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Hadnot Point, Camp Lejeune, NC

Reconstruction of Historical Contaminant Concentrations: A Computational Method

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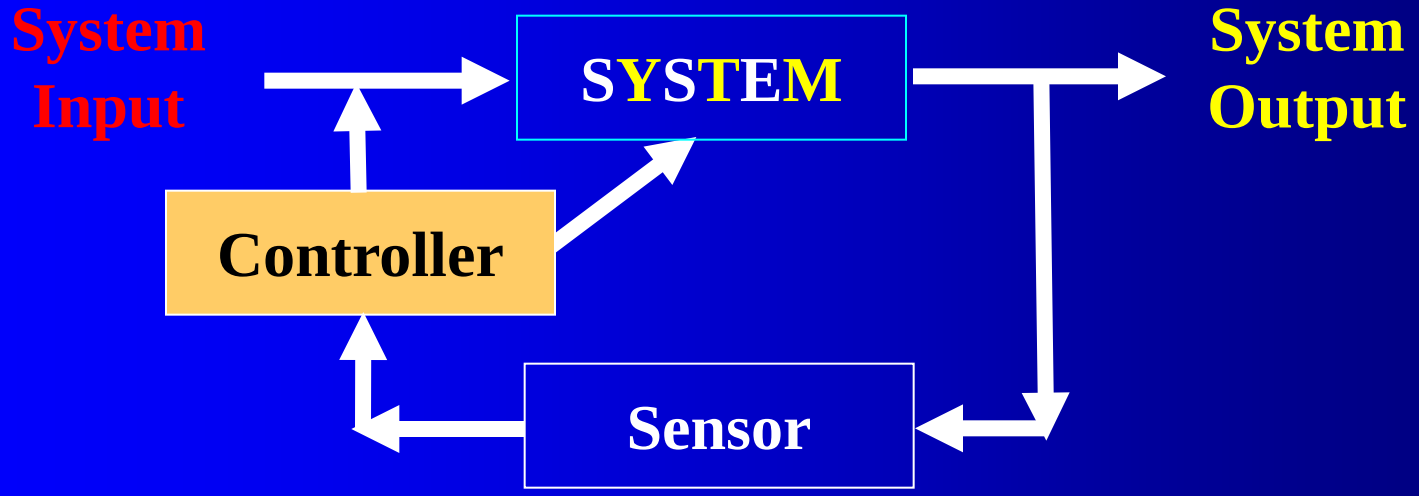
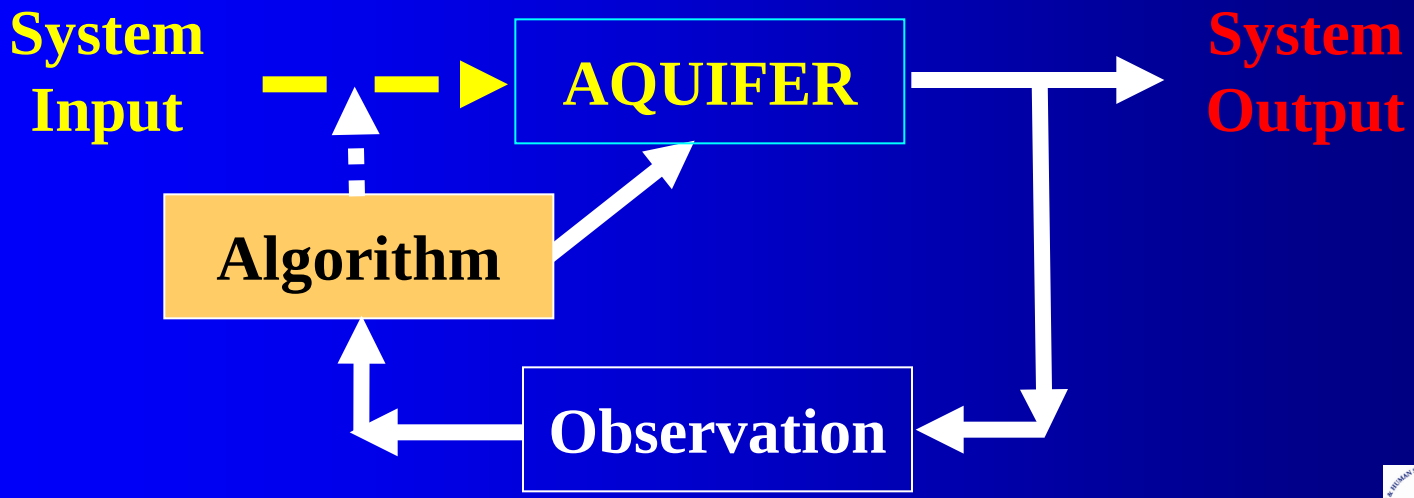
Control Theory Based Time Series Analysis





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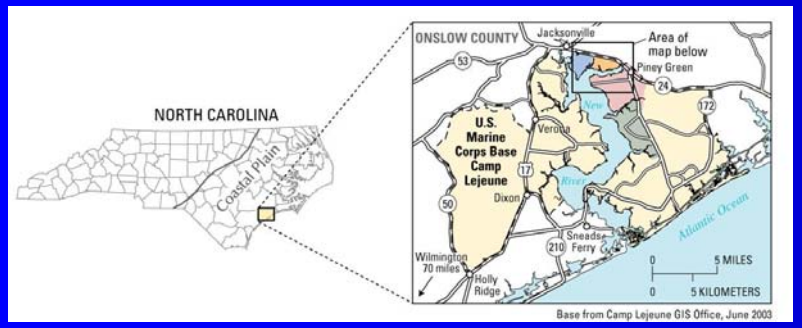
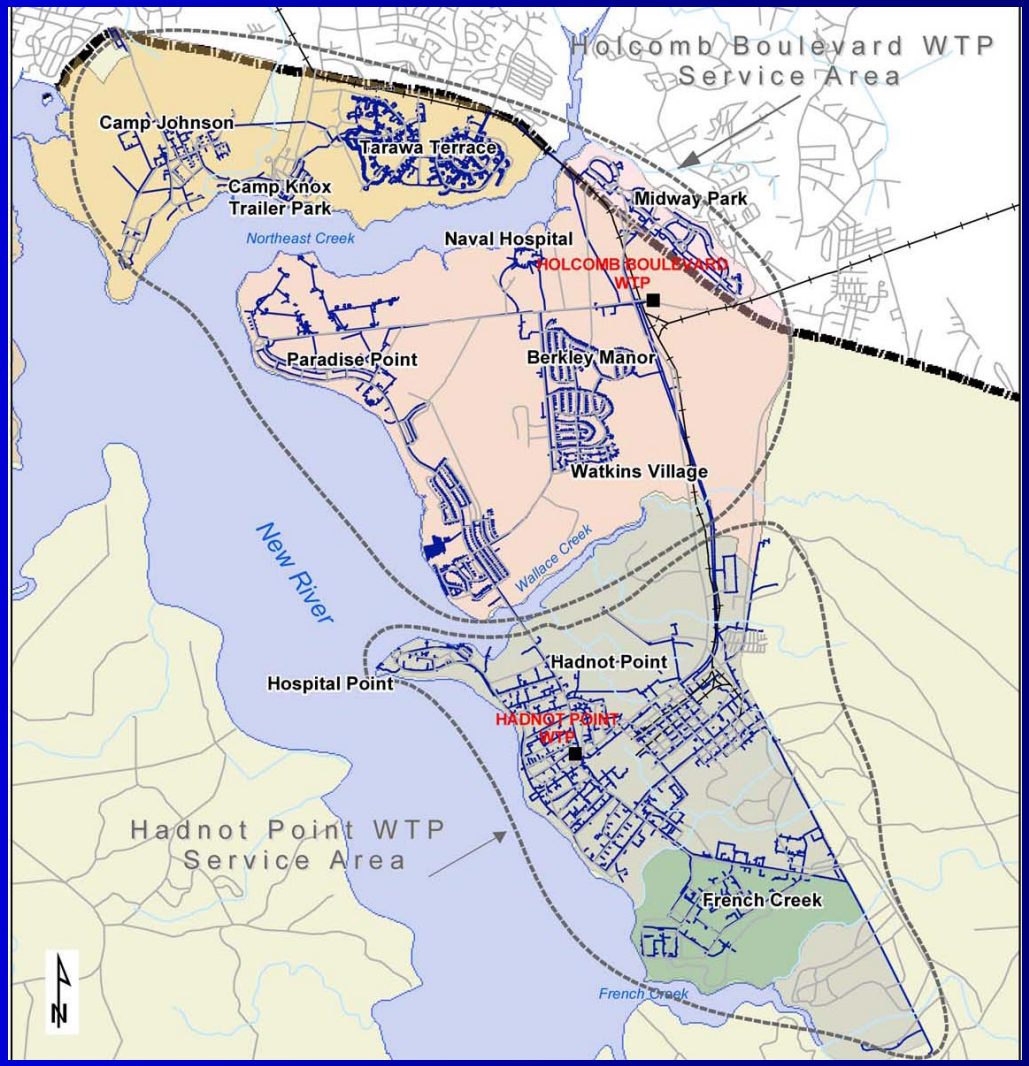
Camp Lejeune, NC Site:





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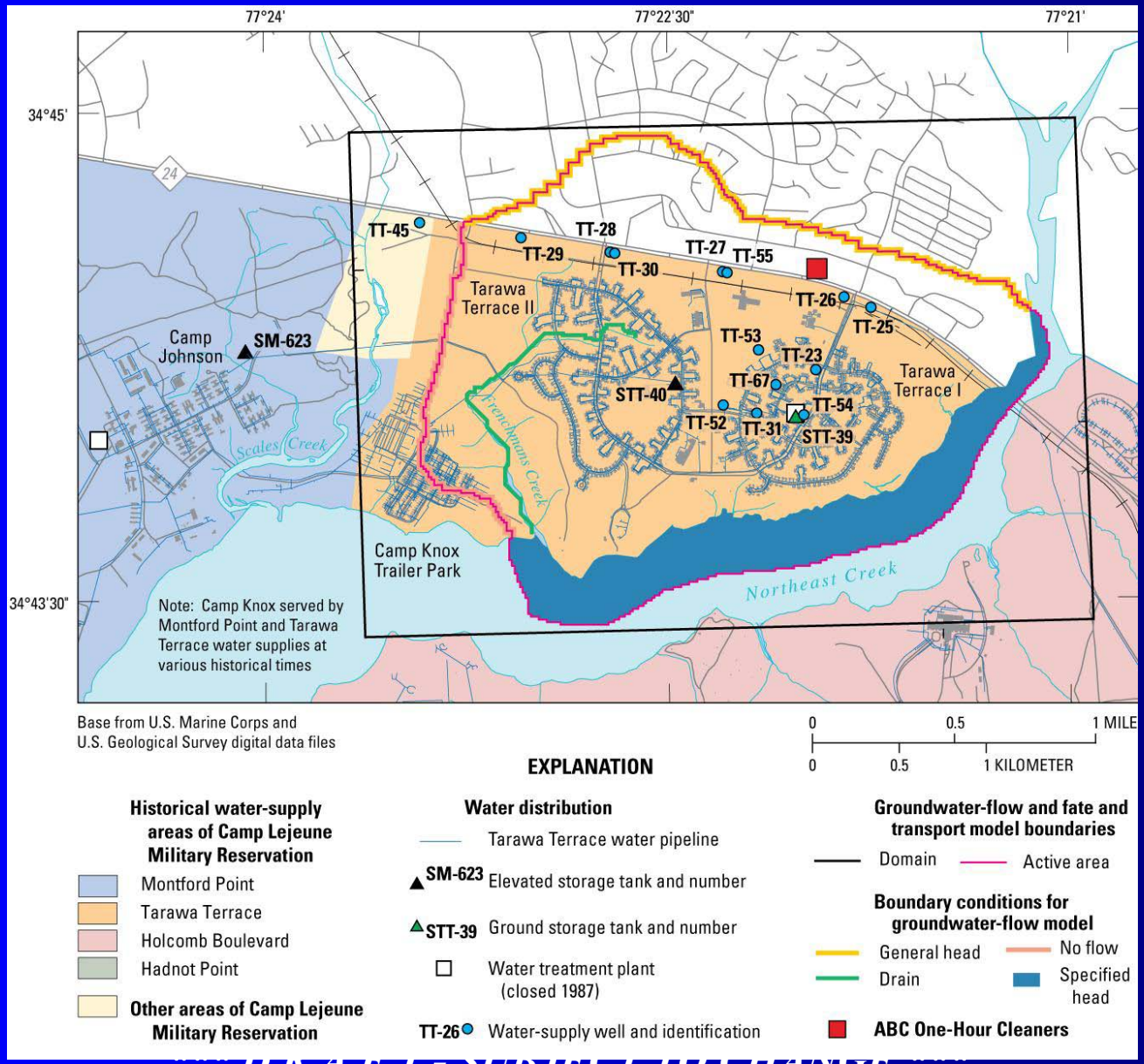
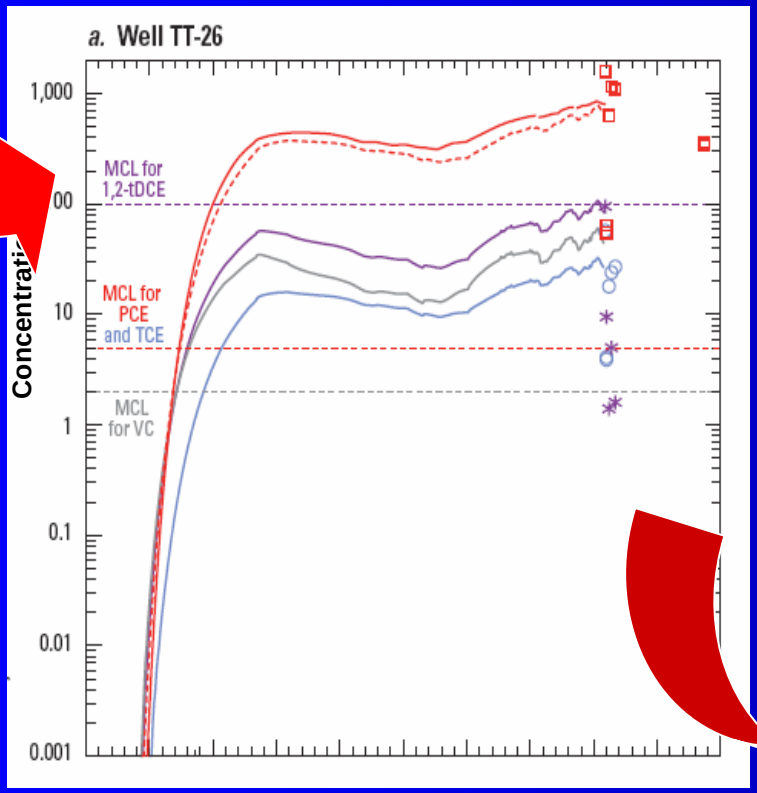
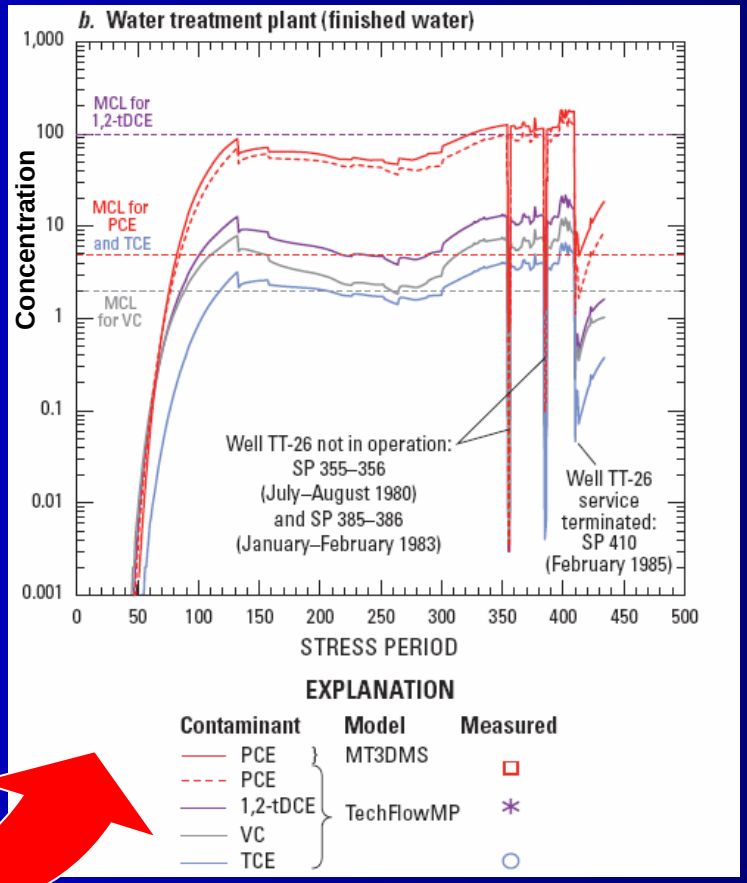
Tarawa Terrace Study:



Tarawa Terrace Study:

- Site data collection;
- Groundwater flow and contaminant fate and transport modeling;
- Mixing model; and,
- WDS system studies;





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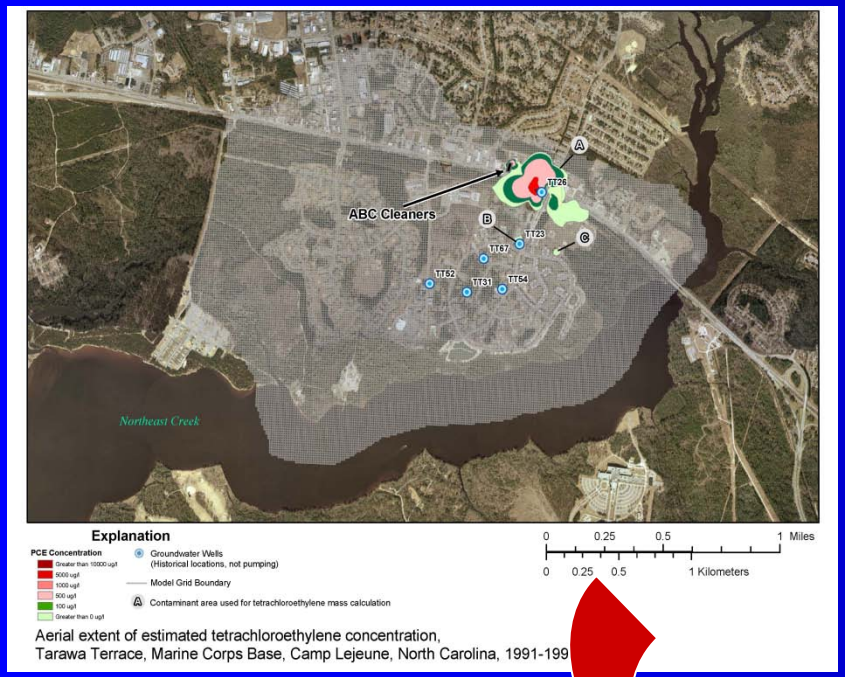
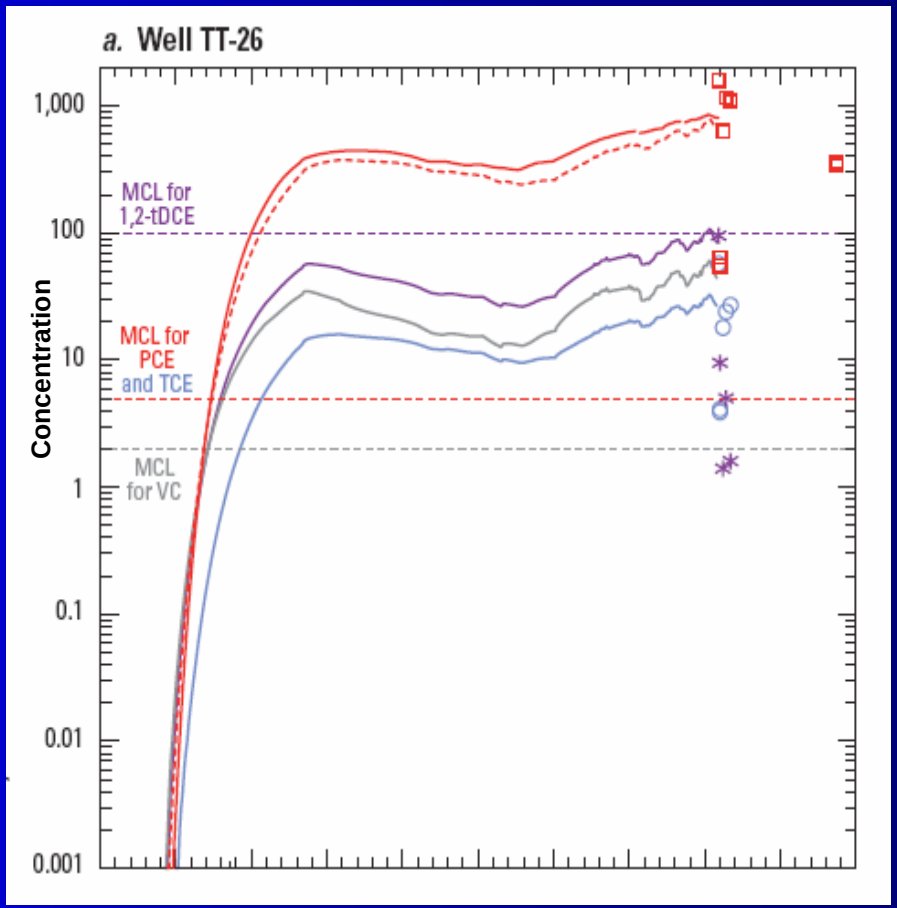
The Purpose of the current study:



Aerial extent of estimated tetrachloroethylene concentration,
Tarawa Terrace, Marine Corps Base, Camp Lejeune, North Carolina, 1991-199



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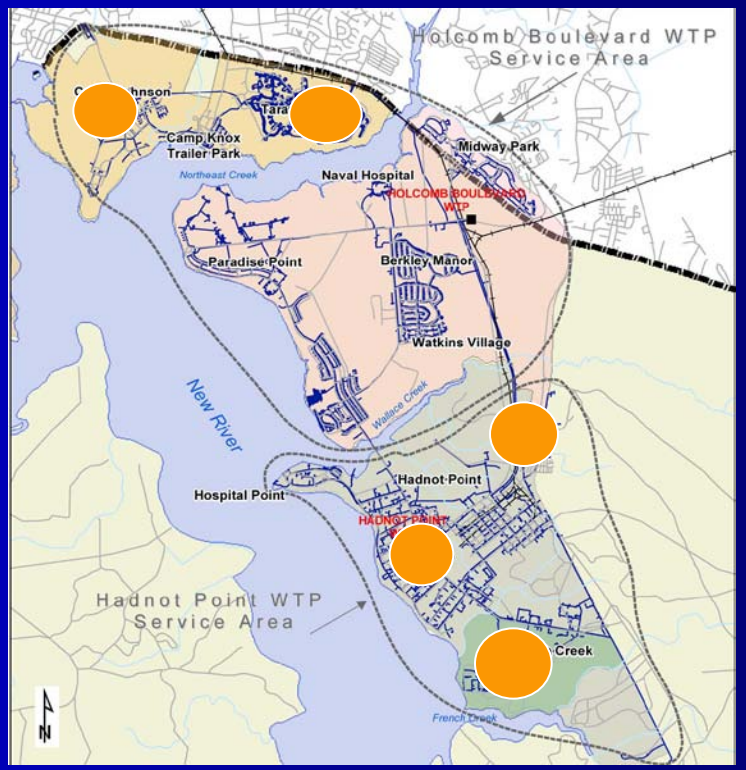
Starting Point and Expectations:

- The proposed method will be a screening level procedure;
- The proposed method is to be used locally (Source, GW Hyd. and fate);
- The accuracy expected from the model is a function of the quality and quantity of the data available at the locale;
- In locations where sufficient data is not available the proposed model cannot be used;



- The proposed model can be used for distinct multiple chemical sources (PCE, Benzene) at a locale by repeating the process for each chemical which shows a different finger print at the observation points.

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Our Understanding of GW Modeling:

- Groundwater flow modeling:

$$\frac{\partial\left(\phi s_f \rho_f\right)}{\partial t} = \nabla \cdot \left[\frac{\rho_f k_m k_{rf}}{\mu_f}\left(\nabla P_f + \rho_f g \nabla z\right)\right] + I_f \quad ; \quad f = w, g$$

- Groundwater fate and transport modeling:

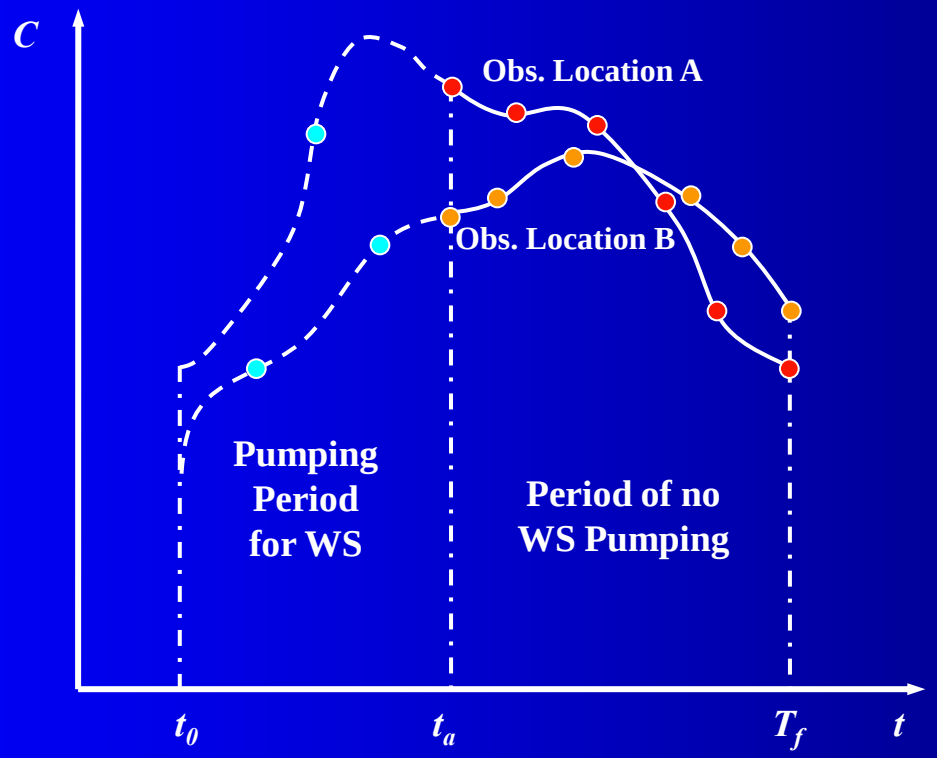
$$\frac{\partial\left(\phi s_f C_f^i\right)}{\partial t} = \nabla\left(\phi s_f D_f^i \nabla C_f^i\right) - \nabla\left(q_f C_f^i\right) \pm I_{f,MT}^i + I_{f,BT}^i + I_{source} \quad ; \quad f = w, g$$

$$i = 1, 2, 3, \ldots$$

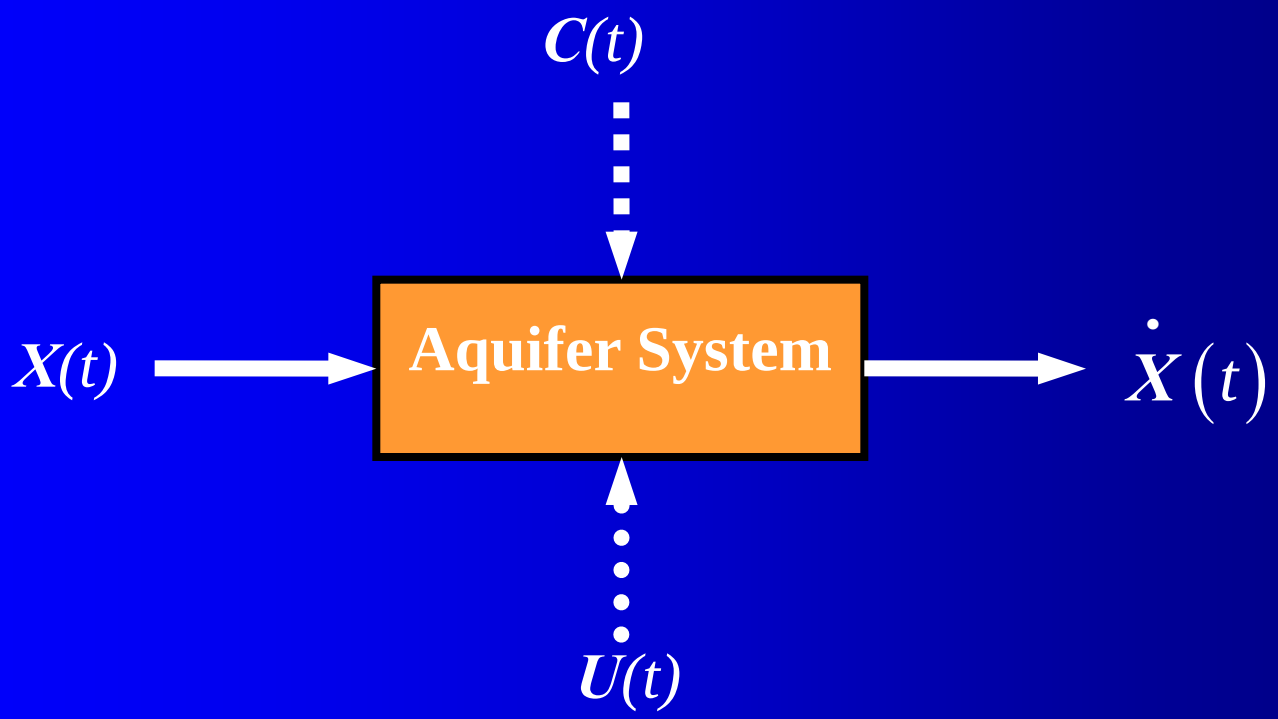
- Matrix equations:

$$[\mathbf{M}]\frac{\partial\left\{C_f^i\right\}}{\partial t} = [\mathbf{S}]\left\{C_f^i\right\} \pm \sum\left\{\boldsymbol{F}_f^i\right\} \quad ; \quad f = w, g$$

Available Data for Historical Reconstruction:



Idealization of the System:



$$\boldsymbol{X}(t) = \left[x_1(t), x_2(t), x_3(t), ..., x_i(t), ..., x_n(t) \right]^T$$ **Contaminant concentrations at monitoring wells**

$$\boldsymbol{U}(t) = \left[u_1(t), u_2(t), u_3(t), ..., u_j(t), ..., u_m(t) \right]^T$$ **Pumping rates at wells**

$$\boldsymbol{C}(t) = \left[C_1(t), C_2(t), C_3(t), ..., C_i(t), ..., C_\ell(t) \right]^T$$ **Contaminant concentrations at the source**

The Proposed Method:

$$\left.\begin{aligned} \dot{X}(t) &= \Phi X(t) + \Psi U(t) \\ X(t_0) &= X_0 \end{aligned}\right\}$$

Φ is the $(n \times n)$ matrix associated with aquifer parameters, BCs, contaminant sources & fate etc.

Ψ is the $(n \times m)$ matrix associated with pumping rates at extraction wells

$\dot{X}(t)$ is the time derivative of the contaminant concentration vector at observation points

X_0 is the initial contaminant concentration vector at the observation locations

$U(t)$ is the pumping rate vector at pumping locations

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The Proposed Method:

- If we use forward time integration:

$$\left.\begin{aligned} \boldsymbol{X}(k+1) &= \left[\boldsymbol{\Phi}\,\Delta t + \mathbf{I}\right]\boldsymbol{X}(k) + \Delta t\,\boldsymbol{\Psi}\boldsymbol{U}(k) \\ \boldsymbol{X}(t_0) &= \boldsymbol{X}_0 \end{aligned}\right\}$$

$$\mathbf{A} = \left[\boldsymbol{\Phi}\,\Delta t + \mathbf{I}\right]; \quad \mathbf{B} = \boldsymbol{\Psi}\,\Delta t$$

$$\left.\begin{aligned} \boldsymbol{X}(k+1) &= \mathbf{A}\boldsymbol{X}(k) + \mathbf{B}\boldsymbol{U}(k) \\ \boldsymbol{X}(t_0) &= \boldsymbol{X}_0 \end{aligned}\right\}$$

The Proposed Method:

- If we use backward time integration:

$$\left.\begin{aligned} X(k) &= \mathbf{A}^{-1}X(k+1) - \mathbf{A}^{-1}\mathbf{B}U(k) \\ X(T_f) &= X_f \end{aligned}\right\}$$

$$\mathbf{A}_b = \mathbf{A}\mathbf{B}^{-1}; \quad \mathbf{B}_b = -\mathbf{A}^{-1}$$

$$\left.\begin{aligned} X(k) &= \mathbf{A}_b X(k+1) + \mathbf{B}_b U(k) \\ X(T_f) &= X_f \end{aligned}\right\}$$

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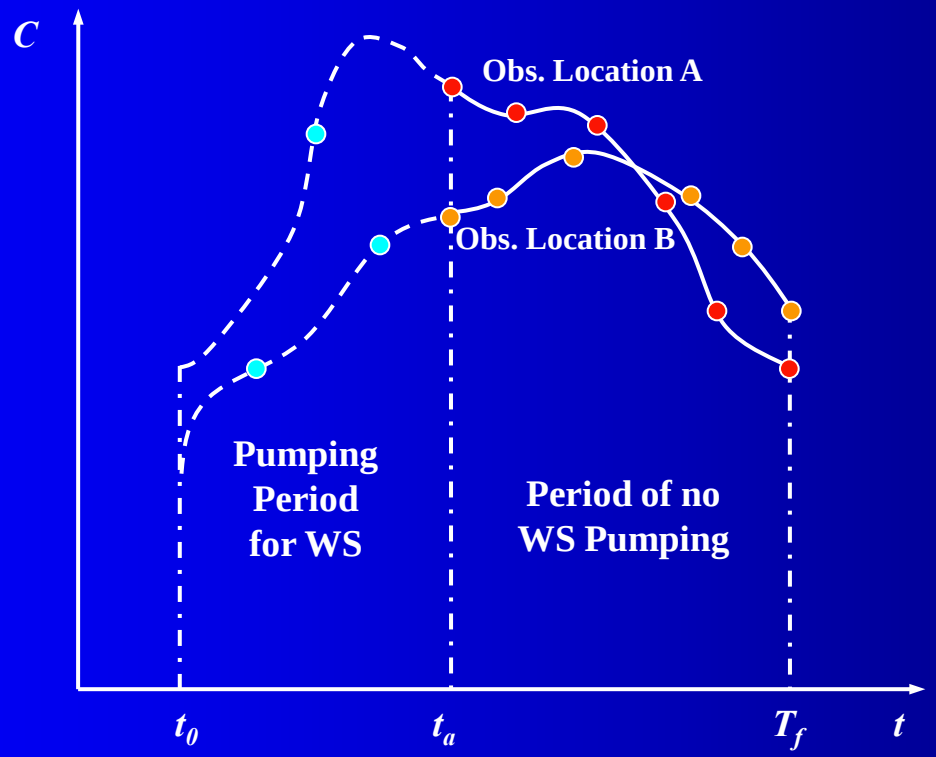
Data Available for Historical Reconstruction:



The Proposed Method: First Step

$$\left.\begin{aligned} X(k+1) &= AX(k) + BU(k) \\ X(t_0) &= X_0 \end{aligned}\right\}$$

- For the period we have data from MW, we have *U(k) = 0*:

$$X(k+1) = A\,X(k)$$

- Least squares method can be used to determine the coefficients of the matrix A

The Proposed Method: Second Step

- Given the matrix **A** optimization methods can be used to determine the coefficients of the matrix **B**

$$X(k+1) = \mathbf{A}X(k) + \mathbf{B}U(k)$$

$$X(t_0) = 0$$

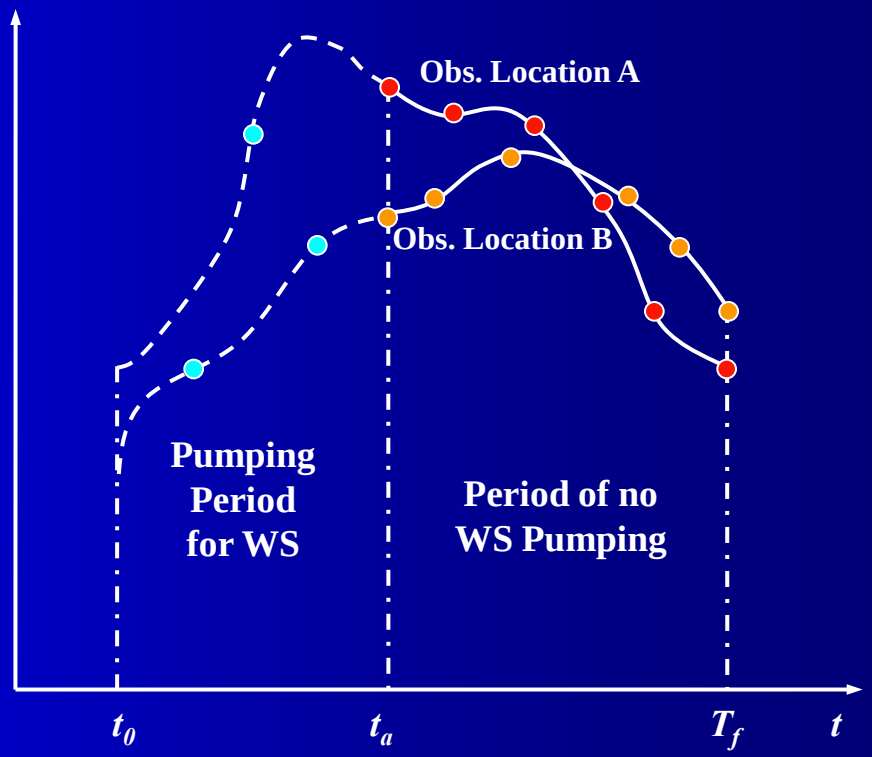
$$f = \min_{\mathbf{B}} \left\{ \frac{1}{2} \sum_{i=1}^{n} \left(x_i(t_a) - x_i^*(t_a) \right)^2 \right\}$$

subject to:

$$X(k+1) = \mathbf{A}X(k) + \mathbf{B}U(k)$$

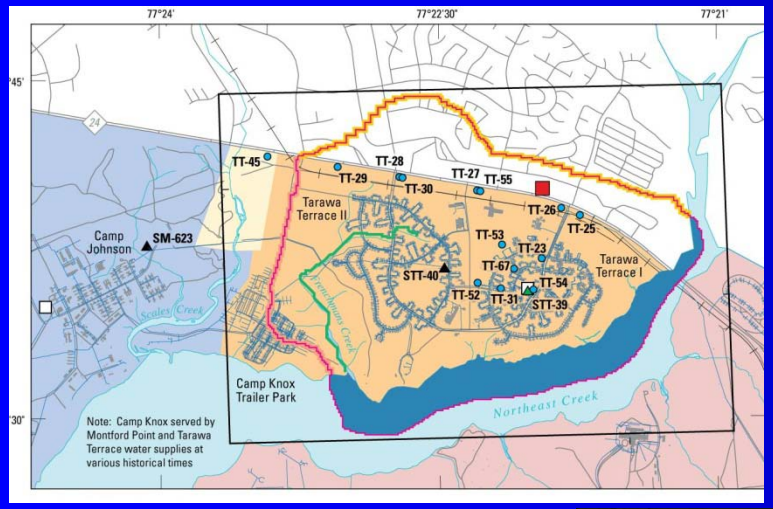
$$X(k) \geq 0$$

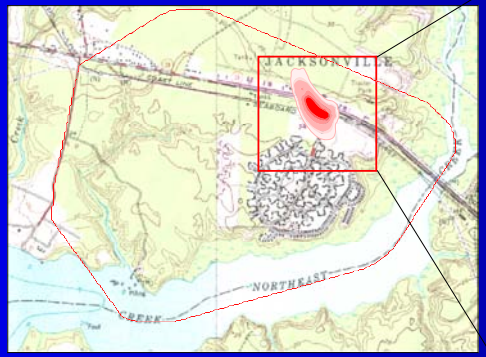
$$X(t_0) = 0$$

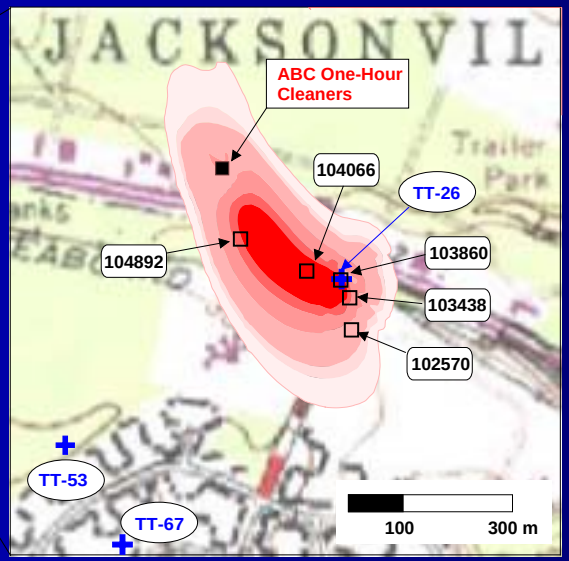


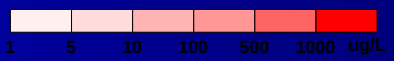
- The proposed method is based on these principles and algorithms

Verification Study: Tarawa Terrace Data









- Three pumping wells: TT-26, TT-53, and TT-67
- Five monitoring points.

PCE concentration distribution in December 1984 (stress period=408) at z = -24 m.

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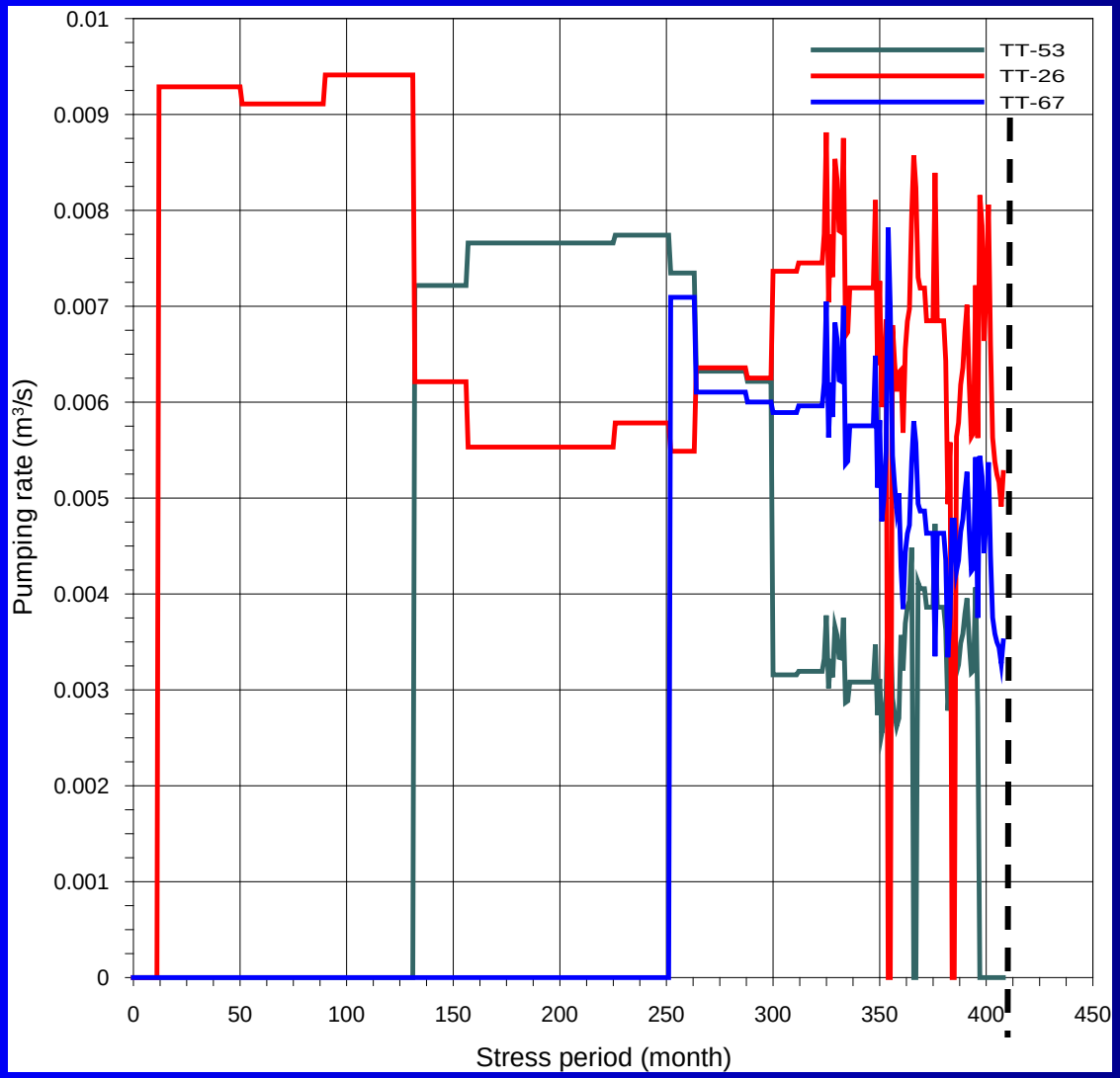
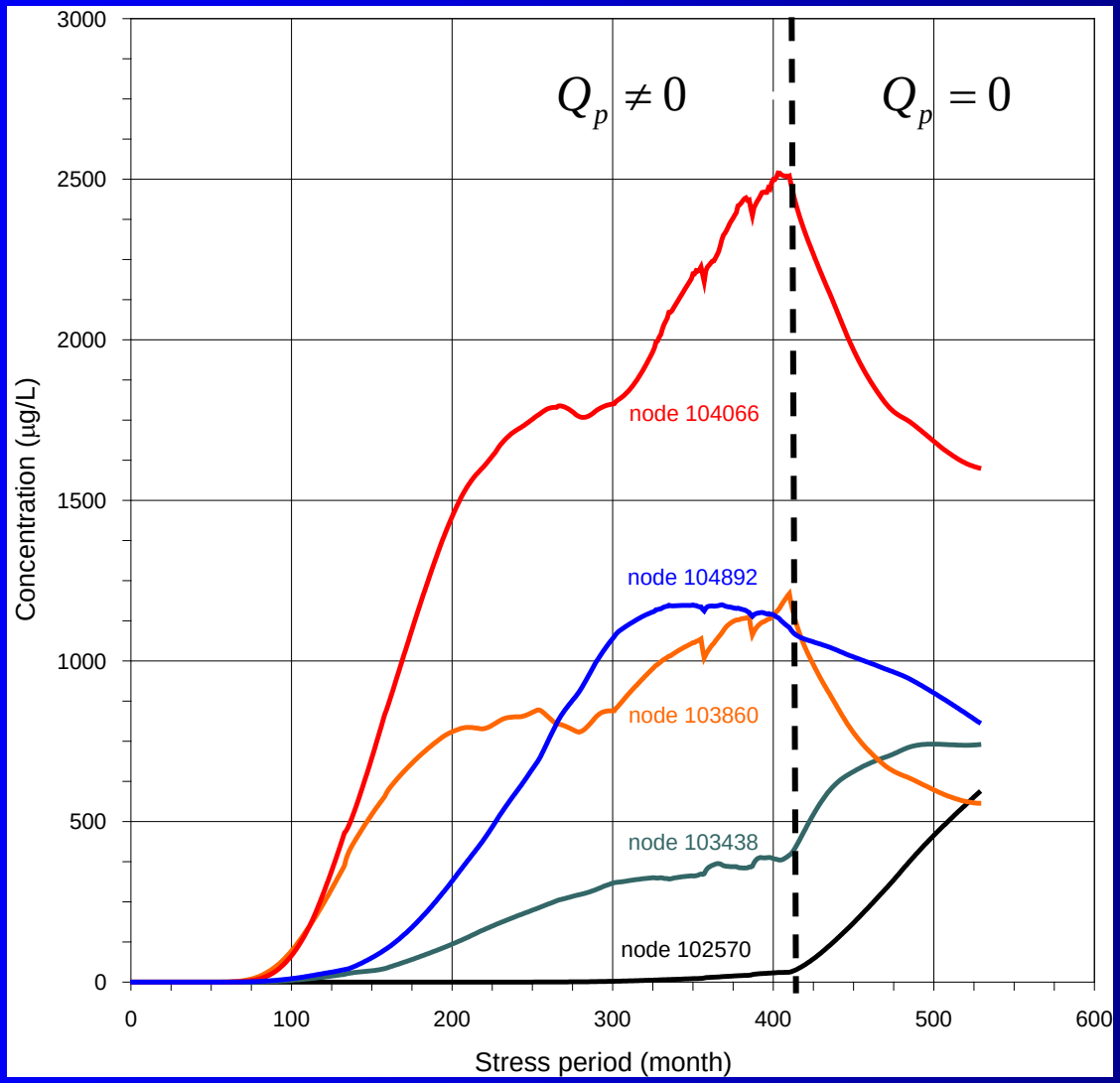
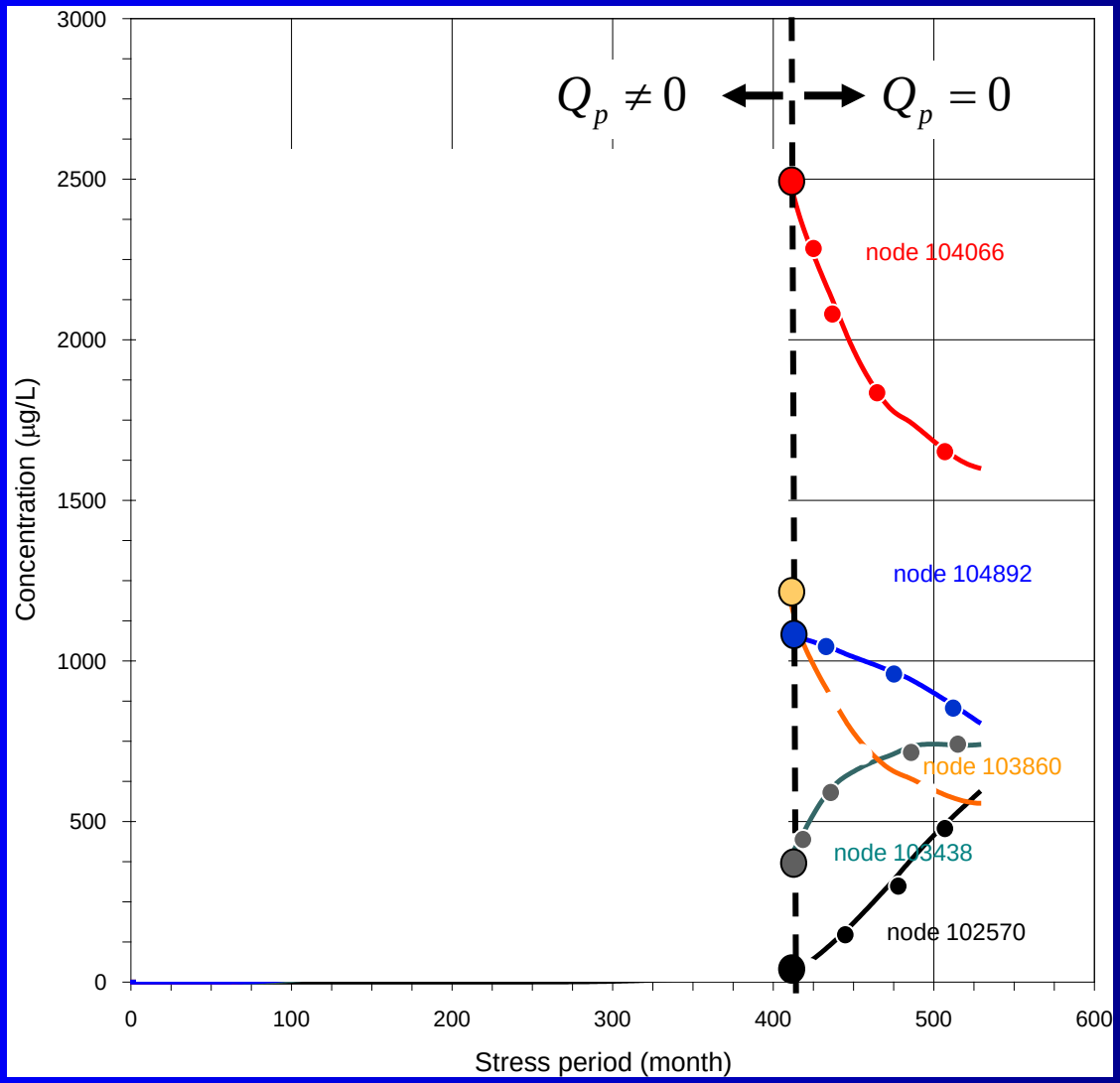
Tarawa Terrace Pumping Schedule:



Tarawa Terrace Data:

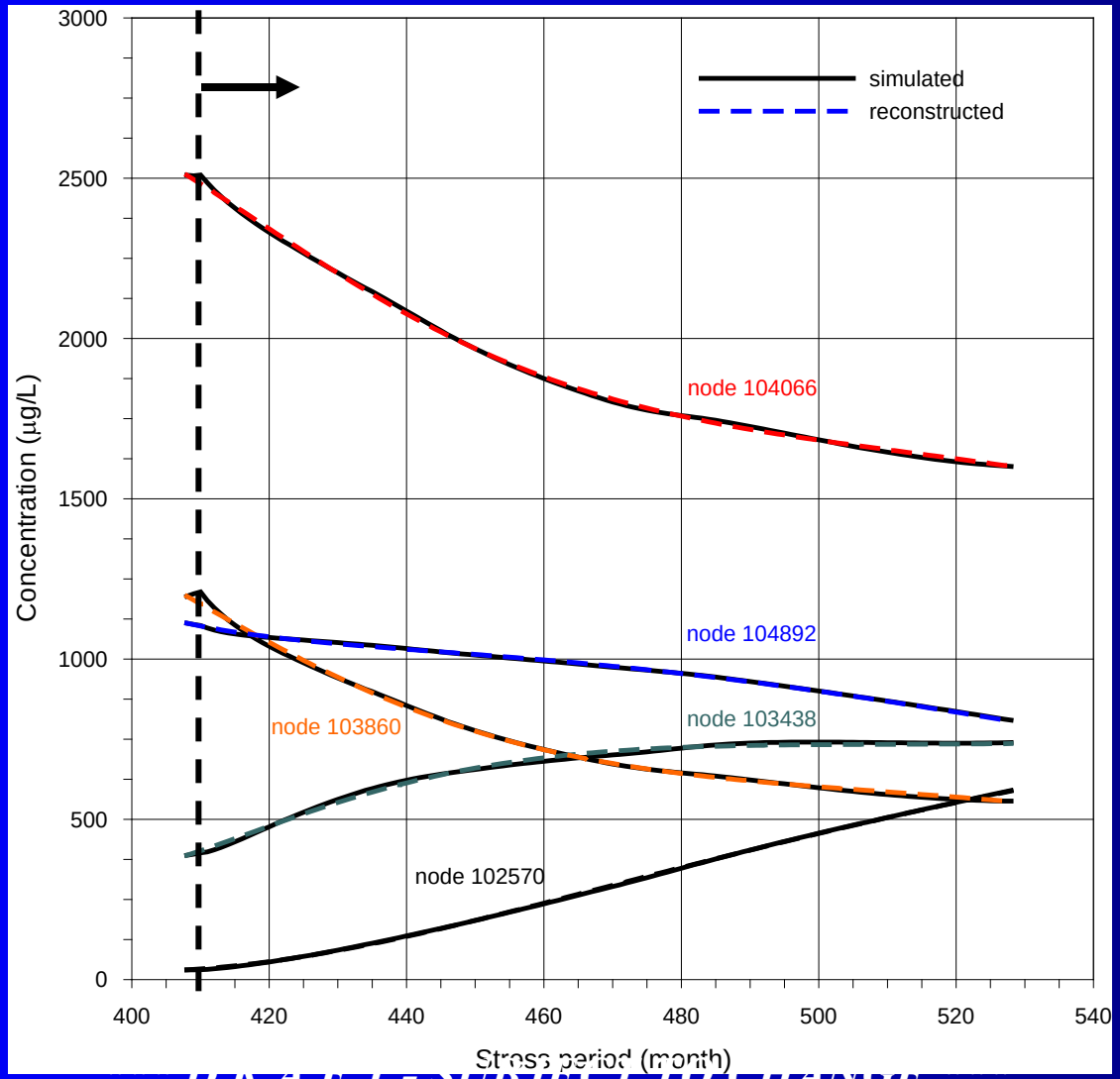


Tarawa Terrace Data:



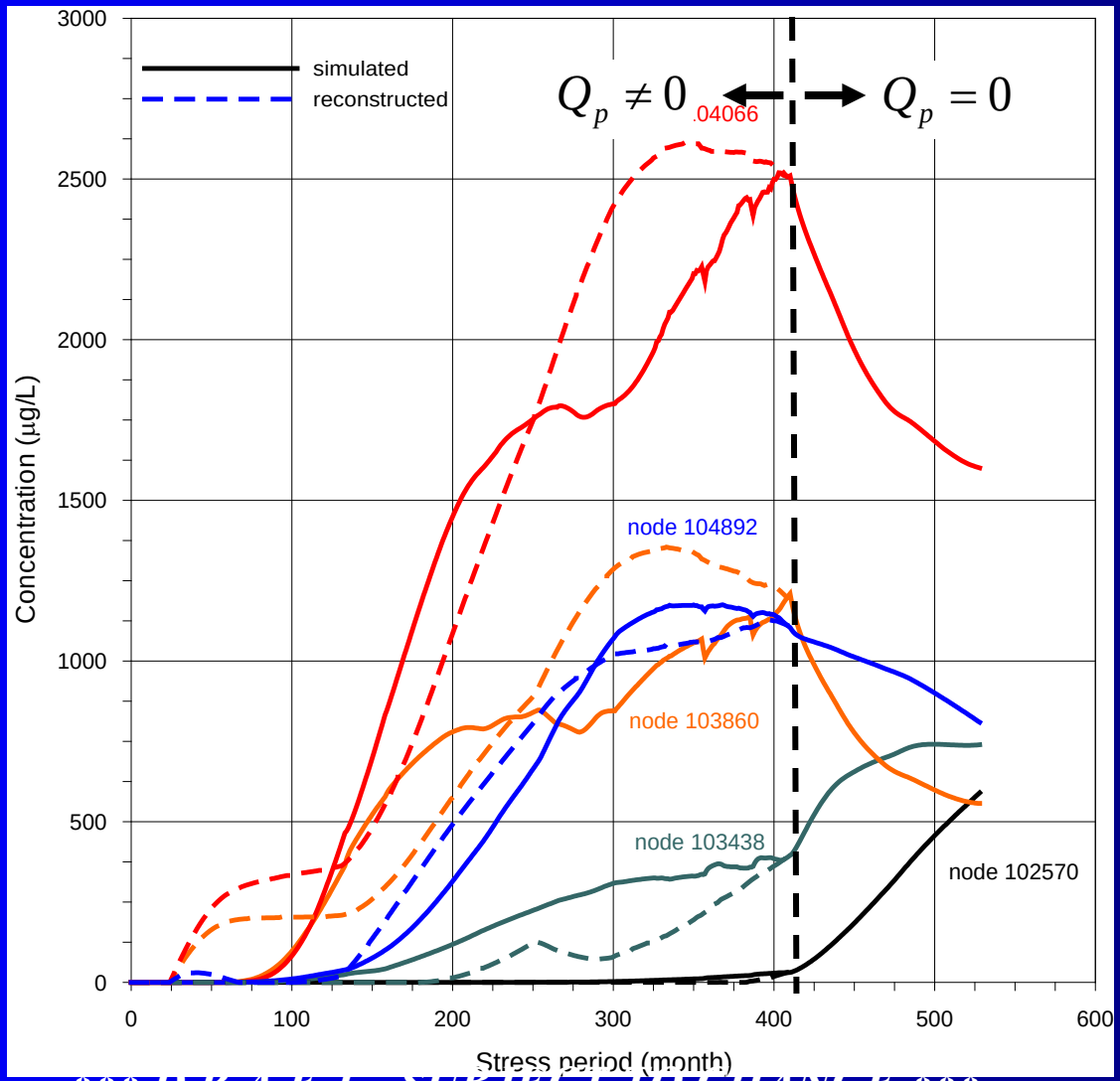
Tarawa Terrace Outcome: LSM

$$X(k+1) = \mathbf{A}\,X(k)$$

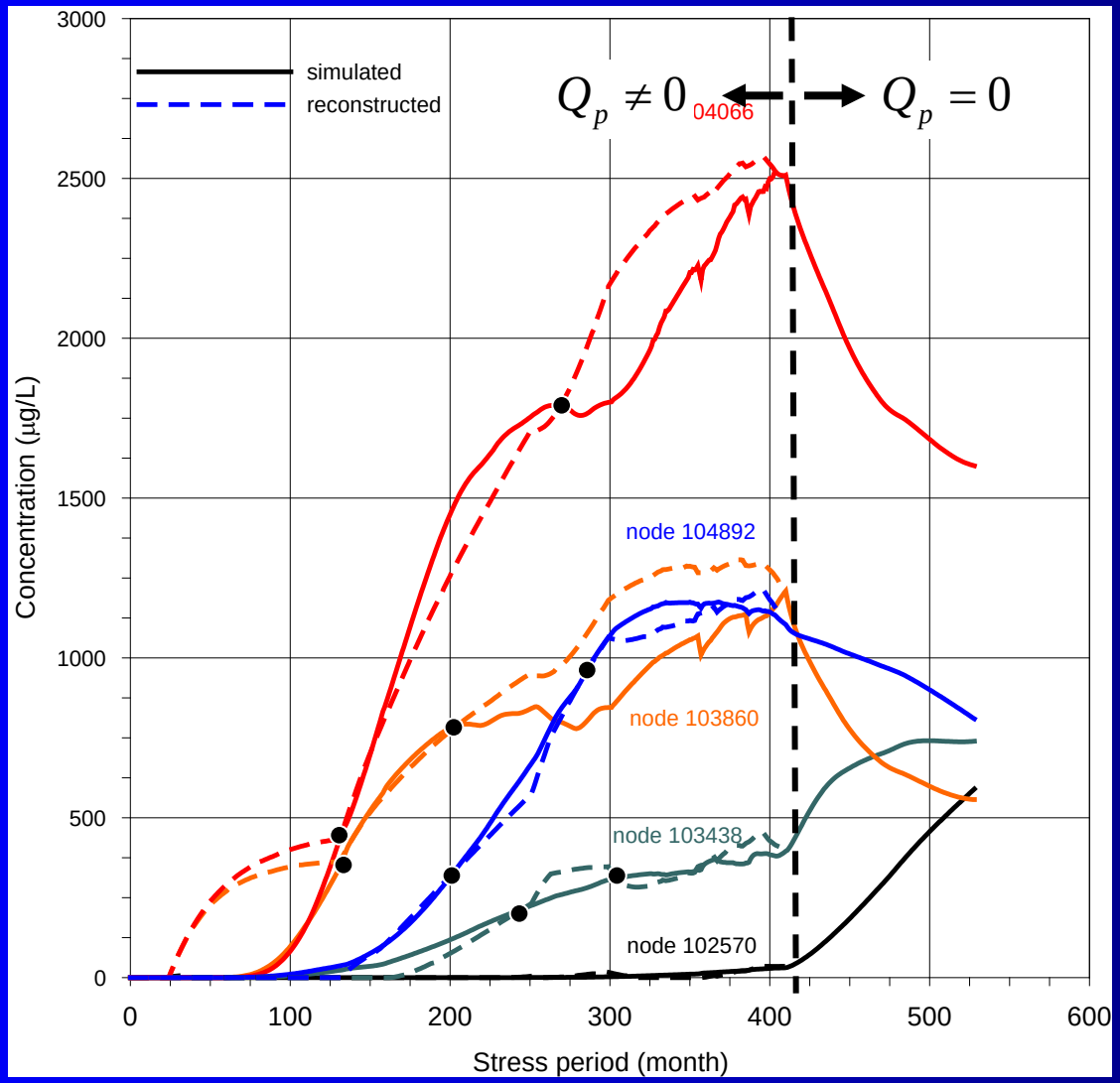


Tarawa Terrace Outcome *(FW – 0 Int. points):*

$$X(k+1) = AX(k) + BU(k)$$



Tarawa Terrace Outcome *(FW – 8 Int. Points):*



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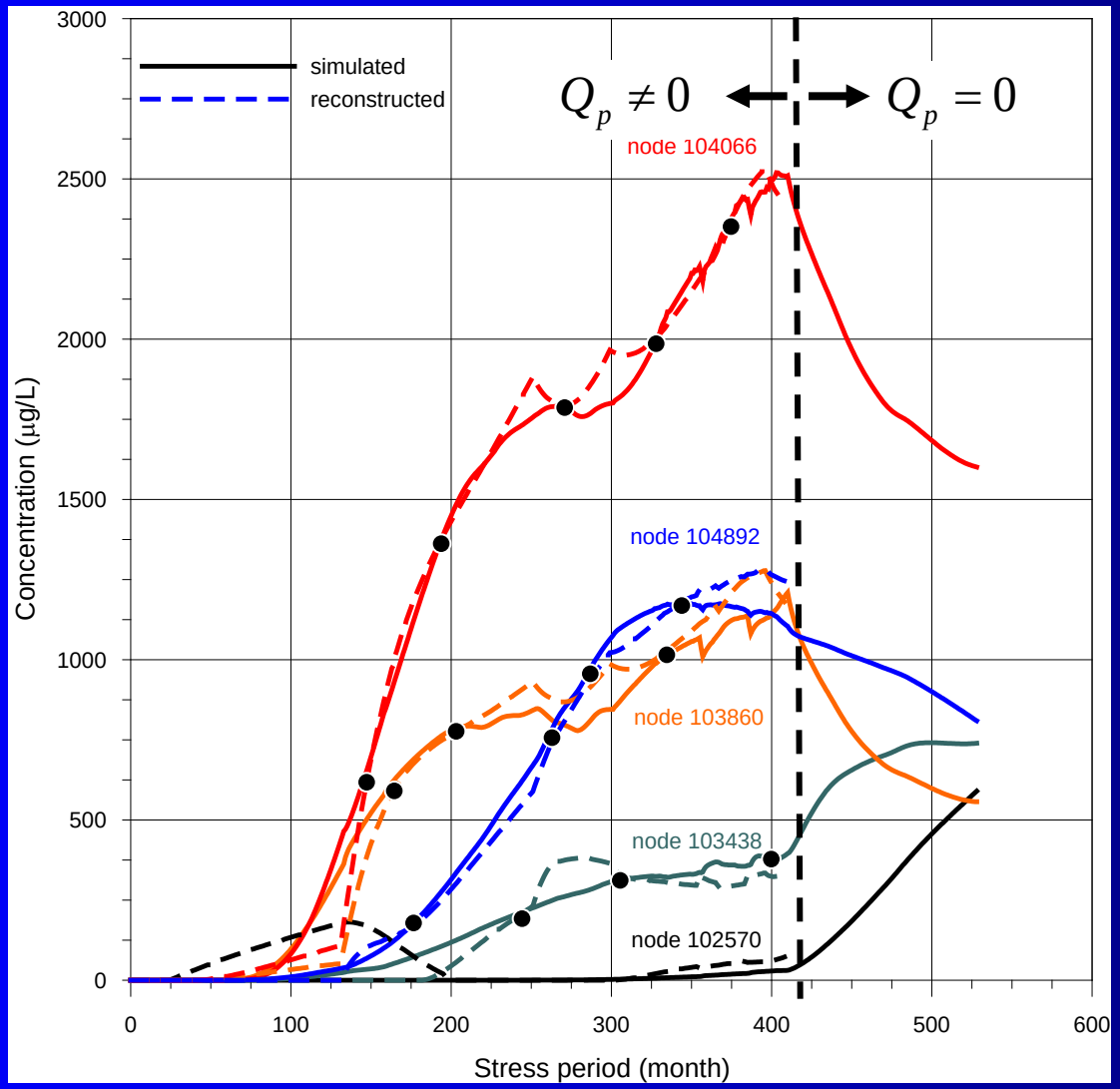
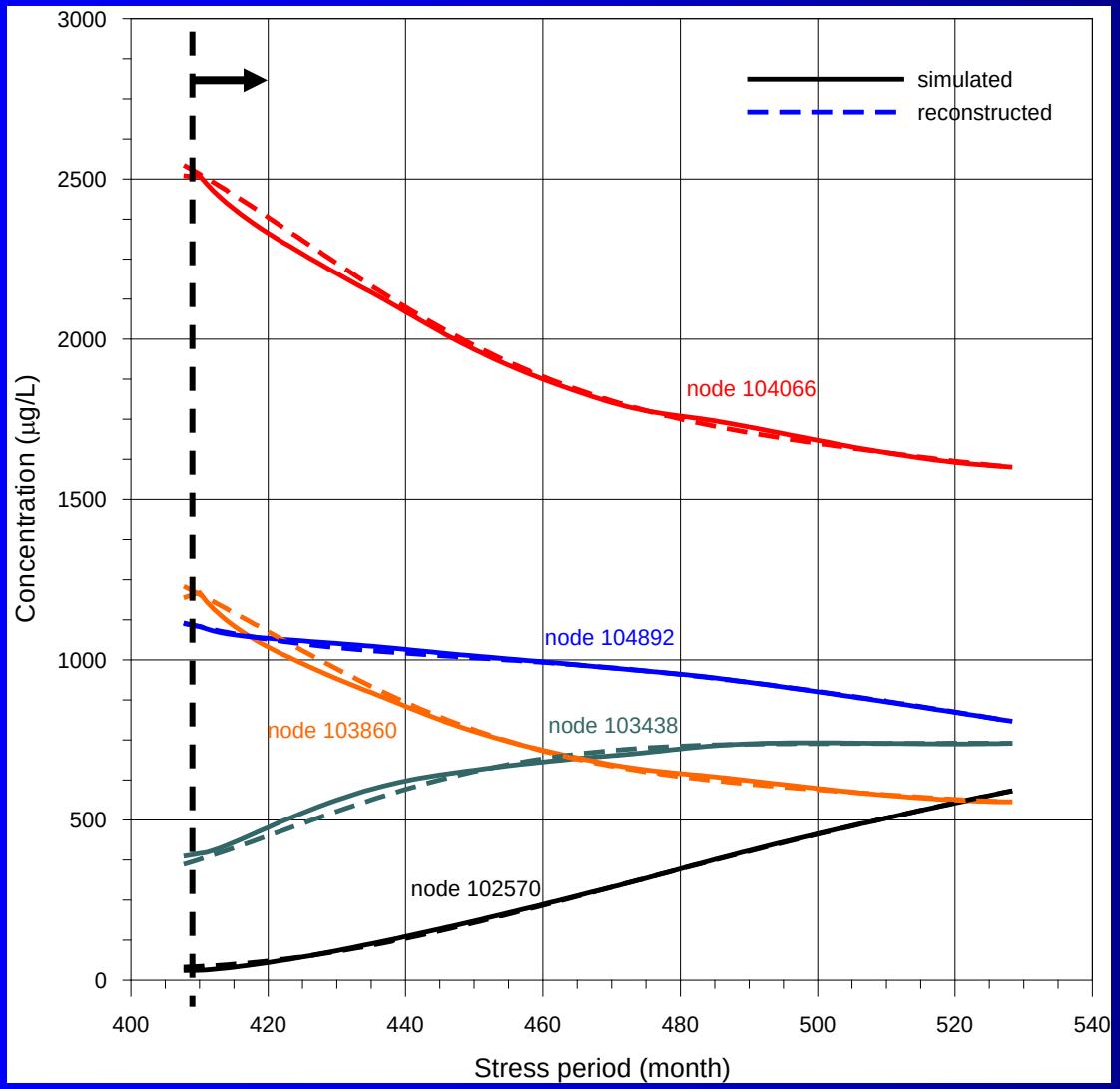
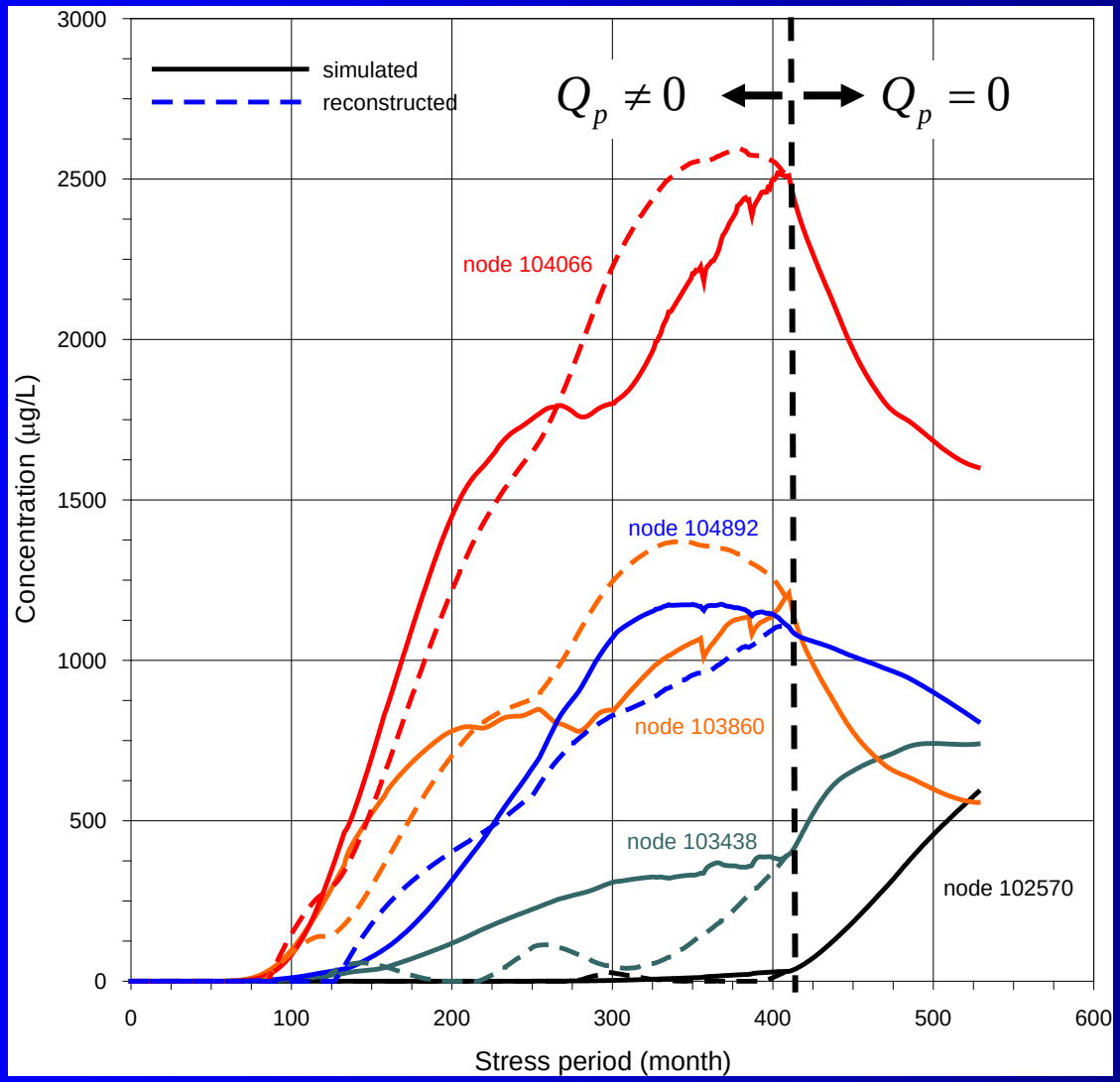
Tarawa Terrace Outcome *(FW – 15 Int. Points):*



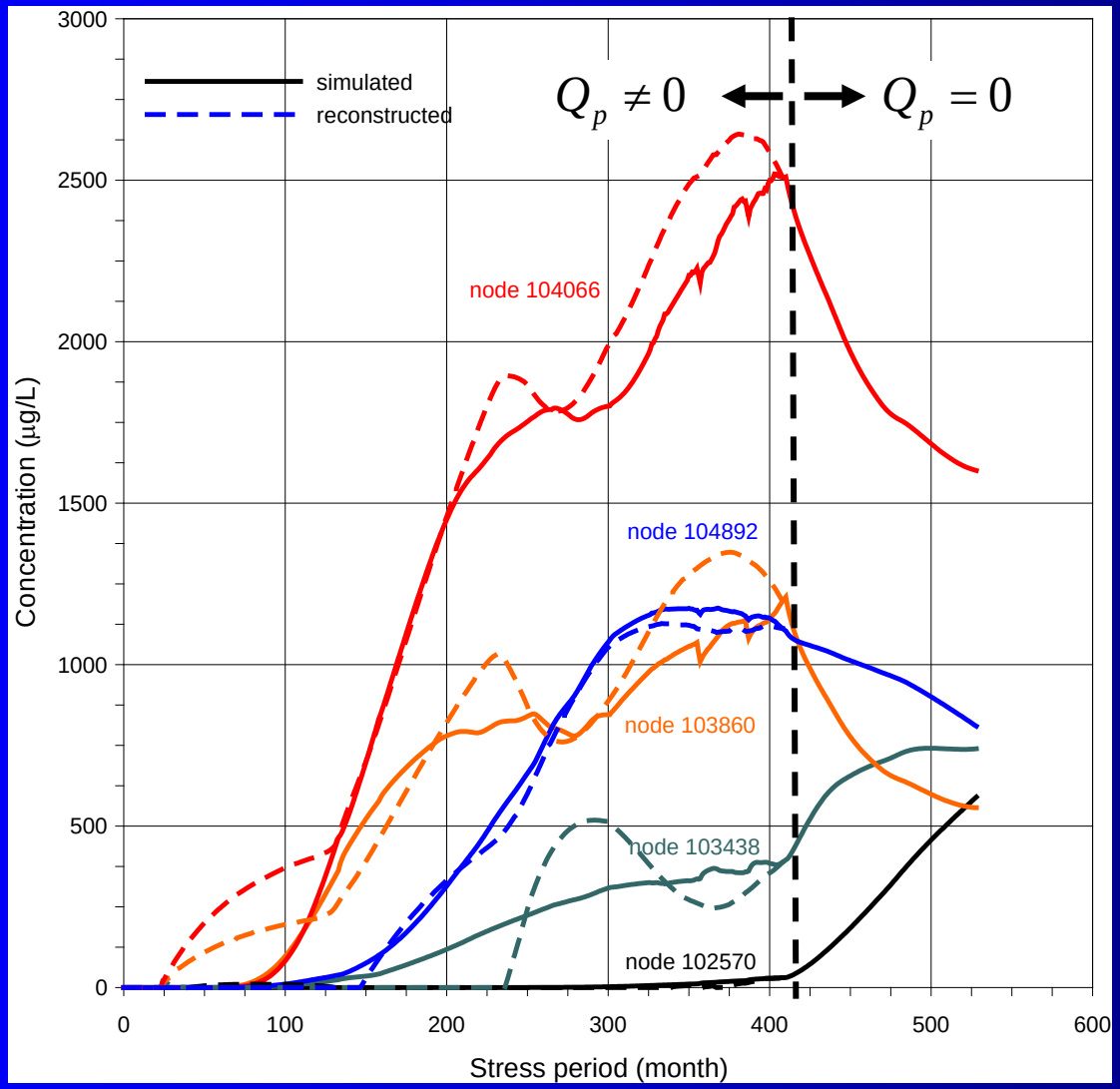
Tarawa Terrace Outcome (BW):



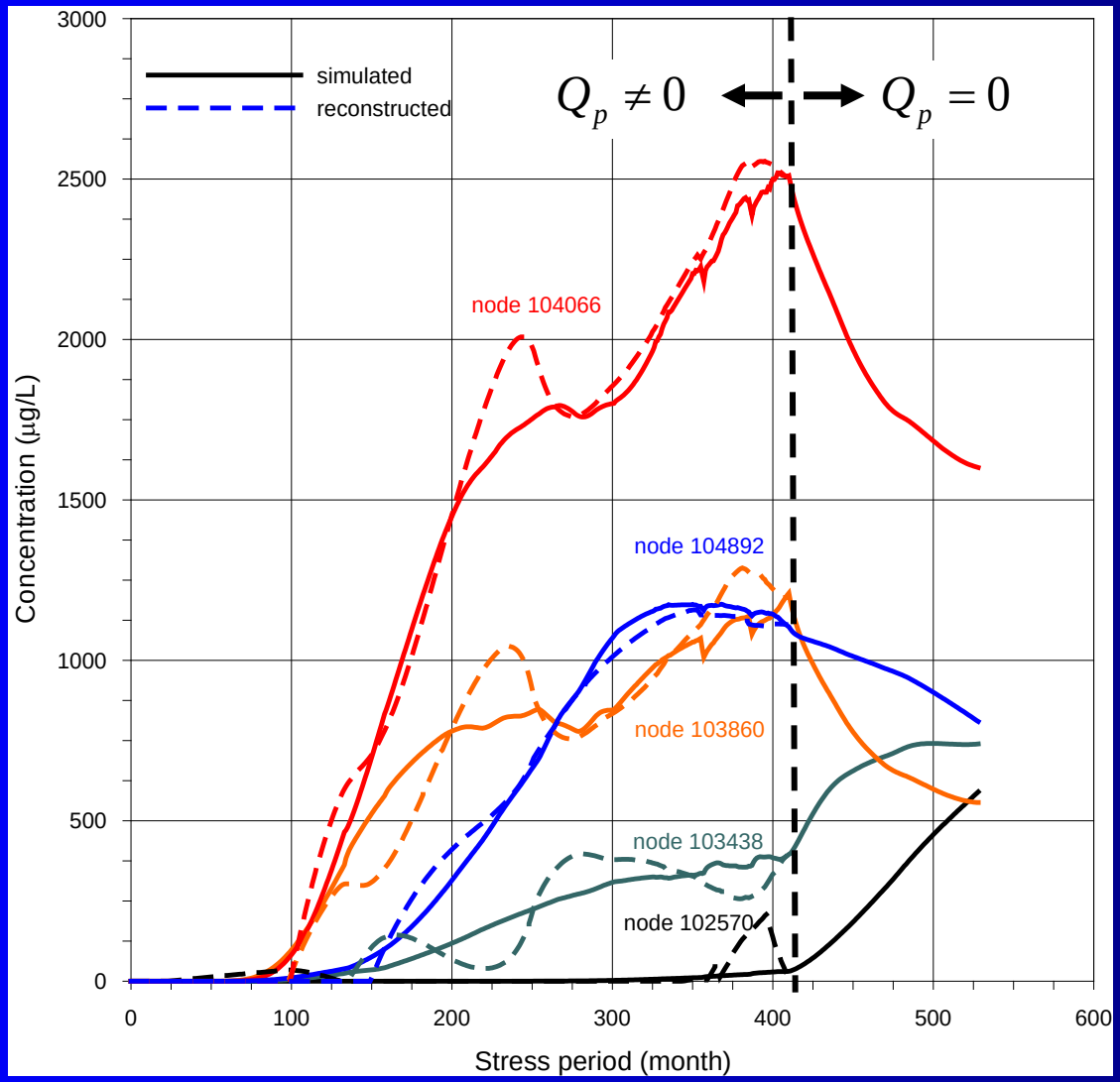
Tarawa Terrace Outcome *(BW – 0 Int. Points):*



Tarawa Terrace Outcome *(BW – 8 Int. Points):*

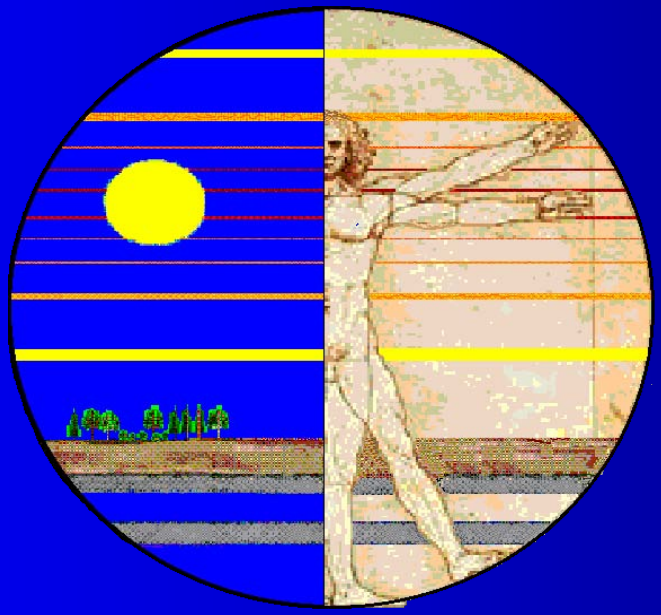


Tarawa Terrace Outcome *(BW – 15 Int. Points):*



Extended procedures:

- **The use of FW and BW procedures iteratively to improve on solution;**
- **Introduction of Kalman filtering method to evaluate the effect of data and computational algorithm error on the solution and establish confidence bands on the solutions obtained; and,**
- **Application of the method to Hadnot Point aquifers.**



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