HHS Public Access
Author manuscript
Radio Sci. Author manuscript; available in PMC 2023 May 11.

Published in final edited form as:
Radio Sci. 2022 May 12; 57(5): . doi:10.1029/2021rs007388.

Magnetic Field Above Stratified Earth in Magnetic Loop Through-the-Earth Wireless Communications

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Abstract

Evaluation of the very-low frequency, ultralow frequency, or extremely low frequency magnetic field (H-field) due to a buried or on-surface magnetic dipole or antenna is important for applications such as geophysical exploration and through-the-Earth (TTE) wireless communications. In this study, we develop an explicit form of magnetic field over multi-layer Earth medium ($N > 3$). The generalized solution is derived for operating frequency, mine depth, and Earth conductivity that would be typically related to TTE applications.

1. Introduction

The Mine Improvement and New Emergency Response Act of 2006 requires the installation of post-accident two-way communications and electronic tracking for all coal mines (MSHA, 2006). The through-the-Earth (TTE) wireless communication system, which operates at very-low frequency (VLF), ultralow frequency (ULF), or extremely low frequency (ELF), is considered more survivable after a mine disaster because its signal penetrates the Earth directly and does not use wires connecting the surface and underground components. There are two types of through-the-Earth (TTE) wireless communication in the mining industry: magnetic loop through-the-Earth (TTE) and electrode-based (or linear) TTE. While the electrode based TTE system sends a signal directly through the mine overburden by driving an extremely low frequency (ELF) or ultralow frequency (ULF) AC current into the Earth (Yan et al., 2017), the magnetic loop system sends a signal through magnetic fields. The receiver at the other end (underground or surface) detects the resultant magnetic fields and receives it as a voltage. A wireless communication link between surface and underground is then established.

For TTE communication, one of the biggest challenges lies in the signal attenuation which is largely controlled by the conductivity of the Earth's medium (Durkin, 1984; Moore, 1951; Shope, 1982; Sommerfeld, 1909; Wait, 1972). The performance and reliability of such a system is also highly dependent on additional properties of the Earth, such as

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the strata layer structure and conductivity associated with each layer. To understand the performance limitations of TTE systems, we need to understand the impact of these layers on the TTE propagation. Typically, the transmit (Tx) and receive (Rx) antennas are vertically separated by the Earth's overburden, having a coaxial arrangement. The underground unit can be integrated with a refuge alternative for communication. In this paper, we present a multi-layer ($N > 3$) model in which the magnetic fields are calculated based on potential theory. The analytical results of magnetic field distribution at the Rx loop are presented for various conditions, such as Earth conductivity or conductivity contrast between layers.

Research has been conducted on the understanding of wave propagation through the Earth for decades. Wait (1972) derived expressions for the H-fields of a small loop buried in a dissipative homogeneous medium, the Earth ($N = 1$). Wait and Spies (Wait & Spies, 1971) also investigated the case of a small loop radiating in the Earth which was considered to be stratified ($N = 2$). Shope extended the solution for a three-layer model ($N = 3$) in the form of infinite integrals (Shope, 1982). In this study, we will derive the H-field solution in detail for a more general case—a multi-layer Earth model with the number of layers greater than three ($N > 3$).

2. The H-Field Due To a Magnetic Dipole in a Stratified Medium

A descriptive heading about methodsThe Earth model is comprised of multi-layer regions of varying conductivity. A magnetic dipole, or current dipole, is located in the deepest layer (the Nth) with a depth of h and aligned with its axis in the z direction of a cylindrical coordinate system (ρ, ϕ, z). In each layer, the Earth medium can be characterized by σ_n, ε_n, and μ, the conductivity, dielectric constant, and the magnetic permeability, respectively. For reasons mentioned above, the magnetic permeability is assumed to be the same everywhere as in the air, μ_0. The situation is illustrated in Figure 1, along with the coordinate system for the problem. Note that due to the azimuthal symmetry, the H-field will be independent of ϕ.

The related electromagnetic (EM) field distribution of interest can be obtained by utilizing Maxwell's equations and the application of appropriate boundary conditions and charge distribution due to the source.

$$\nabla \cdot \boldsymbol{E} = \rho/\varepsilon, \tag{1}$$

$$\nabla \cdot \boldsymbol{B} = 0, \tag{2}$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \tag{3}$$

$$\nabla \times \boldsymbol{B} = \mu_0\left(\sigma \boldsymbol{E} + \varepsilon \frac{\partial \boldsymbol{E}}{\partial t}\right) \tag{4}$$

While the behavior of electromagnetic (EM) fields at high frequencies is due partly to propagation effects and is characteristic of wave theory, the fields at low frequencies like

TTE are more easily addressed using potential theory. Schelkunoff showed that it is feasible to define the electromagnetic (EM) field in terms of a single vector quantity, $\boldsymbol{F}$, the electric vector potential, or Π^*, the magnetic Hertzian potential (Schelkunoff, 1943). They are related to each other by

$$\boldsymbol{F} = j\mu\omega\boldsymbol{\Pi}^* \tag{5}$$

The EM fields produced by a given current source are then expressed as

$$\boldsymbol{E} = -\nabla \times \boldsymbol{F} \tag{6}$$

$$\boldsymbol{H} = -\nabla U - (\sigma + j\omega\varepsilon)\boldsymbol{F} \tag{7}$$

The magnetic scalar potential U in Equation 7 relates to electric vector potential $\boldsymbol{F}$ by $U = -\frac{1}{j\omega\mu}\nabla \cdot \boldsymbol{F}$ (Schelkunoff, 1943). Substituting Equation 5 for Equations 6 and 7, we have

$$\boldsymbol{E} = -j\mu\omega\boldsymbol{\nabla} \times \boldsymbol{\Pi}^* \tag{8}$$

$$\boldsymbol{H} = \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \boldsymbol{\Pi}^*) - \left(j\mu\omega\sigma - \varepsilon\mu\omega^2\right)\boldsymbol{\Pi}^* \tag{9}$$

In a cylindrical coordinate system, the electric and H-field can be expressed as below. Since there is only a ϕ component of the loop current, the corresponding magnetic Hertzian potential or electric vector potential has only a z component, that is, $\boldsymbol{\Pi}^* = \Pi^*\hat{z}$.

$$\boldsymbol{E} = j\mu\omega\frac{\partial \Pi^*}{\partial \rho}\widehat{\varnothing} \tag{10}$$

$$\boldsymbol{H} = \frac{\partial^2 \Pi^*}{\partial \rho \partial z}\hat{\rho} - \left(j\mu\omega\sigma - \varepsilon\mu\omega^2 - \frac{\partial^2}{\partial z^2}\right)\Pi^*\hat{z} \tag{11}$$

The vector notation will be eliminated hereafter. In general, the magnetic Hertzian potential Π^* in each layer i satisfies the wave equation

$$\left(\nabla^2 - \gamma_i^2\right)\Pi_i^* = 0 \tag{12}$$

where

$$\gamma_i^2 = \mu\omega(j\sigma_i - \omega\varepsilon_i) \tag{13}$$

is the intrinsic propagation constant. In the cylindrical coordinate system, Equation 12 can be expanded as below.

$$\frac{\partial^2 \Pi_i^*}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial \Pi_i^*}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 \Pi_i^*}{\partial \varnothing^2} + \frac{\partial^2 \Pi_i^*}{\partial z^2} - \gamma_i^2 \Pi_i^* = 0 \tag{14}$$

Solutions to those equations can be obtained by the separation-of-variables technique or the theory of cylindrical harmonics. The solutions are of the form

$$\Pi_i^* = (\mathrm{Cos}(n\varnothing) + \mathrm{Sin}(n\varnothing))(\mathrm{J}_n(\lambda\rho) + \mathrm{Y}_n(\lambda\rho))\left(e^{\mathrm{k}_i z} + e^{-\mathrm{k}_i z}\right), n = 0, 1, 2\ldots \tag{15}$$

where $J_n(*)$ and $Y_n(*)$ are the Bessel functions of the first kind and the Bessel functions of the second kind, respectively, and

$$\mathrm{k}_i = \sqrt{\lambda^2 + \gamma_i^2} \tag{16}$$

the wavenumber of the ith layer. The Neumann functions $Y_n(*)$ are not permissible in Equation 15 since Π_i^* must have a finite value as $\rho \rightarrow 0$. The axial symmetry to the problem also demands Π_i^* be independent of $\varnothing$. It requires

$$\frac{\partial \Pi_i^*}{\partial \phi} = 0,\ \text{or}\ n\,\mathrm{Cos}(n\varnothing) - n\,\mathrm{Sin}(n\varnothing) = 0 \tag{17}$$

So $n = 0$ for all i. Then Equation 15 reduces to

$$\Pi_i^* = \mathrm{J}_0(\lambda\rho)\left(e^{\mathrm{k}_i z} + e^{-\mathrm{k}_i z}\right) \tag{18}$$

Meanwhile, the plus or minus sign in the exponential terms in Equation 18 must be chosen so that the solutions are finite as z tends to plus or minus infinity in the upper-most (i = 0) and the lower-most ($i = N$) layers. For generality, Equation 18 can be rewritten as below by introducing amplitude coefficients, $T_i'(\lambda)$ and $R_i'(\lambda)$, associated with positive and negative propagation, respectively.

$$\Pi_i^* = \mathrm{J}_0(\lambda\rho)\left(R_i'(\lambda)e^{\mathrm{k}_i z} + T_i'(\lambda)e^{-\mathrm{k}_i z}\right) \tag{19}$$

The complete expressions for the magnetic Hertzian potentials are then the superposition of all solutions (by integration over all values of λ):

$$\Pi_i^* = \int_0^\infty \mathrm{J}_0(\lambda\rho)\left(R_i'(\lambda)e^{\mathrm{k}_i z} + T_i'(\lambda)e^{-\mathrm{k}_i z}\right)d\lambda \tag{20}$$

except for the layer containing the exciting source. $T_i'(\lambda)$ and $R_i'(\lambda)$ in Equation 20 are associated with positive and negative propagation directions and can be viewed as transmission and reflection coefficients based on the positive z direction, respectively.

To obtain the total potential for the layer that contains the exciting source (layer N), the vector potential due to the exciting source needs to be derived. Balanis showed that for a current distribution the vector potential **A** is (Balanis, 1989)

$$\Pi_N^* = \underbrace{\frac{IAe^{-\gamma_N r}}{4\pi r}}_{A} + \underbrace{\int_0^\infty \mathrm{J}_0(\lambda\rho)R_N{}'(\lambda)e^{\mathrm{k}_N z}d\lambda}_{B} \tag{21}$$

$$\mathbf{A}(x, y, z) = \frac{\mu}{4\pi}\int\int\int_V \boldsymbol{J}(x',\ y',\ z')\frac{e^{-jkR}}{R}dv' \tag{22}$$

Consider a current distribution J(x′, y′, z′) which is assumed to be within a linear, isotropic, and homogeneous media, as depicted in Figure 2. If the field point distance is small compared to the wavelength, then we have

$$e^{-jkR} = e^{-jk(R-r)}e^{-jkr} \approx (1 + jkr - jkR)e^{-jkr} \tag{23}$$

For an electric dipole vertically oriented with current I in the z direction, substituting the above back into Equation 22 and replacing the volume integral with the appropriate linear integral leads to

$$\mathbf{A}(x, y, z) = \frac{\mu}{4\pi}e^{-jkr}\left((1 + jkr)\oint\frac{\mathbf{I}dl}{R} - jk\oint\mathbf{I}dl\right) \tag{24}$$

Note that for very-low frequency (VLF), ultralow frequency (ULF), or extremely low frequency (ELF), the loop perimeter is much smaller than a wavelength, so any term containing k in the equation above is essentially trivial and can be neglected. The current carried by the dipole also can be assumed to be uniform and taken out of the integral. In addition, if the field point is sufficiently far from the source domain for it to look like a point source, then we have the approximation $R \rightarrow r$. Equation 24 can then be simplified to

$$\mathrm{A}\hat{z} = \frac{\mu}{4\pi}\frac{e^{-jkr}}{r}\oint\mathbf{I}dl = \frac{Il}{4\pi}\frac{e^{-jkr}}{r}\hat{z} \tag{25}$$

The vector potential of a magnetic dipole then can be obtained by applying a "duality" substitution of

$$Il \rightarrow j\omega M \tag{26}$$

where $M = NIA$ is the magnetic dipole moment, N is the number of turns of the loop, and A is the loop area. Therefore, the electric vector potential of a loop current is given by

$$\mathrm{F}\hat{z} = \frac{j\omega\mu M}{4\pi}\frac{e^{-jkr}}{r}\hat{z} = \frac{j\omega\mu M}{4\pi}\frac{e^{-\gamma r}}{r}\hat{z} \tag{27}$$

Note that jk is replaced with γ in the equation above for dissipative media. This is exactly the same as the formula given by Schelkunoff (Schelkunoff, 1943).

The magnetic Hertzian potential in layer N is then obtained through the superposition of all solutions. Since layer N is at the most negative position, only terms propagating in the $+z$ direction exists in part B in Equation 21. Utilizing the Sommerfeld Identity (Sommerfeld, 1964, Partial Differential Equations in Physics, p. 242), we have

$$\frac{e^{jk_0 r}}{r} = j\int_0^\infty \frac{k_\rho}{k_z} J_0(k_\rho \rho) e^{jk_z|z|} dk_\rho \tag{28}$$

where $k_z = \left(k_0^2 - k_\rho^2\right)^{1/2}$ (Chew, 1990). Using the replacements $k_0 \rightarrow j\gamma_N$ and $k_\rho \rightarrow \lambda$, term (A) in Equation 21, it can be rewritten as:

$$\frac{IAe^{-\gamma_N r}}{4\pi r} = \frac{IA}{4\pi}\int_0^\infty \frac{\lambda}{k_N} J_0(\lambda\rho) e^{-k_N|z+h|} d\lambda \tag{29}$$

in which γ_N, λ *and* k_N are restricted by Equation 16. Substituting Equation 29 into Equation 27, we can rearrange Equation 27 as

$$\Pi_N^* = \frac{IA}{4\pi}\int_0^\infty J_0(\lambda\rho)\left(\frac{\lambda}{k_N} e^{-k_N|z+h|} + R_N(\lambda) e^{k_N z}\right) d\lambda \tag{30}$$

For layer 0, the free space media (air) has a conductivity of zero. Now we have potentials Π_i^* for every layer.

$$\Pi_0^* = \frac{IA_{wire}}{4\pi}\int_0^\infty T_0(\lambda) e^{-\mathrm{k}_0 z} J_0(\lambda\rho) d\lambda \tag{31}$$

$$\Pi_1^* = \frac{IA_{wire}}{4\pi}\int_0^\infty \left(T_1(\lambda) e^{-\mathrm{k}_1 z} + R_1(\lambda) e^{k_1 z}\right) J_0(\lambda\rho) d\lambda \tag{32}$$

$$\Pi_i^* = \frac{IA_{wire}}{4\pi}\int_0^\infty \left(T_i(\lambda) e^{-\mathrm{k}_i z} + R_i(\lambda) e^{\mathrm{k}_i z}\right) J_0(\lambda\rho) d\lambda \tag{33}$$

$$\Pi_N^* = \frac{IA_{wire}}{4\pi}\int_0^\infty J_0(\lambda\rho)\left(\frac{\lambda}{k_N} e^{-k_N|z+h|} + R_N(\lambda) e^{k_N z}\right) d\lambda \tag{34}$$

where $R_N(\lambda) = \frac{4\pi}{IA} R_N{}'(\lambda)$ and $T_N(\lambda) = \frac{4\pi}{IA} T_N{}'(\lambda)$ are the reflection coefficient and transmission coefficient in layer N, respectively. They can be determined from the boundary conditions that require tangential E-fields and H-fields to be continuous across all layer interfaces, that is, $E_{\varnothing i} = E_{\varnothing, i+1}$ and $H_{\rho i} = H_{\rho, i+1}$ at $z = -H_i'$, in which $H_i' = \sum_{n=1}^{i} h_n$ the depth of the lower boundary of layer i, i = 1,2,…,N−1. By using Equations 10 and 11, the boundary conditions of the tangential EM field are then given via the magnetic Hertzian potential as:

$$\Pi^*_0 = \Pi^*_1, \quad \frac{\partial \Pi^*_0}{\partial z} = \frac{\partial \Pi^*_1}{\partial z}, \quad \text{at } z = 0 \tag{35}$$

$$\Pi^*_1 = \Pi^*_2, \quad \frac{\partial \Pi^*_1}{\partial z} = \frac{\partial \Pi^*_2}{\partial z}, \quad \text{at } z = -h_1 \tag{36}$$

$$\Pi^*_i = \Pi^*_{i+1}, \quad \frac{\partial \Pi^*_i}{\partial z} = \frac{\partial \Pi^*_{i+1}}{\partial z}, \quad \text{at } z = -H'_i \tag{37}$$

$$\Pi^*_{N-1} = \Pi^*_N, \quad \frac{\partial \Pi^*_{N-1}}{\partial z} = \frac{\partial \Pi^*_N}{\partial z}, \quad \text{at } z = -H'_{N-1} \tag{38}$$

After some rearrangement and simplification, $T_0(\lambda)$ can be expressed as shown below for any number N.

$$T_0(\lambda) = \frac{\lambda 2^N \left(\prod_{i=1}^{N-1} k_i\right)\left(e^{-k_N(h - H'_N) - \sum_{i=1}^{N-1} k_i h_i}\right)}{L_0 + L_1 + L_2 + \ldots + L_{N-1}} \tag{39}$$

The L_i's$(i = 0, 1, 2, \ldots, \mathrm{N} - 1)$ in the denominator are given in Appendix A.

Now we have a set of complete solutions for the H-field for the region in which we are interested. For layer 0, the conductivity of air is zero. Any displacement current can be dismissed. So $\gamma_0 = 0$ and $k_0 = \lambda$ by Equation 16. The H-fields in region 0 are given by Equation 11 and can be expressed as

$$\boldsymbol{H}_0 = H_{\rho 0}\hat{\rho} + H_{z0}\hat{z} = \frac{\partial^2 \Pi_0^*}{\partial \rho \partial z}\hat{\rho} + \frac{\partial^2}{\partial z^2}\Pi_0^*\hat{z} = b\left(P_0\hat{\rho} + Q_0\hat{z}\right) \tag{40}$$

where $b = M/2\pi h^3$. For the observation directly above the buried loop, b is the vertical H-field strength in free space. P_0 and Q_0 can be viewed as the transmission loss due to the presence of the lossy medium—the conductive Earth. Utilizing the Bessel function identity

$$\frac{\partial J_0(\lambda \rho)}{\partial \rho} = -\lambda J_1(\lambda \rho) \tag{41}$$

and Equation 40, we have the expression for P_0 and Q_0,

$$\{P_0, Q_0\} = \frac{h^3}{2}\int_0^\infty \lambda^2 T_0(\lambda) e^{-\lambda z}\{J_1(\lambda \rho), J_0(\lambda \rho)\} d\lambda \tag{42}$$

where $T_0(\lambda)$ is given by Equation 39.

From Equations 39 and A1 through Equation A6, it is not difficult to validate that the explicit H-field form of Equation 42 will reduce to Wait's half-space homogenous model while setting $N = 1$ (Wait, 1972), to Wait's two-layer formula while setting $N = 2$ (Wait & Spies, 1971), and to Shope's three-layer formula while setting $N = 3$ (Shope, 1982).

The numerical integrations of Equations 31–34 and 42 are straight given the property of each layer (such as the depth, conductivity, magnetic permeability) is known. However, the closed analytical form or the numerical technique for the integrals for two-layered media can also be found in Arutaki and Chiba (1980), King et al. (1981), and King et al. (2012).

3. Conclusions

It is usually hard to obtain all the information (especially the conductivity) for each layer in the multi-layer model. However, a multi-layer model (above three layers), instead of a two-layer or three-layer model, is closer to the reality since the Earth overburden is usually multiple stratified; hence, predicting the magnetic field over that multi-layered Earth is essential for TTE communication. In this study, the general solution of magnetic field above stratified Earth (number of layers greater than 3) for magnetic-loop TTE wireless communications was derived in detail based on potential theory. The analytical solution then can be numerically evaluated with selected parameters given. Since the signal propagation is reciprocal for TTE communication, the solution is also valid for the underground magnetic field while the loop antenna located at surface. These results can be used to predict and improve the performance of a TTE loop communication system.

Acknowledgments

The authors would like to thank all whose comments improved the paper.

Data Availability Statement

Data were not used, nor created for this research.

Appendix A

$$L_0 = \prod_{i=0}^{N-1} (k_i + k_{i+1}) \tag{A1}$$

$$L_1 = \sum_{p=1}^{N-1} \left\{ e^{-2k_p h_p} \prod_{i=0}^{N-1} \left(k_i + (-1)^{\delta 1} k_{i+1} \right) \right\} \tag{A2}$$

$$L_2 = \sum_{p=1,\, q=2,\, p<q}^{N-1} \left\{ \begin{array}{c} e^{(-2k_p h_p - 2k_q h_q)} \\ \times \prod_{i=0}^{N-1} \left(k_i + (-1)^{\delta 2} k_{i+1} \right) \end{array} \right\} \tag{A3}$$

$$L_3 = \sum_{p = 1, q = 2, r = 3, p < q < r}^{N - 1} \left\{ \begin{array}{c} e^{(-2k_p h_p - 2k_q h_q - 2k_r h_r)} \\ \times \prod_{i = 0}^{N - 1} \left(k_i + (-1)^{\delta 3} k_{i + 1}\right) \end{array} \right\} \tag{A4}$$

$$L_m = \sum_{p = 1, q = 2, r = 3, \ldots, t = m, p < q < r \ldots < t}^{N - 1} \left\{ \begin{array}{c} e^{(-2k_p h_p - 2k_q h_q - 2k_r h_r - \cdots - 2k_t h_t)} \\ \times \prod_{i = 0}^{N - 1} \left(k_i + (-1)^{\delta m} k_{i + 1}\right) \end{array} \right\} \tag{A5}$$

$$L_{N - 1} = e^{(-2k_1 h_1 - 2k_2 h_2 - 2k_3 h_3 - \cdots - 2k_{N - 1} h_{N - 1})} \times \prod_{i = 0}^{N - 1} \left(k_i + (-1)^{\delta(N - 1)} k_{i + 1}\right) \tag{A6}$$

Although there are no obvious physical meanings, the L_m can be loosely interpreted as the interaction within an arbitrary combination of m layers. The δ's in the equations above can be determined in a particular manner. In general, δ_m for L_m in Equation A5 is the sum of the occurrence of k_i and k_{i+1} in the exponential term of $e^{(-2k_p h_p - 2k_q h_q - 2k_r h_r - \cdots - 2k_t h_t)}$.

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Key Points:

- The general solution of magnetic field above stratified Earth for magnetic-loop through-the-Earth (TTE) wireless communications was obtained
- The solution is also valid for the underground magnetic field while the loop antenna located at surface due to the reciprocity of TTE signal propagation
- The result can be used to predict and improve the performance of a TTE loop communication system

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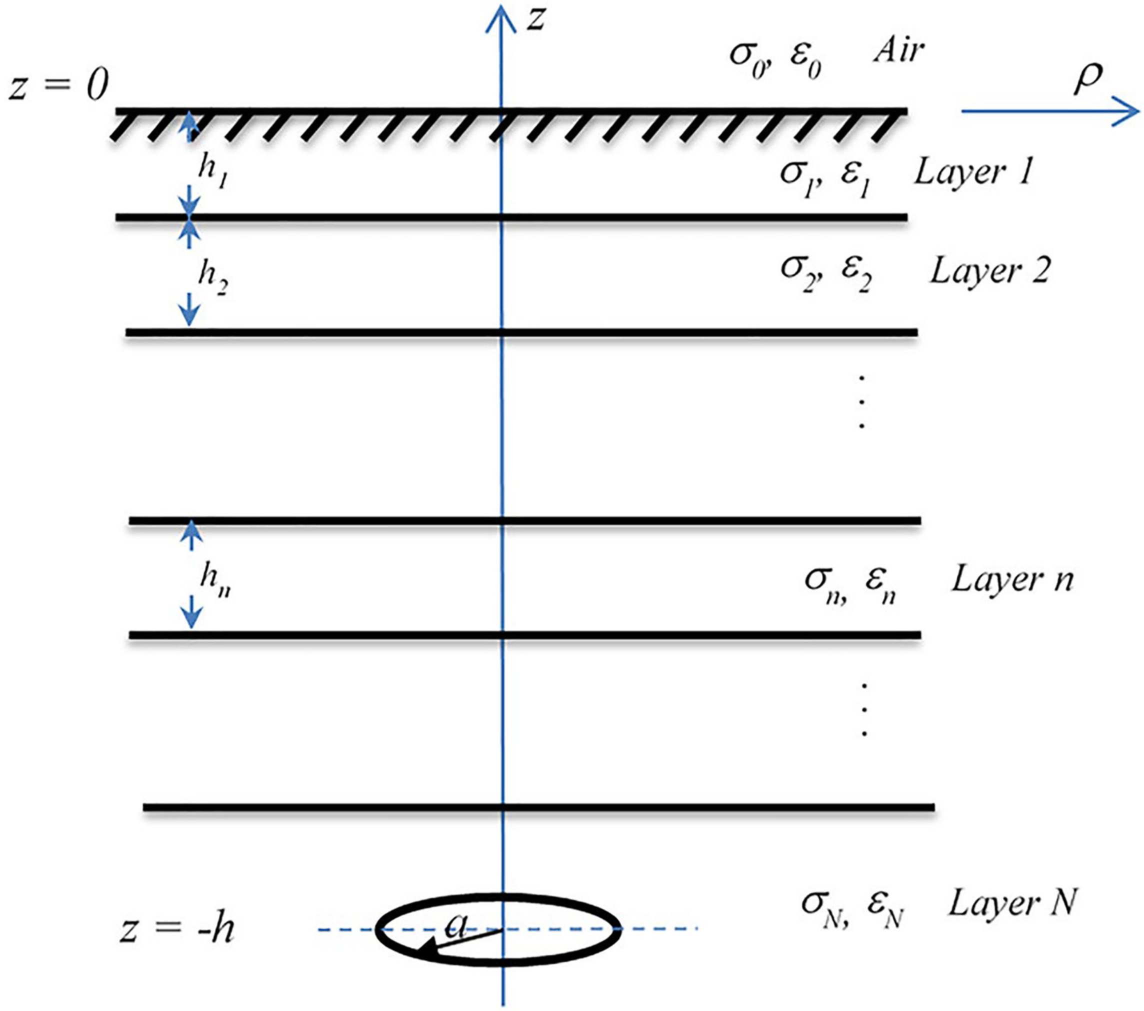

Figure 1.
Magnetic dipole buried in a semi-infinite region which is comprised of N ($N > 3$) layered Earth.

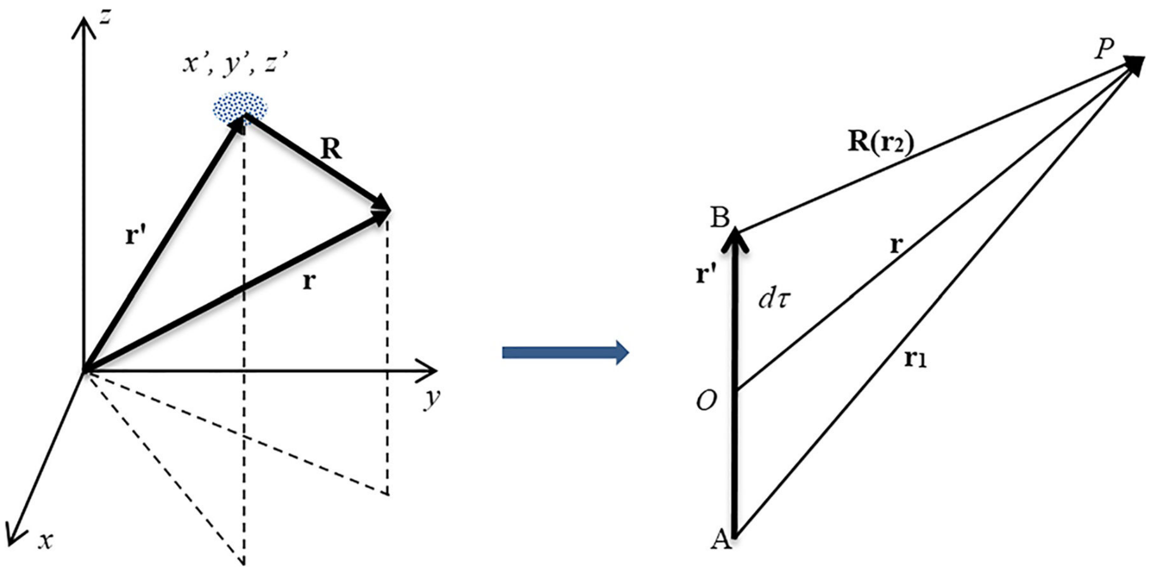

Figure 2.

Coordinate system for computing radiation fields due to source domain not at origin (left); and vector potential A(r) at point P is calculated by integrating the current distribution J(r') throughout region $d\tau$. The source region is an electric dipole AB (right).