Appendix 1. The Model

Model Variables

S_n – The number of susceptible persons at time t = n.

H_n – The number of hospitalized suspected case-patients at time t = n.

I_n – The number of living probable SARS case-patients at time t = n.

D_n – The cumulative number of SARS deaths at time t = n.

Note that time unit is in days.

Assumptions

1. A person is moved out of susceptible class only after onset of symptoms and/or having a close contact with a probable case-patient.
2. An infective person can infect others at either suspected or probable stages.
3. A hospitalized suspected case-patient is removed from the suspected class either by reclassification to a probable SARS case-patient or by returning to susceptible class with no immunity. (If there is immunity, one can always add a new class of persons with immunity. For the present model this assumption is not important for our estimation result.)

Parameters

l_n – Admission rate due to contact with probable SARS case-patient at time n–3.

b_n – Admission rate due to contact with suspected case-patient at time n–3.

x_n – Admission rate due to contacts with probable case-patient at time n.

a_n – Rule-out rate of uninfected hospitalized persons at time n.

g_n – Reclassification rate of suspected SARS case-patients to probable at time n.

s_n – Discharge rate of probable SARS patients at time n.

r_n – Fatality rate of probable SARS patients at time n.

d_n – Proportion of infected persons among all suspected case-patients at time n.
Note that $a_n$, $g_n$, $s_n$, $r_n$, and $d_n$ are proportions between 0 and 1.

The model equations, which describe the change in the model variables from time $n$ to $n+1$, are as follows:

$$
S_{n+1} = S_n - \lambda_n I_{n-3} - \beta_n H_{n-3} - \delta_n I_n + \alpha_n (1 - \delta_n) H_n + \sigma_n I_n
$$

$$
H_{n+1} = \lambda_n I_{n-3} + \xi_n I_n + \beta_n H_{n-3} + (1 - \gamma_n) \delta_n H_n + (1 - \alpha_n) (1 - \delta_n) H_n
$$

$$
I_{n+1} = I_n - (\alpha_n + \rho_n) I_n + \gamma_n \delta_n H_n
$$

$$
D_{n+1} = D_n + \rho_n I_n
$$

with

$$
S_n + H_n + I_n + D_n = S_n + H_n + I_n + D_n.
$$

The flow diagram for the dynamics is given in Figure 2.

Since the equations for $H_{n+1}$, $I_{n+1}$ and $D_{n+1}$ involve only $H_n$, $I_n$ and $D_n$, we can consider these three equations in a simple model

$$
H_{n+1} = \lambda_n I_{n-3} + \xi_n I_n + \beta_n H_{n-3} + [(1 - \gamma_n) \delta_n + (1 - \alpha_n) (1 - \delta_n)] H_n
$$

$$
I_{n+1} = (1 - \alpha_n - \rho_n) I_n + \gamma_n \delta_n H_n
$$

$$
D_{n+1} = D_n + \rho_n I_n
$$

which can be put in the following matrix form:

$$
\begin{bmatrix}
H_{n+1} \\
I_{n+1} \\
D_{n+1}
\end{bmatrix} =
\begin{bmatrix}
(1 - \gamma_n) \delta_n + (1 - \alpha_n) (1 - \delta_n) & \xi_n & 0 \\
\gamma_n \delta_n & (1 - \alpha_n - \rho_n) & 0 \\
0 & \rho_n & 1
\end{bmatrix}
\begin{bmatrix}
H_n \\
I_n \\
D_n
\end{bmatrix} +
\begin{bmatrix}
\beta_n & \lambda_n & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
H_{n-3} \\
I_{n-3} \\
D_{n-3}
\end{bmatrix}
$$

The data for $H_n$, $I_n$, and $D_n$, the respective numbers of admitted suspected case-patients, reported probable SARS case-patients, and SARS deaths, are available for parameter estimation.