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Three-Dimensional Shield Mechanics

By Thomas M. Barczak and W. Scott Burton

UNITED STATES DEPARTMENT OF THE INTERIOR

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THREE-DIMENSIONAL SHIELD MECHANICS

By Thomas M. Barczak[1] and W. Scott Burton[2]

ABSTRACT

This Bureau of Mines report describes research on developing technology to utilize shields as roof load monitors. Three-dimensional equations of static quilibrium are presented for a generic two-legged longwall shield. The equations are presented in the most general form, assuming a force and moment vector at each reaction by rigid-body analysis of the shield mechanics. The resulting system of equations was found to be indeterminate to a very high degree. A three-dimensional solution to measurement of the resultant reaction on the canopy by the strata activity and shield response required the elimination of several forces or moments that were thought to be nonparticipating or to not significantly affect overall shield mechanics. Required measurements for solution of the equilibrium equations were leg, canopy capsule, and lemniscate link forces. Two-dimensional analysis, which considered forces acting only in the roof-to-floor and face-to-waste plane, was also provided by reduction of the three-dimensional equations.

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INTRODUCTION

The Bureau of Mines is conducting research to evaluate the use of longwall roof supports as roof load monitors in an effort to develop a better understanding of the support requirements associated with strata loading and support-strata interaction.

Past efforts to use roof supports as load-sensing devices have generally been limited to one-dimensional analysis by summation of measured leg forces to obtain a vertical (roof-to-floor) reaction to describe support resistance to applied strata loading. Modern longwall supports, such as the shield, are designed to resist both roof-to-floor (vertical) and face-to-waste (horizontal) loading, requiring two-dimensional analyses to determine vertical and horizontal support reactions. The Bureau has demonstrated that vertical and horizontal support resistance can be reasonably determined from two-dimensional modeling of the support structure by static rigid-body analysis.

The two-dimensional model assumes there are no lateral (parallel to the face) loads acting on the support and does not consider the effect of moment loading due to imbalances in leg and lemniscate link forces. These limitations are overcome by three-dimensional modeling of the support structure in which participation of out-of-plane forces and associated moments is considered, in theory permitting determination of reactions in three dimensions, i.e., vertical (roof-to-floor), horizontal (face-to-waste), and lateral (parallel to face). Unfortunately, the advancement of a three-dimensional model is not without difficulties. In the derivation of rigid-body static models in three dimensions, the accumulation of unknowns far outpaces the available force and moment equilibrium equations, making the system statically indeterminate. Assumptions are therefore required regarding the participation of some forces and moments to reduce the system's indeterminacy.

This report first presents the model in full form, depicting all the forces and moments theoretically acting on a shield structure. It then proceeds to eliminate forces and moments that realistically do not significantly participate to provide a determinate solution for three-dimensional resultant force vector determinations. The model is also reduced to a two-dimensional state by elimination of out-of-plane forces and moments.

THREE-DIMENSIONAL RIGID-BODY SHIELD MODEL

Figure 1 shows the configuration of a generic, two-legged shield, illustrating major shield components. In the following pages, the static, rigid-body equations of equilibrium are presented for the canopy, caving shield, and base components of the shield; tables 1-3 give the respective nomenclature descriptions. Each component is considered separately with illustrations of component forces and moments. Equations are provided in both vector and scaler component form. Although the shield is a pin-jointed structure, the equations are presented in the most general form by assuming that reaction moments occur at all joints due to pin friction. Hence a joint is considered to have six degrees of freedom, consisting of three force components and three moment components. With 2 pin contacts, 1 capsule contact, 2 leg contacts, and 1 assumed reaction contact, there are 18 force and moment vectors that must act in force and moment equilibrium on the canopy alone. The only simplification provided in this analysis is the combination of like forces and moments, such as left and right leg forces and left and right link forces, into single, equivalent force-couple systems acting at the component (canopy, caving shield, or base) centerline.

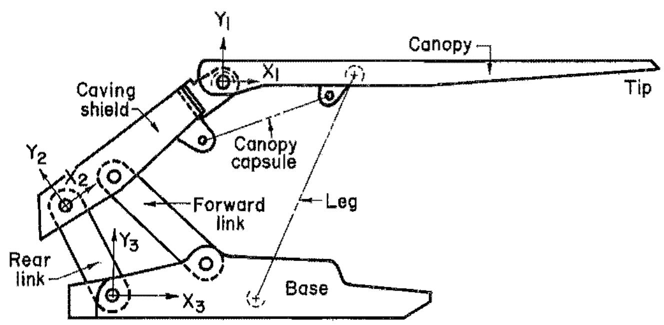

FIGURE 1.—Shield components and coordinate reference systems.

CANOPY

TABLE 1. - Canopy nomenclature description

Symbol	Vector represented	Scaler components
$\tilde{P}$	Canopy hinge pin force..................................	P_{x1}, P_{y1}, P_{z1}
$\tilde{Q}$	Canopy capsule force..................................	Q_{x1}, Q_{y1}, Q_{z1}
$\tilde{L}$	Leg cylinder force..................................	L_{x1}, L_{y1}, L_{z1}
$\tilde{R}$	Resultant force..................................	R_{x1}, R_{y1}, R_{z1}
$\tilde{M}_P$	Canopy hinge pin moments..................................	M_{Px1}, M_{Py1}, M_{Pz1}
$\tilde{M}_Q$	Canopy capsule moment..................................	M_{Qx1}, M_{Qy1}, M_{Qz1}
$\tilde{M}_L$	Leg cylinder moment..................................	M_{Lx1}, M_{Ly1}, M_{Lz1}
$\tilde{M}_R$	Canopy resultant force moment..................................	M_{Rx1}, M_{Ry1}, M_{Rz1}
$\tilde{r}_Q$	Canopy capsule moment vector distance..................................	x_{1Q}, y_{1Q} z_{1Q}
$\tilde{r}_L$	Leg cylinder moment vector distance..................................	x_{1L}, y_{1L}, z_{1L}
$\tilde{r}_R$	Resultant moment vector distance..................................	x_{1R}, y_{1R}, z_{1R}

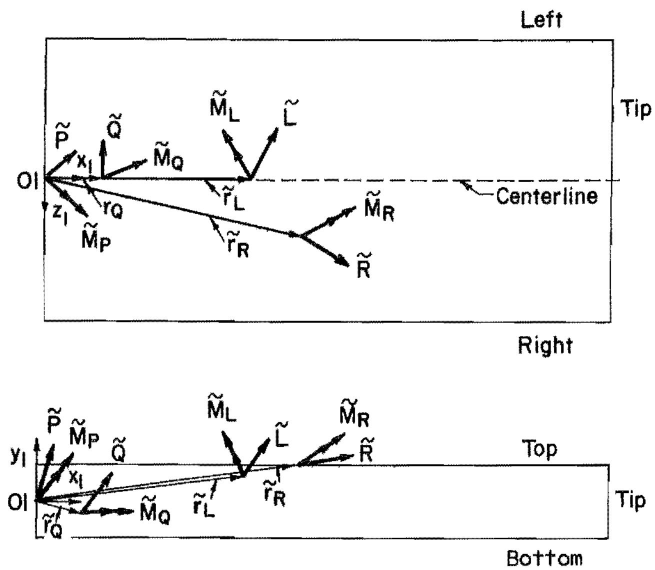

FIGURE 2.—Forces and moments on canopy.

Summation of Forces (fig. 2)

$$\sum\tilde{F} = \tilde{P} + \tilde{Q} + \tilde{L} + \tilde{R} = \tilde{0}. \tag{1}$$

$$\sum F_{x1} = P_{x1} + Q_{x1} + L_{x1} + R_{x1} = 0. \tag{2}$$

$$\sum F_{y1} = P_{y1} + Q_{y1} + L_{y1} + R_{y1} = 0. \tag{3}$$

$$\sum F_{z1} = P_{z1} + Q_{z1} + L_{z1} + R_{z1} = 0. \tag{4}$$

$$\sum\tilde{M}\,(01) = \tilde{M}_P + \tilde{M}_Q + \tilde{M}_L + \tilde{M}_R + (\tilde{r}_Q \times \tilde{Q}) + (\tilde{r}_L \times \tilde{L}) + (\tilde{r}_R \times \tilde{R}) = \tilde{0}. \tag{5}$$

$$\sum M_{x1}(01) = M_{Px1} + M_{Qx1} + M_{Lx1} + M_{Rx1} + (y_{1Q}Q_{z1} - z_{1Q}Q_{y1}) + (y_{1L}L_{z1} - z_{1L}L_{y1}) + (y_{1R}R_{z1} - z_{1R}R_{y1}) = 0. \tag{6}$$

$$\sum M_{y1}(01) = M_{Py1} + M_{Qy1} + M_{Ly1} + M_{Ry1} + (z_{1Q}Q_{x1} - x_{1Q}Q_{z1}) + (z_{1L}L_{x1} - x_{1L}L_{z1}) + (z_{1R}R_{x1} - x_{1R}R_{z1}) = 0. \tag{7}$$

$$\sum M_{z1}(01) = M_{Pz1} + M_{Qz1} + M_{Lz1} + M_{Rz1} + (x_{1Q}Q_{y1} - y_{1Q}Q_{x1}) + (x_{1L}L_{y1} - y_{1L}L_{x1}) + (x_{1R}R_{y1} - y_{1R}R_{x1}) = 0. \tag{8}$$

CAVING SHIELD

TABLE 2. - Caving shield nomenclature description

Symbol	Vector represented	Scaler components
$\tilde{E}$	Canopy hinge pin force..................................	E_{x2}, E_{y2}, E_{z2}
$\tilde{U}$	Capsule force..	U_{x2}, U_{y2}, U_{z2}
$\tilde{T}$	Compression lemniscate link force......................	T_{x2}, T_{y2}, T_{z2}
$\tilde{S}$	Tension lemniscate link force..........................	S_{x2}, S_{y2}, S_{z2}
$\tilde{R}_C$	Caving shield gob reaction force.......................	R_{Cx2}, R_{Cy2}, R_{Cz2}
$\tilde{M}_E$	Canopy hinge pin moment................................	M_{Ex2}, M_{Ey2}, M_{Ez2}
$\tilde{M}_U$	Capsule moment...	M_{Ux2}, M_{Uy2}, M_{Uz2}
$\tilde{M}_T$	Compression lemniscate link moment.....................	M_{Tx2}, M_{Ty2}, M_{Tz2}
$\tilde{M}_S$	Tension lemniscate link moment.........................	M_{Sx2}, M_{Sy2}, M_{Sz2}
$\tilde{M}_{RC}$	Gob reaction moment....................................	M_{RCx2}, M_{RCy2}, M_{RCz2}
$\tilde{r}_E$	Canopy hinge pin moment vector distance................	x_{2E}, y_{2E}, z_{2E}
$\tilde{r}_U$	Capsule moment vector distance.........................	x_{2U}, y_{2U}, z_{2U}
$\tilde{r}_T$	Compression lemniscate link moment vector distance......	x_{2T}, y_{2T}, z_{2T}
$\tilde{r}_{RC}$	Gob reaction moment vector distance....................	x_{2RC}, y_{2RC}, z_{2RC}

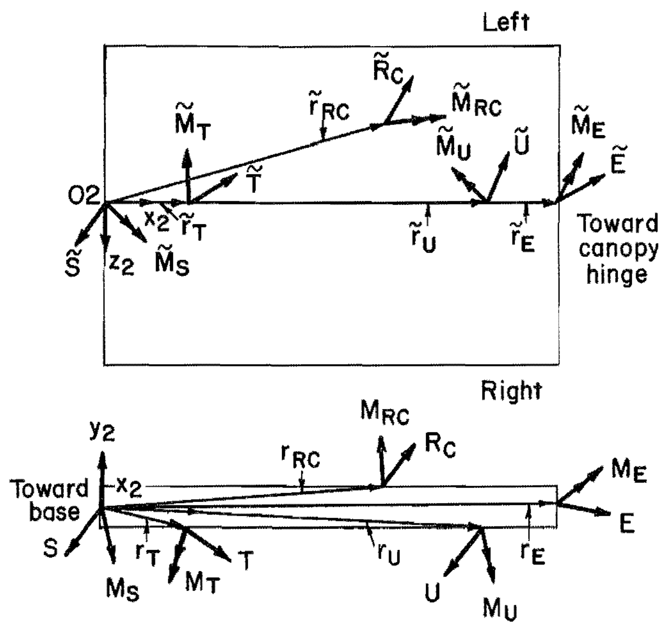

FIGURE 3.—Forces and moments on caving shield.

Summation of Forces (fig. 3)

$$\sum\tilde{F} = \tilde{E} + \tilde{U} + \tilde{T} + \tilde{S} + \tilde{R}_C = \tilde{0}. \tag{9}$$

$$\sum F_{x2} = E_{x2} + U_{x2} + T_{x2} + S_{x2} + R_{Cx2} = 0. \tag{10}$$

$$\sum F_{y2} = E_{y2} + U_{y2} + T_{y2} + S_{y2} + R_{Cy2} = 0. \tag{11}$$

$$\sum F_{z2} = E_{z2} + U_{z2} + T_{z2} + S_{z2} + R_{Cz2} = 0. \tag{12}$$

Summation of Moments (fig. 3)

$$\begin{aligned}\sum\tilde{M}\ (02) &= \tilde{M}_E + \tilde{M}_U + \tilde{M}_T + \tilde{M}_S + \tilde{M}_{RC} \\ &\quad + (\tilde{r}_E \times \tilde{E}) + (\tilde{r}_U \times \tilde{U}) + (\tilde{r}_T \times \tilde{T}) + (\tilde{r}_{RC} \times \tilde{R}_C) = \tilde{0}.\end{aligned} \tag{13}$$

$$\begin{aligned}\sum M_{x2}(02) &= M_{Ex2} + M_{Ux2} + M_{Tx2} + M_{Sx2} + M_{RCx2} \\ &\quad + (y_{2E}E_{z2} - z_{2E}E_{y2}) + (y_{2U}U_{z2} - z_{2U}U_{y2}) + (y_{2T}T_{z2} - z_{2T}T_{y2}) \\ &\quad + (y_{2RC}R_{Cz2} - z_{2RC}R_{Cy2}) = 0.\end{aligned} \tag{14}$$

$$\begin{aligned}\sum M_{y2}(02) &= M_{Ey2} + M_{Uy2} + M_{Ty2} + M_{Sy2} + M_{RCy2} \\ &\quad + (z_{2E}E_{x2} - x_{2E}E_{z2}) + (z_{2U}U_{x2} - x_{2U}U_{z2}) + (z_{2T}T_{x2} - x_{2T}T_{z2}) \\ &\quad + (z_{2RC}R_{Cx2} - x_{2RC}R_{Cz2}) = 0.\end{aligned} \tag{15}$$

$$\begin{aligned}\sum M_{z2}(02) &= M_{Ez2} + M_{Uz2} + M_{Tz2} + M_{Sz2} + M_{RCz2} \\ &\quad + (x_{2E}E_{y2} - y_{2E}E_{x2}) + (x_{2U}U_{y2} - y_{2U}U_{x2}) + (x_{2T}T_{y2} - y_{2T}T_{x2}) \\ &\quad + (x_{2RC}R_{Cy2} - y_{2RC}R_{Cx2}) = 0.\end{aligned} \tag{16}$$

BASE

TABLE 3. - Base structure nomenclature description

Symbol	Vector represented	Scaler components
$\tilde{R}_B$	Base resultant reaction force............................	R_{Bx3}, R_{By3}, R_{Bz3}
$\tilde{B}$	Leg force..	B_{x3}, B_{y3}, B_{z3}
$\tilde{C}$	Compression lemniscate link force........................	C_{x3}, C_{y3}, C_{z3}
$\tilde{D}$	Tension lemniscate link force............................	D_{x3}, D_{y3}, D_{z3}
$\tilde{r}_{RB}$	Base resultant reaction moment distance..................	x_{3RB}, y_{3RB}, z_{3RB}
$\tilde{r}_B$	Leg moment vector distance...............................	x_{3B}, y_{3B}, z_{3B}
$\tilde{r}_C$	Compression lemniscate link moment distance..............	x_{3C}, y_{3C}, z_{3C}

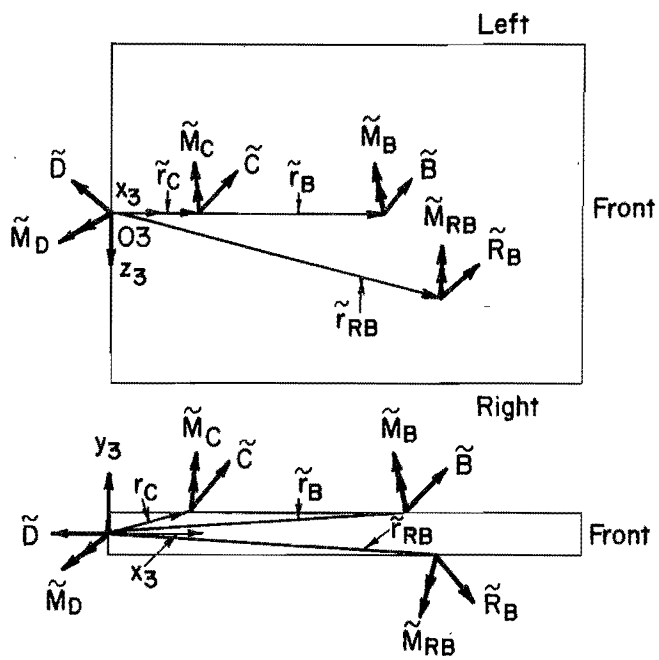

FIGURE 4.—Forces and moments on base.

Summation of Forces (fig. 4)

$$\sum \tilde{F} = \tilde{R}_B + \tilde{B} + \tilde{C} + \tilde{D} = \tilde{0}. \tag{17}$$

$$\sum F_{x3} = R_{Bx3} + B_{x3} + C_{x3} + D_{x3} = 0. \tag{18}$$

$$\sum F_{y3} = R_{By3} + B_{y3} + C_{y3} + D_{y3} = 0. \tag{19}$$

$$\sum F_{z3} = R_{Bz3} + B_{z3} + C_{z3} + D_{z3} = 0. \tag{20}$$

Summation of Moments (fig. 4)

$$\begin{aligned}\sum \tilde{M}\ (03) = {} & \tilde{M}_{RB} + \tilde{M}_B + \tilde{M}_C + \tilde{M}_D \\ & + (\tilde{r}_{RB} \times \tilde{R}_B) + (\tilde{r}_B \times \tilde{B}) + (\tilde{r}_C \times \tilde{C}) = \tilde{0}.\end{aligned} \tag{21}$$

$$\begin{aligned}\sum M_{x3}(03) = {} & M_{RBx3} + M_{Bx3} + M_{Cx3} + M_{Dx3} \\ & + (y_{3RB}R_{Bz3} - z_{3RB}R_{By3}) + (y_{3B}B_{z3} - z_{3B}B_{y3}) \\ & + (y_{3C}C_{z3} - z_{3C}C_{y3}) = 0.\end{aligned} \tag{22}$$

$$\begin{aligned}\sum M_{y3}(03) = {} & M_{RBy3} + M_{By3} + M_{Cy3} + M_{Dy3} \\ & + (z_{3RB}R_{Bx3} - x_{3RB}R_{Bz3}) + (z_{3B}B_{x3} - x_{3B}B_{z3}) \\ & + (z_{3C}C_{x3} - x_{3C}C_{z3}) = 0.\end{aligned} \tag{23}$$

$$\begin{aligned}\sum M_{z3}(03) = {} & M_{RBz3} + M_{Bz3} + M_{Cz3} + M_{Dz3} \\ & + (x_{3RB}R_{By3} - y_{3RB}R_{Bx3}) + (x_{3B}B_{y3} - y_{3B}B_{x3}) \\ & + (x_{3C}C_{y3} - y_{3C}C_{x3}) = 0.\end{aligned} \tag{24}$$

TRANSFORMATION OF COORDINATE SYSTEMS

The static equilibrium equations presented in the previous section utilized three different coordinate systems to take advantage of the relative orientation of major shield components (canopy, caving shield, base). Reduction of these equations requires transformation of these three coordinate systems into a single coordinate system. Since the desired solution is the resultant force acting on the canopy, the caving shield and the base equilibrium equations are transferred to the coordinate system designated for the canopy.

The transformation of the caving shield coordinate system (x_2, y_2, z_2) into the canopy coordinate system (x_1, y_1, z_1) is achieved by rotation of the (x_2, y_2, z_2) system as illustrated in figure 5. Mathematically this rotation is accomplished by the orthogonal transformation:

$$V_{x1} = V_{x2}\cos\theta - V_{y2}\sin\theta, \qquad (25)$$

$$V_{y1} = V_{x2}\sin\theta + V_{y2}\cos\theta, \qquad (26)$$

$$V_{z1} = V_{z2}, \qquad (27)$$

where α is the angle from the x_1 axis to the x_2 axis and $\tilde{V}$ is a generic vector.

The coordinate transformation of the caving shield coordinate system (x_2, y_2, z_2) into the global canopy system (x_1, y_1, z_1) eliminates several unknowns by equating common forces and moments.

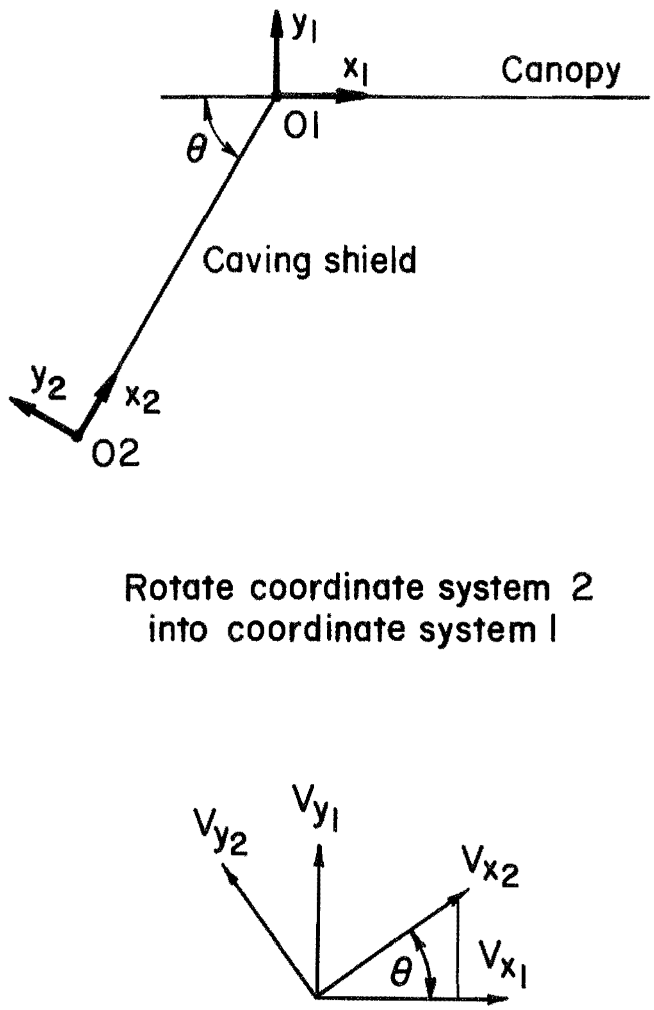

FIGURE 5.—Coordinate system transformations.

$$\tilde{E} = -\tilde{P} \quad \text{or} \quad E_{x1} = -P_{x1} \quad \text{and} \quad M_{Ex1} = -M_{Px1}.$$

$$\tilde{M}_E = -\tilde{M}_P \quad \text{or} \quad E_{y1} = -P_{y1} \quad \text{and} \quad M_{Ey1} = -M_{Py2}.$$

$$E_{z1} = -P_{z1} \quad \text{and} \quad M_{Ez1} = -M_{Pz2}.$$

$$\tilde{U} = -\tilde{Q} \quad \text{or} \quad U_{x1} = -Q_{x1} \quad \text{and} \quad M_{Ux1} = -M_{Qx1}.$$

$$\tilde{M}_U = -\tilde{M}_Q \quad U_{y1} = -Q_{y1} \quad \text{and} \quad M_{Uy1} = -M_{Qy2}.$$

$$U_{z1} = -Q_{z1} \quad \text{and} \quad M_{Uz1} = -M_{Qz2}.$$

Using the transformation presented in equations 25 through 27 and the substitution for equivalent forces and moments discussed previously, the transformed force and moment equations for the caving shield in the global (x_1, y_1, z_1) coordinate system are as follows:

Summation of Forces

$$\sum F_{x1} = -P_{x1} - Q_{x1} + (T_{x2}\cos\alpha - T_{y2}\sin\alpha) + (S_{x2}\cos\alpha - S_{y2}\sin\alpha) + (R_{Cx2}\cos\alpha - R_{Cy2}\sin\alpha) = 0. \tag{28}$$

$$\sum F_{y1} = -P_{y1} - Q_{y1} + (T_{x2}\sin\alpha + T_{y2}\cos\alpha) + (S_{x2}\sin\alpha + S_{y2}\cos\alpha) + (R_{Cx2}\sin\alpha + R_{Cy2}\cos\alpha) = 0. \tag{29}$$

$$\sum F_{z1} = -P_{z1} - Q_{z1} + T_{z2} + S_{z2} + R_{Cz2} = 0. \tag{30}$$

Summation of Moments

$$\begin{aligned}\sum M_{x1}(02) = {} & -M_{Px1} - M_{Qx1} + (M_{Tx2}\cos\alpha - M_{Ty2}\sin\alpha) \\ & + (M_{Sx2}\cos\alpha - M_{Sy2}\sin\alpha) + (M_{RCx2}\cos\alpha - M_{RCy2}\sin\alpha) \\ & + [-P_{z1}(x_{2E}\sin\alpha + y_{2E}\cos\alpha) + (P_{y1})(z_{2E})] \\ & + [-Q_{z1}(x_{2U}\sin\alpha + y_{2U}\cos\alpha) + (Q_{y1})(z_{2U})] \\ & + [T_{z2}(x_{2T}\sin\alpha + y_{2T}\cos\alpha) - (T_{x2}\sin\alpha + T_{y2}\cos\alpha)(z_{2T})] \\ & + [R_{Cz2}(x_{2RC}\sin\alpha + y_{2RC}\cos\alpha) \\ & - (z_{2RC})(R_{Cx2}\sin\alpha + R_{Cy2}\cos\alpha)] = 0. \end{aligned} \tag{31}$$

$$\begin{aligned}\sum M_{y1}(02) = {} & -M_{Py1} - M_{Qy1} + (M_{Tx2}\sin\alpha + M_{Ty2}\cos\alpha) \\ & + (M_{Sx2}\sin\alpha + M_{Sy2}\cos\alpha) + (M_{RCx2}\sin\alpha + M_{RCy2}\cos\alpha) \\ & + [z_{2E}\ (-P_{x1}) + (P_{z1})(x_{2E}\cos\alpha - y_{2E}\sin\alpha)] \\ & + [z_{2U}\ (-Q_{x1}) + (Q_{z1})(x_{2Q}\cos\alpha - y_{2Q}\sin\alpha)] \\ & + [z_{2T}(T_{x2}\cos\alpha - T_{y2}\sin\alpha) - T_{z2}(x_{2T}\cos\alpha - y_{2T}\sin\alpha)] \\ & + [z_{2RC}(R_{Cx2}\cos\alpha - R_{Cy2}\sin\alpha) \\ & - RC_{z2}(x_{2RC}\cos\alpha - y_{2RC}\sin\alpha)] = 0. \end{aligned} \tag{32}$$

$$\sum M_{z1}(02) = -M_{Pz1} - M_{Qz1} + M_{Tz2} + M_{Sz2} + M_{RCz2}$$
$$+ [x_{2E}E_{y2} - y_{2E}E_{x2}] + [x_{2U}U_{y2} - y_{2U}U_{x2}] + [x_{2T}T_{y2} - y_{2T}T_{x2}]$$
$$+ [x_{2RC}R_{Cy2} - y_{2RC}R_{Cx2}] = 0. \quad (33)$$

As indicated by the coordinate transformation (equations 25-27), there is no rotation of the (x_2, y_2, z_2) coordinate system for the z-axis transformation, resulting in a direct transformation $z_1 = z_2$. A further simplification of the $M_{z1}(02)$ equation can be achieved by substituting terms expressed in the (x_2, y_2, z_2) coordinate system by equivalent terms in the (x_1, y_1, z_1) system where

$$x_2 = x_1\cos\alpha + y_1\sin\alpha \quad (34)$$

$$y_2 = y_1\cos\alpha - x_1\sin\alpha \quad (35)$$

and by substitution of $-\tilde{P}$ for $\tilde{E}$ and $-\tilde{Q}$ for $\tilde{U}$. Remember that these are equivalent expressions and not transformations.

The equation for $\sum M_{z1}(02)$ then becomes

$$\sum M_{z1}(02) = -M_{Pz1} - M_{Qz1} + M_{Tz2} + M_{Sz2} + M_{RCx2}$$
$$+ [x_{2E}(P_{x1}\sin\alpha - P_{y1}\cos\alpha) + y_{2E}(P_{x1}\cos\alpha + P_{y1}\sin\alpha)]$$
$$+ [x_{2U}(Q_{x1}\sin\alpha - Q_{y1}\cos\alpha) + y_{2U}(Q_{x1}\cos\alpha + Q_{y1}\sin\alpha)]$$
$$+ [x_{2T}(T_{y2}) - y_{2T}(T_{x2})] + [x_{2RC}(R_{Cy2}) - y_{2RC}(R_{Cx2})] = 0. \quad (36)$$

Since there is no rotation of the base coordinate system, the equilibrium equations for the base result in a direct transformation and therefore are the same as those presented previously, where

$$V_{x3} = V_{x1}, \quad (37)$$

$$V_{y3} = V_{y1}, \quad (38)$$

and $$V_{z3} = V_{z1}. \quad (39)$$

SOLUTION OF THREE-DIMENSIONAL RESULTANT SHIELD LOADING

The resultant force reaction on the canopy is described in six parameters identifying force and location parameters as follows:

R_{x1} = Horizontal resultant canopy reaction force.

R_{y1} = Vertical resultant canopy reaction force.

R_{z1} = Lateral resultant canopy reaction force.

x_{1R} = Location of resultant force in horizontal axis.

y_{1R} = Location of resultant force in vertical axis.

z_{1R} = Location of resultant force in lateral axis.

Direct solution of the equilibrium equations for these six unknown parameters requires static determinacy. A field instrumentation array developed and tested in the Bureau's mine roof simulator[3] enables determination of leg, canopy capsule, and lemniscate link forces. The resulting system of equations is indeterminate to a high degree, requiring reduction of the number of unknowns for a closed-form solution. While various combinations of unknowns are possible as candidates for elimination without endangering the accuracy of the final solution, table 4 presents proposed parameters with justification for elimination. These reductions allow solution of three-dimensional resultant force from measurement of leg, canopy capsule, and lemniscate link forces.

With deletions of nonparticipating forces and moments identified in table 4, solution of the three-dimensional resultant load parameters is as follows:

TABLE 4. - Parameter elimination identification[1]

Parameter	Identification	Justification
L_{z1}.............	Lateral leg load.............	Leg is pin jointed acting in x-y plane incapable of transmitting lateral (z) load without structural deformation of pin joint.
Q_{z1}.............	Lateral canopy capsule load..	Do.
$\tilde{R}_C$.............	Caving shield gob reaction load.	Major loading is to canopy and base. For cantilevered strata, there is likely to be little gob loading.
$\tilde{M}_{RC}$............	Gob reaction moment..........	No moment due to no load assumption (item 3).
$\tilde{M}_R$.............	Canopy resultant force moment	The canopy resultant reaction force couple in pitch and roll is dissolved by allowing the resultant force to be positioned at the proper location to provide an equivalent force reaction at the required distance from the canopy hinge pin. The yaw moment is also assumed zero.
$\tilde{M}_Q$.............	Canopy capsule reaction moment.	No canopy capsule moment by assuming frictionless pin joint which cannot transmit torque.

[1]All z's are zero except for moments since forces act along centerline.

[3]Barczak, T. M., R. C. Garson, P. M. Yavorsky, and F. S. Maayeh. State-of-the-Art Testing of Powered Roof Supports. Paper in Proceedings of Second Conference on Ground Control in Mining (Morgantown, WV, July 19-21, 1982). WV Univ., 1982, pp. 64-77.

HORIZONTAL RESULTANT LOAD (R_{x1})

1. From summation of forces on caving shield (equation 28), determine horizontal reaction at canopy hinge pin (P_{x1}).

$$P_{x1} = -Q_{x1} + (T_{x2}\cos\alpha - T_{y2}\sin\alpha) + (S_{x2}\cos\alpha - S_{y2}\sin\alpha).$$

2. From summation of forces on canopy (equation 2), substitute (P_{x1}) into equation and solve for R_{x1}.

$$R_{x1} = -P_{x1} - Q_{x1} - L_{x1}$$

$$= - [-Q_{x1} + (T_{x2}\cos\alpha - T_{y2}\sin\alpha) + (S_{x2}\cos\alpha - S_{y2}\sin\alpha)]$$

$$- L_{x1} - Q_{x1}.$$

$$R_{x1} = -L_{x1} - (T_{x2}\cos\alpha - T_{y2}\sin\alpha) - (S_{x2}\cos\alpha - S_{y2}\sin\alpha).$$

VERTICAL RESULTANT LOAD (R_{y1})

1. From summation of moments on caving shield (equation 29), determine the vertical reaction at the canopy hinge pin (P_{y1}).

$$P_{y1} = -Q_{y1} + (T_{x2}\sin\alpha + T_{y2}\cos\alpha) + (S_{x2}\sin\alpha + S_{y2}\cos\alpha).$$

2. From summation of forces on canopy (equation 3), substitute P_{y1} into equation and solve for the vertical resultant load (R_{y1}).

$$R_{y1} = -P_{y1} - Q_{y1} - L_{y1}$$

$$= - [-Q_{y1} + (T_{x2}\sin\alpha + T_{y2}\cos\alpha) + (S_{x2}\sin\alpha + S_{y2}\cos\alpha)]$$

$$- Q_{y1} - L_{y1}.$$

$$R_{y1} = -L_{y1} - (T_{x2}\sin\alpha + T_{y2}\cos\alpha) - (S_{x2}\sin\alpha + S_{y2}\cos\alpha).$$

LATERAL RESULTANT LOAD (R_{z1})

1. From summation of moments about x-axis on canopy (equation 6), solve for moment at canopy hinge pin (M_{Px1}).

$$M_{Px1} = -M_{Lx1} - y_{1R}R_{z1} + z_{1R}R_{y1}.$$

2. From summation at moments about y-axis on canopy (equation 7), solve for moment at canopy hinge pin (M_{Py1}):

$$M_{Py1} = -M_{Ly1} - z_{1R}R_{x1} + x_{1R}R_{z1}.$$

3. Substitute M_{Px1} into equation for summation of moments about x-axis (equation 31) on caving shield, and solve for resultant reaction location (z_{1R}).

$$- [-M_{Lx1} - y_{1R}R_{z1} + z_{1R}R_{y1}] + (M_{Tx2}\cos\alpha - M_{Ty2}\sin\alpha)$$

$$+ (M_{Sx2}\cos\alpha - M_{Sy2}\sin\alpha) - P_{z1}(x_{2E}\sin\alpha + y_{2E}\cos\alpha)$$

$$+ T_{z2}(x_{2T}\sin\alpha + y_{2T}\cos\alpha) = 0.$$

or

$$z_{1R} = [M_{Lx1} + y_{1R}R_{z1} + (M_{Tx2}\cos\alpha - M_{Ty2}\sin\alpha)$$

$$+ (M_{Sx2}\cos\alpha - M_{Sy2}\sin\alpha) - P_{z1}(x_{2E}\sin\alpha + y_{2E}\cos\alpha)$$

$$+ T_{z2}(x_{2T}\sin\alpha + y_{2T}\cos\alpha)] / R_{y1}.$$

4. Substitute M_{Py1} into equation for moments about y-axis on caving shield (equation 32), and solve for resultant reaction location z_{1R}.

$$- [-M_{Ly1} - z_{1R}R_{x1} + x_{1R}R_{z1}] + (M_{Tx2}\sin\alpha + M_{Ty2}\cos\alpha)$$

$$+ (M_{Sx2}\sin\alpha + M_{Sy2}\cos\alpha) + P_{z1}(x_{2E}\cos\alpha - y_{2E}\sin\alpha)$$

$$- T_{z2}(x_{2T}\cos\alpha - y_{2T}\sin\alpha)] = 0.$$

$$z_{1R} = [-M_{Ly1} + x_{1R}R_{z1} - (M_{Tx2}\sin\alpha + M_{Ty2}\cos\alpha)$$

$$- (M_{Sx2}\sin\alpha + M_{Sy2}\cos\alpha)$$

$$- P_{z1}(x_{2E}\cos\alpha - y_{2E}\sin\alpha)$$

$$+ T_{z2}(x_{2T}\cos\alpha - y_{2T}\sin\alpha)] / R_{x1}.$$

5. Set expressions for z_{1R} equal to each other.

$$R_{x1}[M_{Lx1} + y_{1R}R_{z1} + (M_{Tx2}\cos\alpha - M_{Ty2}\sin\alpha)$$

$$+ (M_{Sx2}\cos\alpha - M_{Sy2}\sin\alpha) - P_{z1}(x_{2E}\sin\alpha + y_{2E}\cos\alpha)$$

$$+ T_{z2}(x_{2T}\sin\alpha + y_{2T}\cos\alpha)]$$

$$= R_{y1}[-M_{Ly1} + x_{1R}R_{z1} - (M_{Tx2}\sin\alpha + M_{Ty2}\cos\alpha)$$

$$- (M_{Sy2}\sin\alpha + M_{Sy2}\cos\alpha) - P_{z1}(x_{2E}\cos\alpha - y_{2E}\sin\alpha)$$

$$+ T_{z2}(x_{2T}\cos\alpha - y_{2T}\sin\alpha)].$$

Substitute $P_{z1} = -R_{z1}$.

$$R_{z1}[R_{x1}(+y_{1R} + (x_{2E}\sin\alpha + y_{2E}\cos\alpha))]$$

$$- R_{z1}[R_{y1}(x_{1R} + (x_{2E}\cos\alpha - y_{2E}\sin\alpha))]$$

$$= R_{x1}\ [-M_{Lx1} - (M_{Tx2}\cos\alpha - M_{Ty2}\sin\alpha)$$

$$- (M_{Sx2}\cos\alpha - M_{Sy2}\sin\alpha) - T_{z2}(x_{2T}\sin\alpha + y_{2T}\cos\alpha)]$$

$$+ R_{y1}[-M_{Ly1} - (M_{Tx2}\sin\alpha + M_{Ty2}\cos\alpha)$$

$$- (M_{Sx2}\sin\alpha + M_{Sy2}\cos\alpha) + T_{z2}(x_{2T}\cos\alpha - y_{2T}\sin\alpha)].$$

Solve for R_{z1}.

$$R_{z1} = \{R_{y1}[-M_{Ly1} - (M_{Tx2}\sin\alpha + M_{Ty2}\cos\alpha) - (M_{Sx2}\sin\alpha + M_{Sy2}\cos\alpha) + T_{z2}(x_{2T}\cos\alpha - y_{2T}\sin\alpha)] - R_{x1}[M_{Lx1} + (M_{Tx2}\cos\alpha - M_{Ty2}\sin\alpha) + (M_{Sx2}\cos\alpha - M_{Sy2}\sin\alpha) + T_{z2}(x_{2T}\sin\alpha + y_{2T}\cos\alpha)]\} / \{R_{x1}[y_{1R} + (x_{2E}\sin\alpha + y_{2E}\cos\alpha)] - R_{y1}[x_{1R} + (x_{2E}\cos\alpha - y_{2E}\sin\alpha)]\}.$$

VERTICAL RESULTANT LOCATION (y_{1R})

y_{1R} is fixed by the plane of the canopy with the resultant defined to act on the surface of canopy.

HORIZONTAL RESULTANT LOCATION (x_{1R})

From summation of moments about z-axis on the canopy (equation 8), solve for x_{1R}.

$$x_{1R} = [-M_{Pz1} - (x_{1Q}Q_{y1} - y_{1Q}Q_{x1}) - (x_{1L}L_{y1} - y_{1L}L_{x1}) + y_{1R}R_{x1}] / R_{y1}$$

where M_{Pz1} is found from summation of moments about the z-axis on the caving shield (equation 36).

$$M_{Pz1} = M_{Tz2} + M_{Sz2} + [x_{2E}(P_{x1}\sin\alpha - P_{y1}\cos\alpha) + y_{2E}(P_{x1}\cos\alpha + P_{y1}\sin\alpha)] + [x_{2U}(Q_{x1}\sin\alpha - Q_{y1}\cos\alpha) + y_{2U}(Q_{x1}\cos\alpha + Q_{y1}\sin\alpha)] + [x_{2T}(T_{y2}) - y_{2T}(T_{x2})].$$

LATERAL RESULTANT LOCATION

Lateral resultant location was found in step 4 of the solution for lateral resultant load from summation of moments about the y-axis on the caving shield.

$$z_{1R} = [-ML_{y1} + x_{1R}R_{z1} - (M_{Sx2}\sin\alpha + M_{Sy2}\cos\alpha) - (M_{Tx2}\sin\alpha + M_{Ty2}\cos\alpha) - P_{z1}(x_{2E}\cos\alpha - y_{2E}\sin\alpha) + T_{z2}(x_{2T}\cos\alpha - y_{2T}\sin\alpha)] / R_{x1}.$$

Inputs to this set of equations just developed are determined from measuring leg, canopy capsule, and lemniscate link forces. A review of this reduction begins with leg forces and moments. The left and right leg forces are obtained by multiplying the leg pressures by the piston area:

$$L^L = P^L A_{LL} \quad (40)$$

$$L^R = P^R A_{LR} \quad (41)$$

where L^L, L^R, P^L, P^R, A_{LL}, and A_{LR} are the left and right leg forces, and leg pressures and areas in pounds (force), pounds per square inch, and square inch, respectively. The components of these leg forces in the (x1, y1, z1) coordinate frame are

$$L_x^L = L^L \cos \alpha_1, \quad (42)$$

$$L_y^L = L^L \sin \alpha_1, \quad (43)$$

$$L_x^R = L^R \cos \alpha_1, \quad (44)$$

$$L_y^R = L^R \sin \alpha_1, \quad (45)$$

where α_1 is the angle from a line parallel to x1 (the horizontal) to the leg. (This angle is less than 90°.)

Combining the left and right leg forces at the center of the legs, the L_{x1} and L_{y1} are obtained as

$$L_{x1} = {}^L L_x + {}^R L_x, \quad (46)$$

$$L_{y1} = {}^L L_y + {}^R L_y, \quad (47)$$

and the moments created by their imbalance, M_{Lx1} and M_{Ly1}, are

$$M_{Lx1} = -z_L\, L_y^R + z_L\, L_y^L, \quad (48)$$

$$M_{Ly1} = z_L\, L_x^R - z_L\, L_x^R, \quad (49)$$

where z_L is half the distance between the leg axis.

The capsule force (in pounds force) is determined by multiplying the capsule extend pressure by the extend piston area and subtracting the capsule retract pressure multiplied by the retract piston area:

$$Q = (P^{CE}A^{CE} - P^{CR}\, A^{CR}), \quad (50)$$

where P^{CE}, P^{CR}, A^{CE}, and A^{CR} are capsule extend and retract pressures and areas in pounds (force) per square inch and square inch respectively. The capsule inputs, Q_{x1} and Q_{y1}, are then

$$Q_{x1} = Q \cos \alpha_2, \quad (51)$$

$$Q_{y1} = Q \sin \alpha_2, \quad (52)$$

where α_2 is the angle from a line under the capsule, parallel to the x1 axis (horizontal), to the capsule axis. (When the capsule is level, this angle is 0°.)

The link forces are the most difficult to reduce. The first step is to define the geometry. Link forces can be deduced from strain measurements. Three-element rosette gauges provide stress-strain profiles for each structure. For any link rosette the local gauge geometry is shown in figure 6. The principal strains ε_1 and ε_2 and the angle ϕ of the principal frame from the $x^1\ y^1$ frame can be determined from the three rosette strains ε_A, ε_B, ε_C by[4]

$$\varepsilon_{1,2} = 1/2\ (\varepsilon_A + \varepsilon_C) \pm 1/2\sqrt{(\varepsilon_A - \varepsilon_C)^2 + (2\varepsilon_B - \varepsilon_A - \varepsilon_C)^2} \quad (53)$$

$$2\phi = \tan^{-1} \frac{2\varepsilon_B - \varepsilon_A - \varepsilon_C}{\varepsilon_A - \varepsilon_C}. \quad (54)$$

[4]Dally, J. W., and W. F. Riley. Experimental Stress Analysis. McGraw-Hill, 1978, p. 322.

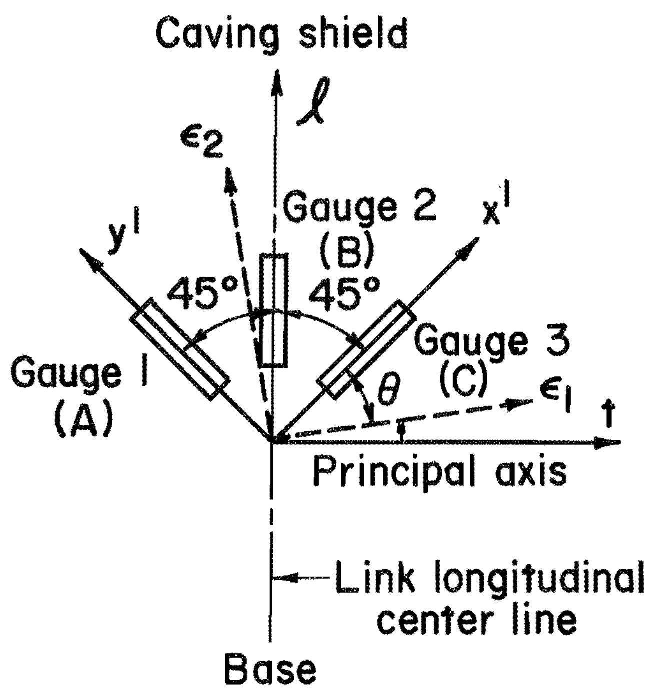

FIGURE 6.—Local forward link strain rosette geometry.

For plane stress, the principal stresses follow:

$$\sigma_1 = [E/(1 - \nu^2)](\varepsilon_1 + \nu\varepsilon_2), \quad (55)$$

$$\sigma_2 = [E/(1 - \nu^2)](\varepsilon_2 + \nu\varepsilon_1), \quad (56)$$

where E is Young's modulus and ν is Poisson's ratio.

These stresses are then rotated into the t, ℓ (transverse, longitudinal) coordinate frame (fig. 6) by

$$\sigma_\ell = \sigma_2 \cos^2 (45° - \phi) + \sigma_1 \sin^2 (45° - \phi), \quad (57)$$

$$\sigma_t = \sigma_2 \sin^2 (45° - \phi) + \sigma_1 \cos^2 (45° - \phi), \quad (58)$$

$$\tau_{t\ell} = (s_1 - \sigma_2) \sin (45° - \phi) \cos (45° - \phi), \quad (59)$$

where σ_ℓ is the longitudinal stress, σ_t is the transverse stress, and $\tau_{t\ell}$ is the transverse, longitudinal shear stress.

Having calculated σ_ℓ, σ_t, and $\tau_{t\ell}$ for each rosette on a link, three forces and three moments can be calculated for the link end connected to the caving shield. Again, the first step is to define the geometry. The link is modeled by a box section of length ℓ from pin center to pin center. The rectangular coordinate system (ξ, η, ζ) is used with an origin at the center of the base pin axis (fig. 7). The forces and moments P_ξ^{LF}, $V_{\eta\xi}^{LF}$, $V_{\zeta\xi}^{LF}$, and $M_{\xi\ell}^{LF}$, $M_{\eta\ell}^{LF}$, $M_{\zeta\ell}^{LF}$, respectively, which act on the caving shield pin, are shown relative to the coordinate system (ξ, η, ζ) in figure 8. The superscript LF refers to left forward link. For the right forward link the analysis is the same, and RF may be exchanged for LF. The relationships among stresses, forces, and moments for the box beam link model are available in most textbooks on simple strength of materials.[5]

[5]Timoshenko, S. P., and J. N. Goodier. Theory of Elasticity. McGraw-Hill, 1970, pp. 112-155.

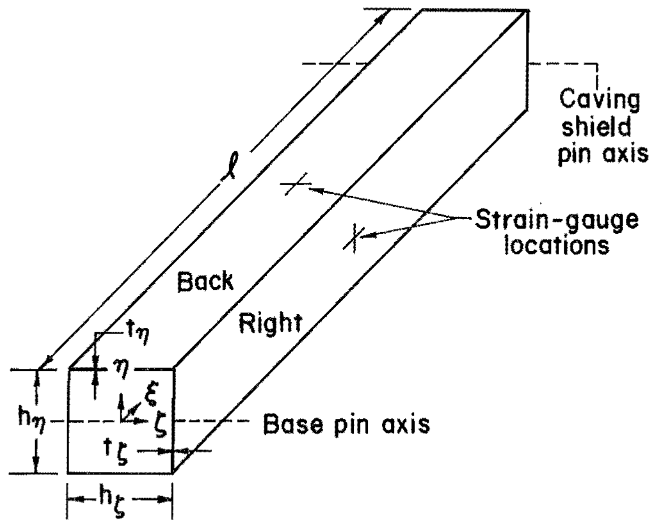

FIGURE 7.—Forward link coordinate system.

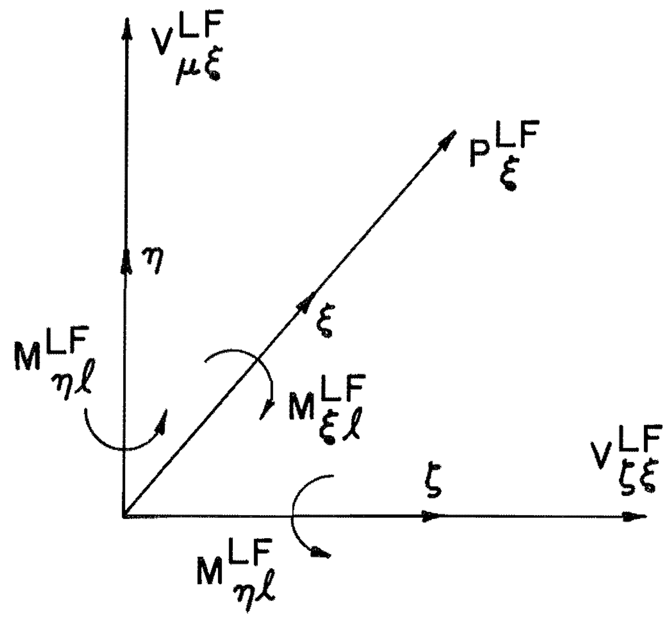

FIGURE 8.—Forces acting on forward link at caving shield pin.

In summary, the following equations were used.

<u>In tension,</u>

$$P_\xi^{LF} = (\sigma_\ell^{LFF} + \sigma_\ell^{LFB}) A_1/2 + (\sigma_\ell^{LFL} + \sigma_\ell^{LFR}) A_2/2, \quad (60)$$

where A_1 is the cross-sectional area of the top and bottom link plates, and A_2 is the cross-sectional area of the left and right link plates.

The superscripts on the stresses are the rosette designations.

In shear,

$$V^{LE}_{\eta\xi} = (\tau^{LFL}_{t\ell} - \tau^{LFR}_{t\ell})\, I_\zeta t_\zeta / Q_\zeta, \qquad (61)$$

where Q_ζ is the first moment of area of the cross section above the ζaxis, I_ζ is the second moment or bending moment of inertia about the ζaxis, t_ζ is the thickness of the top and bottom link plate, and

$$V^{LF}_{\zeta\xi} = (\tau^{LFB}_{t\ell} - \tau^{LFL}_{t\ell})\, I_\eta t_\eta / Q_\eta, \qquad (62)$$

where I_η, t_η, and Q_η are analogous to the above (I_ζ, t_ζ, and Q_ζ).

In torsion,

$$M^{LF}_{\xi\ell} = -1/2\ (h_\eta - t_\eta)(h_\zeta - t_\zeta) \left\{(\tau^{LFB}_{t\ell} + \tau^{LFF}_{t\ell})\, t_\eta + (\tau^{LFL}_{t\ell} + \tau^{LFR}_{t\ell}) t_\zeta\right\}. \qquad (63)$$

For bending, the moment at the caving shield pin is a combination of bending moments at the center and bending effects of $V^{LF}_{\eta\xi}$ and $V^{LF}_{\zeta\eta}$. Bending at the center is given by

$$M^{LF}_{\eta} = (\sigma^{LFR}_{\ell} - \sigma^{LFL}_{\ell}) I_\eta / 4h_\zeta, \qquad (64)$$

$$M^{LF}_{\zeta} = (\sigma_\ell^{LFB}) I_\zeta / 4h_\eta. \qquad (65)$$

Then the bending at the end $\xi = \ell$ is given by

$$M^{LF}_{\eta\ell} = M^{LF}_{\eta} - V^{LF}_{\zeta\xi}\ \ell/2, \qquad (66)$$

and

$$M^{LF}_{\zeta\ell} = M^{LF}_{\zeta} - V^{LF}_{\eta\xi}\ \ell/2. \qquad (67)$$

The simple uniaxial strain-gauge placement on the front and back of the link provides the link extensional force (P^{LR}_{ζ}) and a pure bending moment loading ($M^{LR}_{\zeta\ell}$) on the pin axes. With the uniaxial gauges, only one stress is assumed to exist, i.e., longitudinal stress, σ_ℓ, which is obtained from the strains by

$$\sigma_{\ell\ L}^{LFR} = E\varepsilon^{LRF}, \qquad (68)$$

and

$$\sigma^{LRB}_{\ell} = E\varepsilon^{LRB}. \qquad (69)$$

The superscripts in equations 68 and 69 indicate the uniaxial gauge. The force and moment for each link can now be calculated directly, as follows:

For tension:

$$P^{LR}_{\xi} = (\sigma^{LRF}_{\ell} + \sigma^{LRB}_{\ell})\ A/2, \qquad (70)$$

where $A = A_1 + A_2$ the total cross section of each rear link.

For bending:

$$M^{LR}_{\zeta\ell} = (\sigma^{LRB}_{\ell} - \sigma^{LRF}_{\ell}) I_\zeta / 4h_\eta. \qquad (71)$$

With local link moments and forces defined in terms of strains, they may be combined to form equivalent force-couple systems at the midpoints of their respective caving shield pin axes. First, though, local frames (ξ, η, ζ) must be rotated into the (x1, y1, z1) orientation. This is done with the following transformation (fig. 9):

$$A_{x1} = A_\xi \cos\theta,\ - A_\eta \sin\theta, \qquad (72)$$

$$A_{y1} = A_\xi \sin\phi + A_\eta \cos\theta, \qquad (73)$$

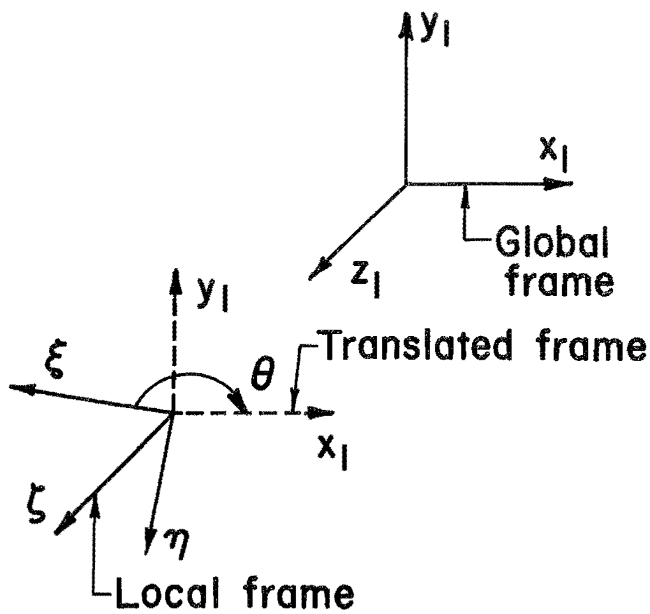

FIGURE 9.—Link rotation angle.

$$A_{z1} = A_\zeta, \tag{74}$$

where the ξ, η, ζ components of the $\tilde{A}$ in equations 72-74 may be any of those just derived.

Then, for links gauges with rosettes,

$$T^{LF}_{x1} = P^{LF}_{\xi} \cos \alpha_3 - V^{LF}_{\eta\xi} \sin \alpha_3, \tag{75}$$

$$T^{LF}_{y1} = P^{LF}_{\xi} \sin \alpha_3 + V^{LF}_{\eta\xi} \cos \alpha, \tag{76}$$

$$T^{LF}_{z1} = V^{LF}_{\zeta\xi}, \tag{77}$$

and

$$M^{LF}_{Tx1} = M^{LF}_{\xi\ell} \cos \alpha_3 - M^{LF}_{\eta\ell} \sin \alpha_3, \tag{78}$$

$$M^{LF}_{Ty1} = M^{LF}_{\xi\ell} \sin \alpha_3 + M^{LF}_{\eta\ell} \cos \alpha_3, \tag{79}$$

$$M^{LF}_{Tz1} = M^{LF}_{\zeta\ell}. \tag{80}$$

For the right link, simply substitute RF for LF in equations 75-80. In all cases α_3 is the angle from the forward link ξ-axis clockwise to the xl-axis. If the links are assumed to be in pure bending (i.e., uniaxial gauge applications), the forces and moments are found as follows:

$$S^{LR}_{x1} = P^{LR}_{\xi} \cos \alpha_4, \tag{81}$$

$$S^{LR}_{y1} = P^{LR}_{\xi} \sin \alpha_4, \tag{82}$$

$$S^{LR}_{z1} = V^{LR}_{\zeta}. \tag{83}$$

The moments M^{LR}_{Sx1} and M^{LR}_{Sy1} are assumed to be zero. This assumption was necessary to make the static three-dimensional equations determinant. The relationship for the remaining moment is

$$M^{LR}_{z1} = M^{LR}_{\zeta\ell}. \tag{84}$$

Then the forces and moments from equations 75-80 and equation 84 were joined with their right link counterparts, as were the leg forces in equations 46-49.

For rosette-gauged links:

$$T_{x1} = T^{LF}_{x1} + T^{RF}_{x1}, \tag{85}$$

$$T_{y1} = T^{LF}_{y1} + T^{RF}_{y1}, \tag{86}$$

$$T_{z1} = T^{LF}_{z1} + T^{RF}_{z1}, \tag{87}$$

$$M_{Tx1} = M^{LF}_{Tx1} + M^{RF}_{Tx1} - z_T T^{RF}_{y1} + z_T T^{LF}_{y1}, \tag{88}$$

$$M_{Ty1} = M^{LF}_{Ty1} + M^{RF}_{Ty1} + z_T T^{RF}_{z1} - z_T T^{LF}_{z1}, \tag{89}$$

$$M_{Tz1} = M^{LF}_{Tz1} + M^{RF}_{Tz1}. \tag{90}$$

For uniaxial-gauged links:

$$S_{x1} = S^{LR}_{x1} + S^{RR}_{x1}, \tag{91}$$

$$S_{y1} = S^{LR}_{y1} + S^{RR}_{y1}, \tag{92}$$

$$S_{z1} = S^{LR}_{z1} + S^{RR}_{z1}, \tag{93}$$

$$M_{Sx1} = -z_S S^{RR}_{y1} + z_S S^{LR}_{y1}, \tag{94}$$

$$M_{Sy1} = z_S S^{RR}_{x1} - z_S S^{LR}_{x1}, \tag{95}$$

$$M_{Sz1} = M^{LR}_{Sz1} + M^{RR}_{Sz1}. \tag{96}$$

In equations 91-93, z_T is half the distance between the forward link longitudinal center lines. In equations 94-96, z_S is analogous to z_T for the rear links.

TWO-DIMENSIONAL REDUCTION

The three-dimensional model previously presented can be reduced to two dimensions by eliminating chosen out-of-plane forces. Since the primary loading occurs in the vertical (roof-to-floor) and horizontal (face-to-waste) axis, forces and moments in the lateral axis (subscript z for forces and x and y for moments) will be eliminated. The equations of static equilibrium, using the same nomenclature as before, then become

FOR CANOPY (fig. 2)

$$\sum F_{x1} = P_{x1} + Q_{x1} + L_{x1} + R_{x1} = 0. \quad (97)$$

$$\sum F_{y1} = P_{y1} + Q_{y1} + L_{y1} + R_{y1} = 0. \quad (98)$$

$$\sum M_{z1}(01) = M_{Pz1} + M_{Qz1} + M_{Lz1} + M_{Lz1} + M_{Rz1}$$
$$+ (x_{1Q}Q_{y1} - y_{1Q}Q_{x1}) + (x_{1L}L_{y1} - y_{1L}L_{x1})$$
$$+ (x_{1R}R_{y1} - y_{1R}R_{x1}) = 0. \quad (99)$$

FOR CAVING SHIELD

$$\sum F_{x1} = -P_{x1} - Q_{x1} + (T_{x2}\cos \alpha - T_{y2}\sin \alpha) + (S_{x2}\cos \alpha - S_{y2}\sin \alpha)$$
$$+ (R_{Cx2}\cos \alpha - R_{Cy2}\sin \alpha) = 0. \quad (100)$$

$$\sum F_{y1} = -P_{y1} - Q_{y1} + (T_{x2}\sin \alpha + T_{y2}\cos \alpha) + (S_{x2}\sin \alpha + S_{y2}\cos \alpha)$$
$$+ (R_{Cx2}\sin \alpha + R_{Cy2}\cos \alpha) = 0. \quad (101)$$

$$\sum M_{z1}(02) + [x_{2E}\ (P_{x1}\sin \alpha - P_{y1}\cos \alpha) + y_{2E}(P_{x1}\cos \alpha + P_{y1}\sin \alpha)]$$
$$+ [x_{2U}\ (Q_{x1}\sin \alpha - Q_{y1}\cos \alpha) + y_{2U}(Q_{x1}\cos \alpha + Q_{y1}\sin \alpha)]$$
$$+ [x_{2T}\ (T_{y2}) - y_{2T}(T_{x2})] + [x_{2RC}(R_{Cy2}) - y_{2RC}(R_{Cx2})] = 0. \quad (102)$$

Solution of the two-dimensional resultant load in the x-y (horizontal-vertical) plane is then found as follows, using the same elimination of forces as in the three-dimensional model.

HORIZONTAL RESULTANT LOAD (R_{x1})

1. From summation of forces acting on caving shield, determine reactions at canopy hinge pin.

$$P_{x1} = -Q_{x1} + (T_{x2}\cos \alpha - T_{y2}\sin \alpha) + (S_{x2}\cos \alpha - S_{y2}\sin \alpha).$$

$$P_{y1} = -Q_{y1} + (T_{x2}\sin \alpha + T_{y2}\cos \alpha) + (S_{x2}\sin \alpha + S_{y2}\cos \alpha).$$

2. Substitute these expressions into the summation of moment equation for the caving shield.

$$x_{2E}\ [-Q_{x1}\sin \alpha + \sin \alpha(T_{x2}\cos \alpha - T_{y2}\sin \alpha) + \sin \alpha(S_{x2}\cos \alpha - S_{y2}\sin \alpha)$$
$$+ Q_{y1}\cos \alpha - \cos \alpha(T_{x2}\sin \alpha + T_{y2}\cos \alpha) - \cos \alpha(S_{x2}\sin \alpha + S_{y2}\cos \alpha)]$$

$$-y_{2E}\ [Q_{x1}\cos\alpha - \cos\alpha(T_{x2}\cos\alpha - T_{y2}\sin\alpha) - \cos\alpha(S_{x2}\cos\alpha - S_{y2}\sin\alpha)$$

$$+ (Q_{y1}\sin\alpha - \sin\alpha(T_{x2}\sin\alpha + T_{y2}\cos\alpha) - \sin\alpha(S_{x2}\cos\alpha + S_{y2}\sin\alpha)]$$

$$+ x_{2U}\ (Q_{x1}\sin\alpha - Q_{y1}\cos\alpha) + y_{2U}(Q_{x1}\cos\alpha + Q_{y1}\sin\alpha)$$

$$+ x_{2T}\ (T_{y2}) - y_{2T}(T_{x2}) = 0.$$

$$x_{2E}\ [-Q_{x1}\sin\alpha - T_{y2} - S_{y2} + Q_{y1}\cos\alpha]$$

$$-\ y_{2E}\ [Q_{x1}\cos\alpha - T_{x2} - S_{x2} + Q_{y1}\sin\alpha]$$

$$+ x_{2U}\ [Q_{x1}\sin\alpha - Q_{y1}\cos\alpha] + y_{2U}[Q_{x1}\cos\alpha + Q_{y1}\sin\alpha]$$

$$+ x_{2T}\ (T_{y2}) - y_{2T}(T_{x2}) = 0.$$

3. Solve for tension link force S where

$$S_{y2} = S\ \sin\beta \text{ and } S_{x2} = S\ \cos\beta$$

where β equals angle between link and y-axis in (x_2, y_2, z_2) coordinate frame.

$$x_{2E}\ [-Q_{x1}\sin\alpha - T_{y2} - S\ \sin\beta + Q_{y1}\cos\alpha]$$

$$-\ y_{2E}\ [Q_{x1}\cos\alpha - T_{x2} - S\ \cos\beta + Q_{y1}\sin\alpha]$$

$$+ x_{2U}\ [Q_{x1}\sin\alpha - Q_{y1}\cos\alpha] + y_{2U}\ [Q_{x1}\cos\alpha + Q_{y1}\sin\alpha]$$

$$+ q_{2T}\ (T_{y2}) - y_{2T}(T_{x2}) = 0.$$

$$S = \{ x_{2E}\ [-Q_{x1}\sin\alpha - T_{y2} + Q_{y1}\cos\alpha] - y_{2E}[Q_{x1}\cos\alpha - T_{x2} + Q_{y1}\sin\alpha]$$

$$+ x\ \ [Q_{x1}\sin\alpha - Q_{y1}\cos\alpha] + y_{2U}[Q_{x1}\cos\alpha + Q_{y1}\sin\alpha]$$

$$+ x_{2T}\ (T_{y2}) - y_{2T}(T_{x2})\}/[x_{2E}\sin\beta - y_{2E}\cos\beta].$$

4. From summation of forces on caving shield, determine horizontal reaction at canopy hinge pin.

$$P_{x1} = -Q_{x1} + (T_{x2}\cos\alpha - T_{y2}\sin\alpha) + (S_{x2}\cos\alpha - S_{y2}\sin\alpha).$$

5. Substitute results of steps 1-4 into equation for summation of forces on canopy and solve for (R_{x1}).

$$R_{x1} = -P_{x1} - Q_{x1} - L_{x1}.$$

VERTICAL RESULTANT LOAD (R_{y1})

1. Solve for vertical reaction at canopy hinge pin from summation of forces on caving shield.

$$P_{y1} = -Q_{y1} + (T_{x2}\sin\alpha + T_{y2}\cos\alpha) + (S_{x2}\sin\alpha + S_{y2}\cos\alpha).$$

2. Substitute P_{y1} and results from step 3 previously into expression for summation of forces on canopy and solve for R_{y1}.

$$R_{y1} = -P_{y1} - Q_{y1} - L_{y1}.$$

VERTICAL RESULTANT LOCATION (x_{1R})

Solve for x_{1R} from summation of moments on canopy.

$$x_{1R} = [-(x_{1Q}Q_{y1} - y_{1Q}Q_{x1}) - (x_{1L}L_{y1} - y_{1L}L_{x1}) + y_{1R}R_x] / R_{y1}.$$

CONCLUSIONS

The solution of shield mechanics from a rigid-body static analysis of the shield structure is fundamental to an understanding of shield behavior. A three-dimensional analysis considering three forces and three moments at each reaction reveals the shield structure is indeterminate to a high degree. A determinate solution is produced by eliminating several unknowns which are nonparticipating or thought to have little effect on overall shield mechanics, and by measuring leg, canopy capsule, and lemniscate link forces. Even with these reductions, the three-dimensional static equilibrium equations are cumbersome. A two-dimensional analysis of the shield is obtained by further reduction of the three-dimensional equilibrium equations by elimination of out-of-plane forces. The two-dimensional planar model, defined by the face-to-waste and roof-to-floor plane, should be adequate for most shield analyses, as the primary response of the shield structure is to resist roof-to-floor and face-to-waste strata displacement.

The research presented in this report is a first step in the solution of shield mechanics. The approach taken in these initial efforts is to provide closed-form solutions of statically determinate models by rigid-body analysis of the shield structure. As shown in the analysis, the determinate solution requires several assumptions as to the participation of forces and moments. A more prudent approach, especially in the longer term, is to achieve an indeterminate solution utilizing a work-energy or stiffness evaluation. Such models will be evaluated as part of the Bureau's continuing efforts in mine roof support research.

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