**Supplementary Material for “Quantile Regression for Exposure Data with Repeated Measures in the Presence of Non-Detects”**

We now present simulation results that supplement the results presented in the manuscript. In Table S1, we examine the regression parameter estimation performances of the existing and proposed approaches. Empirical biases corresponding to these methods, empirical mean squared errors of estimates for , and relative efficiencies (REs) are provided in the table. Three scenarios are considered for . Scenario 1 for independence models without repeated measures assumes that are independent identically distributed and follows a log-chi-squared distribution with one degree of freedom (d.f.), . Scenarios 2 and 3 incorporate correlated errors for models with repeated measures and assume that the random error follows a multivariate log-chi-squared distribution with one d.f., , where the correlation parameter, , is 0.3 or 0.7.

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When the exposure outcome data had no repeated measures, RE results corresponding to the exposure outcome data had no repeated measures were similar to results of the highly skewed chi-squared distribution with one d.f. Overall, the quantile approach had relatively greater performances regardless of correlation, censoring proportion, and sample size.

**Table S1.** Results for case 6 in which a chi-squared distribution with one degree of

freedom was created for the outcome data with three repeated measures.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | |  | *α* = 0*.*3 |  |  |  | *α* = 0*.*7 |  |
| *n* | % Censor |  | LOD/2 | MLE | Quantile |  | LOD/2 | MLE | Quantile |
| 100 | 10 | Bias | -0.0693 | -0.0630 | -0.0001 |  | -0.0808 | -0.0472 | 0.0006 |
|  |  | MSE | 0.0052 | 0.0043 | 0.0001 |  | 0.0069 | 0.0025 | 0.0002 |
|  |  | *RE* | *1.000* | *1.201* | *39.85* |  | *1.000* | *2.718* | *33.98* |
|  | 20 | Bias | -0.0793 | -0.0646 | -0.0003 |  | -0.1152 | -0.0553 | -0.0011 |
|  |  | MSE | 0.0067 | 0.0045 | 0.0001 |  | 0.0137 | 0.0034 | 0.0002 |
|  |  | *RE* | *1.000* | *1.479* | *49.58* |  | *1.000* | *4.054* | *72.71* |
|  | 30 | Bias | -0.0857 | -0.0659 | -0.0022 |  | -0.1398 | -0.0651 | -0.0015 |
|  |  | MSE | 0.0078 | 0.0047 | 0.0001 |  | 0.0200 | 0.0046 | 0.0002 |
|  |  | *RE* | *1.000* | *1.661* | *57.71* |  | *1.000* | *4.357* | *100.0* |
|  | 40 | Bias | -0.0869 | -0.0665 | -0.0062 |  | -0.1542 | -0.0768 | -0.0056 |
|  |  | MSE | 0.0081 | 0.0048 | 0.0002 |  | 0.0244 | 0.0063 | 0.0002 |
|  |  | *RE* | *1.000* | *1.701* | *46.19* |  | *1.000* | *3.888* | *104.4* |
|  |  |  |  |  |  |  |  |  |  |
| 500 | 10 | Bias | -0.0703 | -0.0639 | -0.0011 |  | -0.0814 | -0.0475 | -0.0010 |
|  |  | MSE | 0.0050 | 0.0042 | .00003 |  | 0.0067 | 0.0023 | .00004 |
|  |  | *RE* | *1.000* | *1.209* | *167.3* |  | *1.000* | *2.890* | *167.9* |
|  | 20 | Bias | -0.0800 | -0.0654 | -0.0019 |  | -0.1156 | -0.0556 | -0.0020 |
|  |  | MSE | 0.0065 | 0.0043 | .00003 |  | 0.0135 | 0.0032 | .00004 |
|  |  | *RE* | *1.000* | *1.491* | *216.0* |  | *1.000* | *4.262* | *336.7* |
|  | 30 | Bias | -0.0861 | -0.0664 | -0.0040 |  | -0.1399 | -0.0654 | -0.0042 |
|  |  | MSE | 0.0075 | 0.0045 | .00004 |  | 0.0197 | 0.0043 | 0.0001 |
|  |  | *RE* | *1.000* | *1.675* | *187.4* |  | *1.000* | *4.527* | *328.1* |
|  | 40 | Bias | -0.0868 | -0.0669 | -0.0073 |  | -0.1547 | -0.0776 | -0.0073 |
|  |  | MSE | 0.0076 | 0.0046 | 0.0001 |  | 0.0241 | 0.0061 | 0.0001 |
|  |  | *RE* | *1.000* | *1.677* | *95.41* |  | *1.000* | *3.945* | *267.3* |

*a* Bias - empirical bias.

*b* MSE - empirical mean squared error.

*c RE* - relative efficiency. These are the italicized ratios that, for each setting (*n*), compare the empirical MSE from the LOD/2 substitution method to the MSE from the use of MLE method or quantile regression model.