

HHS Public Access

Author manuscript *Saf Sci.* Author manuscript; available in PMC 2021 September 21.

Published in final edited form as:

Saf Sci. 2021 January ; 133: . doi:10.1016/j.ssci.2020.105026.

An alternative method for analyzing the slip potential of workers on sloped surfaces

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Abstract

Slips and falls on sloped roof surfaces remain an important safety issue among construction workers. The slip potential has been conventionally analyzed and assessed primarily based on ground reaction forces, which cannot differentiate the specific roles of each of the force factors (e.g., workers' motions-induced dynamic forces and slope-induced static forces) contributing to the slip potential. Their differentiation may enhance the understanding of the slip mechanisms on the sloped roof surfaces and help develop effective walking and working strategies/tactics to minimize the dangerous slips on the elevated roofs. Hence, the objective of this study is to develop a biodynamic method as an additional tool for analyzing the slip potential of a worker walking or working on sloped roof surfaces. A whole-body biodynamic model is proposed and used to develop the alternative method, in which the slip potential is expressed as an analytical function of its major controlling factors including coefficient of friction, slope angle, and biodynamic forces. Some experimental data available in the literature are used to demonstrate the application of the proposed method. The results suggest that the slope may not change the basic trends of the biodynamic forces, but the slope may affect their magnitudes, which can be explained using the system's energy equation also derived from the whole-body biodynamic model. The analytical results suggest that reducing the body acceleration in uphill direction or the deceleration in downhill direction can reduce the slip potential. 'Zigging' and 'zagging' walking on a sloped surface may also reduce the slip potential, as it reduces the effective slope angle. The proposed biodynamic theory can be used to enhance the safety guidelines not only for roofers but also for people walking on ramps, inclined walkways, and mountain terrains.

Keywords

Slip; Fall; Sloped surface; Roofer; Walking safety; Roofer safety

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1. Introduction

Slips and falls remain very important safety issues among roof construction workers (Dong et al., 2014; BLS, 2016a; 2016b), which may happen while walking or working on sloped roof surfaces. A majority of the fatal and non-fatal injuries result from slips on sloped roof surfaces (Parsons et al., 1986; Hsiao and Simeonov, 2001; Hsiao, 2016). Slips and falls may also occur when walking on other sloped surfaces such as road ramps and inclined walkways both at work and public places (Lund, 1984; Pollard et al., 2015), especially when these surfaces are covered with contaminants or ice (Grönqvist and Hirvonen 1995; Gard and Lundborg, 2000). Sloped terrains have also caused hikers to suffer from injuries due to slipping and falling (Gardner and Hill, 2002; Boulware et al., 2003).

A large number of investigations on the slips and falls have been conducted, especially on level surfaces, and their basic mechanisms and characteristics have been understood (Li et al., 2019). In principle, a slip occurs at a foot of a person when the ground reaction force in the shear direction (tangential to contact surface) is greater than the available friction force that depends on the coefficient of friction at the foot-floor interface and the ground reaction force in normal direction (Redfern et al., 2001; Chang et al., 2016). The shear force can be either a traction force (in the walking direction) or a resistant force (opposite to walking direction); hence, the slip could occur in any direction with the maximum ratio of the shear force and normal force. While a minor slip for a short period of time may not result in a fall, a major slip may lead to a fall if the slip results in the loss of body balance and it cannot be recovered during the slip reaction (Hanson et al., 1999; Redfern et al., 2001; Allin et al., 2018).

On a level floor or surface, the shear force results fully from the human motions. Theoretically, it can be quantified from the distributed mass and acceleration of the human body according to Newton's second law; hence, it is termed as biodynamic force in this study. On a sloped surface, the shear force includes not only the biodynamic force but also a portion of the gravitational force. The addition must increase the slip potential. The slope also increases the slip potential through reducing the ground reaction force in the normal direction, as the gravitational portion of the normal force reduces with the increase in the slope angle. In addition to these physical effects, the slope may also change the characteristics of the gait and ground reaction forces (Harper et al., 1967; Kawamura et al., 1991; Sun et al., 1996; Breloff et al., 2019), which may also influence the potential of the slip and fall, especially in downhill walking (Cham and Redfern, 2002; Redfern et al., 2001). Roofers may also carry some tools and/or roofing materials when they walk on the roof surfaces, which may add substantial external static and dynamic forces on them. The wind on the elevated roof surfaces could also make the external forces worse.

The same as the assessment of the slip and fall potentials on a level surface, the risk assessment on a sloped surface has been conventionally performed primarily through measuring the ground reaction forces in the shear and normal directions, calculating their ratio, and comparing the ratio with the dynamic coefficient of friction (CF) between the foot and the contact surface (Harper et al., 1967; Grönqvist et al., 2001). While the ratio is usually termed as required CF (RCF or RCOF), the CF itself is termed as available CF

(ACF or ACOF) (Redfern et al., 2001). Some statistical relationship between their difference (ACF-RCF) and the probabilities of slip and fall have been reported (Hanson et al., 1999; Chang, 2004; Burnfield and Powers, 2006). The conventional method provides an effective tool for studying the instantaneous slip on each foot resulting from the combined effect of the slope and biodynamic forces. It, however, cannot differentiate the specific roles of each of these factors contributing to the slip potential. While a person may not be able to select or control the slope angle, he/she may use a good walking strategy and/or some tactics to reduce the biodynamic forces for minimizing his/her slip potential. The developments or identifications of such strategy and tactics require sufficiently understanding the biodynamic forces, their influencing factors, and their interactions with the slope angle and walking direction. However, no study has parceled out the biodynamic forces and their influencing factors present in sloped slipping risk and studied them individually. Furthermore, the reported studies only considered walking on a surface with a slope equal to or less than 20° in the straight uphill and downhill directions. A roofer may walk in any direction on a roof surface and the pitch may be greater than 20° . A theory for analyzing the slip potential in a general direction on the roof surface has not been established. Although some static models of a person walking on a sloped surface have been proposed and helped understand the ground reaction forces (Kawamura et al., 1991; Sun et al., 1996), a biodynamic model of the system has not been reported. Although biomechanics and kinematics have been increasingly considered in the analyses and assessments of the slips and falls in recent years (Chambers et al., 2003; Yang and Pai, 2014; Liu and Lockhart, 2014; Chang and Xu, 2018; Allin et al., 2018; Breloff et al., 2019), a kinetic theory of slip and fall has not been well established. Kinetics should be considered to study roofers' slip potential, especially when external dynamic forces acting on the roofers will be considered in the analyses and assessments.

Some of these issues and scientific gaps can be addressed through developing biodynamic force-focused theory and method for the analysis and assessment of the slip potential. Because the slope-induced changes actually reduce the available friction force for supporting the human motions on a sloped surface, the slip potential on the sloped surface can be studied by examining the biodynamic shear force required for supporting the human dynamic motions, and comparing it with the maximum friction force available for the human dynamic motions. The objective of this study is to implement this concept to establish the basic biodynamic theory and to develop an alternative method for helping analyze and assess the slip potential on a sloped surface. Some experimental data available in the literature were used to demonstrate the application of the developed method. A general mechanical energy equation is also proposed to help analyze and understand the biodynamic forces on the sloped surfaces. Based on the proposed theory, method, and results, some hypotheses for further studies are also proposed and discussed.

2. Method

2.1. The derivation of the basic equations for calculating the biodynamic forces

Fig. 1 illustrates a system model of a person walking in a direction (λ) on a sloped surface, which was proposed and used in the current study for developing the biodynamic theory

and alternative method for helping analyze the slip potential. A slip usually initiates on one foot in realistic scenarios; ideally, the slip potential for each of the feet should be considered separately in the biodynamic analysis. This, however, requires isolating the biodynamic forces from the ground reaction forces distributed on each of the two feet. Technically, this is extremely difficult, as the human body responds to all external forces collectively and the distribution of the biodynamic forces is very complex, which may explain why the biodynamic approach has not been seriously considered and the biodynamic forces have not been separately examined in previous studies of slips and falls. As an initial effort for developing the biodynamic approach, this study simplified the complex problem by considering the sum of the forces on both feet in the biodynamic analysis. As gravity on the entire human body is a constant static force, the sum of the biodynamic forces can be separated from the combined ground reaction forces acting on the feet. Although such a biodynamic method cannot be used to predict the instantaneous slip event on each foot within the stance phase with both feet in contact with the surface, it can be used to analyze the dependence of the overall slip potential of the human body on the coefficient of friction, slope angle, and biodynamic forces. The knowledge of the overall slip potential is considered the most important information guiding the workers to safely walking and

Using the symbols shown and defined in Fig. 1, the sum of the ground reaction forces in each direction can be written as follows:

working on actual roofs.

$$\begin{cases}
F_X = F_{X-L} + F_{X-R} \\
F_Y = F_{Y-L} + F_{Y-R} \\
F_Z = F_{Z-L} + F_{Z-R}
\end{cases}$$
(1)

As mentioned earlier, these ground reaction forces generally include the human biodynamic forces and a portion of the static gravitational force acting on the human body. Their specific formulas can be derived from the equations of motions written based on Newton's second law and the model shown in Fig. 1, which are as follows:

$$\begin{cases} F_{DX} = F_{\beta} \cdot \cos\beta = F_{X} \\ F_{DY} = F_{\beta} \cdot \sin\beta = F_{Y} - Mg \cdot \sin\alpha & \text{or} \\ F_{DZ} = F_{Z} - Mg \cdot \cos\alpha \end{cases} \begin{cases} F_{X} = F_{DX} \\ F_{Y} = F_{DY} + Mg \cdot \sin\alpha \\ F_{Z} = F_{DZ} + Mg \cdot \cos\alpha \end{cases}$$
(2)

where F_{DX} , F_{DY} , and F_{DZ} are the biodynamic forces in the three orthogonal directions. The vector sum of the biodynamic forces distributed in the X and Y directions is defined as the total biodynamic shear force (F_{β}). Its magnitude and direction can be determined using the following formulas:

$$\begin{cases} F_{\beta} = \sqrt{F_{DX}^2 + F_{DY}^2} \\ Tan\beta = F_{DY}/F_{DX} \end{cases}$$
(3)

It should be noted that although the walking direction (λ) is associated with the biodynamic forces, it is generally different from the direction of the biodynamic shear force (β). While the walking direction can be fixed, the β value may vary in the entire range from -180° to 180° in each walking step, as each of the shear forces in the X and Y directions may change from a positive value to a negative value or vice versa.

As above-mentioned, the human biodynamic forces in the three directions can also be calculated using the mass and acceleration distributed in the body (including carried tools and materials) in the three directions from the following formulas:

$$F_{DX} = \int a_X dm = M A_X$$

$$F_{DY} = \int a_Y dm = M A_Y$$

$$F_{DZ} = \int a_Z dm = M A_Z$$
(4)

where a_X , a_Y , and a_Z are the distributed accelerations in the three axial directions, dm is the distributed mass, and A_X , A_Y , and A_Z are the overall equivalent accelerations in the three directions.

2.2. The derivation of the equations for analyzing slip potential

The total ground shear force (F_S) required for standing or walking on a sloped surface is the vector sum of the ground reaction forces measured in the X and Y directions, and its magnitude can be calculated from

$$F_S = \sqrt{F_X^2 + F_Y^2} \tag{5}$$

For the purpose of this study, the dynamic coefficient of friction (μ) was assumed uniform at each foot in each direction. It is well known that the available or maximum dynamic friction force that can be used to support standing or walking is expressed as follow:

$$F_{Max} = F_Z \cdot \mu \tag{6}$$

In principle, the entire body mass of a person will not be at risk of slipping if $F_S < F_{MAX}$. Using Eqs. (5) and (6), this relationship can be expressed as follows:

$$\sqrt{F_X^2 + F_Y^2} < F_Z \cdot \mu \tag{7}$$

This equation can be alternatively written as follows:

$$(\sqrt{F_X^2 + F_Y^2})/F_Z < \mu \tag{8}$$

If the total ground reaction forces in this equation are replaced with those distributed on each foot, it becomes the equation conventionally used in the assessment of the slip and fall potentials, i.e.

$$\begin{cases} RCF_{Foot} = \frac{\sqrt{F_{X-Foot}^2 + F_{Y-Foot}^2}}{F_{Z-Foot}} & Foot = L, R \\ ACF = \mu \end{cases}$$
(9)

where RCF denotes required coefficient of friction and ACF denotes available coefficient of friction.

For the whole-body method considered in the current study, the RCF_{Total} ($=\sqrt{F_X^2 + F_Y^2}/F_Z$) for the entire body can be expressed as a function of the total biodynamic forces (F_{β} , F_{DZ}), slope angle (α), and vector direction of the biodynamic force (β) using Eq.(2), which is written as follows:

$$RCF_{Total} = \frac{\sqrt{\left(F_{\beta}cos\beta\right)^{2} + \left(F_{\beta}sin\beta + Mgsin\alpha\right)^{2}}}{\left(Mgcos\alpha + F_{DZ}\right)}$$
(10)

In this equation, the human biodynamic forces and slope geometric factors are coupled together. It is very difficult to analyze and understand the specific role of each factor in determining the slip potential from this equation. This is a limitation of the conventional approach.

This difficulty can be alleviated by using the biodynamic approach proposed in this study. The required formulas for the proposed method can be derived from Eq. (7). First, replacing the forces in Eq. (7) with those expressed in Eq. (2), we have the following equation:

$$\sqrt{\left(F_{\beta}cos\beta\right)^{2} + \left(F_{\beta}sin\beta + Mgsin\alpha\right)^{2}} < \left(Mgcos\alpha + F_{DZ}\right) \cdot \mu \tag{11}$$

Dividing this equation by the gravity force (Mg), Eq. (11) becomes

$$\sqrt{\left[\frac{F_{\beta}}{Mg} \cdot \cos\beta\right]^2 + \left[\frac{F_{\beta}}{Mg} \cdot \sin\beta + \sin\alpha\right]^2} < \left[\cos\alpha + \frac{F_{DZ}}{Mg}\right] \cdot \mu \tag{12}$$

In this equation, F_{β}/Mg is the biodynamic shear force normalized with respect to the body gravity. In order to express it as a function of the remaining factors, it can be resolved from Eq. (12) by assuming the remaining factors act as constants in the equation. Then, we have the following equation:

$$\left(F_{\beta}/Mg\right) < \sqrt{\left(\sin\beta\sin\alpha\right)^2 - \left(\sin\alpha\right)^2 + \mu^2 \left(\cos\alpha + F_{DZ}/Mg\right)^2} - \sin\beta\sin\alpha \tag{13}$$

This equation can be interpreted as follows: a slip will not occur if the normalized biodynamic shear force (on left side of the equation) is less than the normalized maximum friction force (on the right side of the equation) available for supporting the human motions on the surface. With Eq. (13), the slope problem is virtually transformed to a level problem, as the normalized biodynamic shear force becomes the focus of the analysis, similar to the analysis of the slip potential on a level surface. In fact, Eq. (13) is applicable to both sloped and level surfaces in any walking direction, as *a* can be any value in the range of 0° to 90°, and β can be any value in the range of -180° to 180° . Similar to that used in the conventional method, the normalized biodynamic shear force is termed as required biodynamic coefficient of friction (RBCF) in this study, and the normalized maximum biodynamic friction force is termed as available biodynamic coefficient of friction (ABCF), i.e.

$$\begin{cases} RBCF = \frac{F_{\beta}}{Mg} \\ ABCF = \sqrt{(sin\beta sin\alpha)^2 - (sin\alpha)^2 + \mu^2 \left(cos\alpha + \frac{F_{DZ}}{Mg}\right)^2} - sin\beta sin\alpha \end{cases}$$
(14)

If RBCF ABCF, the slip will occur in the direction (θ : measured from X axis) that can be determined using the following formula:

$$Tan\theta = F_Y/F_X = (F_{DY} + Mg \cdot sin\alpha)/F_{DX}$$
(15)

2.3. The mechanical energy of the human whole body

For each stance, the height change (H) in the travel direction (β_T), kinetic energy change (E_k), and potential energy change (E_p) can be expressed as follows:

$$\Delta H = L \cdot \sin\beta_T \cdot \sin\alpha,\tag{16}$$

$$\Delta E_k = \frac{1}{2}M \cdot \left(V_e^2 - V_i^2\right),\tag{17}$$

$$\Delta E_P = Mg \cdot \Delta H = Mg \cdot L \cdot \sin\beta_T \cdot \sin\alpha, \tag{18}$$

where L is step length, V_i is initial step speed, and V_e is end step speed.

In each stance, the mechanical energy ($E_{Traction}$) gained from the traction force ($F_{\beta-T}$: along walking direction) and the mechanical energy ($E_{Resistance}$) consumed by the friction resistant force ($F_{\beta-R}$: opposite to walking direction) are expressed as follows:

$$\Delta E_{Traction} = \int_{L} F_{\beta - T} \cdot dL = L \cdot \overline{F}_{\beta - T}$$
⁽¹⁹⁾

$$\Delta E_{Resistance} = \int_{L} F_{\beta - R} \cdot dL = L \cdot \overline{F}_{\beta - R}$$
(20)

where $\overline{F}_{\beta-T}$ is equivalent average traction force, and $\overline{F}_{\beta-R}$ is equivalent average resistant force.

The biodynamic forces result from the human motions, but the resulted mechanical energy cannot be converted back to the biological energy stored in the human body. Then, the four types of mechanical energy expressed in Eqs. (16)–(19) should comply with the conservation law of mechanical energy, which can be written as follows:

$$\Delta E_{Traction} = \Delta E_{Resistance} + \Delta E_k + \Delta E_P, \tag{21}$$

or

$$L \cdot \left(\overline{F}_{\beta-T} - \overline{F}_{\beta-R}\right) = 0.5M \left(V_e^2 - V_i^2\right) + Mg \cdot L \cdot \sin\beta_T \cdot \sin\alpha \tag{22}$$

Dividing Eq. (22) by *Mg*, the general mechanical energy equation of the body can be alternatively expressed as follows:

$$\left(\overline{F}_{\beta-T} - \overline{F}_{\beta-R}\right)/Mg = 0.5 \cdot \left[\left(V_e^2 - V_i^2\right)/L\right]/g + \sin\beta_T \cdot \sin\alpha$$
⁽²³⁾

Because the step length or distance (*L*) can be estimated from the time (*t*) consumed in each step and its corresponding average speed { $\bar{v} = 0.5(V_e + V_i)$ }, or $L = 0.5(V_e + V_i)$ } *t*, Eq. (23) can be alternatively expressed as follows:

$$(\overline{F}_{\beta-T} - \overline{F}_{\beta-R})/Mg = \overline{A}/g + \sin\beta_T \cdot \sin\alpha, \qquad (24)$$

in which \overline{A} is the average acceleration for a step and it can be calculated using the following formula:

$$\overline{A} = (V_e - V_i)/\Delta t = [0.5 \cdot (V_e + V_i) \cdot (V_e - V_i)]/[0.5 \cdot (V_e + V_i)\Delta t]$$

= 0.5 \cdot (V_e^2 - V_i^2)/L (25)

2.4. The measurement of the CF values for shoes on sloped roof surfaces

While the CF values for the roofers' footwear on the roof surfaces were not found in the current literature, a preliminary experiment was conducted to explore the possible range of the CF values for parametric study of the slope effects on the slip potential on the roof surface. For this purpose, a simple tilt testing method (Angle of Repose Method) shown in Fig. 2A was used in the measurement of the CFs for six pairs of shoes shown in Fig. 2B. The inclined angle was gradually increased by manually increasing the height of a piece of roof

wood board at its one end until the shoe on the board surface started to slide. The height (H) at which the shoe started to slide and the length (S = 813 mm) of the wood board were used to determine the CF value: $\mu = Tan(H/S)$. Besides the wood board itself (Fig. 2C), a piece of new asphalt shingle shown in Fig. 2D was clamped on the wood board to simulate a shingled roof surface in the CF measurement. Two trials were performed for each test treatment.

2.5. Ground reaction forces used in this study

As an example, a set of experimental data reported by Redfern and DiPasquale (1997) was used to demonstrate the application of the proposed alternative method. Besides a level surface (0°), four slope angles (5°, 10°, 15°, and 20°) were considered in the reported experiment with 15 healthy human subjects (20–30 years, 7 males and 8 females). The study measured the ground reaction shear and normal forces of the left foot during downhill walking for one step. In the current study, we assumed that the average ground reaction forces on the two feet were identical, except that they had a different stance phase. We also assumed that: (i) the reported ground reaction normal force was in the Z direction defined in Fig. 1; (ii) the shear force was in the Y direction; and (iii) the shear force in the X direction was negligible. Furthermore, the forces on the two feet were assumed to have 18% overlap at the beginning and end of each stance phase, as illustrated in Fig. 3. With these assumptions, the total ground reaction force of the two feet in each direction was calculated using Eq. (1), which is also illustrated in Fig. 3. The illustrated total ground reaction forces are comparable to those previously reported (Bake, 2013), suggesting the calculation method used in this study is valid.

3. Results

3.1. Possible CF values of shoes on roof surfaces

Table 1 lists the static CFs of the six shoes measured in the preliminary experiment on the two roof surfaces. The CF values varied in a large range (0.63 to 1.03) and specific values depended on both shoe model and surface material. In the preliminary experiment, the footwear advertised for roofers (Fig. 2B: 1,2,3) generally had a higher CF value than the non-roofer footwear (Fig. 2B: 4,5,6) tested in this study.

3.2. Results of parametric studies

As shown in Eq. (14), the RBCF is fully separated from the slope geometrical factors and they are included in the ABCF formula. Although the ABCF includes the biodynamic normal force (F_{DZ}), it has no interaction term with the slope factors in the equation; therefore, it can be ignored (or $F_{DZ} = 0$) in the initial parametric study for identifying and understanding the basic roles of the CF, slope angle, and biodynamic force direction in determining the slip potential. Then, the ABCF formula expressed in Eq. (14) can be simplified as follows:

$$ABCF = \sqrt{(\sin\beta\sin\alpha)^2 - (\sin\alpha)^2 + \mu^2(\cos\alpha)^2} - \sin\beta\sin\alpha$$
⁽²⁶⁾

For demonstration purposes, Fig. 4 shows the ABCF as a function of the slope angle (*a*) calculated using Eq. (26) for several special cases (CF or $\mu = 0.6, 0.7, 0.8; \beta = -90^{\circ}, -30^{\circ}, 0^{\circ}, 30^{\circ}, 90^{\circ}$). The CF values in these cases are in the possible range of CF values seen on roof surfaces, as shown in Table 1. A roofer may also walk in any direction on the roof surface considered in the parametric study. As expected, ABCF = μ for a level surface (*a* = 0). As also expected, the ABCF increases proportionally with the increase in the CF value. For a given CF value, the effect of the slope on the ABCF depends on the direction of the biodynamic shear force or β value.

When $\beta = 90^\circ$, the biodynamic shear force (F_β) is in the straight uphill direction, as defined in Fig. 1. It provides the full resistance for downhill walking or the full traction for uphill walking. For this direction, Eq. (26) can be simplified as follows:

$$ABCF_{\beta = 90^{\circ}} = \mu \cos\alpha - \sin\alpha = \cos\alpha(\mu - \tan\alpha) \tag{27}$$

The corresponding $ABCF_{\beta=90^{\circ}}$ is the lowest among all the possible directions of the biodynamic shear force for each given slope angle, as shown in Fig. 4. It decreases almost linearly with the increase of the slope angle measured in degrees. According to Eq. (13), the less the ABCF is, the higher the slip probability or the less the safety margin will be. Fig. 4 also indicates that no one can stand on the sloped surface when the slope angle is beyond a certain value that depends on the CF value.

When $\beta = -90^\circ$, the biodynamic shear force is in the straight downhill direction. It provides a portion of the resistance for uphill walking or a portion of the traction for downhill walking. The remaining portion is provided by the gravity on the slope. As a result, the corresponding ABCF_{$\beta = -90^\circ$} is the largest one at each slope angle, as also shown in Fig. 4. For this case, Eq. (26) can be simplified as follows:

$$ABCF_{\beta} = -90^{\circ} = \mu \cos\alpha + \sin\alpha = \cos\alpha(\mu + \tan\alpha)$$
(28)

When $\beta = 0^{\circ}$ (or 180°), Eq. (26) can be simplified as follows:

$$ABCF_{\beta=0} = \cos\alpha\sqrt{\mu^2 - (\tan\alpha)^2}$$
⁽²⁹⁾

As also shown in Fig. 4, the corresponding $ABCF_{\beta=0^{\circ}}$ is substantially larger than that for $\beta = 90^{\circ}$ but it reduces exponentially with the increase in the slope angle.

When $0^{\circ} < \beta < 90^{\circ}$ (or $90^{\circ} < \beta < 180^{\circ}$), the ABCF is usually for the biodynamic shear force in a cross-slope walking for uphill traction or downhill resistance. The corresponding ABCF value is in the range between the two curves for $\beta = 0^{\circ}$ and $\beta = 90^{\circ}$, as also shown in Fig. 4. When $-90^{\circ} < \beta < 0^{\circ}$ (or $-180^{\circ} < \beta < -90^{\circ}$), the ABCF is for the biodynamic shear force in a cross-slope walk for uphill resistance or downhill traction. The corresponding ABCF value is in the range between the two curves for $\beta = -90^{\circ}$ and $\beta = 0^{\circ}$.

To demonstrate the effect of the normalized biodynamic normal force (F_{DZ}/Mg) on the ABCF, two different values (0.2 and -0.2) were assumed in the calculation using Eq. (14), which are close to the peak values observed in the data presented in the next subsection. For demonstration purpose, CF was taken as 0.7, which is close to the CF values for many shoes on a regular floor. The results are illustrated in Fig. 5. The ABCF increases nonlinearly with the increase in the value of F_{DZ}/Mg . The nonlinearity becomes more obvious with the increase in the slope angle. The normalized biodynamic normal force (F_{DZ}/Mg) , however, does not substantially change basic trends of the effects of the slope and walking direction on the ABCF.

3.3. The results calculated from the reported experimental data

Fig. 6 shows the ground reaction forces of the two feet for the five slope angles, which were calculated using the above-described method illustrated in Fig. 3. The positive value of the shear force is a resistant force for the downhill walking (Redfern and DiPasquale, 1997). Therefore, it is along the Y direction defined in Fig. 1. As the shear force along the X direction is assumed negligible, the β angle is equal to 90°. On the other hand, $\beta = -90^{\circ}$ for the negative shear force value.

The two forces for each slope angle can be input to the formulas in Eqs. (2) and (3) to calculate the biodynamic force values. Then, Eq. (14) can be used to calculate the RBCF value and the ABCF values in the positive ($\beta = 90^{\circ}$) and negative ($\beta = -90^{\circ}$) force directions for each of the five slope angles for a given CF value (μ). As examples, the results calculated with $\mu = 0.9$ are plotted in Fig. 7. As expected, the RBCF or normalized shear force for each slope angle was approximately centered at the zero line, similar to that on a level surface. Their basic trends are also similar to those on the level surface. However, the range of the shear force peak values (the maximum peak shear - the minimum peak shear) generally increased with the increase in the slop angle. This was further confirmed from the identification of the peak-to-peak value of the RBCF for each angle, which are listed in Table 2. The basic observations for RBCF also generally held true for the peak values of the normalized biodynamic normal force (F_{DZ}/Mg), which are also listed in Table 2.

As shown in Fig. 7, the ABCF for $\beta = 90^{\circ}$ is taken as positive value. The minimum difference between the ABCF and the positive RBCF [Min (ABCF - RBCF)] can be used to represent the safety margin for downhill slip. Similarly, the ABCF for $\beta = -90^{\circ}$ is taken as negative value. Its minimum difference with the negative RBCF value represents the safety margin for uphill slip. Besides the minimum differences for $\mu = 0.9$, those for several other CF values (0.6, 0.7, and 0.8) are also listed in Table 2. As expected, the differences or safety margins for each slope angle increase with the increase in the CF value.

As also shown in Fig. 7, the two ABCFs are symmetrical to the zero line for the level surface (0 degree of slope). However, the positive ABCF curve generally moves towards the RBCF curve with the increase in the slope angle, but the negative ABCF moves away from it. This is also reflected in their minimum differences listed in Table 2. This means the safety margin for downhill slip reduces with the increase in the slope angle but there is a lower possibility for uphill slip when the slope angle is increased.

4. Discussion

Before this study, it has been generally understood that the slip potential on a sloped surface depends on the dynamic CF, slope angle, ground reaction shear force, and ground reaction normal force (Redfern et al., 2001). The current study replaced the ground reaction forces with biodynamic forces and made it clear that the direction of the biodynamic shear force is also an important factor in determining the slip potential on a sloped surface, as shown in Figs. 4 and 5. Also importantly, this study incorporated all these factors into the proposed theory and method, which makes it possible to quantitatively analyze their specific roles in determining the slip potential. These new developments enhance the understanding of the biomechanics of slips, which may be useful for further developing more objective methods and technologies for preventing slips and falls on sloped surfaces at workplaces and public places. Also elaborated and discussed in this section, these new developments brought about new topics and hypotheses for further studies.

4.1. The role of coefficient of friction in determining the slip potential

This study confirmed that the dynamic CF is certainly the most important factor determining the slip potential, as reflected in Eqs. (13), (14). The CF values listed in Table 1 suggest that the CF can vary in a fairly large range on the same surface but with different models of footwear. Therefore, wearing appropriate shoes or boots with sufficient CF should be considered as the first practical and affordable measure for minimizing the slip potential, especially for people working on the sloped roof surfaces. Besides footwear, the roof materials can affect the CF, as also shown in Table 1. Other factors such as a wet roof condition, temperature, friction direction, footwear orientation, individual weight and load may also affect the dynamic CF. Unfortunately, although several methods and devices for the CF measurement have been developed and extensive studies have reported the footwear CFs on various floors (Chang et al., 2016; Gronqvirst et al., 1989; Pollard et al., 2015; Beschorner et al., 2019, 2020), a systematical investigation on the dynamic CF under these conditions has not been reported in a sloped roofing environment. This is obviously an urgent and important research project. It should be noted that the CF values listed in Table 1 may not be used for any serious risk assessment because they are static CF and they may not be accurate. A more rigorous and reliable method should be considered to measure the in-situ dynamic CF of the shoes and boots on various roof surfaces in further studies.

4.2. The roles of slope angle and walking direction in determining the slip potential

As shown Fig. 4, the available biodynamic coefficient of friction (ABCF) for supporting the human motions on the sloped surface depends on the direction of the biodynamic shear force. In the worst case ($\beta = 90^\circ$), the ABCF reduces almost linearly with the increase in the slope angle. This suggests that the slope angle is almost as important as the CF in determining the slip potential.

The worst case, however, usually occurs only when walking along a straight line in the uphill or downhill direction, when the shear force in the X direction is negligible. In such a scenario, the direction of the major traction or resistant force required for walking is aligned with the direction of the slope-induced additional biodynamic force in the Y direction.

In any other walking direction, their directions must be different, as the required walking traction or resistant force should be approximately aligned with the working direction. This indicates that the β angle in any other walking direction is usually not equal to 90° and the difference should also increase with the change of the walking angle (λ) from the Y direction (Fig. 1). This suggests that the slope effect can be reduced by using a 'zigging' and 'zagging' walking strategy, according to the effect of the β angle on the ABCF shown in Fig. 4. This walking strategy, however, need further studies to help optimize its application for the following reasons: (1) the β angle in the critical walking phase is unlikely to be reduced to zero even when walking along the X direction, as the biodynamic force in the Y direction may not be negligible on a sloped surface; (2) the cross-slope walking may increase the injury potential of some substructures of the human body (Breloff et al., 2019); and (3) this walking strategy will not have much value when the slope angle reaches a certain value determined by the CF value, which is also shown in Fig. 4. The vast majorities of residential roofs in the USA have a pitch of 18° (4/12) or higher. The results shown in Fig. 4 suggest that it is very important to use some fall protection devices when walking on a high pitch roof.

4.3. The roles of biodynamic forces in determining the slip potential

The magnitudes of the RBCF shown in Fig. 7 suggest that the biodynamic forces usually require less than 0.2 CF during a regular walking on a surface with a slope angle at less than 20°. This is unlikely to be a critical issue if the selected shoes have a dynamic CF at more than 0.8 on such surfaces. However, the biodynamic forces on a high pitch roof can become critical as the safety margin is substantially reduced, especially when a worker transports some heavy materials (e.g., shingles, heavy tools) on the sloped roofs. Also critically, some high transient shear force peaks may occur in the heel striking phase on the front foot or the toe lifting phase on the back foot, which may initiate the slips and cause falls (Redfern et al., 2001). Such slips are considered the riskiest in the downhill walking. There are also some potentials to reduce the biodynamic forces using walking strategies and/or through optimized designs of footwear (Redfern et al., 2001).

In the experiment reported by Redfern and DiPasquale (1997), the walking speed did not change during the measurement of the ground reaction forces. The kinetic energy should not have significantly changed; Eq. (24) for this case can be simplified as follows: $(\overline{F}_R - \overline{F}_T)/Mg = sin\alpha$. This equation means that the average resistant force (F_R) must be greater than the average traction force (F_T) if the slope angle is not equal to zero in downhill walking. Also, their difference must increase with the increase in the slope angle. This is consistent with the effect of the slope on the RBCF's peak-to-peak values listed in Table 2. As also shown in Table 2, the biodynamic normal force increased with the increase in the slope angle; this was likely to be because the slope-induced dropping height also increases with the slope angle and the increased dropping height should increase the normal force (Redfern et al., 2001).

As shown Eq. (4), the biodynamic forces are directly related to the accelerations of the human body. Hence, reducing the body accelerations is equivalent to reducing the biodynamic forces or slip potential. The effect of the β angle on the ABCF shown in

Fig. 4 suggest that reducing the body acceleration in the Y or uphill direction or the deceleration in the –Y or downhill direction can most effectively reduce the slip potential. The body acceleration or deceleration of the workers may be affected by many factors. As above-mentioned, the footwear may play an important role in determining the biodynamic forces and/or the stability of the human body on the sloped surfaces. Besides the CF, the influences of the footwear on the biodynamic forces and body stability, together with its comfort and other safety features, should also be considered to obtain an optimized selection of the footwear for the workers.

According to the energy equation (Eq. (23)), the walking speed itself should not be an issue as long as it will not involve in any change of the speed or acceleration. In fact, the acceleration in downhill walking is helpful to reduce the slip potential. One can prove this by testing following the hypothesis: one may slip and fall on a sloped terrain even if he/she walks down slowly and carefully; however, he/she can avoid the slip by walking running down on the same terrain with an increasing speed, then, gradually slowing down on a flat area. This is because the potential energy and traction force are transformed into the kinetic energy such that the resistant force was reduced to avoid the slip, as dictated by Eq. (23) or Eq. (24). This hypothesis, however, should not be tested on a sloped roof surface because there is no flat or uphill area following the slope area for one to gradually slow down on the roof.

The energy equation (Eq. (24)) suggests that the starting or stopping walking process usually corresponds to a larger biodynamic shear force than that during the walking with a constant speed, because there is certain acceleration involved in the process. This mechanism may be very important for the roofers, as they frequently start and stop walking on a sloped surface. It could also be one of the possible reasons that some slip and fall incidents happened during the transition from a ladder to the sloped roof surface (Hsiao, 2016). These observations suggest that it is very important to examine the body accelerations and the biodynamic forces in the starting and stopping processes in further studies of the roofers' safety.

4.4. Other implications

As dictated by Eqs. (3) and (15), generally, $\theta \quad \beta \quad \lambda$ on a sloped surface. This indicates that the slip direction (θ) could be substantially different from the intended walking direction (λ) and the overall acceleration direction of the body (β). This may be one of the mechanisms that may increase the potential of slip injury on a sloped surface.

4.5. Some special notes

While the ground reaction forces simultaneously measured on both feet were not available for this study, the total ground forces assembled in this study may not accurately represent any real situation. While this should not affect the purpose of the application example, the values listed in Table 2 may only be used to help understand the general trends of the effects of the major factors on the slip potential.

It should also be emphasized that the purpose of the proposed biodynamic method is not to replace the conventional method but to provide an additional tool to help analyze and understand the overall slip potential. In fact, these two methods can be complementary

to each other. For example, while the conventional method is effective for detecting the potential instantaneous slip event on each foot that may occur during the heel striking phase during a normal walking, the proposed biodynamic method may provide a reasonable evaluation of the overall slip potential by considering the walking or working conditions (CF, slope angle, and possible range of biodynamic forces) without conducting any experiment. While the knowledge on the heel striking slip can help design a better heel of the footwear, the knowledge on the overall slip potential can provide a guidance to improve workers' safe practices at workplaces. The conventional method may overestimate the risk of slip, as the single-foot slip probability is usually much larger than the fall probability (Hanson et al., 1999); on the other hand, the biodynamic method may underestimate the risk, as it cannot detect the slip event on one foot during the stance phase with two feet in contact with the surface. These observations suggest that both methods should be used in further studies.

5. Conclusion

The current study proposed a novel biodynamic method for analyzing and assessing slip potential on a slope surface. It enhanced the understanding of the biomechanics of slips in the following aspects: (1) it formulated a basic biodynamic theory for studying and understanding the slips on a sloped surface; (2) it developed an alternative method for analyzing the slip potential in any walking direction on the sloped surface; (3) it clearly identified mechanical effects of the slope angle and biodynamic force direction on the slip potential; and (4) it proposed to analyze and understand the biodynamical forces and their related slip potential from a view of mechanical energy. The proposed biodynamic method can be complementary to the conventional method for the analysis and assessment of slip potential.

This study confirmed that the most important physical factor that determines the slip potential on a sloped surface is the CF. This suggests that the selection of appropriate footwear with a high CF value should be considered as the first intervention method for workers working on sloped surfaces, as it is probably the least expensive, easily applicable, and most effective method to reduce slipping risk. As the CFs of footwear on various roof surfaces have been far from sufficiently studied, it is recommended to consider their measurements in further studies. The results of the study suggest that the slope angle as the second most important factor in determining the slip potential. A zigging and zagging walking strategy is likely to reduce the slip potential, as it reduces the effective slope angle. This walking strategy, however, may need further studies to help optimize its application, as the cross-slope walking may increase the injury potential of some substructures of the human body. Furthermore, this walking strategy becomes ineffective when the slope angle reaches a certain level. It is essential to use some fall protection devices when walking on a high pitch roof. The biodynamic shear force is generally ranked as the third important factor in determining the slip potential on sloped surfaces, but it may become a critical factor on a high pitch roof. The results of this study revealed some interactions between the slope angle and the biodynamic forces. While the slope may not change the basic trends of the human biodynamic forces, the slope may influence their peak magnitudes. Theoretically, the biodynamic forces are directly related to the accelerations of the human body. Any measure

that can reduce the biodynamic forces or body accelerations can reduce the slip potential. The footwear may play an important role in determining the biodynamic forces and the body stability on the sloped surfaces, which should also be further studied to optimize the selection of the footwear.

6. Disclaimers

The findings and conclusions in this manuscript are those of the authors and do not necessarily represent the views of the National Institute for Occupational Safety and Health, Centers for Disease Control and Prevention.

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Fig. 1.

A whole-body biodynamic model of a person walking in a direction on a sloped surface: its global coordinate system has its X along the cross-slope direction, Y in the straight uphill direction, and Z perpendicular to the contact surface; A_X , A_Y , and A_Z are the overall accelerations of the person in the three coordinate directions; F_X , F_Y , and F_Z represent the total ground forces in the three directions; R and L represent the right foot and left foot, respectively; a is slope angle; β represents the direction of the biodynamic shear force (F_β); λ is the walking direction; M is the mass of the person and carried tools or materials; and g (9.801 m/s²) is gravity acceleration.

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(C)





Fig. 2.

The measurement of the coefficients of friction of six shoes on simulated roof surfaces: (A) Measurement principle; (B) Footwear tested for CF: 1-3 are roofer footwear and 4-6 are not considered roofer footwear; (C) Non-roofer footwear on oriented strand board (OSB); and (D) Roofer footwear on a standard asphalt shingle.

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Fig. 3.

The method for calculating the total ground reaction force in each direction: the measured left foot ground force reported by Redfern and DiPasquale (1997) + the right foot ground force that was assumed to be the same as that on the left foot except that its phase difference (with 18% overlap at the beginning and end of that for the left foot force); the forces were normalized with respect to body mass, the same as that in the original data.





The ABCF as a function of the slope angle for several special cases ($F_{DZ} = 0$; CF = 0.6, 0.7, 0.8; and $\beta = -90^{\circ}, -30^{\circ}, 0^{\circ}, 30^{\circ}, 90^{\circ}$).



Fig. 5.

The effect of the biodynamic normal force (F_{DZ}) on the ABCF as a function of the slope angle for several special cases (CF = 0.7; and $\beta = -90^{\circ}, -30^{\circ}, 0^{\circ}, 30^{\circ}, 90^{\circ}$).



Fig. 6.

The total ground forces for different slope angles $(0^{\circ}, 5^{\circ}, 10^{\circ}, 15^{\circ}, \text{and } 20^{\circ})$, which were derived using the method illustrated in Fig. 3 from the experimental data reported by Redfern and DiPasquale (1997).



Fig. 7.

The required biodynamic coefficient of friction (RBCF) and its corresponding available biodynamic coefficient of friction (ABCF) with $\mu = 0.85$ for resisting downhill slip ($\beta = 90^{\circ}$) or uphill slip ($\beta = -90^{\circ}$) for each of the slope angles (0° , 5° , 10° , 15° , and 20°).

Table 1

The coefficients of friction of six models of shoes measured in the preliminary experiment.

Shoe ID	Description	Wood board	Shingle
1	Models 1, 2, & 3 are advertised as roofers'	1.03	0.95
2	shoes.	0.87	0.95
3		0.74	0.97
4	Models 4, 5, & 6 are ordinary walking shoes.	0.80	0.87
5		0.65	0.95
6		0.63	0.78

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Table 2

margin (the minimum difference between ABCF and RBCF) calculated using the data reported by Redfern and DiPasquale (1997), by assuming the CF The normalized biodynamic forces in shear direction (RBCF: peak-to-peak value) and normal direction ($F_{DZ}Mg$: peak-to-peak value), and the safety $(\mu) = 0.6, 0.7, 0.8, \text{ and } 0.9.$

Slope angle (deg.)	RBCF (Peak-to-peak)	F_{DZ}/Mg (Peak-to-peak)	Min(AB(CF-RBCF)						
			$\mu = 0.6$ $\beta = 90^{\circ}$	∘06- = ¢	$\mu=0.7$ $\beta=90^{\circ}$	∘06-= ¢	$ \begin{array}{c} \mu = 0.8 \\ \beta = 90^{\circ} \\ \beta = -90^{\circ} \end{array} $		$\mu = 0.9$ $\beta = 90^{\circ}$ $\beta = -90^{\circ}$	
0	0.32	0.29	0.42	0.44	0.51	0.54	0.61	0.64	0.70	0.74
5	0.39	0.44	0.38	0.48	0.46	0.58	0.54	0.68	0.62	0.78
10	0.45	0.51	0.30	0.52	0.38	0.62	0.46	0.71	0.53	0.78
15	0.44	0.54	0.25	0.58	0.32	0.68	0.40	0.76	0.48	0.83
20	0.47	0.53	0.16	0.60	0.25	0.69	0.32	0.77	0.39	0.84