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The purpose of this article is to compare six procedures for comparing the estimated mean of data drawn from a lognormal distribution to a long-term average occupational exposure limit (LTA OEL): the standard *t*-test, the American Industrial Hygiene Association's (AIHA's) mean test, the modified Cox mean test, Rappaport and Selvin's mean test, Lyles and Kupper's mean test, and Land's mean test (a procedure based on Land's exact confidence intervals). In principle, all of these procedures, with one exception, can be used as either an employer's test or an inspector's test. Computer simulation was used to determine (1) the actual confidence level for each procedure for the situation where exposures are lognormally distributed and the true mean equals an LTA OEL, and (2) the power of each procedure when the true mean is different from the LTA OEL. Land's mean test consistently provided confidence levels near the nominal confidence level for all sample sizes and geometric standard deviations (GSDs) evaluated. Furthermore, at any given true long-term average above the LTA OEL the alternative procedures were, in general, more likely to result in the false conclusion that the work environment was acceptable. For the employer's test, Lyles and Kupper's test closely approximated Land's test. Regarding the inspector's test, Rappaport and Selvin's mean test and the AIHA mean test came closest to Land's test. The author recommends Land's mean test because it delivers the advertised confidence level regardless of sample size and underlying GSD, and for most situations it is the more powerful test for detecting true mean exposures greater than the LTA OEL. Hewett, P.: Mean Testing: II. Comparison of Several Alternative Procedures. Appl. Occup. Environ. Hyg. 12(5):347–355; 1997. Published 1997 by AIH.

Typical workplace exposure data are skewed toward the right and are usually adequately described by the two-parameter lognormal distribution. The mean of measurements drawn from such distributions also exhibits a skewed distribution, particularly for small sample sizes and large geometric standard deviations (GSDs). Consequently, the upper and lower confidence limits (UCL and LCL, respectively) will not be symmetric about the estimated mean. Several procedures have been proposed that allow the calculation of asymmetric confidence limits and, at least conceptually, can be used to compare an estimate of the true mean to a long-term average occupational exposure limit (LTA OEL):

1. the American Industrial Hygiene Association (AIHA) mean test[1]
2. the modified Cox mean test[2]
3. Rappaport and Selvin's mean test[3]
4. Lyles and Kupper's mean test[4]
5. Land's mean test (a procedure based on Land's procedure[5] for calculating exact confidence limits for lognormally distributed data).

The objective of this article is to present the results of a computer simulation of the operating characteristic curves for the above mean tests and the simple *t*-test. A companion article[6] examines the concept of mean testing from the viewpoints of the employer, inspector, and employee.

Background

Several authors have discussed the concept of testing the estimated mean calculated from data drawn from a lognormal occupation exposure distribution against an LTA OEL.[1,3,4,7–10] Rappaport and Selvin,[3] in hopes that the Occupational Safety and Health Administration would promulgate a revised permissible exposure limit for benzene that would be defined as a limit on the long-term average exposure (as opposed to the usual daily time-weighted average), proposed a modified *t*-test procedure for mean testing.

Evans and Hawkins[10] compared, using computer simulation, the performance of several mean testing procedures: the simple *t*-test, a variation on Rappaport and Selvin's mean test, and a third alternative which they called the lognormal *t*-test. They concluded that their variation on Rappaport and Selvin's mean test was the best of the three alternatives, but that the actual confidence level of this test will be less than the nominal confidence level and that the actual power will be less than that reported by Rappaport and Selvin.

In 1991 the AIHA Exposure Assessment Strategies Committee[1] presented a mean testing procedure, based on a modified *t*-test, in their guide to occupational exposure assessment. Armstrong[2] evaluated the ability of several procedures to generate accurate confidence limits around the estimated mean for exposure data drawn from a lognormal distribution: the simple *t*-test, a procedure identical to the AIHA modified *t*-test, the Cox method, and a modification of the Cox method. Armstrong used the confidence limits computed using Land's exact confidence interval procedure[5] as a gold standard. He concluded that the "poor performance of the approximate methods . . . [for small sample sizes and GSDs larger than 2] suggests caution in their use," and in these instances recommended the use of Land's exact procedure. However, Armstrong did not test Land's exact confidence limits to determine if they are, indeed, exact. Since Land's exact procedure is laborious to compute, Armstrong recommended the modified Cox procedure as the best alternative procedure, particularly when estimating the UCL.

More recently Lyles and Kupper[4] proposed a new procedure for testing the UCL for the estimated mean of lognormally distributed data against a long-term OEL. They designed this procedure to closely mimic Land's procedure while being readily computable. Their computer simulations showed that the performance of their alternative procedure is similar to that of Land's mean test, but that Land's test is slightly preferable for sample sizes less than 10 or where the GSD is greater than 5.7.

Hypotheses

In those situations where a true LTA OEL exists (see Hewett[6] for a related discussion), it is logical to compare the estimated mean (for example, the simple arithmetic mean would be the average of n exposure measurements collected across multiple work shifts) to the LTA OEL. If the investigator wishes to demonstrate, with a specified level of confidence, that the true mean exposure (μ) is less than the LTA OEL, then the following hypotheses are appropriate:

H_o: $\mu \geq$ LTA OEL (unacceptable exposure condition)

H_a: $\mu <$ LTA OEL (acceptable exposure condition)

There are situations where the investigator may wish to demonstrate, with a specified level of confidence, that the true mean exposure is greater than the LTA OEL. For example, an inspector may wish to justify a citation for overexposures (relative to the LTA OEL) or a company industrial hygienist may wish to support, with compelling evidence, a recommendation for additional engineering controls. In either situation, the following hypotheses are appropriate:

H_o: $\mu \leq$ LTA OEL (acceptable exposure condition)

H_a: $\mu >$ LTA OEL (unacceptable exposure condition)

For the first set of hypotheses, which will be referred to as the employer's test, the investigator will conclude, with, say, 95 percent confidence, that the alternative hypothesis is the correct hypothesis if and only if the 95 percent UCL is less than the LTA OEL. For the second set of hypotheses, which, for lack of something better, will be referred to as the (company or government) inspector's test, the investigator will conclude, with, say, 95 percent confidence, that the alternative hypothesis is the correct hypothesis if and only if the 95 percent LCL is greater than the LTA OEL.

The variable of interest here is the arithmetic mean calculated from a collection (sample) of exposure measurements where it is assumed that the underlying distribution is lognormal. While the central limit theorem states that, with relatively few restrictions, the distribution of simple arithmetic means drawn from any distribution tends to approach a normal distribution as the sample size increases, the distribution of the sample means of size n drawn from a lognormal distribution is skewed for realistic sample sizes and typically encountered GSDs. Consequently, more than 50 percent of the sample means will be below the true arithmetic average of the distribution and the correct LCL and UCL will not be symmetric. Unfortunately, the correct LCL and UCL for the estimated mean for a data set drawn from a lognormal distribution cannot easily be determined, which is why various approximate procedures and one so-called exact procedure have been devised.

Methods

The questions to be addressed using computer simulation are (1) Does each procedure provide the nominal α-error when the true long-term mean equals the null hypothesis long-term mean? and (2) What is the power of each procedure to detect a specific long-term mean?

A computer simulation was used to generate artificial data sets containing n measurements (n = 5, 10, or 20) drawn from a lognormal distribution having a specific GSD (GSD = 1.5, 2, 3, or 4) and a specific true arithmetic mean. One-sided 95 percent confidence limits were calculated using the simple *t*-test procedure, the AIHA mean test,[1] the modified Cox mean test,[2] Rappaport and Selvin's mean test,[3] Lyles and Kupper's mean test[4] (UCL only), and Land's mean test.[5] Each of these procedures are described below. The resulting LCL and UCL for each procedure were compared to an LTA OEL of 1. This procedure was repeated 5000 times for each combination of test, sample size, GSD, and true mean. (True mean values ranged between 0.1 and 3 in increments of 0.05). The probability of concluding that the work environment is acceptable, P(acceptable), for the employer's or inspector's test was simply the fraction of the 5000 simulations where the UCL or LCL was less than the LTA OEL.

Simple t-Test Procedure

In principle, the simple *t*-test is used when the data are known to be normally distributed. However, this procedure is fairly robust (i.e., it works well for many nonnormal distributions, especially as the sample size increases). The procedure is as follows:

1. Calculate the sample (simple arithmetic) mean ($\bar{x}$) and sample standard deviation(s).
2. Calculate the UCL or LCL:

$$CL_{1-\alpha} = \bar{x} + t \cdot \frac{s}{\sqrt{n}}$$

where:

$t = t_{1-\alpha,n-1}$ for the UCL
$t = t_{\alpha,n-1}$ for the LCL

3. Compare the UCL or LCL to the LTA OEL.

AIHA Mean Test

The Exposure Assessment Strategies Committee[1] of the AIHA proposed an approximate procedure that is based on the reasonable assumption that the distribution of estimated means calculated using Equation 1 is similar in shape to the distribution of sample geometric means:

1. Calculate the sample mean ($\bar{y}$) and sample variance (s_y^2) of the log-transformed data, where y = ln(x).
2. Calculate an estimate of the true mean:

$$m = \exp\left(\bar{y} + \frac{1}{2} s_y^2\right) \quad (1)$$

3. Calculate the UCL or LCL:

$$CL_{1-\alpha} = \exp\left(\ln(m) + t\cdot\frac{s_y}{\sqrt{n}}\right) = m\cdot\exp\left(t\cdot\frac{s_y}{\sqrt{n}}\right)$$

where:

$t = t_{1-\alpha,n-1}$ for the UCL
$t = t_{\alpha,n-1}$ for the LCL

4. Compare the UCL or LCL to the LTA OEL.

Modified Cox Mean Test

The modified Cox interval was recommended by Armstrong[2] as an alternative to Land's exact procedure.

1. Calculate the sample mean ($\bar{y}$) and sample variance (s_y^2) of the log-transformed data, where $y = \ln(x)$.
2. Calculate an estimate of the true mean:

$$m = \exp\left[\bar{y} + \frac{1}{2}\left(\frac{n-1}{n}\right)s_y^2\right] \quad (2)$$

3. Estimate the standard deviation for the log-transformed estimated mean:

$$s^\star = \sqrt{\frac{s_y^2}{n} + \frac{s_y^4}{2(n-1)}}$$

4. Calculate the UCL or LCL:

$$CL_{1-\alpha} = \exp\left[\ln(m) + t\cdot s^\star\right] = m\cdot\exp\left(t\cdot s^\star\right)$$

where:

$t = t_{1-\alpha,n-1}$ for the UCL
$t = t_{\alpha,n-1}$ for the LCL

5. Compare the UCL or LCL to the LTA OEL.

Rappaport and Selvin's Mean Test

Rappaport and Selvin[3] proposed an approximate procedure where the variance of the distribution of estimated means is estimated using the sample GSD and one assumes that the true mean equals the LTA OEL:

1. Calculate the sample mean ($\bar{y}$) and sample variance (s_y^2) of the log-transformed data, where $y = \ln(x)$.
2. Using Equation 2, calculate an estimate of the true mean. (Rappaport and Selvin originally used Equation 1 to estimate the true arithmetic mean. For this simulation Equation 2, as recommended by Rappaport,[9] was used.)
3. Estimate the standard deviation for the estimated mean of data drawn from a lognormal distribution (assuming that the null hypothesis, H_o, is true):

$$s_m = \sqrt{\frac{(LTA)^2}{n-2}\left(s_y^2 + \frac{1}{2}s_y^4\right)}$$

4. Calculate T (a statistic expected to be distributed similar to a standard *t*-value):

$$T = \frac{m - LTA}{s_m}$$

5. If ($T > t_{1-\alpha,n-1}$), then conclude that the true mean concentration is greater than the LTA OEL. If ($T < t_{\alpha,n-1}$), then conclude that the true mean concentration is less than the LTA OEL.

Lyles and Kupper's Mean Test

Lyles and Kupper[4] proposed an alternative to Land's exact procedure. Their procedure can only be used for estimating the UCL.[11]

1. Calculate the sample mean ($\bar{y}$) and sample variance (s_y^2) of the log-transformed data, where $y = \ln(x)$.
2. Estimate a noncentrality parameter δ:

$$\hat{\delta} = \frac{-\sqrt{n}\cdot s_y}{2}$$

3. Estimate a constant C:

$$\hat{c}_{1-\alpha,n} = \frac{-\hat{\delta}\sqrt{\frac{n-1}{n}}}{\chi_{n-1,\alpha}} + \frac{t_{1-\alpha,n-1}}{\sqrt{n}}$$

($\chi_{n-1,\alpha}$ refers to the square root of a chi square value.)

4. Calculate the UCL:

$$UCL_{1-\alpha} = \exp\left(\bar{y} + \hat{c}_{1-\alpha,n}\cdot s_y\right)$$

5. Compare the UCL to the LTA OEL.

Land's Mean Test[5,12]

1. Using Equation 1, calculate an estimate of the true mean.
2. Estimate the standard deviation (s_y) of the log-transformed data.
3. Obtain a C-factor for the LCL and UCL: $C(s_y; n-1, \alpha)$ and $C(s_y; n-1, 1-\alpha)$, respectively. These factors can be obtained from Land's tables[5] or from Gilbert.[12] Interpolation between the tabular values of s_y is almost always required with each of Land's tables and sometimes between tables as well if the sample size is not one of those listed. (An alternative procedure where the C-factors are simply read from graphs is described by Hewett.[13]). For the computer simulation, the C-factors in Land's tables were accurately approximated using combinations of linear equations and fourth degree polynomials fit to the table values for each sample size.
4. Calculate the UCL and LCL:

$$CL_{1-\alpha} = \exp\left[\ln(m) + C\frac{s_y}{\sqrt{n-1}}\right]$$

where:

$C = C(s_y; n-1, 1-\alpha)$ for the UCL, where $\alpha = 0.05$
$= C(s_y; n-1, \alpha)$ for the LCL, where $\alpha = 0.05$.

5. Compare the UCL or LCL to the LTA OEL.

Simulation Results

Figures 1 through 4 show the operating characteristic curves derived from the simulation data. The curves on the left of

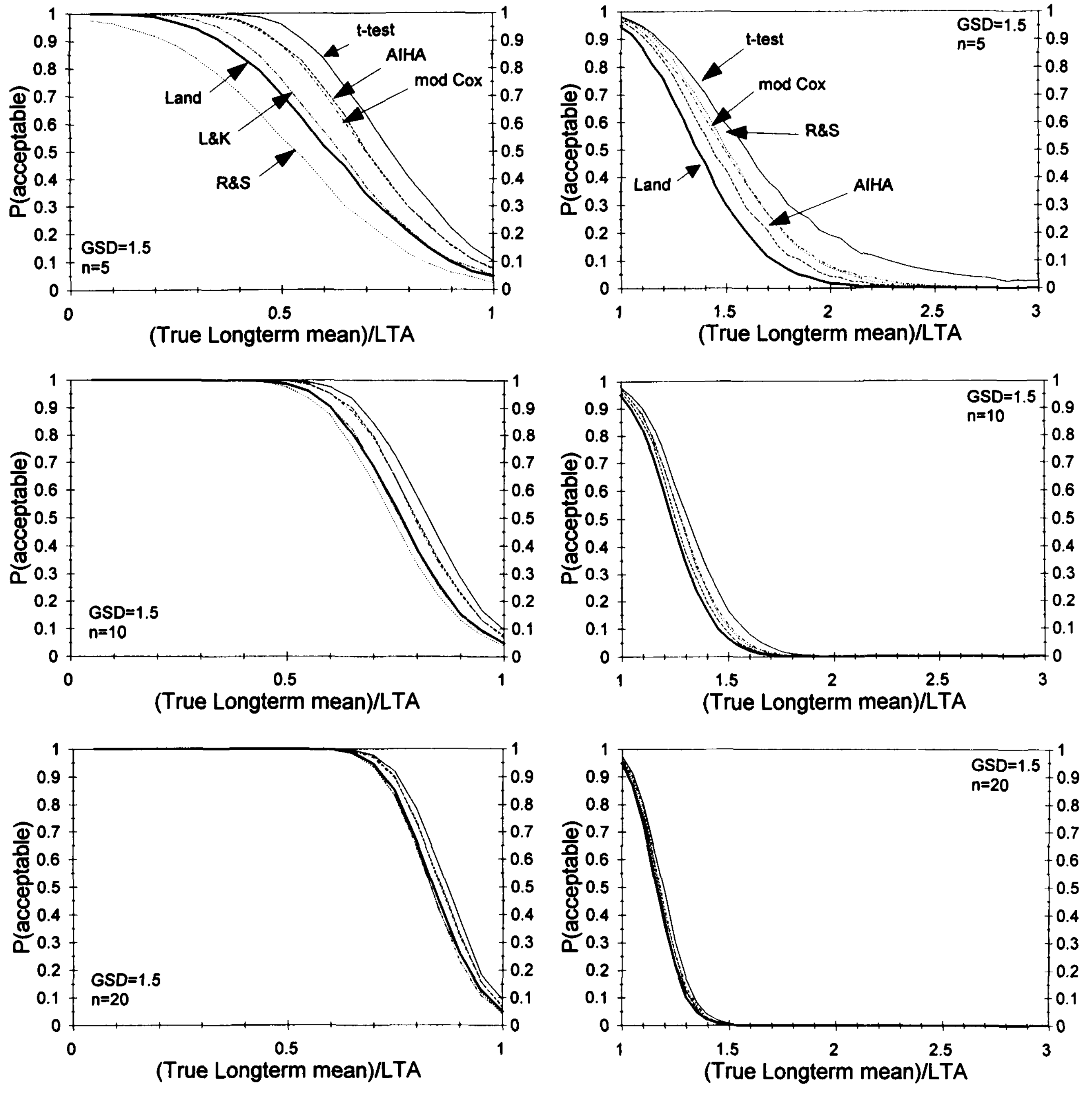

FIGURE 1. Operating characteristic curves for the employer's mean test (left side) and the inspector's mean test (right side) for GSD = 1.5 and n = 5, 10, and 20.

each figure represent the employer's tests while those on the right represent the inspector's tests. An LTA OEL of 1 should be assumed for all figures. The y axis, P(acceptable), should be read as the probability of deciding that the work environment is acceptable. For the employer's test P(acceptable) corresponds to the probability of accepting the alternative hypothesis: H_a:μ < LTA OEL = 1. For the inspector's test P(acceptable) corresponds to the probability of accepting the null hypothesis: H_0:$\mu \leq$ LTA OEL = 1. The α-error for the employer's test is simply P(acceptable), where μ = LTA OEL. The α-error for the inspector's test is 1-P(acceptable), where μ = LTA OEL. Power at any μ other than the LTA OEL is simply P(acceptable) for the employer's test and 1-P(acceptable) for the inspector's test. [There are two attributes that should be considered before adopting a statistical test with a specific null and alternative hypothesis: the α-error and β-error. The α-error (or type I error) is the probability of accepting the alternative hypothesis given that the null hypothesis is true (e.g.,

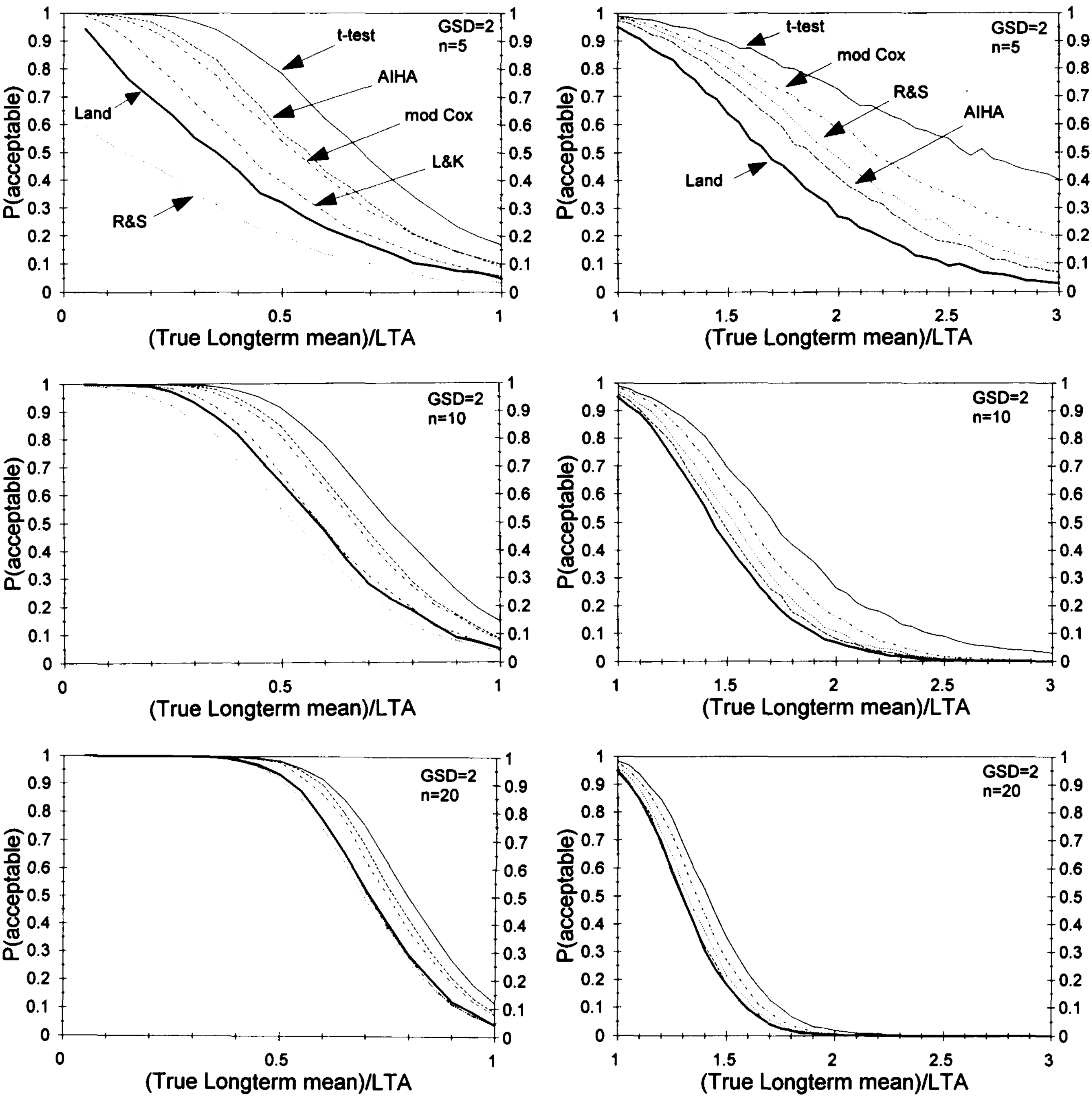

FIGURE 2. Operating characteristic curves for the employer's mean test (left side) and the inspector's mean test (right side) for GSD = 2 and n = 5, 10, and 20.

when μ = LTA OEL). The β-error (or type II error) represents the probability of accepting the null hypothesis given a specific alternative hypothesis (e.g., for $\mu = 2 \cdot$ LTA OEL). The power of a test is simply 1-β and represents the probability of accepting the alternative hypothesis given a specific alternative hypothesis. Ideally, one prefers to have a small α-error (typically around 0.05) and a small β-error (typically around 0.1). If the β-error is around 0.1, then the power of the test at some clearly unacceptable long-term mean (specific alternative) will be around 0.9.]

Only Land's mean test consistently yielded α-errors close to the nominal α-error of 0.05. For the inspector's mean test the alternative procedures usually yielded α-errors less than the expected value of 0.05. For the employer's test the alternative procedures usually yielded α-errors greater than 0.05. Regarding the inspector's test, the power of Land's mean test to detect

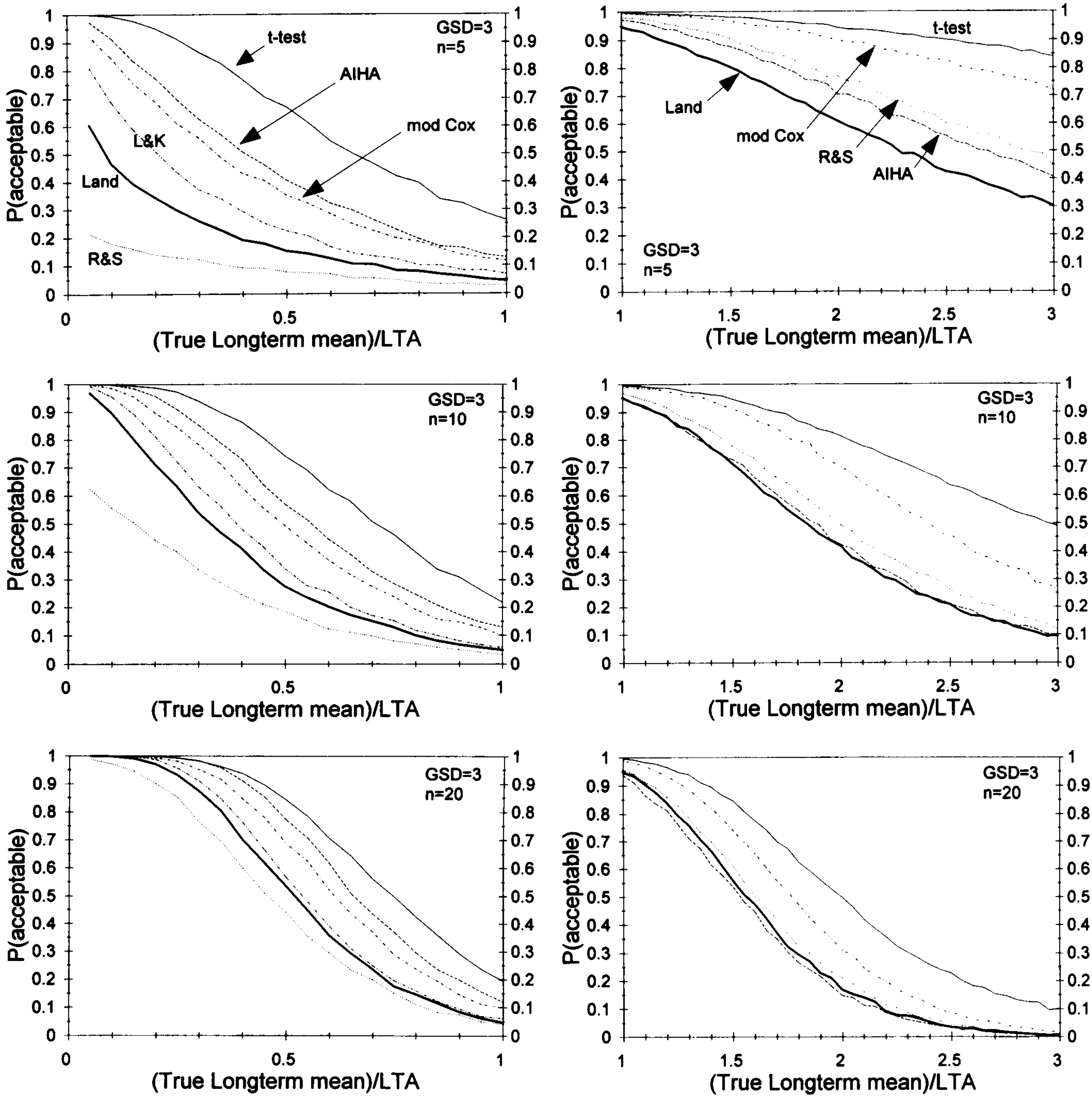

FIGURE 3. Operating characteristic curves for the employer's mean test (left side) and the inspector's mean test (right side) for GSD = 3 and n = 5, 10, and 20.

a true mean greater than the LTA OEL was usually greater than the alternative procedures, particularly the simple *t*-test and the modified Cox mean test. The power provided by either the AIHA mean test or Rappaport and Selvin's mean test was similar to or greater than that of Land's mean test for sample sizes of ten or more. Regarding the employer's test, most of the alternative tests to Land's test were more likely to detect a true mean less than the LTA OEL. Rappaport and Selvin's mean test consistently had less power than Land's mean test and probably should not be used for the employer's test. Lyles and Kupper's (UCL) mean test performed nearly as well as Land's mean test for most combinations of sample size and GSD used in the computer simulations. However, for larger GSDs and small sample sizes the difference between Land's test and Lyles and Kupper's test widens.

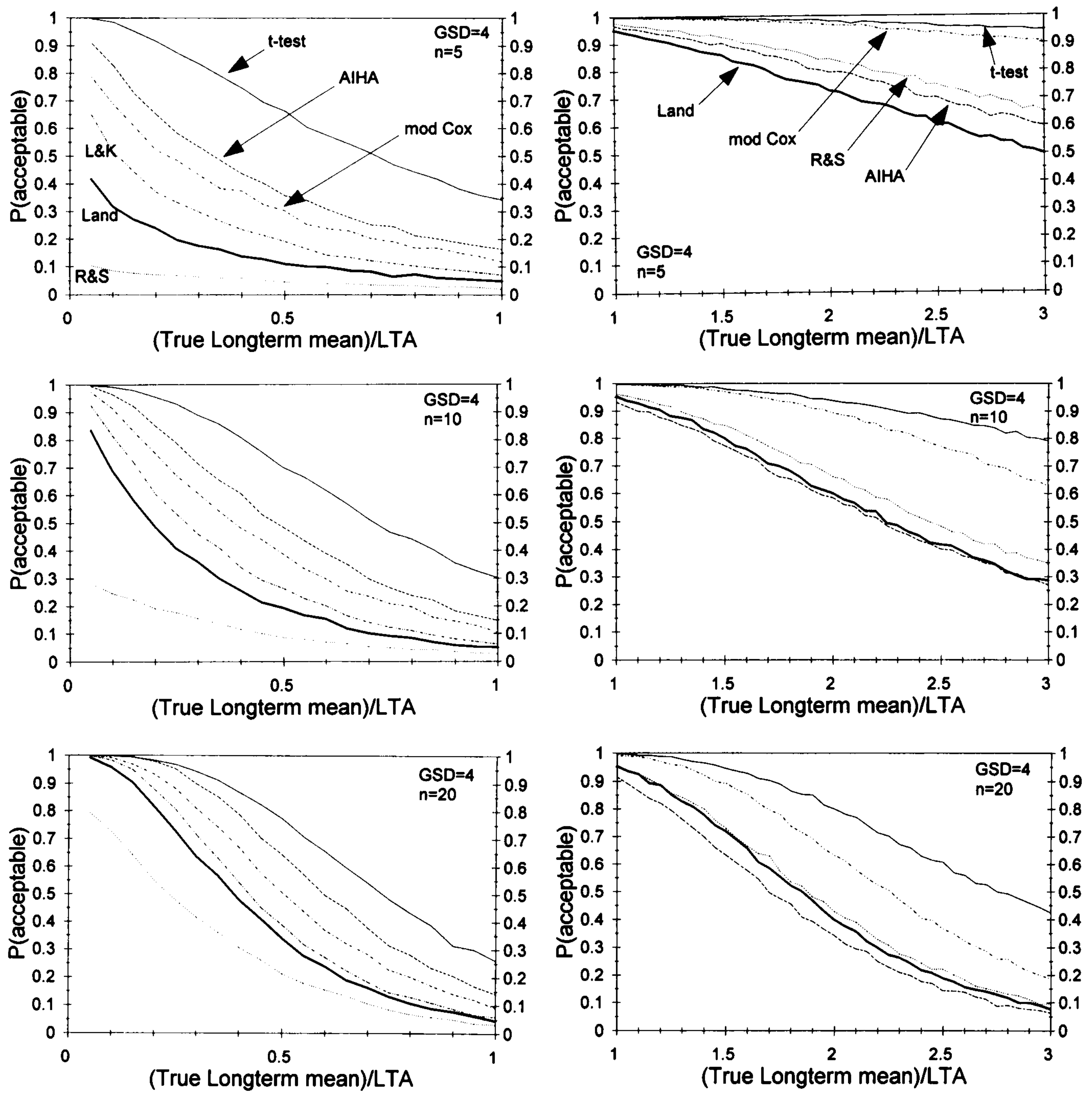

FIGURE 4. Operating characteristic curves for the employer's mean test (left side) and the inspector's mean test (right side) for GSD = 4 and n = 5, 10, and 20.

Discussion

Applications of Mean Testing

Mean testing could be used (1) to justify the reduction of sampling for an employee or exposure group (by comparing the UCL to the LTA OEL), (2) to justify additional controls (by comparing the LCL to the LTA OEL), or (3) as a compliance test (by comparing the LCL to the LTA OEL).

The employer's test is appropriate when a company industrial hygienist suspects, based on past data or experience, that exposures are, in general, well controlled and wishes to demonstrate this with compelling evidence. The investigator then collects n valid, representative measurements from the employee in question (or, if the focus is on an exposure group, from one or more maximum risk employees) and calculates the $(1 - \alpha)100$ percent UCL. [Exposure measurements should be valid: (1) the measurements were collected and analyzed using a reasonably accurate sampling and analytical method, and (2)

TABLE 1. Approximate Sample Sizes for the Employer's Mean Test and the Inspector's Mean Test When Using Land's Confidence Interval Procedure

	μ/(LTA OEL)	Underlying GSD: GSD = 1.5	GSD = 2.0	GSD = 3.0	GSD = 4.0
Employer's mean test	0.1	4	5–6	10	15
	0.5	7	17	49	90
	0.75	23	73	235	435
Inspector's mean test	1.5	8–9	27	85	170
	2	3–4	8–9	26	51
	3	3	4	9–10	18

The α-error when the true mean equals the LTA OEL is approximately 0.05, and the power of the test when the true mean is a fraction or multiple of the LTA OEL is approximately 0.9.

the measurements adequately represent personal exposure. Measurements should be representative: (1) production levels, environmental controls, and work practices were not manipulated or optimized for the benefit of the survey, and (2) measurements were collected in a random fashion (i.e., the sample days were randomly selected). See Hewett[6] for additional details.] In this article it is assumed that α is the customary 0.05. If the UCL is less than the LTA OEL, then the employer can accept the alternative hypothesis and conclude, with at least 95 percent certainty, that the true mean is less than the LTA OEL. It would then be reasonable to reduce the sampling effort for this work environment. If the UCL is greater than the LTA OEL, then one can accept the null hypothesis, H_o:$\mu \geq$ LTA OEL, and conclude that the true mean is equal to or possibly greater than the LTA OEL and act accordingly. In reality, however, if the estimated mean is less than the LTA OEL, then one has no compelling evidence that exposures need additional controls. One has merely failed to reject the null hypothesis and should continue to monitor this particular work environment, seeking ways to reduce exposures further.

The inspector's mean test can be used by either a compliance officer or a company industrial hygienist. For example, a company industrial hygienist may suspect, based on past data or experience, that exposures are poorly controlled, but needs compelling evidence to justify additional controls. The inspector or company industrial hygienist collects n valid, representative measurements and calculates the $(1 - \alpha)100$ percent LCL. If the LCL is greater than the LTA OEL, then the investigator can accept the alternative hypothesis and conclude, with at least 95 percent certainty, that the true mean is greater than the LTA OEL. If, on the other hand, the point estimate of the true mean is greater than the LTA OEL, but the LCL is less than the LTA OEL, then one should reject the alternative hypothesis and accept the null hypothesis. For the inspector this means that a citation cannot be justified, using statistical analysis, with this data set and that additional measurements may be necessary. For the company industrial hygienist this means that there is evidence that exposures need additional controls, but that the existing data do not provide compelling evidence that this is indeed the case. The industrial hygienist should evaluate, and modify if necessary, the existing controls and work practices until additional measurements verify that the average exposure is less than the LTA OEL.

Appropriate Sample Sizes for Mean Testing

Table 1 contains approximate sample sizes based on the operating characteristic curves for Land's test and the following assumptions: the significance level is 0.05 ($\alpha = 0.05$) and the power of the procedure at the indicated true mean (expressed as a fraction or multiple of the LTA OEL) is approximately 0.90. The smallest sample size is n = 3 as this is the smallest size given in Land's tables. (Land's procedure requires that two parameters be estimated from the data, resulting in the loss of two degrees of freedom.) These sample sizes are similar to those calculated by Rappaport and Selvin[3] and Lyles and Kupper.[4]

Conclusions

The description provided for each of the six procedures can be used to compare the ease of use of each procedure. The operating characteristic curves can then be compared to determine the level of performance of each procedure at both the null hypothesis long-term mean where the α-error should be, for example, 0.05, and an alternative hypothesis long-term mean where the power of the test should be, for example, 0.9. It is the author's opinion that procedures that come closest to providing the nominal (null hypothesis) α-error over the range of interest should be preferred.

Only Land's mean test[5] consistently provided the nominal confidence level across the range of sample sizes and GSDs studied. Since none of the alternative procedures consistently matched the performance of Land's mean test, and Land's mean test is based on a published procedure for calculating exact confidence intervals, it is probably the preferred test for both the employer's or inspector's mean test. The Lyles and Kupper mean test[4] approached the performance of Land's procedure for the employer's test and is easy to program, but has the slight disadvantage of being restricted to the estimation of only the UCL, thus being useful only for the employer's test. Regarding the inspector's test, both Rappaport and Selvin's mean test and the AIHA mean test closely emulated Land's procedure for several combinations of sample size and GSD. These procedures are easy to compute; however, since for any data set the true GSD will be unknown, the actual α-error may be considerably different from the nominal level, resulting in a basically inaccurate test procedure [i.e., the confidence level will essentially be unknown, but will be in the vicinity of $(1 - \alpha)100\%$].

Acknowledgment

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