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Journal of Statistical Planning and Inference 128 (2005) 219–229

journal of statistical planning and inference

www.elsevier.com/locate/jspi

Assessing occupational exposure via the one-way random effects model with unbalanced data

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Received 24 January 2003; accepted 16 September 2003

Abstract

This article considers a one-way random effects model for assessing the proportion of workers whose mean exposures exceed the occupational exposure limit based on exposure measurements from a random sample of workers. Hypothesis testing and interval estimation for the relevant parameter of interest are proposed when the exposure data are unbalanced. The methods are based on the generalized p-value approach, and simplify to the ones in Krishnamoorthy and Mathews (J. Agri. Biol. Environ. Statist. 7 (2002) 440) when the data are balanced. The sizes and powers of the test are evaluated numerically. The numerical studies show that the proposed inferential procedures are satisfactory even for small samples. The results are illustrated using practical examples.

MSC: 62F03; 62F25

Keywords: Generalized p-value; Generalized confidence limits; Overall mean; Power; Random effects; Variance components

1. Introduction

Exposure levels at workplaces are commonly assessed by the proportion of exposure measurements exceeding a limit, and sometimes by the long-term mean of the exposure data. To estimate these quantities, parametric statistical models are often used to fit exposure data collected from a sample of workers. Among other models, the appropriateness of a random effects model for fitting a sample of logged shift-long

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0378-3758/$ - see front matter
doi:10.1016/j.jspi.2003.09.025

exposure data (consisting of repeated measurements on each of the sample of workers) has been well addressed in the papers by Rappaport et al. (1993), Rappaport et al. (1995), Lyles et al. (1997a, b) and Maxim et al. (2000). The one-way random effects model incorporates the between workers and within worker sources of variability. When it is used to analyze logged exposure data, the parameter of interest (the proportion of measurements exceeding a limit) is a function of overall mean effect and the two variance components in the model. This makes the problem more complicated, and exact methods are difficult to obtain. Lyles et al. (1997a, b) proposed some large sample approximate methods which are essentially based on the classical central limit theorem. Specifically, Lyles et al. (1997a) considered the hypothesis testing of the relevant parameter when the data are balanced (equal number of measurements on each of sample workers), and Lyles et al. (1997b) addressed the same problem when the data are unbalanced.

Nowadays, modern computing technology allows researchers to investigate simple yet computationally intensive methods for non-standard problems such as the present one. Among other methods, the generalized p-value approach seems to be appropriate for the present problem. Tsui and Weerahandi (1989) introduced the concept of the generalized p-value for hypothesis testing, and Weerahandi (1993) extended the idea to interval estimation. Using these approaches, Krishnamoorthy and Mathew (2002) addressed the present problem with balanced data. These authors provided solutions for the hypothesis testing and interval estimation problems with balanced data which are satisfactory even for small samples. As pointed out by Lyles et al. (1997b), collecting balanced data in the context of the present problem seems to be unrealistic because of the workers burden, and unavailability of the workers during the sampling period. Furthermore, exposure data sets are typically small, and hence methods that are applicable for small samples unbalanced data are really warranted.

In this article we propose generalized p-value methods for interval estimation and hypothesis testing about the proportion of workers whose mean exposures exceed the occupational exposure limit (OEL). The description of the one-way random effects model, and the problem of hypothesis testing of the relevant parameters are outlined in the following section. In Section 3, we give a generalized pivot statistic, an interval estimate and test for the proportion of exposure measurements. The sizes and powers of the test are numerically evaluated and presented in Section 4. These numerical results indicate that our generalized test procedures are satisfactory even for small samples. The proposed testing and interval estimation procedures are illustrated using three data sets (consisting of nickel dust exposures from a nickel producing facility) in Section 5. Some concluding remarks are given in Section 6.

2. Description of the random effects model for exposure data

We shall describe the one-way random effects model to analyze the exposure data as given in Rappaport et al. (1995) and Lyles et al. (1997a). Let X_{ij} denote the jth shift-long exposure measurement for the ith worker, $j = 1, 2, \ldots, n_i$, $i = 1, 2, \ldots, k$. It is assumed that X_{ij} follows a lognormal distribution so that $Y_{ij} = \ln(X_{ij})$ follows a normal distribution. Then the one-way random effects model for the Y_{ij}'s is given by

$$Y_{ij} = \mu + \tau_i + e_{ij}, \quad j = 1, \ldots, n_i, \ \ i = 1, \ldots, k, \tag{2.1}$$

where μ is the overall mean, τ_i is the random effect due to the ith worker that follows a $\mathrm{N}(0, \sigma_\tau^2)$ distribution, and e_{ij}'s are random errors following a $\mathrm{N}(0, \sigma_e^2)$ distribution. All the random variables are mutually independent. The mean exposure of the ith worker is given by

$$\mu_{x_i} = E(X_{ij}|\tau_i) = E[\exp(Y_{ij})|\tau_i] = \exp\{\mu + \tau_i + \sigma_e^2/2\}. \tag{2.2}$$

Let θ denote the proportion of workers whose mean exposures exceed the OEL. Then, noting that $\ln(\mu_{x_i}) \sim \mathrm{N}(\mu + \sigma_e^2/2, \sigma_\tau^2)$, we see that

$$\theta = P(\mu_{x_i} > \mathrm{OEL}) = P(\ln(\mu_{x_i}) > \ln(\mathrm{OEL})) = 1 - \Phi\left(\frac{\ln(\mathrm{OEL}) - \mu - \sigma_e^2/2}{\sigma_\tau}\right), \tag{2.3}$$

where $\Phi(.)$ denotes the standard normal cumulative distribution function. The hypotheses of interest in our problem are

$$\mathrm{H}_0 : \theta \geqslant A \quad \text{vs.} \quad \mathrm{H}_1 : \theta < A, \tag{2.4}$$

where A is a specified quantity, usually between 0.01 and 0.10. It follows from (2.3) that the hypotheses in (2.4) are equivalent to

$$\mathrm{H}_0 : \mu + z_{1-A}\sigma_\tau + \sigma_e^2/2 \geqslant \ln(\mathrm{OEL}) \ \text{ vs. } \ \mathrm{H}_1 : \mu + z_{1-A}\sigma_\tau + \sigma_e^2/2 < \ln(\mathrm{OEL}). \tag{2.5}$$

In the following section, we shall present a generalized pivot variable for testing (2.5), and for constructing upper limits for $\eta = \mu + z_{1-A}\sigma_\tau + \sigma_e^2/2$.

3. Confidence limits and test for the proportion of the mean exposures exceeding the OEL

We shall now derive a generalized pivot variable (see Appendix for constructing a generalized pivot variable in a general setup) for hypothesis testing and interval estimation of $\eta = \mu + z_{1-A}\sigma_\tau + \sigma_e^2/2$. Let SS_e denote the error sums of squares and SS_τ denote the between sums of squares. For the case of unbalanced data, we have $SS_e = \sum_{i=1}^k \sum_{j=1}^{n_i} (Y_{ij} - \bar{Y}_{i.})^2$, where $\bar{Y}_{i.} = 1/n_i \sum_{j=1}^{n_i} Y_{ij}$, $i = 1, \ldots, k$. Furthermore,

$$\frac{SS_e}{\sigma_e^2} \sim \chi^2_{N-k},$$

where $N = \sum_{i=1}^k n_i$ and χ^2_a denotes the chi-square random variable with df$=a$. Now define

$$\tilde{n} = \frac{1}{k}\sum_{i=1}^k n_i^{-1}, \quad \bar{\bar{Y}} = \frac{1}{k}\sum_{i=1}^k \bar{Y}_{i.} \quad \text{and} \quad SS_{\bar{y}} = \sum_{i=1}^k (\bar{Y}_{i.} - \bar{\bar{Y}})^2.$$

Then

$$\bar{\bar{Y}} \sim \mathrm{N}\left(\mu, \frac{\sigma_\tau^2 + \tilde{n}\sigma_e^2}{k}\right).$$

It is well-known that $E(SS_{\bar{y}}) = (k-1)(\sigma_\tau^2 + \tilde{n}\sigma_e^2)$, and

$$\frac{SS_{\bar{y}}}{\sigma_\tau^2 + \tilde{n}\sigma_e^2} \sim \chi_{k-1}^2 \quad \text{approximately.}$$

The above approximation is due to Thomas and Hultquist (1978). Let $\bar{\bar{y}}$, $ss_{\bar{y}}$ and ss_e denote, respectively, the observed values of $\bar{\bar{Y}}$, $SS_{\bar{y}}$ and SS_e. The generalized pivot variable T is given by

$$\begin{aligned} T &= \bar{\bar{y}} + \frac{\sqrt{k}(\mu - \bar{\bar{Y}})}{\sqrt{SS_{\bar{y}}}} \frac{\sqrt{ss_{\bar{y}}}}{\sqrt{k}} + z_{1-A}\sqrt{\left[\frac{\sigma_\tau^2 + \tilde{n}\sigma_e^2}{SS_{\bar{y}}} ss_{\bar{y}} - \frac{\tilde{n}\sigma_e^2}{SS_e} ss_e\right]_+} + \frac{1}{2}\frac{\sigma_e^2}{SS_e} ss_e - \eta \\ &\overset{\mathrm{d}}{\sim} \bar{\bar{y}} + \frac{Z}{\sqrt{\chi_{k-1}^2}}\sqrt{\frac{ss_{\bar{y}}}{k}} + z_{1-A}\sqrt{\left[\frac{ss_{\bar{y}}}{\chi_{k-1}^2} - \frac{\tilde{n} ss_e}{\chi_{N-k}^2}\right]_+} + \frac{1}{2}\frac{ss_e}{\chi_{N-k}^2} - \eta, \\ &= T^* - \eta, \quad \text{say,} \end{aligned} \tag{3.1}$$

where $\overset{\mathrm{d}}{\sim}$ denotes “approximately distributed”, $\chi_{k-1}^2 = SS_{\bar{y}}/(\sigma_\tau^2 + \tilde{n}\sigma_e^2)$ and $\chi_{N-k}^2 = SS_e/\sigma_e^2$ are independent chi-square random variables and $Z = \sqrt{k}(\bar{\bar{Y}} - \mu)/\sqrt{\sigma_\tau^2 + \tilde{n}\sigma_e^2} \sim \mathrm{N}(0,1)$, and $[x]_+ = \max\{0, x\}$.

If all the n_i’s are equal, then it can be easily verified that the T in (3.1) simplifies to the generalized pivot variable given in Krishnamoorthy and Mathew (2002) for the balanced case. We also observe from (3.1) that, for a given $\bar{\bar{y}}$, $ss_{\bar{y}}$, ss_e and a specified value of η, the distribution of T is free of any unknown parameters. Using the first expression in (3.1), we see that the value of T at $(\bar{\bar{Y}}, SS_{\bar{y}}, SS_e) = (\bar{\bar{y}}, ss_{\bar{y}}, ss_e)$ is zero. Furthermore, we see that $P(T > a) = P(T^* > a + \eta)$ is decreasing with respect to η for every fixed a. That is, T is stochastically decreasing in η. Thus, T satisfies all three requirements (see Appendix) for it to be a generalized pivot variable.

3.1. Hypothesis testing about θ

We have already shown in Section 2 that testing θ is equivalent to testing $\eta = \mu + z_{1-A}\sigma_\tau + \sigma_e^2/2$. Since T in (3.1) is stochastically decreasing, the generalized p-value for testing hypotheses in (2.4) or in (2.5) is defined as

$$P(T > 0 | \eta = \ln(\mathrm{OEL})) = P(T^* > \ln(\mathrm{OEL}) | \eta = \ln(\mathrm{OEL})). \tag{3.2}$$

The null hypothesis in (2.4) will be rejected whenever the generalized p-value is less than the nominal level α.

3.2. Interval estimation of $\exp(\eta)$

For a given $0 < c < 1$, let T_c^* denote the 100cth percentile of the variable T^* in (3.1). Then, $(T_{\alpha/2}^*, T_{1-\alpha/2}^*)$ is a $100(1-\alpha)\%$ confidence interval for $\eta = \mu + z_{1-A}\sigma_\tau + \sigma_e^2/2$, and hence $(\exp(T^*_{\alpha/2}), \exp(T^*_{1-\alpha/2}))$ is a $100(1-\alpha)\%$ confidence interval for $\exp(\eta)$ (see (2.2)). Note that $\exp(T^*_{1-\alpha})$ is a $100(1-\alpha)\%$ one-sided upper limit for $\exp(\eta)$—the $(1-A)$th quantile of the distribution of the mean exposures. Therefore, at least $100(1-A)\%$ of the mean exposures are less than $\exp(T^*_{1-\alpha})$ with $100(1-\alpha)\%$ confidence. The null hypothesis in (2.5) will be rejected whenever the upper limit $\exp(T^*_{1-\alpha})$ is less than the OEL.

Since the distribution of T^*, given $\bar{\bar{y}}$, $ss_{\bar{y}}$ and ss_e, is free of any unknown parameters, the Monte Carlo method given in the following algorithm can be used to compute the generalized p-value and interval estimates for η.

Algorithm 1

For a given data set, compute $\bar{\bar{y}}$, $ss_{\bar{y}}$ and ss_e
Compute $\tilde{n} = \frac{1}{k}\sum_{i=1}^{k}\frac{1}{n_i}$, z_{1-A}, and $c = \sqrt{\frac{ss_{\bar{y}}}{k}}$, $N = \sum_{i=1}^{k} n_i$
For $i = 1, m$:
Generate $Z \sim N(0,1)$, χ^2_{k-1}, and χ^2_{N-k} variates
Compute: $D = \frac{ss_{\bar{y}}}{\chi^2_{k-1}} - \frac{\tilde{n}ss_e}{\chi^2_{N-k}}$
If $D < 0$, set $D = 0$
Compute $T_i^* = \bar{\bar{y}} + \frac{Z}{\sqrt{\chi^2_{k-1}}}c + z_{1-A}\sqrt{D} + \frac{0.5ss_e}{\chi^2_{N-k}}$
(end i loop)

Let $T^*_{1-\alpha}$ denotes the $100(1-\alpha)$th percentile of T^* in (3.1). Then the Monte Carlo estimate $\hat{T}^*_{1-\alpha}$ of $T^*_{1-\alpha}$ is the $100(1-\alpha)$th percentile of $\{T_1^*, \ldots, T_m^*\}$. For hypothesis testing in (2.5), the Monte Carlo estimate of the generalized p-value in (3.2) is [Number of $T_i^* > \ln(\mathrm{OEL})]/m$.

4. Size and power studies of the test

We shall now study the size and power properties of the generalized test proposed in Section 3.1. A Monte Carlo method of evaluating the size and power is given in the following algorithm.

Algorithm 2

For a given μ, σ_τ^2, σ_e^2, k, A, and $n_1, \ldots, n_k$:
Compute $\eta = \mu + z_{1-A}\sigma_\tau + \sigma_e^2/2$
For $j = 1, n$:
    Generate χ^2_{N-k}, $Z \sim N(0,1)$ and $\bar{y}_{l.} \sim N(\mu, (n_l\sigma_\tau^2 + \sigma_e^2)/n_l)$, $l = 1, \ldots, k$
    Compute $\bar{\bar{y}} = \sum_{l=1}^{k}\bar{y}_{l.}/k$, $ss_{\bar{y}} = \sum_{l=1}^{k}(\bar{y}_{l.} - \bar{\bar{y}})^2$ and $ss_e = \sigma_e^2\chi^2_{N-k}$
    (Use Algorithm 1 to compute the p-value)
(end j loop)

If the parameters μ, σ_τ^2, σ_e^2 and the value of A are chosen such that $\eta = \ln(\mathrm{OEL})$, then the proportion of p-values which are less than the specified nominal level α is a Monte Carlo estimate of the size; if the parameters and A are chosen so that $\eta < \ln(\mathrm{OEL})$,

Table 1
Monte Carlo estimates of the sizes and powers of the test for (2.5)

σ_τ^2/σ_e^2	σ_e^2	k	$(n_1,\ldots,n_k)$	Size	Power
0.1	0.5	5	(2, 1, 3, 4, 3)	0.019	0.059
0.1	0.5	5	(9, 8, 10, 6, 7)	0.051	0.149
0.1	0.5	5	(20, 21, 16, 22, 19)	0.047	0.182
0.1	0.5	10	(2, 3, 2, 3, 4, 3, 2, 3, 1, 2)	0.025	0.10
0.1	0.5	10	(5, 6, 7, 8, 9, 6, 7, 8, 9, 6)	0.056	0.248
0.1	0.5	10	(10, 20, 12, 13, 14, 15, 16, 17, 18, 19)	0.054	0.331
0.1	0.5	20	(5 5s, 5 10s, 5 15s,5 20s)	0.052	0.514
0.1	0.5	30	(5 6s, 5 8s, 5 10s, 5 12s, 5 14s, 5 16s)	0.051	0.701
0.25	0.5	5	(5, 7, 6, 8, 9)	0.046	0.171
0.25	0.5	5	(30, 25, 22, 29, 20)	0.049	0.214
0.25	0.5	10	(9, 7, 10, 8, 9, 6, 7, 9, 8, 10)	0.056	0.365
0.25	0.5	10	(20, 18, 15, 19, 14, 17, 12, 11, 29, 26)	0.053	0.462
0.25	0.5	20	(5 1s, 5 2s, 5 3s, 5 4s)	0.057	0.343
0.25	0.5	20	(5 3s, 5 4s, 5 5s, 5 6s)	0.048	0.537
0.5	1	5	(9, 8, 6, 5, 3)	0.055	0.191
0.5	1	5	(20, 18, 15, 12, 11)	0.049	0.228
0.5	1	10	(5, 6, 8, 6, 7, 9, 8, 9, 6, 5)	0.050	0.402
0.5	1	10	(20, 19, 11, 18, 17, 13, 14, 10, 16, 17)	0.045	0.484
0.5	1	15	(3 8s, 3 6s, 3 4s, 3 2s, 3 10s)	0.053	0.553
0.5	1	20	(4 8s, 4 6s, 4 4s, 4 2s, 4 1s)	0.054	0.587
0.5	1	20	(4 8s, 4 6s, 4 4s, 4 2s, 4 10s)	0.051	0.684
0.75	0.5	10	(2 2s, 2 4s, 3 3s, 3 5s)	0.050	0.372
0.75	0.5	16	(4 2s, 4 4s, 4 3s, 4 5s)	0.054	0.600
0.75	1	10	(2 2s, 2 4s, 3 3s, 3 5s)	0.049	0.378
0.75	1	16	(4 2s, 4 4s, 4 3s, 4 5s)	0.054	0.590
0.75	3	10	(2 2s, 2 4s, 3 3s, 3 5s)	0.049	0.349
0.75	3	18	(5 2s, 4 4s, 4 3s, 5 5s)	0.058	0.590
1	0.5	8	(2 2s, 2 4s, 2 3s, 2 5s)	0.052	0.334
1	0.5	14	(4 2s, 3 4s, 3 3s, 4 5s)	0.056	0.565
1	1	8	(2 2s, 2 4s, 2 3s, 2 5s)	0.043	0.310
1	1	14	(3 2s, 3 3s, 4 4s, 4 5s)	0.053	0.565
2.5	0.5	10	(2 2s, 3 3s, 2 4s, 3 5s)	0.056	0.486
2.5	0.5	12	(3 2s, 3 3s, 3 4s, 3 5s)	0.054	0.565
2.5	1	12	(3 2s, 3 3s, 3 4s, 3 5s)	0.048	0.559
5	1	8	(2 2s, 2 3s, 2 4s, 2 5s)	0.050	0.397
5	1	11	(2 2s, 3 3s, 3 4s, 3 5s)	0.053	0.548
5	3	8	(2 2s, 2 3s, 2 4s, 2 5s)	0.053	0.387
5	3	11	(2 2s, 3 3s, 3 4s, 3 5s)	0.045	0.529

Note: $\mu = 0$; Nominal level $\alpha = 0.05$; the size is computed at $\theta = A = 0.05$; the power is computed when $\theta = 0.002$ and $A = 0.05$.

then the proportion of p-values which are less than α is a Monte Carlo estimate of the power.

In Tables 1 and 2, we present the sizes and powers of the test for various parameter configurations considered respectively in Tables 1 and 2 of Lyles et al. (1997a). Noting that $\bar{\bar{y}}$ follows a normal distribution with mean μ, we see that the size and power of the

Table 2
Monte Carlo estimates of the sizes and powers of the test for (2.5)

σ_τ^2/σ_e^2	σ_e^2	k	$(n_1, \ldots, n_k)$	Size	Power
0.1	0.5	5	(2, 3, 4, 2, 3)	0.018	0.614
0.1	1	5	(2, 3, 4, 2, 3)	0.013	0.278
0.1	1	8	(2, 3, 4, 2, 3, 4, 2, 3)	0.022	0.601
0.1	3	9	(2 4s, 2 5s, 2 6s, 3 7s)	0.041	0.373
0.1	3	13	(3 3s, 3 4s, 3 5s, 4 6s)	0.034	0.474
0.1	3	13	(3 4s, 3 5s, 3 6s, 4 7s)	0.039	0.534
0.25	0.5	5	(2, 3, 4, 2, 3)	0.034	0.496
0.25	0.5	5	(3, 4, 5, 3, 4)	0.039	0.587
0.25	1	6	(2 4s, 2 5s, 2 6s)	0.044	0.450
0.25	1	10	(4 2s, 3 3s, 3 4s)	0.037	0.596
0.25	3	10	(4 2s, 3 3s, 3 4s)	0.033	0.166
0.25	3	18	(5 6s, 5 7s, 5 8s, 3 9s)	0.048	0.527
0.5	0.5	5	(2, 3, 4, 2, 3)	0.034	0.356
0.5	0.5	5	(2, 7, 9, 8, 6)	0.050	0.443
0.5	1	12	(3 2s, 3 3s, 3 4s, 3 5s)	0.047	0.547
0.5	1	18	(4 2s, 4 3s, 5 4s, 5 5s)	0.051	0.752
0.75	0.5	8	(2, 3, 4, 2, 3, 4, 2, 3)	0.045	0.530
0.75	1	12	(3 2s, 3 3s, 3 4s, 3 1s)	0.052	0.372
0.75	1	20	(5 2s, 5 3s, 5 4s, 5 1s)	0.051	0.570
0.75	3	50	(10 2s, 10 3s, 10 4s, 10 1s 10 5s)	0.052	0.59
0.75	3	50	(10 2s, 10 3s, 10 4s, 10 6s 10 5s)	0.043	0.574
1	0.5	8	(4, 5, 6, 4, 5, 6, 4, 5)	0.050	0.485
1	0.5	10	(3 2s, 3 3s, 2 4s, 2 1s)	0.054	0.529
1	1	20	(5 2s, 5 3s, 5 4s, 5 5s)	0.051	0.560
1	3	50	(10 2s, 10 3s, 10 4s, 10 5s 10 6s)	0.057	0.590
2.5	0.5	20	(5 2s, 5 3s, 5 4s, 5 5s)	0.053	0.559
2.5	1	35	(7 2s, 7 3s, 7 4s, 7 5s 7 6s)	0.052	0.570
2.5	3	25	(5 2s, 5 3s, 5 4s, 5 5s 5 6s)	0.050	0.567
5	1	35	(7 2s, 7 3s, 7 4s, 7 5s 7 1s)	0.053	0.630
5	3	10	(2 2s, 2 3s, 2 4s, 2 5s 2 1s)	0.047	0.494

Note: $\mu = 0$; Nominal level $\alpha = 0.05$; the size is computed at $\theta = A = 0.10$; the power is computed when $\mu_x/OEL = 0.2$, where $E(X_{ij}) = \mu_x = \exp\{\mu + (\sigma_\tau^2 + \sigma_e^2)/2\}$ is the overall mean, and $A = 0.10$.

test do not depend on μ, and hence without loss of generality we can take μ to be zero in our simulation studies. We used Algorithm 2 with $n = 2500$ and Algorithm 1 with $m = 5000$ to compute the sizes and powers. We observe from the table values that the sizes of the test are always close to the nominal level 0.05 except for the cases where σ_e^2 is much larger than σ_τ^2. In these cases, our present test seems to be conservative. The power of the test is increasing as the values of k (sample size in one-way random effects model) increasing, which is a desirable property. In general, if the values of k are from 20 to 25, and the n_i's are greater than or equal to 2, then power is expected to be around 0.60. The power of 0.60 is chosen in order to be consistent with Tables 1 and 2 of Lyles et al. (1997a). If $\sigma_\tau^2 > 1.5\sigma_e^2$, then a sample of 15 to 20 units with $1 \leqslant n_i \leqslant 5$, $i = 1, \ldots, k$, would be enough to attain a power of about 0.60.

The properties of the confidence limit in Section 3.2 are evident from the size properties of the test. The confidence limits are expected to be conservative (coverage probability is more than the confidence level) when σ_τ^2 is much smaller than σ_e^2 and/or sample size k is very small. Otherwise the coverage probabilities are expected to be close to the confidence level.

5. Examples

We shall now illustrate the methods of the preceding sections using three sets of shift-long exposure data reported in Tables D1, D2, and D3 of Lyles et al. (1997b). The data in Tables D1, D2, and D3 represent nickel dust exposure measurements on a sample of furnacemen from a refinery, on a sample of maintenance mechanics from a smelter, and on a sample of maintenance mechanics from a mill respectively. These data were collected from these three groups of workers from a nickel producing facility. For these data sets, we computed the values of $\bar{\bar{y}}$, $ss_{\bar{y}}$, ss_e and $\tilde{n}$ using the formulas given in Section 3, and reported them in Tables 3, 4 and 5. The p-values and the one-sided limits are computed using Algorithm 1 (for testing (2.5) and constructing limits for $\exp(\eta)$ in Section 3.2). For each case, we used 100,000 simulation runs. For all these examples, we take $A = 0.10$ and $\alpha = 0.05$.

Table 3
Test and 95% one-sided upper limit for nickel exposure data in Table D1 of Lyles et al. (1997b) $k = 12$, $N = 27$

$\bar{\bar{y}}$	$\tilde{n}$	$ss_{\bar{y}}$	ss_e	p-value for (2.5)	U lim for $\exp(\eta)$
−0.660	0.649	10.850	22.196	0.886	6.1410

Table 4
Test and 95% one-sided upper limit for nickel exposure data in Table D2 of Lyles et al. (1997b) $k = 23$, $N = 34$

$\bar{\bar{y}}$	$\tilde{n}$	$ss_{\bar{y}}$	ss_e	p-value for (2.5)	U lim for $\exp(\eta)$
−3.683	0.855	16.081	2.699	0	0.1125

Table 5
Test and 95% one-sided upper limit for nickel exposure data in Table D3 of Lyles et al. (1997b) $k = 20$, $N = 28$

$\bar{\bar{y}}$	$\tilde{n}$	$ss_{\bar{y}}$	ss_e	p-value for (2.5)	U lim for $\exp(\eta)$
−4.087	0.854	19.681	9.801	0.004	0.1480

Note: To compute the p-values and confidence limits in Tables 3–5, we used $A = 0.10$, $\alpha = 0.05$, OEL = 1 mg m^{-3}.

We present the results for the furnacemen workers in Table 3. The OEL for nickel dust exposure is 1 mg m^{-3}. The p-value for testing the proportion of mean exposures is 0.886, and hence the data do not provide sufficient evidence to indicate that 90% of the mean exposures are less than the OEL. Furthermore, we see that 90% of the mean exposures of this work group are less than 6.141 mg m^{-3} (with 95% confidence).

The results for the group of maintenance mechanics from a smelter are given in Table 4. Here, we see that exposure assessment strategy based on the proportion of exposure data shows that the exposure levels are within the OEL=1 mg m^{-3} (p-value= 0). We also see that at least 90% of the exposure data are less than 0.1225 mg m^{-3} for this group of workers. We also computed 95% upper limit as 0.748 mg m^{-3} when $A = 0.001$. This means that 99.9% of the mean exposures are less than 0.748 mg m^{-3} with confidence 0.95.

In Table 5 we present the results for the exposure data collected from a group of maintenance mechanics from a mill. The OEL is 1 mg m^{-3}. We observe that the 90% of the mean exposures are within 0.148 mg m^{-3} with 95% confidence. When $A = 0.001$, the 95% upper limit is 0.644 mg m^{-3}, which indicates that 99.9% of the mean exposures in this group are less than 0.644 mg m^{-3}.

We note that our conclusions are in agreement with those based on large sample tests given in Lyles et al. (1997b) for all three examples.

6. Concluding remarks

In this article we provided generalized p-value and confidence interval methods for computing one-sided limits and hypothesis testing about the proportion of the mean exposures exceeding the OEL. As already mentioned, if the data are balanced, then our methods simplify to the ones given in Krishnamoorthy and Mathew (2002) for the balanced case. Our Algorithms 1 and 2 can be used to compute the interval estimates and p-values using softwares such as SAS, S-PLUS and Fortran with IMSL. Our numerical studies clearly indicate that the proposed tests are satisfactory in controlling Type I error rates, and are applicable for small samples. The power calculations reported in Tables 1 and 2 can be used as guideline in determining the sample of individuals needed (for exposure evaluation) to attain a power of 0.60 at the level of 0.05. The results of this article are presented in the context of occupational exposure data analysis so that industrial hygienists and other safety staffs can easily adopt these methods for assessing exposure levels in workplaces.

Acknowledgements

The work of the first author was supported by grant R01-0H03628-01A1 from the National Institute of Occupational Safety and Health (NIOSH).

Appendix

We here describe the method of constructing a generalized pivot variable and the definition of the generalized p-value in a general setup. Let X be a random variable (could be a vector) whose distribution depends on the parameters (θ, δ), where θ is a scalar parameter of interest, and δ is a nuisance parameter. Suppose we are interested in testing the hypotheses

$$\mathrm{H}_0 : \theta \leqslant \theta_0 \quad \text{vs.} \quad \mathrm{H}_a : \theta > \theta_0, \tag{A.1}$$

where θ_0 is a specified quantity. Let x denote the observed value of X. In other words, x is known after the data have been collected. A generalized pivot variable, denoted by $T_1(X; x, \theta, \delta)$, is a function of X, x, θ and δ, and satisfies the following conditions:

(i) For a fixed x, the distribution of $T_1(X; x, \theta, \delta)$ is free of the nuisance parameter δ.
(ii) The value of $T_1(X; x, \theta, \delta)$ at $X = x$ is free of any unknown parameters.
(iii) For fixed x and δ, the distribution of $T_1(X; x, \theta, \delta$ is stochastically increasing or stochastically decreasing in θ. That is, $P(T_1(X; x, \theta, \delta) \geqslant a)$ is an increasing function of θ, or is a decreasing function of θ, for every a.

Let $t_1 = T_1(x; x, \theta, \delta)$, the value of $T_1(X; x, \theta, \delta)$ at $X = x$. If $T_1(X; x, \theta, \delta)$ is stochastically increasing in θ, the generalized p-value for testing the hypotheses in (A.1) is given by

$$\sup_{H_0} P(T_1(X; x, \theta, \delta) \geqslant t_1) = P(T_1(X; x, \theta_0, \delta) \geqslant t_1),$$

and if $T_1(X; x, \theta, \delta)$ is stochastically decreasing in θ, the generalized p-value for testing the hypotheses in (A.1) is given by

$$\sup_{H_0} P(T_1(X; x, \theta, \delta) \leqslant t_1) = P(T_1(X; x, \theta_0, \delta) \leqslant t_1).$$

Note that the computation of the generalized p-value is possible because the distribution of $T_1(X; x, \theta, \delta)$ is free of the nuisance parameter δ and $t_1 = T_1(x; x, \theta, \delta)$ is free of any unknown parameters (see (i) and (ii) in (A.2)).

A generalized confidence interval for θ is computed using the percentiles of a generalized pivot variable, say $T_2(X; x, \theta, \delta)$, satisfying the following conditions:

(i) For a fixed x, the distribution of $T_2(X; x, \theta, \delta)$ is free of all unknown parameters.
(ii) The value of $T_2(X; x, \theta, \delta)$ at $X = x$ is θ, the parameter of interest.

Appropriate quantiles of T_2 form a $1-\alpha$ confidence limit for θ. For example, if $T_2(x; p)$ is the pth quantile of $T_2(X; x, \theta, \delta)$, then $(T_2(x; \alpha/2), T_2(x; 1-\alpha/2))$ is a 95% confidence interval for θ. Usually, $T_2(X; x, \theta, \delta) = T_1(X; x, \theta, \delta) + \theta$.

For further details on the concepts of generalized p-values and generalized confidence intervals, we refer to the book by Weerahandi (1995, Chapters 5 and 6).

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