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SLOPE STABILITY ANALYSIS BY THE FINITE ELEMENT STRESS ANALYSIS AND LIMITING EQUILIBRIUM METHOD

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Bureau of Mines
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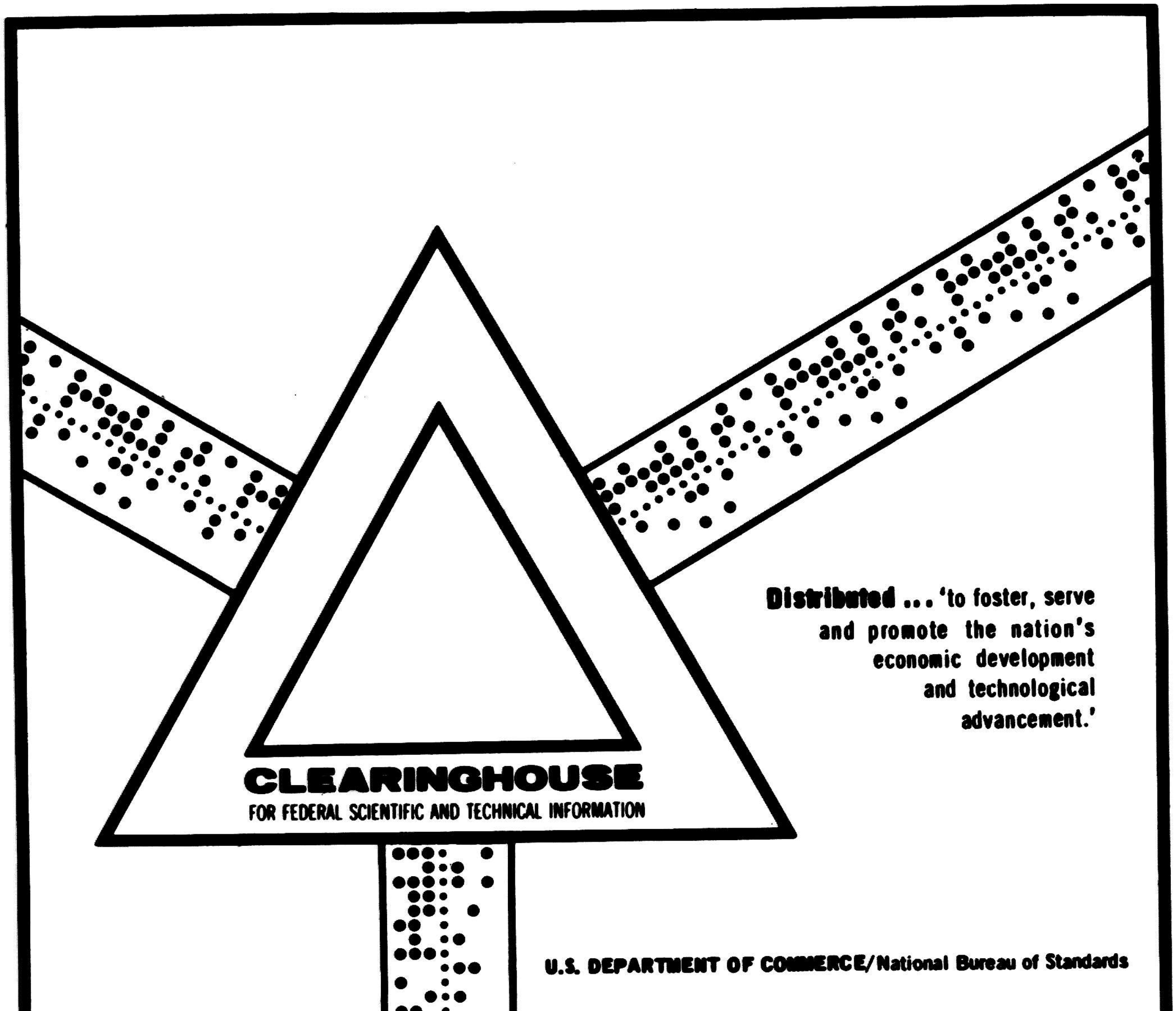

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By Fun-Den Wang and Meng-Cherng Sun

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SLOPE STABILITY ANALYSIS BY THE FINITE ELEMENT STRESS ANALYSIS AND LIMITING EQUILIBRIUM METHOD

by

Fun-Den Wang[1] and Meng-Cherng Sun[1]

ABSTRACT

A new approach for evaluating pit slope stability by the limiting equilibrium method has been developed by the Bureau of Mines. The finite element method of plane strain analysis is used to find the stress field of the pit slope structure. This stress field is then used to find the normal and shear stresses along a potential failure surface. The normal stresses are used with the Coulomb equation to calculate the shear resistance. The shear stresses are integrated to determine the total driving force. The factor of safety is evaluated from the ratio of the resisting force to the driving force along a failure surface, and is found to be affected by the elastic material properties, modulus of elasticity, and Poisson's ratio.

INTRODUCTION

In open-pit mining, pit slope design is of primary concern because the economics and safety of the operation depend on the slope angle and stability of the excavation. Open-pit structures may be designed by conventional methods of slope stability analysis. In this report, a new method of pit slope stability analysis by the finite element approach is presented. This research work is part of the overall ground control research program conducted by the Bureau of Mines to improve the economic design and stability analysis of rock structures.

Several conventional methods of slope stability analysis by the limiting equilibrium mechanics technique have been used for slope design. Because of the indeterminancy of the pit slope structure, these methods require the stresses along a failure surface to be found by the method of slices. In the Swedish circle method (or Fellenius method), each slice is considered to be in equilibrium without forces on the sides of the slice. In the Bishop method (2),[2] each slice is considered to be in equilibrium, with horizontal resultant forces acting on the sides of the slice. A more accurate method, by Morgenstern and Price (9), considers the slice to be in complete static equilibrium (nonhorizontal resultant forces on the sides of the slice).

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[2] Underlined numbers in parentheses refer to items in the list of references at the end of this report.

Because static equilibrium is not completely satisfied in the Fellenius method or the Bishop method, the error in the calculated factor of safety may amount to 60 pct and 7 pct, respectively, for the two methods, as pointed out by Whitman and Bailey (13-14). The Morgenstern-Price method gives physically reasonable solutions, as was demonstrated by Whitman and Bailey (14), but the method requires application of judgment as to the side-force distribution. In all the methods of slices, the elastic properties are not included in the analysis.

The stress field of a pit slope structure can be found by the use of the finite element method of stress analysis (3); then the normal and shear stresses at every point along the failure surface may be found from this stress field. With the normal and shear stresses known, the resisting strength can be found from the Coulomb equation, and the total shear force can be found by integrating the shear stresses along the failure surface. From the mechanics of limiting equilibrium, the factor of safety may be evaluated as the ratio of the resisting strength to the total shear force along the failure surface.

As compared with a method of slices, this method requires a knowledge of additional material properties--namely, the modulus of elasticity and Poisson's ratio--in order to calculate the stress field, which must precede calculation of the factor of safety. The purposes of this Bureau of Mines investigation are (1) to establish the method of stability analysis by the finite element method of stress analysis and limiting equilibrium mechanics, (2) to investigate the influence of the elastic properties, nonhomogeneity, and geometry on the factor of safety, and (3) to compare the results with the Morgenstern-Price method and find the potential and implications of the latter method of analysis.

METHOD OF ANALYSIS

Two-dimensional plane strain analysis by the finite element method is used for finding the stress field of the pit slope structure. The finite element method of stress analysis has been well documented in many publications (4, 15), and only a brief description of the essential features of this method is given in this report. The basic concept of the finite element method is the idealization of an elastic continuum as an assemblage of discrete elements interconnected at their nodal points. The stiffness properties of the elements can be found from the force-deflection relationship and the requirements for compatibility and equilibrium. The stiffness of the structure [k] is obtained by superposing the appropriate stiffness coefficients of the individual elements connected to each nodal point. The equilibrium equations expressing the relationship between the applied nodal forces {F} and the resulting nodal displacements {x} for the complete structure are

$$\{F\} = [k] \cdot \{x\}. \tag{1}$$

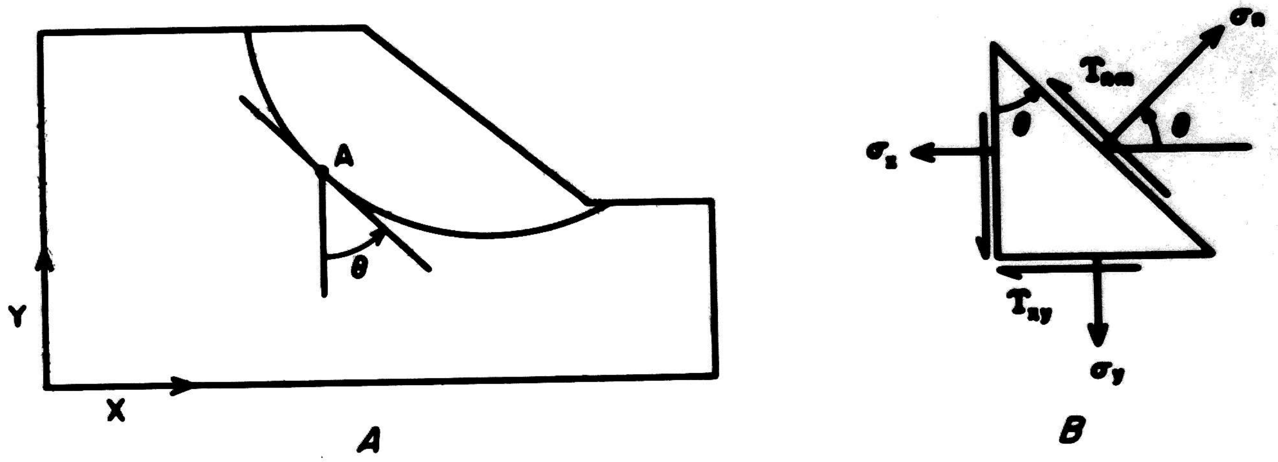

FIGURE 1. - Stresses at a Point on a Failure Surface.

These linear equations are solved for the nodal displacements. Then the stresses $\{\sigma\}$ in all of the elements are calculated from the nodal displacements through a stress transformation matrix $[m]$

$$\{\sigma\} = [m] \cdot \{x\}. \tag{2}$$

From the stresses obtained by the finite element method, the normal stresses and the shear stresses along the failure surface can be calculated.

Consider an assumed failure surface in a pit slope structure, figure 1*A*, along which the stresses, σ_x, σ_y, τ_{xy} are calculated by the finite element method. The directions of these stresses at a point A are shown in figure 1*B*, together with the components of normal stress, σ_n, and shear stress, τ_{nm}, with respect to the tangent to the failure surface. The angle θ is the angle of the tangent from the y-axis or the angle of the normal from the x-axis. The normal stress and shear stress on the failure surface at point A can be calculated from the following two equations (8):

$$\sigma_n = 1/2(\sigma_x + \sigma_y) - 1/2(\sigma_x - \sigma_y) \cos 2\theta + \tau_{xy} \sin 2\theta. \tag{3}$$

$$\tau_{nm} = \tau_{xy} \cos 2\theta - 1/2(\sigma_x - \sigma_y) \sin 2\theta. \tag{4}$$

Because all the stresses along the failure surface are known from the finite element method, the normal and shear stresses at every point along the failure surface can be calculated from equations 3 and 4. From calculated normal stresses, the shear resistance, S_R, at all points may be obtained from the Coulomb equation

$$S_R = C + \sigma_n \tan \Phi, \tag{5}$$

in which C is the cohesion and Φ is the coefficient of internal friction. The total shear resistance (strength) and total shear force can be found by integrating the shear resistances and shear stresses at all points along the failure surface. The factor of safety is

$$\text{factor of safety} = \frac{\int (C + \sigma_n \tan \Phi)\,dl}{\int \tau_{nm}\,dl}, \qquad (6)$$

in which dl is an incremental length.

In the computer program, the failure surface is divided into a number of segments of equal length. Each segment consists of two end points, A and B, as shown in figure 2. The angle θ is the angle between the line of the segment AB and a line parallel to the vertical axis. The stresses (horizontal, vertical, and shear) at the midpoint, M, of each segment are found by interpolating the stresses of the surrounding elements. The normal and the shear stresses for each segment are then calculated from the stresses at its midpoint with the angle θ of the segment.

ANALYSIS OF A PIT SLOPE STRUCTURE

A pit slope structure with a slope of 2:1 was chosen for this analysis. Figure 3 shows the structure with the assumed failure surface of circular shape with the radius twice the height of the slope. About 600 to 800

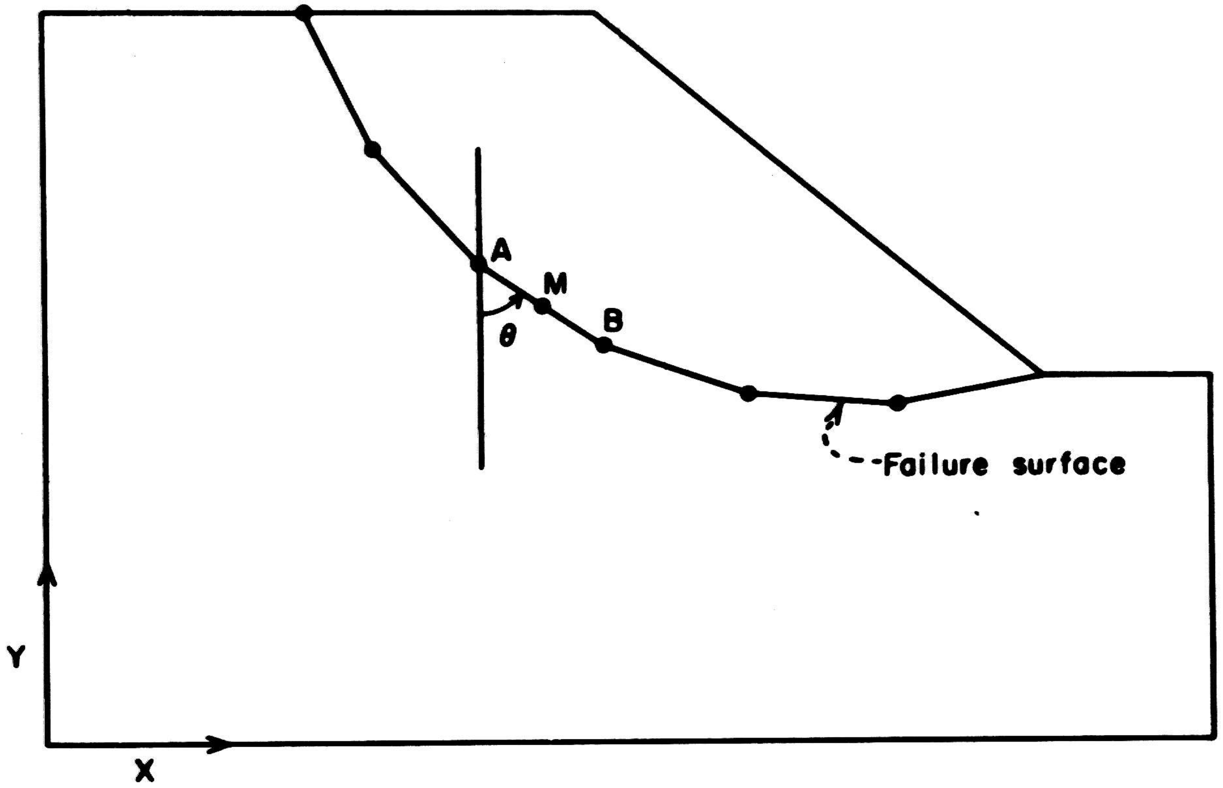

FIGURE 2. - Failure Surface With Its Segments.

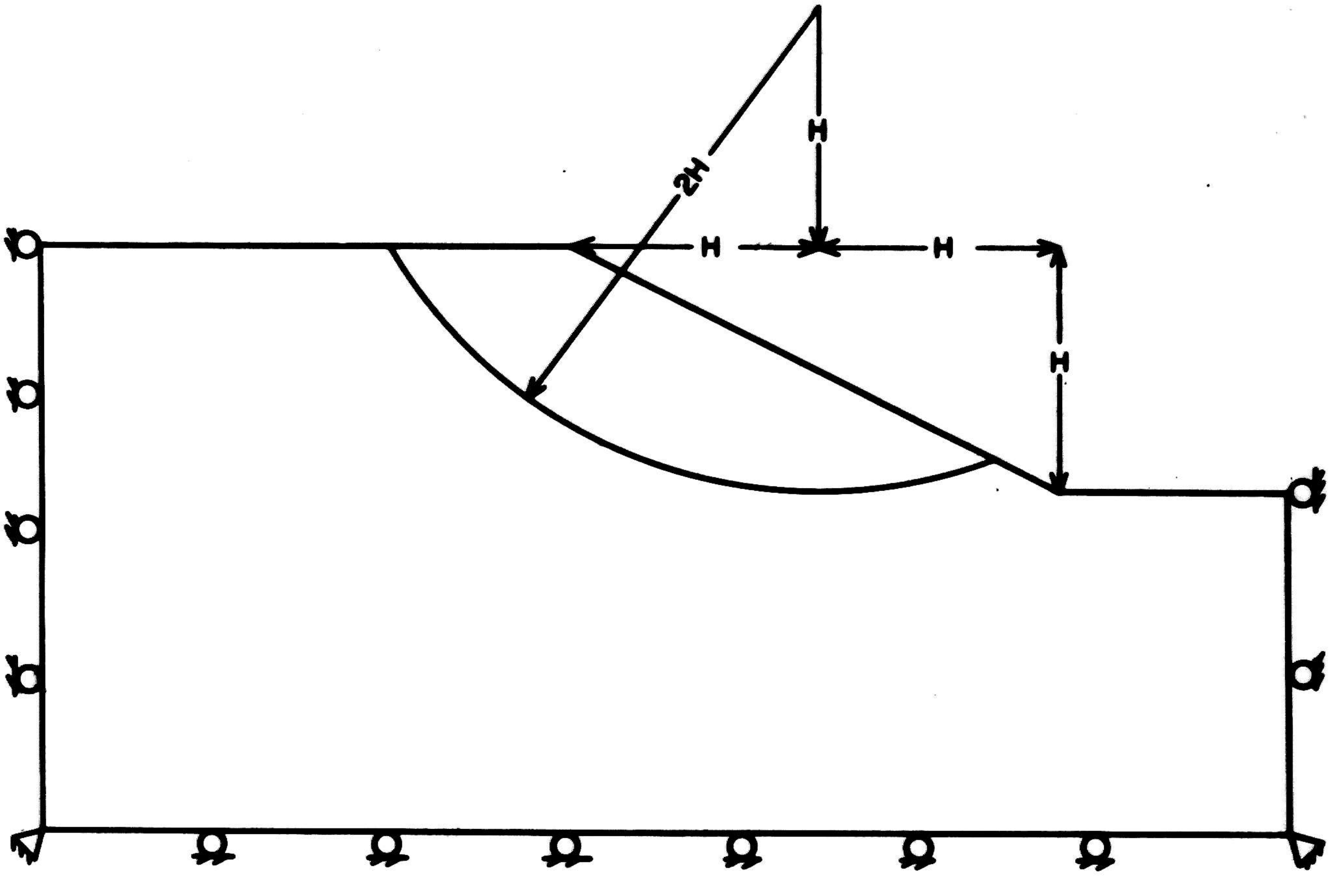

FIGURE 3. - Pit Slope Structure Showing Failure Surface and Boundary Conditions.

elements were used in the finite element model, and 50 segments were used along the failure surface for every case of analysis. Tests were made in regard to the variation of the safety factor with the number of elements in the model and with the number of segments on the failure surface. There was no change in the factor of safety when the number of elements was more than 500 and the number of segments was more than 30.

The structure is loaded by gravity force, with the boundary on both sides constrained in the horizontal direction and the bottom boundary constrained in the vertical direction, as shown in figure 3.

The pit slope structure was assumed to have three different types of material compositions (fig. 4): (1) Homogeneous--the elastic properties and the shear strength properties (cohesion and friction) are uniform through the whole structure, figure 4*A*, (2) nonhomogeneous--the structure is comprised of horizontal layers of uniform thickness with different materials in the layers, figure 4*B*, and (3) nonhomogeneous--the structure has layers of nonuniform thickness and different material properties, figure 4*C*.

Effect of Elastic Constants in a Homogeneous Pit Structure

Pit slopes are solid structures which have small deformations (6), and therefore the elastic modulus does not affect the stress distribution. That is, the elastic modulus of a homogeneous pit structure does not affect the factor of safety, which is calculated from the normal and shear stresses. Two test analyses, one with a modulus of 2.0 × 10^6 psi and the other with 6.0 × 10^6 psi, were performed with other material properties (ν = 0.2, w = 144 lb/ft^3, C = 60 psi, and ϕ = 20°) identical for both analyses. The factors of safety obtained were the same, 2.022, for both cases.

The effect of Poisson's ratio was investigated by performing three analyses with Poisson's ratios of 0.2, 0.3, and 0.4. The other material properties were kept the same for the three analyses, as shown in table 1. The factor of safety increases with increasing Poisson's ratio, but there is only a 0.6-pct change of the factor of safety within the range of Poisson's ratio of 0.2 to 0.4.

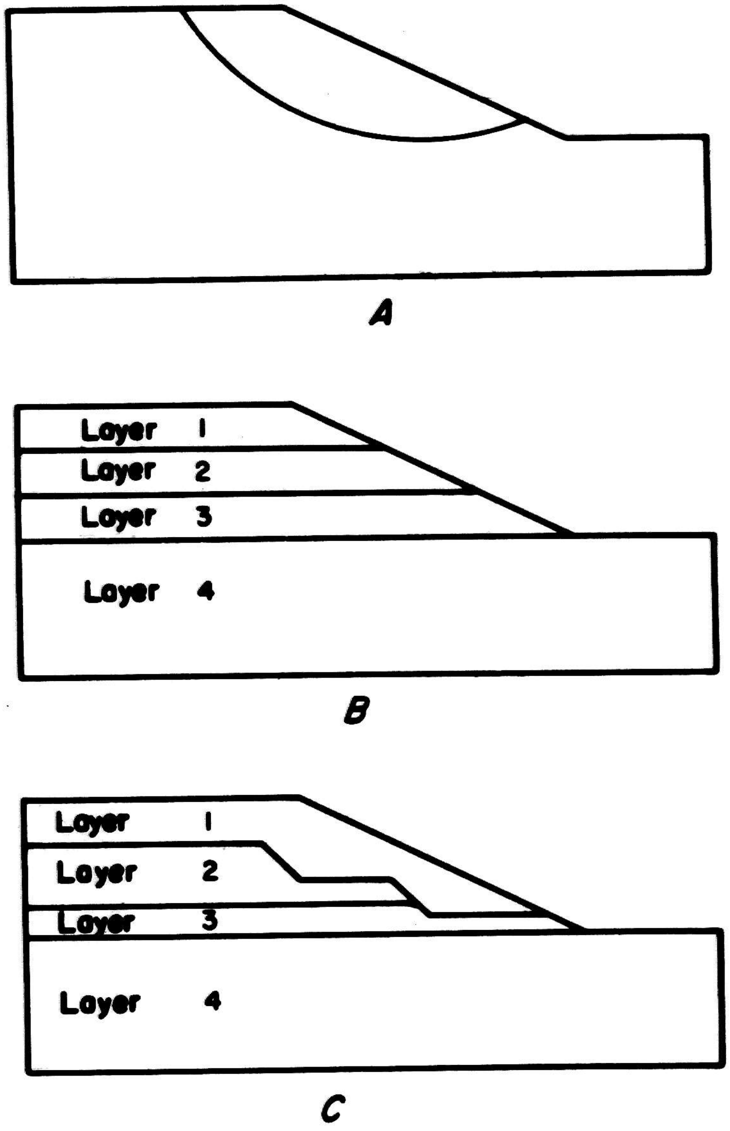

FIGURE 4. - Pit Slope Structures of Layered Material.

TABLE 1. - *Effect of Poisson's ratio, single layer of material*

Analysis	Modulus of elasticity, 10^6 psi	Poisson's ratio	Density, lb/ft^3	Cohesion, psi	Angle of friction, degrees	Factor of safety
1..........	2.0	0.2	144	60	20	2.022
2..........	2.0	.3	144	60	20	2.028
3..........	2.0	.4	144	60	20	2.034

Effect of Layered Material of Uniform Thickness

Three series of analyses were performed to find the effect on the safety factor of layers of unlike material.

In the first series, shown in table 2, density, cohesion, and friction were kept constant through the structure, but the layers had different elastic properties. Analyses were performed for four different combinations of layers. In analyses 4 and 5 the entire structure had the same elastic modulus, but in one there was a decreasing trend of Poisson's ratio from the top layer to the lower layer, in the other an increasing trend. In the third analysis the elastic modulus was different from layer to layer, with Poisson's ratio constant. In the fourth analysis both the elastic modulus and Poisson's ratio varied from layer to layer. For these four analyses, the factor of safety ranged from 2.010 to 2.031; that is, there was a 1-pct variation of factor of safety due to the change of elastic properties in the layers.

TABLE 2. - *Effect of layers with different elastic constants*

(Density, 144 lb/ft^3; cohesion, 60 psi; angle of friction, 20°)

Analysis	Layer	Modulus of elasticity, 10^6 psi	Poisson's ratio	Factor of safety
4................	1	2.0	0.4	2.031
	2	2.0	.3	
	3	2.0	.2	
	4	2.0	.2	
5................	1	2.0	.2	2.025
	2	2.0	.3	
	3	2.0	.4	
	4	2.0	.4	
6................	1	.5	.3	2.012
	2	1.0	.3	
	3	2.0	.3	
	4	2.0	.3	
7................	1	.25	.4	2.010
	2	.50	.3	
	3	1.00	.25	
	4	2.00	.20	

In the second series, shown in table 3, the cohesion and friction were constant and the density was varied from 110 to 150 lb/ft^3 from the top layer to the bottom layer. Seven analyses were performed. Analyses 8, 9, and 10 consider the effect on a homogeneous model of change in Poisson's ratio. Analyses 11 and 12 consider the effect of layers having different values of Poisson's ratio. In analysis 13, the layers had different elastic moduli but equal Poisson's ratios. In analysis 14, both the modulus and Poisson's ratio varied from layer to layer. There was less than a 1-pct variation in the factors of safety in the second series of analysis, which ranged from 2.247 to 2.267.

TABLE 3. - *Effect of layers with different elastic constants and densities*

(Cohesion, 60 psi; angle of friction, 20°)

Analysis	Layer	Modulus of elasticity, 10^6 psi	Poisson's ratio	Density, lb/ft^3	Factor of safety
8..............	1	2.0	0.2	110	2.256
	2	2.0	.2	130	
	3	2.0	.2	140	
	4	2.0	.2	150	
9..............	1	2.0	.3	110	2.261
	2	2.0	.3	130	
	3	2.0	.3	140	
	4	2.0	.3	150	
10..............	1	2.0	.4	110	2.265
	2	2.0	.4	130	
	3	2.0	.4	140	
	4	2.0	.4	150	
11..............	1	2.0	.40	110	2.267
	2	2.0	.30	130	
	3	2.0	.25	140	
	4	2.0	.20	150	
12..............	1	2.0	.2	110	2.257
	2	2.0	.3	130	
	3	2.0	.4	140	
	4	2.0	.4	150	
13..............	1	.5	.3	110	2.247
	2	1.0	.3	130	
	3	2.0	.3	140	
	4	2.0	.3	150	
14..............	1	.5	.4	110	2.250
	2	1.0	.3	130	
	3	2.0	.25	140	
	4	2.0	.20	150	

In the third series, four analyses were performed, the layers having different values of density, cohesion, and friction, as shown in table 4. For analyses 15 and 16, the model had homogeneous elastic properties, but different Poisson's ratios. For analyses 17 and 18, the layers had different elastic properties. In this third series of analyses the factors of safety range from 2.827 to 2.862 (1.3 pct variation).

TABLE 4. - *Effect of layers with different properties*

Analysis	Layer	Modulus of elasticity, 10^6 psi	Poisson's ratio	Density, lb/ft^3	Cohesion, psi	Angle of friction, degrees	Factor of safety
15........	1	2.0	0.25	110	40	20	2.849
	2	2.0	.25	130	50	25	
	3	2.0	.25	140	60	30	
	4	2.0	.25	150	60	30	
16........	1	2.0	.40	110	40	20	2.862
	2	2.0	.40	130	50	25	
	3	2.0	.40	140	60	30	
	4	2.0	.40	150	60	30	
17........	1	.5	.40	110	40	20	2.840
	2	1.0	.30	130	50	25	
	3	2.0	.25	140	60	30	
	4	2.0	.20	150	60	30	
18........	1	.5	.20	110	40	20	2.827
	2	1.0	.25	130	50	25	
	3	2.0	.30	140	60	30	
	4	2.0	.40	150	60	30	

Within two groups of analyses, analyses 8, 9, and 10 as one group and analyses 15 and 16 as another group, the factor of safety increases with increasing Poisson's ratio as was found in the homogeneous case. Neither the effect of elastic properties, nor the combined effect of modulus and Poisson's ratio, on the factor of safety, is over 1.3 pct for the pit structure with either homogeneous or nonhomogeneous material properties.

Effect of Layers of Irregular Shape

The structure with layers of nonuniform thickness, as shown in figure 4*C*, was analyzed to find the effect of layer geometry on the factor of safety. Analyses 19 through 23 were obtained with the same layer density, cohesion, and friction; analysis 24 was performed with a different set of layer friction values (table 5). Homogeneous elastic properties were assigned to the material in analyses 19 and 20; the models had the same modulus but different Poisson's ratios. In analysis 21, the layers had different moduli but constant Poisson's ratio. In analysis 22, the moduli were constant but Poisson's ratio varied from layer to layer. For these five models, in which corresponding layers had the same density, cohesion, and friction, the factors of safety range from 1.519 to 1.577 owing to the variation of the elastic properties; this is a 3.8-pct variation.

Analysis 24, in which cohesion and layer friction values differed from those in analyses 19 through 23, had a factor of safety of 2.222.

TABLE 5. - *Effect of layers of irregular shape*

Analysis	Layer	Modulus of elasticity, 10^6 psi	Poisson's ratio	Density, lb/ft^3	Cohesion, psi	Angle of friction, degrees	Factor of safety
19........	1	2.0	0.4	128	0	35	1.566
	2	2.0	.4	125	0	33	
	3	2.0	.4	110	150	20	
	4	2.0	.4	110	150	20	
20........	1	2.0	.2	128	0	35	1.549
	2	2.0	.2	125	0	33	
	3	2.0	.2	110	150	20	
	4	2.0	.2	110	150	20	
21........	1	.5	.3	128	0	35	1.519
	2	1.0	.3	125	0	33	
	3	1.5	.3	110	150	20	
	4	2.0	.3	110	150	20	
22........	1	2.0	.4	128	0	35	1.577
	2	2.0	.3	125	0	33	
	3	2.0	.2	110	150	20	
	4	2.0	.2	110	150	20	
23........	1	.5	.4	128	0	35	1.531
	2	1.0	.3	125	0	33	
	3	1.5	.2	110	150	20	
	4	2.0	.2	110	150	20	
24........	1	2.0	.4	110	150	20	2.222
	2	2.0	.4	125	0	33	
	3	2.0	.4	128	0	35	
	4	2.0	.4	128	0	35	

Structure Containing a Water Table

Pore-water pressure in pit slope structure may be converted to nodal-point forces in the finite element method of stress analysis. The magnitude of the nodal-point force representing the water pressure may be obtained from the magnitude of the equipotential line which passes through the nodal point, and the direction of the nodal-point force may be found from the flow net. The equipotential line and the flow net may be obtained by a finite element analysis of fluid flow through porous media (11), by an analytical method (12), or by a graphical method (12). With these pore-water pressures entered in the finite element stress analysis by means of nodal-point forces, the normal and shear stresses in equation 6 include the effect of water pressure.

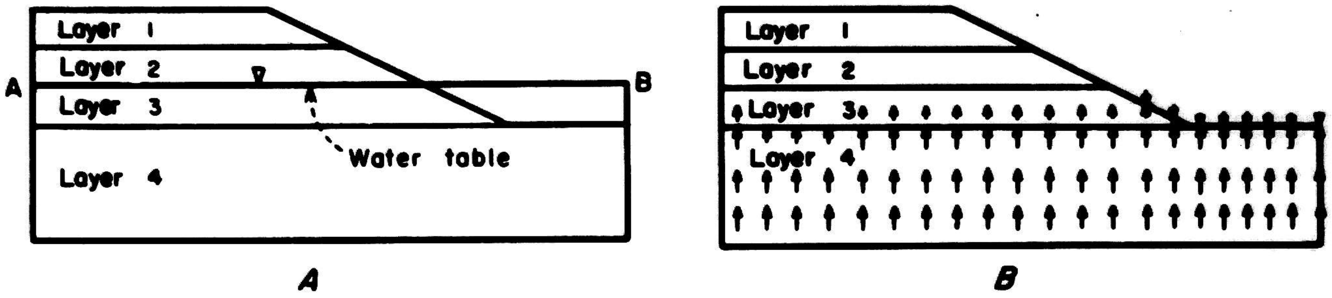

FIGURE 5. - Pit Slope With Water Table.

An example is given here to illustrate this procedure. As shown in figure 5*A*, the pit structure is partially submerged, with the water table indicated by AB. The water pore pressures at various depths are represented by upward nodal-point forces, and the water pressure at the toe due to the water standing in the pit is represented by downward nodal-point forces as shown in figure 5*B*. With the material properties shown in analysis 25, table 6, the factor of safety is 1.536. When the water pressure at the toe is not applied (no water standing in the pit), the factor of safety is 1.508, as shown in analysis 26.

TABLE 6. - *Results from structure with water table*

(Modulus of elasticity, 0.5 × 10^6 psi;
Poisson's ratio, 0.4)

Analysis	Layer	Density, lb/ft^3	Cohesion, psi	Angle of friction, degrees	Factor of safety
25........	1	144	60	20	1.536
	2	144	60	20	
	3	160	40	15	
	4	160	40	15	
26........	1	144	60	20	1.508
	2	144	60	20	
	3	160	40	15	
	4	160	40	15	

STRESSES ALONG A FAILURE SURFACE

Figure 6 shows the normal and shear stresses on the failure surface for analyses 1 and 3. For plotting purposes all points on the failure surface have been projected onto the x-axis. The analyses had the same modulus of elasticity but different Poisson's ratios (ν). The normal stress has a bell-shaped distribution, as shown in figure 6*A*. The maximum values occur at about the midpoint. The closeness of the two curves shows that the influence of Poisson's ratio on the normal stress distribution is not significant. Variation of Poisson's ratio, however, has a significant effect on the shear stress distribution along the failure surface.

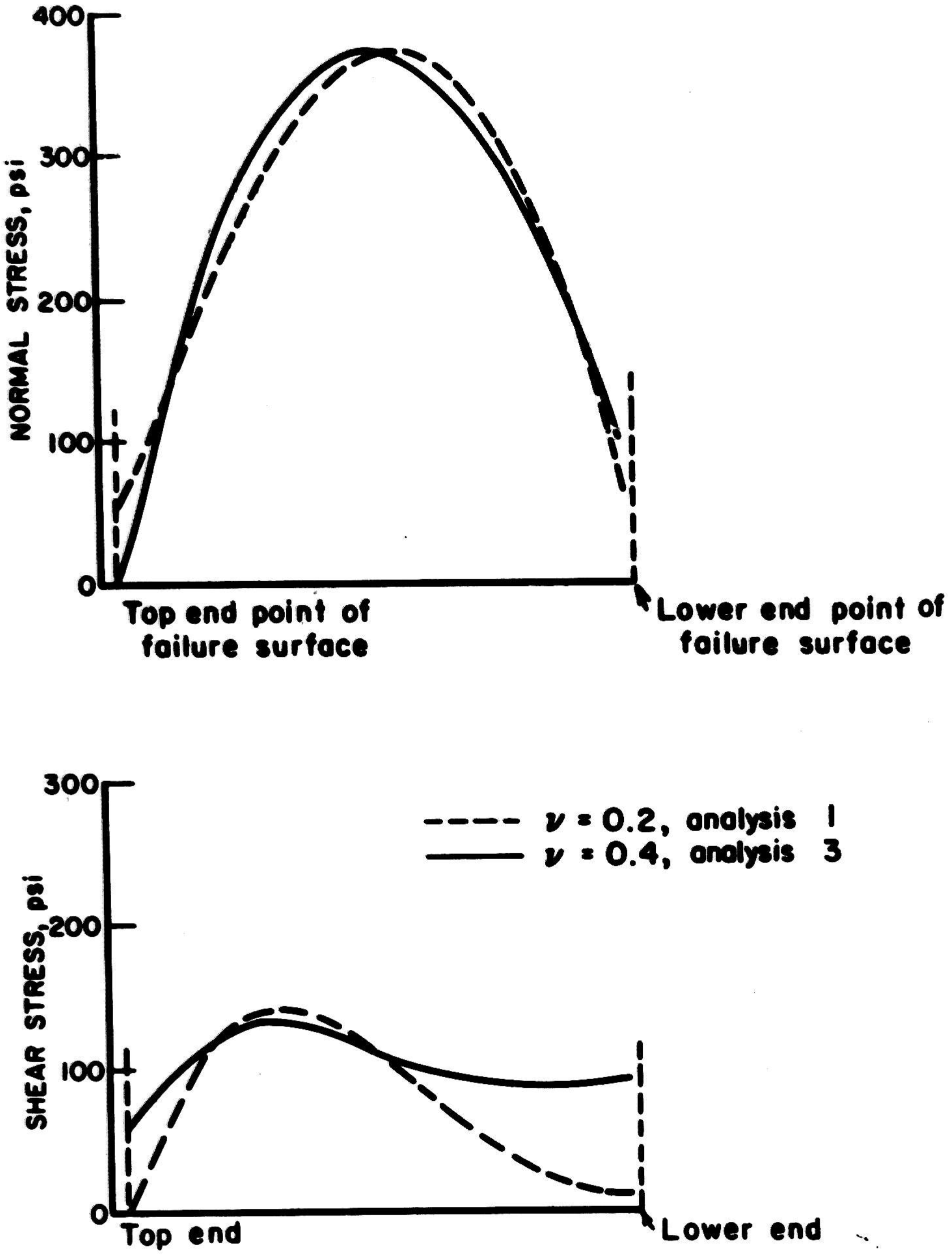

FIGURE 6. - Stresses Along a Failure Surface.

Despite the influence of Poisson's ratio on the normal and shear stress distribution, the variation in factor of safety due to this effect is small; the factor was 2.022 and 2.034, for analyses 1 and 3, respectively.

COMPARISON OF RESULTS WITH MORGENSTERN-PRICE METHOD

The Morgenstern-Price method with side-force function f(x) = 1 was selected to find the factor of safety of the pit slopes with the same geometry, density, cohesion, and angle of friction as the pit slopes in the previous analyses. The computer program given in Bailey's thesis (1) was used to calculate the factor of safety. The factors of safety for the structures treated in analyses 1 to 26 are summarized in table 7, together with the results obtained by the Morgenstern-Price method for the structures with the same density, cohesion, and friction.

It can be seen that the Morgenstern-Price method with side-force function f(x) = 1 gives a slightly higher factor of safety than the finite element method.

TABLE 7. - *Comparison of results with Morgenstern-Price method*

Case	Analyses	Structure	Layer	Density, lb/ft^3	Cohesion, psi	Angle of friction, degrees	Factor of safety	
							Finite element method	Morgenstern method
1	1-7	Constant density, cohesion, friction.	1	144	60	20	2.010-2.034	2.041
2	8-14	Layered uniform thickness.	1	110	60	20	2.247-2.267	2.278
			2	130	60	20		
			3	140	60	20		
			4	150	60	20		
3	15-18	....do.....	1	110	40	20	2.827-2.862	2.910
			2	130	50	25		
			3	140	60	30		
			4	150	60	30		
4	19-23	Layered irregular shape.	1	128	0	35	1.519-1.577	1.574
			2	125	0	33		
			3	110	150	20		
			4	110	150	20		
5	24	....do.....	1	110	150	20	2.222	2.225
			2	125	0	33		
			3	128	0	35		
			4	128	0	35		
6	25-26	With water table.	1	144	60	20	1.508-1.549	1.602
			2	144	60	20		
			3	160	40	15		
			4	160	40	15		

DISCUSSION

A structure with a higher value of homogeneous Poisson's ratio gives a slightly higher factor of safety, whether the structure has a homogeneous or a nonhomogeneous modulus. When two structures have the same modulus, the one with Poisson's ratio decreasing from the top down gives a higher factor of safety than the one with Poisson's ratio increasing from the top down. These facts indicate the factor of safety is dependent on Poisson's ratio. Modulus of elasticity does not affect the factor of safety for a structure which has homogeneous modulus, but comparisons such as analysis 2 versus analysis 6 and analysis 9 versus analysis 13 show that the factor of safety depends on the moduli when the structure contains layers of different moduli.

It is concluded that the elastic properties do influence the factor of safety, although the effect is small. For a pit structure with homogeneous elastic properties, the effect on the factor of safety is less than 0.6 pct; with layered material of uniform thickness, it is less than 1.3 pct; and with layered material of nonuniform thickness, it is 3.8 pct. These examples suggest that (1) for a structure of homogeneous material, arbitrary elastic properties may be assigned without causing significant error in the factor of safety, (2) for a structure of layered material of uniform thickness, the elastic properties of the pit should be included if maximum accuracy of the factor of safety is desired, and (3) for a pit structure with layers of nonuniform thickness or with materials of peculiar shapes and of distinct variation of elastic properties, the elastic properties must be included in the analysis. The requirement for information as to the elastic properties of the rock material for the finite element method of analysis increases the need for testing to determine material parameters, as compared with the method of slices. However, the accuracy in determining the elastic material properties appears not to be critical.

This investigation indicates that ignoring the elastic properties will cause error in the evaluation of the factor of safety by the method of slices.

As to the submerged structure, the finite element method is more logical than the method of slices since the flow characteristic can be included in the analysis. Also, the static water pressure on the outside boundary of the structure may be included, as shown in analysis 25. Without the static water pressure acting at the toe area, as in analysis 26, about 2 pct error in safety factor exists as compared with the previous analysis.

The time required for setting up the model for this analysis and that needed for analysis by the Morgenstern-Price method are about the same. As to the time required for obtaining a set of solutions on the computer, the Morgenstern-Price method takes about 20 to 30 seconds, and the finite element method takes about 2 to 3 minutes. However, the finite element method yields more information; namely, the stress distribution and the displacement field, in addition to the factor of safety.

A failure surface of arbitrary shape is included in this finite element method of analysis; the safety factor is easily evaluated along different paths. Cracks at the crest of the pit can be included in the finite element model. Arbitrary loading, from the surface structure, tectonic forces, or seismic forces, which is difficult to apply in the method of slices, can be easily incorporated by applying equivalent forces at the nodal points.

Elastic stress analysis was used in this investigation. For material having elastic-plastic, nonlinear, or viscoelastic properties, the stress analysis may be achieved by other finite element methods of analysis (5, 7, 10). Then the factor of safety can be obtained as an index of stability in the manner described in this report.

CONCLUSIONS

The finite element method of stress analysis and the method of limiting equilibrium mechanics for the analysis of pit slope stability are presented in this report. The stress is based on linear elastic stress-strain relations for homogeneous and nonhomogeneous materials. The factor of safety is found to be dependent on the elastic properties which are not included in the method of slices.

For a pit slope structure that consists of nonhomogeneous material of uniform or nonuniform thickness, the elastic material properties must be found and included in the analysis. For a pit slope structure of homogeneous material properties, the elastic properties may be assigned arbitrarily without resulting in significant error.

In the case of a pit slope structure including a water table, it is preferable to use the method of analysis given herein since the flow characteristic and the static water pressure due to water standing in the pit may be included.

This method of stability analysis has the potential of including nonlinear or viscoelastic material properties if the finite element method of nonlinear or viscoelastic stress analysis is employed in place of the elastic stress analysis.

This method of analysis is much more versatile than the method of slices since both gravity and arbitrary loading are included and no judgment is required on the part of the user to choose the side-force function as in the Morgenstern-Price method. It is also a much more nearly exact solution than any method of slices because elastic stress analysis by the finite element method is used in the analysis.

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