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# Pressure drop in elbows

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## Introduction

In all ventilation systems, the duct system which provides the air distribution contains a number of elbows and bends. In fact, in most ventilation designs, the pressure loss through elbows constitutes a significant portion of the pressure loss experienced by the system. The estimation of pressure losses through a ventilation system design depends on the two tables provided in the literature<sup>(1,2)</sup> which essentially go back to one estimate.<sup>(3)</sup> Preliminary research reported earlier<sup>(4)</sup> indicates that the two tables of values provided in the standard source of information, *Industrial Ventilation*,<sup>(2)</sup> do not agree with each other and that they do not agree with a constant factor of safety, with the data obtained in this study. Although in each ventilation system designed and installed, there are sufficient design safety factors to take care of any discrepancies, it is important to be able to predict losses accurately so that properly selected design safety factors can be applied to the final design. This process is likely to result in proper design and, hence, savings in operating costs. Although elbows with non-circular cross sections are used frequently, the experimental portion of this study used only elbows with circular cross section. Therefore, the discussions refer only to such elbows. It must be noted that the extension of the semi-empirical equations to elbows of non-circular cross section is not straightforward, and the estimates presented here should not be considered applicable to the common oblong designs. Even though it is tempting to extend the results to square elbows with the adjustment of the size by hydraulic radius, there is no evidence to make that claim.

From a theoretical point of view, the loss at an elbow is expected to have two

The flow of air through elbows and the attendant pressure drop is an integral part of the design of a ventilation system. The traditionally used constants which express such a pressure drop as a function of sharpness of turn and the values of such constants in widely used references were found not to be representative of the experimental values obtained. From the experiments carried out using a wide range of elbows, the results obtained suggest that a single semi-empirical equation may be used to describe pressure drop through elbows of different duct surface characteristics, diameters, and turning ratios. The results obtained for ducts of circular cross section and the resulting equations may be readily used in the design of industrial ventilation systems. **Durr, D.E.; Esmen, N.A.; Stanley, Jr., C.; Weyel, D.A.:** Pressure drop in elbows. *Appl. Ind. Hyg.* 2:57-60; 1987.

components: 1) the skin friction related losses and 2) inertial flow related losses. It is further expected that these losses will diminish as the turning ratio (Tr) of the elbow (the ratio of the radius of the turn to the diameter of the duct) increases. This establishes at least the family of functions which may be appropriately considered for a semi-empirical development of the predictive equations of elbow losses. The frictional losses in turbulent pipe flow may be expressed by the Darcy-Weissbach equation:

$$P_f/p_v = f \cdot (L/D) \quad (1)$$

where:

- $P_1$  = Friction head loss for a straight duct
- $p_v$  = Velocity head
- $f$  = Darcy friction factor
- $L$  = Length of duct
- $D$  = Diameter of duct

If the turning ratio of an elbow is similar to the  $L/D$ , then it may be argued that the frictional component may be represented by an infinite degree polynomial which converges for all values of Tr and diminishes uniformly as Tr approaches infinity. Such a polynomial may be expressed as an exponential decay function. Thus the frictional component may be expressed in the form of:

$$p_f/p_v = f a_1 \exp(-b_1 Tr) \quad (2)$$

where,  $a_1$  and  $b_1$  are empirical constants.

By a similar argument, it may be concluded that the loss due to inertial losses may be expressed as:

$$p_m/p_v = a_2 \exp(-b_2 Tr) \quad (3)$$

It is also reasonable to expect that the pressure loss would be proportional to the completion of the turn. Thus the total equation may be written as:

$$p_e/p_v = a_4(c/360)(\exp[-b_2 Tr] + a_3 f \exp[-b_1 Tr]) \quad (4)$$

This paper reports the determination of the constants for this semi-empirical equation and the application of this equation to design problems.

## Experimental methods

In order to provide a sufficient number of elbow configurations and a wide range of friction factors, two types of flexible ducts were used in the experimental study. The duct designated as smooth was used for 6-, 4-, and 3-inch elbows, and the duct designated as coarse was used for 6-inch and 4-inch elbows. The smooth ducts had absolute roughnesses of 0.00870, 0.00488 and 0.00255 inches for 6-, 4-, and 3-inch ducts, respectively. The coarse ducts had absolute roughnesses of 0.612 and 0.320 inches for 6- and 4-inch ducts, respectively.

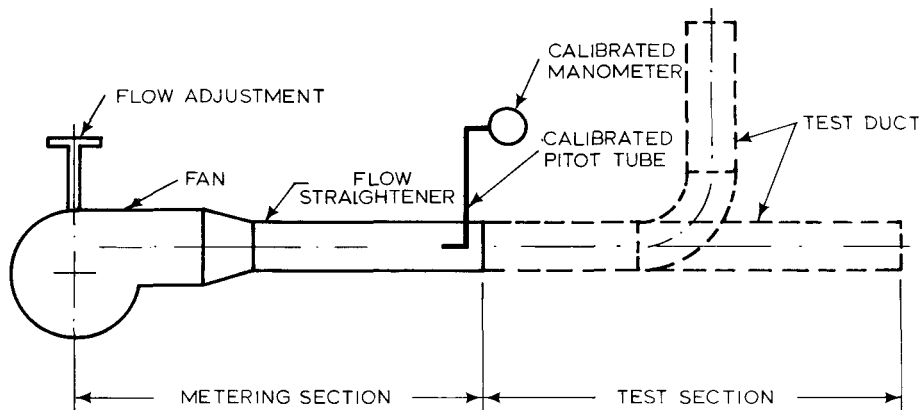


Figure 1—Schematic representation of the experimental system.

The essential attributes of the experimental system are shown in Figure 1. The air flow was monitored by a calibrated impact tube and static pressure tap. The volumetric flow calibration was achieved by a set of 10 point horizontal and vertical velocity traverses. Since the wind tunnel has considerable length-of-flow straightening devices, the velocity profile at the entry of the test section is that of the fully developed turbulent flow. The pressure drop measurements were made using calibrated Magnehelic gages connected to piezometric rings.

In the initial set of tests, 90 degree elbows were generated with turning ratios from 1.5 to 4.5 in steps of 0.25. These elbows were properly fixed to a plane by means of two perpendicular metal troughs so that the inlet and the exits of the elbows were straight and in the plane of the elbows. The elbows tested were located roughly at the middle of ten foot sections of the ducts; thus in each experimental run, the discharge and entry was at least six diameters upstream and downstream of the entry and the exit of the elbow respectively. For each elbow, a minimum of five volumetric flow rates were used to span the Reynolds number spectrum from about 80,000 to about 400,000.

The experimental verification of the influence of the completeness of turning was made by constructing 180 degree turns in the manner described above for six turning ratios of 2.0, 3.0, 3.35, 4.0, 10, and 16 and testing these turns for three or more volumetric flow rates each.

## Results and discussion

A randomly selected portion of the experimental data, constituting about one-fourth of the 420 different conditions studied, were used to determine the constants of equation 4, with the

exception of constant  $c$ , which was set to be 90. The results suggest that  $a_4 = 1.808$ ,  $b_1 = 0.01$ ,  $b_2 = 0.75$  and  $a_3 = 9.956$ . Thus, the predicted equation may be written as:

$$p_c/p_v = 1.808(c/360)(\exp[-3Tr/4] + 9.956 f \exp[-Tr/100]) \quad (5)$$

The predicted pressure drop for the entire length of the duct is:

$$P_{total} = P_i + P_e \quad (6)$$

The first term of the equation above may be calculated using equation 1. The entire data set was used to compare the measured and calculated values of total pressure drop. The mean error in prediction was 1.9 percent with a standard deviation of 6.9 percent. Equation 6 was then used, with appropriate value of  $c = 180$  for the prediction of full 180 degree turns. The results of these

experiments were also satisfactory. For turning ratio range of 2 to 4, the mean error in prediction was 1.3 percent with a standard deviation of 4.9. Thus, it was felt that the confirmation of equation 5 was satisfactorily achieved.

An additional important consideration is the concept that the pressure loss close to the exit of an elbow is greater than further downstream and, in a few diameters downstream, some of this pressure loss is regained. There is no theoretical justification for this statement. In fact, the experiments which show that the second 90 degree elbow in a 180 degree turn behaves the same way as the first elbow may be taken to disprove this hypothesis. The fluid mechanics of the elbow exit suggests that there is a considerable distortion of the velocity profile at the exit of the elbow which affects the values associated with other fluid mechanical processes such as jet formation in discharges, enlargements, mixing in the junctions, and operation of air moving equipment. Thus, it may be claimed that the placement of an elbow close to a discharge fan, or junction may be a poor practice and may affect the pressure loss estimate of the process downstream of the elbow, but it does not change the elbow pressure loss estimate.

The elbow loss values obtained in this study do not agree with the values reported in a widely used reference<sup>(2)</sup>

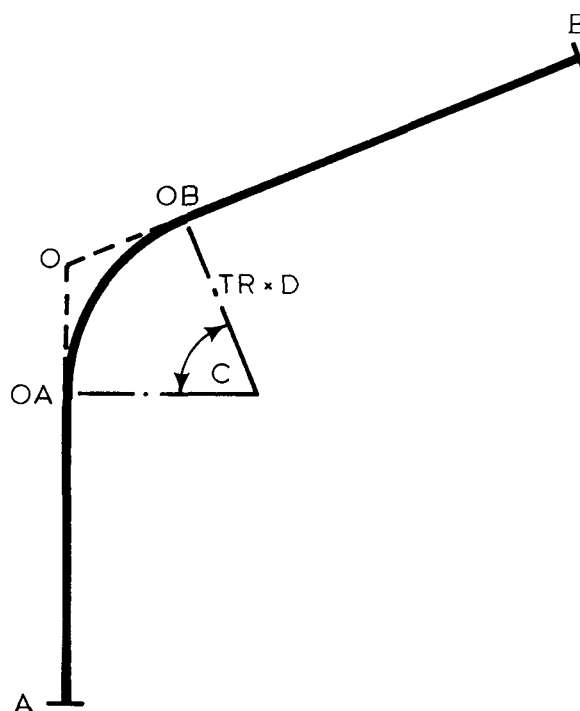


Figure 2—Schematic representation of two straight ducts and an elbow as a design aid.

with a consistent factor of safety. For example, in considering two elbows with 2.5 turning ratio, one with 3 in. and the other with 10 in. diameter, at 4000 fpm air velocity, the values obtained from the velocity pressure table of the *Industrial Ventilation Manual*<sup>(2)</sup> would be 0.22 in. water gauge (wg) for both elbows, whereas the equivalent length table would indicate 0.23 and 0.30 in. wg for 3- and 10-in. diameter elbows, respectively. The values obtained in this study suggest 0.20 and 0.18 in. wg for 3- and 10-in. elbows, respectively. Thus the apparent design safety factors range between 10 and 66 percent.

### The application of the results to design

The equation developed above may be simplified for design applications. Although the equation can apply to ducts of any absolute roughness, this discussion is restricted to standard industrial ventilation system ducts which follow the accepted practice as specified in handbooks such as *Industrial Ventilation*.<sup>(2)</sup> In the estimation of the pressure drop, it is necessary to determine the value of the friction factor. For the ducts under consideration, this may be achieved readily using the pressure drop equation for straight ducts given in *Industrial Ventilation*,<sup>(2)</sup> in conjunction with equation 1. If the duct diameter is given in inches and the velocity is given in feet/min, then the friction factor estimate is:

$$f = 0.07308 D^{-0.22} V^{0.1} \quad (7)$$

**TABLE II**  
Calculated values of the elbow loss function for 90 degree elbows (Fe • 100)

D	Turning Ratio												
	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50
3	0.26	0.24	0.21	0.20	0.18	0.17	0.16	0.15	0.14	0.14	0.13	0.13	0.13
4	0.25	0.23	0.21	0.19	0.18	0.16	0.15	0.14	0.14	0.13	0.13	0.12	0.12
5	0.25	0.22	0.20	0.18	0.17	0.16	0.15	0.14	0.13	0.13	0.12	0.12	0.11
6	0.24	0.22	0.20	0.18	0.17	0.15	0.14	0.14	0.13	0.12	0.12	0.11	0.11
7	0.24	0.22	0.20	0.18	0.16	0.15	0.14	0.13	0.13	0.12	0.11	0.11	0.11
8	0.24	0.21	0.19	0.17	0.16	0.15	0.14	0.13	0.12	0.12	0.11	0.11	0.10
9	0.24	0.21	0.19	0.17	0.16	0.15	0.14	0.13	0.12	0.11	0.11	0.11	0.10
10	0.23	0.21	0.19	0.17	0.16	0.14	0.13	0.13	0.12	0.11	0.11	0.10	0.10
11	0.23	0.21	0.19	0.17	0.15	0.14	0.13	0.12	0.12	0.11	0.11	0.10	0.10
12	0.23	0.21	0.18	0.17	0.15	0.14	0.13	0.12	0.12	0.11	0.10	0.10	0.10
15	0.23	0.20	0.18	0.16	0.15	0.14	0.13	0.12	0.11	0.11	0.10	0.10	0.09
20	0.22	0.20	0.18	0.16	0.14	0.13	0.12	0.11	0.11	0.10	0.10	0.09	0.09
25	0.22	0.19	0.17	0.15	0.14	0.13	0.12	0.11	0.10	0.10	0.09	0.09	0.09
30	0.22	0.19	0.17	0.15	0.14	0.13	0.12	0.11	0.10	0.09	0.09	0.09	0.08
35	0.21	0.19	0.17	0.15	0.14	0.12	0.11	0.10	0.10	0.09	0.09	0.08	0.08
40	0.21	0.19	0.17	0.15	0.13	0.12	0.11	0.10	0.10	0.09	0.09	0.08	0.08
45	0.21	0.18	0.16	0.15	0.13	0.12	0.11	0.10	0.09	0.09	0.08	0.08	0.08
50	0.21	0.18	0.16	0.14	0.13	0.12	0.11	0.10	0.09	0.09	0.08	0.08	0.08

Note: The Diameters are given in inches.

It must be noted that between 1000 to 8000 feet/min duct velocities, the choice of 3000 feet/min for velocity will introduce about 10 percent error in f. Such an error will result in about 3 percent error in the final calculation. Thus equation 7 may be reduced to:

$$f = 0.0328 D^{-0.22} \quad (8)$$

Similarly, if the diameter is expressed in centimeters, then equation 8 becomes:

$$f = 0.0403 D^{-0.22} \quad (9)$$

Consider an elbow-duct system as shown in Figure 2. A straight duct of length AA:OA is connected to another

straight duct of length BB:OB with an elbow of turning ratio Tr and turning of c degrees, and the entire system has a duct diameter D. Clearly, we can sum the losses of this system as elbow loss plus duct loss from AA to O plus from O to B plus duct loss along the arc OA:OB minus duct losses from OA to O to OB. Let x be length OA and y be length OB expressed in feet. Then:

$$p_t = p_e + \text{loss}(x + y - \text{Tr} D \sin[c/2]/6 + 0.0014544 \text{Tr} D c) \quad (10)$$

If x + y is larger than 2 diameters and turning angle, c, is less than 150 degrees, then for turning ratios up to 4.75 the second and third terms of the straight duct loss are insignificant. Thus the total loss can be expressed in terms of an elbow loss and a straight duct loss for a duct length equal to AO + OB. This consideration simplifies the calculation because these duct lengths can be readily picked from the initial line drawings of the ventilation system to be designed. Furthermore, another simplification for the elbow loss can be introduced in a tabular form. With the assumption of air velocity in the range specified for equations 8 and 9, the elbow loss may be written as:

$$p_e/(p_v c) = Fe \quad (11)$$

Fe is a function of the duct diameter and turning ratio, and Table I shows its calculated values. For the designers who prefer to use equivalent length method of calculating losses, the total loss from point A to B may be given as:

$$p_t = 2.54 Fe c D^{1.22} + x + y \quad (12)$$

**TABLE I**  
Calculated values of the elbow loss function (Fe • 100)

D	Turning Ratio												
	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50
3	0.29	0.26	0.24	0.22	0.20	0.19	0.18	0.17	0.16	0.15	0.15	0.14	0.14
4	0.28	0.25	0.23	0.21	0.19	0.18	0.17	0.16	0.15	0.15	0.14	0.14	0.13
5	0.28	0.25	0.22	0.21	0.19	0.18	0.16	0.16	0.15	0.14	0.14	0.13	0.13
6	0.27	0.24	0.22	0.20	0.18	0.17	0.16	0.15	0.14	0.14	0.13	0.13	0.12
7	0.27	0.24	0.22	0.20	0.18	0.17	0.16	0.15	0.14	0.13	0.13	0.12	0.12
8	0.27	0.24	0.21	0.19	0.18	0.16	0.15	0.14	0.14	0.13	0.12	0.12	0.12
9	0.26	0.23	0.21	0.19	0.18	0.16	0.15	0.14	0.13	0.13	0.12	0.12	0.11
10	0.26	0.23	0.21	0.19	0.17	0.16	0.15	0.14	0.13	0.13	0.12	0.12	0.11
11	0.26	0.23	0.21	0.19	0.17	0.16	0.15	0.14	0.13	0.12	0.12	0.11	0.11
12	0.26	0.23	0.21	0.19	0.17	0.16	0.15	0.14	0.13	0.12	0.12	0.11	0.11
15	0.25	0.22	0.20	0.18	0.17	0.15	0.14	0.13	0.12	0.12	0.11	0.11	0.10
20	0.25	0.22	0.20	0.18	0.16	0.15	0.14	0.13	0.12	0.11	0.11	0.10	0.10
25	0.24	0.21	0.19	0.17	0.16	0.14	0.13	0.12	0.11	0.11	0.10	0.10	0.09
30	0.24	0.21	0.19	0.17	0.15	0.14	0.13	0.12	0.11	0.10	0.10	0.10	0.09
35	0.24	0.21	0.19	0.17	0.15	0.14	0.13	0.12	0.11	0.10	0.10	0.09	0.09
40	0.23	0.21	0.18	0.16	0.15	0.13	0.12	0.11	0.11	0.10	0.10	0.09	0.09
45	0.23	0.20	0.18	0.16	0.15	0.13	0.12	0.11	0.10	0.10	0.09	0.09	0.09
50	0.23	0.20	0.18	0.16	0.14	0.13	0.12	0.11	0.10	0.10	0.09	0.09	0.08

Note: The Diameters are given in inches.

where D is in inches and x, y, p<sub>t</sub> are in feet. Similarly:

$$p_t = 0.2482 Fe c D^{1.22} + x + y \quad (13)$$

where D is in centimeters, x, y, p<sub>t</sub> are in meters.

Although it is possible to develop extensive tables and charts relating the parameters of loss to a set of diameters, turning ratios, and volumetric flow rates, we did not undertake such an endeavour with the exception of preparing a table (Table II) for Fe × c when c is 90 degrees. The results shown in Table II can be used directly in the calculation of elbow losses. The calculations presented here are sufficiently easy; therefore, the need for extensive tables is not apparent. Such a task would be useful only in the preparation of future handbooks or practice guides.

## Recommendations

The industrial hygienist who is responsible for the design of ventilation systems can use equations 12 and 13 or equivalently Tables I and II directly in the calculation of the elbow losses in the design of ventilation systems.

## Acknowledgments

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Rudolph G. Mortimer at the University of Illinois, Thomas H. Rockwell at Ohio State University, Robert M. Nicholson, director of the Office of Crash Avoidance Research at the DOT National Highway Traffic Safety Administration (NHTSA), and Robert Henderson, then on Bob Nicholson's staff at NHTSA.

In early 1977, Mark Kirkpatrick and I directed a large-scale human factors field assessment of several brake-light configurations at Essex Corporation. The stated purpose of the study, which was sponsored by Bob Nicholson, was to determine if any of several proposed automobile rear lighting systems would result in fewer rear-end collisions when compared under actual traffic conditions with a typical existing configuration. The configurations tested included a centered high-mounted brake light, dual separated high-mounted brake lights, and a separated function condition in which a wiring change was made to existing vehicle lamps so as to separate the presence (tail light) function from the stop/turn functions.

A total of 2100 taxicabs in the Washington, D.C. area were partitioned into four equally sized groups: three experimental groups and one control group. After the rear lights on the cabs in the experimental groups were modified or the high-mounted lamps were installed according to the experimental protocol, a data collection phase was implemented wherein data on the rear-end collisions, other types of collisions, and mileage for all test vehicles were recorded over a one-year period. The four test groups accumulated nearly 60 million vehicle miles under a wide variety of weather and road conditions. Drivers in the several groups had been matched for age, sex, and prior accident history.

The primary finding of the study was that cabs equipped with a centered high-mounted brake light experienced an accident rate that was 54% lower than that for cabs in the control group. This finding was statistically significant at the 0.0001 level. Neither the configuration of dual high-mounted brake lights nor the separated function condition differed significantly from the control group, although both configurations produced apparent reductions on the order of 20%. An analysis was conducted of rates for accidents other than rear-enders during the data collection period, and these rates were found to be essentially equal across the groups. The rear-end rate effects were, therefore, not due to differences in general

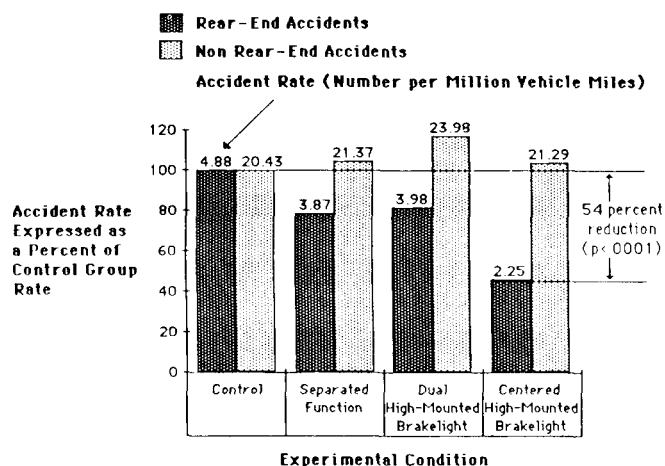


Figure 1—Accident rates by experimental condition.

safety performance between groups. These results are presented in Figure 1, which shows rear-end and non-rear-end accident rates plotted as a percentage of the control group rate. The experimental group non-rear-end accident rates were greater than those for the control group, while all brake light treatments showed an apparent reduction in rear-end accident rates.

Not only did the centered high-mounted brake light reduce the incidence of accidents by more than half, it was also found to reduce the extent of damage to vehicles involved in accidents by 38%. This led to the conclusion that the centered high-mounted tail light results in faster brake application in the following vehicle, resulting in a slower speed at impact and consequently less damage to both vehicles.

An attempt was made by NHTSA to calculate the cost-benefit ratio or return on investment for the centered high-mounted brake light based on vehicle damage costs. It was estimated that there were 3.2 million accidents in 1977 that could have been affected by the centered high-mounted brake light. If the additional light had been in use, this figure would have been

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