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Accelerated evaluation of automated vehicles using extracted naturalistic driving data

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ABSTRACT: It is important to rigorously and comprehensively evaluate the safety performance of Automated Vehicles before they are produced and deployed. Under naturalistic driving conditions, field operational tests and simulations may take a long time to finish, because of the low exposure to safety-critical scenarios. In this paper, we propose an accelerated evaluation approach. Statistics of the motion of the primary other vehicle (POV) were built using extracted naturalistic driving data then modified to present higher risk interactions. Two surrogate automated vehicles obtained from observed production vehicle behaviors were evaluated in car-following scenarios. Results show that the proposed method can accelerate the evaluation process by 5 orders of magnitude.

1 INTRODUCTION

Automated Vehicle (AV) technologies have the potential to change the future of ground mobility significantly. AVs could save fuel, reduce traffic accidents, ease traffic congestion, and increase mobility for the elderly, disabled and blind. In 2030, it is estimated that the consumer demand for AVs will reach $87 billion (Laslau 2014). Nearly every major car company has initiated research and development of AVs.

As the automation level of vehicles increases, the sensing and control systems will become more complex, making the evaluation of AV more challenging. Today's high-end cars may have 100 million lines of code, while the Boeing 787 only has 6.5 million (Information Is Beautiful 2014). AV functions will add to the system complexity. It is desirable to shorten the evaluation and validation process for faster product development iterations. This is consistent with NHTSA (National Highway Traffic Safety Administration)'s view of main challenges in the development of automated driving:

> *"Development of test and evaluation methods - Based on the real world scenarios (use cases) that map to the functional description of the automated system, develop test track tests and/or simulation approaches that can evaluate the performance of the level 2 or level 3 systems relative to these use cases."*(NHTSA 2013, p.9)

A successful design of AV requires accurate detection of the surrounding environment and the ability to handle complex scenarios posed by other vehicles, especially vehicles controlled by human drivers. AVs will penetrate the market gradually and will co-exist with non-AVs for decades (Litman 2014). During this transition period, AVs will encounter the same issue we are facing today driving human-controlled vehicles (HV) – the imperfection of human drivers in the surroundings act as disturbances to the AVs. It is estimated that today seventy to ninety percent of the motor vehicle crashes are mainly contributed to human errors (Barfield & Dingus 2014; Treat et al. 1979). To reduce crashes, AVs must deal with the risky maneuvers initiated by human drivers. A practical and effective evaluation approach that accounts for the imperfect human-driven traffic is essential for the development and evaluation of AVs. In this study, we focus on the interactions between AVs and HVs because this is the most common scenario in the near-term.

Two types of evaluation methods are commonly used to evaluate vehicles and vehicle control systems. Government agencies, such as the U.S. Department of Transportation and Euro-NCAP (New Car Assessment Program), frequently use test matrix evaluation methods, which use a series of pre-defined scenarios (Aust 2012). The main benefits of these methods are repeatability, reliability and time to finish. However, because all the test scenarios are fixed and predefined, control systems can be adjusted to achieve good scores, but their behaviors under broader conditions may not be adequately assessed.

Another evaluation approach is Naturalistic Field Operational Tests (N-FOTs). In an N-FOT, a number of test vehicles are driven in daily traffic over an extended period of time (Aust 2012). Real-world testing is crucial for the final validation of a product. However, it is a very inefficient approach due to the low level of exposure to safety critical events in daily driving. Statistics shows that one would need to drive for 38 years, on average, to be involved in a police-reported crash and 6,877 years for a fatal crash (NHTSA 2012). Thus, this is not a practical approach for production-intent safety products.

In this paper, an accelerated evaluation approach is developed to assess the performance of AVs. The approach will use challenging scenarios representative of real-world driving and reduce the evaluation time. A microscopic simulation environment was built with higher rates of challenging driving scenarios to achieve this goal. Based on the statistical analysis of a large-scale naturalistic driving database, stochastic driver models were built to simulate the actions of the vehicles surrounding the AV. Such driver models should emulate the diverse driving behaviors and risky actions by human drivers, such as unexpected abrupt braking, tailgating, and distraction. By eliminating the normal (safe) driving situations and focusing on the unsafe behaviors of other vehicles, we can accelerate the evaluation procedure.

2 CAR-FOLLOWING MODEL BASED ON NATURALISTIC DRIVING

2.1 *Car-following scenarios*

A microscopic simulation environment was built to evaluate the AV performance including interacting with other (human-controlled) vehicles in car-following scenarios. As shown in Figure 1, there are three vehicles: the lead human controlled vehicle, automated vehicle to be evaluated, and trailing human controlled vehicle.

A few assumptions were made for this research:

- We focus on the interactions between AVs and HVs, as that will be the most common scenario in the near-term. The interactions between AVs or between HVs are not discussed (except when used as a benchmark).
- The sensors and actuators of AVs are assumed to work perfectly.
- Only primary crash events were considered. Secondary impacts may occur but they are outside of the scope of this study.
- Human drivers react to AVs in the same way as they do to other HVs. Many automakers work hard to hide sensors and make their prototype AVs look "normal" (Ackerman 2013), which is the basis for this assumption.

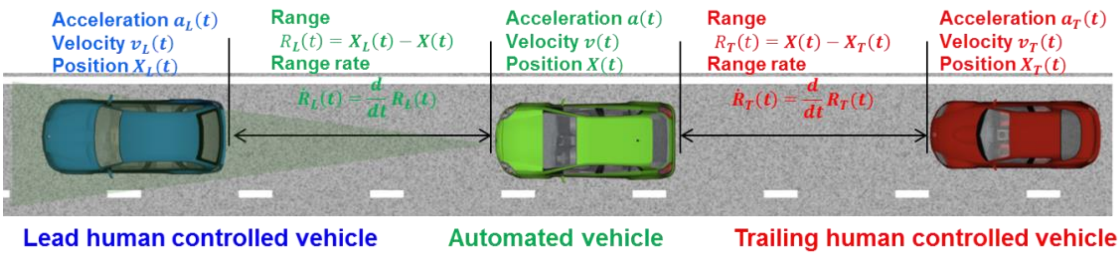
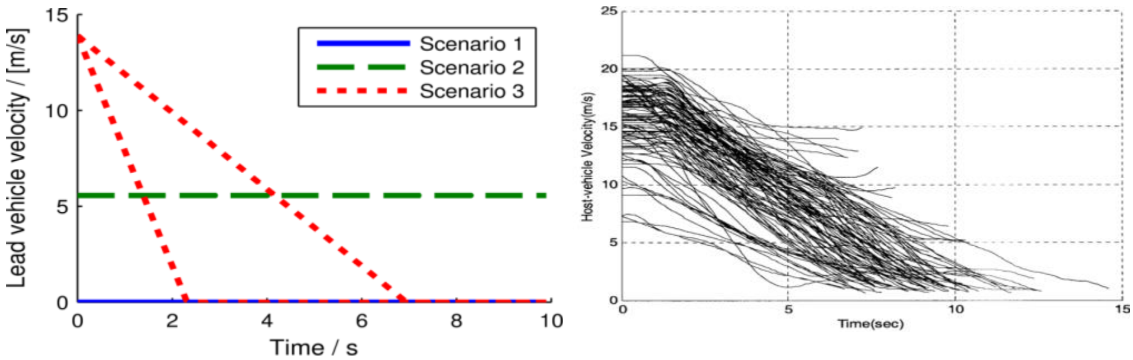


Figure 1. Simulation scenarios for the automated vehicle in car-following

2.2 *Stochastic lead vehicle model*

In the car-following scenarios, it is critical to model the lead vehicle motion. In the test matrix method, the lead vehicle motion is predefined. For example, in the EURO-NCAP Autonomous Emergency Braking (AEB) test protocol, three scenarios are defined: 1) stopped lead vehicle, 2) slow lead vehicle with constant speed, and 3) lead vehicle braking at constant deceleration (Euro NCAP 2013), as shown in Figure 2 a). Lee (Lee 2004) selected 100 challenging cases of lead vehicle motions from a naturalistic driving database as shown in Figure 2 b). As all the scenarios were extracted from the naturalistic driving record, they represent real-world driving scenarios. However, the querying approach (Time To Collision < 11 s) is somewhat ad hoc. It was also not clear how to assign a final performance score based on the simulation results of these 100-case scenarios. Yang (Yang 2010) scanned the whole Road Departure Crash Warning (RDCW) database (LeBlanc et al. 2006) and used all available lead vehicle trajectories to evaluate AVs. This approach avoids the issue of choosing scenarios. However, the exhaustive simulation study takes a long time to finish.



a) EURO-NCAP AEB test b) Speed queried from the SAVEME database

Figure 2. Lead vehicle velocities used in the EURO-NCAP AEB test (a), and selected naturalistic driving data (b)

In this research, a Markov Chain model with the lead vehicle speed and acceleration as the two state variables is used. The transition matrix is obtained from a Gaussian Mixture Model (GMM) (Reynolds 2009). The random acceleration at a time τ second in the future is calculated from

$$p\left(a_{L_{t+\tau}} \middle| a_{L_t}, v_{L_t}\right) = \sum_{i=1}^{M} w_i(v_{L_t})\, G\left(\begin{bmatrix} a_{L_{t+\tau}} \\ a_{L_t} \end{bmatrix} \middle| \boldsymbol{\mu}_i(v_{L_t}), \boldsymbol{\Sigma}_i(v_{L_t})\right) \tag{1}$$

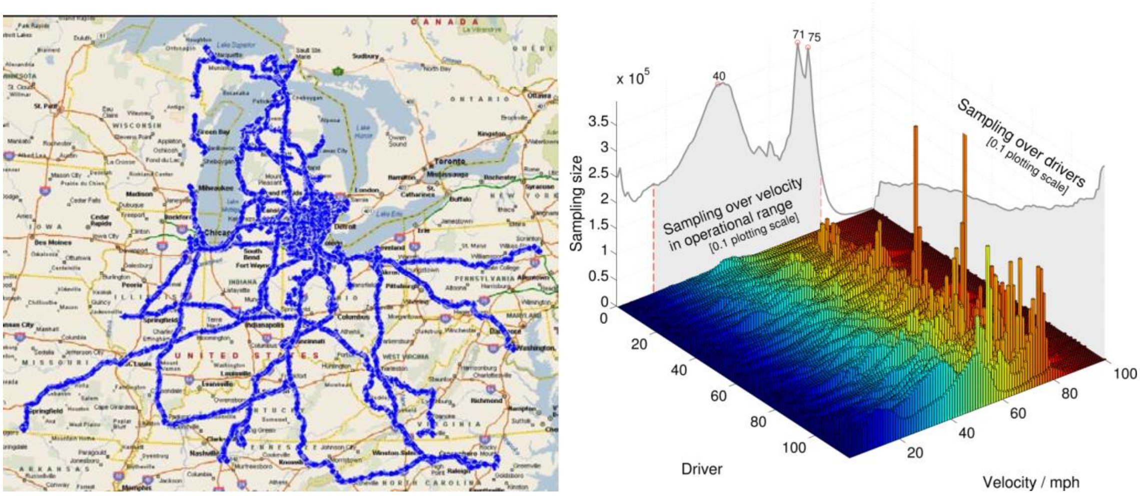
where w_i are the mixture weights satisfying $\sum_{i=1}^{M} w_i = 1$, and $G\left(\begin{bmatrix} a_{L_{t+\tau}} \\ a_{L_t} \end{bmatrix} \middle| \boldsymbol{\mu}_i(v_{L_t}), \boldsymbol{\Sigma}_i(v_{L_t})\right)$ is the component density, in the form of a 2-variate Gaussian model

$$\begin{aligned} &G\left(\begin{bmatrix} a_{L_{t+\tau}} \\ a_{L_t} \end{bmatrix} \middle| \boldsymbol{\mu}_i(v_{L_t}), \boldsymbol{\Sigma}_i(v_{L_t})\right) \\ &= \frac{1}{2\pi \left|\boldsymbol{\Sigma}_i(v_{L_t})\right|^{1/2}} \exp\left\{-\frac{1}{2}\left(\begin{bmatrix} a_{L_{t+\tau}} \\ a_{L_t} \end{bmatrix} - \boldsymbol{\mu}_i(v_{L_t})\right)^T \boldsymbol{\Sigma}_i(v_{L_t})^{-1} \left(\begin{bmatrix} a_{L_{t+\tau}} \\ a_{L_t} \end{bmatrix} - \boldsymbol{\mu}_i(v_{L_t})\right)\right\} \end{aligned} \tag{2}$$

which has a mean vector $\boldsymbol{\mu}_i$ and covariance matrix $\boldsymbol{\Sigma}_i$. The velocity in the next time step is calculated from

$$v_{L_{t+\tau}} = v_{L_t} + \tau \cdot (a_{L_{t+\tau}} + a_{L_\tau})/2 \tag{3}$$

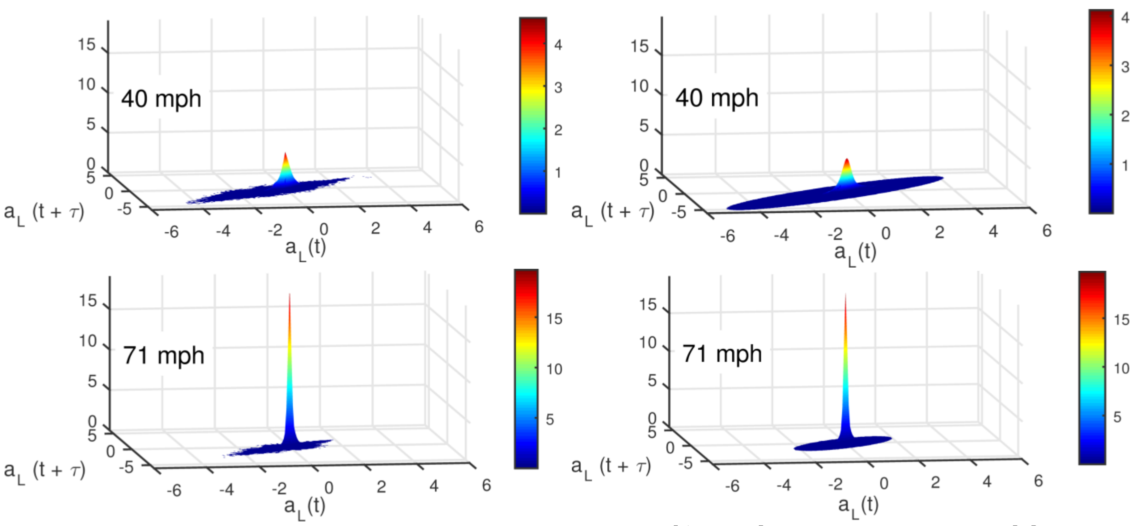
The Integrated Vehicle-Based Safety Systems (IVBSS) database (Sayer et al. 2011) is used in this research. A random sample of 108 licensed drivers was recruited to participate in the study. Participants were in one of three age groups: 20 to 30 (younger), 40 to 50 (middle-aged), and 60 to 70 years old (older), and are gender balanced. Each participant drove a vehicle equipped with the integrated safety system and data acquisition system for approximately 6 weeks. Figure 3 a) shows 23,000 recorded trips of 213,000 miles travel. Vehicle acceleration and velocity were extracted from the IVBSS database at a time step of 0.3 seconds. The data was further discretized based on the vehicle velocity. As shown in Figure 3 b), for each driver and speed interval, the transition pairs $[a_{L_t}, a_{L_{t+\tau}}]$ were collected. The transition pairs were further aggregated for all 108 drivers to represent the diversity of behaviors among drivers. The total amount of data pairs $[a_{L_t}, a_{L_{t+\tau}}]$ used in this research is over 1.5 million with a speed interval of 1 mph.



a) Recorded routes of light vehicles   b) Sampling size for leading vehicle model fitting

Figure 3. Vehicle data in Integrated Vehicle-Based Safety Systems database

The model parameters are estimated using expectation-maximization, also known as maximum likelihood. The number of components for the GMM is set to four to provide an adequate degree of freedom but low enough to avoid over-fitting. Figure 4 a) shows the histogram of the naturalistic data. The fitted GMM model with the same velocity intervals is plotted on the right side in Figure 4 b).



a) Histogram of the naturalistic driving data   b) Fitted Gaussian Mixture Model

Figure 4. Transition matrix modeled by the Gaussian Mixture Model

2.3 *'Errable' trailing vehicle model*

Car-following behaviors have been intensively studied and modeled for more than five decades. Most of the driver models focus on normal driving behaviors. Good fitting results and crash-free simulations are generally considered as model performance metrics (Yang 2010, p.13). However, the majority of crashes occur due to human errors. Therefore, it is important to understand driver errors that lead to unsafe situations. Error-free models are useful to study average traffic flow or human behaviors, but are not very useful for the evaluation of safety systems. A longitudinal 'errable' driver model was suggested in (Yang et al. 2008), which was used to evaluate a number of collision warning systems (Yang & Peng 2010). Tang et al. (Tang et al. 2011), Przybyla et al. (Przybyla et al. 2012) and Bi et al. (Bi et al. 2012) adopted this concept and built other longitudinal or lateral errable models.

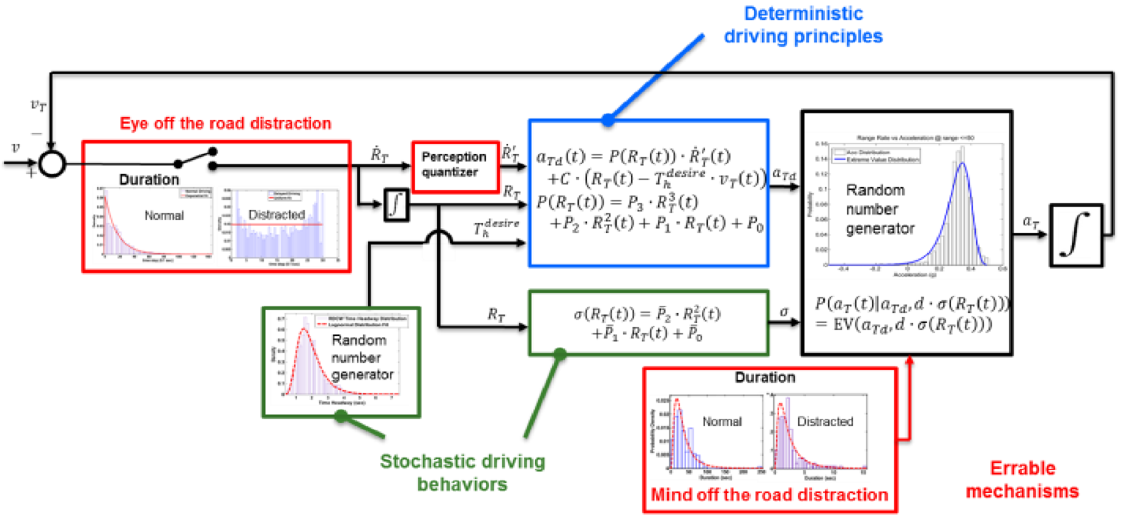


Figure 5. Structure of errable car-following model (Yang et al. 2008)

Errable driver models consist of three components as shown in Figure 5: deterministic driving principles, stochastic driving behaviors, and error mechanisms. The deterministic driving principles establish the basic relationship between the control stimulus variables (such as range and range rate) and the response of the driver (e.g., vehicle acceleration). A stochastic driver behavior block is added to describe driver imperfection. Finally, errable mechanisms based on statistical and psychophysical knowledge of human behavior are included as the third element of the model. In this paper, we use Yang and Peng's errable model to model the trailing vehicle in the car following scenarios. Further explanation errorable driver models can be found in (Yang et al. 2010).

3 ACCELERATED EVALUATION

The normal GMM driver model only emulates normal driving behaviors. An accelerated model is needed to increase the exposure level for risky situations and accelerate the evaluation procedure. Recent findings in naturalistic driving studies indicate that unexpectedness is a key property of critical events (Fitch & Hankey 2012; Aust 2012). In other words, critical situations have low probability by nature. As shown in Figure 6, by removing the most frequent (and boring) situations, the overall exposure rate for critical scenarios is increased.

Figure 7 shows the accelerated and normal lead vehicle velocity profiles generated by the proposed method. It can be seen that the accelerated lead vehicle velocity have more frequent and harsher scenarios than the original (normal) condition.

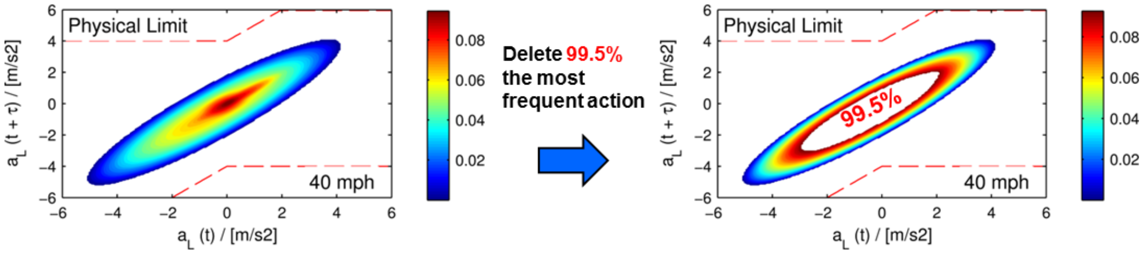


Figure 6. Procedure to generate accelerated transition matrix

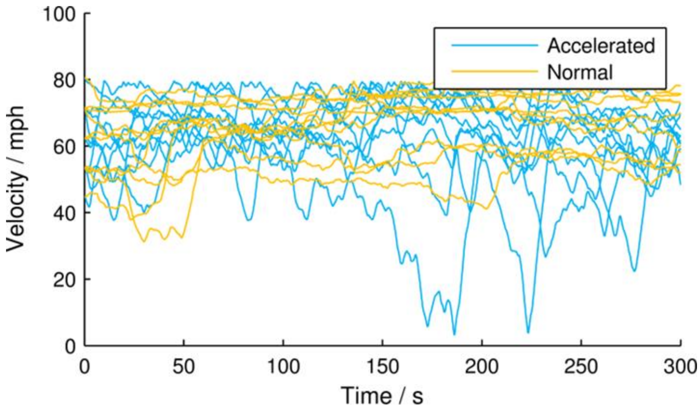


Figure 7. Comparison of accelerated vs. normal lead vehicle speed

4 SIMULATION ANALYSIS

4.1 *Design of automated vehicle models*

Two AVs equipped with ACC (Adaptive Cruise Control) and AEB (Autonomous Emergency Braking) were designed. When the vehicles are not in a frontal crash risk, they are controlled by the same ACC algorithm. The AEB algorithms become active when a threat is detected. The two simplified AEB algorithms were extracted from two production vehicles: Volvo V60 and Infiniti M37S using test track data, data found in owner's manuals, Euro NCAP information, and videos (Gorman 2013). The overall layout of the AV model is shown in Figure 8. Here we name the two AV designs as Design A and Design B.

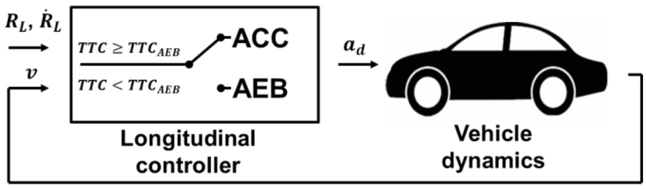
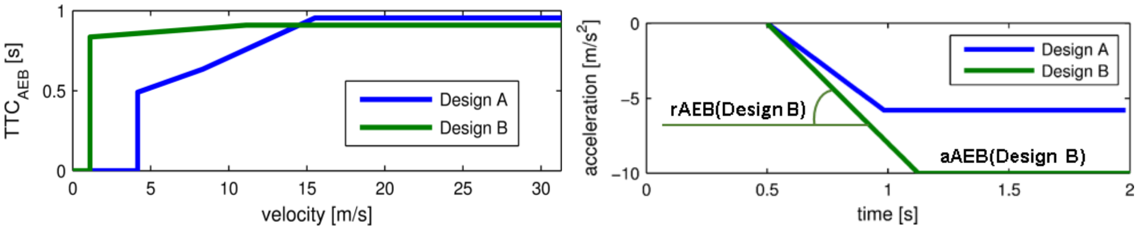


Figure 8. Layout of the AV model

The AEB algorithm becomes active when a risk is detected, and in (Gorman 2013) it was assumed that the risk is only based on a threshold value of "Time-To-Collision", defined as

$$TTC = -\frac{R_L}{\dot{R}_L} < TTC_{AEB} \tag{4}$$

where TTC_{AEB} is the threshold to activate AEB system which is a function of vehicle speed. Figure 9 a) shows the relationship between TTC_{AEB} and vehicle speed. Once triggered, AEB will aim to achieve a high level of braking (aAEB), subject to a rate limit (rAEB), as shown in Figure 9 b). The desired aAEB of Design B reaches -10 m/s^2. However, in reality, the maximum deceleration usually cannot reach this level due to the variance of road conditions. The existence of Anti-lock Braking Systems (ABS) also prevents the longitudinal tire force from reaching its peak value to avoid tires locked-up and losing control of the vehicle. In this research, we set the maximum deceleration to be -8 m/s^2.



a) TTC_{AEB} as a function of vehicle velocities b) Acceleration profiles of the AEB designs

Figure 9. The two AEB designs evaluated in this paper

A PI controller is used to design the ACC algorithm to achieve the desired time headway $T_{HW_d}^{ACC}$.

$$t_{HW}^{Err} = t_{HW} - T_{HW_d}^{ACC} \tag{5}$$

$$a_d(t) = K_p^{ACC} t_{HW}^{Err}(t) + K_i^{ACC} \int_0^t t_{HW}^{Err}(\tau)\, d\tau \tag{6}$$

where t_{HW} is the time headway, defined as

$$t_{HW} = R_L / v \tag{7}$$

a_d is the desired acceleration commanded by ACC controller; gains K_p^{ACC} and K_i^{ACC} are the proportional and integration gains calculated using Matlab Control Toolbox with following requirements:

- Loop bandwidth = 10 rad/s
- Phase margin = 60 degree

4.2 *Simulation and analysis*

The vehicle and control models presented in the previous section may not be good representations of the actual systems in the production vehicles. The results shown below thus should not be interpreted as rigorous evaluation results for the two production vehicles. The roads are assumed to be flat and straight with good adhesion. The effects of tire dynamics, chassis, and braking systems are important but are not analyzed in this paper. Each simulation run will start with the same initial values shown in Table 1 and end when a crash occurs.

Table 1. Initial conditions for the car-following scenario.

Variable	v_L	a_L	$T_{HW_d}^{ACC}$	R_L	$\dot{R}_L$	a	R_T	$\dot{R}_T$	a_T
	mph	m/s^2	s	m	m/s	m/s^2	m	m/s	m/s^2
Value	40	0	1	$T_{HW_d}^{ACC} * v_L$	0	0	$T_{HW_d}^{ACC} * v$	0	0

Two evaluation metrics were used: crash rate and relative velocity Δv at crash. The crash rate is defined as:

$$r_{c_n} = \frac{1}{n}\sum_{i=1}^{n} d_i \tag{8}$$

where i is the index of simulation tests, n is the total number of crashes. d_i is the distance travelled in test i, defined as:

$$d_i = \int_{t=0}^{t_{crash}} v_i(t)dt \tag{9}$$

As each test is run under the same stochastic condition, based on the Law of Large Numbers and Central limit theorem, when the sample size approaches infinity, the sampling average will converge to the expected value, i.e.,

$$r_{c_n} \to \mu_c := \mathbb{E}(r_{c_i}) \qquad \text{for} \qquad n \to \infty \tag{10}$$

with: $$r_{c_n} \to \mathcal{N}(\mu_c, \frac{{\sigma_c}^2}{n}) \tag{11}$$

The stopping criterion for the simulations is to keep conducting more simulations until the relative error

$$r_{rel} = \left|\frac{\bar{r}_{c_n} - \mu_c}{\bar{r}_{c_n}}\right| < b \tag{12}$$

with a confidence level larger than $1-\alpha$ (both b and α are small positive number to be selected), i.e.,

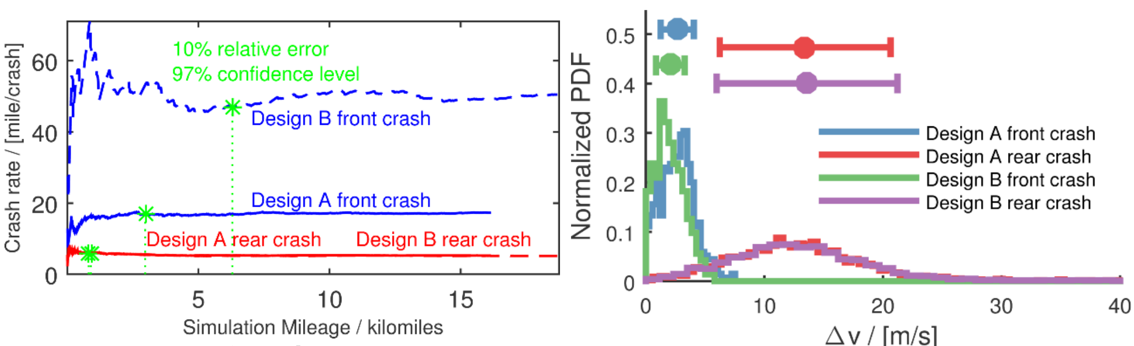
$$P(r_{rel} < b) \geq 1-\alpha \tag{13}$$

The test number n needed to achieve the required confidence level must satisfy

$$n \geq \frac{\hat{\sigma}_{c_n}^2 \ \mu_{\alpha/2}^2}{r_{c_n}^2 \ b^2} \tag{14}$$

where $\hat{\sigma}_{c_n}^2$ is the variance of $\{\bar{r}_{c_1}, \bar{r}_{c_2}, \dots, \bar{r}_{c_n}\}$, $\mu_{\alpha/2}$ is the quantile of the normal distribution at $\alpha/2$.

The estimated crash rates are shown in Figure 10 a). The histograms of relative velocities are shown in Figure 10 b) with means and $\pm\sigma$ error bars. Design B, equipped with a more aggressive algorithm, traveled a longer distance to encounter a frontal crash between the AV and the lead human controlled vehicle than Design A on average and had smaller average Δv. Both designs showed similar rear crash rates. It could be explained that the main reason for crashes from behind are caused by driver error (97.5% for Design A and 96.0% for Design B).



a) Crash rate

b) Relative velocity

Figure 10. Simulation results for car-following situation

Under real-world traffic conditions, it is estimated that the average chance to encounter a crash is 0.27 million miles (NHTSA 2012). By using the same lead and trailing human controlled vehicles in an aforementioned accelerated setting as shown in Figure 11, this distance reduces to 6.98 miles. Therefore, the overall time accelerates roughly 38,000 times.

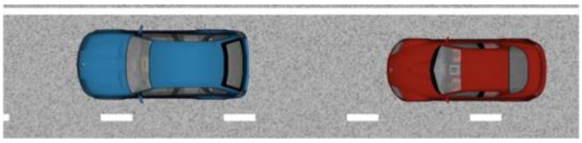


Figure 11. Simulation layout for human controlled vehicles

5 CONCLUSION

A procedure to accelerate the evaluation procedure of AVs using naturalistic driving data was developed in this manuscript. Stochastic driver models were built to construct the microscopic driving environment surrounding the AV in the car-following situation. By eliminating the common but boring driving situation, a higher level of exposure to critical scenarios was created. Two AVs equipped with ACC and AEB were designed based on production vehicles, to demonstrate the accelerated evaluation approach. The simulation showed that the overall evaluation time was reduced by a factor of 38,000.

ACKNOWLEDGEMENT

The authors would like to acknowledge the generous sponsorship of the National Institute for Occupational Safety and Health (NIOSH).

DISCLAIMERS

The findings and conclusions in the report are those of the authors and do not necessarily represent the views of the National Institute for Occupational Safety and Health (NIOSH). Mention of company names or products does not imply endorsement by NIOSH.

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