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# Estimates of Lifetime Risk of Occupational Fatal Injury from Age-Specific Rates

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#### **ABSTRACT**

The lifetime risk of fatal workplace injury is a critical issue in the evaluation of occupational hazards. Recently, Fosbroke, Kisner, and Myers (1997) described a metric for working lifetime risk (WLTR) to determine the probability that a worker will die due to a work-related fatal injury in a year over a certain number of years of employment. This quantity was defined assuming that the annual rate of fatal injuries will be the same each year during employment. Recognizing the fact that annual fatal injury rates differ with the age of the worker along with other factors, modification of the definition of working lifetime risk is derived. We obtain the estimates of the lifetime risk using agecategorized annual fatality rates and derive an estimate of the standard error of the WLTR estimator and a confidence interval for the WLTR. We illustrate these calculations by estimating the lifetime risk for work-related fatal injuries for workers in four high-risk industries: agriculture-forestry-fishing, mining, construction, and transportation-public utilities. The estimates are based on employment data from the Bureau of Labor Statistics and an updated version of fatality data from the National Traumatic Occupational Fatalities surveillance system.

Key Words: confidence interval, delta method, risk assessment

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#### INTRODUCTION

The assessment of hazards associated with occupational fatal injuries is a broad public health concern with thousands of workers dying each year in the United States. A first step in the assessment of these hazards is to quantify risk or rates of fatal injuries. While rates are commonly employed in epidemiology studies, the lifetime probability of the occurrence of a certain event, *i.e.*, the *risk* of the event, is a more natural endpoint for risk estimation purposes. Lifetime risks of occupational fatal injury are of interest for comparison with the assessment of hazards associated with occupational disease or illness.

In an attempt to estimate the lifetime risk for workers in a specific industry, Fosbroke, Kisner, and Myers (1997) used the lifetime risk formula:

WLTR<sub>F</sub> = 
$$\left[1 - \left(1 - R\right)^{y}\right] \times 1000$$
 (1.1)

where R = P (a worker having a work-related fatal injury in a given year),  $0 \le R \le 1$ , and y is the number of years of exposure to work-related injury. Here, y was set at 45 years, which could represent an employment pattern where a worker starts work at age 20 and continues until retirement at age 65. This time period is consistent with work-life values used in many quantitative risk assessments for occupational illnesses (Stayner, 1992). The formula WLTR $_{\scriptscriptstyle \rm E}$ expresses the risk as a rate over a hypothetical group of 1000 individuals who start working at age 20 and work to age 65 in the same industrial division. Fosbroke et al. (1997) estimated R in formula (1.1) from data from the National Institute for Occupational Safety and Health (NIOSH) National Traumatic Occupational Fatal (NTOF) surveillance system for the calendar year 1990 to 1991. The lifetime risk formula (1.1) assumes that the annual risk of fatal injury is constant for a working lifetime. However, the fatality rate R may vary with other covariates, such as number of years at work, age, calendar time, industry, and occupation. For example, as a worker gets more experienced at a job, he or she may experience less risk of a fatal injury; or the risk of fatal injury may increase with the age of the worker. The risk may depend on years of experience and age. Some injury prevention efforts, such as engineering controls, personal protective equipment, and training programs for new workers and new safety equipment may contribute to a further decline of the fatal injury rates. Thus, it may not be reasonable to assume that the annual rate is constant over a working lifetime. This will be illustrated with NTOF data in a subsequent section.

The purpose of this paper is to provide an estimate of the working lifetime risk that allows for the annual risk, R in (1.1), to change at different ages. We illustrate this with calculations that allow for different annual rates of fatal injury at different age intervals and illustrate how the working lifetime risk could be adjusted for a particular covariate. To measure the precision of the estimate, we obtain the standard error and confidence interval for the age-adjusted working lifetime risk estimator that we propose.

#### **METHODS**

# Calculation of Working Lifetime Risk of Death

The working lifetime risk can be estimated by grouping the age scale into a number of categories  $[a_k,a_{k+1})$ . Five-year intervals are often used for this purpose. We assume that the working lifetime corresponds to employment from the age of 20 to 65. The survival times are grouped into intervals of five years  $[a_k,a_{k+1})$  where  $a_k=a_1+5(k-1)$  and  $a_{k+1}=a_k+5$ ,  $k=1,2,\ldots,9$  with  $a_1=20$ . The annual risk of death (*i.e.*, the probability of a worker having a work-related fatal injury in a given year) is assumed to be constant over each categorized age interval  $[a_k,a_{k+1})$ . Let  $R_k$ ,  $0 \le R_k \le 1$ , denote the annual risk of death associated with the age interval  $[a_k,a_{k+1})$ . Then, assuming independence of the occurrence of events between the years, we have

$$\lambda_{k} = 1 - \left(1 - R_{k}\right)^{5} \tag{2.1}$$

 $k = 1, 2, \dots, 9$ . The working lifetime risk (WLTR), the probability of dying over a working lifespan due to work-related injuries for workers in a specific industry (given survival to age  $a_1 = 20$ ) is then

WLTR = 
$$1 - \prod_{k=1}^{9} (1 - \lambda_k) = 1 - \prod_{k=1}^{9} (1 - R_k)^5$$
 (2.2)

assuming independence between the events occurring in two different ageintervals.

# **Interval Estimation of the Working Lifetime Risk**

We now describe an interval estimation method to provide a measure of precision for the estimate of individual working lifetime risk. We demonstrate this by obtaining the standard error of the estimator WLTR in (2.2) and the endpoints of the confidence interval.

Let the random variable  $D_k$  denote the number of deaths in the kth 5-year age period,  $I_k = [a_k, a_{k+1}), k = 1, 2, \ldots, 9$ . We assume that the  $D_k$  are independently distributed with Poisson distribution with mean  $\mu_k = N_k \lambda_k$ .  $N_k$  denotes the number of workers alive at the start of the interval  $[a_k, a_{k+1})$ , and  $\lambda_k$  represents the risk of fatal injury during the 5-year age interval.

Let  $\hat{R}_k$  denote the estimated annual risk of death during the years over the age interval  $I_k = [a_k, a_{k+1})$ . Note that each year in the interval  $I_k$  is assumed to have the same annual fatal injury rate. Suppose these estimates  $\hat{R}_k$  are given for  $k = 1, 2, \ldots, 9$ . Then the estimates of 5-year risks  $\hat{\lambda}_k$  are given by  $\hat{\lambda}_k = 1 - (1 - \hat{R}_k)^5$ . In this case, the estimator of the working lifetime risk (WLTR) is given by

$$1 - \prod_{k=1}^{9} \left( 1 - \hat{\lambda}_k \right) = 1 - \prod_{k=1}^{9} \left( 1 - \hat{R}_k \right)^5$$
 (2.3)

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Under the above assumption, it can be shown that the maximum likelihood estimator for  $\lambda_k$  is

$$\lambda_k$$
 is  $\hat{\lambda}_k = \frac{D_k}{N_k}$ 

To determine the lower and upper bounds of the confidence interval for the WLTR estimator, we obtain approximate standard errors for the estimates of WLTR by applying the delta method (Bishop, Feinberg, and Holland, 1975). If the variance of  $\hat{\lambda}_k$  is known, the method provides an approximation to the variance of the WLTR estimator, which is a function of the  $\hat{\lambda}_k$ , and therefore a function of  $\hat{R}_k$  as in (2.3). We denote this function  $W(\hat{\Lambda}) = W(\hat{\lambda}_1, \hat{\lambda}_2, \ldots, \hat{\lambda}_g)$ . Then, the asymptotic distribution of the estimate of WLTR,  $W(\hat{\Lambda}) = 1 - \prod_{k=1}^g (1 - \hat{\lambda}_k)$ , is given by

$$W(\hat{\Lambda}) \sim N \left( 1 - \prod_{1}^{9} (1 - \lambda_{k}), \prod_{k=1}^{9} (1 - \lambda_{k})^{2} \sum_{k=1}^{9} \frac{\lambda_{k}}{N_{k} (1 - \lambda_{k})^{2}} \right)$$
 (2.4)

Technical details for the delta method to obtain the asymptotic distribution of the WLTR in (2.4) are provided in the Appendix. The standard error of the estimate of WLTR, *s.e.* ( $W(\hat{\Lambda})$ ), is the square root of the estimated variance of  $W(\hat{\Lambda})$ . It follows that an approximate  $100(1 - \alpha)\%$  confidence interval for WLTR,  $W(\hat{\Lambda})$ , is

$$W(\hat{\Lambda}) \pm z_{\alpha/2} s.e.(W(\hat{\Lambda}))$$
 (2.5)

where  $z_{\alpha/2}$  represents the upper tail  $\alpha/2$  critical value from a standard normal distribution.

# **RESULTS**

In this section, we calculate the working lifetime risks for the four most hazardous industries in the United States. We estimate these values (as rates over every 1000 workers) by applying the formula derived in (2.2) based on age-specific annual fatality rates, along with a measure of precision. For comparison purposes, we also provide estimates of the previously studied working lifetime risk based on formula (1.1) for the four industries. We first describe the data that we use in this section.

The data from the NTOF surveillance system are combined with data on employment from the United States Bureau of Labor Statistics (BLS). This updated version of the data provides occupational fatal injury counts and the number of employed over the years from 1983 to 1992, respectively. The NTOF database is a death certificate-based census of occupational fatal injuries in the U.S. containing case data of deaths, including year of death, gender,

race (white, black, other), age (collapsed into 9 intervals for our analysis), industry and occupation. The BLS employment data are based on unpublished data tabulated from the current population survey (U.S. Bureau of the Census, 1978) constructed in response to an interagency agreement between BLS and NIOSH. The fatal injury rates along with a complete description of the classification of industries and occupations are given in Bailer *et al* (1998). The identification of the four most hazardous industries is presented in Table 1. The table also includes annual fatal injury rates along with the fatality counts and number employed in 1992 for these four industries. The industry with the highest fatal injury rate was "mining" with 0.2114 deaths per 1000 workers, followed by "agriculture-forestry-fishing", "construction", and "transportation — public utilities."

The annual mortality rates across the four industries in 5-year age intervals from 20 to 65 based on 1992 data are given in Table 2. The third column of the table presents the age-specific annual fatality rates  $\hat{R}_{\ell}$  which are calculated by

$$\hat{R}_k = \frac{\left( \begin{array}{c} \text{Number of fatal injuries occurring in 1992 among workers} \\ \text{in the $k$th age category} \\ \hline \left( \begin{array}{c} \text{Number of workers in the $k$th age category} \\ \text{at the beginning of 1992} \end{array} \right)$$

Table 1. Annual fatality rates of the four most hazardous industries based on 1992 data.

Industry	Description (e.g.)	Annual fatal. rate $\hat{R}$ (/1000)	No. dead	Person- years
Agriculture, forestry, and fishing	Crops, livestock	0.1519	423	2,785,607
Mining	Metal, coal, oil and gas, sand and grave	0.2114 l	136	643,253
Construction	General and heavy construction, special trade construction	0.1166	786	6,740,071
Transportation and public utilities	Railroad, cabs, trucking, water and air transportation	0.0928	743	8,008,784

This table indicates that the risk of fatal injury varies by age. For these industries, the rates generally decrease or are not changing from the age interval [25,30) through the interval [40,45). Then from age interval [45,50), the rates tend to increase with the highest fatal injury rates occurring in the oldest workers, in the interval [60,65). In the mining industry, the younger age groups [20,25) and [25,30) have the highest annual fatality rates of over 0.32 and 0.33 deaths, respectively, for every 1000 workers.

We now obtain the working lifetime risk for the industries by using the age-specific annual risk of death given in Table 2. We follow the procedures described in the previous section assuming that the annual risk is constant over each given 5-year age interval. The lifetime risk formula (2.3) estimates the average lifetime risk for workers in a given industry. We calculate WLTR based on age-specific fatality rates  $\hat{R}_k$  from Table 2 as a rate over every 1000 workers, by the following formula

WLTR<sub>A</sub> = 
$$\left[1 - \prod_{i=1}^{9} \left(1 - \hat{R}_{k}\right)^{5}\right] \times 1000$$
 (3.1)

The estimates WLTR $_{\rm A}$  for the four industries are reported on the second column of Table 3. Among these four industries, the mining industry has the highest risk with a 95% confidence interval (9.08, 10.84) per 1000 workers while the transportation and public utility industry has the lowest risk (4.28, 4.62) per 1000 among the four industries. The last column of Table 3 provides working lifetime risks, which are calculated by using the formula WLTR $_{\rm F}$  in (1.1) with the annual fatality rates given in Table 1. These estimates are not much far from the estimates by using age-specific annual rates. In fact, the estimates by WLTR $_{\rm F}$  for the agriculture-forestry-fishing and mining industries are within the bounds of the 95% confidence intervals of the corresponding WLTR $_{\rm A}$  values. The estimates for the other two industries are out of the ranges of the 95% confidence intervals for the estimates WLTR $_{\rm A}$ . In all four industries, the estimates WLTR $_{\rm A}$  appear to be slightly higher than the estimates WLTR $_{\rm F}$ .

#### DISCUSSION AND CONCLUSION

The lifetime fatal injury risks WLTR<sub>A</sub> presented in Table 3 are similar to the estimates WLTR<sub>F</sub> based on formula (1.1), although the estimates WLTR<sub>A</sub> are always greater, ranging from 3 to 9 percent. Our proposed modification for estimating WLTR accommodates the possibility of age-related effects. We also illustrated how to determine a standard error and confidence interval for a WLTR estimator.

A projection of risk for a lifetime is clearly an extrapolation well beyond available data. We chose to base our risk extrapolations on the 1992 data since it was the most recent year observed. Given that 10 years of data were available for this analysis, we could have used regression estimates of the annual fatality

Table 2. Summary calculations for age-categorized annual risk for four high-risk industries based on 1992 data.

		Annual fatal. rate	No.	No.
Industry	Age category	$\hat{R}_k$ (/1000)	dead	employed
Agriculture,	20 to 24	0.1162	39	335,615
forestry,	25 to 29	0.1535	55	358,341
and fishing	30 to 34	0.1322	57	341,034
	35 to 39	0.1395	56	401545
	40 to 44	0.1591	52	326,926
	45 to 49	0.1018	27	265,354
	50 to 54	0.2125	49	230,566
	55 to 59	0.2158	48	222,449
	60 to 64	0.1871	40	213,377
Mining	20 to 24	0.3233	11	34,024
, and the second	25 to 29	0.3370	21	62,323
	30 to 34	0.2147	24	111,766
	35 to 39	0.1992	27	135,576
	40 to 44	0.1856	20	107,769
	45 to 49	0.1693	13	76,781
	50 to 54	0.1329	8	60,208
	55 to 59	0.2151	7	32,542
	60 to 64	0.2246	5	22,264
Construction	20 to 24	0.1086	76	700,123
	25 to 29	0.1176	119	1,012,109
	30 to 34	0.0934	115	1,231,214
	35 to 39	0.0997	111	1,113,819
	40 to 44	0.1160	105	905,213
	45 to 49	0.1393	89	639,058
	50 to 54	0.1397	72	515,492
	55 to 59	0.1339	56	389,036
	60 to 64	0.1838	43	234,007
Transportation	n 20 to 24	0.0754	42	557,170
and public	25 to 29	0.0868	83	956,553
utilities	30 to 34	0.0820	108	1,317,553
	35 to 39	0.0932	122	1,309,324
	40 to 44	0.0827	110	1,330,118
	45 to 49	0.0966	103	1,066,726
	50 to 54	0.1196	84	702,150
	55 to 59	0.0984	50	508,382
	60 to 64	0.1572	41	260,808

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Table 3. Working lifetime risks WLTR $_{\rm A}$  and WLTRF and the confidence intervals of WLTR $_{\rm A}$  for the four industries based on 1992 data. All risks are per 1000 workers.

Industry	Lower bound of a 95% CI	WLTR <sub>A</sub> <sup>a</sup>	Upper bound of a 95% CI	WLTR <sub>F</sub> <sup>b</sup> (Diff. in %)
Agriculture, forestry, and fishing	6.75	7.06	7.37	6.81 (3.72%)
Mining	9.08	9.96	10.84	9.47 (5.17%)
Construction	5.49	5.69	5.90	5.23 (8.77%)
Transportation and public utilities	4.28	4.45	4.62	4.17 (6.79%)

<sup>&</sup>lt;sup>a</sup> Estimated WLTR<sub>A</sub> with age-specific annual risk  $\hat{R}_k$  (from Table 2) based on 1992 data.

rates based on the data over the 10-year period from 1983 to 1992. However, a projection of a lifetime risk is an extrapolation that should be calculated with extreme caution; the potential problem surfaces if we attempt to extrapolate this prediction into a distant future that is far beyond the range of the data. We were concerned that using such a year-adjusted prediction of annual fatal injury risks would underestimate the true risk. Thus, to avoid the possibility of danger of projecting the rates to future calendar years, we suggest a conservative approach by using the most recent data available. While this may not be ideal, it may provide a conservative upper bound on the annual risk of occupational fatal injury.

In our calculation of WLTR $_{\rm A}$ , we assumed that the occupational fatal injury risk was constant over 5-year age intervals. The estimates WLTR $_{\rm A}$  did not differ dramatically from the estimates WLTR $_{\rm F}$ . The greatest difference between the two estimates was observed for construction while the smallest difference was observed for agriculture-forestry-fishing. Examining Table 1 gives insight into why this is observed. Note that fatal injury risk generally increases with age for construction workers, while the risk by age category in workers in agriculture-forestry-fishing is more variable with the smallest risk observed in 45- to 49-year-old workers. The WLTR $_{\rm F}$  essentially pools all of the data across all age categories which weights the data towards the age categories with a larger

Estimated previously studied WLTR<sub>F</sub> (formula (1.1)) with annual risk  $\hat{R}_k$  (from Table 1) based on 1992 data.

number of workers. In construction, the younger workers at lower risk of fatal injury are weighted more heavily in the calculation of  $WLTR_F$  because most construction workers are younger than 44 years of age (68%). This leads to a difference from the estimate by  $WLTR_A$  because of the increasing age-related trends in fatal injuries for the construction industry. In contrast, little difference between the two estimates would be expected for agriculture-forestry-fishing since the fatal injury risk does not vary systematically with age category.

As a final observation, the calculation of the WLTR of occupational injury might be incorporated in a comparative risk exercise. For example, the comparison of occupational fatal injuries and death due to lung cancer associated with exposure to some occupational carcinogen might be conducted. This calculation has an inherent shortcoming in that death due to lung cancer is generally a disease of older ages while occupational fatal injuries usually occur in younger workers (median age of death for workers in the NTOF data registry over the years 1983 through 1992 was 35 years). For a comparative risk calculation, other measures of injury have impact; for example, the years of potential life lost should be considered in addition to WLTR. In addition, lifetime risk assessments for illness are often adjusted for competing risks (e.g., death due to heart disease is a competing risk for lung cancer death in smokers) which could be incorporated into a future extension of the methods we describe.

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# **APPENDIX**

# **Asymptotic Distribution of WLTR**

The asymptotic distribution of the estimates of WLTR given in (2.4) is obtained as follows:

1. For  $D_k \sim Poisson(\mu_k = N_k \lambda_k)$  such that  $Var(D_k) = N_k \lambda_k$  and

$$Var(\hat{\lambda}_k) = Var(\frac{1}{N_k} \times D_k) = \frac{1}{N_k^2} \times N_k \lambda_k = \frac{\lambda_k}{N_k}$$

To find the variance of  $W(\hat{\Lambda})$ , we apply the delta method twice — first to  $\ln(1 - W(\hat{\Lambda}))$  and then to  $\exp(\ln(1 - W(\hat{\Lambda})))$ :

2. Let 
$$\hat{\theta} = \ln(1 - W(\hat{\Lambda}))$$
 and  $g(\hat{\lambda}_k) = \ln(1 - \hat{\lambda}_k)$ .

Then 
$$\hat{\theta} = \sum_{k=1}^{9} \ln(1 - \hat{\lambda}_k) = \sum_{k=1}^{9} g(\hat{\lambda}_k).$$

Applying the delta method, we obtain the variance of  $g(\hat{\lambda}_k)$  by

$$Var(g(\hat{\lambda}_k)) \approx Var(\hat{\lambda}_k)(g'(\lambda_k))^2 = \frac{\lambda_k}{N_k} \left(\frac{1}{1 - \lambda_k}\right)^2$$

Thus, the asymptotic distribution of  $\hat{\theta}$  is given by

$$\hat{\theta} = \ln\left(1 - W(\hat{\Lambda})\right) \sim N\left(\sum_{k=1}^{9} \ln\left(1 - \lambda_{k}\right), \sum_{k=1}^{9} \frac{\lambda_{k}}{N_{k}\left(1 - \lambda_{k}\right)^{2}}\right)$$

3. Since  $W(\hat{\Lambda}) = 1 - \exp(\hat{\theta})$ , by using the delta method again, we have

$$Var(W(\hat{\Lambda})) \approx (\exp(\theta))^{2} Var(\hat{\theta})$$

$$= \prod_{k=1}^{9} (1 - \lambda_{k})^{2} \sum_{i=1}^{9} \frac{\lambda_{k}}{N_{k} (1 - \lambda_{k})^{2}}$$

$$as \left(\frac{d}{d\theta} \exp(\theta)\right)^{2} = (\exp(\theta))^{2} = \left[\exp\left(\sum_{k=1}^{9} \ln(1 - \lambda_{k})\right)\right]^{2} = \prod_{k=1}^{9} (1 - \lambda_{k})^{2}$$

Thus, the asymptotic distribution of WLTR,  $W(\hat{\Lambda}) = 1 - \prod_{k=1}^{9} (1 - \hat{\lambda}_k)$ , is given by (2.4).