Strain Rate Effects in Similitude Modelling of Plastic Deformation of Structures Subject to Impact Loading

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IT has been accepted that roll over protection structures (ROPS) are a feasible means of protecting the tractor operator in the event of roll over. Proper design is critical to protection potential of any ROPS. Test procedures have been developed to determine deformation of frames and cabs in case of an actual overturn. These tests, based on actual overturns of tractors, were developed basically in Europe and accepted in this country with some modifications and additions (ASAE Standard S305.3). Controversy over the relative severity of various tests and the severity of each test relative to accidental overturns continues to prevent universal acceptance of any one testing procedure (US Dept. of Transportation 1971).

Adequacy of any test depends primarily on its ability to impact the ROPS with a loading similar to that encountered in acutal overturns. The loading that any ROPS is subjected to in actual overturn depends on a multitude of factors. It is difficult to determine these loading conditions for every significant combination of affecting factors by actual overturns because of time, cost and, more importantly lack of control on test conditions. It is desired that a method be developed to obtain reproducible engineering data needed for developing appropriate test procedures. If tests are performed on scale models a greater control on the test conditions is possible with considerably less cost and time.

To model a tractor-ROPS overturn situation it is important that all pertinent variables be modelled correctly including the properties of ROPS material. It has been shown that modelling of ROPS material properties imposes conditions that can only be satisfied if the same material is used for model and prototype (Davis 1971). It has also been shown that rate of strain can not be precisely modelled and the corresponding design condition is distorted (Davis 1971 and Brown et al. 1974). It is imperative that effects of rate of strain be determined and implications of distorting the corresponding design condition be clearly understood.

The objective of research reported in this paper was, therefore, to study the effects of distortion in the strain rate design condition. This objective was accomplished by studying such effects on a simplified cantilevered beam subject to impact loading by:

1 Identifying and studying the effects of material strain-rate dependent pi-term using dimensional analysis.

2 Establishing distortion of the design condition related to rate of strain and determining a prediction factor using principles of similitude modelling.

3 Developing an analytical expression for strain-rate effects and comparing it with the experimental results.

PREVIOUS WORK

Other work done in similitude modelling of plastic deformation of structures was reported by Brown et al. (1974). They modelled the dynamic portion of SAE J334a, Protective Frame Test Procedures and Performance Requirements. They conducted two analyses and derived two sets of design conditions. Rate of strain was included as one of the variables in both of the analyses. In the first analysis, rate of strain was higher in the model than the prototype by a factor of square root of the length scale if the same material was used for model as for prototype ROPS. In view of the strain rate dependence of yield stress in steel, a second analysis was developed in which rate of strain in the model frame was kept the same as that in prototype. Time, velocity of pendulum impact and mass density were distorted in the second model system. According to the first analysis, the mass scale was the square of length scale, whereas in the second analysis it was the same as the length scale. Impact energy was an additional variable included in the second analysis.

Tests were conducted based on the two analyses and deflection of ROPS and force on the pendulum were measured as a continuous function of time. Tests with lighter pendulum resulted in lesser total deflection by higher impact force. However, amplitude of maximum deflection was within 10 percent for both the analyses. The maximum force for both the tests was within 5 percent. They preferred the second analysis to test a model (1/4 scale) of a two post ROPS. Comparison of test results of prototype revealed that the impact force was within 3.6 percent, deflection within 2.5 percent and the energy absorption within 10 percent.

From the two analyses it becomes evident that if the rate of strain is kept the same in the model as the prototype, better predictions can be made. However, effects of varying rates of strain were not explicitly determined.

EFFECTS OF RATE OF STRAIN

Fig. 1 shows an idealization of a stress-strain diagram illustrating the elevation of yield stress as the rate of strain was increased. Significance of this phenomenon has been realized by many researchers working in the area of dynamic plastic deformation of structures. Elevation of dynamic yield stress as a function of rate of strain has been studied experimentally and a model has been developed. This model, reported by Perrone (1971) is presented below:

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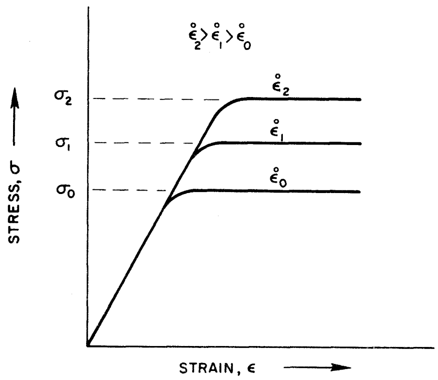

FIG. 1 An idealized stress-strain diagram showing the effects of rate of strain on yield stress.

$$\frac{\sigma_y}{\sigma_o} = 1 + (\dot{\epsilon}/c)^{1/p} \quad \ldots\ldots\ldots\ldots\ldots [1]$$

Values of c and p have been found to be 40.4 per sec and 5 respectively, for mild steel. This model requires knowledge of rate of strain which, in many situations is not known explicitly and may not be uniform across a cross section as encountered in bending of cantilver beams. The following model as reported by Ashburner (1972) does not include strain rate explicitly, instead it requires knowledge of time before yielding.

$$\frac{M_{po}}{M_{pso}} = \left(\frac{t_o}{D}\right)^A \quad \ldots\ldots\ldots\ldots\ldots [2]$$

Values of D and A are 0.17 sec and -0.1, respectively, for mild steel. He also reported that,

$$\frac{M_{po}}{M_{pso}} = \frac{\sigma_y}{\sigma_o} \text{ if } \frac{M_{po}}{M_{pso}} < 1.5 \quad \ldots\ldots\ldots [3]$$

Obviously, equation [2] and [3] have limitations, however, they provide good estimates for most of the conditions encountered in pendulum tests of ROPS. The model used in this study consisted of equations [2] and [3] mainly because an analytical estimate of strain-rate effects was sought from the mathematical model developed by Ashburner for predicting ROPS deformation.

DIMENSIONAL ANALYSIS

The variables pertinent to the system of a cantilver beam as struck by a pendulum are: height of beam, l; width of beam, b; thickness of beam, h; distance of impact point to pivot point, l_i; size of semi-cylindrical impact piece, s; and mass of impact pendulum, M. Properties of the materials included in the analysis were modulus of elasticity, E; static yield stress, σ_o; and strain rate parameters D and A. The operating variables were impact velocity, v; angle of impact, Φ; acceleration due to gravity, g; and time, t. The observed variable was beam deflection, d. The variables were used to form linearly independent dimensionless parameters normally called π terms. (d/l), can be expressed as a function of all the independent π terms as follows:

$$\frac{d}{l} = \phi\left\{\frac{b}{l}, \frac{h}{b}, \frac{l_i}{l}, \frac{s}{l}, \frac{E}{\sigma_o}, \frac{l}{D^2 g}, A, \frac{El^2}{M_g}, \frac{v^2}{gl}, \phi, \frac{gt}{v}\right\} \quad \ldots\ldots\ldots\ldots\ldots [4]$$

In this study only maximum and permanent deflections, independent of time, were of interest. Therefore, time, t (and consequently gt/v) was omitted. E/σ_0, A, s/l, l_i/l and Φ were held constant throughout the entire experimental investigation. Therefore, the following equation represents a generalized relationship between the dependent and independent π-terms.

$$\frac{d}{l} = \phi\left\{\frac{b}{l}, \frac{h}{b}, \frac{l}{D^2 g}, \frac{El^2}{Mg}, \frac{v^2}{gl}\right\} \quad \ldots [5]$$

The term $1/D^2g$ contains a strain-rate variable and represents a property of the material. Change in the value of this term can be brought about by a change in the value of D. But, a change in this value corresponds to change in the right-hand side of equation [2] which in turn results in a change in yield stress according to the same equation. A similar change can be caused by changing time to yielding, t_o, which corresponds to changing the rate of strain. Therefore, by studying the effects of changing $1/D^2g$ on the dependent π-term, sufficient information can be obtained to determine strain rate effects in similitude modelling.

EXPERIMENTAL DETAILS

A compound pendulum was designed to apply impact force to test beams. The length and the weight of the pendulum could be changed to follow similitude design conditions for various length scales. The height of suspension point of pendulum could be adjusted. Cantilever beams were held vertically in a vise which was rigidly bolted to a common heavy steel base plate. A semi-cylindrical rod, mounted horizontally across the face of the pendulum block at the center of percussion, was used to strike the beams. A solenoid actuated release mechanism was used to hold the pendulum at a predetermined angle until it was released. An angular displacement transducer was mounted at the pivot axis and continuously

TABLE 1. VALUES OF THE SYSTEM VARIABLES.

Deflection, d	Beam length, l,cm	Beam width, b,cm	Beam depth, h,cm	Pendulum weight Mg(kg)	Pendulum velocity, v(cm/sec)
Observed variable	12.70	1.270	0.508	20.52	109.22
		1.587	0.635		
		1.905	0.762		
		2.540	1.016		
	12.70	1.587	0.476	20.52	109.22
			0.635		
			0.794		
			0.952		
	6.34	0.794	0.317	5.13	77.22
	12.70	1.587	0.635	20.52	109.22
	19.05	2.381	0.952	46.19	133.76
	25.40	3.175	1.219	82.10	154.46
	12.70	1.587	0.635	34.72	109.22
				32.18	
				27.50	
				20.52	
	12.70	1.587	0.635	20.52	88.89
					109.22
					126.99
					143.25

ϕ = 90 deg; D = 0.17 sec^{-1}

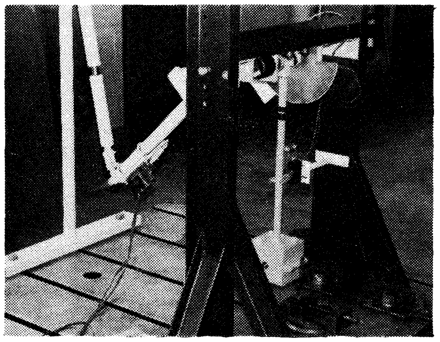

FIG. 2 A general view of the experimental equipment showing the impact pendulum, the release mechanism, an angular displacement transducer and a cantilever beam.

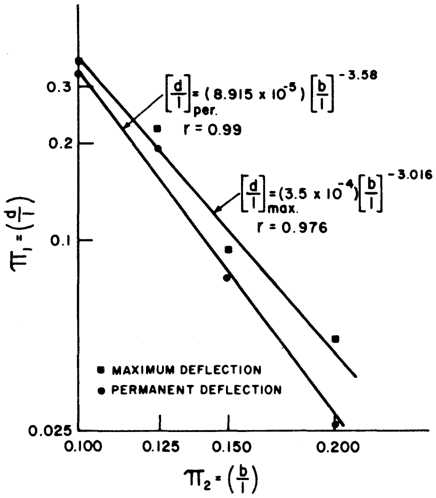

FIG. 3 Effects of changing b/l on beam deflection. Other π-terms were held constant (π_3 = 0.4; π_4 = 0.448; π_5 = 16.34; π_6 = 0.96).

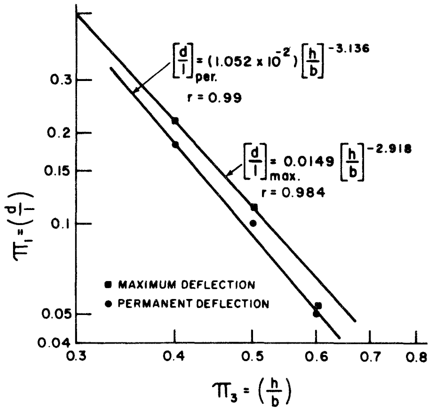

FIG. 4 Effects of changing h/b on beam deflection. Other π-terms were held constant (π_2 = 0.125; π_4 = 0.448; π_5 = 16.34; π_6 = 0.96).

recorded the position of the pendulum during a test. Test beams were made of cold rolled mild steel and were machined to desired sizes. The beams were annealed by heating them to 900 °C and then leaving them in the furnace to cool slowly. Cantilever beams were tested at the various levels of independent π-terms. One term was varied while the other terms were held constant. Table 1 presents values of dimensional variables corresponding to the experimental schedule. The general experimental set up is shown in Fig. 2. Permanent plastic and maximum instantaneous deflections of cantilever beams were measured.

RESULTS AND DISCUSSION

Experimental results are shown in Figs. 3 through 7. These figures present both dependent π-terms representing maximum and permanent deflections. Effects of (b/l) are shown in Fig. 3 and Fig. 4 shows the effects of changing (h/b). These π-terms are related to section modulus of the beams and an increase in these π-terms is related to increase in the value of section modulus which causes a rapid decline in the deflection values. Regression equations were fitted to the data and are shown in the respective figures. Fig. 5 shows the effects of changing (l/D^2g). It can be seen that increasing the value of this variable causes a slight increase in both maximum and permanent deflections. Increase in maximum deflection is greater than that in the permanent deflection. It is also seen that data for maximum delfection show a greater scatter. This may be due to an indirect method of measuring the maximum deflection. A 400 percent increase in the value (l/D^2g) causes a 1.48 percent increase in dimensionless permanent deflection and a 19.1 percent increase in the value of maximum dimensionless deflection.

If theory of similitude modelling is applied to the system represented by the generalized equation [5], the following design conditions for the model can be developed:

$$b_m = \left(\frac{l_m}{l}\right) b \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad [6]$$

$$h_m = \left(\frac{b_m}{b}\right) h \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad [7]$$

$$D_m = \left(\frac{l_m g}{l g_m}\right)^{1/2} \cdot D \quad \ldots\ldots\ldots \quad [8]$$

$$M_m = \left(\frac{E_m l_m^2 g}{E\, l^2\, g_m}\right) M \quad \ldots\ldots\ldots\ldots \quad [9]$$

and

$$v_m = \left(\frac{g_m l_m}{g\ \ l}\right)^{1/2} \cdot v \quad \ldots\ldots\ldots\ldots [10]$$

If the same material is used for the model and the prototype structure, (E_m = E and D_m = D) and if the experiments are performed in the same gravational field (g_m = g), all of

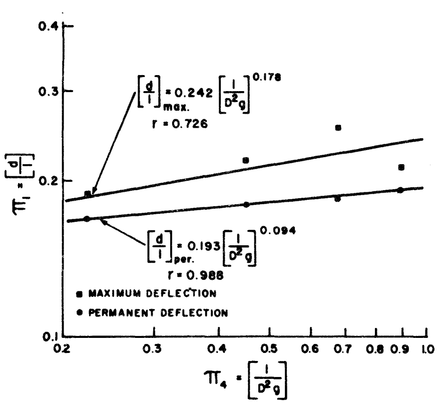

FIG. 5 Effects of changing l/D^2g on beam deflection. Other π-terms were held constant (π_2 = 0.125; π_3 = 0.4; π_5 = 16.34; π_6 = 0.96).

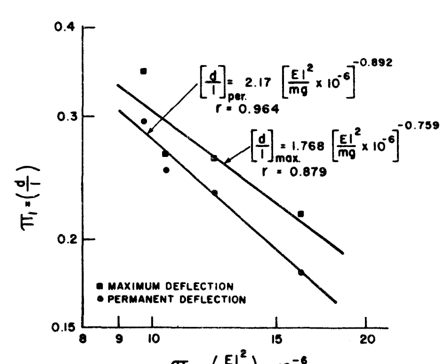

FIG. 6 Effects of changing El^2/Mg on beam deflection. Other π-terms were held constant (π_2 = 0.125; π_3 = 0.4; π_4 = 0.448; π_6 = 0.96).

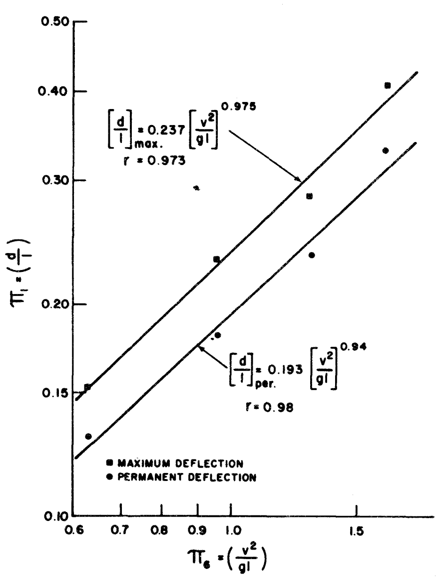

FIG. 7 Effects of changing v^2/gl on beam deflection. Other π-terms were held constant (π_2 = 0.125; π_3 = 0.4; π_4 = 0.448, π_5 = 16.34).

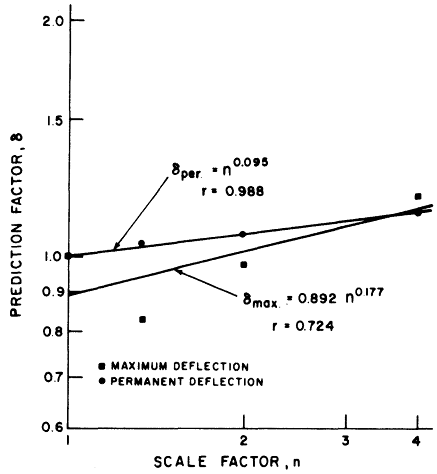

FIG. 8 Effects of distortion in the strain rate design condition on prediction factors for dimensionless maximum and permanent deflections.

the above design conditions can be satisfied except equation [8]. Distortion factor, a, can be calculated as follows:

$$a = \left(\frac{l_m/D_m^2\ g_m}{l/D^2 g}\right)$$

or,

$$a = \left(\frac{l_m}{l}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots [11]$$

Regarding the beam of largest dimensions as a prototype and others as models, prediction factor, δ, can be developed as a function of the distortion factor. Fig. 8 presents a plot of prediction factor vs. scale factor, n, for both dimensionless maximum and permanent deflections. It can be interpreted from this plot that if strain rate effects are disregarded in structural modelling of dynamic plastic deflections with the same materials then prediction errors of 12.9 percent and 16 percent for permanent and maximum deflections can be expected, respectively, for a scale factor up to 4. Appropriate correction can be made by using a prediction factor. These results can also be interpreted by considering distortion of rate of strain in a model system made of the same material as the prototype. This should, however, be noted that Fig. 8 present the effects of distorting a particular design condition. The distortion can be caused by any of the parameters involved in establishing the design condition.

Before accurate conclusions can be drawn, it was necessary to estimate the experimental error. The error in determining maximum and permanent dimensionless deflections is contributed by errors in each of the system variables. The dimensional system variables had the following maximum error.

$\Delta l = \pm 2.5$ mm
$\Delta b = \pm 0.125$ mm
$\Delta h = \pm 0.125$ mm
$\Delta l_{c.g.} = \pm 6.25$ mm
$\Delta Mg = \pm 0.5$ kg
$\Delta \Theta_b = \pm 0.5$ deg.

Maximum error was calculated by taking summation of absolute values of each term of equation [12].

$$\Delta\pi_1 = \frac{\partial\pi_1}{\partial\pi_2}\Delta\pi_2 + \frac{\partial\pi_1}{\partial\pi_3}\Delta\pi_3 + \frac{\partial\pi_1}{\partial\pi_4}\Delta\pi_4 + \frac{\partial\pi_1}{\partial\pi_5}\Delta\pi_5 + \frac{\partial\pi_1}{\partial\pi_6}\Delta\pi_6 \quad \ldots\ldots\ldots\ldots [12]$$

Max. $\Delta\pi_1 = 0.018$

Maximum error was calculated to be 9 percent for permanent deflection. Therefore, the results presented here are subjected to a maximum error of 9 percent.

Theoretical Analysis of Strain Rate Effects

Ashburner (1972) developed an analytical method to predict deformation of ROPS subject to impact by a pendulum. He predicted maximum and permanent deflections of frames and peak deceleration of an impacting pendulum. The model included elastic and plastic behavior of frames as well as elevation of yield stress due to dynamic loading. The following set of equations constitute the model:

$$\text{Maximum acceleration} = \frac{M_{po}}{Wl} \quad \ldots\ldots [13]$$

$$\text{Maximum deflection} = \left(\frac{E_w l}{M_{po}} + \frac{M_{po}}{2Kl}\right) \quad \ldots\ldots\ldots\ldots [14]$$

$$\text{Final Deflection} = \left(\frac{E_w l}{M_{po}} - \frac{M_{po}}{2Kl}\right) \quad \ldots\ldots\ldots\ldots [15]$$

$$\frac{M_{po}}{M_{pso}} = \left(\frac{t_o}{D}\right)^A \text{ if } \frac{M_{po}}{M_{pso}} < 1.5 \ldots [16]$$

$$M_{pso} = \frac{bh^2}{6}\sigma_0 \quad \ldots\ldots\ldots\ldots [17]$$

$$K = \frac{Ebh^3}{4l^3} \quad \ldots\ldots\ldots\ldots [18]$$

$$E_w = \frac{1}{2}Mv^2 \quad \ldots\ldots\ldots\ldots [19]$$

The model is used to derive a generalized prediction equation in a dimensionless form with non-dimensional parameters being the same as expressed in equation [5]. This derivation constituted a two part derivation of equations [14] and [15]. Separate expressions were derived for the first term, $(E_w l/M_{po})$ and for the second term, $(M_{po}/2Kl)$.

Let,

$$d_s = \frac{E_w l}{M_{po}} \quad \ldots\ldots\ldots\ldots [20]$$

and

$$d_c = \frac{M_{po}}{Kl} \quad \ldots\ldots\ldots\ldots [21]$$

Equations [14] and [15] can be rewritten as:

$$\text{Max. Def.} = d_s + \frac{d_c}{2} \quad \ldots\ldots\ldots [22]$$

$$\text{Final Def.} = d_s - \frac{d_c}{2} \quad \ldots\ldots\ldots [23]$$

Time to reach yielding, t_o, in equation [22] can be estimated from the following equation:

$$t_o = \frac{d_c}{v} \quad \ldots\ldots\ldots\ldots [24]$$

Equations [20] and [21] were expressed in the following non-dimensional form. Numerical exponents were calculated using the experimentally determined value of A (Ashburner 1972).

$$\left(\frac{d_s}{l}\right) = K_1 \left(\frac{b}{l}\right)^{-3.09} \left(\frac{h}{b}\right)^{-2.09} \left(\frac{l}{D^2 g}\right)^{0.045} \left(\frac{El^2}{Mg}\right)^{-1} \left(\frac{v^2}{gl}\right)^{0.954} \quad \ldots [25]$$

$$\left(\frac{d_c}{l}\right) = K_2 \left(\frac{b}{l}\right)^{-0.909} \left(\frac{h}{b}\right)^{-0.909} \left(\frac{l}{D^2 g}\right)^{-0.045} \left(\frac{El^2}{Mg}\right)^{0} \left(\frac{v^2}{gl}\right) \quad \ldots [26]$$

Equations [25] and [26] constitute generalized equations derived from an analytical model. In order to compare the experimental results with theoretical results, a multiplicative relationship was assumed for the component equations to constitute experimentally determined generalized expressions. d_s and d_c can be written in dimensionless form in the following manner:

$$\left(\frac{d_s}{l}\right) = \frac{1}{2}\left\{\left(\frac{d_{max}}{l}\right) + \left(\frac{d_{per}}{l}\right)\right\} \quad \ldots [27]$$

$$\left(\frac{d_c}{l}\right) = \left\{\left(\frac{d_{max.}}{l}\right) - \left(\frac{d_{per}}{l}\right)\right\} \quad \ldots [28]$$

(d_s/l) and (d_c/l) were calculated from the data of (d_{max}/l) and (d_{per}/l). Component equations were developed for (d_s/l) and (d_c/l) for each dimensionless variable by fitting a power curve, $y = ax^b$, using a least squares analysis.

$$\left(\frac{d_s}{l}\right) = K_1' \left(\frac{b}{l}\right)^{-3.24} \left(\frac{h}{b}\right)^{-3.0} \left(\frac{l}{D^2 g}\right)^{0.14} \left(\frac{Fl^2}{Mg}\right)^{-0.87} \left(\frac{v^2}{gl}\right)^{0.96} \quad \ldots [29]$$

$$\left(\frac{d_c}{l}\right) = K_2' \left(\frac{b}{l}\right)^{-1.53} \left(\frac{h}{b}\right)^{-2.48} \left(\frac{l}{D^2 g}\right)^{0.77} \left(\frac{El^2}{Mg}\right)^{-0.05} \left(\frac{v^2}{gl}\right)^{1.15} \quad \ldots [30]$$

A comparison of equations [25] and [26] with equations [29] and [30], respectively, can be made to establish agreement between the experimental data and the theoretical results. It can be seen that there is a good agreement among the exponents of expressions for (d_s/l). However, results for (d_c/l) do not compare well with that of derived expression. Also correlation coefficients for regression analysis of component equations for (d_c/l) are much lower than that of (d_s/l). d_s is a combination of plastic and elastic deformations, whereas d_c is the elastic component of the total deformation. Low correlation coefficients for d_c and poor comparison of exponents in its expression could be attributed to the inaccuracy in measuring maximum deflection or an improper form for the component equation.

CONCLUSION

1 In similitude modelling of large elastic and plastic deformation of structures made of strain rate sensitive materials, it is not possible to build a true model of like material because of the distortion of the design conditions related to strain rate property of material.

2 Effects of rate of strain dependent π-term are small. A 400 percent increase in its value causes a 14.9 and a 19 percent change in normalized permanent and maximum delfections, respectively.

3 A prediction factor developed to calculate the effect of distortion of the strain rate sensitive design condition based on distorted model theory.

4 Analytical derivations for d_s (representing an average permanent and maximum deflection) were found to be in good agreement with the experimental results. However, similar derivation for elastic component of total deflection, d_c, did not compare well with the experimental results.

5 According to estimated maximum experimental error, these conclusions are subject to an error of no more than 9 percent.

LIST OF SYMBOLS

A,p	Strain rate parameters
C,D	Strain rate parameters, T^{-1}
E	Modulus of elasticity, FL^{-2}
E_w	Kinetic energy, FL
F	Dimension of force
L	Dimension of length
M	Pendulum mass, $FL^{-1}T^2$
M_{po}	Dynamic yield moment, FL
M_{pso}	Static yield moment, FL
T	Dimension of time
W	Pendulum weight, F
d	Beam deflection, L
d_{max}	Maximum beam deflection, L
d_{per}	Permanent beam deflection, L
g	Acceleration due to gravity, LT^{-2}
h	Beam thickness, L
k	Frame stiffness, FL^{-1}
l	Height of beam, L
$l_{c.g.}$	Distance between pivot point and center of gravity of the pendulum, L
l_i	Distance between pivot point and center of percussion of the pendulum, L
m	Subscript, denotes a variable in model system
s	Size of impact piece, L
t	Significant time, T
t_o	Time to reach yield, T
α	Distortion factor
δ	Prediction factor, $(d/l)p/(d/l)m$
Δ	Prefix, denotes error in the variable
ϵ	Strain
$\overset{o}{\epsilon}$	Rate of strain, T^{-1}
Θ	Angular displacement of pendulum
Θ_b	Pendulum release angular
π	A dimensional parameter
σ_o	Static yield stress, FL^{-2}
σ_y	Dynamic yield stress, FL^{-2}
Φ	Angle of impact

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