ANALYTICAL METHODS APPLICATIONS IN SAFETY ENGINEERING

A TRAINING MONOGRAPH

U.S. DEPARTMENT OF HEALTH AND HUMAN SERVICES
Public Health Service
Centers for Disease Control
National Institute for Occupational Safety and Health

october 1982

ANALYTICAL METHODS APPLICATIONS IN SAFETY ENGINEERING

A TRAINING MONOGRAPH

Project Officer: James B. Walters, Safety Manager
Division of Training and Manpower Development
Curriculum Development Branch

Contractor: Dr. Shimon Y. Nof
Purdue University, School of Industrial Engineering
West Lafayette, Indiana

U.S. DEPARTMENT OF HEALTH AND HUMAN SERVICES
Public Health Service
Centers for Disease Control
National Institute for Occupational Safety and Health
Cincinnati, Ohio

October 1982

DISCLAIMER

The opinions, findings, and conclusions expressed herein are not necessarily those of the National Institute for Occupational Safety and Health, nor does mention of company names or products constitute endorsement by the National Institute for Occupational Safety and Health.

PREFACE

PURPOSE AND SCOPE

The purpose of this monograph is to serve as an educational resource for instructors and trainees of safety engineering in the area of analytical methods. Analytical methods can be defined as:

> Techniques used to represent complex functional relationships as mathematical models that can be analyzed or solved so as to provide a quantitative basis for the decision making.

The use of analytical approach in the analysis and design of innumerable engineering issues has become a common practice in recent decades. Major developments in applied mathematics, operations research, and computer science have provided effective, practical solutions that are objective and precise rather than subjective and imprecise; that are predictive, a priori, and useful for correct planning rather than operating by trial and error after the fact; and that seek to eliminate and avoid the unwanted, rather than identifying it after it has occurred and attempting to rectify it. For these reasons, safety engineers should apply analytical methods wherever possible.

The lack of organized material in this area and the need to train people to apply analytical methods for safety problems warranted the development of this monograph. Its intent is to make safety engineers aware of the usefulness of analytical methods and to prepare them to apply these methods in their work.

The material covered here focuses on typical industrial occupational health and safety issues. Problems associated with highway and air traffic safety, pollution, health care, and the like are not included, even though analytical methods have also been applied in these areas.

PREREQUISITES

Users of this monograph are expected to have a general background in the techniques that are discussed. A brief review should be sufficient for them to understand the particular applications that are described.

OBJECTIVES

Upon completion of the six lessons included in this monograph, the user should be able to:

1. Understand the differences between analytical, quantitive, empirical, judgmental, and qualitative methods.
2. Know the advantages, limitations, and context of analytical methods as applied to safety engineering.
3. Know how to apply particular analytical methods to solve specific safety problems.
4. Read about and be able to conduct simple technical discussions on applications of analytical methods in the solution of safety problems.
5. Identify when analytical approaches are or are not appropriate for use on safety problems.

SOME INSTRUCTIONAL NOTES

1. Case studies appearing in this monograph can be distributed to students as case studies, if the students do not receive the complete manual.
2. The figures can be distributed to students as handouts, or can be made into visual presentations.
3. Answers to questions and exercises appear at the end of the monograph. These can be given to students with the assignment or used to check the student's work.
4. It is recommended that students be provided with easy access to the reading materials that are specified in the exercises (Lesson 1: E1, E2, E3; Lesson 4:E1).

Feb. 1980 Shimon Y. Nof
West Lafayette, Indiana

CONTENTS

3. TRANSPORTATION MODELS

4. LOCATION METHODS

5. SIMULATION

6. ANSWER GUIDE TO QUESTIONS AND EXERCISES

1. FUNDAMENTAL CONCEPTS OF ANALYTICAL METHODS

LESSON OBJECTIVES

1. Provide an introduction to the set of subsequent lessons.
2. Explain the differences between various analysis approaches.
3. Discuss the advantages of limitations of analytical methods in safety engineering.
4. Illustrate quantitative measures and analytic performance evaluation in safety engineering.
5. Define certain important terms in analytical methods.

1. FUNDAMENTAL CONCEPTS OF ANALYTICAL METHODS

ANALYSIS AND ANALYTICAL METHODS

Before we explore the details of analytical methods and their applications for safety engineering, let us first examine the purpose of analysis in general and of analytical methods in particular.

In most safety engineering activities, as in most other engineering areas, the engineer has to evaluate specific situations and seek a course of action among alternatives. Typically, certain overall objectives must be achieved with limited resources and budgets. The engineer has to identify and propose feasible alternative actions; measure, evaluate, and compare these actions; and then recommend a preferred one among them. For instance, in reviewing the emergency equipment needs of a factory the engineer must decide between conflicting issues. On one hand, more equipment will improve safety levels, which is the engineer's objective, particularly if it is spread all over the factory. On the other hand, the budget is limited, and also, having too much equipment can sometimes cause confusion during an emergency. Furthermore, a lesser amount of equipment may well be sufficient if enough people are trained to use if effectively. Thus, it may be wise to divert some of the available budget towards training. The engineer must decide how much equipment to purchase and how much of the budget to divert to training.

The solution to this problem may be approached by judgmental and qualitative methods, empirical methods, quantitative methods, or analytical methods.

JUDGMENTAL AND QUALITATIVE METHODS

Judgmental and qualitative methods describe the details of a problem and of various possible solutions. A description, which can be verbal, symbolic, or graphic (but not mathematical), explains the properties and qualities of the issue. Based on this description, a decision maker is expected to evaluate and judge alternatives and make decisions. In a safety problem, a qualitative analysis may seek to eliminate all hazards without regard to their probability.

Although the descriptive part of every problem is essential, it is clear that any judgment that is based on description alone cannot be free of subjective biases. To illustrate this point, let us consider again the emergency equipment problem:

Assume that installing a complete fire station costs $500, whereas installing a smaller partial station costs only $200. A qualitative description would probably add that a complete station is "very good in a case of a blaze and that the smaller station is just adequate for small to medium fires." The decision maker has to judge (guess?) if and where a blaze, a medium fire, or a small fire could occur, and figure out how many stations of each type to install. Because the quality of this decision would be questionable, we often hear people refer to such decisions as having been made "by the seat of the pants."

On the other hand, qualitative methods can be quite successful for the solution to other types of safety problems. A good example is the behavioral approach, which describes human behavior and its relationship to safety. One such approach is called Error Analysis (DeGreene 1970). It involves describing work tasks and their activities, and identifying the potential human errors that are likely to be made by an operator. Task engineering is another qualitative technique (Altman 1970) in which behavioral aspects and human factors are checked in an effort to eliminate accident-causing circumstances from a task method. The qualitative results of such methods point up which situations may be dangerous, what should be avoid-ed, and which actions are better than others. Such methods do not, however, usually produce quantitative answers about *how to solve problems*.

EMPIRICAL METHODS

"Empirical" means "based on experience and observations." This method is usually combined with a problem description, just as judgmental and qualitative methods are. An experienced engineer would recognize a problem and, based on his past experience, would recommend a solution. For example, we could expect a seasoned, experienced safety engineer to come up with a satisfactory solution to the emergency equipment problem just discussed. An example of a beneficial empirical technique is the Critical Incident Technique (CIT) (Tarrants 1970), in which operators are encouraged to report "near-miss" instances, situations that almost led to an accident. The information is evaluated subjectively, and the potential severity and frequency of the "almost accidents" lead to preventive measures to control the hazards.

There are two major disadvantages to the empirical approach. First, new problems, different circumstances, and changing technologies are always occurring, for which no experience exists. Second, relying excessively on experience may lead an engineer to overconfidence and neglect. In summary, while past experience is invaluable, it is not enough.

QUANTITATIVE METHODS

Quantitative methods rely on quantified measures and their manipulation. Quantified measures include statistics, probabilities, cost figures, various process rates, and so on. The numerical values of relevant quantities are often obtained by direct measurement, reference tables, and other forms of data collection. Sometimes, however, values must be estimated; for example, when measurement is impractical or when prediction or ranking is involved.

Most quantitative methods rely on a model that involves mathematical formulations and equations. Some methods apply simple equations of functional relationships and use quantified measures to perform some calculation or logical analysis. One such example is the fault-tree analysis (Brown 1976), in which extensive logic networks with failure probabilities are used to represent the operation of a complex system. By following these networks according to certain algebraic and logic rules and considering combined probabilities, one can determine the probability of occurrence of various undesirable events.

Other examples of quantitative techniques include the use of statistical control charts (Greenberg 1971) to follow the safety performance of a plant in an approach similar to quality control; and safety sampling (Johnson and Rogers 1975), which is applied like work sampling to determine the percentage of safe work in a given department.

A formula for justification rating in accident control (Fine 1971) is an example of a quantitative approach that relies on estimated measures. The formula is: $J = (C \cdot E \cdot P)/CF \cdot DC$. It relates subjective quantities that measure hazard exposure (E), accident consequences (C), probability of occurrence (P), a cost factor (CF), and a degree of correction (DC) to evaluate a measure of justification (J). This example, which depends on sensitive subjective estimates, illustrates a case where quantities are difficult to measure.

ANALYTICAL METHODS

This special type of analysis is a subset of quantitative methods.

As defined earlier in the introduction, analytical methods have the following features:

1. They provide an unbiased, systematic, mathematical representation of a problem.
2. They concentrate on the significant, meaningful aspects and variables of a problem; thus they generally lead the analyst to effective preparation of the necessary data and simplify the solutions to complex problems.
3. They apply a quantitative, mathematical model, often called "Analytic Model."
4. They provide a precise procedure, an algorithm to analyze the model and obtain quantitative measures, information, or solutions that are clear and objective.

5. They permit repeated use of the analysis with different quantities and values to check and measure the sensitivity of results.
6. They often can be applied with computer packages that can analyze very large amounts of data quickly and accurately.

Analytical methods have provided engineers and managers with powerful tools for solving most difficult planning, design, and control problems. To date, however, their role in safety engineering has been quite limited. Some of the reasons for this limited application are discussed under the next heading.

ADVANTAGES AND DISADVANTAGES OF ANALYTICAL METHODS

Some safety engineers object to the use of analytical methods and quantitative techniques. Their general objections can be summarized in the statements of some of those who object:

1. "Quantitative measures and data which are the basis for any analytic application are inaccurate and they are often based on subjective estimates. Therefore, the results of the analysis may not be dependable."
2. "Analytic models require certain simplifying assumptions that may be unrealistic. For instance, modeling assumptions of linear relationships and of exponential probability distributions are frequently made in order to provide a mathematical solution, even though these assumptions are not correct."
3. "Analytical methods involve mathematics and are too complicated to explain to nonexperts. As a result, decision makers have to trust the analysts, and often are forced to accept the results without really understanding how they are derived."

Although these statements may be partly true, people experienced in the application of analytical methods know that such statements do not tell the whole story. It is indeed true that data sometimes have to be derived by estimation, subjective ranking and rating, and other imprecise methods. Because some of the data are nonmeasurable, however, does not mean that the majority of data, which are precise and reliable, should be ignored. Of course, we must ensure the high quality and correctness of all data and attempt to eliminate errors, but there are methods of minimizing individual biases in estimates. One such method is to get estimates from several experienced people.

Another important factor regarding the problem of data accuracy is that the analytical method can always include a sensitivity analysis. In this approach, values and quantities that are used as input to a problem can be varied within reasonable ranges. The computed results represent a range that points to the sensitivity of various measures and results with respect to the magnitude of different parameters. Those parameters that are considered critical may then require further investigation by the analyst, who will attempt to obtain as accurate a value for each of them as possible.

To some extent the same approach can also be used to check the sensitivity of results to various assumptions in the analytic model. The fact that some modeling assumptions (e. g., linear relationships) may be unrealistic, however, does not necessarily mean that the results are not useful. On the contrary, numerous cases are documented in which simplifying assumptions have led to excellent results because those assumptions were not critical to the behavior of the complete system. Of course, this is not always true, and each case should be considered carefully. It may often be practical, however, to apply an analytical method to get at least an approximate indication of the direction the solution will take.

Analytical methods do not produce miracles, but when used by experts they can be an excellent tool that can yield precise, quantitative solutions to complicated problems and objectively guide us to exact, "how to" decisions. The fact that some of these methods are complicated should not discourage us; their advantages surely outweigh the effort needed to apply them and to overcome their limitations. Nor should we abandon the application of these methods because some professionals do not feel comfortable with them. Rather, we should encourage these people to learn more about the methods, what they can do, and how they can benefit the role of the safety engineer.

A RECOMMENDED PROCEDURE TO THE APPLICATION OF ANALYTICAL METHODS

In general, every analytical method can be viewed as consisting of four major parts:

1. Problem description and data collection
2. Modeling
3. Analysis and computation
4. Recommended solution.

The first part includes the definition of the problem, identification of objectives and constraints, and background information. In practice, a broad problem description is sufficient initially, followed only by essential quantitative data. At this stage, some qualitative discussion of the problem, possibly with several experts, may be necessary to clarify salient components and variables of possible approaches to the solution.

An appropriate model is then applied or developed, which describes mathematically the major components of the problem and the relationships among them. Subsequently, it will become clear which particular data are required for the analysis and computation. Frequently, standard computer programs are available for computation of typical models. Based on the numerical results and measures obtained by the computation, an engineer or a team of analysts (depending on the complexity and importance of the problem) can prepare cost computation, performance comparison, checks, and so on. Finally, a conclusive recommendation can be obtained by combining the results with considerations based on experience, judgment, and a thorough qualitative evaluation.

In summary, a sound approach involves a combination of qualitative exploration, collaboration with experts, application of analytical methods, and judgment of the solution by experienced professionals.

SUMMARY

Several analysis approaches are suitable for solving safety engineering problems involving planning, design, and control. Analytical methods, combined with good judgment and experience, can provide safety engineers with excellent tools, including clear models, mathematical procedures, and objective, quantitative information.

QUESTIONS

Q1. Specify three typical safety problems and suggest which analysis approach will be most suitable for them. In each case identify:
- a. the objective
- b. the constraints
- c. examples of possible recommended actions

Q2. Recall an unsafe situation from your experience and suggest if and how a quantitative approach could have provided a basis for improvement. Explain your answer.

Q3. Since the application of analytical methods requires expertise, data collection efforts, and often the use of computer programs, does it mean that they should be applied only for large, important problems? (Refer specifically to safety engineering problems.)

EXERCISES

E1. Read the article "Systems Hazard Analysis Applied to Production," by Robert J. Firenze, National Safety News, June 1971, pages 48-55, and answer the following:
- a. What are four basic methods of information acquisition?
- b. What are three logic processes that can be used by analysts?
- c. The article describes a method for system hazard analysis. In your opinion, what type of analysis approach is it? Explain.

E2. Read the article "Mathematical Evaluation for Controlling Hazards," by William T. Fine, in the Journal of Safety Research, December 1971, Vol. 3, No. 4, pages 157-166, and answer:
- a. What are the two significant needs in hazard control?
- b. Briefly, how can we calculate:
  - (1) Risk scores?
  - (2) Justification scores?
- c. In example No. 2 in the article, a risk score of 300 was calculated, based on parameters C, E, P. Evaluate the sensitivity of this score to ±10% and

then to ±25% variations in each of the values of the risk parameters.

d. In the article, example No. 3 of justification scores is concerned with the location of a propane storage tank. Identify in this example:
   (1) The objective
   (2) The model
   (3) The parameters
   (4) The solution procedure
   (5) The recommendation

E3. Read the article "Two approaches to a Non-Accident Measure for Continuous Assessment of Safety Performance," by T. H. Rockwell and V. D. Bhise, in the Journal of Safety Research, September 1970, Vol. 2, No. 3, pages 176-187, and answer the following:

a. What are the active and passive procedures of the incident technique as defined by this article?
b. The procedures developed in the article involve more than one analysis approach. Identify which type of analysis approach is followed by each component of the procedures.
c. Discuss the advantages and disadvantages of the assessment process that is specified by the article.

REFERENCES

Altman, J. W. 1970. Behavior and Accidents. J. of Safety Research, September 1970.

Brown, D. B. 1976. Systems Analysis and Design for Safety. Prentice Hall.

DeGreene, K. B. 1970. Systems Psychology. McGraw Hill.

Fine, W. T. 1971. Mathematical Evaluation for Controlling Hazards. J. of Safety Research, Vol. 3, No. 4, December 1971, pages 157-166.

Firenze, R. J. 1971. Systems Hazard Analysis Applied to Production. National Safety News. June 1971, pages 48-55.

Greenberg, L. 1971. A Quantitative Approach to Safety Inspections. Hazard Prevention, Vol. 8, No. 1, September/October 1971.

Johnson, W. L., and T. R. Rogers. 1975. Measuring Safety Performance, J. of Industrial Engineering, Vol. 7, No. 12, December 1975, pages 19-23.

Rockwell, T. H., and V. D. Bhise. 1970. Two Approaches to a Non-Accident Measure for Continuous Assessment of Safety Peformance, J. of Safety Research, Vol. 2, No. 3, September 1970, pages 176-187.

Tarrants, W. E. 1970. Utilizing the Critical Incident Technique as a Method for Identifying Potential Accident Causes. National Highway Safety Bureau, U. S. Department of Transportation, Washington, D. C.

2. LINEAR PROGRAMMING APPLICATIONS

LESSON OBJECTIVES

1. Understand the fundamental concepts and assumptions of linear programming.
2. Know the structure of a linear programming model.
3. Be able to specify simple objectives and constraints.
4. Be familiar with some typical applications of linear programming in safety engineering.

2. LINEAR PROGRAMMING APPLICATIONS

INTRODUCTION

In general, mathematical programming models are models in which a certain objective is to be optimized (i. e., maximized or minimized), subject to a set of specific constraints, by applying a well-defined mathematical procedure. Linear programming (LP) is one branch of mathematical programming in which we assume that the objective and the constraints have a linear form.

Linear programming is an optimization method; that is, its purpose is to find the optimal strategy among alternative strategies that are all influenced by conflicting effects. For instance, a typical problem for which LP is applicable is making a decision on an optimal mix – specific quantities of different products, specific ratios of various activities, etc. On one hand, each element in the mix makes a different contribution, either positive or negative. On the other hand, each element differs in cost or requires a different capacity of material, process, energy, etc. The optimal strategy will be the one that optimizes some measure, e.g., maximizes utilization or profit or minimizes cost or damage. The optimal strategy must, of course, comply with all the imposed limitations and constraints. An example of such a problem is the question of fire station allocation discussed in Chapter 1. The problem is to determine how many stations of each type should be established, or in other words, the optimal mix of stations.

Another type of problem for which LP is applicable is scheduling. Here the question is how to assign certain activities to time periods in the planning process. In effect, the purpose again is to find the optimal mix, now with a consideration of time. The planning of a safety training schedule is an example of this type of problem.

Now let us review the major components of the LP method.

OBJECTIVE

An objective can be modeled either to maximize or to minimize some measure, e. g., to maximize a safety score or to minimize costs. Assume that on a scoring system of 0 and 10 the fire stations in our example were scored 9 for the full station, and 4 for the small station. In other words, each full station contributes 9 points to the fire safety score of the factory, and each small station contributes 4 points. With an objective of maximizing the total safety score, we would designate x_1 as the number of full stations and x_2 as the number of small stations; then we can write:

$$\text{Maximize } Z = 9x_1 + 4x_2$$

CONSTRAINTS

Constraints can be written as equalities or as inequalities. The various types of constraints include technological, budget, legal, resource, and others. Let us assume in our example that the total budget for fire extinguishing equipment is limited to $4,000. If all the money has to be spent, the budget constraint can be written as:

$$500x_1 + 200x_2 = 4000$$

Usually, we will write it as an inequality because not all the money may be necessary. In this case:

$$(1) \quad 500x_1 + 200x_2 \leq 4000$$

Let us further assume that because of space limitations in the factory a maximum of four full stations can be established. This constraint will be written as:

$$(2) \quad x_1 \leq 4$$

Also,

(3) $x_1 \geq 0$, and (4) $x_2 \geq 0$

To summarize the LP formulation of the example problem:

Maximize $Z = 9x_1 + 4x_2$

Subject to:

$$
\begin{aligned}
&(1)\quad 500x_1 + 200x_2 \leq 4000\\
&(2)\quad x_1 \leq 4\\
&(3)\quad x_1 \geq 0\\
&(4)\quad x_2 \geq 0
\end{aligned}
$$

FEASIBLE REGION AND SOLUTION

The feasible region is defined as the range of solutions or decisions that satisfies the constraints. Because in this simple example we have only two variables, x_1 and x_2, we can plot the constraints graphically (see Figure 1). A line for the objective function is also shown for an arbitrary $z = 36$, to indicate its slope. Since Z increases as that line is shifted to the right, and upward (as x_1 and x_2 increase), an optimal solution will be obtained when $x_1^* = 0$, $x_2^* = 20$, and $Z^* = 80$. Note, that in cases of minimizing the objective function, the objective line should be shifted left and downward (for decreasing x_1 and x_2) until the last extreme point is reached.

THE CANONICAL FORM OF LP MODELS

Any LP model can be written as:

$$(1)\quad \text{Maximize} \sum_{j=1}^{n} C_j x_j$$

Subject to:

$$(2)\quad \sum_{j=1}^{n} a_{ij} x_j \leq b_i \quad \text{for } i = 1, 2, \ldots m$$

$$(3)\quad x_j \geq 0 \quad \text{for } j = 1, 2, \ldots n$$

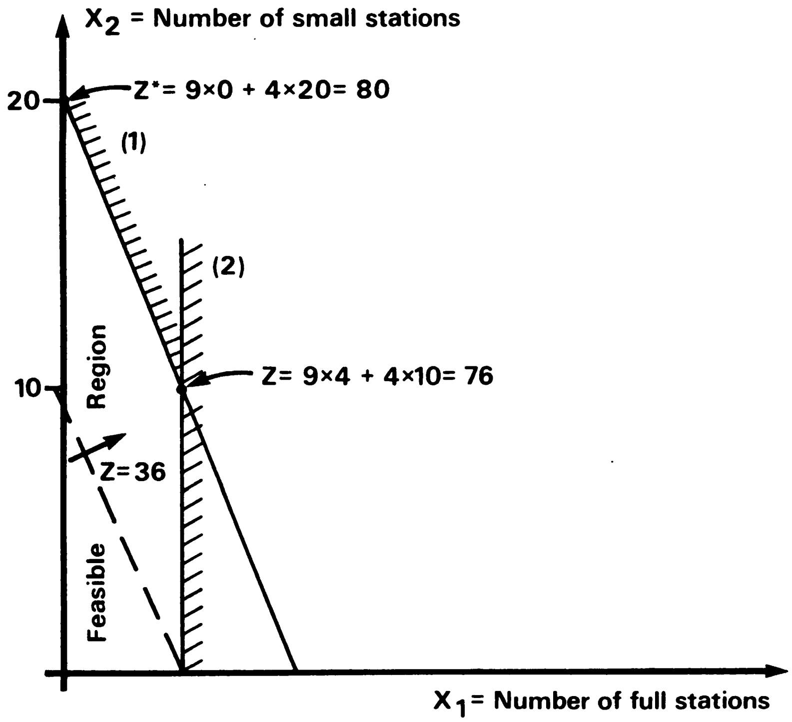


Figure 1. The plotting of constraints based on two variables shows that an optimal decision will be reached when $X_1 = 0$, $X_2 = 20$, and $Z = 80$.

Similarly, any LP model can be written as:

$$(1)\ \text{Minimize} \sum_{j=1}^{n} c_j x_j$$

Subject to:

$$(2)\ \sum_{j=1}^{n} a_{ij} x_j = b_i$$

for $i = 1, 2, \ldots m$, with $b_i \geq 0$

$x_j \geq 0$ for $j = 1, 2, \ldots n$

Note, that the objective

$$\text{Maximize} \sum_{j=1}^{n} c_j x_j \text{ can be translated to}$$

$$\text{Minimize} \sum_{j=1}^{n} (-c_j x_j)$$

SURPLUS AND SLACK VARIABLES

Any inequality can be converted to an equality by adding (subtracting) an artificial variable. Examples:

$7x_1 + 3x_2 \leq 180$ can be written as $7x_1 + 3x_2 + 1y = 180$ where $y \geq 0$ is called a *slack variable.*

$9x_1 + 12x_2 \geq 65$ can be written as $9x_1 + 12x_2 - 1y = 65$ where $y \geq 0$ is called a *surplus variable.*

THE NUMBER OPTIMAL SOLUTIONS AVAILABLE

Figures 2a, 2b, 2c, and 2d illustrate four typical problems:

- One optimal solution – when at the optimum the objective function line touches only one extreme point (2a).
- An infinite number of optimal solutions – when at the optimum the objective function line is parallel to a border line of the feasible region (2b).
- Unbounded optimal solution – the objective function is not constrained and can be made as large (or as small) as desired (2c).
- Infeasible problem – there is no optimal solution (2d).

THE SIMPLEX METHOD

The simplex method is an iterative technique that can handle practically any number of decision variables (x_i's). Some standard computer packages can use this solution method.

ASSUMPTIONS OF LINEAR PROGRAMMING

A discussion of the assumptions of linear programming is useful to find out when this method is and is not applicable.

Assumption 1: Proportionality

Proportionality means that all quantities are directly proportional; that is, they increase and decrease by a constant factor throughout the whole range of their levels of activity. This assumption will not hold if inconstant, different rates of change occur within the activity range.

Assumption 2: Additivity

This assumption is valid if when quantities of different measures, resources, etc. are added (or subtracted) together, the total measure is equal to their sum. This assumption is not valid in cases where interactions between activities produce a nonlinear result (e. g., a chemical process).

Assumption 3: Divisability

This assumption means that values of decision variables can be divided into any fractional levels. This assumption is not valid when the solution must be in integer values only. Usually LP can still be used, when decision variables are large enough, by rounding off the results. As an alternative, integer programming can be used.

Assumption 4: Deterministic Behavior

In this assumption, all the parameters and coefficients are constant over time. This assumption is usually not valid because planning problems involve the future. A sensitivity analysis can reveal how significant an error can be in this assumption.

A CASE STUDY: ALLOCATING SAFETY RESOURCES

Several companies have used linear programming to plan the allocation of safety resources. An article by Ayoub (1975) describes such an application in the furniture industry. In this example a table (as shown in Table 1) is used to evaluate and rate safety levels on the

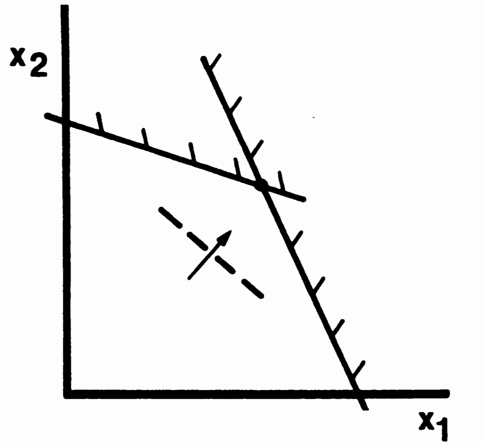


Figure 2a. **One solution. At the optimum, the objective function line touches only one extreme point.**

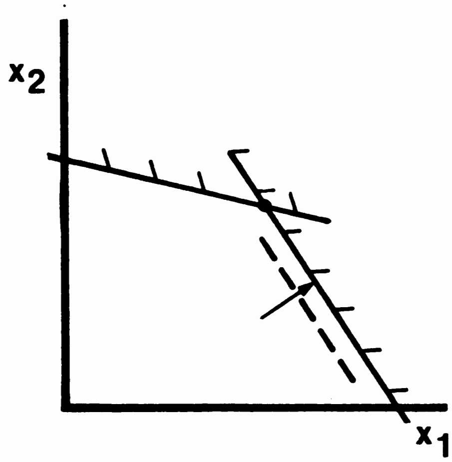


Figure 2b. **Infinite solutions. At the optimum, the objective function line is parallel to a border line of the feasible region.**

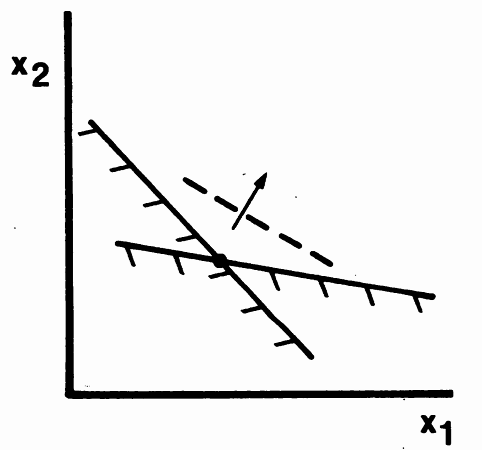


Figure 2c. **Unbound solution. The objective function is not constrained.**

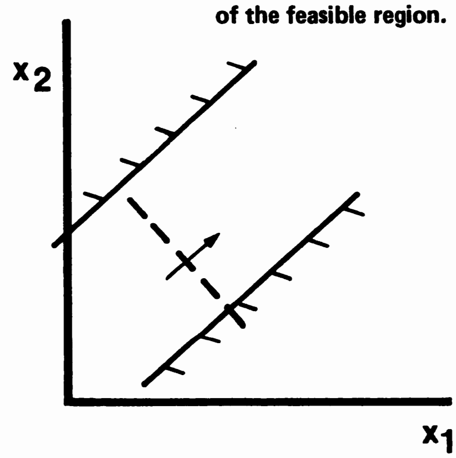


Figure 2d. **Infeasible solution. There is no optimal solution.**

basis of four aspects: management, engineering, supervision, and machine guarding. The following is a similar example of such an application.

The Problem

An OSHA inspection resulted in the following ratings of the listed activities according to some rating scheme similar to the one given in Table 1:

1. Safety aspects of layout and remodeling $(x_1) = 5$.
2. Periodic inspections for OSHA compliance $(x_2) = 3$.
3. Hazard control: Machine guarding and safety equipment $(x_3) = 10$.

Although no penalties were assessed as the result of this inspection, the company was told that its rate of injuries per 10^6 hours of employee exposure must be reduced to not more than 10 during the following six months.

The management of the company decided that on top of its current safety budget of $5000 it could allocate an additional $2000 for this purpose. What should the company do to achieve the necessary safety compliance?

Table 1. A rating scheme to evaluate safety performance

Rate on the scale from 0 (Poor) to 5 (very good).

A. Management Activities		
1. Safety policy	______	
2. Safety training	______	
3. Safety promotion and incentive	______	
4. Specific safety management positions	______	
5. Periodic safety meetings	______	
Total Management involvement and support		______
B. Safety Engineering		
1. Safety planning activities	______	
2. Periodic plantwide safety inspections	______	
3. Development of safety awareness programs	______	
4. Development of safety training programs	______	
5. Maintenance and utilization of current safety information	______	
6. Maintenance and utilization of accident and injury data	______	
Total safety engineering effectiveness		______
C. Safety Supervision		
1. Accountability	______	
2. Frequent inspections	______	
3. Enforcement of safety regulations and procedures	______	
4. Accident investigations	______	
5. Safety meetings	______	
Total safety supervision effectiveness		______
D. Safety Equipment		
1. Machine guarding	______	
2. Personal protective equipment	______	
3. Emergency equipment	______	
4. First aid facilities and personnel	______	
Total Safety equipment effectiveness		______

Modeling

Because this is a problem of allocation or finding the optimal mix of activities, linear programming is applicable (provided all the relationships can be assumed linear, as we shall assume).

First, let us define the objective function as minimizing the total safety cost per employee, including the costs of operating the safety program and the cost of accidents. For that function we need to find the cost coefficient per each one-level unit in the ratings of the three activities. For example, how much will it cost (per employee) to increase the rating of hazard control by 1 from its present level of 10?

A simple way to find this value is to divide the current expenditures on hazard control by the current rating of 10, and then by the number of employees. The relationship of each activity to accident costs in the company also has to be computed, which can be done as follows:

From previous accident investigations find the percentages of accidents attributed to layout issues, to compliance inspections, and to hazard control. Then to compute the portion of each activity in accident cost, divide it by the current ratings and by the number of employees.

A Numerical Example

There are 885 employees in the company. The recent total annual cost of x_1, safety-related changes in layout and remodeling, has been \$1770. The portion of layout-dependent accidents = 15%. Total accident costs = \$8900.

1. Coefficient component for x_1 due to operating cost:

$$\frac{1770}{5 \cdot 885} = 0.40$$

2. Coefficient component for x_1 due to accident cost:

$$\frac{(0.15) \cdot 8900}{5 \cdot 885} = 0.30$$

The total coefficient is then 0.3 + 0.4 = 0.70. We will assume that the coeffiecient remains the same even when we increase the level of the activities.

Following this procedure for the other activities, the company found:

Minimize Z = (Fixed current safety costs per employee) $+ 0.70x_1 + 0.45x_2 + 0.30x_3$

Since the first part of the objective function does not depend on the decision variables (x_1, x_2, x_3), it can be ignored in the analysis without affecting the optimal solution.

To summarize the model:

Minimize $Z = 0.70x_1 + 0.45x_2 + 0.30x_3$

Subject to:

$$350x_1 + 300x_2 + 130x_3 = 7000$$
$$5 \leq x_1 \leq 10$$
$$3 \leq x_2 \leq 10$$
$$x_3 = 10$$

A solution can be obtained graphically, as shown in Figure 3. The optimal solution is:

$$\overset{*}{x}_1 = 7.5$$
$$\overset{*}{x}_2 = 10$$
$$\overset{*}{x}_3 = 10$$

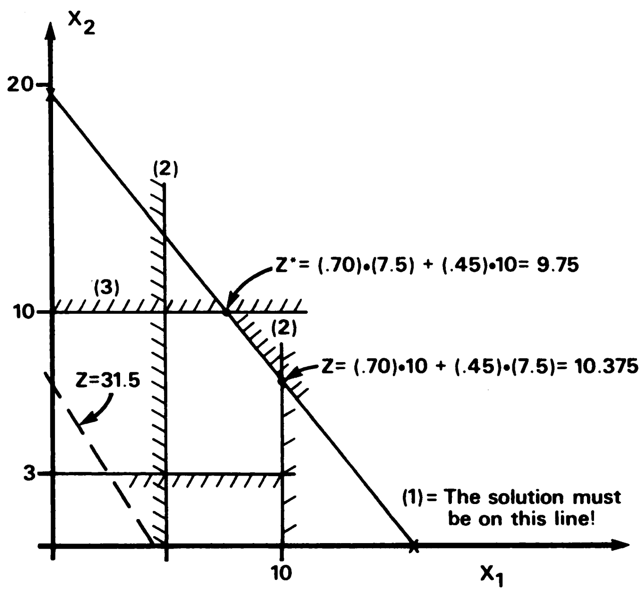


Figure 3. This graphic presentation shows an optimal solution of X_1 = 7.5, X_2 = 10, and X_3 = 10.

Recommendation

The optimal solution indicates that layout-related safety activities should be increased from a rating of 5 to 8 and 350 • (7.5) = $2625 should be spent to accomplish this. Activity in the inspection area should jump from a rating of 3 to 10, at a cost of 300·10 = $3000. Hazard control may remain at the same level because it is already at the maximum compliance rate.

The implementation of the recommended solution requires that the additional activities be extended according to the specific items that are included under each topic, i.e., layout and remodeling and inspections.

SUMMARY

This review of linear programming has shown that its typical applications are in planning allocation of resources, finding an optimal mix of activities, and scheduling activities over certain periods. A case study of allocating safety resources to three types of activities of safety in a company was discussed in detail.

QUESTIONS

Q1. Considering the allocation example that is presented in the case study, discuss the linearity assumptions. Refer to the linearity of the objective function and of the constraints.

Q2. The coefficients of the objective function in the allocation example are all positive. Can they ever become negative?

Q3. Can the allocation problem in the example be solved by a non-analytical approach? Explain and compare the resulting decision to the one obtained in the example.

EXERCISES

E1. Consider the allocation case study and solve it each time with the listed assumption. Interpret the meaning of your results.

a. The objective is:

Minimize $Z = 1400 - 3X_1 - 5X_2 + 9X_3$

The constraints are:

(1) $x_1 \leqq 4$

(2) $2x_2 \leqq 12$

(3) $3x_1 + 2x_2 \leqq 18$

(4) $x_3 = 9$

(5) $x_1 \geqq 0, x_2 \geqq 0, x_3 \geqq 0$

b. The objective function is:
Minimize $Z = 2x_1 + 3x_2 + x_3$

The constraints are:

(1) $x_1 + 4x_2 + 2x_3 \geqq 8$

(2) $3x_1 + 2x_2 \geqq 6$

(3) $x_3 = 7$

(4) $x_1 \geqq 0, x_2 \geqq 0, x_3 \geqq 0$

E2. Develop a simple rating scheme similar to the one in Table 1, but with the following guidelines:

a. Rating is from 0 to 10.
b. The rating represents a safety score of each activity relative to its contribution to the overall safety level of the company.

Using your scheme, prepare an LP model of the problem specified in E1, b. Note that the objective function now has to be maximized. (Pick your own coefficients for the objective function.) Interpret the meaning of your results.

E3. The following is an example of a dynamic problem for which LP is applicable. The safety director of a company has to schedule the training of employees in safety procedures. Assume that a special training course has been organized that requires one full week. For simplicity, let us assume that one week out of each of the next three months (periods) will be designated to this safety training course.

A total of 60 people must be sent to the course, but there are restrictions regarding how many can attend in any one period. First, the course cannot handle more than 30 people at one time. On the other hand, a group of less than 6 people will be impractical. Further, the production manager requires that no more than 50 people be away from production activity during any two consecutive periods.

Several costs are associated with this training program. Every trainee incurs an actual training cost, which depends on the period. Training is most expensive in the first period – \$200 per person. In the second period this cost decreases to \$150, and in the third period, the cost is only \$100. There is an additional cost associated with having to operate the production equipment without the missing trainee. This "no-production" cost also varies by period: \$200 the first period, \$300 the second, and \$350 the third.

After consideration, the safety director decides to send exactly 15 people in the first period, on the premise that "15 is not too many, but sufficient to get the course rolling." The problem is to decide how many people to send in each of the other two periods.

a. Formulate the problem by a LP model.
b. Calculate the optimal solution.
c. Interpret your result.
d. How sensitive is the solution to a variation of ± 10% in the estimate of training costs?

BIBLIOGRAPHY

Ayoub, M. A. April 1975. The Problem of Occupational Safety. *IE*, Vol. 7.

Kwak, N. K. 1973. Mathematical Programming With Business Applications. McGraw-Hill.

Wagner, H. M. 1975. Principles of Management Science. 2nd ed. Prentice Hall, Inc.

3. TRANSPORTATION MODELS

LESSON OBJECTIVES

1. Introduce transportation models, including transshipment and assignment models that are used for optimal assignment of given resources to given requirements.

2. Discuss typical situations in which transportation models can be applied.

3. Describe some specific safety engineering applications of transportation models.

4. Familiarize the student with important terms and concepts of transportation models.

3. TRANSPORTATION MODELS

INTRODUCTION

Transportation models are a special subset of general LP models. They are called "transportation models" because they represent cases in which certain resources at given locations are to be transported to certain destinations in a way that minimizes transportation costs.

The transportation context should be viewed in an abstract form, however, as the same model can be applied whenever certain resources must be allocated, shipped, or assigned to certain targets, goals, or objectives, as described later.

In general, we say there are m origins, each designated i (i = 1,2,...,m). Each origin i possesses a_i units. There are also n destinations, each designated j (j = 1,2,...,n), which require b_j units. Although m does not have to equal n, the sum of all units available must equal the total requirement (see Figure 4).

To n destinations (n=6)

From m orgins (m=4)

j \ i	1	2	3	4	5	6	a_i:
1	4						40
2	3						20
3							10
4				8			30
b_j:	10	20	30	20	10	10	100

Examples: $e_{11} = 4$; $a_1 = 40$; $b_2 = 20$; $x_{34} = 8$; $\sum_{i=1}^{4} a_i = \sum_{j=1} b_j = 100$

An empty cell implies x = 0.

Figure 4. This is an example of a transportation model.

The allocation of one unit from each origin to each destination is associated with a certain coefficient – cost-effectiveness, safety score, etc. Let e_{ij} designate the coefficient of allocating one unit from origin i to destination j. Let x_{ij} be the number of units allocated from origin i to destination j.

Then the transportation model can be stated as:

$$\text{Optimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} e_{ij} \cdot x_{ij}$$

Subject to:

$$\sum_{i=1}^{m} x_{ij} = b_j \qquad j = 1, 2, \ldots, n$$

$$\sum_{j=1}^{n} x_{ij} = a_i \qquad i = 1, 2, \ldots, m$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

The decision variables are the quantities shipped from i to j, which are under the control of the decision maker. Additional details and solution approaches of the transportation model are discussed in Lesson 4, Location Methods. Two special types of transportation models are discussed in this lesson.

THE ASSIGNMENT MODEL

The assignment model deals with problems that arise when there is a need to assign or allocate each of a number of means or resources to an equal number of requirements on a one-for-one basis. A very typical application is the assignment of people to tasks with the objective of optimizing their performance. In the safety context, for example, the accident proneness of each individual for different types of tasks can be estimated, with the objective being to minimize the overall score by an optimal assignment. (An example of testing accident proneness is described by Dr. Alexander (1978). He defines a technique used in California to evaluate candidates for climbing telephone poles. Another evaluation technique for assigning manual material handlers that is based on safety considerations is described by D. B. Chaffin (1977).

The assignment model can be described as follows: There are n means (e. g. , people) that can satisfy n requirements (e. g. tasks). The effectiveness of assigning means i to requirement j is designated by e_{ij}. Since the assignment is on a one-for-one basis, $x_{ij} = 1$ implies that means i is assigned to j; $x_{ij} = 0$ implies that i is not assigned to j.

Note that in this model each means can be assigned to only one requirement.

The model can be stated:

$$\text{Optimize } Z = \sum_{i=1}^{n} \sum_{j=1}^{n} e_{ij} x_{ij}$$

Subject to:

$$\sum_{i=1}^{n} x_{ij} = 1 \qquad j = 1, 2, \ldots, n$$

$$\sum_{j=1}^{n} x_{ij} = 1 \qquad i = 1, 2, \ldots, n$$

As an illustration on how to use the assignment model, consider the example mentioned earlier, in which the objective is to minimize the accident-proneness score. The matrix below shows that four employees (1 to 4) are to be assigned to four tasks (A through D).

	Tasks:	A	B	C	D
Employee:	1	6	4	5	8
	2	2	3	6	4
	3	4	3	1	2
	4	6	8	5	7

The accident-proneness scores for each task have been established on a scale of 0 to 10 by testing the employees. These scores are indicated in the matrix. For example, the riskiest matches would be to assign Employee 1 to Task D or Employee 4 to Task B. On the other hand, Employees 2 and 3 show relatively low scores, 2 in 2-A and 1 in 3-C.

When the objective is to minimize the total score, the first step of the solution method is usually the reduction of all the matrix coefficients. This is accomplished by subtracting the minimum element in each row from each element in the row, and then subtracting the minimum element in each column from all elements in the column. In our example, we would subtract 4 from row 1, 2 from 2, 1 from 3, and 5 from 4; then we would subtract 0 from columns 1, 2, 3, and 1 from column 4. The result:

	Tasks:	A	B	C	D
Employee:	1	2	0	1	3
	2	0	1	4	1
	3	3	2	0	0
	4	1	3	0	1

An assignment that minimizes the total of a matrix that is reduced in this manner will also minimize the total score for the original matrix of coefficients. If we assign employees to tasks according to cells that contain a zero in the reduced matrix, the resulting total score will be the minimum. In this case, the optimal solution is indicated by the following assignment: 1-B, 2-A, 3-D, 4-C. The associated minimum value of the total score is 4 + 2 + 2 +5 = 13.

In simple cases such as the one in this example, an optimal solution may be found in the reduced matrix. In general, however, further steps must be taken to generate additional zero cells in order to complete an assignment. The referenced material offers a student review of general solution techniques.

ASSIGNING EMERGENCY RESPONSIBILITIES

Let us now apply the assignment model to formulate the problem of assigning responsibilities to several emergency centers. Consider the case of a company that has 13 plants located in a certain area. Emergency services, including fire and chemical hazard control and first aid services, are scattered in five centers over the area. For maximum efficiency of services at a time of emergency, it is proposed that the plants be divided into five groups. Each emergency center would then be responsible for only one given group, except in extraordinary situations.

The problem is to establish the optimal way for centers to be assigned to groups. In practice, an effective assignment must include consideration of the type of hazards to be controlled, the distance between centers and plants, the type of emergency services available, and so on. For the sake of simplicity, let us consider only the distance between centers and plant groups and attempt to minimize the total distance. Such an approach reflects the minimization of time required to reach either the emergency site or the center. The following matrix contains the distance data in miles.

		To Plant Group				
		A	B	C	D	E
From Emergency Center	1	5	1	8	15	1
	2	1	7	16	6	3
	3	9	6	1	6	7
	4	1	7	3	8	7
	5	2	4	5	1	4

The solution techniques for this problem require several iterations that are not shown here; however, the final solution is given below, with a minimum total distance of 8 miles.

Assignment Matrix of Emergency Centers

Plant Group:		A	B	C	D	E
Center	1		x			
	2					x
	3			x		
	4	x				
	5				x	

THE TRANSSHIPMENT MODEL

The transportation model that was described at the beginning of this lesson deals with problems of optimizing the shipment of goods or resources from certain origins directly to certain destinations. Often the movement of resources or people involves transit through intermediate points. In these cases the transshipment model is more appropriate. We will first describe this model and then show its applicability to the important issue of emergency evacuation.

The transshipment model is associated with a network of locations. At some of the locations there is a surplus of items (or people), and at others there is a demand (see Figure 5).

A positive value at a location implies a surplus that is to be redistributed to the rest of the network. A negative value implies the additional quantity that is required at a location. In Figure 5, Locations 1 and 5 are termed *sources* because they have excess quantities; Locations 2, 3, 4, and 5 are termed *intermediate points* because items can be shipped through them; Location 6 is called a *sink* because it is not an intermediate point and because it has the required quantity.

In the network shown in Figure 5, it would cost $c_{13} + c_{34}$ to ship one item from Location 1 to Location 4, with Location 3 serving as an intermediate point.

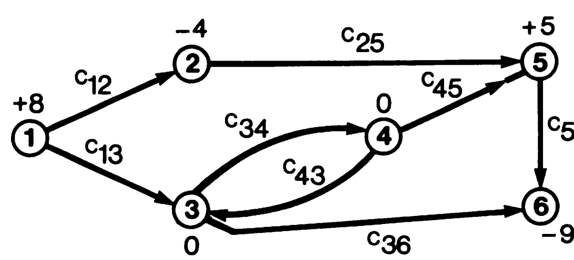


Figure 5. This is an example of a transshipment network.

Usually, the objective is to minimize the total cost or total time to perform the shipment or distribution. The solution technique for the transshipment problem is similar to the solution of the transportation model.

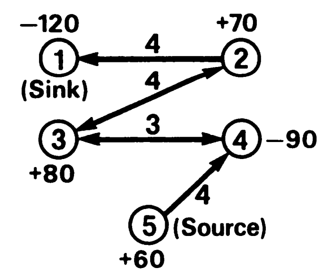
A CASE STUDY: EMERGENCY EVACUATION

A very useful application of the transshipment analytic model has been demonstrated by Francis and Saunders (1979) in a paper that describes the emergency evacuation of hundreds of employees from a high-rise building. The basic idea is to represent all floors that have to be evacuated as sources or intermediate points, and to represent safe exits from the building, either ground-floor or other emergency exits, as sinks. The cost associated with "shipping" (evacuating) employees is actually the time to get from one floor to the next. Obviously, the objective is to minimize the total evacuation time for all employees. Francis and Saunders explain that the optimal evacuation routes that were yielded by this technique were used to plan evacuation procedures, and to train employees periodically in following these procedures.

Let us now examine a simple example as an illustration of how to use the transshipment model for an emergency evacuation case. Let us consider a company located in a five-story building in which the normal occupancy during the work day is distributed as follows:

Floor	No. of Occupants (employees, visitors)
1	100
2	70
3	80
4	50
5	60

During an emergency, evacuation is possible either through the main floor (Floor 1) or by special evacuation ladders from Floor 4. Because it is assumed that people from Floor 4 will immediately evacuate from there and that people on the main floor will evacuate from there, these "local" people are not shown in the diagram. Figure 6 shows that 70, 80, and 60 people (a total of 210) are to be evacuated from Floors 2, 3, and 5, respectively. Floor 4, however, can accommodate only 90 evacuees in addition to the local people; thus Floor 1 has



From Floor \ To Floor	1	2	3	4	Supply
2	[4] 120	[0] 160	[4]	X	280 (70+80+60) + 70= 280
3	X	[4] 50	[0] 210	[3] 30	290 (70+80+60) + 80= 290
4	X	X	[3]	[0] 120	120 (70+80+60)− 90= 120
5	X	X	X	[4] 60	60 (actual)
Demand	120 (actual)	210	210	210	

Figure 6. This is an example of a transshipment model for emergency evacuation.

to evacuate 120 (210-90 = 120). The average time to move between floors is also shown in the figure.

The transshipment matrix in Figure 6 is developed as follows. Both a row and a column are constructed for each intermediate point; demand represents the total number of people to evacuate (210 here); supply represents the algebraic sum of this total number and the demand or supply at each given location. Each sink appears only once, as a column; each source appears only once, as a row. Sinks and sources appear with their actual supply or demand.

The solution procedure of the transshipment problem is not shown here, but the optimal solution is (see Figure 6): From Floor 2 to Floor 1 a total of 120, including 70 from Floor 2 and 50 from Floor 3; from Floor 3 to Floor 4, 30; from Floor 5 to 4, 60. The total evacuation time is 120.4 + 50.4 + 30.3 + 60.4 = 1010 min. Although the solution is not surprising in this example, it serves to illustrate the potential contribution of the technique in complex situations.

SUMMARY

Two important variants of the transportation method have been discussed: assignment and transshipment. Both assign resources to destinations. The assignment model can be applied for optimal assignment of workers to tasks based on safety considerations, or assignment of safety resources to targets. The transshipment model can be useful in the planning and establishment of optimal emergency evacuation routes, or optimal shipments of safety equipment.

QUESTIONS

Q1. Could the problem of assigning emergency responsibilities as described in the lesson be solved by a non-quantitative approach? Explain.

Q2. Is the assignment method applicable for matching groups of employees to particular tasks? Particular employees to given departments?

Q3a. Describe a safety problem that can be handled by the transshipment method.

b. For the above problem define:
   1) The objective function
   2) Sources
   3) Sinks
   4) Intermediate point
   5) Coefficient of effectiveness

c. What would the optimal value of the objective function of this transshipment problem be useful for?

EXERCISES

E1. If an assignment matrix contains coefficients of a positive nature, e.g., profit per unit or safety performance per unit, the objective may be to maximize a particular function. In such cases the procedure is as follows:
   a. Replace each element in the matrix by its negative.
   b. Subtract the most negative in each row from all elements in the row. Repeat for each column. This results in a reduced matrix. Note: Minimizing according to the negative elements is equivalent to maximizing by the original values.

Consider the following assignment matrix, which contains the safety score of five employees on five tasks.

	A	B	C	D	E
1	14	8	12	11	9
2	12	9	13	13	13
3	8	9	12	12	11
4	13	13	11	12	10
5	11	10	12	11	13

   a. Find the optimal assignment and interpret it.
   b. Compute the optimal value of the objective function.
   c. How sensitive is your solution to ± 10% variations in the safety scores? Answer by examining several examples.

E2. In the case of assigning emergency responsibilities that was described in the lesson, distance data were used to find an optimal solution. Using your own arbitrary numbers, show how considerations of the type of emergency services could be incorporated into the model, *in addition to* the distance data. Then find how this changes the solution and interpret your results.

E3. The company that was described in the emergency evacuation example is faced with the following decision problem: An additional floor, No. 6, has to be added to the building. It is expected to provide space for another 70 people. Should the evacuation capacity of Floor 4 be increased? By how much? Should an alternative evacuation facility similar to the one on Floor 4 be installed on Floor 5 or 6? Outline a detailed solution procedure to solve this problem. Describe what results could be obtained and how the decision can be made.

REFERENCES

Alexander, Dr. 1978. Journal of Occupational Medicine.

Chaffin, D. B., et al. 1977. Pre-Employment Strength Testing. NIOSH CDC-99-74-62.

Francis, R. L., and P. B. Saunders. 1979. Network Optimization Models of Building Evacuation. TIMS/ORSA Joint National Meeting, New Orleans.

BIBLIOGRAPHY

Shore, B., *Operations Management*, McGraw Hill, 1973.

Sivazlian, B. D., and L. E. Stanfel. 1975. *Optimization Techniques in Operations Research*. Prentice-Hall.

Wagner, H. M., 1975. *Principles of Management Science*. 2nd ed. Prentice-Hall.

4. LOCATION METHODS

LESSON OBJECTIVES

1. Describe certain analytical methods that can be useful in solving location problems that involve safety considerations.
2. Introduce the students to certain safety-related location problems.
3. Review some terms of location methods.
4. Discuss the applicability of location methods to safety engineering.

4. LOCATION METHODS

LOCATION PROBLEMS

Location problems arise when a facility is expanded or a system is redesigned. In both cases, sites have to be selected for new or added equipment, or for complete installations. Once the need arises, alternative sites and site arrangements must be identified, and both tangible and intangible costs have to be evaluated. In this evaluation, safety enclosures also must be considered. Of particular importance are costs and other location-dependent measures. Location methods are applied to compare alternative sites and site arrangements.

Single Site Selection Problems

In a single site selection problem one optimal location is sought among several alternatives. For example, consider the problems of locating an emergency stop button on a very large control board, or locating a storage tank of explosive material. Usually a single site selection problem can be solved by collecting data about all relevant aspects and the costs of each alternative location, and comparing them on the basis of their individual merits. When the same measure (typically dollars) can be used for all location factors, then a breakeven analysis can be applied. When intangible variables are included, however, as is the case in most safety-related problems, then figures of merit and significance weights have to be considered.

Multifacility Location Decisions

In multifacility location decisions, the objective is to find an optimal arrangement of units or devices in a network of facilities. Often the network is already in existence, and a new unit is added because of some change in the operation. For example, consider the problem of adding an inspection center to a company complex because of increased demand for safety inspection. Another type of multifacility decision involves a layout arrangement of a facility. The layout and materials handling within that layout have a significant effect on the safety of employees.

CASE STUDY: LOCATING AN INSPECTION CENTER

Suppose a manufacturing company has to inspect all of its equipment periodically for compliance with OSHA regulations. The equipment is scattered among three large complexes, and inspectors use special instruments to sample data and later analyze these data in a laboratory. Certain critical devices are also taken to the laboratory for testing. Two laboratories provide inspection centers for the inspectors. Recently, additional equipment has been added because of an increased demand for the company's products. As a result, instead of a previous total of 260 pieces of equipment, 440 pieces now have to be inspected routinely. The company decided to add another lab/inspection center, and has identified two potential locations for the new lab, say X1 and X2. The major issue in deciding where to locate the lab is inspection time, which is dependent on the distance between the labs and the equipment. Inspected equipment is idle until the inspection process is over; therefore, the objective is to minimize the total inspection time. In other words, the decision as to whether the new lab should be located at X1 or X2 will be determined by how long it takes all three labs to inspect all the equipment. See Figure 7 for the location of the three complexes, the two existing lab/inspection centers,and the two possible locations of a third center.

Additional data for the problem are tabulated below.

Complex:	A	B	C
Pieces of equipment	100	130	210

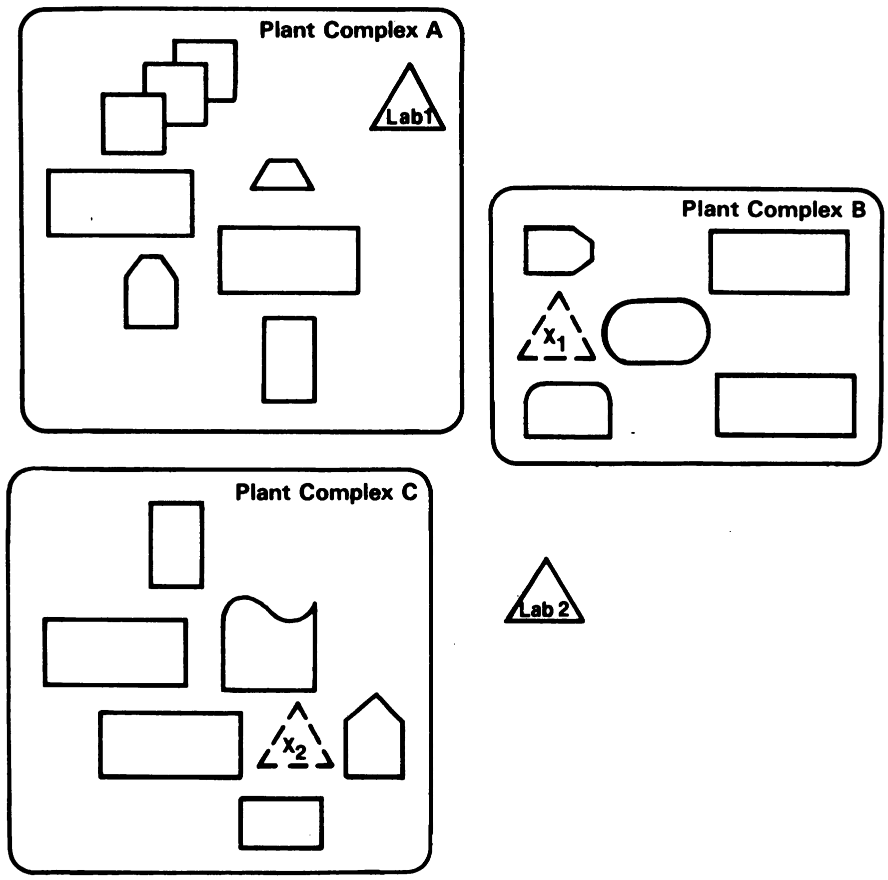


Figure 7. The map shows the layout of three plant complexes, two existing laboratory/inspection centers, and two possible locations for a third laboratory/inspection center (X1 and X2).

Laboratory:	1	2	3(new)
Pieces that can be inspected during the designated inspection period	90	170	180

Average inspection time values in minutes are given in the following matrix:

		Equipment at Complex		
		A	B	C
Inspection at lab location	Lab 1	4	6	12
	Lab 2	7	5	6
	x1	8	2	5
	x2	13	7	3

This problem can be solved by the transportation method that was discussed in Lesson 3. Two matrix that follow the transportation model formulation are shown, (a) for Location X1 and (b) for Location X2. Each optimal solution indicates the minimum total inspection

(a) The new lab at location X1

	Complex A	Complex B	Complex C	Inspection Capacity
Inspection: Lab 1	4	6	12	90
Lab: Lab 2	7	5	6	170
x1	8	2	5	180
Pieces of Equipment:	100	130	210	

(b) The new lab at location X2

Complex

		A	B	C	Inspection Capacity
Inspection:	Lab 1	[4]	[6]	[12]	90
Lab:	Lab 2	[7]	[5]	[6]	170
	x2	[13]	[7]	[3]	180
Pieces of Equipment:		100	130	210	

time that can be achieved with the new lab at the given location. The final solution, or the recommended location, will be the one for which the total inspection time is lower.

Solution Procedure

Step 1. Initial allocation by Vogel's approximation.

Refer back to matrix (a) and for each row and for each column find the difference between the two lowest values (in a maximization problem–between the two highest values). Write the results on the right and lower margin of the table. For laboratory location X1 we obtain the following:

	A	B	C		
Lab 1	[4] 90	[6]	[12]	90	2;8;–
Lab 2	[7] 10	[5]	[6] 160	170	1;1;1;2
X1	[8]	[2] 130	[5] 50		3;3;3;–
	100	130	210		
	3	3	1		
	3	–	1		
	1		1		
	1		–		

Identify the largest difference. This is 3, which appears in row X1, in Column A, and in Column B, so pick one *arbitrarily*, say row X1. This is the first candidate for allocation. Allocate the maximum allowed by the row and column totals, in the cell with the lowest time value, 2. Mark out all the cells that cannot be filled because of column and row totals. Here Column B is marked out. Recompute the differences in each row and column, ignoring the cells that are marked out, and write them on the margins again. The maximum is 8 for the Lab 1 row, so allocate 90 in the cell, with a minimum time value of 4, and so on until all allocations are completed.

Step 2. Testing for Optimality.

For each empty cell, evaluate the consequence of allocating one unit to it. That one unit would add the time coefficient of that particular cell, but it would also result in subtracting the time coefficient from the cell from which it is removed in the same row or column, and so on. A complete loop, which starts at the empty cell that is being evaluated and goes through full cells, must be followed in order to maintain the balance of row and column totals. For instance, evaluating the consequence of reallocating one unit to cell X1-A will add +8 min.; subtract −5 for cell X1-C; add +6 to cell Lab 2-C; and subtract −7 for cell Lab 2-A. The net result is $+8 - 5 + 6 - 7 = +2$, or an increase (+2) to the total. That means that allocating one unit to this empty cell will not improve (decrease) the total time. Other empty cells can be evaluated similarly. If all yield positive net values, the current allocation is optimal. Indeed, the solution indicated here by Vogel's approximation is an optimal solution. (If some cells show a negative net value, the one with the most negative value should be selected for reallocation. The reallocation quantity will be decided according to the closed loop that was used for the evaluation of that cell.)

In this example, the optimal solution for matrix (a) implies that Lab 1 will inspect the 90 pieces of equipment in Complex A, Lab 2 will inspect the other 10 pieces in Complex A, and 160 pieces in Complex C; the new lab at X1 will inspect all 130 pieces of equipment in Complex B, and 50 pieces in Complex C. Total inspection time is $90 \cdot 4 + 10 \cdot 7 + 160 \cdot 6 + 130 \cdot 2 + 50 \cdot 5 = 1900$ min.

The optimal solution for matrix (b) is: Lab 1 inspects 90 pieces of equipment in Complex A, Lab 2 inspects the other 10 pieces in Complex A and 130 pieces in Complex B, and 30 pieces in Complex C; the new lab at X2 will inspect 180 pieces in Complex C. Total inspection time = 1800 min; therefore, location X2 is the preferred one.

OTHER METHODS FOR LOCATION PROBLEMS

Because location problems can involve a variety of considerations and measurement variables, a variety of methods can be used for solving them. One approach that is particularly applicable when important measures are intangible is the assignment method. Careful ranking of each location alternative, including scores for positive and negative effects, will yield a set of effectiveness coefficients. An optimal assignment will then match available locations to particular units.

A mixed-integer programming formulation can be used to solve a multi-facility problem with fixed and variable costs (Shore 1975). A mixed-integer programming formulation is similar to linear programming except that some of the variables can assume only integer values.

A problem requiring this type of programming would involve how many additional units are required as well as where to locate them. In the context of the last case study, the questions might be: How many new labs are required? Where should they be located: On one hand, more laboratories would provide closer service and shorten total inspection time. On the other hand, the overhead cost would be much higher for setting and running additional labs.

Queueing models have also been applied to some location problems. These models capture the probabilistic nature of operations and attempt to measure phenomena rather than optimize solutions. Typically, the question of locating certain services (e.g., emergency services) involves an attempt to set the response time to any level below an upper limit. Queueing models would provide information on the probability that people in need will wait beyond that acceptance limit. Chaiken and Larson (1972), for instance, describe queueing models as useful for analyzing and determining the number and location of emergency ambulance services.

Simulation methods have been applied to study more complex questions of location and allocation. In one such study, E. S. Savas, (undated) analyzed the emergency ambulance services at New York City in an attempt to determine whether locating ambulances in satellite garages would improve service. Generally, the criterion used was the response time, which is defined as the period between the receipt of a call at the ambulance station and the arrival of the ambulance at the scene.

SUMMARY

The two most typical location problems involve (1) selection of a location for a new unit, e.g., a safety device or a hazardous station; (2) selection of a location for one or several additional units, where similar units already exist, e.g., locating a new emergency service unit or arranging a facility layout. In the first type of problem a comparison between alternatives, such as a breakeven analysis, is usually applicable, whereas the second type is more complicated. Because adding another unit may change the operation of the existing units, the transportation method may be required. In more complicated cases, the simulation method is usually applied, as described in Lesson 5.

QUESTIONS

Q1. Classify the following problems to those that refer to single site versus multi-facility:
  a. Locating fire doors
  b. Placement of exits in a building
  c. Locating noise producers
  d. Placing a guard on a machine
  e. Adding two emergency water sprinklers to wash eyes

Q2. Could we use the transportation method for the case study if we had to consider, in addition to time coefficients, the fact that the people in the new lab will be less experienced (at least in the beginning)?

Q3. Discuss the advantages and disadvantages of a large central emergency service facility as opposed to those of dispersed, decentralized facilities. Do the same for safety inspection centers.

EXERCISES

E1. As indicated in the lesson, an important type of location problem is the design of systems layout. Safety considerations in layout problems are discussed in the referenced article by Tompkins (1976). Read this article, and answer the following questions:

a. Describe the common objectives of safety and of facility design.
b. Explain how departmental hazards are evaluated. Devise a different method for quantitatively evaluating departmental hazards.
c. An important safety consideration in layout is the proximity of various departments. Is it handled in the article? Explain how it can be handled.
d. Explain in detail how the COSFAD method is applied.

E2. Solve the original case study problem of selecting a location for a new inspection center. This time suppose that *three* potential locations for the new lab are identified. The third location, x_3, has the following time coefficients: 10 for each of Complex A's units; 5 for each of Complex B's units, and 2 for each of Complex C's units. Solve the problem and recommend the one preferred location.

E3. Solve the original case study problem of selecting a location for a new inspection center, but assume that Complex A has 180 units, Complex B has 170 units, and Complex C has 90 units. Find the optimal location. Is it different from the original solution? Explain why.

REFERENCES

Chaiken, J. M., and R. C. Larson. 1972. Methods and Allocating Urban Emergency Units: A Survey. *Management Science*, Vol. 19, No. 4, December 1972.

Hillier, F. S., and G. J. Lieberman. 1974. *Introduction to Operations Research*. 2nd ed. Holden-Day.

Savas, E. S. Undated. Simulation and Cost Effectiveness Analysis of New York's Emergency Ambulance Service. Management Science, Vol. 15, pp. 608-627.

Shore, B. 1975. Operations Management, McGraw Hill.

Tompkins, J. A. 1976. Safety and Facilities Design. *IE Journal*, January 1976, pp. 38-42.

5. SIMULATION

LESSON OBJECTIVES

1. Review basic concepts and terms of simulation.
2. Describe certain applications of simulation to safety engineering problems.
3. Discuss advantages and limitations of simulation.
4. Identify when the simulation method is preferred over other analytical methods.

5. SIMULATION

INTRODUCTION

Simulation is duplication of the important components of a system without having to operate or even build the system itself. The use of a simulation model provides a means of observing the performance of a complex system or operation based on conditions defined by the analyst. Simulation is preferred to the operation of a system for several reasons. First, when a new system or a new arrangement is planned, several alternatives must be tried, evaluated, and compared. Building each of the alternatives could be costly and time consuming. The simulation process itself also saves considerable time because an imitated long time can be compressed into a few days or even seconds on a computer. A most significant advantage of using simulation is that the analysis is performed in abstract; therefore, any accidents or disasters indicated by the simulation analysis are harmless.

The two major types of simulation are physical simulation, in which an actual model or prototype of the system is constructed, and computer simulation. A good example of physical simulation is described by Rubinsky and Smith (1971) who used models of machine tools to experiment and find the proper design features that would prevent accidents. Our emphasis will be on computer simulation, specifically event simulation, which can be used to analyze events occurring in complex systems.

Simulation is very useful when a problem involves complicated relationships that cannot be modeled directly by models such as those described in previous lessons. A simulation analyst can easily incorporate uncertainties and dynamic variations into the model. Furthermore, the simulation analyst usually can choose the level of detail of the model. While this ability lends much power to the analysis, it also introduces the danger of attempting to capture too many details, which can result in confusion and wasted efforts. Another disadvantage of this analytical method is that simulation models may be costly to construct and validate. An advantage of simulation is that it does not require advanced mathematical knowledge and therefore can be applied by non-mathematicians.

Simulation is particularly important in safety engineering because it can be used *to identify and diagnose* risks before accidents can occur. Because a system, a situation, or a process is analyzed in abstract, any potential accident also occurs in abstract, and consequently without injuries or damages.

Simulation is also useful as a design and planning technique to avoid safety-related problems. For example, a complex layout with safety consideration is usually designed with the aid of simulation. Typical planning of preventive maintenance to avoid dangerous equipment breakdowns would also be performed with the aid of simulation.

The Process of Simulation

The process of a simulation involves repeated observations of the behavior of the studied system. Experimenting with the simulation model, the analyst tries different states of the system and estimates measures of effectiveness, such as the number of occurrences of an event of interest. The simulation model describes the interrelationship between different system components, which usually follow a different probability distribution.

For example, consider a chemical process that infrequently discharges certain dangerous gases to a special treatment reservoir. When the gas has been treated, it can be released safely, but batches of only 200 gallons can be treated at a time. An accident may occur, however, if the accumulated quantity of untreated gas in the reservoir exceeds 1000 gallons at any one time. Two major factors influence the buildup of the gas in the reservoir: 1) the time between consecutive discharges, and 2) the temperature of the discharged gas, which affects the treatment time. It can be assumed that the quantity discharged during each occurrence is about 200 gallons. It can also be assumed that the time between discharges and the treatment process time are independent.

If the treatment process is fast enough, there is no danger of an accident, however, if the treatment process cannot cope with the frequency and temperature of discharge, either a larger, more costly reservoir is needed, or a better treatment process must be developed. Simulation of the system will be based on observations of accident occurrence and treatment process rates.

For characterization of the discharge behavior, a sample of 60 observations is taken in a laboratory and the time between discharges is recorded as shown below.

Time Between Discharges

Time, min.	No. of Occurrences	Frequency of Occurrences, %
1	5	8
2	8	14
3	20	33
4	11	18
5	16	27

By observing the treatment process time for the 60 observed discharges, we can summarize its behavior as follows:

Treatment Process Time

Time, min.	No. of Occurrences	Frequency of Occurrences, %
1	5	8
2	11	14
3	16	33
4	20	18
5	8	27

The discharge process due to the chemical process can be simulated by use of the statistical patterns that have been observed. In the simulation we have to generate discharge events, with in-between times distributed as shown in the first minitable. Each discharge will be followed by a simulated treatment process, which requires the time to be distributed as shown in the second minitable. During the simulation we will record the total quantity in the reservoir during each period, or every time there is a new discharge.

Random Numbers

Random numbers are numbers that have the same likelihood to be chosen as any other number and are used to sample from probability distribution. Their use in simulation guarantees that independent events occur without biases. So that they will represent a given distribution, numbers are preassigned to events according to the frequency of the event. The computer then generates a random number, using a mathematical formula. The simulated event is the one to which the random number had been preassigned. In our example, preassigned two-digit numbers are assigned as follows:

Time Between Discharges	Frequency	Assigned Numbers
1	8	00-07
2	14	08-21
3	33	22-54
4	18	55-72
5	27	73-99

Treatment Time	Frequency	Assigned Numbers
1	8	00-07
2	18	08-25
3	27	26-52
4	33	53-85
5	14	86-99

For example, if a time between discharges has to be sampled, a random number is generated. Say it is 39; 39 falls between 22 and 54, and the simulated time is determined as 3 min. Note that between 22 and 54 there are 33 possible numbers (including 22 and 54), which is the frequency of the value of 3 minutes.

A sequence of random numbers can be prepared (by use of random number tables) for both the time between discharges and the treatment process time, as shown below.

	Random Number	Corresponding Time to next Discharge	Discharge Clock Time
a.	72	4	0:04
b.	11	2	0:06
c.	62	4	0:10
d.	07	1	0:11
e.	65	4	0:15
f.	86	5	0:20
g.	13	2	0:22
h.	99	5	0:27

	Random Number	Corresponding Time of Treatment Process
a.	08	2
b.	75	4
c.	92	5
d.	81	4
e.	97	5
f.	82	4
g.	99	5
h.	45	3

Simulation Example

The complete system, Combining discharge times, treatment times, and the resulting gas buildup in the reservoir are shown in Table 2 and Figure 8.

In the example in Table 2, only eight entries are considered. In practice, the number of observations and the number of times the whole simulation process has to be repeated must be determined by thorough statistical analyses. In our simple example, the maximun quantity of untreated gas reached only 600 gallons; therefore no danger is indicated.

During a simulation, information can be gathered about various performance measures. For instance, means and variance maximum and minimum values can be computed for a variety of measures of effectiveness. In our example, the waiting time before treatment can begin could also be observed in case it had any useful meaning.

Although a simulation is somewhat of an experimental measurement technique, it can be applied as an aid for optimization. By varying the values of decision variables and then estimating the system performance that results, an analyst can identify trends and influences. Through repeated experimentation, a near optimal solution can be achieved.

CASE STUDY

Hartman, Rubinsky, and Smith (1971) report an interesting use of simulation in the study of noise exposure. The investigators applied the simulation model to predict noise exposure hazards that would exceed OSHA limits before em-

Table 2. Simulation example of the complete process system.

	Clock time	Can start treatment	Treatment time	Release time	Accumulated untreated quantity
a.	0:04	0:04	2	0:06	400
b.	0:06	0:06	4	0:10	200
c.	0:10	0:10	5	0:15	200
d.	0:11	0:15	4	0:19	400 (from 0:11)
e.	0:15	0:19	5	0:24	400
f.	0:20	0:24	4	0:28	400
g.	0:22	0:28	5	0:33	600
h.	0:27	0:33	3	0:36	600

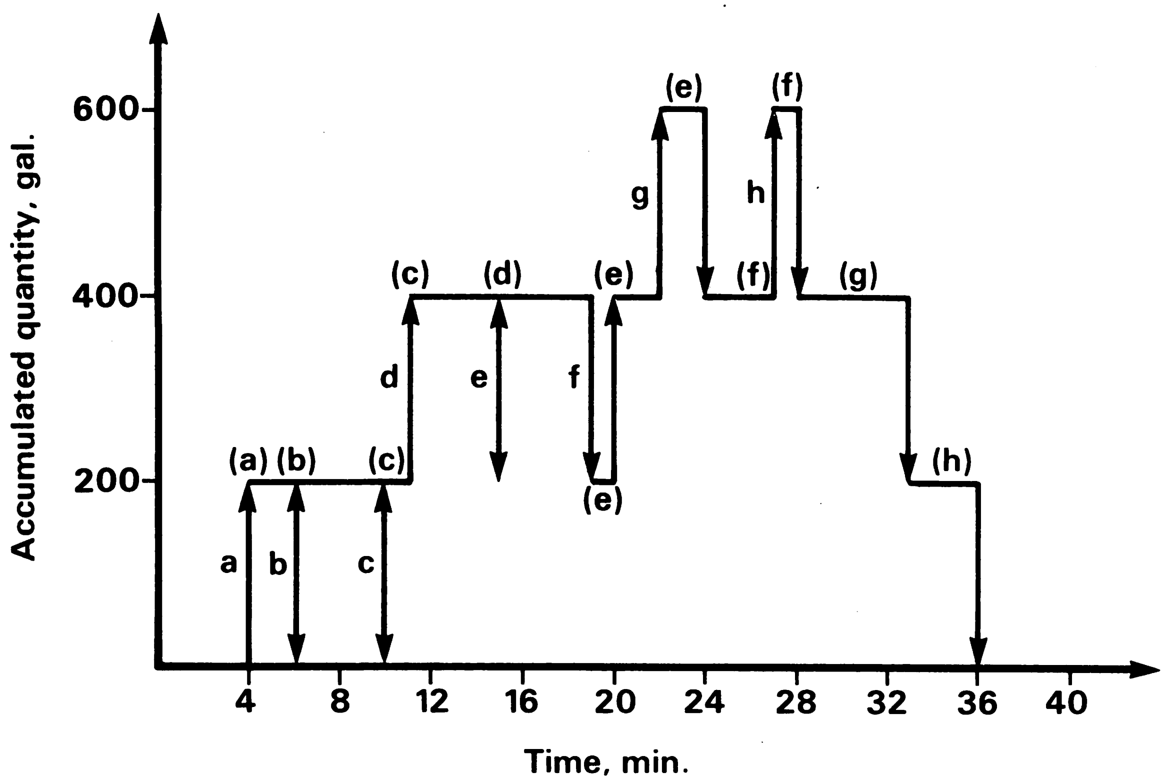


Figure 8. This is a graphic description of the simulation example: new discharge; end of one treatment and release.

ployees were subject to the hazard. A simplified version of this case is described here.

A practical problem of noise exposure arises when an employee is not restricted to one given area. Whereas in any given area noise levels can be measured by instruments such as a dosimeter and effective steps can be taken to comply with OSHA regulations, the situation is different when an employee has to move from station to station. An expeditor or a supervisor, for example, must take various routes through a factory, and is thus exposed to variable levels of noise. The cumulative daily noise exposure may be computed according to OSHA's calculation of the combined effect as:

$$D = \sum_{i=1}^{n} (Ci/Ti)$$

where

D = Cumulative damage
Ci = Time of exposure at noise level i (hours)
Ti = Permitted exposure time at noise level i (hours)
n = Total number of noise levels

If the employee's route and schedule vary from day to day, however, it is cumbersome to measure and compute daily exposure. Furthermore, a simulation of different routes and schedules can establish a priori unacceptable levels of cumulative noise exposure and hence the need to change the tour plan.

NOISE EXPOSURE SIMULATION MODEL

A graph published by OSHA specifies the permissible exposure time at different noise levels (in dBA) during an eight-hour day. For example, employee should not be exposed to a noise level of 100 dBA for more than two hours.

Suppose a factory has 20 departments with various sources of noise. Each source can be measured and the department can be classified according to its noise level, say from a level of 1 to 9. The two major factors affecting noise exposure are the departments that an employee has to visit, and the duration of those visits. A simulation model can sample values for both departments and visit duration, and the cumulative noise exposure can be calculated according to the summation formula given.

In the study mentioned earlier (Hartman, et al. 1976), daily noise exposures were simulated once with and once without hearing protection. Hearing protection was assumed to reduce local noise exposure by 10 dBA. In the case without hearing protection, the mean cumulative daily exposure was about 4 times greater than the maximum allowable. In the case with hearing protection, the mean cumulative daily exposure decreased to about 1.33 times greater than the daily maximum. The study also established the particular noise sources that caused the greatest exposure hazard. These results indicate that some daily tours in the studied factory may have to be replanned to comply with OSHA regulations.

SUMMARY

Simulation is a powerful tool for the analysis of complex systems or processes because it permits the gathering of information through observation of the simulated behavior of these systems. Simulation is particularly relevant for safety engineering because it enables the analyst to study a system in various experimental states without actually risking anything in anyone. Therefore, simulation is suitable for identification of hazards and hazardous circumstances and for design and planning of new, potentially dangerous systems or procedures.

QUESTIONS

Q1. Describe three types of safety engineer applications in which simulation is useful. Give one specific example for each type.

Q2. Summarize the advantages and disadvantages of simulation as a method, referring specifically to safety engineering aspects.

Q3. Could the case study of noise exposure be analyzed qualitatively? Explain.

Q4. Identify in the example of the gas discharge problem:
- a. Decision variables
- b. Parameters
- c. Objective
- d. Model
- e. Controlled variables

EXERCISES

E1. Continue the simulation of the gas discharge problem until you have reached 60 minutes of simulated time. Estimate:
- a. Maximum quantity of untreated gas in the reservoir
- b. Mean quantity of untreated gas in the reservoir
- c. Maximum waiting time for treatment process start
- d. Mean waiting time for treatment process start

E2. Simulate the cumulative noise exposure, as described in the case study, for five days with the following additional hypothetical data:

Dept.	Noise Level	Permissible Exposure, h	Visit Frequency, %
1	2	8	15
2	4	4	20
3	4	4	10
4	6	2	20
5	8	1	15
6	8	1	20

Visit Duration, h	Frequency, %
1	40
2	30
3	20
4	10

Develop the table for the complete simulation, and then estimate:
- a. Maximum daily noise exposure
- b. Mean daily noise exposure
- c. Standard deviation of daily noise exposure

E3. Using the data of E2, evaluate whether the following daily tour is acceptable:

Department	Visit Duration, h
3	½
2	1½
1	1½
6	2
5	2
3	½

6. ANSWER GUIDE TO QUESTIONS AND EXERCISES

6. ANSWER GUIDE TO QUESTIONS AND EXERCISES

Lesson 1

Q1. For example, consider the problem of planning emergency evacuation routes from a large building. An analytic technique can be used as explained later in lesson 3. The objective: to plan the routes that result in the fastest evacuation of all employees from the building. The constraints: the given building structure and layout. The possible alternatives: all combinations of assigning routes to individuals.

Q2. Consider problems such as insufficient training in safety procedures, or poor selection of employees to hazardous operations. In each case, quantitative measures can be applied to improve planning.

Q3. All safety problems should be regarded as important; "small" may mean that they are small-scale or that they involve a small number of people.

Analytical methods can be applied to small problems too; e. g., the safest layout of a small department. The main issues are safety engineering resources and training safety engineers in the application of analytical methods.

E1. a. Experience, testing, conjecture, analysis
  b. Intuition, induction, deduction
  c. Qualitative analysis

E2. a. To determine the relative seriousness of hazards; to measure the justification for recommended corrective actions
  b.1) By quantifying accident consequences, exposure factors, and probability. A rating table is provided.
  b.2) By quantifying the cost factor and the degree of correction. A rating table is provided.
  c. Consider the extremes. For instance:

  $R=C \times E \times P=5 \times 10 \times 6=300$
  $R(+10\%)=5.5 \times 11 \times 6.6=399$ (not 330!)
  $R(-10\%)=4.5 \times 9 \times 5.4=219$ (not 270!)

  d.1) Several objectives can be defined, such as to minimize damage costs, or to minimize probability of accidents. (Note that to "eliminate hazard" is a wish rather than an objective here.) A well-defined objective is to minimize the risk of damage to the storage tank.
  d.2) The model is the risk formula on one hand and the justification formula on the other hand.
  d.3) The values of C, E, P, CF, and DC.
  d.4) Computation of J by the risk and justification formulas for alternative protection and location strategies of the tank.
  d.5) The strategy that achieves the highest value of J.

E3. a. Both procedures assess safety performance by evaluating unsafe acts and unsafe conditions through eight steps. In the active procedure, employees' participation and cooperation are required; whereas in the passive procedure, only past data are being used.
  b. Most of the analyses are empirical and collect statistics on past unsafe acts, unsafe conditions, and actual accidents. Some are qualitative, e.g., evaluation of programs; and some are quantitative, e.g., calculation of scores and frequencies.
  c. Advantages: a systematic, partly quantitative approach involves companywide participation; attempts to predict

and prevent. Disadvantages: subjective evaluations by employees, possibly untrained or wrongly motivated; based on empirical data, which are very sensitive to the definition of what is unsafe; may be difficult to maintain on an ongoing basis.

LESSON 2

Q1. The assumption of linearity is necessary for application of LP; however, it is quite possible that accident and operating costs in the objective function are not linear with the amount of safety resources that are allocated. It is also possible that some of the constraints are not linear.

Q2. Not in practical situations.

Q3. Yes, for instance, by an empirical or qualitative approach; however, it is difficult to decide and justify in the absence of objective, analytical measures.

E1 a. $x_1^* = 2$ $x_2^* = 6$ $x_3^* = 8$

b. $x_1^* = 3.6$ $x_2^* = -2.4$ $x_3^* = 7$

E2. Rating now is from 0 (poor) to 10 (very good), and the objective is to maximize total rating; $z = ax_1 + bx_2 + cx_3$, where a, b, c are coefficients of contribution to score rather than cost of activities.

E3 x_i = No. sent each period, i = 1, 2, 3

$x_1 + x_2 + x_3 \leq 60$

$x_i \leq 30$

$x_i \geq 6$

$x_1 = 15$

$x_1 + x_2 \leq 50$; $x_2 + x_3 \leq 50$

so $x_2 \leq 35$; $x_3 \leq 35$

$$\text{Min. } Z = 200x_1 + 150x_2 + 100x_3 + 200x_1 + 300x_2 + 350x_3$$

The problem reduces to:

$$\text{Min. } Z = 450x_2 + 450x_3$$

S.T. $x_2 + x_3 \leq 45$

$6 \leq x_2 \leq 30$

$6 \leq x_3 \leq 30$

To minimize the cost, the optimal solution is:

$$x_1^* = 15, x_2^* = 6, x_3^* = 6.$$

To send all 60 people:

$x_2 + x_3 = 45$

and the optimal solution is:

$x_1^* = 15$, and any combination that yields $x_2 + x_3 = 45$ (e.g., $x_2 = 30$, $x_3 = 15$; $x_2 = 29$, $x_3 = 16$, etc.)

LESSON 3

Q1. Not successfully. An analytic method would further improve the solution.

Q2. Yes to both

Q3. a. Shipment of safety instruments

b. (1) To minimize the total shipment cost

(2) Original sources of the instruments

(3) Final destinations of the instruments

(4) Points at which the instruments are needed, but from which they will be shipped on.

(5) Cost or benefits per instrument along the shipping routes.

c. To decide if the optimal shipment plan could be further improved by a different strategy; if the optimal is within the budget; etc..

E1. First reduction-subtract the most negative from each row

0	6	2	3	5	(-14)
1	4	0	0	0	(-13)
4	3	0	0	1	(-12)
0	0	2	1	3	(-13)
2	3	1	2	0	(-13)

An optimal (max.) solution is 1-A, 2-C, 3-D, 4-B, 5-E

E2. In the example distance data was the basis to compute the time to move from floor to floor. One example of an emergency service could be evacuation directly from the roof. In this example another sink may be defined, say No. 6, and the potential number of evacuees there can be specified. A time coefficient to move from Floor 5 to the roof also will be needed. Now the model will include three sinks.

E3. The solution procedure will be:

a. Solve the evacuation problem after adding a new source (Floor 6) with +70. Try once with all of them going to Floor 1 (i.e., a sink with −190).

b. Now solve with all 70 new people evacuating through Sink 4 (now with a demand of −160). This strategy will require investment in added evacuation capacity at Floor 4.

c. Solve with all 70 leaving through Floor 5 (or through Floor 6, or the roof); estimate the added cost of evacuation capacity there.

Now you have solutions for three strategies, each with the optimal evacuation time, and the cost estimate of new evacuation capacity. In the decision consideration must be given to one of the above strategies, or a combination provided the evacuation time is reasonable and the costs are within the budget.

E2.

	A	B	C	
Lab 1	90 [4]	[6]	[12]	90
Lab 2	10 [7]	130 [5]	30 [6]	170
x_3	[10]	[5]	180 [2]	180
	100	130	210	

$Z_3^* = 1620$ min

The optimal solution is shown above (it is obtained in the first Vogel's iteration). $Z_3^* = 90 \cdot 4 + 10 \cdot 7 + 130 \cdot 5 + 30 \cdot 6 + 180 \cdot 2 =$ 1620 min. This is the preferred location: less than $Z_1^* = 1900$, and $Z_2^* = 1800$ min.

LESSON 4

Q1. a. Multi
b. Multi
c. Multi
d. Single
e. Multi

Q2. Yes, by adjusting coefficients.

Q3. Advantages of centralized facilities: good control, less overhead cost, ability to train specialized people. Main disadvantage: no quick response to remote locations. Advantages of decentralized facilities: on location; can specialize for local requirements. Main disadvantage: costly.

E1. a. Minimizing material handling; minimizing backtracking; good housekeeping; ease of maintenance.

b. Departmental hazards can be evaluated by applying Fine's method, which quantifies risks, consequences, exposure, and accident likelihood of particular materials handling and other facilities. Another method could be to develop departmental hazards based on past accident statistics in similar facilities.

c. The proximity issue is handled by specifying from-to charts for each piece of materials handling equipment.

E3.

	A	B	C	
Lab 1	90 [4]	[6]	[12]	90
Lab 2	90 [7]	[5]	80 [6]	170
x_1	[8]	170 [2]	10 [5]	180
	180	170	90	

$Z_2^* = 1860$ min

	A	B	C	
Lab 1	90 [4]	[6]	[12]	90
Lab 2	90 [7]	80 [5]	[6]	170
x_2	[13]	90 [7]	90 [3]	180
	180	170	90	

$Z_1^* = 2290$ min

Now location x_1 is preferred. $Z^* = 1860$ is larger than the original Z_2^* because the requirements have changed.

LESSON 5

Q1 and Q2. (Material is detailed in the chapter)

Q3. Yes, but with less accuracy and therefore open to more errors and subjective judgement.

Q4. a Size of reservoir;
b. Treatment duration;
c. To never overflow the reservoir;
d. Simulation of events: discharge, treatment, and release;
e. Size of reservoir.

EXERCISES

E1. a. 800
b. Mean computed over the period 0 through 60 min:

$$\frac{1}{60}[0 \cdot (4+4)+200\,(7+6)$$
$$+400\,(8+2+3+2+1+2+1+2)$$
$$+600\,(2+1+3+2+1+1+2+2+2)$$
$$+800\,(1+3+1)]$$
$$=\frac{21000}{60}=350 \text{ Gal.}$$

c. 8 min for ℓ, 8
d. Mean waiting time:

$$\frac{1}{17}(0+0+0+4+4+4+6+6+$$
$$6+6+4+\frac{\ell}{8}+\frac{m}{8}+6+4+0+0)$$
$$=\frac{66}{17}=3.9 \text{ min.}$$

TABLE 3

	Clock time	Can start treatment	Treatment time	Release time	Acc. untreated quantity
h	0:27	0:33	3	0:36	600
i	0:30	0:36	4	0:40	600
j	0:34	0:40	4	0:44	600
k	0:38	0:44	3	0:47	600
l	0:39	0:47	2	0:49	800
m	0:41	0:49	3	0:52	800
n	0:46	0:52	2	0:54	800
o	0:50	0:54	1	0:55	600
p	0:55	0:55	1	0:56	200
q	0:60	0:60	3	0:63	200
r	0:64	0:64	4	0:68	200

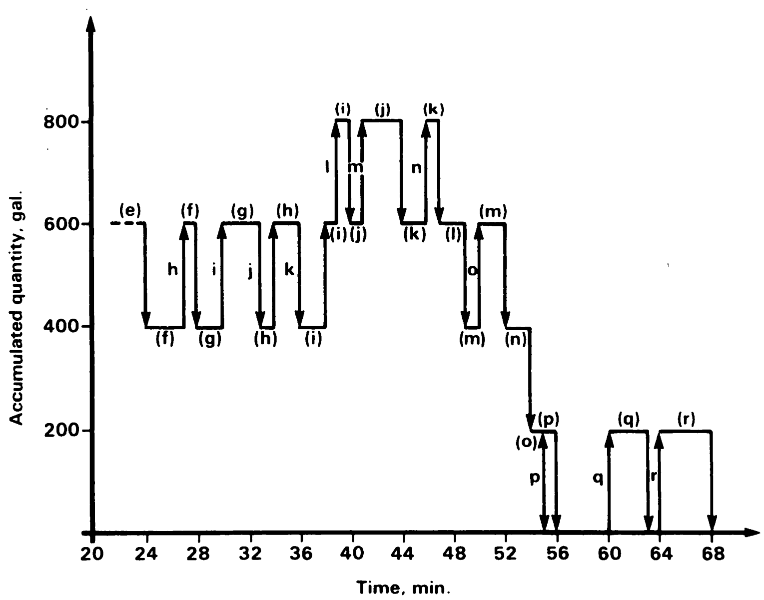


Figure 9. Chart for Exercise E1.

E2. Randomly Generated data

Day	Tour (day/hours)
1	2/3, 3/1, 6/2, 5/2
2	3/2, 4/3, 1/1, 5/2
3	4/2, 6/2, 5/2, 1/2
4	1/3, 2/3, 6/2
5	5/3, 6/1, 2/4

Day	Cumulative Damage (D), dBA
1	$\frac{3}{4} + \frac{1}{4} + \frac{2}{1} + \frac{2}{1} = 5.0$
2	4.1
3	5.3
4	3.1
5	5.0

a. Maximum daily exposure is 5.3 dBA, maximum overexposure occurs on day 5 in Dept. 5, 300% of permissible amount.

b. 4.5 dBA

c. 0.9 dBA

E3. Visits in Departments 5 and 6 are twice as long as permissible, and the tour must be replanned.

GLOSSARY

Following are definitions of terms commonly used in analytical methods.

Analytic Models: Mathematical equations that represent the relevant components of a situation or a system by variables that are related in such a way that there is a mathematical procedure to use the model for an optimal solution (in the case of optimization models), to compute values of the variables, or to calculate the outputs that will result from given inputs to the model.

Constraints: Limitations on the range of activities that a decision maker can implement. Examples are budget, legal, resource, or capacity constraints.

Inputs: Different types of resources that are introduced into a system and are transformed into the system's products. Inputs have to be constantly controlled to avoid undesirable results.

Linear: Of a straight line form, e. g., $Y = Ax + B$. A linear function is one in which a constant amount of change in x (an independent variable) will produce a constant amount of change in Y (the dependent variable). In graphic form, a linear function is represented by a straight line, whereas a nonlinear function is represented by a curved line.

Model: A representation of reality that is meant to explain the behavior of certain aspects of that reality. Qualitative models use descriptive tools such as figures, verbal statements, or physical models; quantitative models use numbers, equations, and mathematical relationships.

Operations: Actions involving technological and information transformations, which are performed to accomplish certain desired effects.

Optimal: The absolute best decision or solution that can be obtained towards a given objective and within given limitations.

Outputs: Products of a system that are obtained by the transformation of inputs. When a system gets out of control undesirable outputs, such as accidents, may result.

Parameters: Constants that explain the behavior of a system. The behavior of a system can be controlled by changing the values of these constants.

Prediction: A process that attempts to specify future values or future events. Predictive models use statistical methods to predict the future based on past experience.

Strategy: A set of activities that comprise a plan of action a decision maker can follow to achieve certain goals.

Variables: Factors of a system that can assume different values at different times. The variables in a model represent relevant characteristics of the modeled system.