Assessing Occupational Exposure Via the One-Way Random Effects Model With Balanced Data

K. KRISHNAMOORTHY and Thomas MATHEW

A one-way random effects model is considered for the log-transformed shift-long personal exposure measurements, where the random effect in the model represents an effect due to the worker. Following a previous approach, we address a hypothesis-testing problem involving the proportion of workers for whom the mean exposure exceeds the occupational exposure limit. A confidence interval is constructed for the relevant parameter of interest, following the idea of a previously presented generalized confidence interval. The confidence bound is used for the purpose of testing hypotheses, and the performance of the test is numerically investigated. It turns out that the test exhibits satisfactory performance regardless of the sample size, in particular, for small samples. A similar procedure is then employed for testing hypotheses concerning the overall mean exposure. The results are illustrated using examples.

Key Words: Generalized confidence interval; Generalized p-value; Random effect; Variance components.

1. INTRODUCTION

For the assessment of occupational exposure to contaminants, several authors have argued about the need to use models that involve random effects (see, e.g., Rappaport, Kromhout, and Symanski 1993; Heederik and Hurley 1994). Some of the recent work on exposure monitoring has focused on the use of the one-way random effects model for the log-transformed shift-long exposures in order to incorporate the between- and within-worker sources of variability (see Lyles, Kupper, and Rappaport 1997a,b). A parameter of interest in this context is the proportion of workers in a job group for whom the long-term mean exposure exceeds the occupational exposure limit (OEL). A hypothesis-testing problem involving this parameter is addressed in Lyles et al. (1997a) for the case of balanced data and in Lyles et al. (1997b) for unbalanced data. The problems that come up in this context

K. Krishnamoorthy is Professor, Department of Mathematics, University of Louisiana at Lafayette, Lafayette, LA 70504. Thomas Mathew is Professor, Department of Mathematics and Statistics, University of Maryland, 1000 Hilltop Circle, Baltimore, MD 21250 (E-mail: mathew@math.umbc.edu)

Journal of Agricultural, Biological, and Environmental Statistics, Volume 7, Number 3, Pages 440–451
DOI: 10.1198/108571102311

are quite different from the traditional problems encountered in the analysis of mixed and random models. In fact, the above hypothesis involves all the parameters in the one-way random effects model, the mean as well as the two variance components. Deriving an exact test for such a hypothesis appears difficult, and Lyles et al. (1997a,b) propose large-sample tests (Wald, likelihood ratio, and score-type tests). These authors also suggest suitable modifications to control the type I error probability of the tests.

The concepts of generalized p-values and generalized confidence intervals appear to be attractive options to address the above hypothesis-testing problem. The major appeal of these concepts is that they provide procedures applicable to small samples. The concept of the generalized p-value was introduced by Tsui and Weerahandi (1989) and that of the generalized confidence interval by Weerahandi (1993) [see the book by Weerahandi (1995) for a detailed discussion along with examples]. Some applications to mixed and random models are given in Weerahandi (1991) and Zhou and Mathew (1994). In the present article, we use the generalized confidence interval approach to compute an upper confidence bound for the parameter of interest and then use it for testing hypotheses. This is also equivalent to carrying out a test based on the generalized p-value.

The article is organized as follows. A brief review of the generalized confidence interval and the generalized p-value is given in the next section. In Section 3, we introduce the one-way random effects model for exposure assessment in the setup of Lyles et al. (1997a). The shift-long exposures are assumed to have a log-normal distribution, and the one-way random effects model applies to the log-transformed shift-long exposures. The hypothesis-testing problem along with a solution based on the generalized confidence interval is described in Section 3. Simulation results concerning the type I error probability and power of the test are given in Section 4. Our procedure is then applied to the examples taken from Lyles et al. (1997a), and the results are given in Section 5. It is also possible to apply our approach for obtaining confidence bounds and for testing hypotheses for the overall mean exposure, and this is explained in Section 6. In a simpler setting, this problem has been considered by Lyles and Kupper (1996). Finally, some concluding remarks appear in Section 7.

2. THE GENERALIZED CONFIDENCE REGION AND THE GENERALIZED P-VALUE

We shall define the generalized p-value and the generalized confidence interval in the following setup. Let X be a random variable whose distribution depends on a parameter of interest, θ, and a nuisance parameter, δ. Suppose we want to obtain a confidence interval for θ. Let x denote the observed value of X. Let $T(X; x, \theta, \delta)$ be a pivot quantity that depends on the random variable X, its observed value x, and the parameters, and suppose $T(X; x, \theta, \delta)$ satisfies the following:

(a) The distribution of $T(X; x, \theta, \delta)$ is free of any unknown parameters.

(b) The observed value of $T(X; x, \theta, \delta)$, i.e., $T(x; x, \theta, \delta)$, is free of the nuisance parameter δ (2.1)

Let C_α be a region satisfying

$$P[T(X; x, \theta, \delta) \in C_\alpha] = 1 - \alpha.$$

Then $\{\theta : T(x; x, \theta, \delta) \in C_\alpha\}$ is called a $100(1-\alpha)\%$ generalized confidence region for θ.

The quantity $T(X; x, \theta, \delta)$ is called a generalized pivot statistic. The condition (2.1a) guarantees that C_α does not depend on θ or δ. It should, however, be noted that the usual repeated sampling property of confidence regions may not hold for generalized confidence regions, i.e., the actual coverage probability of the generalized confidence region may not be $1-\alpha$. In fact, the coverage probability could depend on the nuisance parameter, δ.

In the above setup, suppose we are interested in testing the hypotheses

$$\mathrm{H}_0: \theta \le \theta_0 \quad \text{vs.} \quad \mathrm{H}_1: \theta > \theta_0, \tag{2.2}$$

for a specified θ_0. Suppose $T^*(X; x, \theta, \delta)$ satisfies the following:

(a) The observed value of $T^*(X; x, \theta, \delta)$, i.e., $T^*(x; x, \theta, \delta)$, is free of any unknown parameters,

(b) The distribution of $T^*(X; x, \theta, \delta)$ is stochastically monotone (i.e., stochastically increasing or stochastically decreasing) in θ for any fixed x and δ,

(c) The distribution of $T^*(X; x, \theta_0, \delta)$ is free of any unknown parameters. (2.3)

Let $t^* = T^*(x; x, \theta, \delta)$, the observed value of $T^*(X; x, \theta, \delta)$. When the conditions in (2.3) hold, the generalized p-value for testing the hypotheses in (2.2) is defined as

$$P\left[T^*(X; x, \theta_0, \delta) \ge t^*\right]$$

if $T^*(X; x, \theta, \delta)$ is stochastically increasing in θ. On the other hand, if $T^*(X; x, \theta, \delta)$ is stochastically decreasing in θ, the generalized p-value is defined as

$$P\left[T^*(X; x, \theta_0, \delta) \le t^*\right].$$

The test based on the generalized p-value consists of rejecting H_0 when the generalized p-value is small, say, less than α. However, the size and power of such a test may depend on the nuisance parameters.

For further details on the above, along with examples, we refer to Weerahandi (1995, chaps. 5 and 6).

3. THE ONE-WAY RANDOM EFFECTS MODEL FOR EXPOSURE ASSESSMENT

The setup described here is exactly the same as that in Lyles et al. (1997a). Let X_{ij} denote the jth shift-long exposure measurement for the ith worker, assumed to be distributed as lognormal, $j = 1, 2, \ldots, n$, $i = 1, 2, \ldots, k$. Let $Y_{ij} = \ln(X_{ij})$ so that Y_{ij} follows a normal distribution. The assumed one-way random effects model is, for the Y_{ij}'s,

$$Y_{ij} = \mu + \tau_i + e_{ij}, \qquad i = 1, \ldots, k;\ j = 1, \ldots, n, \tag{3.1}$$

where μ is the general mean, $\tau_i \sim \mathrm{N}(0, \sigma_\tau^2)$, and $e_{ij} \sim \mathrm{N}(0, \sigma_e^2)$. All the random variables are mutually independent. Here τ_i represents the random effect due to the ith worker. Let

$$\mu_{x_i} = \mathrm{E}(X_{ij} \mid \tau_i) = \mathrm{E}[\exp(Y_{ij}) \mid \tau_i] = \exp\left\{\mu + \tau_i + \sigma_e^2/2\right\}. \tag{3.2}$$

Note that μ_{x_i} is the mean exposure for the ith worker and $\ln(\mu_{x_i}) \sim \mathrm{N}(\mu + \sigma_e^2/2, \sigma_\tau^2)$. Let θ denote the probability that μ_{x_i} exceeds the occupational exposure limit (OEL). Then

$$\theta = P(\mu_{x_i} > \mathrm{OEL}) = P(\ln(\mu_{x_i}) > \ln(\mathrm{OEL})) = 1 - \Phi\left(\frac{\ln(\mathrm{OEL}) - \mu - \sigma_e^2/2}{\sigma_\tau}\right), \tag{3.3}$$

where $\Phi(\cdot)$ denotes the standard normal c.d.f. Consider the hypotheses

$$\mathrm{H}_0\colon \theta \geq A \quad \text{vs.} \quad \mathrm{H}_1\colon \theta < A, \tag{3.4}$$

where A is a specified quantity, usually small. It follows from (3.3) that the hypotheses in (3.4) are equivalent to

$$\mathrm{H}_0\colon \mu + z_{1-A}\sigma_\tau + \sigma_e^2/2 \geq \ln(\mathrm{OEL}) \quad \text{vs.} \quad \mathrm{H}_1\colon \mu + z_{1-A}\sigma_\tau + \sigma_e^2/2 < \ln(\mathrm{OEL}), \tag{3.5}$$

where z_{1-A} denotes the $100(1 - A)$th percentile of the standard normal. For a given significance level α, the null hypothesis in (3.4) will be rejected if a $100(1 - \alpha)\%$ upper confidence limit for $\eta = \mu + z_{1-A}\sigma_\tau + \sigma_e^2/2$ is less than ln(OEL). In other words, we can carry out a test of the hypotheses in (3.4), or equivalently in (3.5), once we compute an upper confidence limit for η.

We shall now derive an upper confidence limit for $\eta = \mu + z_{1-A}\sigma_\tau + \sigma_e^2/2$ using the idea of a generalized confidence interval. Let SS_e denote the error sums of squares, SS_τ denote the between sums of squares, and $\bar{Y}$ denote the average of all of the Y_{ij}'s. Note that

$$\hat{\sigma}_e^2 = \frac{SS_e}{k(n-1)} \quad \text{and} \quad \hat{\sigma}_\tau^2 = \frac{1}{n}\left(\frac{SS_\tau}{k-1} - \frac{SS_e}{k(n-1)}\right) \tag{3.6}$$

are, respectively, the ANOVA estimators of σ_e^2 and σ_τ^2. Furthermore,

$$\bar{Y} \sim \mathrm{N}\left(\mu, \frac{\sigma_e^2 + n\sigma_\tau^2}{kn}\right), \qquad U_e = \frac{SS_e}{\sigma_e^2} \sim \chi^2_{k(n-1)}, \qquad U_\tau = \frac{SS_\tau}{\sigma_e^2 + n\sigma_\tau^2} \sim \chi^2_{k-1} \tag{3.7}$$

and $\bar{Y}$, U_e, and U_τ are independent. Here χ^2_r denotes a central chi-square distribution with d.f. $= r$. Let $\bar{y}$, ss_τ, and ss_e denote the observed values of $\bar{Y}$, SS_τ, and SS_e, respectively. Now consider the generalized pivot statistic T given by

$$\begin{aligned} T &= \bar{y} - \frac{\bar{Y} - \mu}{\sqrt{SS_\tau}}\sqrt{kn}\frac{\sqrt{ss_\tau}}{\sqrt{kn}} + z_{1-A}\sqrt{\frac{1}{n}\left[\frac{\sigma_e^2 + n\sigma_\tau^2}{SS_\tau}ss_\tau - \frac{\sigma_e^2}{SS_e}ss_e\right]_+} + \frac{1}{2}\frac{\sigma_e^2}{SS_e}ss_e \\ &= \bar{y} + \frac{Z}{\sqrt{U_\tau}}\sqrt{\frac{ss_\tau}{kn}} + z_{1-A}\sqrt{\frac{1}{n}\left[\frac{ss_\tau}{U_\tau} - \frac{ss_e}{U_e}\right]_+} + \frac{1}{2}\frac{ss_e}{U_e}, \end{aligned} \tag{3.8}$$

where Z is a standard normal random variable and $x_+ = \max(x, 0)$. The observed value of T is obtained by replacing $\bar{Y}$, SS_τ, and SS_e by their respective observed values $\bar{y}$, ss_τ, and ss_e. Using the first expression for T in (3.8), it is readily verified that the observed value of T is $\eta = \mu + z_{1-A}\sigma_\tau + \sigma_e^2/2$, the parameter of interest to us. Furthermore, from the second expression for T given in (3.8), it should be clear that the distribution of T depends only on the data (via the observed values $\bar{y}$, ss_τ, and ss_e) and is free of any unknown parameters. Hence, for a given data set, the percentiles of T can be computed using Monte Carlo simulation. Let $T_{1-\alpha}$ denote the $100(1-\alpha)$th percentile of T so obtained. Then a $100(1-\alpha)\%$ generalized upper confidence interval for $\eta = \mu + z_{1-A}\sigma_\tau + \sigma_e^2/2$ is given by $\{\eta : \eta \le T_{1-\alpha}\}$.

The following algorithm can be used for estimating $T_{1-\alpha}$.

Algorithm 1

(1) For a given data set, compute ss_τ, ss_e, and $\bar{y}$.

(2) For $i = 1, m$, generate

$Z \sim N(0,1)$,

$U_\tau \sim \chi^2_{k-1}$,

$U_e \sim \chi^2_{k(n-1)}$.

(3) Compute

$$D = \frac{ss_\tau}{U_\tau} - \frac{ss_e}{U_e}.$$

(4) If $D < 0$, set $D = 0$.

(5) Set

$$T_i = \bar{y} + \frac{Z}{\sqrt{U_\tau}}\sqrt{\frac{ss_\tau}{kn}} + z_{1-A}\sqrt{\frac{1}{n}D} + \frac{1}{2}\frac{ss_e}{U_e}.$$

(6) End i loop.

(7) The Monte Carlo estimate of $T_{1-\alpha}$ is the $100(1-\alpha)$th percentile of $\{T_1, \ldots, T_m\}$.

Once $T_{1-\alpha}$ is computed, a test procedure for the hypotheses in (3.5) consists of rejecting H_0 when $T_{1-\alpha} < \ln(\text{OEL})$.

The above procedure is actually equivalent to a test based on the generalized p-value. To see this, define

$$T^* = T - \eta, \tag{3.9}$$

where T is defined in (3.8) and $\eta = \mu + z_{1-A}\sigma_\tau + \sigma_e^2/2$, as defined earlier. Thus, the hypotheses (3.5) can be expressed as

$$H_0 : \eta \ge \ln(\text{OEL}) \quad \text{vs.} \quad H_1 : \eta < \ln(\text{OEL}).$$

We shall now verify that T^* satisfies the conditions in (2.3). Clearly, the observed value of T^* is zero. From the definition of T^* in (3.9), it is obvious that T^* is stochastically decreasing in η. Also, at $\eta = \ln(\text{OEL})$, the distribution of T^* is free of any unknown parameters.

In other words, T^* satisfies the three conditions in (2.3). Hence, the generalized p-value is defined as $P[T^* > 0 \mid \eta = \ln(\text{OEL})]$, i.e., $P[T > \ln(\text{OEL})]$, where T is defined in (3.8). Suppose we reject H_0 when the generalized p-value is less than α, i.e., when $P[T \le \ln(\text{OEL}) \mid \eta = \ln(\text{OEL})] > 1 - \alpha$. This is clearly equivalent to our earlier procedure, i.e., reject H_0 when $T_{1-\alpha} < \ln(\text{OEL})$.

4. SIMULATION STUDY

We shall now study the type I error rate and power of the proposed test. To compute the size of the test for (3.4) when H_0: $\theta = A = p_0$, where p_0 is a specified number in (0, 1), we note from (3.3) that

$$\theta = 1 - \Phi\left(\frac{\ln(\text{OEL}) - \mu - \sigma_e^2/2}{\sigma_\tau}\right) = p_0$$

and hence

$$\ln(\text{OEL}) = z_{1-p_0}\sigma_\tau + \mu + \sigma_e^2/2. \tag{4.1}$$

The size of the test is the proportion of times $T_{1-\alpha} < \ln(\text{OEL})$ in (4.1). For computing the power when $\theta = p_1 < A$, we note that

$$\ln(\text{OEL}) = z_{1-p_1}\sigma_\tau + \mu + \sigma_e^2/2. \tag{4.2}$$

The power of the test at $\theta = p_1$ is the proportion of times $T_{1-\alpha} < \ln(\text{OEL})$ in (4.2), where $T_{1-\alpha}$ can be estimated by algorithm 1 using $A = p_0$. The following algorithm can be used to obtain Monte Carlo estimates of the size and power of the proposed test.

Algorithm 2

(1) For given $\mu, \sigma_\tau^2, \sigma_e^2, n$, and k, compute $\eta = \mu + z_{1-A}\sigma_\tau + \sigma_e^2/2$, $\ln(\text{OEL})$ defined in (4.1) and $\ln(\text{OEL})$ defined in (4.2).

(2) For $i = 1$ to m, generate χ^2_{k-1}, $\chi^2_{k(n-1)}$, and $Z \sim N(0, 1)$ variates.

(3) Set $ss_\tau = (\sigma_e^2 + n\sigma_\tau^2)\chi^2_{k-1}$, $ss_e = \sigma_e^2\chi^2_{k(n-1)}$, and

$$\bar{y} = \mu + Z\sqrt{\frac{\sigma_e^2 + n\sigma_\tau^2}{kn}}.$$

(4) For $j = 1$ to s, generate $Z \sim N(0, 1)$, $U_e \sim \chi^2_{k(n-1)}$, and $U_\tau \sim \chi^2_{k-1}$.

(5) Compute T_j using the formula for T in (3.8).

(6) End j loop.

(7) Find the $100(1-\alpha)$th percentile $T_{1-\alpha}$ of $T_1, \ldots, T_s$.

(8) If $T_{1-\alpha} < \ln(\text{OEL})$ in (4.1), set $c_i = 1$; else set $c_i = 0$.

(9) If $T_{1-\alpha} < \ln(\text{OEL})$ in (4.2), set $d_i = 1$; else set $d_i = 0$.

(10) End i loop.

(11) $(1/m)\sum_{i=1}^{m} c_i$ is an estimate of the type I error rate and $(1/m)\sum_{i=1}^{m} d_i$ is an estimate of the power.

Table 1. Monte Carlo Estimates of the Type I Errors and Powers of the Test for (3.4)

σ_τ^2/σ_e^2	σ_e^2	n	k	*Type I Error*	*Power*
0.1	0.5	7	26	0.051	0.556
0.1	1	7	26	0.051	0.575
0.1	3	6	32	0.053	0.579
0.25	0.5	3	25	0.048	0.561
0.25	1	3	24	0.051	0.563
0.25	3	2	37	0.049	0.561
0.5	0.5	2	26	0.050	0.648
0.5	1	2	25	0.050	0.645
0.5	3	2	26	0.051	0.609
0.75	0.5	2	19	0.050	0.594
0.75	1	2	19	0.055	0.594
0.75	3	2	20	0.048	0.573
1	0.5	2	16	0.049	0.565
1	1	2	16	0.051	0.579
1	3	2	17	0.051	0.549
2.5	0.5	2	12	0.050	0.570
2.5	1	2	12	0.049	0.551
2.5	3	2	13	0.052	0.535
5	0.5	2	10	0.051	0.514
5	1	2	10	0.048	0.510
5	3	2	11	0.051	0.519

NOTE: $\mu = -2$ and $\alpha = 0.05$; the type I error is computed at $\theta = A = 0.05$; the power is computed when $\theta = 0.002$ and $A = 0.05$.

Table 1 gives the type I error rates and the power values obtained using algorithm 2 with $m = s = 10,000$. We used the IMSL subroutine RNCHI to generate chi-square random variates and function subroutine RNNOR() to generate normal random variates. The sample size and parameter configurations are taken from Table 4 of Lyles et al. (1997a). Thus, we choose $\theta = A = 0.05$ in (3.4), and for computing the power, we shall choose $\theta = 0.002$. The sample sizes considered in Table 1 are the approximate values required to attain a power of 0.60 at the level of significance $\alpha = 0.05$ (see Lyle set al. 1997a).

For Table 2, we used the setting of Table 5 of Lyles et al. (1997a). Here the power is computed when $\gamma = \mu_x/\text{OEL} = 0.2$, where the mean exposure $\mu_x = \exp\{\mu + (\sigma_\tau^2 + \sigma_e^2)/2\}$, $A = 0.10$, and $\alpha = 0.05$. In this case, $\ln(\text{OEL}) = \mu + (\sigma_\tau^2 + \sigma_e^2)/2 - \ln(\gamma)$. The reason for computing the power as a function of γ instead of θ is that, in some situations, it is easier for a practitioner to hypothesize the value of γ than to hypothesize the value of θ. The sample sizes are approximate values to attain a power of 0.60 at the level 0.05; see Table 5 of Lyles et al. (1997a).

The numerical results in Table 4 of Lyles et al. (1997a) show that both the Wald and the likelihood ratio-type test can have type I error probabilities exceeding the nominal level. However, for the sample sizes and parameter combinations considered in Table 5 of their paper, the Wald test turns out to be quite satisfactory, both in terms of type I error probability and power. Nevertheless, among the three tests considered by them, only the score test provides a type I error probability below the nominal level across all the sample sizes and parameter combinations considered by them. However, the score test appears to be too conservative, resulting in poor power. The numerical results in Table 1 and Table

Table 2. Monte Carlo Estimates of the Type I Errors and Powers of the Test for (3.4)

σ_τ^2/σ_e^2	σ_e^2	n	k	*Type I Error*	*Power*
0.1	0.5	3	5	0.018	0.648
0.1	1	3	9	0.023	0.719
0.1	3	4	25	0.044	0.748
0.25	0.5	2	7	0.019	0.606
0.25	1	2	15	0.028	0.712
0.25	3	2	67	0.053	0.774
0.5	0.5	2	10	0.041	0.710
0.5	1	2	24	0.051	0.780
0.5	3	2	114	0.052	0.826
0.75	0.5	2	13	0.053	0.729
0.75	1	2	32	0.056	0.787
0.75	3	2	125	0.046	0.827
1	0.5	2	16	0.054	0.741
1	1	2	41	0.052	0.807
1	3	2	119	0.052	0.822
2.5	0.5	2	38	0.053	0.792
2.5	1	2	71	0.047	0.819
2.5	3	2	49	0.048	0.782
5	0.5	2	61	0.050	0.809
5	1	2	56	0.045	0.804
5	3	2	17	0.037	0.698

NOTE: $\mu = -2$ and $\alpha = 0.05$; the type I error is computed at $\theta = A = 0.10$; the power is computed when $\gamma = \mu_x/\text{OLE} = 0.2$, where $\mu_x = \exp(\mu + (\sigma_\tau^2 + \sigma_e^2)/2)$ and $A = 0.10$.

2 of this article correspond to the sample size and parameter configurations considered in Tables 4 and Table 5, respectively, of Lyles et al. (1997a). From our numerical results, it is clear that our proposed test is quite satisfactory for controlling the type I error probability. Furthermore, in situations where it is conservative, it turns out to be less conservative compared with the score test. Perhaps as a consequence of this, the proposed test also has a larger power compared with the score test in most cases. Thus, it appears that the proposed test can be recommended for practical use regardless of the sample sizes and regardless of the parameter values. Furthermore, the test is also quite easy to carry out.

5. EXAMPLES

We shall now apply our test procedure to the three examples discussed in Lyles et al. (1997a).

Example 5.1. This example deals with styrene exposures on laminators at a boat manufacturing plant, and the data are given in Table C.1 of Lyles et al. (1997a). For this data, $k = 13$ and $n = 3$. Furthermore, the observed values of SS_τ, SS_e, and $\bar{Y}$ are, respectively, $ss_\tau = 11.426$, $ss_e = 14.711$, and $\bar{y} = 4.810$. Also, for styrene exposure, OEL = 213 mg/m^3. When $\alpha = 0.05$ and $A = 0.10$, the value of $T_{0.95}$ is 6.144, which is greater than $\ln(213) = 5.3613$. Therefore, the null hypothesis will be retained. In other words, the data do not provide enough evidence to indicate that less than 10% of the workers have mean exposures below the OEL. A similar conclusion was also arrived at by Lyles et al. (1997a) based on the Wald, likelihood ratio, and score tests; see Section 5 of their article.

Example 5.2. The data for this example is taken from Table C.2 of Lyles et al. (1997a) and deals with nickel exposures on maintenance mechanics at a nickel-producing plant. For this example, $k = 14$, $n = 2$, $ss_\tau = 14.349$, $ss_e = 12.682$, and $\bar{y} = -3.393$. For nickel exposure, OEL $= 1$ mg/m^3. For $\alpha = 0.05$ and $A = 0.05$, $T_{0.95} = -1.415$. Note that -1.415 is less than $\ln(1) = 0$ and hence the null hypothesis will be rejected, i.e., we conclude that less than 5% of the workers have mean exposures exceeding the OEL. This is consistent with the conclusion in Lyles et al. (1997a) based on the Wald, likelihood ratio, and score tests.

Example 5.3. This example also deals with nickel exposures, and the data are given in Table C.3 of Lyles et al. (1997a). Here we have $k = 11$, $n = 2$, $ss_\tau = 8.91$, $ss_e = 12.05$, and $\bar{y} = -0.892$. When $\alpha = 0.05$ and $A = 0.10$, the value of $T_{0.95}$ is 0.866. In this case, $t_{0.95}$ is greater than $\ln(1) = 0$, and so the null hypothesis will be retained. Thus, there is not sufficient evidence to support that less than 10% of the workers have mean exposures exceeding the OEL. As with the previous two examples, our conclusion agrees with that in Lyles et al. (1997a) based on the Wald, likelihood ratio, and score tests.

6. AN UPPER CONFIDENCE BOUND AND TEST FOR THE OVERALL MEAN EXPOSURE

The overall mean exposure is given by $\mu_x = \mathrm{E}(X_{ij})$. Recalling that $Y_{ij} = \ln(X_{ij})$ and using (3.1), we get

$$\mu_x = \exp\left\{\mu + \left(\sigma_\tau^2 + \sigma_e^2\right)/2\right\}. \tag{6.1}$$

We shall consider the problem of deriving an upper confidence bound for μ_x and the problem of testing whether μ_x exceeds the OEL. Note that it is enough to obtain confidence bounds and tests concerning $\eta_0 = \mu + (\sigma_\tau^2 + \sigma_e^2)/2$. Thus, we shall address the problem of testing

$$\mathrm{H}_0\colon \mu + \left(\sigma_\tau^2 + \sigma_e^2\right)/2 \geq \ln(\mathrm{OEL}) \quad \text{vs.} \quad \mathrm{H}_1\colon \mu + \left(\sigma_\tau^2 + \sigma_e^2\right)/2 < \ln(\mathrm{OEL}). \tag{6.2}$$

Our procedure is similar to that in Section 3, i.e., we shall derive a generalized upper confidence bound for $\eta_0 = \mu + (\sigma_\tau^2 + \sigma_e^2)/2$ and then use it for testing the hypotheses in (6.2). Using the notation in Section 3, define the generalized pivot statistic

$$\begin{aligned} T_0 &= \bar{y} - \frac{\sqrt{kn}(\bar{Y} - \mu)}{\sqrt{SS_\tau}}\sqrt{\frac{ss_\tau}{kn}} + \frac{1}{2}\left[\left(\frac{\sigma_e^2 + n\sigma_\tau^2}{nSS_\tau}ss_\tau - \frac{\sigma_e^2}{nSS_e}ss_e\right) + \frac{\sigma_e^2}{SS_e}ss_e\right] \\ &= \bar{y} + \frac{Z}{\sqrt{U_\tau}}\sqrt{\frac{ss_\tau}{kn}} + \frac{1}{2}\left[\frac{1}{n}\frac{ss_\tau}{U_\tau} + \frac{n-1}{n}\frac{ss_e}{U_e}\right]. \end{aligned} \tag{6.3}$$

The distribution of T_0 depends only on $\bar{y}$, ss_τ, and ss_e and is free of any unknown parameters. Furthermore, the observed value of T_0 (obtained by replacing SS_τ, SS_e, and $\bar{Y}$ by their observed values ss_τ, ss_e, and $\bar{y}$, respectively) is η_0. Therefore, for a given data set, the percentiles of T_0 can be computed using the Monte Carlo method as shown in algorithm 1. If $T_{0,1-\alpha}$ is the $100(1-\alpha)$th percentile of T_0, then a $100(1-\alpha)\%$ generalized upper confidence limit for η_0 is $\eta_0 \leq T_{0,1-\alpha}$. Furthermore, a test for the hypotheses in (6.2) consists of rejecting H_0 when $T_{0,1-\alpha} < \ln(\text{OEL})$.

The following algorithm can be used for computing $T_{0,1-\alpha}$.

Algorithm 3

(1) For a given data set, compute ss_τ, ss_e, and $\bar{y}$.

(2) For $i = 1, m$, generate

$$Z \sim N(0, 1),$$
$$U_\tau \sim \chi^2_{k-1},$$
$$U_e \sim \chi^2_{k(n-1)}.$$

(3) Compute

$$T_{0i} = \bar{y} + \frac{Z}{\sqrt{U_\tau}}\sqrt{\frac{ss_\tau}{kn}} + \frac{1}{2}\left[\frac{1}{n}\frac{ss_\tau}{U_\tau} + \frac{n-1}{n}\frac{ss_e}{U_e}\right].$$

(4) End i loop.

(5) The $100(1-\alpha)$th percentile of $\{T_{01}, \ldots, T_{0m}\}$ is an estimate of $T_{0,1-\alpha}$.

Table 3 gives the Monte Carlo estimates of the type I error probability and power of the test that rejects H_0 when $T_{0,1-\alpha} < \ln(\text{OEL})$. For this, we have chosen the same set of values for n, k, σ^2_τ/σ^2_e, and σ^2_e as in Table 1. Furthermore, we have chosen OEL = 1 so that $\ln(\text{OEL}) = 0$. In other words, for computing the type I error probability, we are choosing $\mu = -(\sigma^2_\tau + \sigma^2_e)/2$. For each choice of n, k, σ^2_τ/σ^2_e, and σ^2_e, two different values η_0 were arbitrarily chosen for computing the power. For example, for $n = 7$, $k = 26$, $\sigma^2_\tau/\sigma^2_e = 0.1$, and $\sigma^2_e = 0.5$, we chose the values $\eta_0 = -0.125$ and $\eta_0 = -0.225$ for the computation of power. Note that, once we specify the values of σ^2_τ/σ^2_e, σ^2_e, and η_0, the value of μ is uniquely determined. The numerical results in Table 3 were obtained using an algorithm similar to algorithm 2.

We shall now illustrate the test for the overall mean exposure using the examples considered in Section 5.

Example 5.1 (continued). When $\alpha = 0.05$, the value of $T_{0,0.95}$ using algorithm 3 is 5.563, which is greater than $\ln(213) = 5.3613$. Therefore, the null hypothesis in (6.2) will be retained.

Example 5.2 (continued). We now have $T_{0,0.95} = -2.289$. Note that -2.289 is less than $\ln(1) = 0$, and hence the null hypothesis will be rejected.

Example 5.3 (continued). The value of $T_{0,0.95}$ using algorithm 3 is 0.307, which is greater than $\ln(1) = 0$, and the null hypothesis will be retained.

7. CONCLUDING REMARKS

For obtaining confidence intervals and tests for nonstandard parameter combinations in the one-way random effects model, the concepts of generalized p-values and general-

Table 3. Monte Carlo Estimates of the Type I Errors and Powers of the Test for (6.2) with OEL = 1.0

σ_τ^2/σ_e^2	σ_e^2	n	k	η_0	Power	σ_τ^2/σ_e^2	σ_e^2	n	k	η_0	Power
0.1	0.5	7	26	0	0.052	0.75	3	2	20	0	0.051
				−0.125	0.471					−1.375	0.352
				−0.225	0.856					−2.375	0.652
0.1	1	7	26	0	0.047	1	0.5	2	16	0	0.051
				−0.200	0.438					−0.500	0.418
				−0.450	0.963					−1.00	0.858
0.1	3	6	32	0	0.049	1	1	2	16	0	0.044
				−0.350	0.331					−0.500	0.209
				−0.850	0.919					−1.50	0.775
0.25	0.5	3	25	0	0.043	1	3	2	17	0	0.050
				−0.390	0.848					−1.00	0.156
				−0.688	0.999					−2.00	0.392
0.25	1	3	14	0	0.047	2.5	0.5	2	12	0	0.047
				−0.375	0.464					−0.625	0.230
				−0.875	0.979					−2.125	0.860
0.25	3	2	37	0	0.048	2.5	1	2	12	0	0.050
				−0.625	0.322					−1.25	0.253
				−1.63	0.910					−3.25	0.739
0.5	0.5	2	26	0	0.048	2.5	3	2	13	0	0.049
				−0.300	0.452					−2	0.148
				−0.600	0.932					−4	0.334
0.5	1	2	25	0	0.048	5	0.5	2	10	0	0.050
				−0.250	0.160					−1.00	0.189
				−1.250	0.969					−3.00	0.670
0.5	3	2	26	0	0.049	5	1	2	10	0	0.050
				−1	0.339					−3.00	0.353
				−3	0.976					−6.00	0.637
0.75	0.5	2	19	0	0.049	5	3	2	11	0	0.052
				−0.400	0.209					−5.00	0.195
				−0.625	0.374					−15.0	0.648
0.75	1	2	20	0	0.051						
				−1.375	0.352						
				−2.375	0.652						

ized confidence intervals appear to be attractive options. We have to deal with such p rameter combinations in the context of analyzing exposure assessment data, where t log-transformed data follows a one-way random effects model. The concepts of gener ized p-values and generalized confidence intervals provide exact procedures applicable small samples. In our problem dealing with occupational exposures, we used the idea the generalized confidence interval in order to compute an upper confidence bound for parameter of interest and used such a confidence bound for testing a one-sided hypothe The computations required to carry out the above procedure turned out to be quite sim and straightforward, and numerical results show that our test performs quite well. N that our procedures are applicable only when we have balanced data. Similar problems currently under investigation for the one-way random effects model with unbalanced d We hope that this article will draw the attention of researchers in occupational hygiene the applicability of generalized p-values and generalized confidence intervals as valua tools for their data analysis.

ACKNOWLEDGMENTS

The authors are thankful to two reviewers for providing useful comments and suggestions. This research has been supported by grant R01-0H03628-01A1 from the National Institute of Occupational Safety and Health (NIOSH).

[Received November 2000. Accepted October 2001.]

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