Response to "Lorentz transformation of a system carrying 'Hidden Momentum,'" by E. Comay [Am. J. Phys. 68, 1007 (2000)]

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We respond to Comay's criticism of the use of covariant definitions of the electromagnetic and mechanical energy–momenta in an analysis of the role of hidden momentum in the total energy–momentum four vector of a macroscopic body. © *2000 American Association of Physics Teachers.*

In his paper,[1] Comay calls the use in Ref. 2 of covariant definitions of the "electromagnetic" and "mechanical" contributions to the total energy and momentum of a macroscopic body "mistakes." He illustrates in detail, using the examples of a free electromagnetic wave, a charged capacitor, and a current-carrying solenoid inside a charged capacitor, the well-known fact that the energy and momentum of the electromagnetic field in a system (i.e., the "electromagnetic" energy–momentum) form a covariant four vector only when the four divergence of the field's energy–momentum tensor vanishes. Similarly, he shows that the hidden mechanical momentum in his third example, obtained as the integral of the simultaneous values of the momentum-density component of the energy–momentum tensor of the charged fluid that is the current carrier in the system, does not transform as the momentum part of an energy–momentum four vector. Here again, the reason for this is that the four divergence of the fluid's energy–momentum tensor can be shown not to vanish. Only the total energy–momentum of the system, i.e., the sum of the electromagnetic and mechanical contributions, is a covariant four vector, as the four divergence of the total energy–momentum tensor of a closed system vanishes.[3]

This is shown also in Ref. 2, where detailed calculations on the examples of finite exactly solvable systems[4] illustrate the fact that only the total energy–momentum is a covariant four vector when the standard definitions are used for the electromagnetic and mechanical contributions. And it is shown in Ref. 5, without relying on any specific example, how the noncovariant "electromagnetic" and "mechanical" energy–momenta calculated using the standard energy–momentum tensors combine to form a covariant four vector of the total energy–momentum of a macroscopic body that carries general stationary macroscopic charge and/or current distributions.

The employment of definitions that separately impose the relativistic four-vector covariance on the electromagnetic and nonelectromagnetic ("mechanical") energy–momenta has been pioneered by Rohrlich[6] as a procedure that needs no explicit consideration of nonelectromagnetic forces to deal with the problem of the noncovariance of the energy–momentum of the electron in classical electron theory. The use of this procedure has been criticized on numerous occasions by proponents of the standard definitions, as these are sufficient to produce a covariant total energy–momentum when the contribution of nonelectromagnetic forces, necessary for the stability of an extended particle, to the energy–momentum of the classical electron is included. (That has been shown already by Poincaré,[7] and the nonelectromagnetic stresses needed for the stability of an extended charged

particle are usually called Poincaré stresses.) Comay's criticism is a relapse into yet another round of that old debate,[8] in regard to which it has now been recognized by several authors[9] that the point here is not that only one of the procedures of Rohrlich's covariant definitions and Poincaré stresses is a "correct" one while the other is "wrong," but that either procedure can be used for the purpose of constructing a covariant *total* energy–momentum of a system that has electromagnetic and nonelectromagnetic components. The recent third edition of Jackson's classic text on classical electrodynamics[10] has a thorough discussion of the problem of the electromagnetic mass, Poincaré stresses, and covariant definitions without expressing any preference for one of the two procedures over the other.

We reiterate here the point emphasized in Refs. 2 and 5, namely that the covariant definitions have a formal character akin to the procedure of the renormalization of mass in quantum electrodynamics, which is carried out in a covariant fashion separately from any nonelectromagnetic contribution to the rest mass of a charged particle. (In fact, it can be argued that a mass renormalization with a negative nonelectromagnetic mass is implied also in classical electrodynamics whenever a charged body is assigned a rest mass that is smaller than the electromagnetic mass due to the body's charge distribution.) In principle, the electromagnetic energy–momentum arising from a macroscopic distribution of charge and current is measurable separately from the body's "mechanical" energy–momentum (unlike in the electron or any other "elementary" particle), and the standard electromagnetic and mechanical energy–momenta would agree observationally with the covariantly defined quantities only in one inertial frame of reference, namely the reference frame in which the standard and covariant definitions coincide.

The purpose of a covariant-definition procedure for a macroscopic system is that of the construction of a covariant *total* energy–momentum, and the separately covariant "electromagnetic" and "mechanical" energy–momenta obtained in such a procedure serve only that purpose. The covariant definitions of the electromagnetic and mechanical energy–momenta were used in Ref. 2 not because they are the only "correct" definitions of such quantities for the systems in question, but to show how the covariant-definition procedure, along with the Poincaré stresses procedure of the standard definitions, would consistently take into account the existence of hidden mechanical momentum.

[1] E. Comay, "Lorentz transformation of a system carrying 'Hidden Momentum,'" Am. J. Phys. **68**, 1007 (2000).

[2] V. Hnizdo, "Hidden momentum and the electromagnetic mass of a charge

and current carrying body," Am. J. Phys. 65, 55–65 (1997).

[3]Comay cites in detail from the proof of this statement in L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Pergamon, Oxford, 1975). There are several such proofs in the literature, the oldest going back some 80 years: H. Weyl, *Space-Time-Matter* (Dover, New York, 1950), first American printing of the 4th edition of 1922, Sec. 33; W. Pauli, *Theory of Relativity* (Pergamon, London, 1958), Sec. 21; C. Møller, *The Theory of Relativity* (Clarendon, Oxford, 1972), 2nd ed., Sec. 6.2; C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973), Sec. 5.8, case (c).

[4]Comay's examples are systems of infinite extension, and also of infinite energy and, except in his second example, infinite momentum. While the use of such systems simplifies the requisite integrations, infinite systems are, strictly speaking, unphysical; moreover, the general theorem that guarantees that the total momentum of a stationary macroscopic system vanishes [see, e.g., L. Vaidman, "Torque and force on a magnetic dipole," Am. J. Phys. 58, 978–983 (1990); V. Hnizdo, "Hidden mechanical momentum and the field momentum in stationary electromagnetic and gravitational systems," *ibid.* 65, 515–518 (1997)] cannot be applied to an infinite system. Presumably, the assumption here is that when the systems Comay considers are finite, the relative contribution of the fringing fields (and of the wave-packet "tails" in the case of a free electromagnetic wave) to the quantities of interest can be shown to be arbitrarily small when suitable dimensions of the systems are sufficiently large.

[5]V. Hnizdo, "Covariance of the total energy-momentum four vector of a charge and current carrying macroscopic body," Am. J. Phys. 66, 414–418 (1998).

[6]F. Rohrlich, "Self-energy and stability of the classical electron," Am. J. Phys. 28, 639–643 (1960); "Electromagnetic momentum, energy, and mass," *ibid.* 38, 1310–1316 (1970); *Classical Charged Particles* (Addison–Wesley, Reading, MA, 1965 and 1990).

[7]There is an English, modernized presentation of Poincaré's 1906 paper on the electron by H. M. Schwartz, "Poincaré's Rendiconti paper on relativity. I," Am. J. Phys. 39, 1287–1294 (1971); "II," 40, 862–872 (1972); "III," 40, 1282–1287 (1972).

[8]Reference 5 gives several references to the debate, both in the context of classical electron theory and the Trouton–Noble experiment (on the latter, see S. A. Teukolsky, Ref. 9).

[9]D. J. Griffiths and R. E. Owen, "Mass renormalization in classical electrodynamics," Am. J. Phys. 51, 1120–1126 (1983); S. A. Teukolsky, "The explanation of the Trouton–Noble experiment revisited," *ibid.* 64, 1104–1109 (1996); F. Rohrlich, "The dynamics of a charged sphere and the electron," *ibid.* 65, 1051–1056 (1997).

[10]J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1999), 3rd ed., Secs. 16.4–16.6.