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Short Communication

Applying the Discrete Vortex Method in Environmental Fluid Mechanics: A Study of the Time-Averaged Near Wake behind a Circular Cylinder

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Abstract. This work illustrates the discrete vortex method (DVM) as a tool for simulating environmental fluid mechanics problems involving transport in the wake of a bluff body. The DVM was used to model both the long-time-averaged and instantaneous features of flow past a circular cylinder. Simulations were performed for Re $= 140,000$. Verification testing was accomplished by refining time-step and vortex element circulation. The DVM was validated through comparison with experimental data from Cantwell and Coles. Verification testing demonstrated that, while global convergence is not possible for an unsteady flow simulation, it is possible to have ‘convergence to physical results’. This verification entails identifying a range of parameters in which a flow containing coherent structures and matching physical conditions is attainable. Validation tests demonstrated excellent agreement between experimental and simulated results for time-averaged velocity and shear stress profiles, as well as Strouhal number.

Key words: bluff bodies, computational fluid dynamics, environmental fluid mechanics, near wake, vortex methods

Abbreviations: CFL – Courant–Friedrichs–Lewy; DVM – Discrete Vortex Method; LES – large eddy simulation; C – constant; D – cylinder diameter; f – vortex shedding frequency; h – vortex sheet length; L_B – recirculation zone length; N – number of vortex elements; n_e – number of vortex elements; n_{gen} – number of vortex sheet generation points; Re – Reynolds number; Str – Strouhal number; t – time; U – velocity; Δt – time-step; ξ – vortex circulation.

1. Introduction

Many important problems in environmental fluid mechanics involve the transport of contaminants in air and water. For high Reynolds number flows, partic-

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ularly in wakes, an important part of the transport is governed by large-scale, time-dependent eddies. Numerical simulations of such phenomena are challenging since time dependent resolution of the flow is required at scales that are small compared to the averaging time required. In addition, the specification of empirical turbulence closures leads to uncertainty and loss of generality. Vortex methods are a particularly appealing tool for dealing with such problems. They offer a direct simulation of the Navier–Stokes equations in which a turbulence model is not required; and, they are spatially adaptive because vortex elements automatically cluster in regions of steep gradients.

This brief communication presents the results of verification and validation studies on the long-time-averaged flow past a two-dimensional circular cylinder using the Discrete Vortex Method (DVM). Although the DVM has been in use for over thirty years, there has not been a validation for this important, and now frequently cited, test case. A well-referenced experimental data set at Reynolds number, Re = 140,000 [1] is used as the validation data set for comparison with the simulated velocity field. Time-averaged and instantaneous features of the solution are examined. This work employs the original form of the DVM with a numerical boundary layer [2, 3]. Details of the DVM are presented in these references and, for brevity, are not repeated here. This work illustrates that the DVM produces a reasonably accurate representation of true physical behavior over an $x - y$ plane for uniform airflow past the two-dimensional cylinder. These results are comparable to three-dimensional LES simulations.

The code employed here was used originally by Flynn and Miller [4] and has been adapted for this work. The verification studies follow the method of Sethian and Ghoniem [5] by simultaneously refining the time-step, Δt, and number of vortex elements, n_e, until coherent eddies appear, and the vortex shedding frequency, f, and recirculation zone length, L_B, stabilize. The solution is then validated with the experimental data obtained by Cantwell and Coles [1], as presented in the results section. The discussion section lends an analysis of strengths and weaknesses of the simulation in light of assumptions made with this simulation and in comparison with other available simulations.

Although there have been many improvements to the original DVM, including the use of multipole accelerators for the N-body problem, higher order core functions, and improved boundary layer models, our objective here is to capture accurately the downstream velocity field in the near-wake. In particular we are concerned with applications of the DVM to problems involving contaminant transport in the near wakes of people and buildings. Thus vortex shedding and the correct long-time behavior of the flow field are critical. The use of multipole accelerators, while computationally efficient, introduces additional error into the calculations and they are not employed here. These simulations may thus provide information on the impact of these errors in long time calculations. The DVM used here is not optimal for situations where it is important to determine drag.

2. Verification of Vortex Methods

Verification is difficult for vortex methods for two reasons. When the flow exhibits vortex shedding, the instantaneous velocity field cannot be compared on a point-wise basis with experimental results [6]. However, when input parameters are sufficiently refined, as shown in Anderson *et al.* [7], a symmetric, steady solution arises in the absence of perturbation. Hence, continued refinement can produce a non-physical solution, as cautioned by Anderson *et al.* [8]. They also warn that an improper designation of the perturbation may result in nonphysical results.

Sethian [9] lists the objectives of a verification and validation study for fluid dynamics modeling:
- predicting qualitative behavior of the fluid mechanism, regardless of the level of refinement of numerical parameters;
- convergence of average characteristics of the flow field, such as vortex shedding frequency and average velocity profiles; and,
- accurate representation of the physical flow.

These criteria will henceforth be called ‘physical convergence’. For the vortex method, this means that (1) numerical convergence is demonstrated with respect to the average characteristics of the flow, and (2) the vortex shedding that would be lost upon further parameter refinement is maintained. Given the Courant–Friedrichs–Lewy (CFL) constraint:

$$\Delta t \leq \frac{h}{U_{\max}} \tag{1}$$

and Puckett’s condition [10].

$$\xi_{\max} \leq \frac{Ch^2}{\Delta t}, \tag{2}$$

convergence of the time-averaged velocity profile can be obtained through refinement of Δt in conjunction with the introduction of more vortex elements. Here, U = velocity, h = length of a vortex sheet in the numerical boundary layer, ξ = vorticity, and C is a constant. This study design follows that made by Sethian and Ghoniem [5] for the backwards-facing step problem. In this case, they were able to demonstrate that refinement of the numerical parameters past a certain level of discretization yielded only small changes in the time-averaged bubble length, L_B.

3. Results

Table I lists the input parameters used for simulations that are designated as A–D. Note that n_{gen} is the number of evenly-spaced vortex sheet generation points distributed around the cylinder surface, and N_1 is the number of vortex elements in the flow at $t = 1$, chosen arbitrarily to compare the relative magnitude of the number of elements in the domain for each simulation.

Table I. Discrete vortex method input parameters. Δt is the time step, n_{gen} is the number of evenly-spaced vortex sheet generation points distributed around the cylinder surface, and N_1 is the number of elements when $t = 1$, chosen arbitrarily to compare the relative magnitude of the number of elements in the domain for each simulation.

Simulation	Δt	n_{gen}	N_1
A	0.0125	160	11,159
B	0.025	80	2,962
C	0.05	40	524
D	0.075	30	444

Table II. Strouhal number output for the discrete vortex method simulations.

Simulation	*Str*
A	0.224
B	0.217
C	0.216
D	0.190

Cantwell and Coles [1] measured a Strouhal number of 0.179 for their set-up but acknowledged it to be slightly low relative to other data sets. They found that values in the literature for *Str* when Re = O (10^5) were around 0.2, in good agreement with the experimentally measured, and generally accepted value of $Str = 0.21$ determined by Roshko [11]. Table II shows good agreement between the DVM simulations of Strouhal number and the range of experimental values.

Results for convergence of the time-averaged recirculation zone length, L_B, were encouraging. Note that L_B was determined as the distance along the centerline from the downstream edge of the cylinder to the downstream stagnation point. Figure 1 plots L_B vs. Δt, once L_B had stabilized for each simulation. Clearly, L_B first decreased with Δt and then approached a constant value of 0.45. The predicted time-averaged bubble length showed good agreement with Cantwell and Coles [1], who found $L_B \sim 0.5$.

Figures 2–5 compare simulation results from run A with experimental data from Cantwell and Coles [1], as well as with three-dimensional large eddy simulation (LES) results computed on coarse and fine grids by Breuer [12]. Figure 2, which shows the comparison for the time-averaged velocity fields in the near-wake, indicates good agreement with perhaps a slightly broader bubble width for the DVM simulation. There is fair comparison between the experimental and DVM

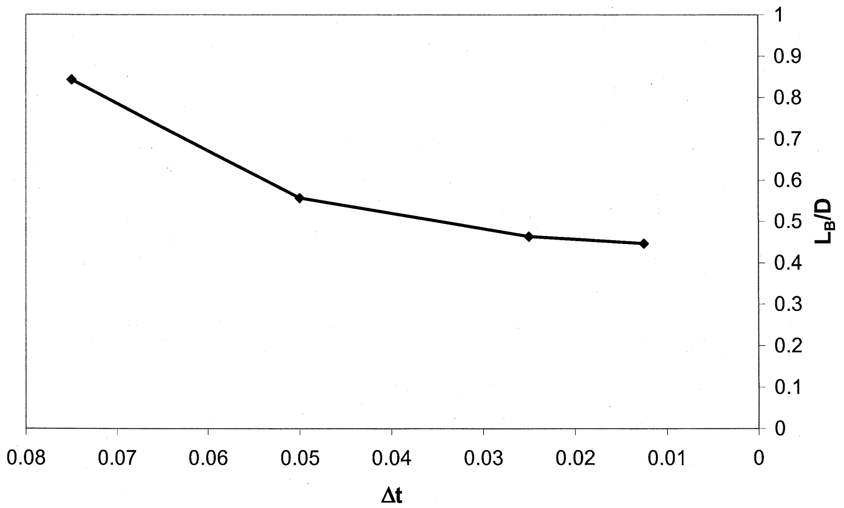

Figure 1. Change in time-averaged bubble length with respect to time-step for Re = 140, 000.

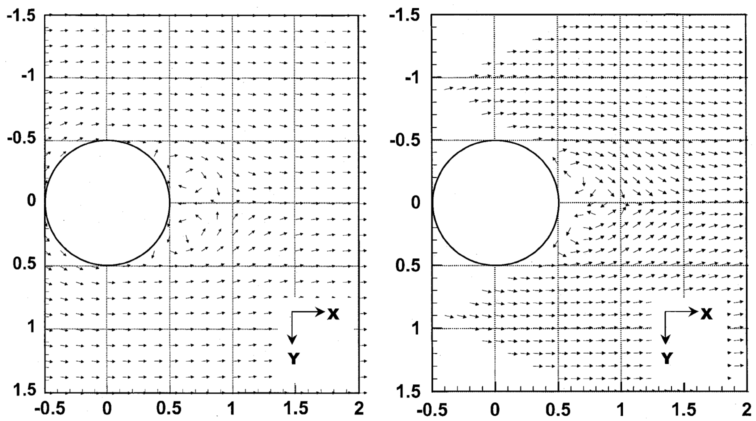

Figure 2. Time averaged near-wake velocity fields: DVM simulation results on left, Cantwell and Coles experimental data on right.

centerline velocity profile (Figure 3). The primary discrepancy between this data occurs for $1 \le x/D \le 3$, where the velocity transitions from near zero to that of the freestream. In this near-wake region, the DVM simulation is outside the 95% confidence intervals of the experimental data. As $x/D \to \infty$, the centerline velocity for the DVM simulation approached that of the experiment. There is excellent agreement between the vortex simulation and experiment for the shear layer velocity profile (Figure 4) and for the shear stress profiles (Figure 5). Simulation results are well within the 95% confidence intervals of the experiment.

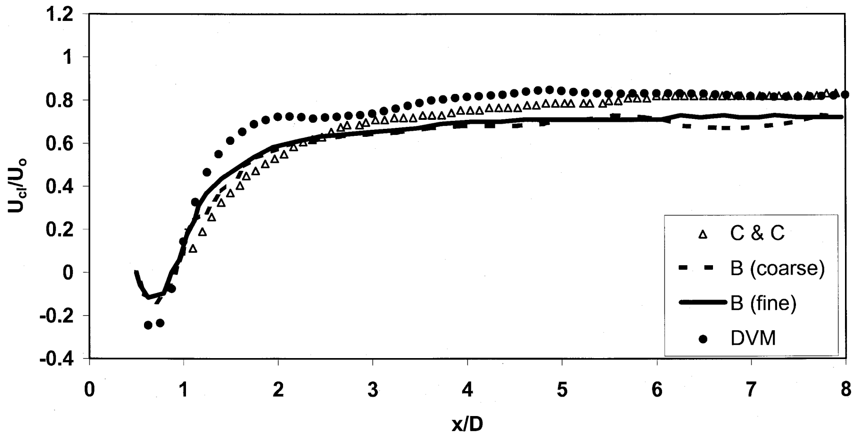

Figure 3. Comparison between centerline velocity profiles (U_{c1}) obtained from the simulation and from Cantwell and Coles [1] (C & C), and Breuer [12] (B).

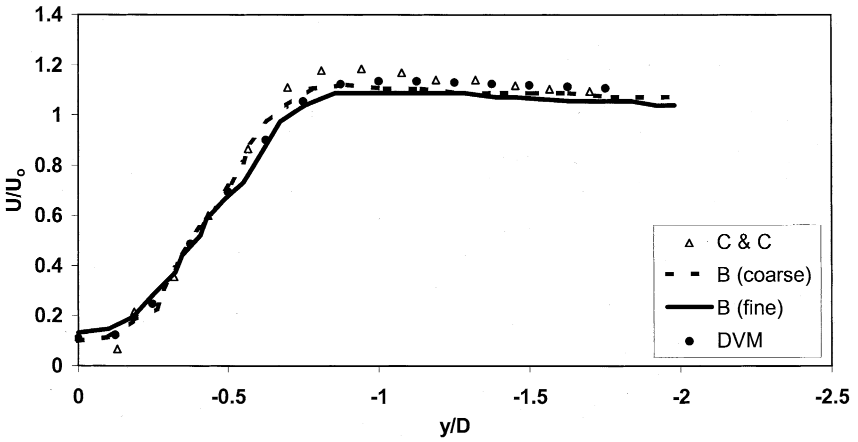

Figure 4. Comparison between shear velocity profiles, obtained at $x/D = 1$, from the simulation and from Cantwell and Coles [1] (C & C) and Breuer [12] (B).

4. Discussion

The qualitative test for convergence to physical results was the appearance of coherent shedding eddies. However, by itself, the Strouhal number is an insufficient criterion for convergence. As shown in Table II, there was little variability in the value of *Str* upon parameter refinement. Rodi *et al.* [13] and Breuer [14] also observed this insensitivity of *Str* to changes in discretization level. The time-averaged length of the recirculation zone downstream of the cylinder, L_B, was considered the best quantitative measure of code verification because it was more sensitive to changes in input parameters. In Figure 1, L_B approaches a constant value of 0.45 as $\Delta t \to 0$. Shedding is maintained at $\Delta t = 0.0125$ while the asymptotic time-

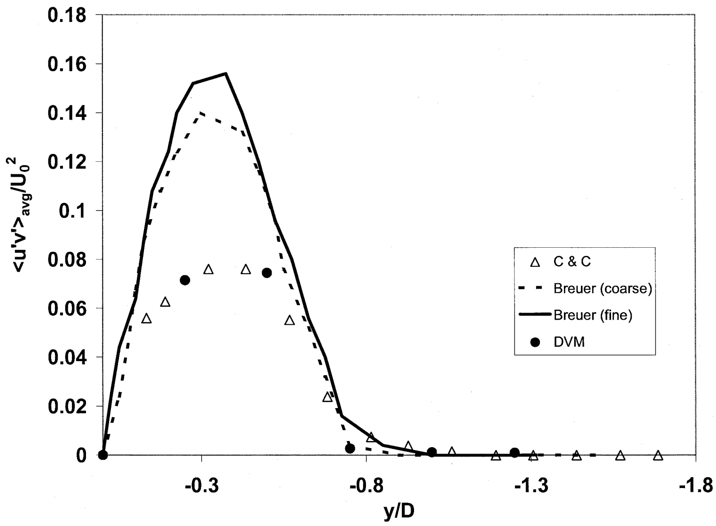

Figure 5. Comparison between shear stress profiles, obtained at $x/D = 1$, from the simulation and from Cantwell and Coles [1] (C & C) and Breuer [12] (B).

averaged value of L_B remains constant. This is strong evidence that a physically converged time-averaged solution can be attained for this DVM simulation.

Overall, good agreement was observed between the physically converged DVM results and Cantwell and Coles' experimental dataset [1] for point-wise time-averaged velocity results as well as for L_B. Some disparities between the DVM simulation and Cantwell and Coles' data [1] were seen for the centerline velocity profile. It is unclear if this discrepancy is related to experimental error or numerical bias. Breuer's [12] results from a three-dimensional large eddy simulation (LES) showed better agreement with those of Cantwell and Coles [1] in this region. In the far wake, the DVM results were much closer to those of Cantwell and Coles [1] than were Breuer's [12]. The DVM simulation also yielded much better agreement with Cantwell and Coles' [1] shear stress profile at $x/D = 1$ than did Breuer's results [12]. Given these observations, the DVM is at least a comparable method for capturing flow in the wake of the circular cylinder at Re $= 140,000$, despite the fact that this implementation of the DVM does not account for three-dimensional flow features.

Interestingly, Breuer [12] noted that he observed worse agreement between his numerical predictions and Cantwell and Coles' data [1] as he refined the numerical parameters; this can be seen in Figures 3–5. This raises a classic verification-validation problem: unrefined simulations may produce good agreement with data while the more accurate numerical solutions provide inferior validation of experimental data. In this case, good agreement between a coarse-grid solution and experiment holds little meaning in the absence of accurate numerics.

5. Conclusions

The discrete vortex method is capable of accurately predicting the evolution of large-scale, time-dependent eddies that are so important in high Reynolds number environmental transport problems. The work presented here indicates that vortex methods also can accurately predict long-time averaged values of the velocity field. Thus, vortex methods have the capability to estimate time-averaged concentrations over extended intervals. At the Reynolds number of 140,000, environmental contamination problems including reentry of pollutants into building air supply vents, air circulation within street canyons, and human exposure to proximal sources of pollution in outdoor environments are amenable to analysis with the vortex method. Given that Re = 140,000 falls within the transitional flow regime of approximately $\mathrm{Re} = 1,000$–$\mathrm{Re} = 300{,}000$, it is likely that environmental flows in this range, such as indoor airflow past a human with $\mathrm{Re} \sim 10,000$, may also be accurately represented. More work is needed to confirm this hypothesis.

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