The Viscoelastic Standard Nonlinear Solid Model: Predicting the Response of the Lumbar Intervertebral Disk to Low-Frequency Vibrations

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Due to the mathematical complexity of current musculoskeletal spine models, there is a need for computationally efficient models of the intervertebral disk (IVD). The aim of this study is to develop a mathematical model that will adequately describe the motion of the IVD under axial cyclic loading as well as maintain computational efficiency for use in future musculoskeletal spine models. Several studies have successfully modeled the creep characteristics of the IVD using the three-parameter viscoelastic standard linear solid (SLS) model. However, when the SLS model is subjected to cyclic loading, it underestimates the load relaxation, the cyclic modulus, and the hysteresis of the human lumbar IVD. A viscoelastic standard nonlinear solid (SNS) model was used to predict the response of the human lumbar IVD subjected to low-frequency vibration. Nonlinear behavior of the SNS model was simulated by a strain-dependent elastic modulus on the SLS model. Parameters of the SNS model were estimated from experimental load deformation and stress-relaxation curves obtained from the literature. The SNS model was able to predict the cyclic modulus of the IVD at frequencies of 0.01 Hz, 0.1 Hz, and 1 Hz. Furthermore, the SNS model was able to quantitatively predict the load relaxation at a frequency of 0.01 Hz. However, model performance was unsatisfactory when predicting load relaxation and hysteresis at higher frequencies (0.1 Hz and 1 Hz). The SLS model of the lumbar IVD may require strain-dependent elastic and viscous behavior to represent the dynamic response to compressive strain. [DOI: 10.1115/1.2904464]

Keywords: spine, lumbar, intervertebral disk, viscoelastic, nonlinear, creep, relaxation, standard, model

Introduction

Although the etiology of low back pain (LBP) is not completely understood, evidence has revealed that prolonged static awkward postures (i.e., sitting) may be a contributing factor to LBP [1–5]. Moreover, studies have suggested that there is an increased risk of LBP when the spine is exposed to seated low-frequency (<6 Hz) vibration (e.g., vibration when operating trucks, buses, etc.) [6–8]. These findings suggest that there is a link between intervertebral joint dynamics and tissue damage leading to LBP. Therefore, it is necessary to further research the dynamics of the intervertebral joint when subjected to prolonged loading and low-frequency vibration.

Musculoskeletal spine models have become a very important tool for predicting the mechanics of the human ligamentous spine [9–13]. However, due to the mathematical complexity of these models, there is a need for more computationally efficient models of the intervertebral disk (IVD) [11–13]. The present study addresses this need by improving upon current lumped parameter models of the IVD.

Several studies have successfully modeled the creep characteristics of the human IVD using the three-parameter viscoelastic standard linear solid (SLS) model [14–17]. Burns et al. [14] tested several rheological models and concluded that the SLS model best simulates the creep response of the IVD. However, when the model is subjected to low-frequency vibration, the model has several limitations. In particular, the model underestimates the load relaxation, the cyclic modulus, and the hysteresis of the human lumbar IVD [16,17].

The standard nonlinear solid (SNS) model introduced within the present study modifies the SLS model by adding a nonlinear component. It is proposed that modeling the nonlinear mechanical behavior of the human lumbar IVD will better capture the dynamic response of the intervertebral joint under compressive loads. It has been shown that the IVD experiences a nonlinear stress behavior when subjected to axial strain [18–23]. The nonlinear behavior is associated with an increase in stiffness as the intervertebral joint is compressed [23]. Further experimental evidence reveals that the stiffness of the intervertebral joint increases after cyclic loading [24]. Because the parameters of the SLS model are strain independent, the model cannot account for the nonlinear behavior of the IVD.

The primary goal of the present study was to develop a novel method of modeling the behavior of the IVD using the SNS model. A secondary goal was to determine whether the SNS model could be used to predict the response of the IVD subjected to prolonged axial loading and to low-frequency vibration. Specifically, we test whether the SNS model can predict the cyclic load relaxation, the cyclic modulus, and the hysteresis of a lumbar motion segment using experimental load deformation and stress-relaxation curves obtained from the literature [23,25].

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Contributed by the Bioengineering Division of ASME for publication in the Journal of Biomechanical Engineering. Manuscript received January 10, 2007; final manuscript received October 17, 2007; published online April 22, 2008. Review conducted by Clark T. Hung.

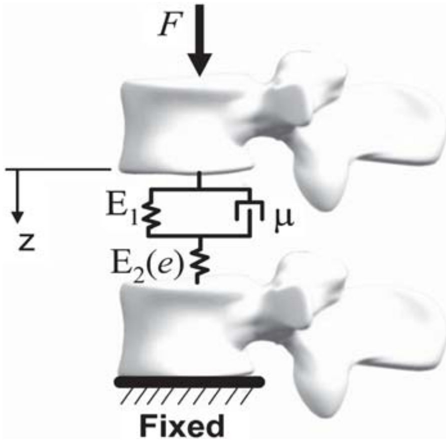

Fig. 1 SNS model. The arrangement of the standard model consists of a Kelvin body in series with a spring. The SNS model in the present study differs from the SLS model in that it replaces the series spring's constant modulus with a strain-dependent modulus.

Methods

Standard Nonlinear Solid Model. Linear elastic springs (Hooke body) and linear viscous elements (dashpots or Newton body) are considered to be the building blocks for most traditional viscoelastic systems. By organizing the springs and dashpots in various arrangements, theoretical models can be constructed to describe empirical results [26]. The Kelvin (also known as Kelvin–Voigt) body is considered to be one of the simplest arrangements that describes the viscoelastic behavior of the IVD [15]. The arrangement of the Kelvin body consists of a spring and dashpot in parallel. However, Burns et al. [14] determined that the SLS model is better when representing the creep characteristics of the IVD subjected to a prolonged axial load.

The arrangement of the SLS model consists of a Kelvin body in series with a spring (Fig. 1). This model can be represented by the following differential equation:

$$p_1\dot{S} + S = q_1\dot{e} + q_0 e \tag{1}$$

where S is the IVD stress and e is the IVD strain, while $\dot{S}$ and $\dot{e}$ represent the rate of change of stress and strain with respect to time. Model coefficients are derived from material properties by the following equations:

$$p_1 = \frac{\mu}{E_1 + E_2}, \quad q_1 = \frac{\mu E_2}{E_1 + E_2}, \quad q_0 = \frac{E_1 E_2}{E_1 + E_2} \tag{2}$$

where E_1 is the elastic modulus of the Kelvin body, E_2 is the elastic modulus of the spring in series with the Kelvin body, and μ is the viscosity coefficient of the Kelvin body [16,17]. The elastic modulus E_2 is also known as the instantaneous elastic modulus because during initial loading, the intervertebral joint experiences an instantaneous deformation that is proportional to this modulus [15].

The SNS model in the present study differs from the SLS model in that it replaces the series spring's constant instantaneous elastic modulus E_2 with a strain-dependent elastic modulus where the elastic modulus is a linear function of strain $E_2(e)$. Substituting the strain-dependent modulus $E_2(e)$ into the model coefficients (Eq. (2)) of the standard model yields

$$p_1 = \frac{\mu}{E_1 + E_2(e)}, \quad q_1 = \frac{\mu E_2(e)}{E_1 + E_2(e)}, \quad q_0 = \frac{E_1 E_2(e)}{E_1 + E_2(e)} \tag{3}$$

Strain-Dependent Elastic Modulus of the SNS Model. The strain-dependent elastic modulus of the intervertebral joint was modeled from experimental load deformation data obtained from the literature [23]. Gardner-Morse and Stokes' [23] study was chosen on the basis of being the most comprehensive experimental stiffness data currently available for human lumbar motion segments. Within this study, Gardner-Morse and Stokes recorded the load deformation behavior of eight female human lumbar motion segments (L2-L3 and L4-L5) under axial preloads of 0 N, 250 N, and 500 N. Specimens were subjected to four sawtooth-waveform axial displacements (± 0.35 mm) for 87 s and a least squares regression routine was used to determine the axial stiffness at each preload (Table 1). Because the data were recorded over a relatively short time duration, it was assumed that the data represent the instantaneous elastic modulus of the SNS model.

Table 1 Gardner-Morse and Stokes et al. [23] reported stiffness coefficients. The * denotes values calculated from reported results. Values in ( ) denote standard deviation.

Axial preload F (N)	Stiffness K (N/mm)	Strain e (%)*	Modulus E_2 (MPa)*
0	438 (92)	0	3.36
250	1700 (67)	1.65	13.03
500	2420 (158)	2.32	18.55

IVD geometry was used to normalize the stiffness coefficients and calculated displacements so that they could be reported as elastic moduli and strains, respectively. The average disk height, disk lateral width, and disk anterior-posterior width were reported as 8.9 mm, 45.5 mm, and 32.5 mm, respectively. The area of the disk (1161 mm^2) was assumed to be the shape of a simple ellipse.

Once the strains and corresponding elastic moduli were determined, the values were plotted and a linear least squares regression was applied to determine a relationship (Eq. (4)) that describes how the IVD's instantaneous elastic modulus E_2 changes with strain e (Fig. 2(*a*)).

$$E_2(e) = 641.4e + 3.14 \tag{4}$$

The strain-dependent instantaneous modulus was used to describe the nonlinear stress behavior of the intervertebral joint (Fig. 2(*b*)), and the nonlinear stress behavior appears to be similar to other experimental and theoretical studies [20,21,27,28].

Constant Parameters of the SNS Model. The constant parameters (E_1 and μ) for the Kelvin body of the SNS model (Eqs. (1) and (3)) were calibrated by minimizing the sum of the squared error between the predicted stress (Eq. (5)) and the experimental results from Holmes and Hukins' [25] stress-relaxation experiment on a male lumbar (L3-L4) motion segment (Fig. 3). Calibrated parameters are reported in the Results section (Table 2). Predicted stress relaxation was determined by analytically solving the SLS model (Eqs. (1) and (2)) for relaxation conditions, in which the applied strain e remained constant and the initial conditions were set as $S(t_0)=S_0$.

$$S = eq_0 + (S_0 - eq_0)\exp\left(\frac{-t + t_0}{p_1}\right) \tag{5}$$

The constant applied strain $e=0.0417$ was determined by displacing the series spring of the SNS model, (Eq. (4)), until the desired initial stress, $S_0=1.25$ MPa, was reached (Fig. 2(*b*)). The applied strain resulted in a constant instantaneous modulus of $E_2=29.97$ MPa. Initial stress S_0 was determined by normalizing the reported maximum force value (2050 N) on the stress-relaxation experiment [25] and dividing the value by the area of the disk.

Cyclic Deformation of the SNS Model. The SNS model with calibrated parameters (E_1, $E_2(e)$, and μ) was used to predict the intervertebral joint response to cyclic axial displacement. For

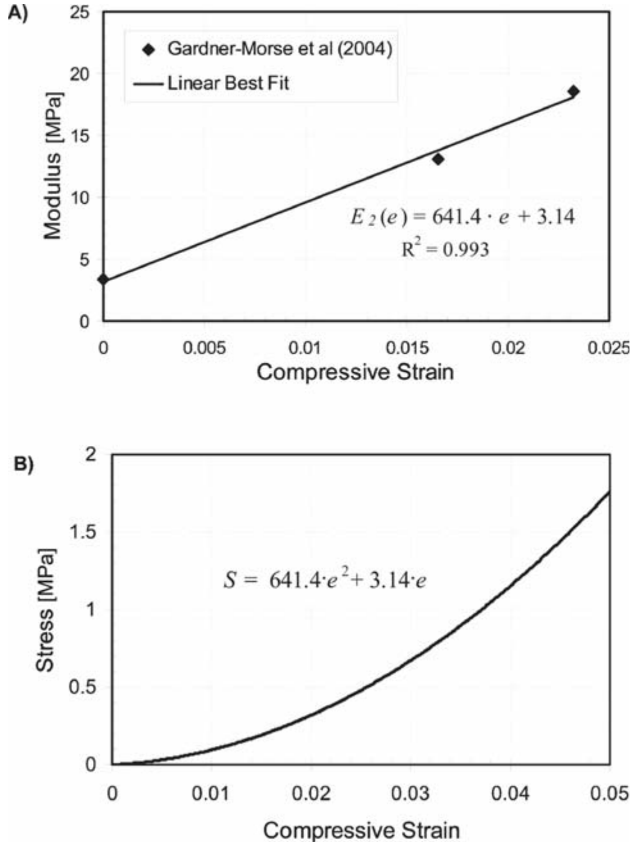

Fig. 2 Instantaneous elastic response of the IVD. (*a*) Relationship for how the elastic modulus $E_2(e)$ changes with axial compressive strain *e* was found through a linear least squares regression. (*b*) The IVD experiences a nonlinear stress when subjected to compressive strain input. The nonlinear stress is due to an increase in stiffness as the intervertebral joint (IVJ) is compressed.

comparison, the SLS model used by Li et al. [16,17] was also investigated. The input displacement function used to simulate the strain was

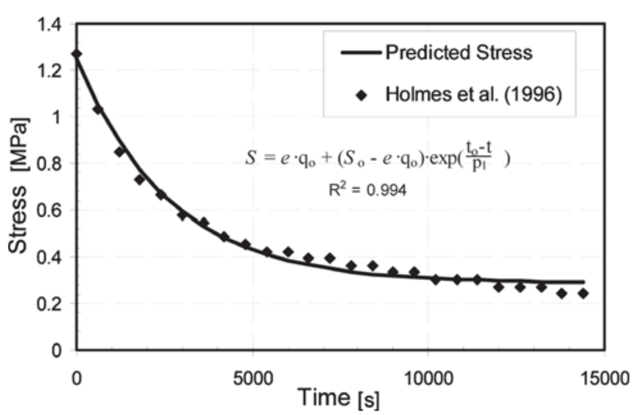

Fig. 3 Stress relaxation of the IVD. The constant parameters (E_1 and μ) for the Kelvin body of the SNS model were determined by least squares regression with the experimental stress-relaxation response of a lumbar IVD. Note that the experimental values used were estimated from graphical results reported by Holmes and Hukins.

$$e = e_0(1 - \cos(\omega t)) + e_1 \tag{6}$$

where e_0 is the cyclic strain amplitude equal to one-half of the peak-to-peak strain input (100 μm/disk height), e_1 is the strain required to reach the desired preload, and $\omega=2\pi f$ is the driving frequency, where f is the desired frequency of vibration [16,17].

Applying the calibrated parameters (E_1, $E_2(e)$, and μ) to the SNS model (Eqs. (1) and (3)), the cyclic relaxation of the IVD was solved for frequencies of 0.01 Hz, 0.1 Hz, and 1 Hz. The cyclic relaxation cannot be analytically solved because of the strain-dependent stiffness in the nonlinear spring. Therefore, a numerical approach was implemented using an ordinary differential equation solver (ODE23; MATLAB 7.0) (Fig. 4).

Response to Cyclic Deformation. The response of the SNS model to cyclic deformation was used to quantify the load relaxation, the cyclic modulus, and the hysteresis of the IVD. Load relaxation R_i was computed as the change in force from the initial value F_0 to the force following the 30th cycle F_f [16,17].

$$R_i = \frac{F_0 - F_f}{F_0} \tag{7}$$

The cyclic modulus, $E(\omega)$, was defined as the ratio of apparent stress versus strain during the 30th cycle of the load relaxation experiment.

$$E(\omega) = \frac{S}{e} \tag{8}$$

where S is the peak-to-peak stress and e is the peak-to-peak strain estimated for each driving frequency ω. Note that the term "cyclic modulus" in the present study is comparable to the term "dynamic modulus" used within the studies by Li et al. [16,17].

The hysteresis loop was characterized by the loading and unloading paths on a load deformation curve. The area within the hysteresis loop is the energy dissipated during 1 cycle of displacement. Because this area depends on the energy input, the hysteresis λ for 1 cycle of displacement was defined as the following ratio:

$$\lambda = \frac{A_L - A_{UL}}{A_L} \tag{9}$$

where A_L is the area under the loading curve and A_{UL} is the area under the unloading curve [16,17,29].

Results

Based on a high R^2 value of 0.993, a linear relationship (Eq. (4)) adequately described the change in the instantaneous elastic modulus $E_2(e)$ with axial strain e of the IVD (Fig. 2(*a*)). The elastic modulus E_1 and viscosity coefficient μ for the Kelvin body of the SNS model were determined to be 8.96 MPa and 102.3 GPa s^{-1}, respectively. The model's fit to the stress-relaxation experimental data [25] had an average error of 6.1% and a maximum error of 20.6% [16,17]. The time constant ($\tau = \mu / E_1$) for the SNS model was calculated to be 190 min. Table 2 compares the SNS model's results with previously reported values for the three-parameter SLS model. Note that previous studies [14,15,17] obtained parameters from creep data, whereas current results were estimated from stress-relaxation data.

The SNS model predicted the relaxation indices to be 0.49, 0.07, and 0.01 for frequencies of 0.01 Hz, 0.1 Hz, and 1 Hz, respectively. The SLS model predicted the relaxation indices to be 0.38, 0.08, and 0.01, respectively. The experimental values for the relaxation index were determined by Li et al. to be 0.51, 0.19, and 0.1, respectively (Table 3). The SNS model underestimated the relaxation by 3.92%, 63.1%, and 90.0%, respectively. The SLS model underestimated the relaxation by 25.5%, 57.8% and 90.0%, respectively. Moreover, the accuracy of both the SLS and the SNS

Table 2 Comparison of parameters found for the three-parameter standard model. Note that previous studies obtained parameters from creep data, whereas current results were estimated from stress-relaxation data.

Reference	Degeneration	E_1 (MPa)	E_2 (MPa)	μ (GPa s)	τ (min)
Adjusted Burns et al.	~	6.65	2.51	~	~
Keller et al.	~	6.26	1.61	5.41	14.4
Li et al.	Combined	7.62	5.06	19.6	42.8
	Mild	9.25	6.35	27.4	49.3
	Severe	6.63	4.29	15.0	37.7
Present study	~	8.96	Eq. (4)	102	190

model declined as the frequency increased. The cyclic relaxation at a frequency of 0.01 Hz for the SLS model and SNS model is shown in Fig. 4.

The SNS predicted the cyclic moduli to be 10.8 MPa, 16.5 MPa, and 17.4 MPa for frequencies of 0.01 Hz, 0.1 Hz, and 1 Hz, respectively. The SLS model predicted the cyclic moduli to be 3.33 MPa, 4.71 MPa, and 5.02 MPa, respectively. The experimental values for cyclic modulus were determined by Li et al. [17] to be 10.5 MPa, 14.3 MPa, and 15.5 MPa, respectively (Table 3).

The SNS model underestimates the hysteresis ratio by 91.2%, 98.7%, and 99.6% for frequencies of 0.01 Hz, 0.1 Hz, and 1 Hz, respectively. The SLS model underestimates the hysteresis ratio of the IVD by 95.0%, 98.7%, and 99.6%, respectively. Furthermore, the calculated hysteresis ratio decreased with increasing frequency over the range of 0.01–1.0 Hz (Table 3).

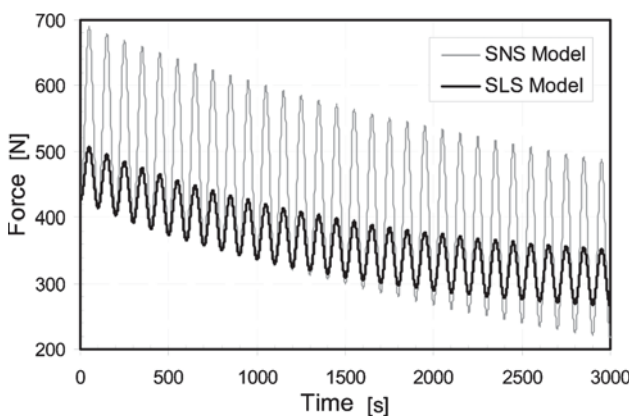

Fig. 4 Cyclic relaxation of the IVD models subjected to a frequency of 0.01 Hz. Note the increase in peak-to-peak force of the SNS model compared to the SLS model.

Table 3 Comparison of the SNS model response. The * indicates an attempt at replication of Li et al. [16,17] model as reported in the literature and was used to create similar conditions of the present model. Values in ( ) denote standard deviation.

	Present study		Li et al.	
Input frequency (Hz)	SNS	SLS*	Experiment	SLS
		Relaxation index R_i (a.u.)		
0.01	0.49	0.38	0.51 (0.05)	0.40 (0.07)
0.1	0.07	0.08	0.19 (0.05)	0.08 (0.02)
1	0.01	0.01	0.1 (0.04)	0.01 (0.00)
		Cyclic modulus $E(\omega)$ (MPa)		
0.01	10.8	3.33	10.5 (3.80)	~
0.1	16.5	4.71	14.3 (5.03)	~
1	17.4	5.02	15.5 (5.47)	~
		Hysteresis λ (%)		
0.01	0.7	0.4	8.0 (5.0)	~
0.1	0.1	0.1	8.0 (4.0)	~
1	0.04	0.04	11.0 (5.0)	~

Discussion

Many studies have noted that the IVD experiences nonlinear stress when subjected to a linear strain input [18–23]. The nonlinear behavior of the IVD is characterized by an increase in stiffness as the intervertebral joint is compressed. Therefore, nonlinear models should be considered when modeling phenomenological behavior of the IVD. A SNS model was introduced and developed in the present study to better represent the cyclic relaxation characteristics of the IVD. Parameters of the SNS model were estimated from load deformation and stress-relaxation data [23,25]. We quantified the performance of the SNS model subjected to cyclic displacements by calculating the relaxation index, the cyclic modulus, and the hysteresis.

The constant elastic modulus E_1 and the viscosity coefficient μ of the Kelvin body were compared to previously reported parameters of the standard model (Table 3). Note that previous results [14,15,17] were obtained through a least squares fit to experimental creep deformation data, while in the present study, the parameters were estimated from experimental stress-relaxation data. Furthermore, it is important to note that Burns et al. [14] used the motion segment height and not disk height when calculating the strain, resulting in lower strain values and higher material properties. Therefore, Keller et al. [15] adjusted the values to more closely match realistic magnitudes of material properties using estimated disk heights.

The viscosity coefficient μ was much larger than any of the values previously reported for creep deformation experiments [15–17]. The large viscosity coefficient resulted in a large time constant, indicating that the intervertebral joint takes longer to reach equilibrium. According to Li et al. [17], a large time constant also suggests that the IVD has mild degeneration. However, Holmes and Hukins [25] did not report the level of degeneration of the disk. One reason for the discrepancy between the current results and the results from others could be due to the extended length (4 h) of the stress-relaxation experiment. Fung [30] acknowledged that the relaxation constant can be affected by the length of the experiment.

The cyclic load relaxation at the end of the 30th cycle was predicted by the SNS model for a frequency of 0.01 Hz. However, neither the SLS nor the SNS model accurately predicted the relaxation at higher frequencies (0.1 Hz and 1 Hz) (Table 3). The underestimation of relaxation is likely attributed to the model's prediction of a decrease in hysteresis at higher frequencies. The decrease in hysteresis may be attributed to the constant viscosity coefficient of the SNS model. Conversely, Li et al. [16,17] experimentally determined that the hysteresis does not change with frequency. This suggests that the viscosity parameter may also be strain and/or rate dependent (i.e., μ may change with load or frequency).

Tissue and fluid flow properties of the IVD may help explain the need for a strain- or frequency-dependent viscosity coefficient. The permeability of biological tissues has been described as having a dependence on strain. Tissue permeability decreases as compressive strain increases due to the drag force increasing as the interstitial spaces decrease in size [31]. In shear stress-relaxation tests, Iatridis et al. [32] noted that the relaxation spectrum varied with frequency, thereby describing the viscous effects as linearly increasing with frequency. An increase in viscosity at higher frequencies would suggest that the ground substance within the nucleus pulposus may have thixotropic properties [30]. Furthermore, increasing frequency increases the rate of change of the IVD's fluid volume [33]. The rate of change in fluid volume may explain a change in the viscous characteristics of the IVD. These studies support the need for a strain- and/or frequency-dependent viscosity coefficient; however, this was beyond the scope of the present study. Future studies should investigate how a strain- or frequency-dependent viscosity parameter could improve prediction of the hysteresis and relaxation of the IVD.

Although the cyclic relaxation cannot be predicted for the higher frequencies, the cyclic moduli determined by the SNS model were within the standard deviation reported by Li et al. [17] for frequencies of 0.01 Hz, 0.1 Hz, and 1 Hz. Therefore, the SNS model was in good agreement with the experimental observations [17]. This indicates that the model does well at all frequencies tested when predicting the force magnitude. However, because only one compressive preload was applied to the SNS model at the start of cyclic displacement, future investigations should be performed to assess how the model performs at different levels of preload.

Several limitations of the present study analyses must be considered. The parameters for this study were determined from two different experimental studies [23,25]. Gardner-Morse and Stokes [23] used female lumbar (L2-L3 and L4-L5) motion segments, while Holmes and Hukins [25] used a male lumbar (L3-L4) motion segment. To help reduce gender differences, segmental height differences, and protocol differences of the two experimental data sets, the experimental values for force, displacement, and stiffness were normalized using reported values of the IVD geometry. However, Holmes and Hukins [25] did not report a value for the area of the disk for the male lumbar (L3-L4) specimen used for the stress-relaxation data. Therefore, the area ($1650\ \text{mm}^2$) reported by Li [16] was used to normalize the force values. This value is comparable to the average value of other male lumbar disks found in the literature [14,15,22].

Gardner-Morse and Stokes' [23] study was chosen on the basis of being the most comprehensive experimental stiffness data currently available for human lumbar motion segments. However, stiffness coefficients were reported for only three different preloads. Therefore, a linear regression was chosen to model the strain-dependent elastic modulus. Stiffness coefficients reported at more preloads would better describe the strain-dependent behavior of the IVD. Furthermore, Holmes and Hukins' [25] experimental study was chosen because it is the most recent axial stress-relaxation experiment of the IVD and falls within physiological limits of the past research [34]. Moreover, the experimental data from the stress-relaxation curve were estimated from graphical results, thereby increasing the error associated with the current study.

Despite these limitations, this paper presents a novel method for modeling the IVD using the SNS model. Additionally, the cyclic modulus was accurately predicted at all frequencies tested and therefore we feel confident that the values used and the reported results are within reason. However, a more thorough experimental validation of the SNS model is required to fine-tune the parameters and to confirm the findings within this study.

Acknowledgment

This study was supported in part by Grant No. R01 AR46111 from NIAMS of the National Institute of Health and Grant No. R01 OH07352 from NIOSH of the Centers of Disease Control. The authors would like to thank Dr. Michael Madigan, Dr. Moshe Solomonow, Dr. Stefan Duma, Dr. Mary Kasarda, and Dr. Clay Gabler for their valuable support, help, and contributions.

Nomenclature

A_L = area under loading curve
A_{UL} = area under unloading curve
E_1 = elastic modulus of the spring on the Kelvin body of the standard model
E_2 = elastic modulus of the series spring on the standard model
$E(\omega)$ = cyclic modulus
F = axial force
F_0 = initial axial force
F_f = final axial force
H = IVD height
K = satic axial stiffness or instantaneous stiffness
R_i = relaxation index
S = stress
$\dot{S}$ = stress rate
e = strain
e_0 = cyclic strain amplitude (one-half peak-to-peak strain)
e_1 = strain required for initial preload
$\dot{e}$ = strain rate
t = time
t_0 = initial time
f = frequency
z = axial displacement
λ = hysteresis ratio
τ = time constant
μ = viscosity coefficient
ω = input or driving frequency

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