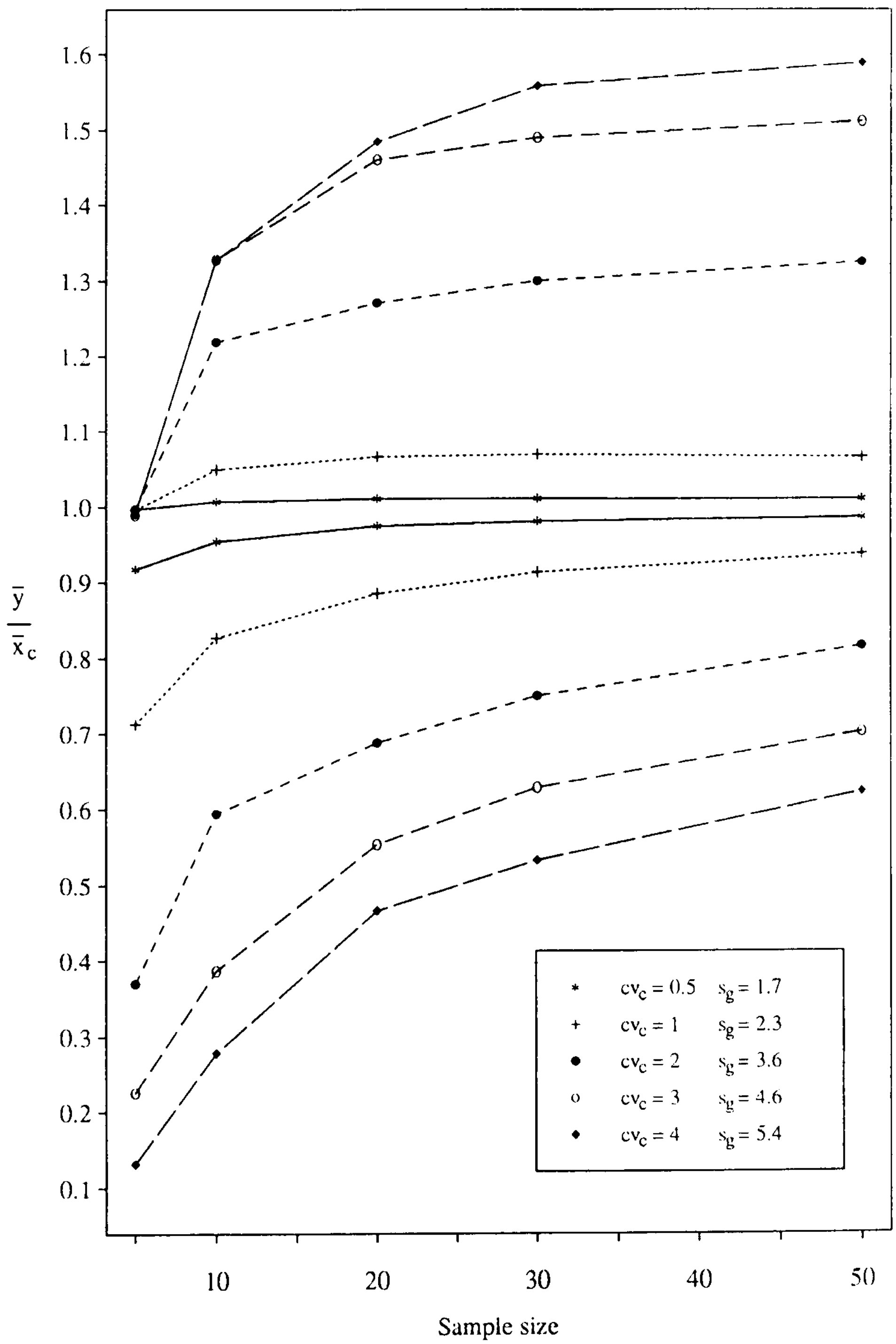

FIGURE 6. Upper 97.5% and lower 2.5% quantiles of the ratio $\bar{y}/\bar{x}_c$

USING THE RATIO METRIC

The ratio metric is simple to apply to exposure data. After the data have been collected, the two estimates of the mean $\bar{y}$ and $\bar{x}_c$ are calculated and their ratio computed. The estimated coefficient of variation $s/\bar{y}$ is also calculated. Table III or Figure 7 can then be used to evaluate the conjecture that the observed ratio arose by chance under the condition that the sampled population of exposures has a lognormal distribution. The estimated coefficient of variation and the sample size dictate which row in Table III is used. Alternately, the appropriate upper and lower bounds are selected in Figure 6 based on the coefficient of variation and sample size. If the ratio is not extreme (lies within an expected range), it is reasonable to conclude that the data are adequately described by a lognormal distribution. If the ratio is extreme (exceeds a prespecified percentile), then a conclusion of lognormality may not be appropriate. A probability plot of the data may yield information about how to proceed with the data analysis. For example, the probability plot may indicate the presence of a bimodal distribution or of one or more outliers.

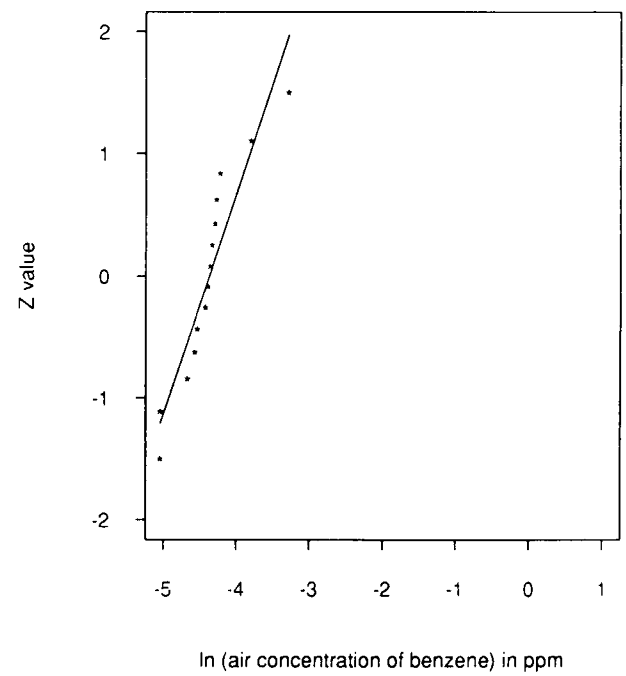

FIGURE 7. Benzene Group 2

THE DATA SETS

Two occupational data sets will be used for illustration. The first consists of 592 full-shift (8-hr) exposures to inorganic mercury measured daily for an 8-week period.[6] Between 26 and 40 measurements were obtained per worker. Random selection of individuals and time periods for monitoring was unnecessary because all workers were monitored for 40 consecutive work periods. Cases with less than 40 measurements per worker reflect either absenteeism or losses of samples.

The second data set consists of randomly collected samples of benzene exposures.[19] Full-shift (12-hr) benzene exposures of 57 refinery operators within six job groups were measured over a 3-month period. (Some operators alternated working in two job categories during this period and were monitored for both groups.) Sampling was performed with 3M organic vapor monitors (3M Company, St. Paul, Minn.) with analysis by capillary gas chromatography with flame ionization detection (HP5790A, Hewlett-Packard, Palo Alto, Calif.). Random selection of workers within job groups was unnecessary because all workers in each job group were monitored. Stratified random sampling was used to select the periods for monitoring, such that shifts were selected randomly for monitoring within each 7-day period. Between 2 and 11 measurements were obtained per worker.

Benzene exposure values that were less than the analytical detection limit (0.0098 ppm) were set at two-thirds of the detection limit (DL) for the purposes of this analysis. That is, all values measured as <0.0098 ppm were treated as if they were measured as 0.0065 ppm. The choice of 2/3 × DL is based on the assumption of a proportional distribution for values below the DL. This procedure was used because it results in a less biased estimate of the worker's mean exposure, $\bar{y}_i$, than elimination of the data or other simple substitution methods in cases of moderate variability ($\sigma_g < 3$) and moderate truncation (less than 30%).[19] The optimal approach for replacement of values below the DL is based on maximum likelihood estimation[20]; however, this is not a simple procedure, and adequate estimates of the mean may be obtained by substitution. For example, Hallez and Derouane[21] compared substitution of 1/2 × DL and 2/3 × DL with the true value of the arithmetic mean of simulated lognormal distributions and determined that substitution of 2/3 × DL provided a better estimate of the arithmetic mean for geometric standard deviation up to 3 and censoring ranging from 0% to 50%. Similarly, Hornung and Reed[22] compared substitution of 1/2 × DL and 0.707 × DL with the maximum likelihood approach for estimating the geometric mean and suggested the use of 0.707 × DL in cases where the geometric standard deviation is less than 3. A separate investigation showed that substitution of 1/2 × DL produces estimates of the geometric mean comparable to or better than complicated replacement techniques when the geometric standard deviation equals 2 for truncation up to 80%.[23]

For the mercury data, goodness-of-fit tests were applied both to the 16 individual (within-worker) distributions of exposures and to the between-worker distribution (distribution of worker's mean exposures, values of $\bar{y}_i$). For the benzene data, goodness-of-fit tests were applied to the between-worker distributions for the six job categories.

Table IV displays the sample estimates of the mean and standard deviation of the log-transformed air concentrations for the mercury and benzene workers. Units for mercury air concentrations are micrograms per cubic meter ($\mu g/m^3$) and for benzene are parts per million (ppm). The number of measurements or workers is also given. Benzene Groups 3A, 4A, and 5A are subsets of Groups 3, 4, and 5.

RESULTS

Table V lists the results of the Lilliefors test, the W test, and the values of the ratio metric for the two data sets. For the two formal

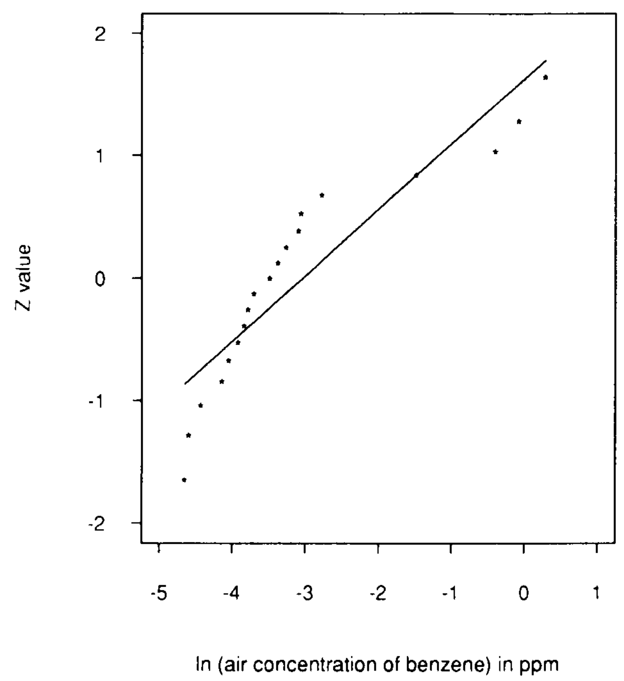

FIGURE 8. Benzene Group 3

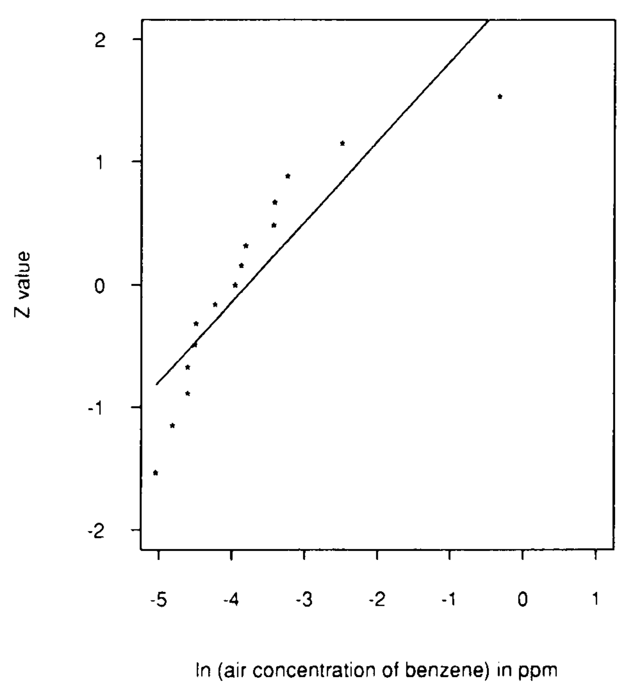

FIGURE 9. Benzene Group 5

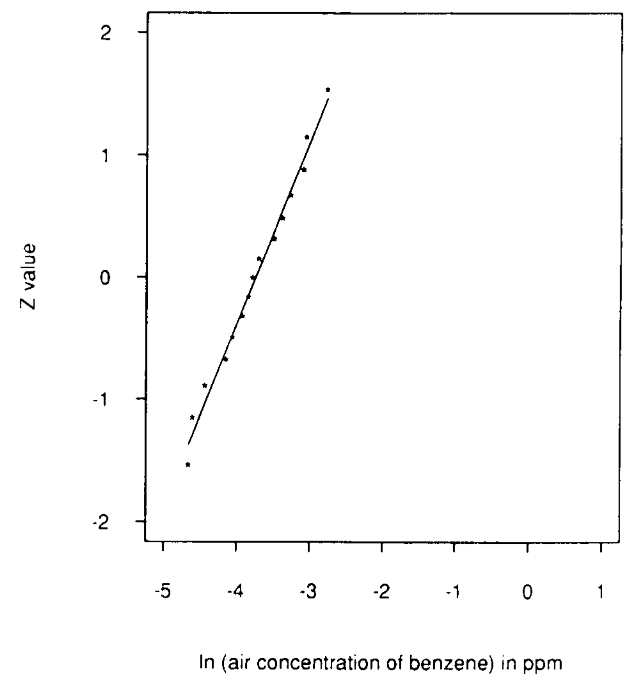

FIGURE 10. Benzene Group 3A

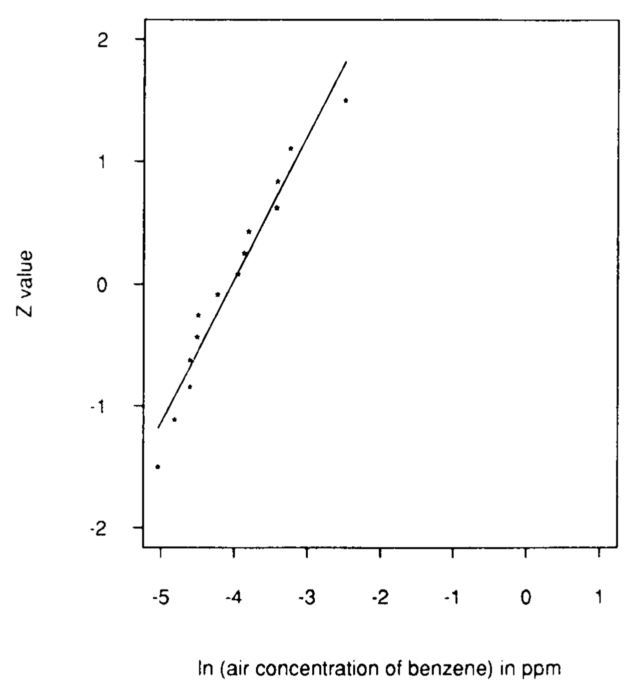

FIGURE 11. Benzene Group 5A

goodness-of-fit procedures, a 5% significance level was used. For the ratio metric the 2.5 and 97.5 percentiles were used from Table III. Interpolation was based upon observed values of $s/\bar{y}$ and n. The columns display the calculated test statistic, the 5% critical value or range, and the decision of whether to consider the underlying distribution as lognormal for each of the three procedures. Table V shows that for all cases except one, the same decision regarding lognormality results from the W test and the ratio metric. The exception (Worker 14) is a borderline case.

Log-probability plots of selected workers and groups are presented in Figures 1–5 and 7–11. Figures 1 and 2 indicate that the plots for Mercury Workers 1 and 4 are approximately linear. Table V indicates that for these workers each of the three goodness-of-fit procedures suggests the underlying distribution to be lognormal.

TABLE IV. Sample Estimates of Parameters of the Distribution of Workers' Exposures to Mercury and Benzene

Worker/Group		n^A	$\bar{x}_L{}^B$	$s_L{}^C$	$s_g{}^D$
Mercury[E]					
worker	1	40	3.19	0.437	1.5
	2	40	3.12	0.576	1.8
	3	30	2.84	0.439	1.6
	4	39	3.60	0.504	1.7
	5	40	2.69	0.509	1.7
	6	38	3.00	0.321	1.4
	7	38	3.04	0.559	1.7
	8	40	2.83	0.460	1.6
	9	26	3.12	0.430	1.5
	10	40	2.97	0.403	1.5
	11	38	2.96	0.748	2.1
	12	40	2.54	0.483	1.6
	13	40	2.74	0.541	1.7
	14	34	3.02	0.514	1.7
	15	39	2.81	0.605	1.8
	16	30	3.11	0.589	1.8
	Group	16	3.11	0.254	1.3
Benzene[F]					
group	1	15	−4.511	0.587	1.8
	2	14	−4.372	0.446	1.6
	3	19	−3.045	1.516	4.6
	4	17	−3.811	1.382	4.0
	5	15	−3.785	1.178	3.2
	6	5	−4.555	0.403	1.5
	3A	15	−3.747	0.575	1.8
	4A	10	−4.614	0.279	1.3
	5A	14	−4.033	0.713	2.0

[A]n = number of measurements or workers.
[B]$\bar{x}_L$ = mean of the logarithms of the air concentrations.
[C]s_L = standard deviation of the logarithms of the air concentrations.
[D]s_g = exp(s_L) = geometric standard deviation.
[E]Mercury concentrations in $\mu g/m^3$.
[F]Benzene concentrations in ppm.

Figure 3 illustrates that for Mercury Worker 11, the log-probability plot is fairly linear in the lower region, but not for the upper eight data points, which appear to come from a different distribution. One plausible explanation for a mixture of two distributions is that the worker performed different tasks on the 8 days that reflect higher exposures, e.g., one day each week might have been spent on maintenance or cleaning tasks. Thus, exposures on those days in which routine duties are performed may constitute one distribution and exposures on workdays with periodic or less routine tasks may constitute a second distribution. In the absence of task-activity descriptions, this explanation remains a conjecture, but the existence of two governing distributions seems clear. As Table V indicates, all three goodness-of-fit procedures lead to the same decision that the underlying distribution should not be considered as lognormal.

The plots in Figures 4 and 5 are linear with the exception of the highest one or two points, which appear to be outliers. For Worker 12, Table V indicates that both the W test and the ratio metric result in rejection of the hypothesis of lognormality at the 5% level; the less discriminating Lilliefors test results in acceptance of the null hypothesis. For Worker 14, the ratio metric indicates that the data are not lognormal, and the Lilliefors and W tests result in acceptance of lognormality. Comparison of the W statistic with the critical value shows that the decision to accept or reject lognormality is a borderline case.

Figure 7 presents the plot for Benzene Group 2. As stated earlier, each point in this plot represents the estimated mean exposure $\bar{y}_i$ for an individual worker. This plot indicates that these workers constitute a monomorphic group, i.e., a group of workers whose mean exposures comprise a single lognormal distribution.[1] All three goodness-of-fit procedures result in acceptance of a decision of lognormality in this case.

The log-probability plot for Benzene Group 3 (Figure 8) reveals that 4 workers have exposures that are significantly higher than the other 15 members of the group. Thus, the first 15 workers appear to comprise a monomorphic group but not the 4 individuals with the highest exposures. The upper four workers are presumed to be either performing different tasks or the same tasks in a different manner than the other members of the job group. The exposures of Benzene Group 3 are not lognormally distributed as judged by either the W test or the ratio metric. As with the case of Mercury Worker 12, the Lilliefors test is not sufficiently discriminating to reject the hypothesis of lognormality.

Figure 9 shows that the mean exposures for 14 of the 15 workers from Benzene Group 5 are approximately lognormally distributed. The highest exposure appears to be an outlier. Again, the W and ratio measures result in a decision that exposures of the group as a whole are not lognormally distributed. The Lilliefors test does not reject the lognormal hypothesis.

The data for Benzene Groups 3 and 5 were reanalyzed after removing the upper points from each group, which appear to be from different distributions. Exposures of workers in the new subgroups, designated as Groups 3A and 5A, are shown in Figures 10 and 11. In both cases, the plots are approximately linear and thus the subgroups appear to constitute monomorphic groups. All three goodness-of-fit procedures lead to the same conclusion that these subgroups can be considered as lognormal.

In designing the sampling strategy for the benzene workers, task and activity patterns were used to initially classify workers into observational groups based on job title, location, and type of work. As shown above, data analysis can allow groups to be reclassified and can identify individuals whose exposures require additional scrutiny and, possibly, control measures.

In summary, these results indicate that both the W test and the ratio metric almost always yielded conclusions that were consistent with the appearances of the probability plots. However, it is evident that the Lilliefors test was less discriminatory than either the W test or ratio metric, and even in cases of obvious deviations from linearity in the log-probability plot (e.g., Benzene Groups 3 and 5), the null hypothesis of lognormality was sometimes not rejected.

It should be noted from Table V and the figures that, for these two data sets, there is no evidence that the lognormal distribution cannot be used to describe both the within- and between-worker distribution.

TABLE V. Comparison of Three Goodness-of-Fit Measures for Distributions of Workers' Exposures to Mercury and Benzene

Worker/ Group	Lilliefors Test^A Lill. Stat.	Crit. Value	Accept H_o?^B	W Test^A W Stat.	Crit. Value	Accept H_o?^B	Ratio Metric^A Ratio	Critical Range Lower	Upper	Accept H_o?^B
Mercury										
worker 1	0.063	0.140	yes	0.986	0.940	yes	0.995	0.980	1.007	yes
2	0.111	0.140	yes	0.965	0.940	yes	1.015	0.980	1.016	yes
3	0.061	0.162	yes	0.970	0.927	yes	0.998	0.984	1.007	yes
4	0.102	0.142	yes	0.957	0.939	yes	1.009	0.986	1.010	yes
5	0.098	0.140	yes	0.962	0.940	yes	0.997	0.986	1.010	yes
6	0.084	0.144	yes	0.971	0.938	yes	0.996	0.979	1.007	yes
7	0.144	0.144	yes	0.950	0.938	yes	0.983	0.979	1.016	yes
8	0.125	0.140	yes	0.927	0.940	no	1.014	0.980	1.007	no
9	0.096	0.173	yes	0.936	0.920	yes	1.006	0.982	1.007	yes
10	0.081	0.140	yes	0.944	0.940	yes	1.005	0.980	1.007	yes
11	0.187	0.144	no	0.841	0.938	no	1.084	0.979	1.037	no
12	0.082	0.140	yes	0.927	0.940	no	1.026	0.980	1.007	no
13	0.080	0.140	yes	0.962	0.940	yes	1.012	0.980	1.016	yes
14	0.089	0.151	yes	0.936	0.933	yes	1.029	0.985	1.011	no
15	0.149	0.142	no	0.940	0.939	yes	1.020	0.980	1.021	yes
16	0.105	0.162	yes	0.909	0.927	no	0.963	0.984	1.020	no
Group	0.087	0.221	yes	0.959	0.887	yes	0.999	0.973	1.006	yes
Benzene										
group 1	0.226	0.229	yes	0.748	0.881	no	1.070	0.953	1.011	no
2	0.152	0.237	yes	0.906	0.874	yes	1.004	0.970	1.005	yes
3	0.188	0.203	yes	0.828	0.901	no	1.250	0.595	1.242	no
4	0.188	0.215	yes	0.769	0.892	no	1.897	0.628	1.192	no
5	0.144	0.229	yes	0.816	0.881	no	1.532	0.745	1.195	no
6	0.119	0.396	yes	0.987	0.762	yes	0.985	0.930	0.996	yes
3A	0.075	0.229	yes	0.970	0.881	yes	0.984	0.953	1.011	yes
4A	0.145	0.280	yes	0.933	0.842	yes	0.998	0.963	1.003	yes
5A	0.097	0.237	yes	0.845	0.803	yes	1.006	0.913	1.022	yes

^A The critical values for each test reflect 5% significance levels.
^B H_o: The data are a random sample from a lognormal distribution.

CONCLUSIONS

Occupational hygienists need to classify individuals into groups in order to manage exposure monitoring for an occupational cohort. One approach is to establish monomorphic groups,[1] that is, workers whose individual mean exposures comprise discrete lognormal distributions. In the past hygienists have relied on the log-probability plot to evaluate distributional assumptions. One reason for the failure to supplement this subjective method has been the inconvenience of performing formal statistical tests. A new method to help in the evaluation of the goodness-of-fit to the lognormal model that is a simple extension of calculations made routinely by many occupational hygienists has been described. Occupational hygienists are encouraged to employ the ratio metric in determining validity of the lognormal assumption.

It is important to note that all goodness-of-fit tests have certain limitations. When a test fails to reject the null hypothesis the conclusions are not always clear-cut. The null hypothesis can be "accepted" for two reasons: either it is true or insufficient evidence exists to disprove it. When the sample size is small, the null hypothesis of model fit will almost always be accepted, except in extreme cases. Thus, a goodness-of-fit test with small sample sizes may not be very useful to determine whether a specific distribution "fits" the data. Only when the test results show that the lognormal model doesn't fit the data are the results unequivocal.

Goodness-of-fit tests also have problems when the sample size is very large because it becomes very difficult to disprove the alternative hypothesis. This failure to fit the distribution again can be attributed to two possible reasons: either the data truly don't fit the model or the test has become so discriminating because of the large sample size that very small quirks in the data cause rejection of the null hypothesis. When this occurs, a probability plot can often distinguish between these two alternatives.

In summary, the probability plot is an extremely useful tool for visualizing data and identifying anomalies or trends, and the authors do not propose abandoning this tool. Rather, occupational hygienists can easily improve on this approach by computing the ratio of two means as a first step in determining lognormality of exposures. Of course, the W test should be used when a formal test of goodness-of-fit is required. However, in the large majority of cases, computation of the ratio metric provides a useful measure of goodness-of-fit to a lognormal model that is both simple and accurate.

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