Statistical methods for determining non-compliance . . .

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In the May, 1975 Journal, an article by Dr. Morton Corn was published dealing with the application of statistical methods to the problems of determination of noncompliance with health standards.[1] Dr. Corn's article and conclusions refer to an early draft paper we had published on the subject.[2] *Trans. ACGIH,* 125-133 (1973). A technically updated version of our work in this area is now available from NIOSH.[3]

Unfortunately, Dr. Corn incorrectly applied the statistical methods of our paper and reached incorrect numerical results. First, the coefficient of variation (CV) for weighing errors was overestimated. It is incorrect to "stack" or add maximum expected errors in a sequential procedure. Doing so leads to unrealistic overestimates of the CV for total (net) error. The correct procedure is as follows:

Estimation of CV for weighing errors (CV_w)

Typically for one brand of filter plus cassette:

let a = final weight of dust + filter + cassette = 402 mg

b = tare weight filter + cassette = 400 mg

w = net weight of dust = (a-b) = 2 mg

estimate s_a = standard deviation of weight a = 2 mg

s_b = standard deviation of weight b = 0.05 mg

s_w = standard deviation of net dust weight

then:

$$s_w = (s_a^2 + s_b^2)^{1/2} = [(0.05 \text{ mg})^2 + (0.05 \text{ mg})^2]^{1/2} = 0.071 \text{ mg}$$

and,

$$CV_w = \frac{s_w}{w} = \frac{0.071 \text{ mg}}{2 \text{ mg}} = 0.035 = 3.5\%$$

Note that this figure of 3.5% for CV_w is 30% (relative) less error than the 5% Dr. Corn estimates.

However, a more important flaw is the calculation of the critical concentration (above the 30 mg/cu m standard) that the sum of 10 samples must exceed before noncompliance could be declared with 95% confidence. The statistical test given is a one-sided comparison of means test using the normal distribution at the 5% Type I error level. Dr. Corn overestimates the CV for the net error in the sampling and analytical sequence for 10 samples.

Calculation of 10 sample net error CV and noncompliance parameter for 10 samples

The CV_{net} for one sample would be:

$$CV_{net} = [(CV_{flow})^2 + (CV_{weight})^2 + (CV_{inst})^2]^{1/2}$$

Dr. Corn calculates this as 8.66% (using his assumed values for component CV_s). We agree with this value in that in our April 1975 report referenced above we give a conservative 9% CV for one respirable dust sample. However, the CV_{10} for the average (or sum in this case) of 10 samples is:

$$CV_{10} = \left[\frac{(CV_{flow})^2}{10} + \frac{CV_{weigh})^2}{10} + \frac{(CV_{inst})^2}{10}\right]^{1/2}$$

$$= \frac{1}{10^{1/2}} [(0.05)^2 + (0.035)^2 + (0.05)^2]^{1/2}$$

$$= \frac{1}{10^{1/2}} [0.0789] = 0.02495 \cong 2.5\% \cong CV_{10}$$

This is less than a third of the 8.66% for CV_{10} Dr. Corn obtained for 10 samples because he neglected to divide by the square root of 10 samples.

Stated differently, the standard deviation (at the concentration level equal to the standard) for the sum of 10 independently obtained and weighed samples is calculated by multiplying the standard for the sum of 10 samples (30 mg/cu m) by the CV for the net error of one sample (CV_{net}) and then dividing by the square root of 10:

S.D. (10 samples) = (30 mg/cu m) (0.0789) $(1/10^{1/2})$.
= 0.747 mg/cp m

Note that we calculated the standard deviation (SD) at the standard because the null hypothesis for our test is that the sum of the 10 samples is equal to the 30 mg/cu m standard.

The parameter for noncompliance is:

parameter = (1.645) ($SD_{10 \text{ samples}}$) + standard
= (1.645) (0.747 mg/cu m) + 30 mg/cu m)
= 1.23 mg/cu m) + (30 mg/cu m)
= 31.23 mg/cu m

This value is considerably different than the 34.4 mg/cu m calculated by Dr. Corn. Note that 31.23 is 4.1% higher than the standard. Dr. Corn's value of 34.3 is 14.3% higher than the standard.

Concerning the parameter above, we can state that if the sum of the 10 samples concentrations exceeds 31.23 mg/cu m, we are 95% confident that the true sum of the concentrations exceed the 30 mg/cu m ten sample cumulative concentration standard and that a condition of noncompliance exists. Thus Dr. Corn has incorrectly testified that (in effect) the sum of the 10 sample concentrations must exceed the standard by over 14% in order to "demonstrate noncompliance" when actually the sum must exceed the standard by about 4%.

As a final comment, these required technical modifications to Dr. Corn's paper do not detract from an overall excellent presentation. He is to be commended for recognizing the value of statistical methods for determining noncompliance.

References

1. CORN, M.: Remarks on Determination of Non-Compliance with the Respirable Dust Standard, Federal Coal Mine Health and Safety Act of 1969. *Amer. Ind. Hyg. J. 36*:404 (1975).
2. LEIDEL, N. A. and K. A. BUSCH: Statistical Methods for the Determination of Non-Compliance. *Trans. ACGIH:* 125 (1973).
3. Statistical Methods for the Determination of Non-Compliance with Occupational Health Standards. (NIOSH) 75-159 (April 1975).

A reappraisal of procedures . . .

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Nelson Leidel and Kenneth Busch focus on two procedures in my article "Remarks on Determination of Non-Compliance with the Respirable Dust Standard, Federal Coal Mine Health and Safety Act of 1969." The criticism of the first procedure, that of estimating the coefficient of variation for the weighing of a dust laden cassette is not major; my estimate of 5% was literally an estimate and I would defer to a better estimate if appropriate supporting data were submitted. However, it is with respect to the second criticism, my procedure for estimating the confidence limits for the total weight of respirable dust associated with a ten sample cycle, that Leidel and Busch correctly fault my procedures. I thank them for better organizing my own thinking on this procedure, one of inestimable importance for determining non-compliance with legislated standards for airborne contaminants.

It may help to recapitulate my reasoning in this matter, so that others will avoid this pitfall. The procedure I utilized is referred to as "compounding of errors" by Wilson.[1] It simply states that variance of a sum is equal to the sum of the variances of the component parts. Equation 1 in my article expresses this relationship in terms of coefficients of variation. Because the U.S. Bureau of Mines procedure in the period of concern was to first normalize all concentrations expressed as mg/m^3 to a 1.0 m^3 basis, and then to add the numerators, i.e., mg, of the ten concentrations to determine if this sum was less than 30*, the Bureau essentially substituted a 10 sample, 30 mg weight standard for a single sample, 3.0 mg/m^3 concentration standard. I estimated the coefficient variation of the substituted weight standard by multiplying the 30 mg total by the single sample coefficient of variation. I correctly determined the coefficient of variation due to errors incorporated into each sample, but then reasoned that the substituted 30 mg standard was a single standard and subject to the single sample coefficient of variation estimator for confidence limits determination. Equation (1) should have been utilized for the 10 sample sum, as indicated by Leidel and Busch.

My article focused on methodology. It would be a mistake to end the discussion by believing that 1.23 mg, rather than 4.3 mg, is the 95% confidence limit on a 30 mg total associated with 10 samples. I utilized errors with good estimators for my example; other sources of error in the procedure were not included because their magnitudes were unknown. The 95% confidence limit on the 30 mg total, while still not completely defined, is undoubtedly greater than $\pm$1.23 mg. The estimates or error in procedures used for regulation of the work environment require better definition. Hopefully this is forthcoming now that statistical procedures used to determine compliance with standards for airborne contaminants are being clarified.

*30 mg was derived from a single sample concentration standard of 3.0 mg/m^3.

Reference

1. E. BRIGHT WILSON, JR. *An Introduction to Scientific Research,* First Ed. McGraw-Hill Book Company, Inc., New York, 1952, p. 272.