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# Procedures for Determining Support of Excavations in Highly Yielding Ground

By J. D. Dixon, M. A. Mahtab, and T. W. Smelser



UNITED STATES DEPARTMENT OF THE INTERIOR



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UNIT OF MEASURE ABBREVIATIONS USED IN THIS REPORT

ft	foot	pct	percent
h	hour	lbf/in <sup>2</sup>	pound (force) per square inch
in	inch	lbf/ft <sup>2</sup>	pound (force) per square foot
in <sup>2</sup>	square inch	lbf/ft <sup>3</sup>	pound (force) per cubic foot
lb	pound		

# PROCEDURES FOR DETERMINING SUPPORT OF EXCAVATIONS IN HIGHLY YIELDING GROUND

By J. D. Dixon,<sup>1</sup> M. A. Mahtab,<sup>2</sup> and T. W. Smelser<sup>3</sup>

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## ABSTRACT

This Bureau of Mines report describes the results of an investigation for developing and applying procedures for stabilizing excavations openings in highly yielding ground. The approach developed in this report involves nonlinear modeling of the progressive relaxation of the zones of rock mass around the excavations opening where Coulomb criterion of failure is exceeded.

Stresses are calculated by using a computer code; Coulomb-failure condition is examined for both intact rock and joints (using strength parameters  $\phi$  and  $c$ ). The rock mass in the failed zones is relaxed by reducing the Young's modulus of the rock based on an empirical formulation that is a function of the shape of the opening and the extent of the failed zone. In addition, the material in the failed zone is assigned reduced strength parameters  $\phi'$  and  $c'$ . The limiting (or convergent) relaxed zone is obtained through an iterative process by manually changing the rock modulus and the strength parameters between iterations. Support pressures are computed by applying forces normal to the excavation surface. The approach is applied to the analysis of stability and support requirements for an entry in a longwall coal mine.

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## INTRODUCTION

An important problem in underground mining is that of stabilizing excavations in highly yielding rock masses. In underground excavations, this behavior is manifested by the intrusion of rock into the opening made by the excavation. Examples of rocks prone to this behavior are coal and coal measure rocks, fault gouge, uranium sands, and highly jointed or fragmented rock. In mining and tunneling situations, these rocks produce relatively large deformations that lead to interference with mining operations and produce safety hazards. This report is concerned with the development of a rational basis for calculating pressures, applied against the underground excavation surface, that are needed to confine the rock, prevent intrusions, and bring about stability. This solution is useful for assessing the minimum requirements for artificial support for excavations in highly yielding rock masses.

In a previous investigation (4),<sup>4</sup> methods were developed for calculating confinement pressures for preventing Coulomb failure along the surface of an underground excavation in rock. The elastic stress distribution around underground excavations, and the Coulomb strength parameters,  $c$ , and  $\phi$ , were used as the basis for making this calculation. In overstressed rock, confinement pressure distribution around the excavation represents a solution for determining artificial support requirements for stabilizing the opening.

For two reasons, the previous solution is conservative and overpredicts the magnitude of confinement pressure actually needed for stability. First, the stress distribution obtained from the elastic analysis overpredicts the magnitude of stresses in areas where the rock material yields. In such areas, stresses are redistributed away from the edge of the excavation, leaving lower stresses and, therefore, lower confinement pressure

needed for stability. Second, the earlier solution does not account for a degradation of the initial rock-strength parameters,  $c$  and  $\phi$ , that occurs in yielding rock. Beyond certain strain limits, these parameters degenerate to their residual values,  $c'$  and  $\phi'$ , thus extending and modifying the plastic zone (or shear zone) that develops around the excavation. This further reduces the stresses at the boundary of the excavation and, therefore, the required confinement pressure. This procedure is overly conservative and predicts much higher confinement pressures than are needed.

By extending the methods developed in the earlier investigation, a solution approach is presented here that takes these factors into account. This approach includes the calculation of the elastic stress distribution around an underground excavation using the finite-element method, a procedure for determining the zone of plastic yielding, a method for reducing Young's modulus in the plastic zone, and a method for determining the confining pressure distribution necessary for bringing about stability.

There are no engineering solutions that adequately treat the problem of stabilizing excavations in highly yielding rock masses, although some approximate solutions are available.

Most investigators recognize that a support system in highly yielding ground must meet two requirements. First, it must supply sufficient confinement or reinforcement to allow stresses in the rock to readjust until stability is achieved; and second, it must comply with the movements of the rock mass without losing its ability to provide confinement and reinforcement.

The stability of underground excavations in yielding rock depends on the shear strength of the surrounding mass. Westergaard (8) published a solution for the stresses around a borehole, assuming that a plastic zone develops around the hole when the material fails according to Coulomb criterion. The results showed

<sup>4</sup>Underlined numbers in parentheses refer to items in the list of references preceding the appendix.

that a small radial confining pressure at the surface of the borehole enabled the radial stresses to increase rapidly with distance, such that the radial pressures at a fraction of the radius (behind the borehole surface) furnished sufficient confinement to support the high circum-

ferential stresses. Hendron and Aiyer (5) advanced a solution to this problem by assuming that the yielding material behaved as an incompressible solid and account for the dilatant behavior of the rock mass.

## DISCUSSION

### PRELIMINARY CONSIDERATIONS

#### Failure and Strength of Rock

The behavior of underground excavations is manifested in many forms. In a weak rock that is considerably overstressed, the behavior is characterized by large rock deformations and intrusions of the rock into the opening. In a predominantly vertical stress field, these deformations occur along the sides of the excavations, resulting in the material falling out of the roof--a rock-loosening type of behavior.

The stress condition on the rock mass, relative to its strength, is a factor that preconditions the behavior of an underground excavation. For instance, if the rock is considered to be a Coulomb material (as it will be throughout this report), then the closer the stress condition is to the Coulomb failure surface (fig. 1) before excavation, the greater is the tendency of the rock toward deformation into the opening after excavation. While the magnitude of stresses in one rock mass may be at the threshold of failure prior to excavation, another may have low-level stresses and be capable of sustaining relatively large increases in stress before reaching this threshold of failure.

The stress-strength condition of rock, usually unknown before excavation, is estimated in the design assumptions and criteria for underground construction. This, in part, is due to the difficulties in determining either the in situ state of stress or the in situ rock strength.

### Stability Requirements

Three mechanisms, stress redistribution, reinforcement, and confinement, singly or collectively, promote a stress condition in the rock that does not exceed the Coulomb failure surface.

The stress redistribution that occurs due to the presence of the excavation is concentrated near the boundary. Tangential stresses are in excess of field stresses, and the radial stress is zero at the boundary. If it is assumed that the circle A in figure 1 is the Mohr diagram for the stress condition on an element of rock near the boundary of the

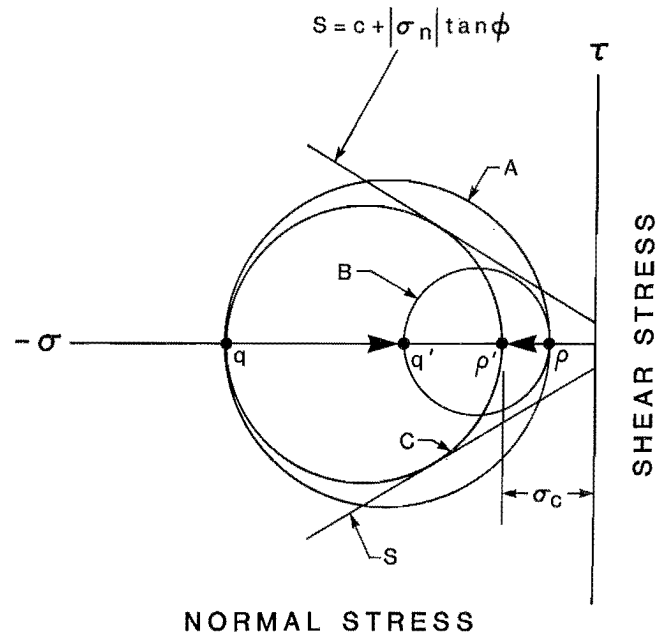


FIGURE 1. - Stress equilibrium in a Coulomb material.



excavation, then the tangential stress is the minimum algebraic principal stress,  $q$ , and the radial stress is the maximum algebraic principal stress,  $p$ . As shown, this stress condition exceeds the Coulomb failure surface and cannot exist indefinitely. Either the material will break down to allow further stress redistribution to occur, or, if artificial support is present, the material can be reinforced or confined sufficiently such that failure will be prevented.

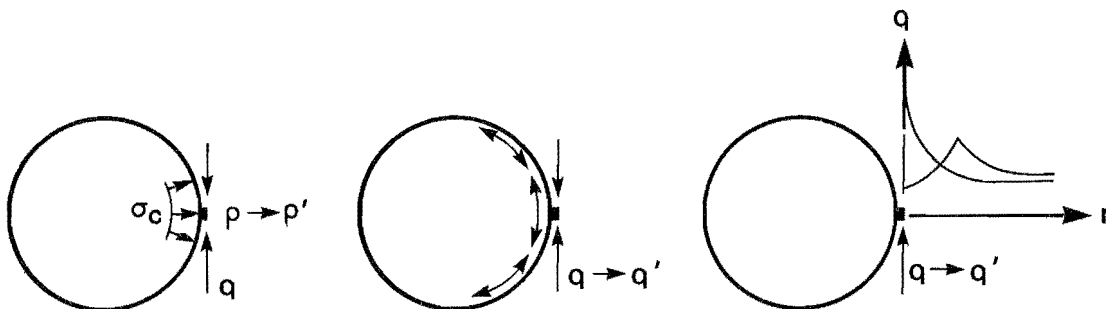
If unsupported, an overstressed excavation can achieve stability only through the mechanism of stress redistribution. In this process, material failure will be necessary. Material properties such as Young's modulus, Poisson's ratio, friction angle, and cohesion will undergo changes as internal fractures and flaws develop, thus allowing material deformations that are necessary for the stress redistributions to occur. High stresses near the boundary will be lowered as excessive loads are transferred deeper into the rock abutments. In figure 1, this process is represented by the reduction of the tangential stress, from  $q$  to  $q'$ , such that the stress circle falls within the Coulomb failure surface. The stress redistribution that is necessary is shown in figure 2, section *C*.

The two other mechanisms, reinforcement and confinement, are associated with the installation of support after excavation of the rock; these two mechanisms are represented in sections *B* and *A*, respectively. In general, artificial supports are not installed until excavation and

mucking operations are completed and may not initially be fitted tightly against the excavated surface. Therefore, in overstressed rock, uninhibited deformations will accompany the stress redistribution until the slack between the artificial support and the rock face is taken up. Reinforcement of the rock face results in lowering the minor principal compressive stress from  $q$  to  $q'$  as indicated in section *B*. Excessive stresses in the rock are shed to artificial reinforcing members placed around the excavations as the overstressed rock yields. Concrete linings, shotcrete, steel columns, and timber posts are examples of such reinforcement.

The third mechanism, confinement, is promoted by the normal pressures that develop between artificial supports in the presence of the inwardly deforming rock. The confinement pressure,  $p$ , is increased until the stress circle is contained within the failure surface, as indicated in section *A*.

After excavation, but before the achievement of stability, the rock may slough if its strain limits are exceeded. Interaction between the support system and the rock mass leads to the development of forces that may or may not stabilize the rock mass. If the support system is poorly adapted, or is inadequate, the induced rock pressures may cause failure in the support system before stability is achieved. When rock bolts are used and the deformability characteristics of the rock mass and bolts are grossly mismatched, the anchorage points



A, By confinement      B, By reinforcement      C, By stress redistribution

FIGURE 2. - Mechanisms for achieving stress equilibrium around underground excavation.

of the bolt may fail, or, in the other extreme, the bolt may not be adequately loaded to bring about stability.

One approach for stabilizing the excavation would be to calculate the required confining forces without consideration of the support system, and then design a system capable of supplying these forces. This approach does not consider other interactions between the support system and the rock mass that may lead to premature local failures, either in the rock or of the supports. These interaction forces, not necessarily related to those required for stability (see discussion above), result from the deformational interactions between the rock mass and the support system that are caused by changes in the stiffness of the two systems. For instance, as the rock mass deforms, the support system must adjust to this deformation, requiring the development of forces between the two systems.

Another approach to stability would be to follow the excavation and installation sequence, thus evaluating the support-rock mass interactions at each step. This may allow a design of the support system that accounts for the interacting forces but does not supply the necessary stability conditions.

The solution process that was evolved to treat the difficulties in the above approaches is discussed in the next section of this report.

#### SOLUTION PROCEDURE

An outline of the solution procedure for stabilizing drifts in highly yielding ground (fig. 3) is given below and is further discussed in subsequent sections.

1. Perform a two-dimensional, linear, elastic stress analysis of the underground excavation (figure 3, section A).

2. Determine the zone(s) of incipient failure of rock around the excavation using the Coulomb-strength parameters  $c$  and  $\phi$  (section B).

3. Reduce the elastic modulus of the rock in the failed zone (section B).

4. Repeat the stress analysis as in step 1.

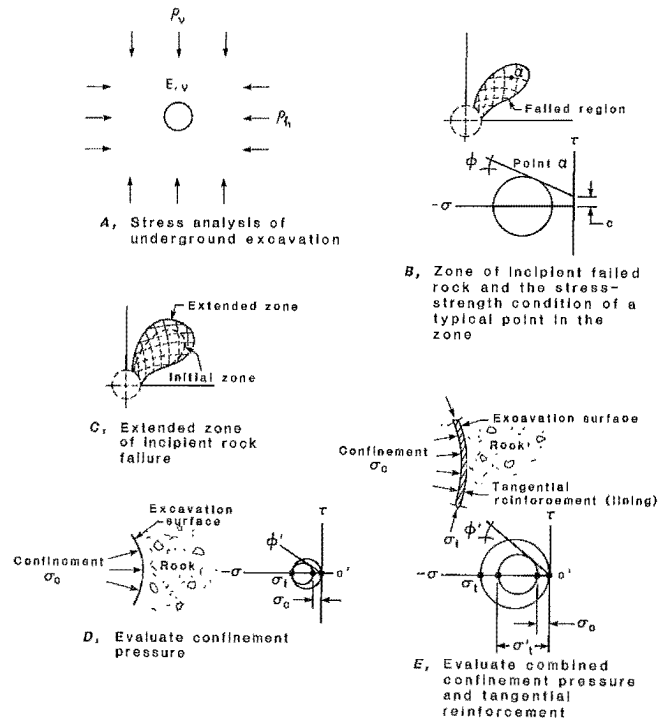


FIGURE 3. - Schematic representing steps of solution process.

5. Determine the extended zone of incipient rock failure using the strength parameters  $c$  and  $\phi$  (section C).

6. Iterate steps 3, 4, and 5 until convergence has been achieved.

7. Determine requirements for achieving ground control based on reduced-strength parameters  $c'$  and  $\phi'$  of the rock elements adjacent to the excavation surface.

a. Evaluate the confinement pressure necessary (section D), or

b. Evaluate the combined confinement pressure and tangential reinforcement (section E).

#### METHOD OF STRESS ANALYSIS

In the solution procedure given above, linear stress analyses are made to approximate the progressive rock-loosening process and determine the stability requirements. The finite-element method (2, 9) was applied in making these analyses. The NONSAP computer code, developed by Bathe, Wilson, and Iding (3), and modified by Dixon and Mahtab (4), was

used in this investigation. An independent two-step procedure is used in performing this analysis. In the first step, the stress redistribution in the rock mass due to progressive rock property degradation is found. In the second step, stabilizing requirements of the support system are determined. In either step, the presence of the tunnel support is not considered as part of the finite-element mesh. The analyses assume two-dimensional plane-strain conditions.

To approximate the progressive loosening process, an iterative technique is used in which the modulus of rock in the loosened zones was reduced manually between computer executions. This analysis is iterated until convergence of the solution is obtained. Convergence is considered to be attained when successive solutions indicate no further extension of the loosened rock zone. At this point, the rock mass is readjusted in compliance with the existing stress and strength conditions.

In the loosened rock zone, the rock strength parameters  $\phi$  and  $c$  are considered to have degraded to residual values,  $c'$  and  $\phi'$ . In the limiting case, cohesion is considered to be lost ( $c' = 0$ ), and the friction angle is reduced to a minimum value ( $\phi' = 30^\circ$ ). Therefore, the excavation may be unstable unless confinement or reinforcement is provided.

In the next step of the analysis, the stabilizing requirements of the support system are determined as necessary to reduce the tangential stresses around the opening to be in compliance with the reduced rock-strength properties,  $c'$  and  $\phi'$ . The stabilizing requirements are the confining pressures and tangential forces that are supplied by the tunnel support system.

In the process of designing a support system, not further described here, the forces due to confinement and reinforcement are applied to the support system in order to determine the internal stress distribution in the support system. This will allow the analyst to compare the existing stress condition against the support material strength, as in the design process.

The procedure for determining confining pressure distribution is provided in the subroutine STABIL (4) used in conjunction with the NONSAP code. The subroutine does not consider the contribution of reinforcement toward stability, but a similar procedure is conceivable. Such a procedure would require additional rationale for assigning the relative contributions of confinement and reinforcement toward the achievement of stability.

Some support systems, such as rock bolts, supply only confinement while others, such as rigid linings, supply both confinement and tangential reinforcement to the adjacent rock. In general, confinement alone is sufficient for bringing about stability. However, reinforcement alone cannot bring about stability if cohesion does not exist.

#### DETERMINATION OF FAILED ZONES

This section presents the methods used to determine failure zones in the rock mass around an underground excavation and used in the computer solutions. The treatment given here is based on graphical and analytical expressions utilizing Mohr's stress diagram for illustrations. The analysis procedures presented are used for comparing the existing shear stress,  $\tau_\eta$ , with the existing shear strength,  $S$ , at any point in the rock mass around the excavation. Where  $\tau_\eta > S$ , the rock is considered to have failed. These procedures are incorporated into the computerized solution technique. Considerations are made for evaluating  $\tau_\eta$  and  $S$  for both intact and jointed rock.

The state of stress at a point is represented in figure 4. An element is subjected to compressive principal stresses  $p$  and  $q$ . Obert and Duvall (6) give sign conventions and methods for constructing a Mohr stress diagram.

The Coulomb shear strength equation is

$$S = c + |\sigma_\eta| \tan \phi, \quad (1)$$

where  $S$  = shear strength of the rock,

$c$  = cohesion of the rock,

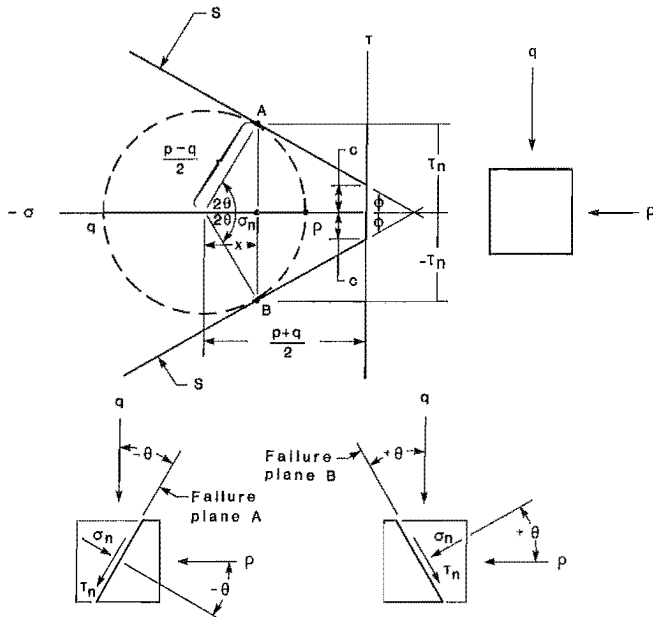


FIGURE 4. - Coulomb failure surface and Mohr stress circle at incipient failure condition.

$\phi$  = friction angle of the rock,

and  $\sigma_n$  = the compressive normal stress on the incipient failure plane.

In this discussion, the quantities  $\phi$  and  $c$  are considered to be positive.

Equation 1 is plotted on the Mohr stress diagram (fig. 4). The failure condition occurs when the Mohr stress circle becomes tangent to the Coulomb failure surface (at points A and B). The principal stresses for this condition are  $p$  and  $q$ . Note that each of the two points of tangency represents a different failure surface. In relation to the unit stress element, these failure surfaces can be found by the usual graphical techniques. The stress condition at point A can be determined by rotating counter-clockwise at  $2\theta$  from the  $p$  axis on the Mohr diagram and, conversely, clockwise at  $\theta$  from the  $p$  axis on the unit stress element. Note, that from trigonometry,

$$|-\theta| = 45^\circ - \phi/2. \quad (2)$$

We may denote this failure surface A. In the same way, we may find the stress condition along the other failure surface B (fig. 4). Note that in this case,

$$|+\theta| = 45^\circ - \phi/2. \quad (3)$$

Along these planes (A and B), the stress conditions that define the Coulomb failure criterion are the normal stress  $\sigma_n$  and the shear stress  $\tau_n$ , which can be determined graphically from the quantities given in figure 4. Here we find that

$$\sigma_n = \frac{p+q}{2} + x.$$

$$\text{Since } x = \frac{p-q}{2} \cos 2\theta,$$

$$\text{hence } \sigma_n = \frac{p+q}{2} + \frac{p-q}{2} \cos 2\theta. \quad (4)$$

Also, we find that

$$\tau_n = -\frac{p-q}{2} \sin 2\theta. \quad (5)$$

The negative sign in equation 5 is necessary to assure that the sign of  $\tau_n$  is positive (note that the angle  $\theta$  is negative in this case, and the quantity  $p-q$  will always be positive).

Since  $\phi$  and  $\theta$  are related, equations 4 and 5 can be expressed in terms of  $\phi$  as follows:

$$\sigma_n = \frac{p+q}{2} + \frac{p-q}{2} \sin \phi, \quad (6)$$

$$|\tau_n| = \frac{p-q}{2} \cos \phi. \quad (7)$$

Equation 7 does not prescribe a sign for  $\tau_n$ , since failure could occur in either of the directions  $\pm\theta$  from  $q$ . If the direction  $\theta$  is known, equation 5 may be used to correctly identify the sign of  $\tau_n$ .

These equations predict the stress condition at failure. An estimate of the severity of an existing stress condition compared with the strength is given by the quotient

$$F = \frac{\tau_n}{S}. \quad (8)$$

If failure were imminent, this quotient would equal or exceed 1.0. The calculation for  $\tau_{\eta}$  is based on the preceding equations 5 or 7; however, the calculation of  $S$  involves an element of subjectivity. This subjectivity arises because the existing stress condition does not necessarily provide a stress circle that has tangency with the failure surface, and there exist multiple possibilities as to what stress change must be made to achieve the tangency condition. Even though the existing normal stress  $\sigma_{\eta}$  is known, it is not generally possible to predetermine what normal stress, say  $\sigma'_{\eta}$ , would exist at the point of incipient failure. This point is illustrated in figure 5. The existing stress condition can be modified in three different ways to bring it to tangency with the failure surface:

1. Decrease the magnitude of  $p$  to  $p'$  (figure 5, section A).
2. Increase the magnitude of  $q$  to  $q'$ , and decrease the magnitude of  $p$  to  $p'$  (panel B).
3. Increase the magnitude of  $q$  to  $q'$  (section C).

As indicated in figure 5,  $\sigma_{\eta}$  changes for each of these possibilities, and  $S$  is likely to be overevaluated or underevaluated. Since  $\sigma_{\eta}$  is available from the existing state of stress and represents an intermediate value of  $\sigma'_{\eta}$ , it is a likely choice for the calculation in equation 8 and is used whenever necessary, thus

$$\sigma'_{\eta} = \sigma_{\eta} \quad (9)$$

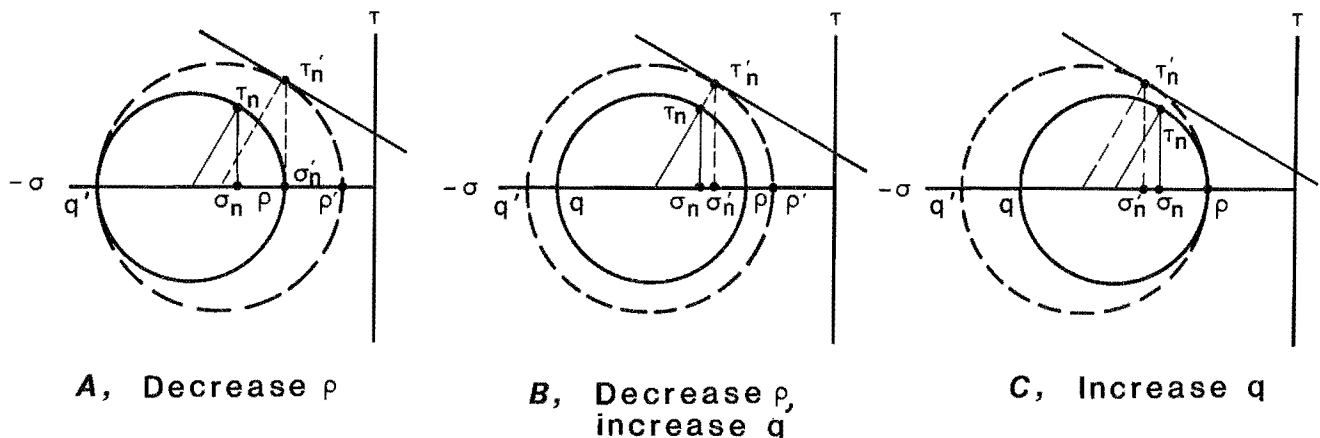


FIGURE 5. - Evaluation of normal stress on failure surface.

When joints exist in a rock mass, shear failure may occur along the joint surface instead of through the intact rock. To evaluate the strength and stress conditions along a joint plane, the same approach (as used for intact rock) can be applied. In this case, failure must occur along the plane of the joint inclined at  $\theta$  from  $q$  (fig. 6). The Coulomb strength equation is as follows:

$$S_j = c_j + |\sigma_{\eta j}| \tan \phi_j \quad (10)$$

where  $S_j$  = shear strength of the joint,

$c_j$  = cohesion of the joint,

$\phi_j$  = friction angle of the joint,

and  $\sigma_{\eta j}$  = compressive normal stress on the plane of the joint.

Equation 10 is evaluated by computing the quantities  $\sigma_{\eta j}$  and  $\tau_{\eta j}$  from a stress analysis of the structure. Given the state of stress at a point, and the orientation of the joint plane, this evaluation is illustrated on the Mohr stress diagram of figure 6. Since the strength of the joint computed from equation 10 depends on the joint inclination  $\theta_j$ , the joint strength may exceed the intact rock strength for certain values of  $\theta_j$ . Therefore, both the intact-rock shear strength and the joint shear strength should be computed and compared with the shear stress along the potential planes of failure.

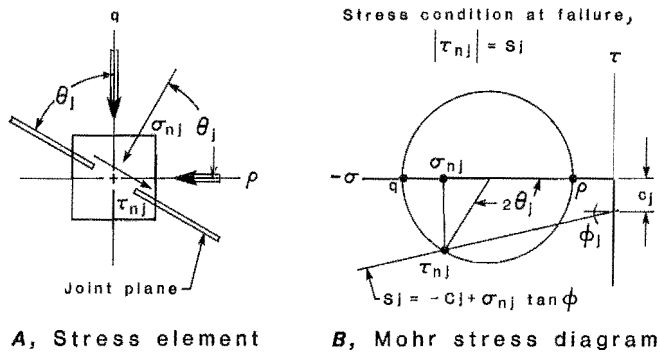


FIGURE 6. - Stress conditions at failure for jointed rock.

#### REDUCTION OF ELASTIC MODULUS IN FAILED ZONE

The failed zones in the rock mass, determined by the procedures given above, are considered to have undergone changes in their effective stiffness. In the analysis procedure (steps 3 and 5), the modulus of the rock mass is decreased in these failure zones. However, a rational method is needed for determining this reduction of modulus. Some finite-element codes, such as BMINES (7) or ADINA (1), provide nonlinear material models, based on Coulomb-type behavior, that reduce the modulus of the material in the failed zones. In this investigation, problems were encountered in obtaining converged solutions.

Consequently, an empirical approximation scheme was developed for reducing the modulus in the failed zones. A basic requirement was that the modulus reduction must be compatible with the failure criterion; that is, the stress distribution in the failure zone must everywhere satisfy the equation

$$\tau_n/S \leq 1.0. \quad (11)$$

Several candidate relationships for estimating the reduced modulus in the failed (plastic) zones were evaluated. The relationships that best met the criterion stated by equation 11 were characterized by having values that decay exponentially with distance from the nonfailed (intact or elastic) rock zones. That is, the reduced modulus,  $E_r$ , increases very slowly

near the edge of the opening (where rock has failed) and then increases nearly exponentially as the elastic-plastic boundary is approached.

A similar relationship between the reduction factor for rock modulus,  $E_r/E_{seis}$ , and quality of rock, RQD, is shown in figure 7 (5). This relationship is based on field tests around underground excavations and suggests that at the boundary of the opening, where RQD is nearly zero, the effective modulus of the broken rock,  $E_r$ , is about 15 pct of the modulus of the intact rock,  $E_{seis}$ . With increasing distance from the boundary (that is, with increasing RQD), the modulus of the rock increases gradually, up to a RQD of 0.65 and very rapidly thereafter, finally becoming equal to the value of the modulus for intact rock when the RQD equals 1.0.

Based on these findings, the formulations for reduced modulus of rock in the broken or plastic zone were developed for this study as a function of distance from the edge of the opening (in the case of simple geometries of plastic zones) or from the centroids of odd-shaped plastic zones.

For circular excavations, the relationship

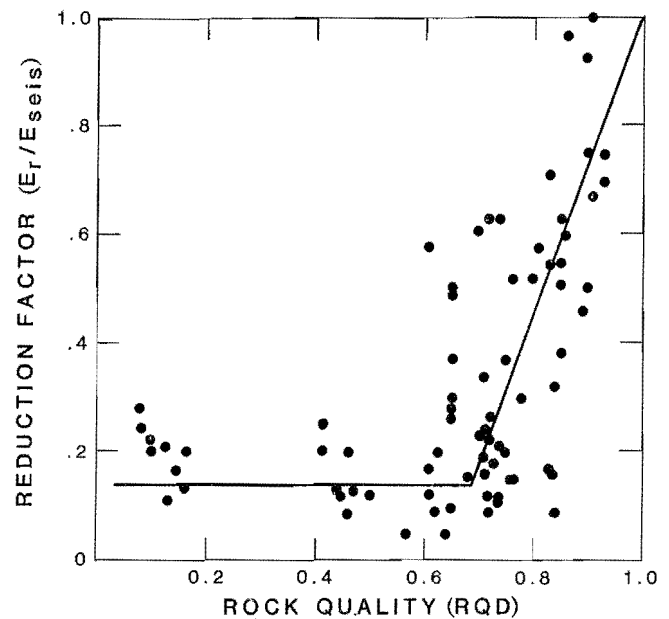


FIGURE 7. - Relation between rock quality and modulus-reduction factor.

$$E_r/E = 0.15 \left[ 3.95 \left( \frac{r-a}{R-a} \right)^3 + e \left( \frac{r-a}{R-a} \right)^3 \right] \quad (12)$$

was formulated. This relationship was plotted in figure 8. In equation 12, the reduced modulus,  $E_r$ , at a radial distance,  $r$ , in the plastic region is related to the initial modulus,  $E$ , by the radius of the excavation,  $a$ , and the radial distance,  $R$ , to the elastic-plastic boundary. The symbol  $e$  is the base of natural logarithm ( $=2.71828\cdots$ ).

In making the modulus reductions, iterations of the stress analysis are required. After the first iteration, an approximation of the failed rock zone is determined. Then, by manual calculation, the modulus is reduced in this zone. The

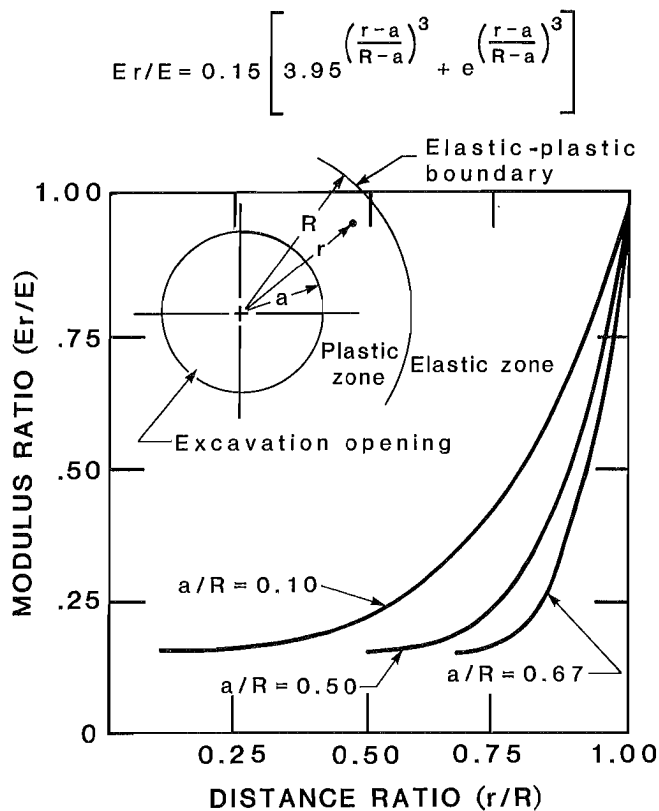


FIGURE 8. - Assumed modulus distribution around circular underground excavation.

second and subsequent iterations are reported in this way. The area of the failed rock zone is extended until no further changes occur between two successive iterations. This method was developed by performing analyses using a circular excavation shape. An example of the progressive development of the plastic zone achieved by this method is given in figure 9.

For noncircular openings, equation 12 cannot be used because of its dependency on the radius,  $a$ . In this case, the following modified version of equation 12 was used:

$$E_r/E = 0.15 \left[ 3.95 \left( \frac{r}{R} \right)^3 + e \left( \frac{r}{R} \right)^3 \right] \quad (13)$$

The use of equation 13 is illustrated by figure 10. This is an example of an excavation of rectangular cross section that contains failed rock zones of two different shapes.

In the zone designated as "failed rock region 1," a focal point is determined from which the values  $r$  and  $R$  are measured to points within and at the boundary, respectively, of the failed zone. The focal point is the approximate centroid of the zone. In the other zone, designated as "failed rock region 2," the values  $r$  and  $R$  are measured from the excavation surface to the points  $c$  (within the failed zone) and  $d$  (boundary of failed zone), respectively.

The reduced modulus,  $E_r$ , is thus computed throughout the failed rock zones and used in the finite-element analyses.

Steps 1 through 6 have defined the extent and properties of the failed zone around the excavation. In step 7 of solution procedure, the confining pressure distribution is computed, based on the reduced strength parameters  $c'$  and  $\phi'$ .

In either instance (intact or jointed rock), the stabilizing pressure  $p$  is obtained by substituting the values of  $\sigma_\eta$  and  $\tau_\eta$  (equation 5 and 6) into the failure law (equation 1 or 10):

$$p = \frac{q[\sin 2\theta - \tan \phi (1 - \cos 2\theta)] + 2c}{\sin 2\theta + \tan \phi (1 + \cos 2\theta)} \quad (14)$$

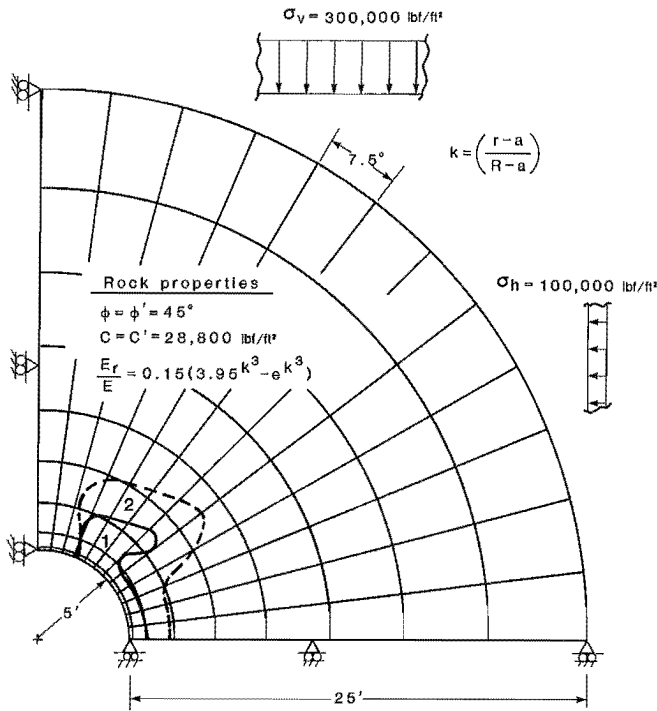


FIGURE 9. - Progressive development of plastic zone around circular excavation.

The angle,  $\theta$ , is the orientation of the failure plane with respect to the minor principal stress axis,  $q$ , as indicated in figure 4. For joints,  $\theta = \theta_j$  as indicated in figure 6.

Equation 14 can further be simplified for the case of intact rock by noting that

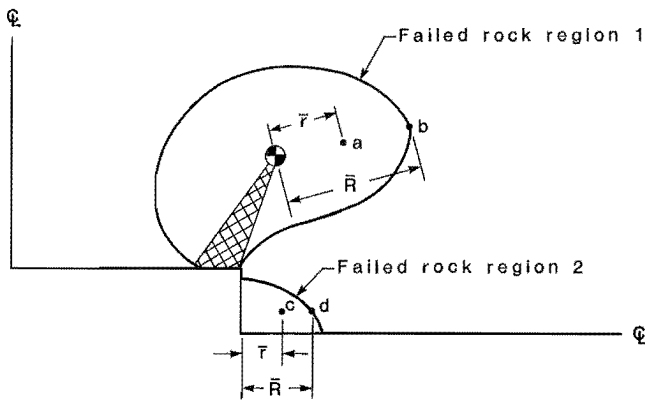
$$|\pm\theta| = 45^\circ - \phi/2, \quad \text{equations 2 and 3}$$

hence, for intact rock

$$p = \frac{1}{1 + \sin \phi} [q(1 - \sin \phi) + 2c \cos \phi].$$

#### APPLICABILITY OF METHOD TO UNDERGROUND SUPPORT SYSTEM

The objective of determining stability requirements for a yielding ground is to provide criteria from which a suitable support system may be designed. The procedure described allows the determination of the confining pressure distribution on the excavation surface that is required to prevent rock intrusions. In addition, the procedure permits this evaluation to be made simultaneously with the evaluation of the effect of tangential reinforcement placed along the excavation surface. The two criteria are thus sufficient to determine the structural requirements of certain types of support systems. For example, the confining pressure distribution can be converted into equivalent concentrated point loads that are spaced on the excavation surface to correspond with the gridwork of a rock-bolt system. The strength requirement of the rock-bolt system then would be known. Similarly, tangential reinforcement can be sized to be equivalent in stiffness to a lining consisting of, for example, concrete or shotcrete. The axial stress computed in the tangential reinforcement members would be the strength requirement of a lining. If the excavation surface is curved, the lining



In general,

$$\frac{E_r}{E} = 0.15 \left[ 3.95 \left( \frac{\bar{r}}{\bar{R}} \right)^3 + e \left( \frac{\bar{r}}{\bar{R}} \right)^3 \right] \quad (13)$$

For the failed zone in the shaded area,

$$\frac{E_r}{E} = 0.15.$$

FIGURE 10. - Method of modulus reduction for odd-shaped, failed rock zones.



would also develop confining pressures against the rock, in addition to the reinforcement.

In the foregoing discussion, it is assumed that a support system so designed could, if required, supply the necessary stabilizing forces. However, there are factors that would prevent the support system from achieving this. Design incompatibilities could develop if the deformability characteristics of the support system mismatched with those of the rock mass. For example, if a rigid support system was used against a highly yielding ground, premature support

failure could occur due to rock movements resulting from relaxation of the failed rock zones. The present method does not address this situation.

The solution procedure developed here is viewed as appropriate for support systems that provide compliance or flexibility in coping with large rock deformations. The systems that qualify under this recommendation are various combinations of rock bolts and a shotcrete lining. A discussion of the requirements and behavior of this type of support design in highly yielding rock masses follows.

#### APPLICATION TO DESIGN OF COAL MINE ENTRY

A multitude of potential ground control problems (relating to roof, floor, and rib control) can be encountered in longwall mining. Some of the factors that enter into the development of ground control problems are the virgin field stresses, coal and rock properties, the plan view geometry of the mine, the relative stiffness of the gob material, and the sequence of excavation.

A longwall mining system is illustrated in figure 11. The sketch shows a panel of coal being mined. The panel is typically dimensioned in the range of 500 ft wide by 5,000 ft long. Before the coal is mined, the panel is accessed by driving entries along both flanks. In the case shown, a two-entry system is used. Each entry is dimensioned 20 ft wide by 6 ft high. The pillars created by the entries and crosscuts are dimensioned 60 by 80 ft. The operation shown implies that the coal on either side of the panel being excavated has not been mined. In longwall mining practice, several panels are normally mined consecutively. After mining the first panel with solid coal abutments, the second and subsequent panels would be mined with one solid coal abutment and one extracted panel (the gob) as the abutment. The sequence of panel extraction would have significant effects on ground stress distributions around the panels.

In the layout of the longwall mining system, consideration is given to the general and local instabilities that can occur. General stability considerations lead to the proper dimensioning of panels, entry systems, number of panels, and proximity of current mining to mined-out areas. This involves the redistribution of field stress (as a result of panel-coal extraction) to the abutments around the panel. The choice of overall

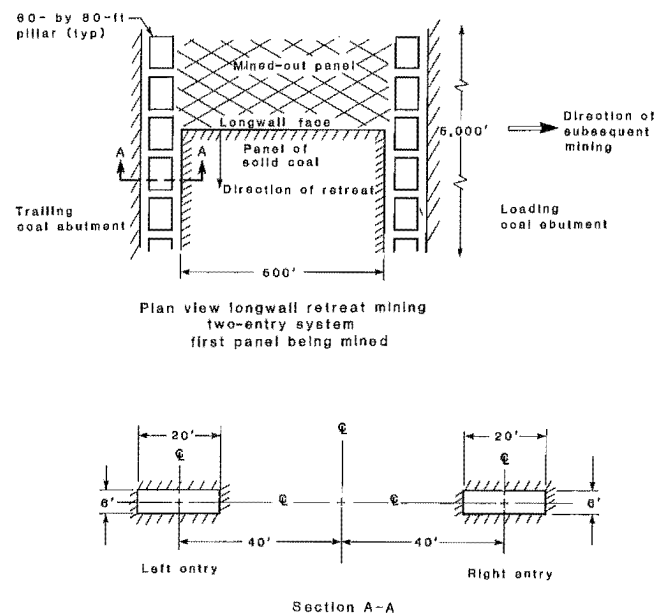


FIGURE 11. - Geometry of coal mine entry.

mine dimensions and the relative stiffnesses of the gob, panel, pillars, and nearby mined-out areas influences the magnitudes of the stress redistribution that occurs. The magnitude of the stress increases, and the strengths of the rock determine whether the system is stable and mineable. In an overall stable mining layout, local instabilities can occur. These are due to stress concentrations on areas of weak rock that result in roof, floor, and rib failures. Proper dimensioning is necessary to avoid general instabilities, whereas artificial support is necessary to reinforce and confine local rock instability.

The problem studied here is that of rib control in an entry being retreated on by the longwall face. The entry under investigation is located near the longwall face. This is indicated in figure 11 by section A-A. Each entry, as shown, is subjected to the following loading conditions:

1. The virgin field stress.
2. The redistribution of stress during excavation of the entry.
3. The redistribution of stress caused during excavation of nearby entries and crosscuts.
4. The redistribution of stress due to the approach of the longwall face.
5. The redistribution of stress due to subsequent mining of nearby panels.

The purpose of this analysis is to demonstrate the method. Dummy material properties, strengths, and geometries are used. Whether or not rib support is required depends on the state of stress compared to the local strength condition and also on whether the strains in the coal reach the tertiary creep level. It is assumed that the overall pillar dimensions and material strength are sufficient to sustain the applied loading. Thus, the only instability being analyzed here is due to sloughing along the rib line.

#### ANALYSIS

It is required that the ribs of the pillars and the coal-panel rib sides must be controlled against potential failures.

This may require the use of rock bolts, tunnel sets, tunnel linings, wire mesh, or some combination of these. It is assumed that supports are constructed just before the effects of the longwall face advance occur. Then, as the increased longwall face loads are applied, support requirements are evaluated.

The solution procedure is itemized below:

1. Apply the far-field stress condition around the entries, and perform a linear elastic stress analysis.
2. Determine the zone of (incipient) failed rock about the excavation, using the strength parameters  $c$  and  $\phi$ .
3. Reduce the elastic modulus of the rock in the failed zones.
4. Repeat the stress analysis of the excavation.
5. Determine the extended zone of failed rock around the excavation, using the strength parameters  $c'$  and  $\phi'$ .
6. Iterate steps 3, 4, and 5 until convergence has been achieved.
7. Increase the field stress condition due to the effects of the approaching longwall face, and perform a linear elastic stress analysis.
8. Iterate steps 3, 4, and 5 until convergence has been achieved.
9. Determine requirements for achieving stability.

In step 9, the necessity for using shotcrete and rock bolts for rib control is evaluated. The rock-bolt system is not modeled using discrete finite elements. Instead, confining pressures are calculated along the edge of the entry in order to restore stress equilibrium under the application of additional field stresses.

The finite-element mesh for the coal mine entry problem is shown in figure 12. The three axes of symmetry allow construction of a model that is considerably simplified. The coal layer in the entry horizon is considered to be 6 ft thick. The entry width is 20 ft. Other coal measure rocks, sand and shale, lie above the coal bed. The material properties needed in the analysis are given in the following tabulation:

Young's modulus, $10^6$ lbf/in <sup>2</sup> :	
Sand stone.....	1.5
Shale.....	2.5
Coal.....	0.5
Poisson's ratio $\nu$ .....	0.20
Initial strength:	
$\phi$ .....	40°
$c$ .....psi..	100
Reduced strength:	
$\phi'$ .....	30°
$c'$ .....psi..	0

The strength given to the coal and the applied loading are represented in the Mohr diagram in figure 13. The stress circles for the initial field stress and the increased field stress (due to the approach of the longwall face) are shown in relationship with the Coulomb failure surfaces. Two loading sequences are considered. The first loading sequence

occurs when the entry is first loaded. It is subjected to the virgin field stresses, which are assumed to be

$$P_{v1} = \gamma h = 165 \times 500 = 82,500 \text{ lbf/ft}^2,$$

and

$$P_{h1} = \frac{\nu}{1-\nu} P_{v1} = (0.25) \times 82,500 = 20,625 \text{ lbf/ft}^2,$$

where  $P_{v1}$  = the initial vertical field stress, lbf/ft<sup>2</sup>,

$P_{h1}$  = the initial horizontal field stress, lbf/ft<sup>2</sup>,

$\gamma$  = the weight density of rock, lbf/ft<sup>3</sup>,

and  $h$  = the vertical depth of the entry, ft.

These field stresses correspond to a mining depth of 500 ft and an effective Poisson's ratio of 0.2. The second loading sequence on the entry is due to the

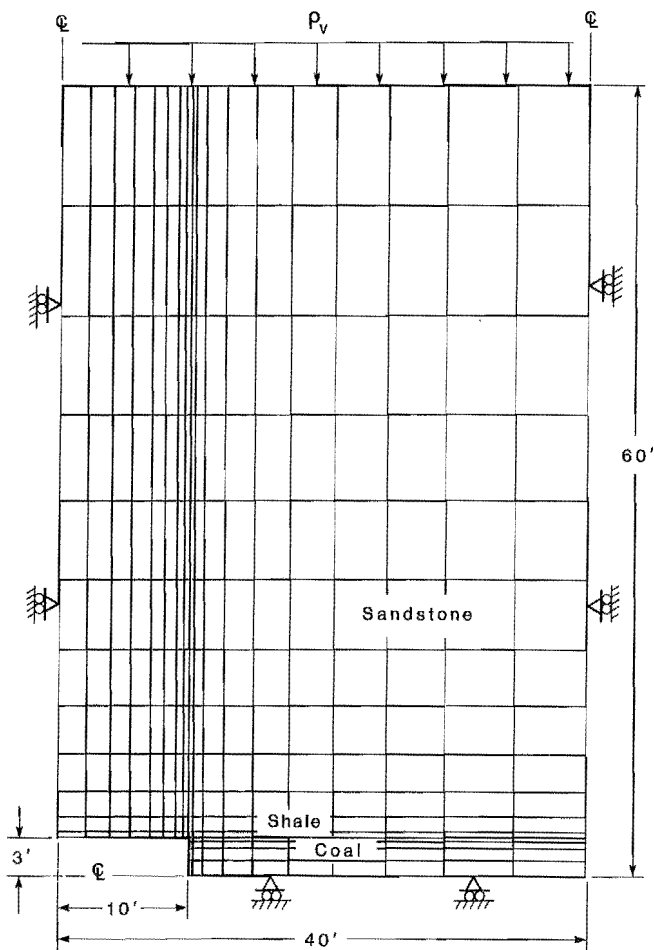


FIGURE 12. - Finite-element mesh for coal mine entry.

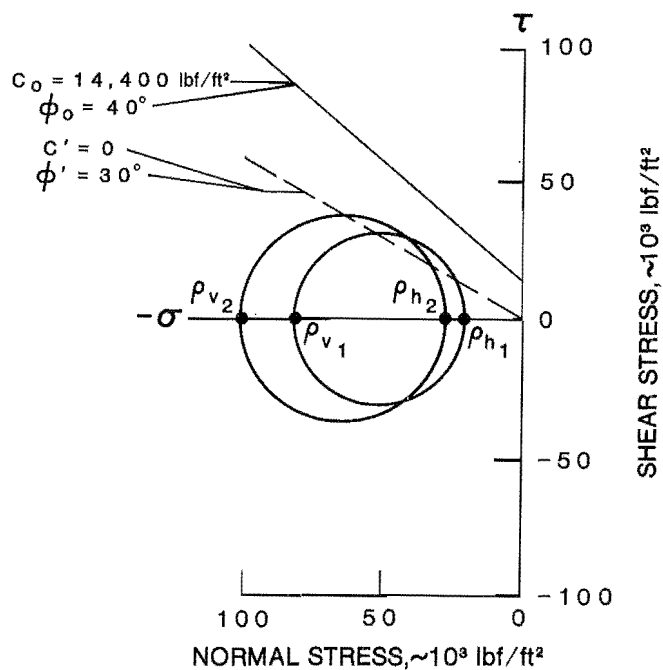


FIGURE 13. - Coulomb strength parameters and field stress conditions for coal mine entry.

increase in field stress that results from the frontal abutment stress concentration of an approaching longwall. The actual increase in the field stress would require a separate calculation in which the overall changes in mining geometries would need to be considered. For this example, this increase is assumed to be 25 pct of the virgin field stresses or

$$P_{v2} = 0.25 P_{v1} = 20,625 \text{ lbf/ft}^2,$$

$$\text{and } P_{h2} = 0.25 P_{h1} = 5,156 \text{ lbf/ft}^2,$$

where  $P_{v2}$  = the increase of vertical stress due to longwall, lbf/ft<sup>2</sup>.

$P_{h2}$  = the increase of horizontal stress due to the longwall, lbf/ft<sup>2</sup>.

## RESULTS

### Initial Loading

With reference to the solution procedure, the stress condition due to the initial field loading (step 1) exceeded the strength conditions in two zones (step 2) near the entry. In accordance with step 3 of the procedure, the elastic modulus in these zones was reduced by applying equation 13, and the process was iterated until no further extensions of the failed zones occurred in two successive iterations.

The failed zones are located above the corner of the entry and at the edges of the pillar. These zones are shown in figure 14 section A-C. Section A illustrates the location of the failed rock zones immediately after application of the initial loading, but before the modulus of the elements in the zones was reduced. Section B shows the same condition after the modulus reduction was made and after the analysis was iterated to convergence.

A zone of tension is also present in the roof of the entry. This indicates that this entry configuration has a potential for roof falls.

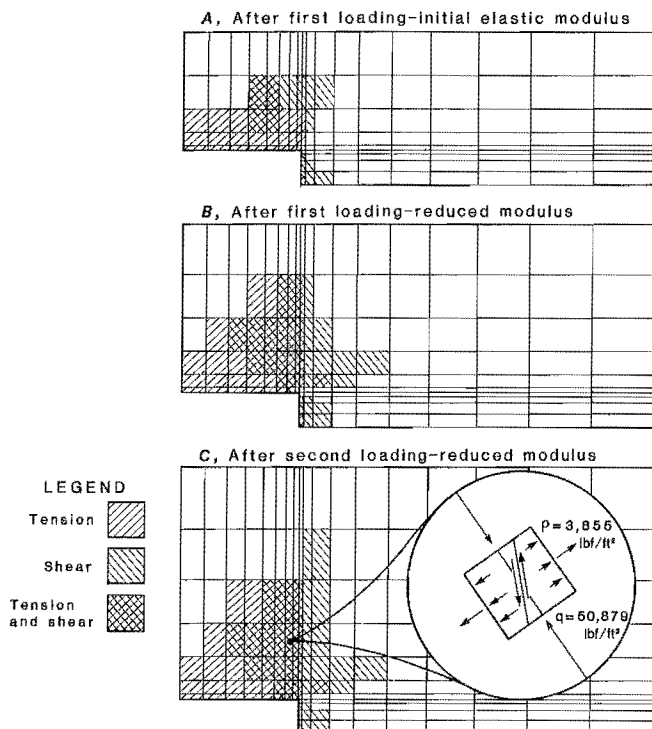


FIGURE 14. - Shear and tensile zones in coal mine entry.

### Final Loading

The analysis procedure indicated by steps 7, 8, and 9 was performed to evaluate the effect of increased field loading due to the approach of the longwall face. The increase in the field stress resulted in expansion of the failure zones caused by the initial loading. This expansion is indicated in figure 14, section C.

The tensile and shear zones that develop in the roof do not primarily influence rib stability. However, in the example, the shear zone developed almost directly over the rib-side abutment. Therefore, the modulus reduction associated with this zone had an important secondary effect. Due to the softening that occurred in this zone, vertical stresses were reduced both in this zone and in the neighborhood of the rib side. Consequently, the zone of rock failure was reduced, both in intensity and magnitude, thus reducing any requirement for rib-side support.

The zone of tension in the roof indicates a potential occurrence of rock

loosening, especially if the roof is weak or favorably jointed. Most roof rocks are weak in tension and cannot withstand tensile stresses and would, therefore, ravel from the roof. This mode of failure is not within the scope of the present analysis.

Also, in the roof, both tensile and shear failure regions overlap in the area over the rib abutment. In these regions, there is a strong potential for initiation of nearly vertical cracks and caving if a low-angle joint set is present. The state of stress at a point in such a region is shown in figure 14, section C.

#### Stability Analysis

The confining pressure analysis was performed using the residual strength parameters  $\phi'$  and  $c'$ . The distribution of confining pressure around the periphery of the excavation is shown in figure 15. The magnitudes of these pressures are the maximum needed to bring about stability,

since not all of the rock would have strength parameters as low as those used. The pressures needed along the rib side are of the order of 80 lbf/in<sup>2</sup>.

The zone of instability extends to the intersection of the rib with the roof where the confining pressure peaks above 200 lbf/in<sup>2</sup>. The confining pressure distribution given in figure 15 can be used as a criterion for evaluating support requirements. In this case, conventional rock bolts alone would not supply the equivalent of 80 lbf/in<sup>2</sup> needed for most of the rib side. For instance, consider that the bolts are spaced in a 5- by 5-ft grid. The force requirement per bolt would be

$$P = A\sigma_c,$$

where  $A = 5 \times 5 \times 144 \text{ in}^2,$

$$\sigma_c = 80 \text{ lbf/in}^2,$$

and  $P = 288,000 \text{ lb.}$



FIGURE 15. - Confining pressure distribution around coal mine entry.

The strength of a 5/8-in-diam bolt, based on the yield strength of A36 steel, is

$$P_y = \frac{\pi(5/8)^2}{4} (36,000) = 11,045 \text{ lb.}$$

Consequently, the rib side would also require tangential reinforcement, which might consist of a shotcrete lining or a rib-side pack wall.

#### CONCLUSIONS AND RECOMMENDATIONS

A rationale has been developed in this investigation that attempts to explain the behavior of excavations made in highly yielding rock and identifies the stability requirements for controlling rock intrusions into the excavation. In this investigation, it was necessary to apply an empirically derived method of approximation to account for the progressive relaxation of rock that occurs in the failed zones. The time-dependent behavior of the rock and the time factors involved with the installation of a support system were ignored in this study. The report shows how the criteria for the design of a support system can be formulated. Because of the difficulties in accounting for the time-dependent deformational interactions that occur between a support and a rock mass, these methods are most suitable for a flexible (or yieldable) type of support system such as rock bolting used in conjunction with a shotcrete lining. This type of support system is most compliant with large deformations that accompany the behavior of highly yielding ground. In many applications in mining, such a support system has been found to achieve good results while having economical advantages over rigid support systems.

The application of this method has been made to the problem of entry development for longwall coal mining. The necessary role of a support system has been shown to be the control of the failed rock adjacent to the excavation surface. This requires relatively small confinement pressures or tangential reinforcement if movement of the rock is allowed to occur. This predicted behavior is often observed in mining practice.

When rigid support systems are installed, rock movements are prevented.

This brings about the development of much higher pressures that often result in failure of the support. Consequently, if the support system is suitably compliant with these deformations and does not fail prematurely, then only relatively small resistance is required for the achievement of stability.

Further efforts are needed to achieve the overall goal of designing support systems in highly yielding ground. The following are suggested as areas requiring further study:

1. Extend the solution capability to consider rock-mass loosening.
2. Modify the analysis to include the excavation and construction sequence.
3. Introduce an analytical model that will simulate the development of failed rock zones.
4. Modify the analysis to provide for the introduction of the support using finite-element representations. The present analysis does this to some degree, but representations of several different types of support elements are needed.
5. Extend the analysis to account for the stresses in the support and along the support-rock interface.
6. Select a computer program better suited for implementing the improvements listed above.
7. Develop criteria that can be used to predict the tertiary-creep strain limits of various rock types. This is needed to predict the standup time that is available after excavation and before a support system must be installed to prevent premature rock disintegration.
8. Verify the methodology thus developed by investigating the field behavior of existing, well-documented support systems.

9. The subroutine STABIL should be extended to include the stability mechanism that controls the magnitude of tangential compressive stress on the excavation

surface. This involves the development of external tangential forces that are required to bring the Mohr stress circle tangent with the failure surface.

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## APPENDIX.--MATHEMATICAL NOMENCLATURE

$\phi$ = angle of friction.	$p_i$ = support pressure along surface of underground excavation.
$c$ = cohesion.	$r$ = radial distance from the center of an excavation of circular shape to a point in the rock.
$r$ = subscript denoting rock.	$a$ = opening radius or dimension.
$j$ = subscript denoting joint.	$R$ = distance from the center of an excavation of circular shape to the elastic-plastic boundary.
$\sigma_\tau$ = tangential stress.	$p_o$ = pressure required to prevent deformation of excavation surface.
$\sigma_c$ = confining pressure.	$\epsilon$ = strain.
$\sigma_\eta$ = normal stress on plane of Coulomb failure.	$\bar{r}$ = distance from "focal point" of the plastic zone to a point in the rock.
$\tau_\eta$ = shear stress on plane of Coulomb failure.	$\bar{R}$ = distance from "focal point" of the plastic zone to the elastic-plastic boundary.
$S$ = Coulomb shear strength.	$E$ = Young's modulus.
$P_v$ = vertical field stress.	$E_r$ = reduced modulus of failed rock.
$P_h$ = horizontal field stress.	$\gamma$ = weight density.
$q$ = major compressive principal stress.	$\nu$ = Poisson's ratio.
$p$ = minor compressive principal stress.	$e$ = base of natural logarithm (=2.71818...).
$\theta$ = orientation of failure plane with $q$ .	
$\tau$ = shear stress.	
$\sigma$ = normal stress.	
$c'$ = cohesion for failed rock.	
$\phi'$ = angle of friction for failed rock.	