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Evaluation of a Proposed Area Equation for Improved Exothermic Process Control

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Our understanding of heat transfer and meteorological theories and their applications for engineering control design have been refined since the collective work in ventilation engineering for manufacturing process was published by Hemeon in 1955. These refined theories were reviewed and used to develop a newly proposed equation to estimate buoyant plume area (A). The area is a key parameter in estimating the plume volumetric flow $(Q = \bar{U}A)$ required for exothermic process control. Subsequent to developing a theoretical equation for plume area (A), plume velocity and area data were collected in the laboratory using a thermal anemometer and a scale-model exothermic process. Laboratory results were compared to solutions provided by the proposed, American Conference of Governmental Industrial Hygienists (ACGIH) and Hemeon plume area equations to determine which equation most closely matched the laboratory data. To make this determination, either *t*-tests or Wilcoxon signed-rank tests were conducted (based on examination of data normality) to determine the difference between collected data and solutions from the proposed, ACGIH and Hemeon equations. Median differences and *P*-values from Wilcoxon signed-rank tests (non-parametric) indicate that the ACGIH and Hemeon plume area equations provide significantly lower values than the laboratory data. However, the proposed equation provided solutions that were not significantly different from the collected data. Results indicate that the plume area equations currently recommended by the ACGIH and Hemeon are not as accurate as the proposed equation over the range of parameters investigated.

Keywords: engineering controls; heat and cold; hot processes; local exhaust; ventilation

INTRODUCTION

The volumetric flow (Q) of the buoyant plume rising from an exothermic process is defined as the product of the plume mean velocity ($\bar{U}$) and area (A) (ACGIH, 1998, 2007; Hemeon 1999). The purpose of the currently accepted equations by the American Conference of Governmental Industrial Hygienists (ACGIH) and Hemeon is to estimate Q at which heat and fine particle effluents are being introduced to the face of the engineering control (i.e. receiving hood or high canopy hood). Determining the plume flow is of critical interest for two reasons: (i) it must provide adequate control of the heat and effluents from the process to prevent spillage back into the workplace air and (2) it should be calculated as accurately as possible to prevent over-ventilating the process, since this wastes conditioned indoor air. Accuracy of the flow estimation is important, and laboratory studies conducted to validate current flow equations provide contradictory results (Siebert and Fraser, 1973; Zhivov *et al.*, 2001).

In calculating Q present above exothermic processes, both $\bar{U}$ and A continually change as the plume rises. The goal of this research was to accurately ascertain the area of the heat and effluent plume being introduced to the face of a receiving hood located above an exothermic process. Hemeon was the first to provide an estimation equation for this purpose (Hemeon, 1955, 1963, 1999). The original Hemeon

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plume area equation continues to be used and is the basis of the plume area equations in the US Public Health Service Air Pollution Engineering Manual and the ACGIH Industrial Ventilation Manual (USPHS, 1973; ACGIH, 1998, 2007). More recently, proposed equations for Q and $\bar{U}$ were developed and published by McKernan *et al.* (McKernan and Ellenbecker, 2007; McKernan *et al.*, 2007).

To determine the plume area, concepts from meteorology, heat transfer and fluid mechanics were used to define the effective plume radius (R) and a 'virtual point source'. Concepts from these disciplines indicate that the plume area orthogonal to the direction of the velocity profile (i.e. cross-sectional area) in the axisymmetric plume is round at distances above the source where the plume flow has become turbulent (Bill and Gebhart, 1975). Application of these concepts also requires the use of a virtual point source some distance below the heated body that provides an origin for the characteristic behavior of the axisymmetric plume (Morton *et al.*, 1956). A diagram of the virtual point source concept is included in Fig. 1.

One of the difficulties of defining R is determining where the plume boundary exists. The ACGIH and Hemeon's plume radius equation defines the plume boundary as 'points in the cloud with velocities that are 10% of the velocity measured at the plume centerline', where the velocity is at a maximum (U_{max}) (Sutton, 1950; Mundt, 1996; Hemeon, 1999). Although this definition is arbitrary, it was established by Sutton and has been perpetuated by Hemeon and the ACGIH (Sutton, 1950; ACGIH, 1998, 2007; Hemeon, 1999).

There are two approaches in the literature to developing plume area equations. The first is to construct

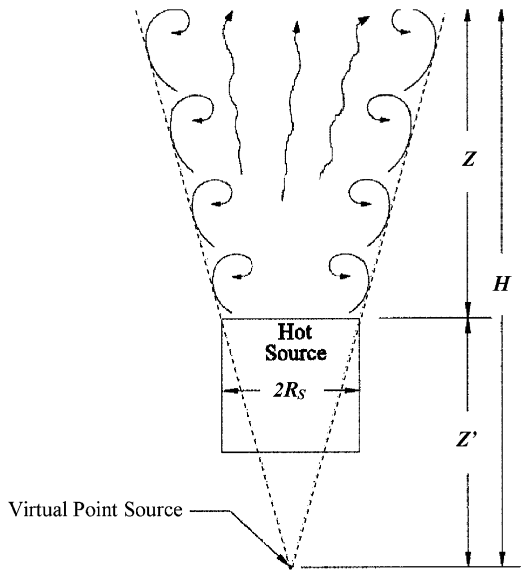


Fig. 1. Diagram of plume parameters based on virtual point source concept (ACGIH, 1998, 2007; Hemeon, 1999; McKernan and Ellenbecker, 2007).

equations based on plume entrainment. Entrainment is characterized through the use of a plume entrainment value (α). Depending on the theory or conducted research, this value is either a variable or a constant. A variable α is used when source or environmental temperatures, or heated source dimensions, are changed. A constant α is produced when environmental temperature, source temperature and dimensions remain unchanged.

Plume entrainment research appears to have begun in applied fluid mechanics (Kuethe, 1935; Schmidt, 1941). Initial research using Prandtl's theory for mixing lengths required the use of α to characterize the inflow of ambient air into jets and plumes at their boundaries (Schmidt, 1941). The entrainment value has been found to vary in proportion with the velocity of the rising plume (Rouse *et al.*, 1952; Priestley and Ball, 1955). The difference in air velocities between heated air rising in the plume and still air in the environment surrounding the plume causes shearing stresses (Priestley and Ball, 1955). These stresses at the plume boundary lead to turbulent eddies with ambient air entrainment. This entrainment increases the volume of the rising head, giving jets and plumes their characteristic conical shape, widening as the distance (Z) from the source increases. Both Kuethe and Schmidt conducted experiments with fixed source and environmental conditions. They based their developed equation for plume radius on a linear relationship between the radius and the product of α and Z, that is $R = \alpha Z$. Values for α from the work of Kuethe and Schmidt were 0.07 and 0.12, respectively.

After these initial developments in applied fluid mechanics, publications on the topic of plume rise and entrainment began appearing in the meteorological literature. Meteorological research applies to the large-scale processes of heat transfer in the ambient (i.e. outdoor) environment. Equations and solutions based on dimensional analysis were common in these publications, and entrainment values continued to be used. Assumptions to describe the characteristics of buoyant air masses in meteorological research were similar to fluid mechanics work in the early stages, with the Gaussian velocity profile being used to characterize velocities in the plume cross-section. A number of researchers have indicated that the Gaussian profile is most accurate in describing the distribution of the velocity profile, since it provides for low velocities at the plume boundary and high velocities at the centerline (Morton *et al.*, 1956; Mundt, 1996). Early meteorological results that used the Gaussian profile indicate α values between 0.08 and 0.12. The radius equation was the same as that used in fluid mechanics research ($R = \alpha Z$) (Taylor, 1945; Rouse *et al.*, 1952; Batchelor, 1954; Priestley and Ball, 1955; Morton *et al.*, 1956).

In recent meteorological research, the top-hat model has been used extensively due to its ease of application and acceptable accuracy for large masses of effluents released in the ambient environment (i.e. stack emissions) (Hanna *et al.*, 1982). Instead of the Gaussian distribution, this model uses a simplified profile that assumes a constant velocity (set equal to the mean velocity) inside the plume at a particular *Z*. The ambient velocity outside the plume is assigned another constant value less than that inside the plume. The discontinuity between the velocity inside and outside the plume defines the plume diameter, or radius when measured from the plume centerline. Illustrations of the Gaussian and top-hat velocity profiles are provided in Fig. 2. The top-hat assumption leads to an estimation of α that is larger than those produced using the Gaussian profile, since entrainment is proportional to the difference in air velocity between the plume and environment. The radius equation was the same as that used in previous meteorological research ($R = \alpha Z$), but the α value is 0.16 (Hanna *et al.*, 1982).

Limited industrial hygiene work that included the evaluation of engineering control techniques for heated processes has been conducted by other authors (Bender, 1979; Goodfellow and Bender, 1980). Empirical equations for use in industrial hygiene that use α values to determine the plume radius above point sources are provided in that literature.

The second approach to developing plume area equations in the literature is to construct equations that are not based on plume entrainment values. Equations that are not based on α tend to require additional variables to characterize the operating conditions; however, the advantage is that the resulting equation is more widely applicable to changing source and environmental conditions. The purpose of this research is to validate a theoretical plume area equation for exothermic process control that is not

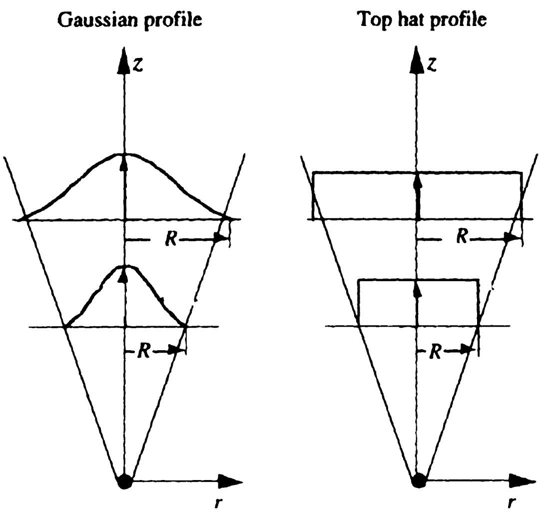


Fig. 2. Gaussian and top-hat plume profiles (ACGIH, 2007).

based on α. This theoretical equation was developed utilizing relationships identified during a more recent review of heat transfer and meteorological theories (McKernan and Ellenbecker, 2007).

This article presents: (i) an overview of the current plume area estimation equations and proposed equation development; (ii) description of the laboratory work conducted and (iii) statistical comparison of the laboratory results to solutions from the proposed and current plume area estimation equations. The latter provided an evaluation of the accuracy of the current and proposed plume area estimation equations based on *P*-value results. With this information, ventilation engineers and practicing industrial hygienists will be better equipped to design ventilation systems for adequate control of heat and contaminant plumes generated by exothermic processes.

EQUATIONS AND METHODS

Plume area equation development

As mentioned previously, the ACGIH and Hemeon's plume radius equation defines the plume boundary as points in the cloud with velocities that are 10% of the velocity measured at the plume centerline, where the velocity is at a maximum ($U_{\max}$) (Sutton, 1950; Mundt, 1996; Hemeon, 1999; ACGIH, 2007). However, the proposed plume radius equation defines the plume boundary as points in the cloud with velocities that are 1% of $U_{\max}$ (McKernan and Ellenbecker, 2007). To determine the proposed plume radius, where the velocity is 1% of $U_{\max}$, Morton's equation for the Gaussian velocity distribution is used (Morton *et al.*, 1956; McKernan and Ellenbecker, 2007).

$$\frac{U_r}{U_{\max}} = e^{-\left(\frac{r^2}{b^2}\right)}, \tag{1}$$

U_r = buoyant plume velocity at a distance r from the axisymmetric centerline (m s^{-1}),
$U_{\max}$ = maximum buoyant plume velocity located at the axisymmetric centerline (m s^{-1}),
r = distance from the axisymmetric centerline where U_r is measured, arbitrary location (m),
b = characteristic horizontal length scale, location where $U_{\max}$ has been reduced by a factor of e^{-1}; equivalent to $6/5\alpha H$ (m) (Morton *et al.*, 1956),
α = plume entrainment value (dimensionless),
H = vertical height above virtual point source (m).

A limitation of this equation is that it cannot provide an accurate estimate for the velocity value at any desired height due to the relationship of the equation on experimentally derived values for the variable α. Therefore, it is desirable to define an equation for b which does not require the use of α. To determine values for b, investigators looked to meteorological theory developed by Sutton (Sutton, 1950). Sutton put forth a theoretical equation defining the radius as points in the buoyant plume where the velocity has been reduced to 10% of the maximum centerline velocity (i.e. a 90% reduction radius):

$$R_T = 1.52C\ \left(H^{0.5m}\right), \tag{2}$$

C = diffusion coefficient for a continuous point source plume (dimensionless),
m = atmospheric diffusion constant (dimensionless).

The utility of the equation depends on the development of diffusion coefficients that characterize the meteorological condition (i.e. stability) of the environment and an atmospheric diffusion constant. In the ambient environment, temperature decreases as height increases. This meteorological condition is defined as neutral or unstable. In indoor environments, temperature increases as height increases. This condition is defined as stable (USAEC, 1968). The generalized diffusion coefficients (C) that have been reported for these conditions are 0.27 and 0.12, respectively (Sutton, 1950; Stewart *et al.*, 1954). The atmospheric diffusion constant (m) is reportedly in the range of 1.70–2 (Schmidt, 1941; Sutton, 1950; Railston, 1954). With substitution of $C = 0.12$ for stable conditions and $m = 1.75$, Sutton's plume radius equation becomes:

$$R_T = 0.18 \cdot H^{0.88}. \tag{3}$$

Subsequent experiments to validate Sutton's plume radius equation were conducted by Railston (Railston, 1954). Railston used an optical system to determine the radius by measuring the temperature differentials inside a buoyant plume created by a small heat source. Railston's results indicated values for C between 0.24 and 0.27 (i.e. unstable conditions), with an experimentally determined value of $m = 1.70$. Reducing the results for C by 10% to adjust for Railston's measurement of the temperature profile instead of the velocity profile, the empirically derived 90% reduction radius is expressed (Rouse *et al.*, 1952; Railston, 1954; Papanicolaou and List, 1988).

$$R_E = 0.36 \cdot H^{0.85}. \tag{4}$$

To determine a reasonable and conservative equation to characterize the behavior of the 90% reduction radius, the average of the theoretical and empirical radius equations were calculated for each height. These solutions were then used to fit an equation of the form $\bar{R}_{TE} = a_1 \cdot H^{a_2} + e$. $\bar{R}_{TE}$ is the mean of R_T and R_E, and H is the height. The least squares estimates of a_1 and a_2 were 0.27 and 0.86, respectively, and the correlation was 1. With substitution, the average of the theoretical and empirical radius equations was as follows:

$$\bar{R}_{TE} = 0.27 \cdot H^{0.86}. \tag{5}$$

Using this equation for the 90% reduction radius, it is possible to develop an equation for b which does not require the use of the empirically derived variable α. An equation for b can now be developed by setting the r value in Morton's Gaussian velocity equation (equation 1) equal to the $\bar{\boldsymbol{R}}_{\boldsymbol{TE}}$, and setting $U_r/U_{max} = 0.10$ (i.e. 90% reduction radius) (McKernan and Ellenbecker, 2007).

$$b = \frac{\bar{R}_{TE}}{1.52} = 0.18 \cdot H^{0.86}. \tag{6}$$

If this equation is used after reducing Morton's Gaussian velocity equation to solve for r in terms of b, constants can be used to express radius equations other than the 90% reduction radius. A solution for the 99% reduction radius (R_P) has been proposed (McKernan and Ellenbecker, 2007).

$$R_P = 2.14\ b, \tag{7}$$

$$R_P = 0.38 \cdot H^{0.86}. \tag{8}$$

From equation 6, it can be seen that α is no longer required to solve for b. However, the vertical height above the virtual point source (H) remains a critical variable. H is the sum of the determined distance between the top surface of the heated source and the virtual point source (Z' [m]) and the distance between the top surface of the heated source and a point of interest on the plume centerline (Z [m]).

$$H = Z + Z'. \tag{9}$$

Although Z is usually known, the equation for Z' changes among the proposed, ACGIH and Hemeon equations for H. The following equations are proposed for the virtual point source distance (Z'_P) and height (H_P) based on existing theory (Sutton, 1950; Batchelor, 1954; Morton *et al.*, 1956; Mundt, 1996; McKernan and Ellenbecker, 2007).

$$Z'_P = 3.03R_V^{1.16}, \tag{10}$$

$$H_P = Z + \left(3.03R_V^{1.16}\right),$$

R_V = projected radius of heated source; $R_S + \delta$ (m),
R_S = physical radius of heated source (m),
δ = boundary layer thickness; $0.05\left(\frac{L}{\Delta T}\right)^{0.25}$ (m),
L = physical length (height) of heated source (m),
ΔT = excess temperature; $T_S - T_\infty$ (K),
T_S = surface temperature of heated source (K),
T_∞ = ambient temperature (K).

For comparison, the respective ACGIH and Hemeon equations for Z' and H are (ACGIH, 1998, 2007; Hemeon, 1999; McKernan and Ellenbecker, 2007):

$$\begin{aligned} Z'_A &= (5.20R_S)^{1.14}\ (\mathrm{m}), \\ H_A &= Z + (5.20R_S)^{1.14}(\mathrm{m}), \end{aligned} \tag{11}$$

$$\begin{aligned} Z'_H &\approx 4R_S\ (\mathrm{m}), \\ H_H &= Z + 4R_S\ (\mathrm{m}). \end{aligned} \tag{12}$$

From the area equation for a circle, the cross-sectional area of the buoyant plume (m^2) can now be expressed.

$$A_P = \pi \cdot R_P^2 = \pi\left(0.38H_P^{0.86}\right)^2 = 0.14\pi \cdot H_P^{1.72} = 0.45 \cdot H_P^{1.72}. \qquad (13)$$

A case study illustrating the use of this proposed plume area equation is provided in Appendix 1. For comparison, currently accepted equations to determine plume area from the ACGIH (A_A) and Hemeon (A_H) for the high canopy hood condition are (ACGIH, 1998, 2007; Hemeon, 1999):

$$A_A = 0.15 \cdot H_A^{1.76}, \qquad (14)$$

$$A_H = 0.15 \cdot H_H^{1.76}. \qquad (15)$$

Experimental design and equipment

Experimental data were collected at the National Institute for Occupational Safety and Health, Hamilton Building Ventilation Laboratory, Cincinnati, OH. A vertical cylinder was selected to represent the basic shape of an exothermic process vessel, such as a reactor or furnace. This model heated source was constructed of round galvanized steel ductwork 0.15 m in diameter (D) and 0.3 m high (L). The cylinder was painted matte black to better approximate the predictable radiant and absorptive properties of a blackbody. The apparatus was heated at the bottom with a 1100 W (maximum output) 0.15 m diameter electric heating element with a built-in proportional voltage controller (Durabrand, Model SBS110-B, Bentonville, AR). Inside the cylinder, heavy-gauge aluminum stock was stacked on top of the heating element with minimal air space between the stock and interior walls of the cylinder. Surface temperatures were measured with a high-temperature immersion mercury thermometer (Fisher Scientific, Model NC9262583, Pittsburgh, PA). The heated vertical cylinder was placed 0.3 m above the floor in a 3.65 by 4.57 by 2.43 m room. The cylinder was placed away from objects that could potentially disturb the buoyant flow around and above the cylinder. An attempt was made to control room drafts by sealing air supply and return ducts, as well as gaps around doors and windows. To determine the ambient temperature, a mercury thermometer (Fisher Scientific, Model 15-041-1A, Pittsburgh, PA) was positioned at 1.5 m from the floor in the center of the room. The heated source was located far enough away

Table 1. Plume radii (r in m) investigated at each unique height (Z) and excess temperature (ΔT) combination

Z (m)	ΔT (K)					
	27	38	52	83	96	124
0.10	0.08	0.08	0.08	0.08	0.08	0.08
0.25	0.08	0.08	0.08	0.08	0.08	0.08
0.40	0.08	0.08	0.08	0.08	0.08	0.08
0.70	0.20	0.20	0.20	0.20	0.20	0.20
1.00	0.25	0.25	0.25	0.25	0.25	0.25

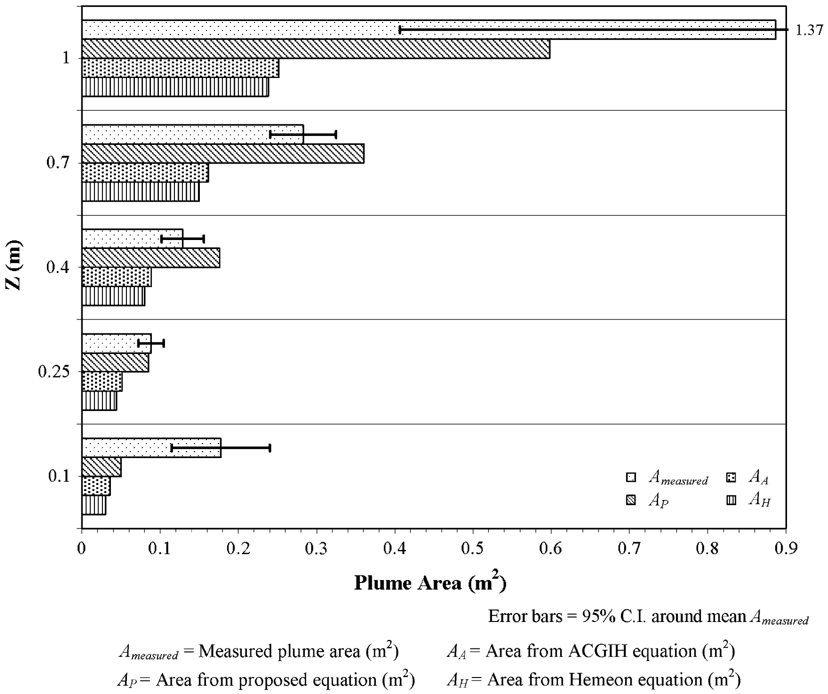
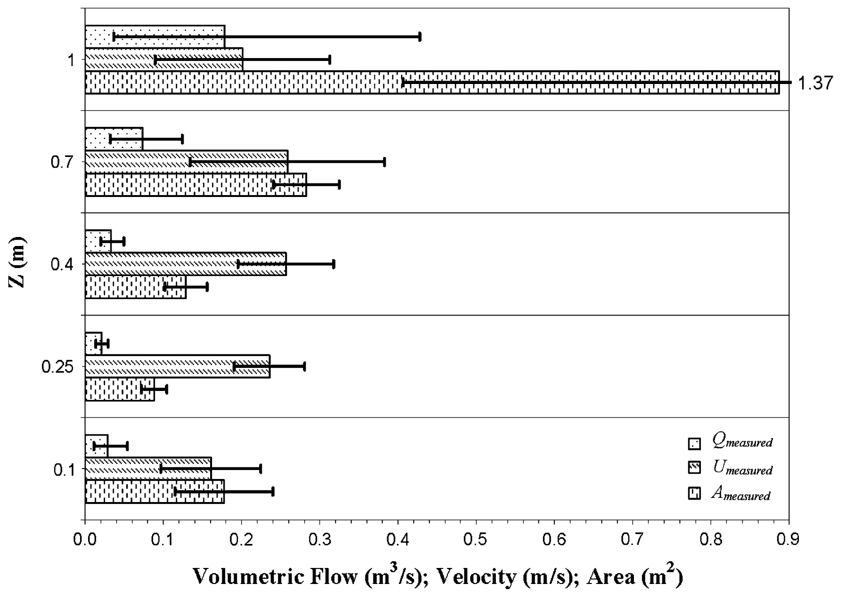


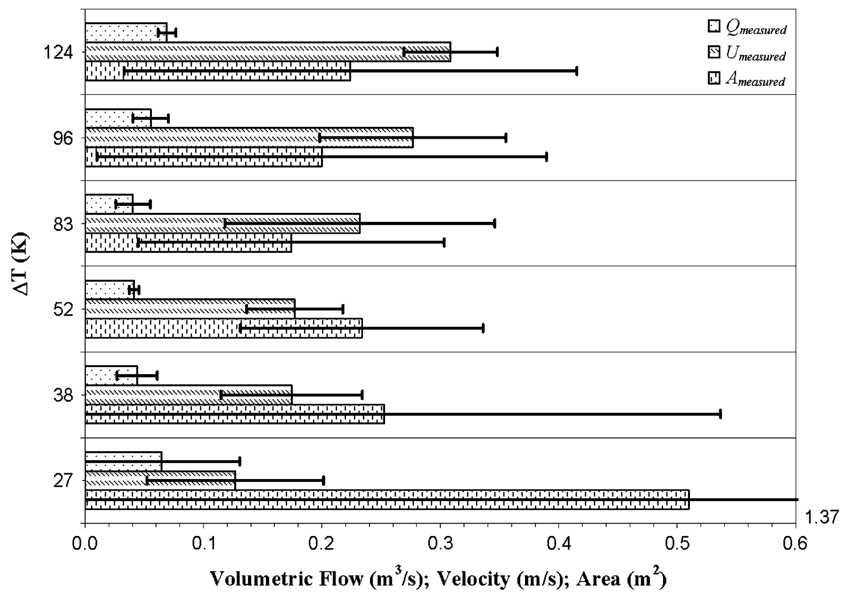
Fig. 3. A_{measured} (95% CI) and solutions from the proposed, ACGIH and Hemeon area equations at heights (Z).



Error bars = 95% C.I. around mean

$Q_{measured}$ = Measured plume flow (m^3/s) $U_{measured}$ = Measured plume velocity (m/s)
$A_{measured}$ = Measured plume area (m^2)

Fig. 4. Measured plume flow, velocity and area with 95% CI at heights (Z).



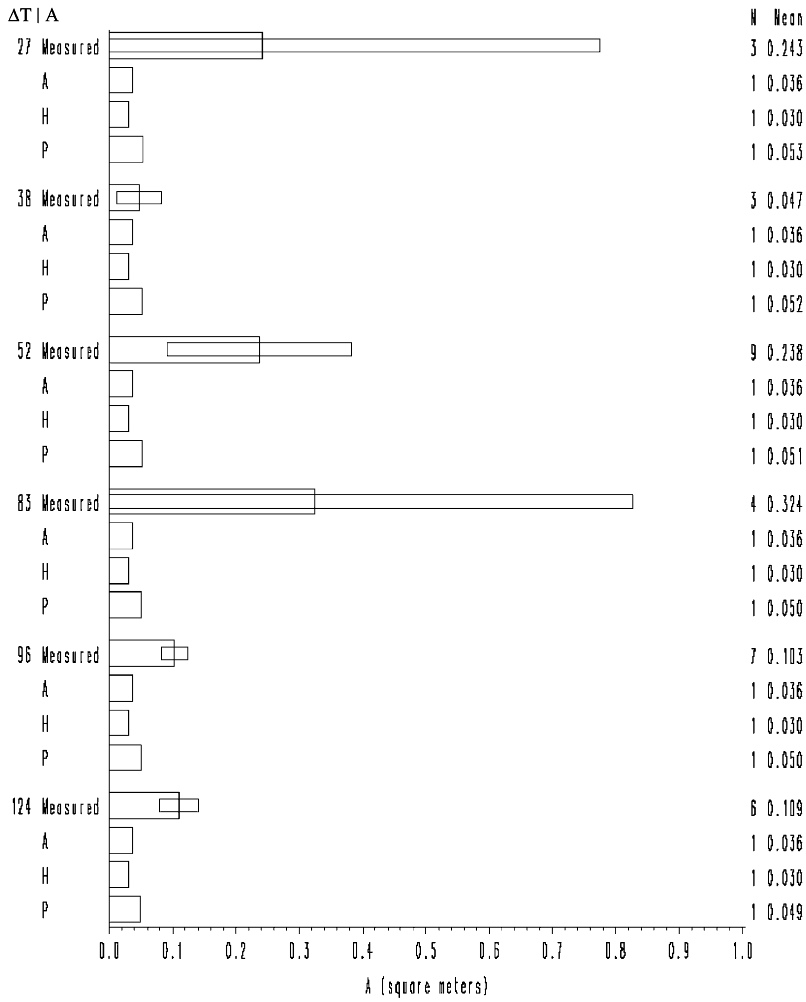
Error bars = 95% C.I. around mean

$Q_{measured}$ = Measured plume flow (m^3/s) $U_{measured}$ = Measured plume velocity (m/s)
$A_{measured}$ = Measured plume area (m^2)

Fig. 5. Measured plume flow, velocity and area with 95% CI at excess temperatures (ΔT).

from the thermometer to ensure temperature measurements were independent of the thermal plume.

A thermal anemometer was used to measure air velocities in the plume cross section at the axisymmetric centerline and at radii (r) of 0.08, 0.2, or 0.25 m from the centerline determined by the height above the heated source (see Table 1). Calibration was conducted by the manufacturer prior to the collection of



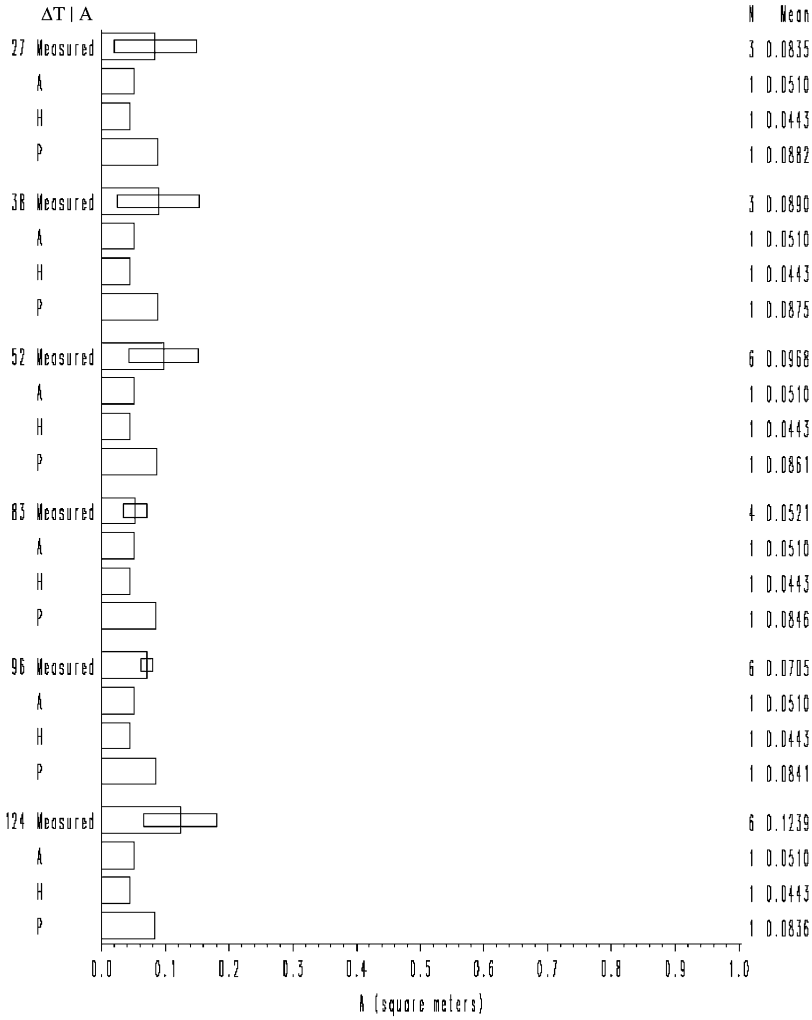
Measured = Measured plume area (m^2) H = Area from Hemeon equation (m^2)
A = Area from ACGIH equation (m^2) P = Area from proposed equation (m^2)

Fig. 6. Excess temperature, A_{measured} (95% CI) and solutions from the ACGIH, Hemeon and proposed area equations at $Z = 0.10$ m.

laboratory data. This instrument allows for the measurement of air velocities in the range of 0–50 m s^{-1}, with an accuracy of ±0.02 m s^{-1} or ±3% of reading, whichever is greater (Velocicalc Plus Model 8386A, TSI Inc., Shoreview, MN). The limit of detection (LOD) for the instrument was established as <0.02 m s^{-1}.

Variables that could be controlled in the experiments included the height above the heated source (Z), radial distance from the axisymmetric centerline (r) and the excess temperature (ΔT). These were pivotal to determine boundary layer thickness (δ), velocity (U_r) and height (H) above the virtual point source. Five height (Z), three radii (r) and six excess temperature (ΔT) combinations were included in the experiments (see Table 1). The maximum Z value selected was 1 m, as this would provide data sufficient for comparing the three high canopy hood equations. High canopy hoods are defined as being located at a distance of one source diameter or 1 m above the top of the hot source, whichever is smaller (ACGIH, 1998, 2007; Hemeon, 1999). Thirty minutes was given between each change in ΔT to provide time for the source to achieve steady-state conditions. At least three repetitions were conducted for each unique combination. The anemometer collected velocity data every 20 s and logged the mean every 2 min. These data were then downloaded into a spreadsheet application for organization and initial analyses (Microsoft Inc., Excel 2003, Redmond, WA).

The characteristic horizontal length scale (b) in the Gaussian velocity distribution equation was used to determine the effective plume radius (R). The b_{measured} was determined from the experiments by measuring the centerline velocity ($U_{\max}$) and a velocity (U_r) at a radius (r) (equation 1).



Measured = Measured plume area (m^2) H = Area from Hemeon equation (m^2)
A = Area from ACGIH equation (m^2) P = Area from proposed equation (m^2)

Fig. 7. Excess temperature, A_{measured} (95% CI) and solutions from the ACGIH, Hemeon and proposed area equations at $Z = 0.25$ m.

$$\frac{U_r}{U_{\max}} = e^{-\left(\frac{r^2}{b^2_{\text{measured}}}\right)}.$$

Rearranging the equation to solve for b_{measured} (m) provides:

$$b_{\text{measured}} = \left[\frac{-r^2}{\ln(U_r/U_{\max})}\right]^{0.5}. \qquad (16)$$

This equation for the horizontal length scale provides the radius where the centerline velocity has been reduced by the factor e^{-1}. This same relationship also allows for extrapolating the location where the plume velocity in the experiments has decreased to 1% $U_{\max}$ (i.e. the 99% reduction radius). As described earlier, if the Gaussian velocity distribution equation is used to solve for r in terms of b_{measured}, constants can be used to express additional reduction radius equations (equation 7).

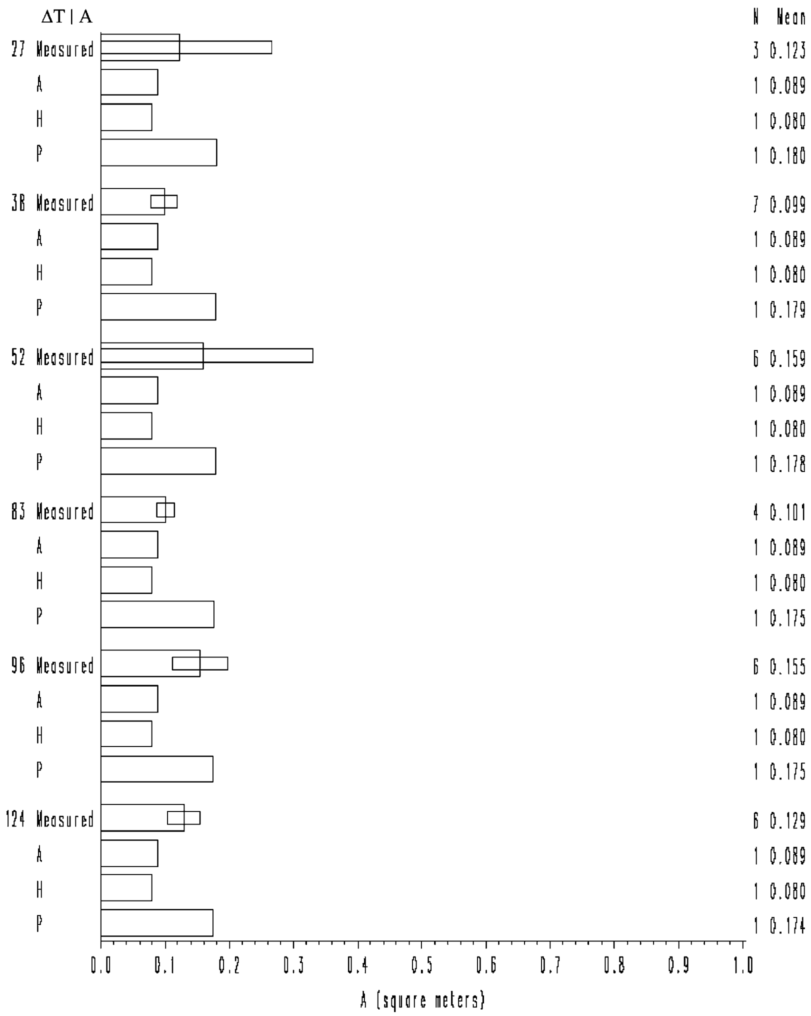
$$R = 2.14 b_{\text{measured}}$$
$$R = 2.14 \cdot \left[\frac{-r^2}{\ln(U_r/U_{\max})}\right]^{0.5}. \qquad (17)$$

The area of the plume (A_{measured}) determined from the experiments is then:

$$A_{\text{measured}} = \pi \cdot R^2. \qquad (18)$$

Substituting R from equation 17 into equation 18 provides the expression for the experimental A_{measured} (m^2).

$$A_{\text{measured}} = \pi(2.14 b_{\text{measured}})^2 = 4.58\pi \cdot \left[\frac{-r^2}{\ln(U_r/U_{\max})}\right]. \qquad (19)$$



Measured = Measured plume area (m^2) H = Area from Hemeon equation (m^2)
A = Area from ACGIH equation (m^2) P = Area from proposed equation (m^2)

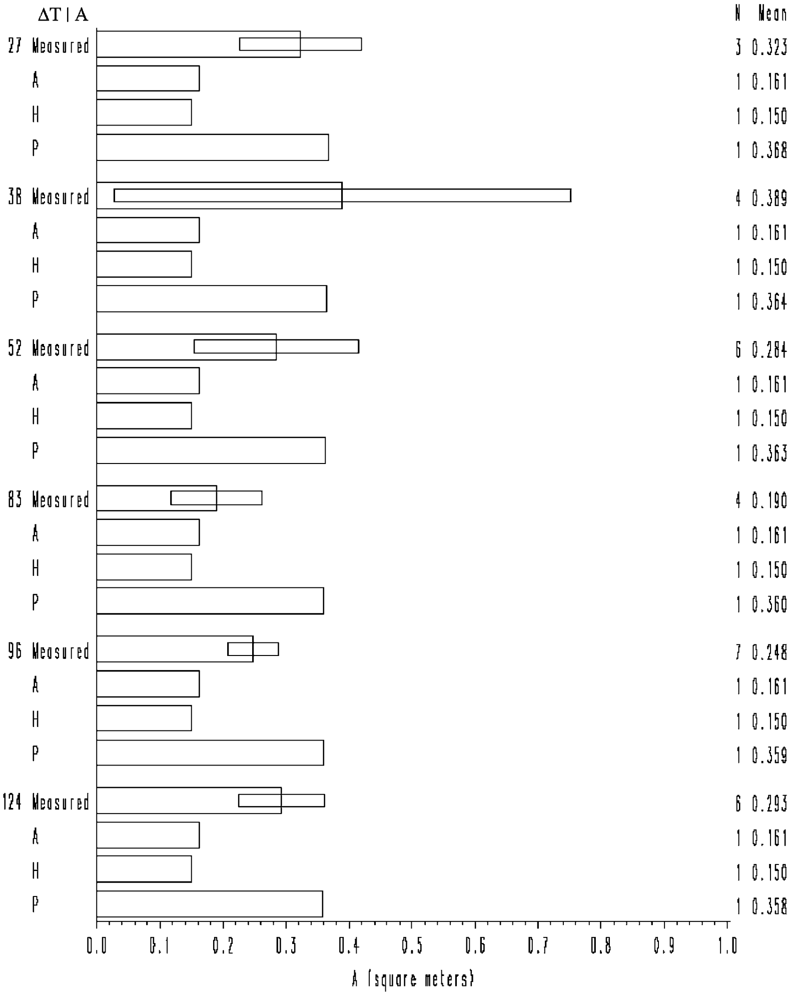
Fig. 8. Excess temperature, A_{measured} (95% CI) and solutions from the ACGIH, Hemeon and proposed area equations at $Z = 0.40$ m.

Statistical analyses

Centerline velocity data ($U_{\max}$), U_r and r data collected at each of 30 unique combinations of ΔT and Z were analyzed. Due to unequal sample sizes for $U_{\max}$ and U_r, b_{measured} was calculated for all combinations of $U_{\max}$ and U_r at each unique ΔT and Z combination. The thermal anemometer's LOD divided by $\sqrt{2}$ was substituted for U_r values that were below the LOD (i.e. $U_r < 0.02$ m s^{-1}) (Hornung and Reed, 1990). The mean b_{measured} was determined for each of the 30 unique combinations of ΔT and Z. This mean value was then used to calculate the A_{measured} at each of the 30 combinations of ΔT and Z.

A comparison was conducted between the A_{measured} data from the laboratory and the plume area values provided by the proposed, ACGIH and Hemeon equations. Examinations to determine the normality of the data distribution were conducted for the difference between the measured data and the three estimation equations within unique Z and ΔT combinations (i.e. 30 data pairs) using the CAPABILITY procedure in SAS (SAS Institute, Version 9.1.3, Cary, NC). If the data were approximately normal, separate paired t-tests were conducted for the difference of means between the measured data within unique Z and ΔT combinations and solutions from each of the three estimation equations using the UNIVARIATE procedure in SAS. If the data were not approximately normally distributed, three separate non-parametric Wilcoxon signed-rank tests were conducted for median differences using the same SAS procedure.

The statistical tests conducted regarded the difference of means (or median differences) as uncorrelated between unique Z and ΔT combinations. Ideal results would be that there were no differences



Measured = Measured plume area (m^2) H = Area from Hemeon equation (m^2)
A = Area from ACGIH equation (m^2) P = Area from proposed equation (m^2)

Fig. 9. Excess temperature, $A_{measured}$ (95% CI) and solutions from the ACGIH, Hemeon and proposed area equations at $Z = 0.70$ m.

between the calculated solution from each of the three estimation equations and the $A_{measured}$ values (i.e. P-values > 0.05). The three t-tests, or Wilcoxon signed-rank tests, provided an overall assessment of the agreement between the $A_{measured}$ and estimated area solutions from each of the three equations. If the individual estimation equation analysis fails to reject the null hypothesis, then that equation can be used to estimate the plume area with insignificant difference from the $A_{measured}$ data. Adjusted coefficients of determination (R^2) values are provided as a measure of association between $A_{measured}$ data and height.

RESULTS

One hundred and fifty-five data points were collected in the laboratory to determine plume radii for the 30 unique combinations of ΔT and Z. Figure 3 provides mean measured plume area ($A_{measured}$) data with 95% confidence intervals (95% CI) and solutions from the proposed (A_P), ACGIH (A_A) and Hemeon (A_H) equations at each of the five Z values investigated. This figure indicates a positive association between increased Z and increased plume area (adjusted $R^2 = 0.35$). Figures 4 and 5 provide measured plume volumetric flow ($Q_{measured}$), velocity ($U_{measured}$), and area ($A_{measured}$) data, with 95% CI, from the laboratory at the five Z values and six excess temperatures investigated. Plume velocity was directly measured. Plume area was based on equation 19, and volumetric flow was determined using the basic flow equation, $Q = \bar{U}A$ (ACGIH, 1998, 2007; McKernan and Ellenbecker, 2007). Figure 4 indicates a positive trend of increased flow and area and a non-linear velocity trend at increased Z values.

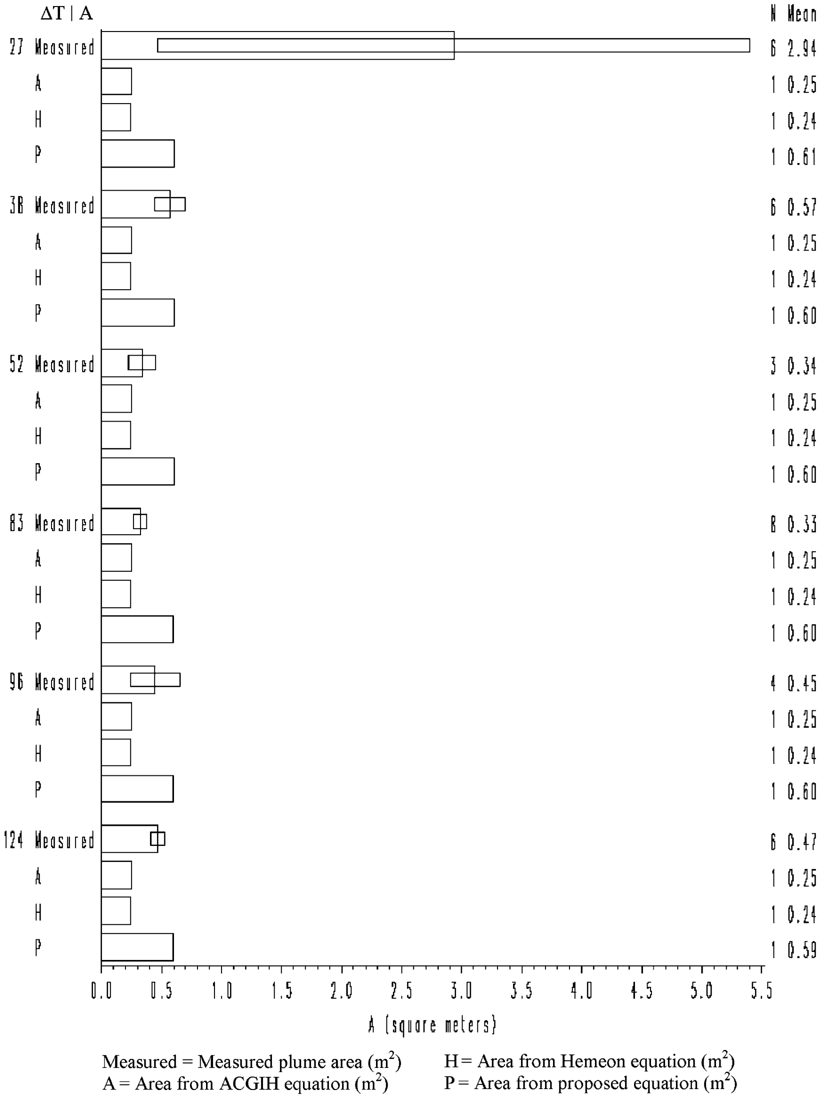


Fig. 10. Excess temperature, A_{measured} (95% CI) and solutions from the ACGIH, Hemeon and proposed area equations at $Z = 1.00$ m.

Figure 5 indicates a positive trend of increased velocity and non-linear trends of flow and area at increased excess temperatures.

Figures 6–10 depict plume area data from each of the five Z values investigated. The figures provide the six ΔT investigated in the left column, with A_{measured} and the three solutions from the estimation equations at each ΔT. The number of samples collected and mean area values are also provided in columns on the right of the figure. Confidence intervals (95% CI) are provided for the A_{measured} data.

The difference of means between A_{measured} and each of the three estimation equations were plotted individually (with all test parameters combined) and were not normally distributed. Therefore, significance for the three estimation equations was determined using the P-value from the Wilcoxon signed-rank test. The signed-rank test results indicate that area estimates provided by the proposed equation were not significantly different from the measured plume area (median difference $= -0.03$ m^2, $\sigma = 0.24$ m^2, P-value $= 0.24$). However, the measured plume area was significantly greater than estimates provided by the ACGIH and Hemeon equations (median difference $= 0.08$ m^2, $\sigma = 0.27$ m^2, P-value <0.01).

DISCUSSION AND CONCLUSIONS

Figure 3 indicates a positive association between increased Z and increased plume area. This result was expected; however, the growth of the plume area is more rapid than the currently recommended equations indicate. Notably, solutions from the ACGIH

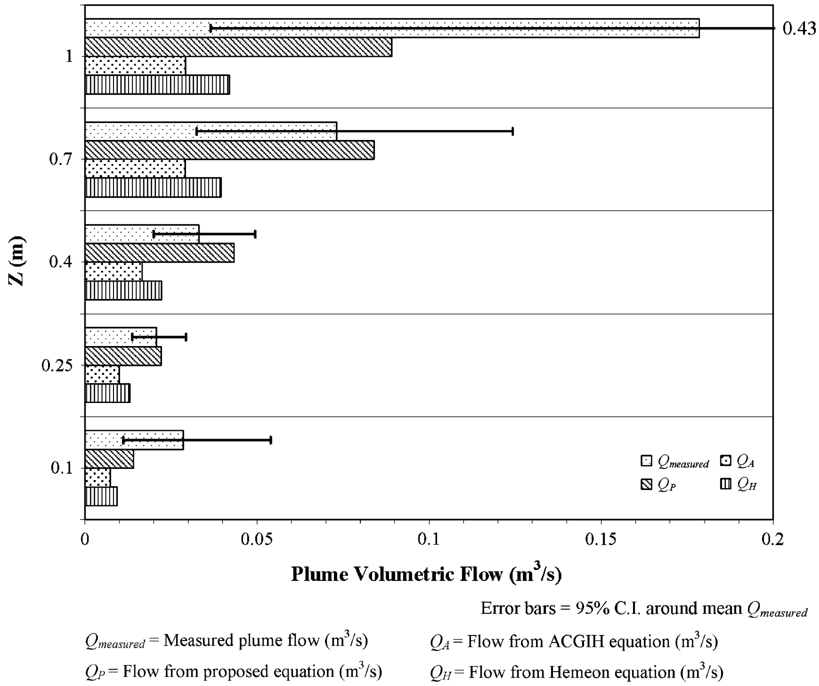


Fig. 11. Q_{measured} (95% CI) and solutions from the proposed, ACGIH and Hemeon flow equations at heights (Z).

and Hemeon equations are similar throughout the range of Z values investigated. There also appears to be a narrow region close to the top of the heated source (Z = 0.10 m) where plume growth is highly variable (see Fig. 6), particularly at $\Delta T \leq 83$ K. A hypothesis for the variability close to the top of the heated source is that the velocity profile at Z = 0.10 m and $\Delta T \leq 83$ K may not be Gaussian in nature. Flow in this region may be better described using laminar flow profiles. This point was also indicated in previous research regarding the study of velocity profiles above heated sources conducted by the authors, as well as by an independent researcher (Mundt, 1996; McKernan *et al.*, 2007).

Results indicate that the plume area based on R_{P} (i.e. 99% reduction radius) provides an ~98% increase in estimated plume area over the plume area based on the $\bar{R}_{\mathrm{TE}}$ (i.e. 90% reduction radius). This is illustrated by the percent difference between the plume areas using the 90% ($\bar{R}_{\mathrm{TE}}$) and 99% (R_{P}) reduction radii:

$$\frac{\pi\left(R_{\mathrm{p}}^{2}-\bar{R}_{\mathrm{TE}}^{2}\right)}{\pi(\bar{R}_{\mathrm{TE}})^{2}}\cdot 100=\left(\frac{2.14^{2}-1.52^{2}}{1.52^{2}}\right)\cdot 100=98\%. \tag{20}$$

Implications for determining the volumetric flow

The proposed (Q_{P}), ACGIH (Q_{A}) and Hemeon (Q_{H}) volumetric flow estimation equations are provided and described elsewhere (ACGIH, 1998' 2007; Hemeon, 1999; McKernan and Ellenbecker, 2007). A case study illustrating the use of the proposed flow equation is provided in Appendix 1. Solutions for Q_{P}, Q_{A} and Q_{H} were calculated and plotted in Fig. 11. For comparison, the measured flow (i.e. Q_{measured}) is also plotted in the same figure, with 95% CI around the mean. A comparison was conducted between the Q_{measured} data from the laboratory, and the volumetric flow estimations provided by the Q_{P}, Q_{A} and Q_{H} equations. As described for the area equation comparison, significance for the three flow estimation equations was determined using the P-value from the Wilcoxon signed-rank test. Results indicate that flow estimates provided by the proposed equation were significantly greater than the measured plume flow (median difference = -0.02 m^3 s^{-1}, σ = 0.03 m^3 s^{-1} and P-value = 0.0002). On the other hand, flow estimates provided by the ACGIH and Hemeon equation were significantly less than measured data (median difference = 0.03 m^3 s^{-1}, σ = 0.03 m^3 s^{-1} and P-value < 0.0001). Although estimates provided by the Q_{P} equation are significantly different than Q_{measured} data, it can be seen in Fig. 11 that Q_{P} provides added accuracy over solutions from the ACGIH and Hemeon equations at Z values >0.40 m over the range of parameters investigated.

These research findings provide valuable information necessary for researchers and practitioners to determine critical parameters to assist in the design and evaluation of engineering controls for exothermic processes. This study indicates that the proposed equation for buoyant plume area (A_P) provides additional accuracy for plume area estimation when compared with solutions from the currently accepted ACGIH (A_A) and Hemeon (A_H) equations over the range of parameters investigated. All three plume flow estimation equations (i.e. Q_P, Q_A and Q_H) were significantly different from the observed flow data. However, the proposed flow equation yielded estimates that were closer to the observed values than those from the ACGIH or Hemeon flow equations.

Additional research is required due to the limited heights (Z), temperature ranges (ΔT) and fixed dimensions of the source investigated. Although the evaluation method conducted provides statistically valid results, additional Z, ΔT and varying source dimensions would augment the results of this research.

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APPENDIX 1: NOMENCLATURE AND CASE STUDY UTILIZING THE PROPOSED AREA AND VOLUMETRIC FLOW EQUATION

Nomenclature

Boundary layer thickness (δ)—thickness of the air layer in contact with the heated source that experiences conductive heat transfer (m).

Laminar flow—flow in which layers of air move smoothly over one another in the direction of movement.

Turbulent flow—flow in which the layers of air are perturbed and mix freely with one another in the direction of movement.

Case study

Given:

1.20 m melting pot diameter (D) [$R_S = 0.60$ m],
2 m melting pot height (L),
600°C melting temperature = 873 K (T_s),
70°C ambient temperature = 343 K (T_∞),
Circular canopy hood located 3 m above pot (Z),
Emissivity of melting pot; $\varepsilon = 0.95$ (dimensionless),
Stefan–Boltzmann's constant; $\sigma = 5.67 \times 10^{-8}$ W m^{-2} · K^4,

Calculate H_P:

$$\delta \approx 0.05\left(\frac{L}{\Delta T}\right)^{0.25} \approx 0.05\left(\frac{2}{873-343}\right)^{0.25} = 0.01\,\text{m},$$

$$R_V = R_S + \delta = 0.60 + 0.01 = 0.61\,\text{m},$$

$$Z'_P = 3.03 R_V^{1.16} = 1.71\,\text{m},$$

$$H_P = Z + Z'_P = 3 + 1.71 = 4.71\,\text{m}.$$

Calculate the plume cross-sectional area (A_P) at the hood face:

$$R_P = 0.38 \cdot H_P^{0.86} = 0.38 \cdot 4.71^{0.86} = 1.44\,\text{m},$$

$$A_P = \pi \cdot R_P^2 = \pi \cdot (1.44)^2 = 6.5\,\text{m}^2.$$

Calculate the plume mean velocity ($\bar{U}_P$) at the hood face:

$$A_S = \pi \cdot R_V^2 = \pi \cdot 0.61^2 = 1.17\,\text{m}^2,$$

$$P_R = \varepsilon\sigma\left(T_S^4 - T_\infty^4\right) = 5.40 \times 10^{-8}\left(873^4 - 343^4\right) = 30\,618\ \text{Wm}^{-2},$$

$$h_P = 1.52(\Delta T)^{0.33}\text{Wm}^{-2} \cdot K,$$

$$P_C = h_P\left(\Delta T\right) = 1.52(\Delta T)^{1.33} = 1.52(873 - 343)^{1.33} = 6385\quad \text{Wm}^{-2},$$

$$P = P_R + P_C = 30\,618 + 6385 = 37\,003\ \text{Wm}^{-2},$$

$$\bar{\boldsymbol{U}}_{\mathbf{P}} = \frac{0.37}{\boldsymbol{H}_{\mathbf{P}}^{0.29}}\left(\frac{\boldsymbol{A_S P}}{\boldsymbol{T}_\infty}\right)^{0.33} = \frac{0.37}{4.71^{0.29}}\left(\frac{1.17 \cdot 37003}{343}\right)^{0.33} = 1.2\,\text{m}\,\text{s}^{-1}$$

Calculate the plume volumetric flow (Q_P) at the hood face:

$$Q_P = \bar{U}_P \cdot A_P = 0.17 H_P^{1.43}\left(\frac{A_S P}{T_\infty}\right)^{0.33} = 0.17(4.71)^{1.43}\left(\frac{1.17 \cdot 37\,003}{343}\right)^{0.33},$$

$$Q_p \approx 7.6\,\text{m}^3\,\text{s}^{-1}.$$

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