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Ventilation Equations for Improved Exothermic Process Control

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Exothermic or heated processes create potentially unsafe work environments for an estimated 5–10 million American workers each year. Excessive heat and process contaminants have the potential to cause acute health effects such as heat stroke, and chronic effects such as manganism in welders. Although millions of workers are exposed to exothermic processes, insufficient attention has been given to continuously improving engineering technologies for these processes to provide effective and efficient control. Currently there is no specific occupational standard established by OSHA regarding exposure to heat from exothermic processes, therefore it is important to investigate techniques that can mitigate known and potential adverse occupational health effects. The current understanding of engineering controls for exothermic processes is primarily based on a book chapter written by W. C. L. Hemeon in 1955. Improvements in heat transfer and meteorological theory necessary to design improved process controls have occurred since this time. The research presented involved a review of the physical properties, heat transfer and meteorological theories governing buoyant air flow created by exothermic processes. These properties and theories were used to identify parameters and develop equations required for the determination of buoyant volumetric flow to assist in improving ventilation controls. Goals of this research were to develop and describe a new (i.e. proposed) flow equation, and compare it to currently accepted ones by Hemeon and the American Conference of Governmental Industrial Hygienists (ACGIH). Numerical assessments were conducted to compare solutions from the proposed equations for plume area, mean velocity and flow to those from the ACGIH and Hemeon. Parameters were varied for the dependent variables and solutions from the proposed, ACGIH, and Hemeon equations for plume area, mean velocity and flow were analyzed using a randomized complete block statistical design (ANOVA). Results indicate that the proposed plume mean velocity equation provides significantly greater means than either the ACGIH or Hemeon equations throughout the range of parameters investigated. The proposed equations for plume area and flow also provide significantly greater means than either the ACGIH or Hemeon equations at distances >1 m above exothermic processes. With an accurate solution for the total volumetric flow, ventilation engineers and practicing industrial hygienists are equipped with the necessary information to design and size hoods, as well as place them at an optimal distance from the source to provide adequate control of the rising plume. The equations developed will allow researchers and practitioners to determine the critical control parameters for exothermic processes, such as the exhaust flow necessary to improve efficacy and efficiency, while ensuring adequate worker protection.

Keywords: engineering controls; hot process; local exhaust ventilation

INTRODUCTION

In general, it is desirable to locate and operate controls at an optimal distance from the heated source to minimize adverse effects on process performance, and at the same time maintain a safe and healthy environment for workers in the area. Designs for

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ventilating exothermic processes are difficult in that they must meet two sometimes competing goals. The design should minimize heat transfer rates to ensure retaining a majority of the heat in the process, thereby reducing operating costs, and at the same time maintain a safe and healthy work environment for employees. If the ventilation system induces excessive flow, it negatively impacts operations in two ways. It decreases the operating efficiency of the process by increasing heat transfer rates, and wastes conditioned indoor air—requiring more energy to maintain process and work environment temperatures. At the same time, exhaust flows must be sufficient to capture airborne contaminants and reduce worker exposures.

Examining the early ventilation design strategies intended to control heat and associated occupational exposures from exothermic processes, it can be seen that an initial attempt was made in the middle of the last century to correlate early heat transfer and empirical meteorological data. These data were used to develop equations to estimate the behavior of heated air masses (i.e. buoyant plumes) in indoor environments, with the intent of assisting engineers in designing ventilation systems to control these plumes. Hemeon was first, and most notable for attempting these data correlations and designing ventilation controls for exothermic processes beginning with his publication, 'Plant and Process Ventilation' in 1955 and again in 1963 and 1999 (Hemeon, 1955, 1963, 1999). Hemeon based his assertions for the behavior of buoyant plumes and effluents from exothermic processes on simplified heat transfer theory as it was understood in the mid-1950s. In his writings, he also utilized an observational study and an atmospheric plume model (Griffiths and Davis, 1931; Sutton, 1950). However, Hemeon did not make use of the numerical heat transfer theory being developed at the time, as described by McAdams and others (McAdams, 1954). Instead, Hemeon used empirically derived equations to characterize the behavior of buoyant plumes rising from exothermic processes. For Hemeon's equations to be valid, canopy hoods must be located more than two source diameters above the source, and temperature differences between the environment and the plume must be <110°C. This ensures that the results obtained from solutions to the three differential equations, namely conservation of mass, momentum and density difference, and their boundary conditions yield results with an acceptable accuracy. Hemeon's equations are still in use, and have been adopted in many publications including influential reference books, such as the American Conference of Governmental Industrial Hygienists (ACGIH) Industrial Ventilation Manual (ACGIH, 2004; Hemeon, 1999).

Other researchers followed Hemeon's example, attempting to correlate an ever increasing body of empirical data to hone his initial equations and ventilation control designs (Baturin, 1972; Goodfellow, 1985; Heinsohn, 1991). These authors provide little guidance to the development of their initial equations and assumptions, an attempt is made in this research to present the heat transfer and meteorological theories applicable to developing estimation equations for buoyant flow generated by exothermic sources.

BACKGROUND AND METHODS

To calculate volumetric flow (Q), the mean velocity ($\bar{U}$) and area (A) of the buoyant plume at the location of the control device is required. If the mean velocity and area at the hood face can be ascertained, then the basic flow equation can be solved [$Q = \bar{U}A$] (ACGIH, 2004). Since both the velocity and area of the buoyant plume emitted by a heated source vary as it rises, applicable heat transfer and meteorological theories were identified to develop proposed equations to accurately characterize the flow.

A review of heat transfer and meteorological theories provide the basic differential, partial differential and integral equations to evaluate flow in the boundary layer close to the heated source, and in the buoyant thermal plume. Attaining accurate results from these equations requires knowledge of the physical characteristics and orientation of the source. However, since the objective of the work is to provide simplified equations for use by researchers and practitioners, correlation equations will be presented in lieu of complex equations with exact solutions. Given that a wide array of processes, such as smelting, chemical and nuclear reactors, and plastic melting operations occur in vessels of roughly rectangular or cylindrical geometry, the equations developed here will assume a simple geometry, specifically a vertical truncated cylinder (Burgess, 1995). The shape selected should be generalizable to cubes and rectangular boxes [right-angled parallelepipeds] (Warrington *et al*., 1988). The applicable heat transfer correlation equations for vertical cylinders are the same as those for the well-characterized condition of the heated vertical flat plate (Bird *et al*., 2002; Incropera and Dewitt, 1996; Jaluria, 1980). Results from past studies support the use of heated vertical flat plate correlation equations in lieu of the heated vertical cylinder equations since transverse curvature influences on boundary layer development are small or negligible (Incropera and Dewitt, 1996). In fact, for the vertical upright cylinder the error introduced by using correlation equations instead of exact partial differential solutions is on the order of 20%. Error from disregarding the influence of transverse curvature present for the case of the vertical truncated cylinder is ≤5% at the Prandtl value investigated (Sparrow and Gregg, 1956).

In this investigation, it is assumed that conditions exist where steady-state and two-dimensional physical properties are conserved. Basic assumptions also include that the immersion fluid is infinite quiescent air at atmospheric pressure and uniform temperature (i.e. isothermal conditions). 'Infinite' implies that the fluid is not restricted in its movement, and is expansive enough to minimize conductive heat transfer. Additionally, in all calculations the proposed exothermic process has uniform, constant temperature (i.e. constant heat flux), and the immersion fluid (i.e. air) is incompressible. The exception to the incompressibility assumption involves accounting for the effect of variable density in the buoyancy force. This exception is explained through the use of the Boussinesq approximation (Bird *et al*., 2002). This approximation simplifies the momentum equation, but retains the relevant terms that represent the variation in the density of the fluid, since this variation induces the buoyant flow.

To utilize heat transfer correlation equations to estimate the plume mean velocity ($\bar{U}$) and area (A), a number of dimensionless equations used to characterize the advective conditions created must first be presented. A treatment of the problem of determining the mean velocity and area of the buoyant plume will follow the presentation of the dimensionless heat transfer equations, as they are related to the equations necessary for the determination of total flow at the control face.

To investigate agreement between the proposed equations developed for buoyant volumetric flow and existing accepted equations, numerical methods were used. Numerical solutions from the proposed equations were compared to solutions from Hemeon and ACGIH equations for plume area, mean velocity and volumetric flow (ACGIH, 2004; Hemeon, 1999). Additionally, a case study illustrating the use of the proposed equations for Q, $\bar{U}$ and A is provided in Appendix 1.

Grashof and Prandtl numbers

The Grashof number is a dimensionless metric used to characterize buoyant flow. The Grashof number (Gr) provides a dimensionless ratio of the buoyancy force to the viscous force in the fluid. The Grashof number plays the same role in natural convection that Reynolds number (Re) does in forced convection, and is proportional to the Reynolds number (Bird *et al*., 2002; Gebhart *et al*., 1988; Pitts and Sissom, 1977). Generally speaking, if the body is oriented such that the major axis is horizontal, the horizontal length (width) is used in calculating the Grashof number. If the body is situated so that the major axis is vertical, the vertical height is used. In developing the following Gr for heat flux at the surface, a combination of these two orientations was used to represent the body. This two-dimensional approach requires the combination of Gr equations from existing heat transfer theory where the length and diameter of the source are substantial (i.e. they both contribute to heat transfer) (Gebhart *et al*., 1988; Holman, 2002). Here, the Gr is solved using a characteristic length X that is based on the physical length (height) and equivalent width of the heated source in meters.

$$Gr_X = \frac{g\beta(\Delta T)}{\nu^2} \times \frac{L^4}{W}, \tag{1}$$

where in air:

Gr_X = Grashof number based on the characteristic length (X) [dimensionless];
X = Characteristic length; L^4/W (m^3);
L = Physical length (height) of heated source (m);
W = Width of the equivalent vertical plate; $\pi \cdot D$(m);
g = Gravitational constant; 9.81 (m/s^2);
β = Coefficient of thermal expansion evaluated at T_F (K^{-1});
T_F = Film temperature; 0.50 ($T_S + T_\infty$) (K);
ΔT = Excess temperature; $T_s - T_\infty$ (K);
T_s = Surface temperature of heated source (K);
T_∞ = Ambient temperature (K);
ν = Kinematic viscosity evaluated at T_F (m^2/s);
D = Diameter of heated source; $2R$: equivalent to hydraulic diameter for polygonal or irregularly shaped objects (m), where

$$\text{Hydraulic diameter} = 4 \times \left(\frac{\mathrm{L \cdot W}}{2(\mathrm{L+W})}\right).$$

For similarity to exist between the upright vertical cylinder and the vertical plate correlations, vertical cylinders must satisfy the equation (Jaluria, 1980):

$$\frac{D}{L} \geqslant \frac{35}{Gr_X^{0.25}}. \tag{2}$$

The Prandtl number (Pr) represents the dimensionless ratio of the momentum diffusion to the thermal diffusion (Bird *et al*., 2002).

$$Pr = \frac{\nu}{\alpha'} = \frac{C_p\mu}{k} = 0.70, \tag{3}$$

where in air:

α' = Thermal diffusivity (m^2/s);
C_p = Heat capacity or specific heat (J/kg·K);
μ = Dynamic viscosity (kg/m·s);
k = Thermal conductivity (W/m·K).

Prandtl numbers for air are relatively stable within a wide range of temperatures (0–1500°C) since the product of the heat capacity and dynamic viscosity change proportionately with the thermal conductivity over this range of temperatures. Therefore, a constant standard Prandtl value of 0.70 is used in the equations developed.

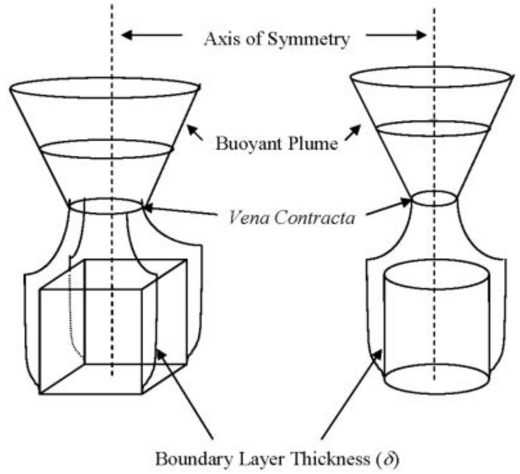


Fig. 1. The convective, thermal and velocity boundary layer with buoyant plume development.

Values of Gr_X can be used to determine if buoyant plume flow exists. For vertical planes and large inclined cylinders in air, it has been shown that if the Grashof number exceeds $\sim 10^3$ then the buoyant force present is sufficient to create a plume. Grashof numbers greater than 10^3 and less than 10^9 indicate laminar flow, and those greater than 10^9 indicate turbulent flow (Incropera and Dewitt, 1996).

Determining boundary layer thickness

To determine the initial plume area at the top of the heated source, the thickness of the boundary layer at the top of the source was calculated. A visual representation of the physical manifestation of heat transfer in the form of the boundary layer and buoyant plume development at and above two heated sources is provided in Figure 1. The curves around the outside of the cubic and cylindrical sources define the edge, or thickness, of the buoyancy and velocity boundary layer. Between the sources and these boundaries, fluid flow is moving upward due to the decreased density of the heated air. In Equation 4, by definition the velocity at this outer edge is 1% of the maximum velocity inside the boundary layer. Applying the constant Pr value for air to the boundary layer thickness (δ in m) equation from heat transfer theory provides (Jaluria, 1980):

$$\delta = 4.87 \cdot L \cdot (Pr \cdot Gr_X)^{-0.25}. \tag{4}$$

At standard indoor conditions, ($T_\infty \approx 20°\text{C}$) the temperature dependent variables reduce the equation to (Mundt, 1996):

$$\delta \approx 0.05 \left(\frac{L}{\Delta T}\right)^{0.25}. \tag{5}$$

Determining plume radius and area

Proposed equations can now be developed to describe the characteristics of the rising hot air after the boundary layer containing that air has separated from the source, and has formed a buoyant plume that is circular in cross-section. It has been noted that buoyant plumes become circular in cross-section as they rise, regardless of the source shape, as turbulence tends to sweep the plume edges inward to a minimal volume (Bill and Gebhart, 1975). To ensure that these equations for buoyant flow downstream from the heated source are constructed using the best available relevant information, it is necessary to move from heat transfer theory to meteorological plume theory to determine the buoyant plume radius and area. The analysis builds on the assertion that the velocity profile in the plume is Gaussian (Batchelor, 1954; Morton *et al.*, 1956; Priestley and Ball, 1955; Sutton, 1950).

Currently accepted equations express the axisymmetric plume radius (i.e. the radius measured from the axis of symmetry) as the product of a constant and a power of vertical height (H) above a virtual point source (ACGIH, 2004; Hemeon, 1999; Sutton, 1950). The virtual point source is fully described, and equations to determine its location are provided in the next section. However, what defines the boundaries of the axisymmetric plume radius is not well-described in the literature. Upon close examination of the equations by Hemeon and the ACGIH, it can be seen that the accepted equation for the plume radius at H is defined by the convention that the buoyant plume boundary is found at points where the velocity has been reduced to $\sim$10% of the maximum centerline velocity. This conclusion is arrived at through examining Sutton and Hemeon's work, and relating it to Morton's expression for relative velocity and plume radius. To determine the plume radius where the velocity is some fraction of U_{Max} at a vertical height (H), Morton's equation for the Gaussian velocity distribution is used (Morton *et al.*, 1956):

$$\frac{U_r}{U_{\text{Max}}} = e^{-\left(\frac{r^2}{b^2}\right)}, \tag{6}$$

U_r = Buoyant plume velocity at a distance r from the axisymmetric centerline (m/s);

U_{Max} = Maximum buoyant plume velocity, located at the axisymmetric centerline (m/s);

r = Distance from the axisymmetric centerline where U_r is measured, arbitrary location (m);

b = Characteristic horizontal length scale, location where U_{Max} has been reduced by a factor of;

e^{-1}; equivalent to (6/5)αH(m) (Morton *et al.*, 1956);

α = Plume entrainment value (dimensionless);

H = Vertical height above virtual point source (m).

This relative velocity calculation can be useful to determine radii when the proportion of one velocity to another is desired or when the distances from the source to the point source are small. However, a limitation to this equation is that it cannot provide an accurate estimate for the velocity value at any desired height, due to the relationship of the equation on experimentally derived values for the variable α. Therefore, it is desirable to define an equation for b, which does not require the use of α. To determine values for b, with the additional intent of developing convenient equations for buoyant plume area based solely on H, the investigators looked to meteorological theory developed by Sutton (Sutton, 1950). Sutton put forth a theoretical equation for defining the radius as the points in the buoyant plume where the velocity has been reduced to 10% of the maximum centerline velocity (i.e. a 90% reduction radius). The equation is based on the continuous point source theory, which includes mechanistic entrainment theory developed by Taylor and Prandtl (Schmidt, 1941). Considering entrainment and continuous point source theories, Sutton provides the following theoretical equation for the 90% reduction radius:

$$R_T = 1.52C\left(H^{0.5m}\right), \tag{7}$$

C = Diffusion coefficient for a continuous point source plume (dimensionless);

m = Atmospheric diffusion constant (dimensionless).

The utility of the equation depends on the development of generalized diffusion coefficients, which are dependent on the meteorological condition (i.e. stability) of the environment, and an atmospheric diffusion constant. In the ambient environment, temperature decreases as height increases. This condition in meteorology is defined as neutral, or unstable. In indoor environments, temperature increases as height increases. This condition is defined as stable (USAEC, 1968). The generalized diffusion coefficients (C) that have been determined and reported are 0.27 and 0.12 for these conditions, respectively (Stewart *et al.*, 1954; Sutton, 1950). The atmospheric diffusion constant (m) is reportedly in the range of 1.70–2 (Railston, 1954; Schmidt, 1941; Sutton, 1950). With substitution of $C = 0.12$ for stable conditions, and $m = 1.75$, Sutton's plume equation becomes:

$$R_T = 0.18 \cdot H^{0.88}. \tag{8}$$

With these substitutions, Sutton's theoretical equation becomes semi-empirical since the generalized diffusion coefficient and atmospheric diffusion constant were determined through experiments. Furthermore, subsequent experiments were conducted by Railston utilizing a precision Schlieren optic system, which measures small variations in the refractive index of a heated fluid (Railston, 1954). Railston used this optical system to measure the radius and absolute temperature differential for a buoyant plume in air created by a small heat source limited to an output of 20 W. Railston's experimental values for C were between 0.24 and 0.27 (i.e. unstable conditions), with an experimentally determined value of $m = 1.70$. Railston's empirically derived 90% reduction radius equation is:

$$R_E = 0.40 \cdot H^{0.85}. \tag{9}$$

These results are not in agreement with Sutton's equation (Equation 8). Explanations for the differences may be due to the fact that Railston's equation was determined by the measurement of the temperature profile. The velocity profile would be better suited for the case of estimating volumetric flow, as the product of the buoyant plume mean velocity and area of the velocity profile would provide the most applicable flow estimates. It has been determined experimentally that in airflows dominated by buoyant forces, the temperature profile is approximately 10% wider than the velocity profile above point sources (Papanicolaou and List, 1988; Rouse *et al.*, 1952). Therefore, reducing the constant value of 0.40 in Railston's empirical equation by 10% provides:

$$R_E = 0.36 \cdot H^{0.85}. \tag{10}$$

To determine a reasonable and conservative equation to characterize the behavior of the 90% reduction radius, the average of the theoretical and empirical radius equations was used. This provides a plume radius equation that mitigates the differences between unstable meteorological conditions (i.e. empirical equation) and stable conditions (i.e. theoretical equation). The equation for this mean 90% reduction radius is:

$$R_{TE} = 0.27 \cdot H^{0.86}. \tag{11}$$

Using these theories and equations for the 90% reduction radius, it is possible to develop an equation for b which does not require the use of the empirically derived variable α. An equation for b can now be developed by setting the r value in Morton's relative velocity equation equal to the R_{TE}, and setting $U_r/U_{\text{Max}} = 0.10$ (i.e. 90% reduction radius), then rearranging for b:

$$0.10 = e^{-\left(\frac{R_{TE}^2}{b^2}\right)}$$

$$b = \frac{R_{TE}}{1.52} = 0.18 \cdot H^{0.86}. \tag{12}$$

Notably, this equation directly relates b to H, not to buoyant plume velocity or any specific reduction radius. Therefore, if this equation is used after reducing Morton's original relative velocity equation to solve for r in terms of b, constants can be used to express radius equations other than the 90% reduction radius. A solution for the 99% reduction radius (R_P) follows:

$$0.01 = e^{-\left(\frac{R_P}{b}\right)^2}$$

$$R_P = 2.14b, \tag{13}$$

$$R_P = 0.38 \cdot H^{0.86}. \tag{14}$$

Utilizing R_P (i.e. the 99% reduction radius) to calculate the buoyant plume area assures that the estimated area present at H, and the boundary layer area equation previously developed from heat transfer theory are directly comparable. Buoyant plume area is calculated as:

$$A_P = \pi \cdot R_P^2 = \pi\left(0.38H^{0.86}\right)^2 = 0.14\pi \cdot H^{1.72}. \tag{15}$$

Substituting the proposed equation for vertical height (H_P), developed in the next section, for H.

$$A_P = 0.45 \cdot H_P^{1.72}. \tag{16}$$

Currently accepted equations to determine plume area from ACGIH (A_A [m^2]) and Hemeon (A_H [m^2]) have the same form, with the exception of the calculation for H provided in the next section (ACGIH, 2004; Hemeon, 1999).

$$A_A = 0.15 \cdot H_A^{1.76}$$

$$A_H = 0.15 \cdot H_H^{1.76}. \tag{17}$$

Determining virtual point source distance and height

The location of the virtual point source is of some controversy, and has been defined by Hemeon and others as the distance below the top surface of the heated source where the presence of a point source could explain the growth of the plume at its fully developed height (Goodfellow, 1985; Hemeon, 1999; Morton *et al.*, 1956; Olander *et al.*, 2001). The location of this virtual point source is important in determining the plume area as it rises to the face of the control. Morton developed an equation involving the use of the characteristic horizontal length scale (b) to solve for the virtual point source distance. Building on Morton's equation and the previously developed equation for b in Equation 12, the proposed distance from the top surface of the heated source to the virtual point source (Z'_P) can be expressed as:

$$Z'_P = \sqrt[0.86]{\frac{b}{0.18}}. \tag{18}$$

The relationship for Z'_P can be simplified through the substitution of the equations for R_p in terms of b, and the projected source radius equation (R_V) for R_P at the top of the heated source. Therefore, the distance to virtual point source can be deduced:

$$Z'_P = \sqrt[0.86]{\frac{R_P/2.14}{0.18}}$$

$$Z'_P = \sqrt[0.86]{\frac{R_V/2.14}{0.18}}$$

$$Z'_P = \sqrt[0.86]{\frac{R_V}{0.34}} = 3.03R_V^{1.16}, \tag{19}$$

R_V = Projected radius of heated source; $R_S + \delta$(m);
R_S = Physical radius of heated source; $D/2$ (m).

The vertical height above the virtual point source (H) is the sum of the determined distance between the top surface of the heated source and the virtual point source (Z' [m]), and the distance between the top surface of the heated source and a point of interest on the plume centerline (Z [m]).

$$H = Z + Z'. \tag{20}$$

Although Z is usually known, the equation for Z' changes among the proposed, ACGIH and Hemeon equations for H. A proposed virtual point source distance (Z'_p) is defined above, therefore the proposed height equation (H_P) is:

$$H_P = Z + Z'_P \tag{20}$$

$$H_P = Z + \left(3.03\,R_V^{1.16}\right). \tag{21}$$

For comparison, the respective ACGIH and Hemeon equations for Z' and H are:

$$\begin{aligned} Z'_A &= (5.20R_S)^{1.14}(\mathrm{m}) \\ H_A &= Z + (5.20R_s)^{1.14}(\mathrm{m}) \end{aligned} \tag{22}$$

$$\begin{aligned} Z'_H &\approx 4(3.28R_s)[\mathrm{ft}];\ \text{or } 4R_s(\mathrm{m}) \\ Z_H &= 3.28Z(\mathrm{ft});\ \text{or } Z(\mathrm{m}) \\ H_H &= Z_H + Z'_H(\mathrm{ft});\ \text{or } Z + Z'_H(\mathrm{m}). \end{aligned} \tag{23}$$

Determining plume mean velocity

The plume mean velocity was derived using meteorological theory. Sutton developed a meteorological equation to describe plume velocity decay above heated sources. This equation is useful in that it relates power consumption to temperature, and provides plume mean velocity. In Sutton's equation, the excess temperature between the heated source and ambient environment are translated into power per unit area (Sutton, 1950). The power per unit area, or heat flux from the source (P [W/m^2]) is expressed as the sum of the radiant (P_R) and convective (P_C) heat flux based on Stefan–Boltzmann's law and Newton's law of cooling (Lide and Frederikse, 1996).

$$P_R = \varepsilon\sigma\left(T_S^4 - T_\infty^4\right)$$
$$P_C = h_P(\Delta T)$$
$$P = P_R + P_C = \left[5.40 \times 10^{-8}\left(T_S^4 - T_\infty^4\right)\right] + \left[1.52(\Delta T)^{1.33}\right]. \qquad (24)$$

ε = Emissivity of heated material; 0.95 (dimensionless) (Lide and Frederikse, 1996);

σ = Stefan–Boltzmann constant; 5.67×10^{-8} ($\mathrm{W/m^2 \cdot K^4}$);

h_P = Natural convection heat loss coefficient for horizontal plates; $1.52(\Delta T)^{0.33}$ [$\mathrm{W/m^2 \cdot K}$].

An emissivity of 0.95 is used in the following equations, due to the proposed use of black body approximations in subsequent experiments (Lide and Frederikse, 1996).

The proposed equation for plume mean velocity is an application of compiled meteorological theory outlined by Sutton (Stewart *et al.*, 1954; Sutton, 1950). The proposed meteorological equation for plume mean velocity decay above heated sources relates gravitation, heat flux and the physical properties of the heated medium to provide a plume mean velocity ($\bar{U}_P$[m/s]), at a given vertical height (H_P):

$$\bar{U}_P = \frac{1}{H_P^{0.29}}\left[\frac{7gA_SP}{3\pi C_P\rho_\infty T_\infty C}\right]^{0.33}. \qquad (25)$$

With substitution and reduction:

$$\bar{U}_p = \frac{0.37}{H_p^{0.29}}\left(\frac{A_sP}{T_\infty}\right)^{0.33} \qquad (26)$$

A_S = Projected area of source; $\pi(R_S + \delta)^2(m^2)$;

P = Total heat flux from the source; P_R + P_C = [$5.40 \times 10^{-8}(T_S^4 - T_\infty^4)$] + [$1.52(\Delta T)^{1.33}$]($\mathrm{W/m^2}$);

ρ_∞ = Density of ambient air at T_∞; $\sim$1.20 kg/m^3.

Currently accepted equations to determine plume mean velocity from ACGIH ($\bar{U}_A$) and Hemeon ($\bar{U}_H$) are (ACGIH, 2004; Hemeon, 1999):

$$\bar{U}_A = \frac{8.50 \times 10^{-2}}{H_A^{0.25}}A_T^{0.33}\Delta T^{\,0.42} \qquad (27)$$

$$\bar{U}_H = \frac{0.19}{H_H^{0.29}}\left(\frac{h_H}{60}A_B\Delta T\right)^{0.33}, \qquad (28)$$

A_T = Area of top surface of body emitting heat; $\pi(R_S)^2(\mathrm{m^2})$;

h_H = Natural convection heat loss coefficient for vertical plates; $0.30(\Delta T)^{0.25}\left(\mathrm{BTU/hr \cdot ft^2 \cdot {}^\circ F}\right)$;

A_B = Area of entire cylinder emitting heat; $\pi(3.28R_S)^2 + 21.5\pi LR_S\left(\mathrm{ft^2};\ L \text{ and } R_S \text{ in m}\right)$;

ΔT = Excess temperature; $(T_s - T_\infty)$ (S.I. units K, or English Units °F).

Table 1. Values for ΔT, L, D and Z included in the numerical assessment

ΔT (K)	L (m)	D (m)	Z (m)
80	0.05	0.01	0.02
230	0.1	0.05	0.1
380	0.75	0.1	0.5
480	3	0.75	1
1101	4	3	5
1490			15

Determining plume volumetric flow

The product of the plume mean velocity and area provides the conventional volumetric flow equation (ACGIH, 2004):

$$Q = \bar{U}A.$$

The product of the proposed plume mean velocity and area equation provides the proposed volumetric flow equation (m^3/s).

$$Q_P = 0.17H_P^{1.43}\left(\frac{A_SP}{T_\infty}\right)^{0.33}. \qquad (29)$$

The plume mean velocity ($\bar{U}$) and area (A) are currently determined through the use of either the ACGIH or Hemeon's equations. With reduction, the current equations from ACGIH (Q_A) and Hemeon (Q_H) are (ACGIH, 2004; Hemeon, 1999):

$$Q_A = 1.30 \times 10^{-2}H_A^{1.51}A_T^{0.33}\Delta T^{0.42}. \qquad (30)$$

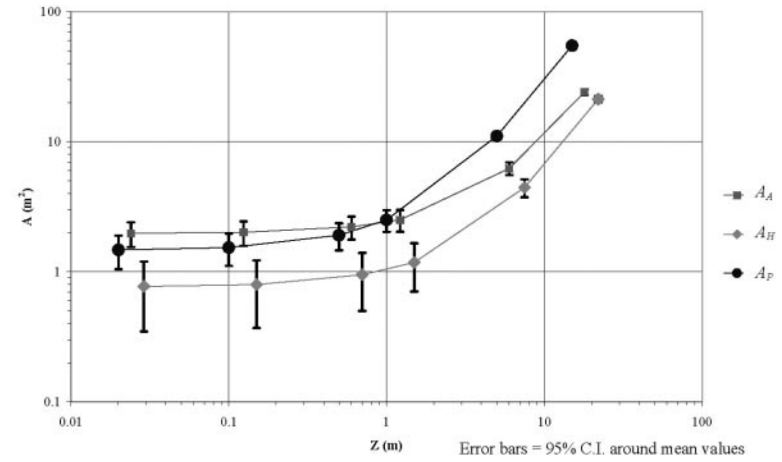
$$Q_H = 2.62 \times 10^{-3}H_H^{1.47}\left(\frac{h_H}{60}A_B\Delta T\right)^{0.33}. \qquad (31)$$

Numerical assessments of the proposed, ACGIH, and Hemeon equations for plume area, mean velocity and volumetric flow were conducted over a range of ΔT, L, D and Z-values with $L > D$ (Table 1). The assessment was conducted by plotting the mean solutions provided by each equation for plume area, mean velocity, and flow with confidence intervals about the means. The purpose of the assessments was to determine trends and differences between the solutions provided by the three equations.

Randomized complete block ANOVA with fixed effects was also conducted using the PROC MIXED procedure in SAS (SAS Version 9.1.3, SAS Institute, Cary, NC) to determine if the proposed, ACGIH and Hemeon equations for $\bar{U}$, A and Q provided solutions that were statistically different from one another. The outcome variable (Y) was defined as the solution to $\bar{U}$, A or Q from each of the three estimation equations, and (X) was defined as the specific estimation equation used (proposed, ACGIH or Hemeon). Blocks were constructed for each unique combination of ΔT, L, D and Z (Table 1). The block analysis provided multiple comparisons to investigate pair-wise differences. P-values were adjusted to correct for multiple comparisons using the Tukey option in SAS.

RESULTS

Numerical assessments of the three estimation equations are provided in Figures 2–4 for plume area, mean velocity and volumetric flow, respectively. Solutions are plotted using the normalized height above the heated source (Z). In Figures 2–4, the Z-values for the proposed, ACGIH and Hemeon equation solutions are intentionally skewed from the base values of 0.02, 0.10, 0.50, 1, 5 and 15 m. The skew provides separation between the mean solutions from the proposed, ACGIH and Hemeon equations with 95% confidence intervals (95% C.I.) around the plotted means in each figure. Trend lines for each flow equation were generated using the least squares fit procedure.

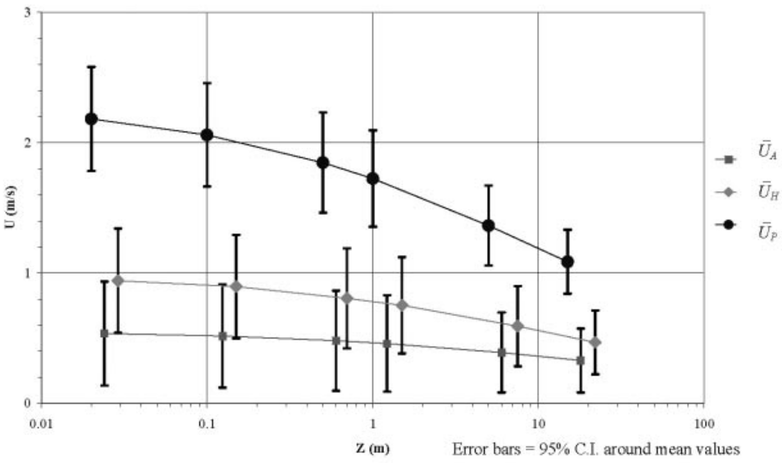


A_A = ACGIH plume area equation

A_H = Hemeon plume area equation

A_P = Proposed plume area equation

Fig. 2. Plume area (A) by normalized height (Z) for varying cylinder dimensions and temperatures. A_A = ACGIH plume area equation. A_H = Hemeon plume area equation. A_P = Proposed plume area equation.



$\bar{U}_A$ = ACGIH plume mean velocity equation

$\bar{U}_H$ = Hemeon plume mean velocity equation

$\bar{U}_P$ = Proposed plume mean velocity equation

Fig. 3. Plume mean velocity ($\bar{U}$) by normalized height (Z) for varying cylinder dimensions and temperatures. $\bar{U}_A$ = ACGIH plume mean velocity equation. $\bar{U}_H$ = Hemeon plume mean velocity equation. $\bar{U}_P$ = Proposed plume mean velocity equation.

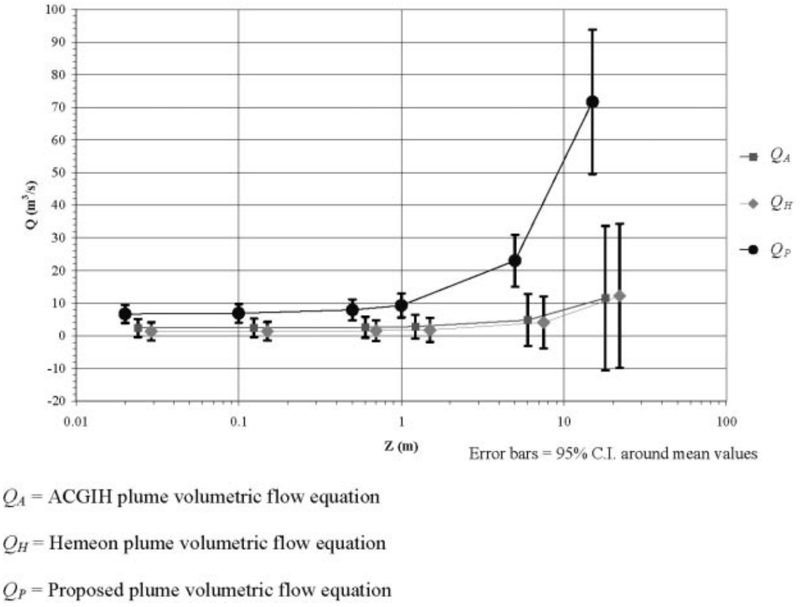


Fig. 4. Plume volumetric flow (Q) by normalized height (Z) for varying cylinder dimensions and temperatures. Q_A = ACGIH plume volumetric flow equation. Q_H = Hemeon plume volumetric flow equation. Q_P = Proposed plume volumetric flow equation.

Figure 2 and the results from the randomized complete block analysis for the plume area equation indicate that the proposed and ACGIH means are not significantly different at $Z \leqslant 1$ m. At $Z > 1$ m, the proposed means are significantly greater than the ACGIH means. The proposed and Hemeon means are not significantly different at $Z \leqslant 0.10$ m. At $Z > 0.10$ m, the proposed means are significantly greater than Hemeon means. The ACGIH and Hemeon means are significantly different throughout the range of parameters examined.

Figure 3 and the results from the randomized complete block analysis for the plume mean velocity equation indicate that the ACGIH and Hemeon means are not significantly different throughout the range of Z investigated. The proposed means are significantly greater than either the ACGIH or Hemeon means throughout the range of parameters examined.

Figure 4 and the results from the randomized complete block analysis for the plume volumetric flow equation indicate that the proposed and ACGIH means are not significantly different at $Z \leqslant 0.50$ m. At $Z > 0.50$ m, the proposed means are significantly larger than the ACGIH means. The ACGIH and Hemeon means are not significantly different throughout the range of Z investigated. The proposed means are significantly greater than the Hemeon means throughout the range of parameters examined.

DISCUSSION AND CONCLUSIONS

Hazards associated with exposure to exothermic processes include heat stress and potential inhalation exposure to chemical vapor, and fine or ultrafine (fume) particulates emitted from the process. Current equations recommended for the design of exothermic process control have limitations that have been outlined in previous publications (Heinsohn, 1991; Siebert and Fraser, 1973; Zhivov *et al.*, 2001). A review of heat transfer and meteorological equations applicable to constructing a proposed equation for buoyant flow estimation (Q_P) were developed and presented. The numerical assessment conducted over varying excess temperatures, source sizes and plume heights, provided solutions to the proposed, ACGIH and Hemeon equations for comparison.

When combining results from Figure 4 and the randomized complete block analysis, it can be seen that the proposed volumetric flow equation (Q_P) provides significantly greater means than those from the ACGIH and Hemeon at Z-values >0.50 m. At $Z \leqslant$ 0.50 m, the proposed equation provides means within the 95% C.I. of the ACGIH means, while remaining significantly greater than the Hemeon means. The differences between the proposed, ACGIH and Hemeon equations for plume flow are likely due to the proposed mean velocity equation ($\bar{U}_P$) providing significantly greater solutions than either the ACGIH or Hemeon equations throughout the range of parameters investigated. Additionally, the difference between the volumetric flow equations at Z-values >1 m are attributable to the use of a proposed plume area equation (A_P) that utilizes a plume radius (R_P) based on a mitigated solution from theoretical and empirical research. The R_P is based on a 99% reduction radius, (i.e. points in the plume where the velocity has been reduced to 1% of the centerline velocity) that provides a larger estimate of the plume area, compared to solutions from the ACGIH or Hemeon equations that are based on a 90% reduction radius.

Subsequent research will include collecting plume mean velocity and area data for model and actual exothermic processes. The numerical assessments conducted indicate that expected mean values for plume area are $\sim$3 m^2, with mean velocity values $\sim$1 m/s at Z-values <5 m. Data collected will be used to validate the proposed plume area, mean velocity and flow equations developed here. Additionally, solutions from the ACGIH and Hemeon equations will be compared to plume area, mean velocity and flow data from the laboratory and field to determine which of the three estimation equations (i.e. proposed, ACGIH or Hemeon) provide solutions that are closest to those measured. This research holds the potential to reduce exposures to particulates, chemical vapors and heat associated with exothermic processes while assuring effective designs to limit ventilation costs.

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Disclaimer—The findings and conclusions in this report are those of the authors and do not necessarily represent the views of the National Institute for Occupational Safety and Health.

APPENDIX 1: CASE STUDY UTILIZING THE PROPOSED EQUATIONS FOR Q, $\bar{U}$, AND A

Given: 1.20 m melting pot diameter (D) [$R_S = 0.60$ m]
2 m melting pot height (L)
600°C melting temperature = 873 K (T_s)
70°C ambient temperature = 343 K (T_∞)
Circular canopy hood located 3 m above pot (Z)
Calculate H_P:

$$H_p = Z + Z'_P = 3 + 1.71 = 4.71 \text{ m}$$

$$Z'_P = 3.03 R_V^{1.16} = 1.71 \text{ m}$$

$$R_V = R_S + \delta = 0.60 + 0.01 = 0.61 \text{ m}$$

$$\delta = 0.05\left(\frac{L}{\Delta T}\right)^{0.25} = 0.05\left(\frac{2}{873-343}\right)^{0.25} = 0.01 \text{ m}$$

Calculate cross-sectional area of plume (A_P) at hood face:

$$R_p = 0.38 \cdot H_P^{0.86} = 0.38 \cdot 4.71^{0.86} = 1.44 \text{ m}$$

$$A_P = \pi \cdot R_p^2 = \pi(1.44)^2 = 6.5 \text{ m}^2\text{; or}$$

$$A_P = 0.45 \cdot H_p^{1.72} = 0.45 \cdot (4.71)^{1.72} = 6.5 \text{ m}^2$$

Calculate the plume mean velocity ($\bar{U}_P$) at hood face:

$$\bar{U}_P = \frac{0.37}{H_P^{0.29}}\left(\frac{A_S P}{T_\infty}\right)^{0.33} = \frac{0.37}{4.71^{0.29}}\left(\frac{1.17 \cdot 37\,003}{343}\right)^{0.33} = 1.2 \text{ m/s}$$

$$A_s = \pi \cdot R_V^2 = \pi \cdot 0.61^2 = 1.17 \text{ m}^2$$

$$P = P_R + P_C = 30\,618 + 6385 = 37\,003 \text{ W/m}^2$$

$$P_R = \varepsilon\sigma\left(T_s^4 - T_\infty^4\right) = 5.40 \times 10^{-8}\left(873^4 - 343^4\right) = 30\,618 \text{ W/m}^2$$

$\varepsilon = 0.95$ (dimensionless)

$\sigma = 5.67 \times 10^{-8}$ W/m^2 ·K^4

$$P_C = h_P(\Delta T) = 1.52(\Delta T)^{1.33} = 1.52(873 - 343)^{1.33} = 6385 \text{ W/m}^2$$

$h_P = 1.52\ (\Delta T)^{0.33}$ W/m^2 ·K

Calculate the plume volumetric flow (Q_P) at hood face:

$$Q_P = \bar{U}_P \cdot A_P = 0.17 H_P^{1.43}\left(\frac{A_S P}{T_\infty}\right)^{0.33} = 0.17(4.71)^{1.43}\left(\frac{1.17 \cdot 37\,003}{343}\right)^{0.33}$$

$$Q_P = 7.6 \text{ m}^3\text{/s}$$

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