OPTIMIZATION OF FREELY SUSPENDED EXTERIOR HOODS IN INDUSTRIAL VENTILATION

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INTRODUCTION

In the design of an exterior hood, the value of the airflow rate through the hood can be found if specification of the "reach" of the hood is given. By the reach of the hood we mean a set of air speeds induced by the hood, to be achieved or exceeded at specified locations in front of the hood. If the velocity profile generated by the hood air flow is known, the problem is simply matching the velocity profile with the specifications to obtain the flow rate which will achieve the correct air speeds. There are a number of expressions which give velocity profiles with about equivalent accuracy.[1-4] Thus, any one of these expressions can be used to design a hood. Clearly, if the air speed specifications are given correctly, then the capture efficiency of the hood is expected to be optimal. It must be noted that the optimum for the capture efficiency can be made independent of cross currents, because the effect of cross currents can be included into the specification of the air speed. This seemingly direct and simple method of computation, which determines the minimum flow rate to accomplish the desired result, is flawed with respect to the mechanical efficiency of the hood. This flaw is due to the a priori selection of the hood geometry and orifice size without a quantitative investigation of the possibilities of achieving the same end result with a hood of different geometry and/or orifice size. Although the experience of the designer may be invoked as an influencing factor in the design, even for an experienced designer it is unlikely that the consequences of such alternatives have ever been a consideration.

In order to simplify the theoretical development, it will be assumed that a specific value of air speed on all points of a regular geometric shape defined on a plane located in front of a hood is given as the design criterion for the hood. It is important to note that the restriction of specification surface to a plane rather than a curved surface will not give a general solution. Therefore, it may be considered to be a limitation of the theoretical development. However, such a specification would be sufficiently common in the industrial applications and more importantly, the methodological approach can be presented without undue complexity of the mathematical formulation so that the results would be useful to a ventilation system designer.

THEORETICAL CONSIDERATIONS

In the investigation of the implications of hood orifice geometry and in the selection of proper size of the orifice, the development of the theory is facilitated if the specification geometry is chosen in a way that the distances measured from the point on the hood is readily accomplished. This will suggest that the shape of the specification surface is symmetric with respect to both of the axes of the plane. An oblong or a circle would satisfy this criterion. Since a square has four extremal points, then the structuring of the optimization problem can be reduced to matching the air speed generated by the hood to the specified air speed at the extrema. This process would be sufficiently general, in the sense that the specification can be in terms of a component of a vector.

Suppose it is necessary to generate air speed of V_C at the surface of an oblong located on a plane parallel to the hood surface and centered on the x-axis with its sides parallel to the xz and yz planes. Furthermore, suppose that it is necessary to keep the hood face velocity equal to or below a specified value V_o. Let A and B be the maxima of the y and z coordinates respectively. For an oblong hood, with sides a fraction c of A and B placed with its center at the origin (Figure I) minimization of the flow rate Q might be sought by the object function:

$$Q = L^2 f(a,b,h)\, V_C \tag{I}$$

Subject to:

$$V_C\, f(a,b,h)/\, 4abc^2 < V_o \tag{2}$$

where,

a = Dimensionless specification oblong side, A/L
b = Dimensionless specification oblong side, B/L
h = Dimensionless distance to the specification surface, H/L

For an oblong orifice, the function f(a,b,h) may be shown to be represented by the non-dimensionalized velocity scaling function (1) multiplied by the hood orifice area:

$$f(a,b,c,h) = \pi(a+b)cr + 2\pi r^2 + 4abc(c+h(a+b)/(a^2+b^2))^{1/2} \tag{3}$$

with,

$$r^2 = h^2 + (1-c)^2 \tag{4}$$

Equations 1 through 4 can be extended directly to a circular orifice by taking c.L to be the radius of the orifice. In such an extension, Equations 2 and 3 will have to be modified to conform to the description of the flow field in front of a circular orifice. The modified equation for a circular orifice hood

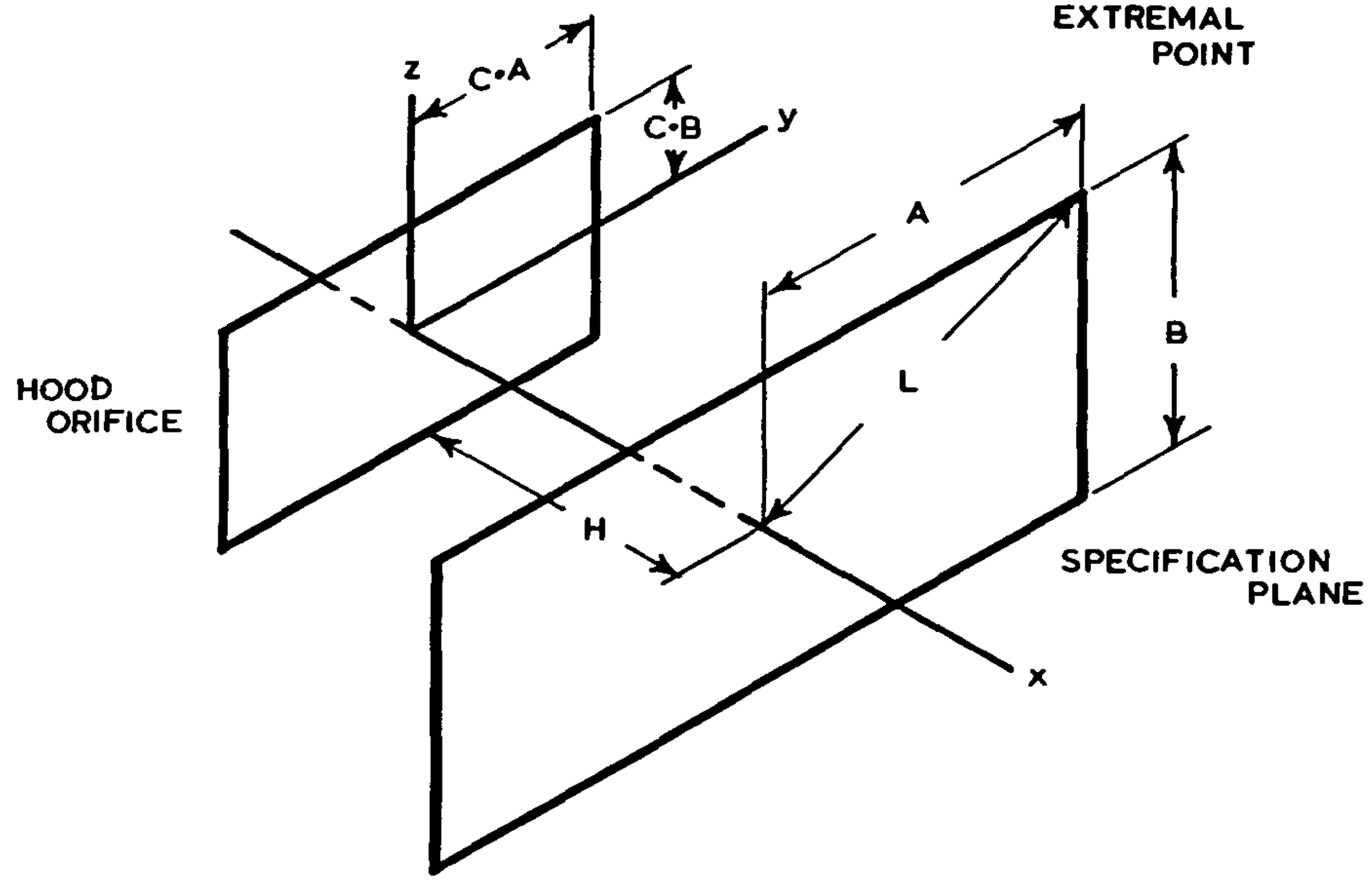

Figure 1. The parameters and the configuration used in the derivation of hood optimization.

may be shown to be 3:

$$f_C(a,b,c,h) = \pi^2(cr+r^2)/2+\pi c(c^2+Kh^2)^{1/2} \qquad (5)$$

$$K = 4/(\pi^2-2\pi)$$

and,

$$V_C\ f(a,b,h)/\pi(a^2+b^2)c^2 < V_o \qquad (6)$$

The minimum sought may be found directly by differentiating either equation 3 or equation 6 with respect to c and finding the root of the resulting equation which is between zero and one. For an oblong orifice, the non linear equation to be solved is:

$$\pi(a+b)\frac{h^2+1-3c+2c^2}{(h^2+1-2c+c^2)^{1/2}}+4\pi(c-1)+4abc\frac{2c^2+h(a+b)}{(c^2+h(a+b))^{1/2}}=0 \qquad (7)$$

and similarly for a circular orifice:

$$\frac{\pi}{2}\bullet\frac{h^2+1-3c+2c^2}{(h^2+1-2c+c^2)^{1/2}}+\pi(c-1)+\frac{2c^2+Kh}{(c^2+Kh)^{1/2}}=0 \qquad (8)$$

If the specification surface, instead of oblong, is a circular one then calculation process may be modified by taking two different values for square or circular hoods. It may be shown that for a square orifice hood the square hood A = B and L = A $\sqrt{2}$ and for a circular orifice hood L = A. With these altered specifications equations 1 through 8 apply to optimization without further change. (Table I)

In general, the process of calculation is straight forward and with the use of a computer presents no significant problems. However, in certain cases no root may exist in the zero to one interval. This suggests that the global optimum design does not exist for that condition. This situation will arise when the dimensionless frontal distance h is sufficiently large. Consequently, the local optimum which is defined by the maximum face velocity specified in equation 2 or 6 and the corresponding orifice size may be used.

For infrequent design problems where the use of a computer is not warranted or for those who do not have ready access to a computer, there are a number of simplifications, albeit limited, that reduce the calculations to simple use of tables. To develop these simplifications, consider the specification surface to be bounded by a square, i.e. the sides are such that A = B. Then the optimization can be carried out utilizing the values shown in Table II. The simplest use of this table may be illustrated by an example. Suppose the specification surface is located 10 cm from the hood plane with A = .0.25 m.

If the air speed desired on this surface is 1 m/sec and the maximum face velocity allowed is 25 m/sec then the optimum hood size for a square hood is calculated as follows:

$L = A\sqrt{2} = 0.35355$ thus $h = H/L = 0.283$. Therefore, interpolating the proper values from Table II, $f(h) = 3.6191$ and $c = 0.784$; consequently, the optimum value of the side of the square hood is 19.6 cm and from equation 1 the a volumetric flow rate is 0.45 m^3/sec. Similarly, for a circular hood, the optimum radius is 22.5 cm and the flow rate is 0.49 m^3/sec. Thus for this simple illustration, a square orifice hood would be an optimum choice.

If the example above is recalculated using a circular specification, the optimum square hood would be the same, but the optimum circular hood would have $L = 0.25$, $h = 0.40$ which results in a hood radius of 13.0 cm and flow rate of 0.31 m^3/sec. In this case a circular hood would be superior.

It is important to note that the theoretical results developed

Table I

Optimization Parameters for Squares and Circles

Dimensionless Distance	Circle C	Circle f(a,h)	Square C	Square f(a,h)
0.05	0.9351	2.2007	0.7734	3.1161
0.10	0.8926	2.4464	0.7344	3.2874
0.15	0.8594	2.7265	0.6953	3.4966
0.20	0.8291	3.0372	0.6641	3.7355
0.25	0.8018	3.3768	0.6211	3.9989
0.30	0.7754	3.7440	0.5859	4.2843
0.35	0.7598	4.1381	0.5508	4.5893
0.40	0.7266	4.5588	0.5156	4.9126
0.45	0.6992	5.0057	0.4805	5.2530
0.50	0.6758	5.4785	0.4414	5.6094
0.55	0.6562	5.9770	0.4062	5.9811
0.60	0.6328	6.5012	0.3672	6.3672
0.65	0.6104	7.0511	0.3203	6.7667
0.70	0.5869	7.6263	0.2634	7.1790
0.75	0.5635	8.2270	0.2266	7.6030
0.80	0.5400	8.8531	0.1719	8.0375
0.85	0.5166	9.5044	0.1094	8.4809
0.90	0.4932	10.1810	0.0312	8.9307
0.95	0.4688	10.8828	------	------
1.00	0.4434	11.6098	------	------
1.05	0.4209	12.3619	------	------
1.10	0.3965	13.1391	------	------
1.15	0.3721	13.9414	------	------
1.20	0.3467	14.7688	------	------
1.25	0.3203	15.6211	------	------
1.30	0.2949	16.4984	------	------
1.35	0.2695	17.4007	------	------
1.40	0.2441	18.3278	------	------
1.45	0.2148	19.2799	------	------
1.50	0.1914	20.2567	------	------

Circular orifices with maximum face velocity:

$$L^2 f_c(a,b,h) V_c - \pi(a^2 + b^2) c V_0 = 0$$

Oblong orifices with maximum face velocity:

$$L^2 f(a,b,h) V_c - 4abc^2 V_0 = 0$$

Table II
Comparison of Traditional and Optimized Designs
(Unit Control Speed)

Design	B vert. cm	A Hor. cm	H cm	Diameter or Height cm	Width cm	Flow m^3/sec
CASE I	15	15	15			
Traditional Square				30	-	0.354
Optimum Square				27	-	0.254
CASE II	20	30	25			
Traditional Oblong				40	60	1.12
Optimum Oblong				33	49	1.01
CASE III	10	20	30			
Traditional Oblong				20	40	0.902
Optimum Oblong				12	24	0.880
CASE IV	20	20	25			
Traditional Square				40	-	0.889
Optimum Square				25	-	0.830
CASE V	20	20	10			
Traditional Square				40	-	0.421
Optimum Square				19	-	0.358
Circle				15	-	0.359

above are not inherently limited to applications which involve specification surfaces assumed in the development. Obviously, if the specification surface is not nearly a square circular or square orifice hoods will be inherently inappropriate but must be replaced by oblong orifice hoods. Finding the roots of the derivative of the objective function may be carried out by hand but such a calculation would be cumbersome. Although the computerized solution is simple, when a computer is not available, the optimization of each dimension of the orifice may be carried out approximately, one at a time by treating each side as an independent imaginary square hood. Although the orifice dimensions determined in this manner may not predict the exact optimum design values, the resulting dimensions are expected to be near the optimum values. The flow rate for such an orifice cannot be calculated directly from Equations 1 to 4.

EXPERIMENTAL RESULTS

The direct experimental verification of the optimization procedure given above is at best cumbersome. Such an experiment would involve the construction of a very large number of hoods. However, an indirect experimental verification of the procedure may be accomplished by showing that a few representative hoods may be constructed and studied. In the experimental study carried out to verify the theoretical calculations indirectly, three oblong hoods were constructed. These hoods were 5 cm by 5 cm square, 3 cm by 5 cm oblong and 4 cm by 8 cm oblong. With hood opening fixed, conditions under which these hoods will be optimum were calculated for different values of frontal distance and for each condition, the optimum flow rate was predicted. The air speed was measured at each, the theoretically determined specification point and the flow rate was adjusted until the air speed specification is fulfilled. This experimentally determined flow rate was then compared to the theoretical flow rate. All air flow and air speed measurements were carried out by hot wire anemometry. The hot film sensor in X configuration was calibrated in our laboratory and it is capable of measuring velocities with good accuracy and reproducibility. The hood airflow measurement was carried out by measuring the air speed by a traverse as close to the orifice plane as possible.

The comparison of the calculated optimum and the measured flow rates are given in Figure 2. The results suggest that the optimization procedure is satisfactory and perhaps slightly pessimistic in the indication of the flow rate required. On the average, about 10 percent less flow was required than it was calculated as necessary.

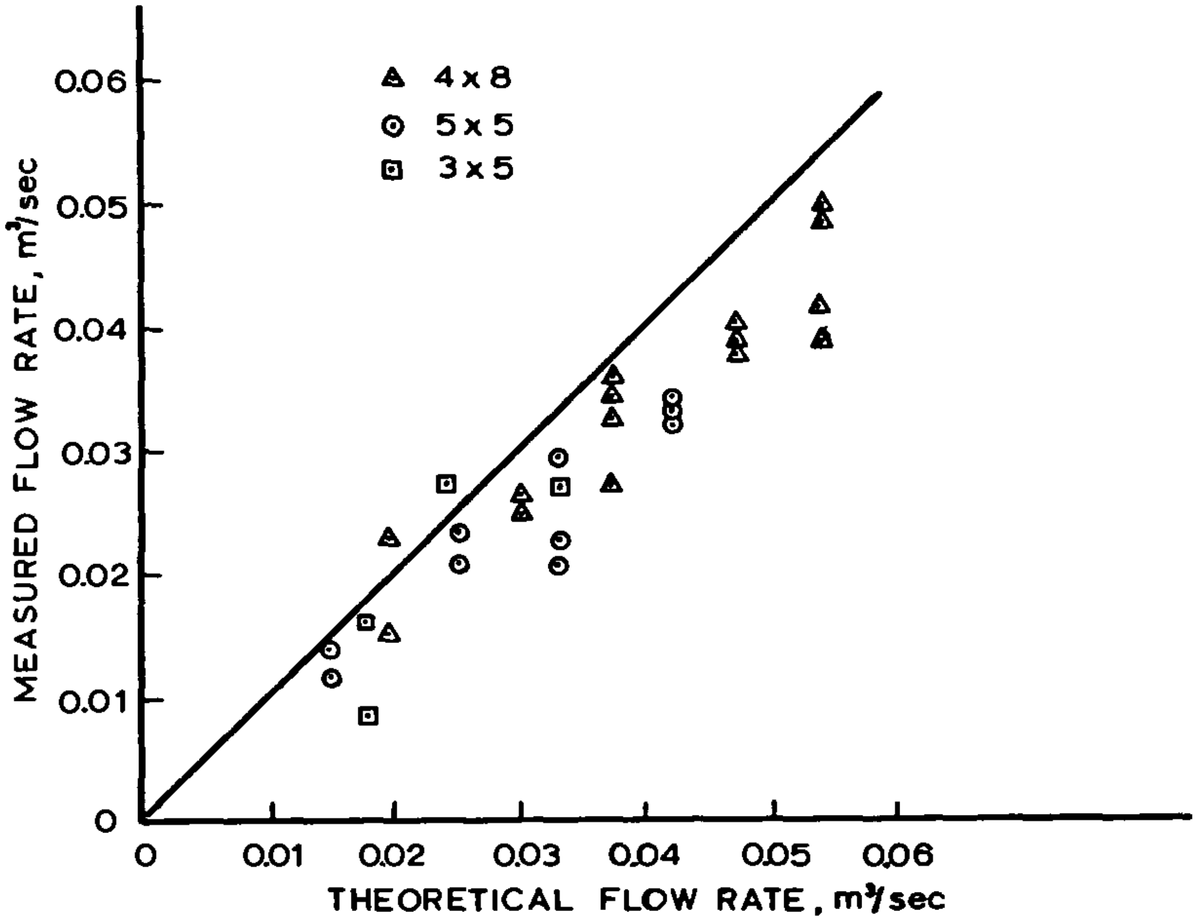

Figure 2. Comparison of theoretically and experimentally obtained flow rates for three hoods.

DESIGN APPLICATIONS AND DISCUSSION

The application of the results presented above to the design of freely suspended hoods with single square, circular or oblong orifices is a straight forward process but it must be recognized that the success of the hood design based on such calculations will ultimately depend upon the correct specification of the velocities to be generated at specific locations. The estimation of these velocities is beyond the scope of this paper and may be found in manuals dealing with currently accepted practice. If the specification surface is judged to be a curved surface rather than a plane or if the vector components of the velocity at specified points are sought, new objective functions following the theoretical development above can be found. Alternately, the hood size may be selected at an external point by considering that point to be one of the vertices of an oblong specification surface and the proper flow rate through the system can be calculated by point matching between the generated flow field and the required flow field.

In order to show the efficacy of the optimization procedure developed here, five hypothetical cases were compared to the traditional design procedure. The results of this comparison are shown in Table II. For the cases shown in Table II, the efficiency gain through optimization is about 13 percent with a range from 2 to 30 percent. These cases were not constructed with a forethought to show the effectiveness of the optimization procedure, but rather they were arbitrarily selected. Since the optimization process is based on the velocity profile in front of the hood, and the traditional design procedure which is based on the adjustment of the centerline velocity of the hood, then the hood designed by the optimization procedure ensures that the air speeds specified on the specification plane are satisfied. On the other hand such a statement would not necessarily be correct for the design based on centerline velocity. Consequently, the hoods designed through the process described above would always have a superior total efficiency as compared to the traditionally designed hoods.

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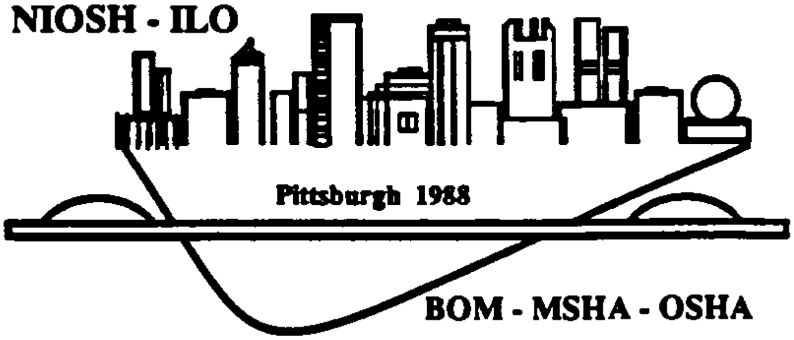

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