RI 9088

Bureau of Mines Report of Investigations/1987

Indices of Mine Safety Resulting From the Application of the Poisson Distribution to Mine Accident Data

By J. C. Kerkering and P. C. McWilliams

UNITED STATES DEPARTMENT OF THE INTERIOR

Report of Investigations 9088

Indices of Mine Safety Resulting From the Application of the Poisson Distribution to Mine Accident Data

By J. C. Kerkering and P. C. McWilliams

UNITED STATES DEPARTMENT OF THE INTERIOR
Donald Paul Hodel, Secretary

BUREAU OF MINES
Robert C. Horton, Director

Library of Congress Cataloging in Publication Data:

Kerkering, J. C. (John C.)
Indices of mine safety resulting from the application of the Poisson distribution to mine accident data.

(Bureau of Mines report of investigations; 9088)

Bibliography: p. 7.

Supt. of Docs. no.: I 28.23: 9088.

1. Mine safety—Mathematical models. 2. Mine accidents—Mathematical models. 3. Poisson distribution. I. McWilliams, P. C. (Paul C.). II. Title. III. Series: Report of investigations (United States. Bureau of Mines); 9088.

TN23.U43 [TN295] 622 s [622'.8'0724] 86-600357

CONTENTS

ILLUSTRATION

TABLES

INDICES OF MINE SAFETY RESULTING FROM THE APPLICATION OF THE POISSON DISTRIBUTION TO MINE ACCIDENT DATA

By J. C. Kerkering[1] and P. C. McWilliams[2]

ABSTRACT

The Bureau of Mines used mathematical reliability theory to define, for use in the mining industry, the concepts of risk, safety, reliability, hazard, and mean time between accidents. In this report, the definitions are explained theoretically and illustrated by application to actual underground mine accident data extending over a 9-1/2-year period.

The theoretical development proceeds along two parallel analytical paths. For each line of reasoning, the data are presumed to be independent. If they are not, the analysis stops, although there are interesting implications when dependency is demonstrated.

The first method uses the elapsed times between accidents as the basic data. These elapsed times are fitted to a negative exponential distribution function using least squares and yielding the function parameter, P_1. The P_1 is used to form the Poisson distribution function for the data, which is then used to define the safety-related concepts. The second method uses the maximum likelihood estimator to formulate a Poisson distribution function for the data. The function is tested for adequacy by a goodness-of-fit test; if the fit is acceptable, the Poison function is used to define the concepts. Both methods were applied to the data base, yielding nearly identical results.

[1]Mathematician.
[2]Mathematical statistician.
Spokane Research Center, Bureau of Mines, Spokane, WA.

INTRODUCTION

There are already a number of methods for describing mine accident rates, such as the incident, severity, and frequency rates, as well as the accident-experience learning curves (A-ELC) proposed by Greenberg as a way of monitoring changes in mine safety performance (1).[3] However, existing methods cannot easily answer in a quantitative way such questions as--How safe is this mine? What are the chances of having no accidents today? This week? This month? What are the chances of having a fatality today? How often can we expect an accident of this type? Are the accidents independent of one another or is there perhaps one common underlying cause?

The Bureau of Mines has developed a different but simple method to measure mine safety and to predict mine accident rates, which mine operators can easily use to answer such questions. The Bureau undertook this work to provide mine managers with an objective method for measuring and predicting the safety of their mines. The information required for application of the method is available at every mine operation. The method enables mine operators to objectively predict the safety of their operations and to rank and justify mining operations, procedures, and equipment for safety improvements. In the Bureau method, either of two mathematical approaches can be used to arrive at a statistical distribution of the accident data, from which quantitative answers can easily be calculated, using the mine accident records kept by every mine or the mine accident records kept by Health Safety and Accident Center (HSAC), a part of the U.S. Mine Safety and Health Administration.

THEORY

If accidents are independent,[4] two approaches can be used to arrive at the measures of mine safety. The first uses elapsed time between accidents and is referred to as the inter-arrival (IA) method. In this approach, a negative exponential distribution function is fitted, using least squares, to the frequency histogram of elapsed times between accidents. The parameter of this fitted negative exponential function is then used to form a Poisson function that will adequately describe the accident distribution. Quantifiable definitions of safety concepts are given in terms of parameters derived from this distribution. There are many statistical distributions that can model the frequency histogram with varying degrees of success. As will be demonstrated later, the negative exponential and, consequently, the Poisson distributions are often the appropriate theoretical choices for modeling this kind of information. For the data investigated in this study, these distribution functions prove to be good choices.

The second approach uses the maximum likelihood estimator (MLE) to arrive at parameter estimates for a possible Poisson distribution. The proposed empirical Poisson function is then checked against a theoretical model using a standard goodness-of-fit test. If the empirical Poisson model is satisfactory, it can be used to define the safety concepts. The theoretical results are applied to mine accident data extending over a 9-1/2-year period in order to illustrate the two methods and the resulting definitions of mine safety and mine hazard rate.

[3]Underlined numbers in parentheses refer to items in the list of references preceding the appendixes.

[4]Two events are statistically independent if the occurrence of one in no way influences the occurrence or nonoccurrence of the other.

INDEPENDENCE

Two tests for statistical independence are used for the data discussed in this report: the chi-squared contingency test and the Runs test. The application of both tests to the data is documented in appendix C. The results indicate that the data are indeed independent, which is a requisite for using the Poisson probability density function (pdf) to describe the data.

If the data are not independent, a Poisson distribution cannot be established, and therefore, a precise definition of the safety concepts for this mining operation cannot be arrived at using the methods given in this report. However, dependent data do indicate the possible existence of a connecting link, an underlying cause, for the accidents and, therefore, would suggest that efforts be made by mine management to identify and eliminate it.

If the data are independent, the analysis continues via either IA or MLE to establish the appropriate parameters for the safety concepts. Both methods aim to establish the Poisson distribution.

INTER-ARRIVAL APPROACH

Poisson Distribution

A distribution is described as a Poisson (2-3) if the data are independent and if the frequency histogram of the data is adequately described by the following equation:

$$f(x) = (m^x e^{-m})/x!, \quad x = 0, 1, 2, \ldots, \qquad (1)$$

where f(x) is the relative frequency, that is, the probability of x = 0 or x = 1, etc., where x is the number of events occurring in a given time period, and m is the mean of the distribution, that is, the expected number of accidents for a given time period for this study. For the Poisson distribution, the standard deviation equals the square root of the mean. Therefore, this is a one-parameter distribution.

Sometimes m is written as λt where λ is the rate of occurrence of the event and t is the basic time period. Two estimates of m are used in this report. The first, $\hat{L}$, is the estimate derived from the least squares fit of the accident IA times to a negative exponential cumulative distribution function. The second, $\tilde{L} = \bar{x}$, is the MLE and equals the arithmetic average of the data, assuming that the accidents form a Poisson distribution. For this report, λ will equal the number of accidents per day and t will equal 1 day.

Thus, f(x) can be rewritten as

$$f(x) = [(\lambda t)^x e^{-\lambda t}]/x!, \quad x = 0, 1, 2, \ldots \qquad (2)$$

For t = 1, this reduces to

$$f(x) = (\lambda^x e^{-\lambda})/x! \qquad (2A)$$

Therefore, the parameter λ must be derived from the mine accident data base, using least squares to fit first-order IA times to the cumulative negative exponential function.

Inter-Arrival Times

This method is based on the fact that if the first-order IA times form a negative exponential distribution (see equation 3), then the events themselves have a Poisson distribution (2-3); that is, the probability of x events occurring in time t is f(x), where f(x) is the Poisson distribution function.

The first-order IA times are the times between accidents over a certain time period (fig. 1). This information is readily available since the date and time of accidents are usually recorded serially. The elapsed time between events can be calculated using one of the many available algorithms (4).

The set of elapsed times, $T = (t_1, t_2, \ldots, t_n)$, is then checked by the method of least squares (5), described in appendix D, to determine if it fits a negative exponential distribution function. (The Kolmogorov-Smirnov (K-S) goodness-of-fit test can then be used to verify the compatibility of the curve fit and the data (6). This test is described in appendix E.)

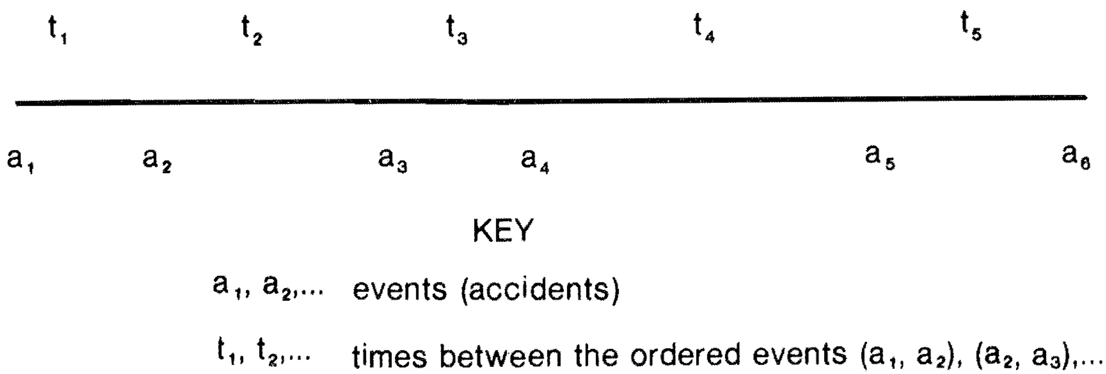

FIGURE 1.—First-order IA times.

Negative Exponential Distribution

The negative exponential distribution function is

$$f(x) = P_1 e^{-P_2 x}, \qquad (3)$$

where P_1 is the parameter to be estimated by the least squares method. If the curve fit is determined to be satisfactory, for example, by the K-S test, then T, the set of IA times, has a negative exponential distribution and P_1 can be estimated by the least squares computer program (7). Furthermore, if T has a negative exponential distribution, then the numbers of accidents form a Poisson distribution with $\lambda = P_1$.

Note the relationship between the exponential distribution (time as the random variable) and the Poisson distribution (number of occurrences per time period). A continuous random variable (time) is related to a discrete random variable (number of occurrences per time interval).

MAXIMUM LIKELIHOOD ESTIMATOR APPROACH

The second approach uses the MLE to derive the Poisson parameter.[5] The MLE is the average number of events per time period (in this study, the number of accidents per day). This average, $\bar{x}$ is now the estimate for the Poisson parameter, m, described in equation 1. A goodness-of-fit test, such as the K-S test, is used to determine if the resulting Poisson distribution adequately models the data.

[5] An MLE is the most desirable choice in using sample data to estimate statistical parameters of interest. The reader should see reference 3 for further information on this topic.

SAFETY INDICES

The use of either approach--IA times or MLE--establishes a Poisson process and the value of λ, the hazard rate, which are then used to define the following concepts:

1. Hazard rate: Equals λ and is the number of accidents in a given time period.
2. Mine reliability or safety: Equals $e^{-\lambda t}$ and is the probability that zero accidents will happen in a given time period, t. Derived from equation 2 with x = 0 accidents.
3. Risk: Equals $1 - e^{-\lambda t}$ and is the probability that at least one accident will happen in the time period, t. This is the complement of 2 above.
4. Mean time between accidents (MTBA): Equals $1/\lambda$ and is the expected time between accidents. Found by computing the mean of the distribution represented by equation 3.

APPLICATION TO REAL MINE ACCIDENT DATA

DATA BASE

The data base consists of all the accidents at one U.S. mine reported to HSAC over a 9-1/2-year period (January 1, 1975, to July 31, 1984). There were 590 accident reports covering 584 separate accidents. The accidents themselves ranged in severity from minor injuries to fatalities. (See appendix A for a listing of the original accident data.)

INTER-ARRIVAL METHOD

First, the elapsed times in days and fractions of days between the accidents were calculated. (See appendix B for a listing of elapsed times.) These elapsed times were then fitted (via a nonlinear least squares computer program (5)) to the negative exponential cumulative distribution model, $1 - e^{-P_1 x}$. The value for the parameter P_1 ($\hat{L}$) was calculated to be 0.18293. (See appendixes D and E for a documentation of the procedure.) The resulting pdf is $f(x) = 0.18293e^{-0.18293x}$, and the corresponding cumulative distribution function is $1 - e^{-0.18293x}$. A cumulative distribution corresponds to the integration of the area under the pdf from the left endpoint to a given value of the random variable (2, 8). The cumulative distribution model was tested against the accident data using the K-S test (6) (see appendix E). The maximum absolute difference between the value predicted by the model and the actual value from the data was 0.03. The critical value at the 0.05 significance level for the K-S test was $1.36/(n)^{1/2} = 1.36/(584)^{1/2} = 0.06$, which is greater than the actual difference of 0.03. Therefore, the hypothesis that the elapsed time data form a negative exponential distribution with a parameter of $P_1 = 0.18293$ was accepted.

Because the first order IA times form a negative exponential distribution with $P_1 = 0.18293$, the accidents form a Poisson distribution with a pdf of

$$f(x) = (\lambda t)^X e^{-\lambda t}/X!$$

$$= 0.18293^X e^{-0.18293}/X!, \qquad (4)$$

because $P_1 = 0.18293$ and $t = 1$ day.

This function, f(x), allows calculation of hazard rate, mine safety, risk, and MTBA, in the following manner.

1. Hazard rate: $\lambda = 0.18293$ accidents per day. Therefore, the number of accidents in a time period t is equal to λt. For example, the number of occurrences in a year equals (0.18293 accidents per day) × (365 days per year) = 66.8 accidents per year.
2. Mine reliability or safety: $e^{-\lambda t} = e^{-0.18293t} = 0.83283$ with t = 1 day. This says that the probability of having zero accidents in a day is 0.83283 at this mine. The safety factor can be calculated for various time periods by using appropriate values for t. For example, the probability of having zero accidents in a week (7 days) is $e^{-0.18293*7} = 0.2779$.
3. Risk: $1 - e^{-\lambda t} = 0.16717$. This is the probability of having at least one accident in a given day. The probability of having at least one accident in a week is $1 - e^{-0.18293*7} = 0.72210$. (The risk values are simply the complement of the mine safety values.)
4. Mean time between accidents (MTBA): $1/\lambda = 1/0.18293 = 5.467$ days. This is the expected time in days between accidents.

Finally, the probability of one accident per day was computed directly from the Poisson distribution, giving:

$$Pr\ (X = 1) = e^{-0.18293}$$

$$* (0.18293) = 0.15235.$$

Similar computations could be made for zero, two, three, four, or more accidents per day. What is important is that a quantitative meaning can be given to these concepts by determining that the accidents follow a Poisson process and by determining the value of λ.

The entire accident set can be partitioned into eight subsets, depending on the severity of the injuries, each subset having its own λ. This makes possible, for example, the application of the above concepts to only fatal accidents, described by the parameter λ_1, and the prediction of the mean time between fatalities. According to an established theorem for Poisson distributed data, if all the accidents were independent of one another and if each of the subsets is Poisson, then the sum of the individual hazard rates will equal the overall hazard rate. This is called the convolution or superimposition property of the Poisson distribution (9).

MAXIMUM LIKELIHOOD ESTIMATOR METHOD

The Poisson pdf can be directly derived from the data, that is, without involving the previously described IA time computations. For the entire data set, the mean (m) = $\bar{x}$ = the average number of accidents per day = 0.16681; the standard deviation (s) = 0.42981; and the variance (s^2) = 0.18474. (It is reassuring to note the closeness of $\bar{x}$ and s^2 because the population mean equals the population variance in a theoretical Poisson distribution. However, there is no statistical test available to measure this relationship further.) Using $m = \bar{x} = \tilde{L}$ = 0.16681, the K-S test (6) can be used to determine if the resulting pdf and its cumulative distribution function accurately model the data. Again, applying the K-S test criteria, the calculations showed that the greatest distance between the Poisson cumulative model and the cumulative data occurs at zero accidents per day and equals 0.00682. The critical value at the 0.05 significance level for 584 accidents is $1.36/(584)^{1/2}$ = 0.05628, which is considerably greater than 0.00682. Therefore, the hypothesis that the data form a Poisson distribution with $\tilde{L}$ = 0.16681 can be accepted.

This value can be used to calculate hazard rate, mine safety, risk, and MTBA. For example, using this L, mine safety computed to 0.84636, compared with 0.83283 computed using $\hat{L}$ = 0.18293, derived from the IA approach. Note that $\hat{L}$ and $\tilde{L}$ are so close that they can be equated for all practical purposes (see appendix F for an example of a confidence interval computation). It is, of course, not surprising that "comparable" answers were obtained by the IA and the MLE approaches. The Poisson nature of the complete data set dictated the consistency of the answers.

With the MLE process, each subset of accidents, as defined by the degree of injury, was found to have Poisson distribution. The values of L_i, s_i, and s_i^2 for each of the eight data subsets are given in table 1.

The sum of the subset means (ΣL_i) = 0.167956, and the mean for the entire data set as calculated in the MLE approach ($\tilde{L}$) = 0.16681, as was predicted by the superimposition or convolution of the Poisson process (9). Summing the subset means actually gives a third estimate of the Poisson parameter.

TABLE 1. - Poisson parameters for Poisson subsets

Subset, degree	Type of injury	Mean (L_i)	Std dev (s_i)	Variance (s_i^2)
0......	No injury, but accident.	0.00314	0.05597	0.00313
1......	Fatality.	.00143	.04	.0016
2......	Permanent disability.	.00200	.04468	.00200
3......	Days off.	.11025	.34035	.11584
4......	Days off and restricted.	.001143	.033783	.001142
5......	Restricted work.	.001143	.033787	.001142
6......	No lost time.	.047986	.227995	.051982
7......	Occupational illness.	.00086	.02926	.00086

SUMMARY

Two theoretical methods for determining if accident data form a Poisson distribution, the IA method and the MLE method, have been discussed and applied to one experimental data set. If it is determined that accident data form a Poisson distribution, then the L calculated from the data can be used to quantify the concepts of mine hazard rate, mine safety, mine risk, and MTBA. The probabilities of having a specific number of accidents per day, week, etc., at the mine can be easily calculated from the ensuing Poisson distribution. If the subsets also form Poisson distributions with hazard rate L, and if the accidents are independent of each other, then the sum of the individual subset hazard rates, L_I, equals the hazard rate for the entire data set.

CONCLUSIONS

The ability to quantify mine hazard rate, safety, risk, and MTBA allows a mining company to compare quantitatively its past and present performances and to quantitatively predict future safety performance. The company can then decide if the predicted values are acceptable or not. If not, appropriate actions can be taken and the success of these actions can be measured against the predictions.

These quantified concepts can be used to compare the safety and incurred risk of one operation or technology with those of another in order to establish norms or rankings.

It is not necessary for the practitioner to use both the IA and the MLE approaches to the Poisson distribution. In this report, both were used for illustrative purposes. The MLE procedure is not difficult to apply: a suitable time subinterval is chosen (for example, a week). The average number of accidents over this time interval is computed, yielding an appropriate $\bar{x}$. This is the required Poisson parameter. Using either method, a test such as the K-S or chi-squared goodness-of-fit test (6) is still required to verify that the distribution is truly Poisson. The preceding arguments were validated for only one experimental data set.

REFERENCES

1. Greenberg, L. Why the Mine Injury Picture Is Out of Focus. Min. Eng. (N.Y.), v. 23, No. 3, 1971, pp. 51-53.

2. Drake, A. W. Fundamentals of Applied Probability Theory. McGraw-Hill, 1967, 283 pp.

3. Brownlee, K. A. Statistical Theory and Methodology in Science and Engineering. Wiley, 2d ed., 1965, 590 pp.

4. Hewlett Packard (Corvallis, OR). HP-41C Financial Decisions Pac., 1980(?), 52 pp.

5. McWilliams, P., and D. Tesarik. Multivariate Analysis Techniques With Applications in Mining. BuMines IC 8787, 1978, 40 pp.

6. Siegel, S. Nonparametric Statistics. McGraw-Hill, 1956, 312 pp.

7. Moore, R., and K. Zeigler. The Solution of the Several Least Square Problem With Special Reference to High-Speed Computers. Los Alamos Sci. Lab. Rep. LA-2367, 1959, 94 pp.

8. Hoel, P. G. Introduction to Mathematical Statistics. Wiley, 3d ed., 1966, 427 pp.

9. Saunders, S., and L. Penas. Statistical Reliability for Engineers. Lecture notes, 1984, pp. 0.1-VIII.9, available from S. Saunders, Dep. Math., WA State Univ., Pullman, WA.

10. Bowers, E. T. Using ADA (Accident Data Analyses) in Mine Safety Research. BuMines OFR 72-86, 1985, 96 pp.

11. Mann, N., R. Schafer, and N. Singpurwalla. Methods for Statistical Analysis of Reliability and Life Data. Wiley, 1974, 564 pp.

APPENDIX A.--HSAC DATA AND COMPUTER PROGRAM

The original mine accident data taken from the HSAC files are given in table A-1. The computer program, SRC/ADA (10),[1] was used to retrieve the data for analysis. A second Fortran program was used to retrieve those pieces of accident information needed for this analysis, namely, the time, year, and degree of injury. The program converted the hour of the accident into hundredths of a day and added this to the date. The first column in table A-1 contains the accident record number. Not all 591 accident records were used in the analysis because some records represent multiple injuries resulting from the same accident. For example, records 89 and 90 are typical listings of multiple injuries resulting from the same accident.

The following is a listing of the Fortran program used to read the selected accident file (generated by ADA) and then to write the accident records listed in table A-2.[2]

[1] Underlined numbers in parentheses refer to items in the list of references preceding this appendix.

[2] The data generated by the program are in a slightly different format from that used in the table.

```
#FILE (JCK)ELAPSED/DAYS/PLUCKED ON SRC4
FILE  1(KIND=DISK, MAXRECSIZE=30, TITLE="LF81 ",
      * FILETYPE=7, MYUSE=IN)
FILE  4 (KIND=DISK,MAXRECSIZE=14,TITLE="LF81/PLUCKED",
      *  FILETYPE=7,MYUSE=OUT)
FILE  8 (KIND=PRINTER,MAXRECSIZE=22, MYUSE=OUT)
FILE  5 (KIND=REMOTE,MAXRECSIZE=14,MYUSE=IO)
        DIMENSION X(100,62)
        OPEN(4)
        FIND(1=2)
        PRINT/, "ENTER YEAR(XX)"
        READ/,YEAR
          I=1
      5 CONTINUE
          READ(1,10,END=20) (X(I,J),J=1,62)
     10 FORMAT (I7,A3,3I2,3I4,I2,I3,I5,I1,2I2,I3,I2,2A4,A3,I4,
```

```
      *2I2,4I3,3A4,I1,10I2,5I3,I2,3I4,I1,3I2,3I3,4I2,2I3,9X)
        X(I,6)=X(I,6)/2400
        X(I,5)=X(I,5) + X(I,6)
        WRITE(4,12) X(I,4),X(I,5) , YEAR,X(I,46)
 12      FORMAT (I2,1X,F5.2,1X,I2,1X,I1)
       I=I+1
       GO TO 5
 20    CONTINUE
        CLOSE (4,DISP=CRUNCH)
           END
#
```

TABLE A-1. - Listing of HSAC data

Record	Month	Time[1]	Degree of injury
1975			
1	1	2.89	3
2	1	6.79	3
3	1	24.38	3
4	1	29.54	3
5	2	12.93	3
6	2	20.93	3
7	3	12.80	3
8	3	13.50	3
9	3	19.73	3
10	3	20.68	3
11	3	21.01	3
12	3	27.89	3
13	4	7.38	3
14	4	11.93	3
15	5	14.43	3
16	5	15.47	3
17	5	24.43	3
18	6	10.80	3
19	6	10.83	3
20	6	18.35	3
21	8	18.35	3
22	8	22.42	3
23	8	25.75	2
24	8	25.75	3
25	10	10.42	3
26	10	16.58	3
27	10	31.43	3
28	11	11.22	3
29	11	18.05	3
30	11	21.75	3
31	11	25.76	3
32	12	9.76	3
33	12	23.21	3
1976			
34	1	2.60	3
35	1	26.54	3
36	1	30.89	3
37	2	14.71	1
38	2	25.30	3
39	2	26.71	3
40	3	9.58	3
41	3	10.42	3
42	3	22.23	3
43	3	31.35	3
1976--Continued			
44	4	5.80	3
45	4	22.89	3
46	5	11.47	3
47	5	14.39	3
48	5	20.93	3
49	5	20.93	3
50	5	26.79	3
51	7	11.89	3
52	8	16.33	3
53	9	17.39	3
54	10	16.83	3
55	12	1.79	3
1977			
56	1	25.46	3
57	3	7.38	3
58	5	4.38	3
59	6	17.09	3
60	7	12.75	3
61	7	20.59	3
62	7	26.75	3
63	8	8.42	3
64	10	4.80	3
65	11	10.96	3
66	11	14.42	3
67	12	29.88	3
68	12	30.58	3
1978			
69	1	5.38	3
70	1	10.42	3
71	1	13.46	3
72	1	17.54	6
73	1	18.76	3
74	1	23.38	3
75	1	23.73	3
76	1	23.79	3
77	1	29.35	6
78	1	31.22	3
79	2	6.05	3
80	2	9.96	6
81	2	10.46	6
82	2	14.42	3
83	2	16.38	3
84	2	24.08	3
85	2	28.21	2

[1]Day and hundredth of a day.

TABLE A-1. - Listing of HSAC data--Continued

Record	Month	Time[1]	Degree of injury
1978--Continued			
86	2	31.17	3
87	4	4.89	6
88	4	5.34	3
89	4	6.46	6
90	4	6.46	3
91	4	18.04	6
92	4	19.71	6
93	5	2.55	6
94	5	5.54	6
95	5	18.43	6
96	6	1.88	3
97	6	5.56	6
98	6	9.81	3
99	6	13.39	3
100	6	21.39	3
101	6	22.68	3
102	6	26.72	3
103	6	27.42	6
104	6	28.42	3
105	6	28.76	3
106	7	10.80	3
107	7	13.10	1
108	7	13.93	3
109	7	17.43	6
110	7	25.01	6
111	8	1.76	2
112	8	4.76	3
113	8	7.77	6
114	8	8.10	3
115	8	14.33	3
116	8	14.33	6
117	8	18.48	3
118	8	22.50	3
119	8	23.42	2
120	9	5.33	3
121	9	9.38	3
122	9	13.30	3
123	9	14.38	3
124	9	15.80	3
125	9	18.39	3
126	9	18.50	6
127	9	22.80	6
128	9	25.48	3
129	9	25.48	3
1978--Continued			
130	9	26.39	3
131	9	26.77	3
132	10	9.38	3
133	10	11.13	6
134	10	11.18	3
135	10	27.04	3
136	11	1.39	3
137	11	9.85	3
138	11	13.72	3
139	11	19.01	6
140	11	21.04	3
141	11	22.96	6
142	11	27.34	3
143	11	29.54	3
144	11	30.38	6
145	12	8.88	3
146	12	12.42	3
1979			
147	1	5.75	3
148	1	6.43	3
149	1	12.51	3
150	1	23.39	3
151	1	24.75	3
152	1	24.92	7
153	1	33.17	7
154	1	34.17	3
155	2	1.39	6
156	2	9.42	3
157	2	10.54	3
158	2	13.51	3
159	2	26.83	6
160	3	5.42	6
161	3	6.38	3
162	3	9.04	3
163	3	13.17	3
164	3	20.85	3
165	4	2.89	3
166	4	5.47	3
167	4	10.58	6
168	4	24.42	6
169	4	24.51	3
170	4	30.39	3
171	4	30.68	3
172	5	2.39	3

[1]Day and hundredth of a day.

TABLE A-1. - Listing of HSAC data--Continued

Record	Month	Time[1]	Degree of injury
1979--Continued			
173	5	4.51	6
174	5	8.33	3
175	5	14.43	3
176	5	15.76	3
177	5	21.33	3
178	5	24.21	3
179	6	5.43	6
180	6	7.89	6
181	6	11.30	3
182	6	11.92	6
183	6	12.54	3
184	6	14.42	3
185	6	21.43	3
186	6	28.38	3
187	7	2.52	6
188	7	10.50	3
189	7	11.46	6
190	7	12.30	3
191	7	12.35	3
192	7	12.52	3
193	7	13.17	3
194	7	18.39	6
195	7	25.51	6
196	7	30.46	6
197	7	30.55	6
198	8	31.92	3
199	8	3.75	3
200	8	6.72	3
201	8	14.17	6
202	8	15.50	3
203	8	30.29	6
204	9	6.51	3
205	9	11.17	3
206	9	12.39	6
207	9	12.54	6
208	9	12.55	3
209	9	18.00	3
210	9	18.54	6
211	9	20.43	3
212	9	27.17	0
213	10	10.17	3
214	10	23.88	3
215	10	26.89	6
216	10	34.17	3
217	11	5.01	3
218	11	10.72	6
219	11	15.54	3
220	11	16.39	6
221	11	27.52	4
222	11	29.47	6
223	11	29.85	6
224	12	3.48	2
225	12	3.79	6
226	12	10.30	3
227	12	10.51	3
228	12	11.38	3
1980			
229	1	11.73	6
230	1	22.42	6
231	1	24.50	6
232	2	7.44	3
233	2	18.88	3
234	2	20.54	3
235	2	25.33	6
236	2	27.92	3
237	2	29.43	3
238	3	3.72	3
239	3	12.29	3
240	3	21.92	3
241	3	21.94	3
242	3	25.43	6
243	3	28.79	3
244	4	1.01	3
245	4	3.43	3
246	4	3.88	6
247	4	7.38	3
248	4	8.50	6
249	4	14.93	3
250	4	29.33	3
251	5	5.17	0
252	5	5.42	3
253	5	8.39	3
254	5	9.47	3
255	5	13.84	3
256	5	18.79	6
257	5	21.79	3
258	5	29.42	6
259	5	34.17	7

[1]Day and hundredth of a day.

TABLE A-1. - Listing of HSAC data--Continued

Record	Month	Time[1]	Degree of injury	Record	Month	Time[1]	Degree of injury
1980--Continued				1980--Continued			
260.............	6	4.63	3	304.............	10	27.46	3
261.............	6	5.55	6	305.............	10	29.42	3
262.............	6	11.79	3	306.............	11	3.43	3
263.............	6	20.35	6	307.............	11	14.51	3
264.............	6	26.89	3	308.............	11	25.55	6
265.............	7	5.38	3	309.............	12	2.35	3
266.............	7	7.39	6	310.............	12	4.55	6
267.............	7	7.42	6	311.............	12	11.58	6
268.............	7	8.56	3	312.............	12	17.75	6
269.............	7	14.35	3	313.............	12	17.93	0
270.............	7	16.30	3	314.............	12	19.46	6
271.............	7	21.93	3	315.............	12	28.80	3
272.............	7	22.72	6	316.............	12	31.38	6
273.............	7	28.46	6	1981			
274.............	7	28.79	3	317.............	1	8.39	1
275.............	7	29.85	3	318.............	1	13.71	3
276.............	7	31.33	3	319.............	1	13.72	3
277.............	8	6.72	3	320.............	1	16.51	6
278.............	8	13.35	6	321.............	1	28.38	6
279.............	8	13.56	6	322.............	1	28.89	3
280.............	8	14.38	3	323.............	2	5.33	3
281.............	8	15.50	6	324.............	2	12.83	6
282.............	8	18.80	3	325.............	2	17.47	6
283.............	8	20.04	3	326.............	2	17.75	3
284.............	8	20.83	3	327.............	2	18.72	6
285.............	8	21.10	3	328.............	2	18.75	3
286.............	8	26.55	6	329.............	2	21.63	6
287.............	8	27.68	6	330.............	2	26.38	3
288.............	8	28.05	6	331.............	3	2.50	3
289.............	9	2.38	3	332.............	3	5.72	6
290.............	9	3.58	3	333.............	3	6.39	6
291.............	9	8.89	3	334.............	3	14.33	6
291.............	9	11.43	3	335.............	3	19.44	6
293.............	9	15.93	3	336.............	3	24.25	6
294.............	9	17.39	3	337.............	5	16.83	3
295.............	9	18.75	3	338.............	6	4.43	3
296.............	9	19.38	3	339.............	6	11.43	6
297.............	9	25.64	3	340.............	6	18.26	6
298.............	9	30.88	3	341.............	6	18.76	3
299.............	9	33.17	3	342.............	6	29.67	6
300.............	10	2.39	3	343.............	6	30.10	0
301.............	10	8.47	3	344.............	7	9.39	3
302.............	10	9.51	3	345.............	7	10.55	3
303.............	10	21.93	3	346.............	7	15.73	3

[1]Day and hundredth of a day.

TABLE A-1. - Listing of HSAC data--Continued

Record	Month	Time[1]	Degree of injury	Record	Month	Time[1]	Degree of injury
1981--Continued				1982--Continued			
347............	7	19.75	3	390............	3	15.35	6
348............	7	22.73	3	391............	3	16.42	6
349............	7	24.85	3	392............	4	2.39	6
350............	8	3.35	3	393............	4	5.59	3
351............	8	14.51	6	394............	4	5.92	3
352............	8	25.76	6	395............	4	5.92	3
353............	8	30.25	6	396............	4	7.33	4
354............	9	4.43	3	397............	4	9.51	6
355............	9	14.80	3	398............	4	13.76	3
356............	9	18.42	6	399............	4	17.67	3
357............	9	18.96	3	400............	5	7.93	6
358............	10	1.34	3	401............	5	11.35	3
359............	10	6.38	3	402............	5	21.17	3
360............	10	7.43	3	403............	5	24.55	3
361............	10	13.50	6	404............	6	1.72	3
362............	10	22.54	6	405............	6	4.59	1
363............	10	27.52	3	406............	6	10.42	3
364............	10	29.71	6	407............	6	21.17	0
365............	11	5.35	6	408............	6	21.35	3
366............	11	6.54	6	409............	6	22.42	3
367............	11	10.88	6	410............	6	24.55	6
368............	11	12.76	3	411............	6	25.93	6
369............	11	12.80	6	412............	7	8.96	3
370............	11	18.35	3	413............	7	12.98	3
371............	11	19.88	3	414............	7	13.38	3
372............	11	23.72	6	415............	7	20.39	3
373............	11	25.55	6	416............	7	30.79	3
374............	12	8.43	3	417............	8	2.79	3
375............	12	9.55	3	418............	8	5.92	3
376............	12	15.35	6	419............	8	10.79	3
377............	12	18.39	6	420............	8	13.92	3
1982				421............	8	16.88	3
378............	1	9.96	3	422............	8	18.35	3
379............	1	10.09	6	423............	8	24.33	6
380............	1	27.38	3	424............	8	26.47	3
381............	1	28.88	6	425............	8	28.14	6
382............	2	2.79	3	426............	9	4.22	3
383............	2	4.92	3	427............	9	13.43	3
384............	2	10.72	5	428............	9	14.79	3
385............	2	16.71	3	429............	9	21.42	3
386............	2	26.35	6	430............	9	24.38	6
387............	3	8.71	6	431............	9	29.08	6
388............	3	10.23	6	432............	10	13.17	6
389............	3	13.34	6	433............	10	13.50	6

[1]Day and hundredth of a day.

TABLE A-1. - Listing of HSAC data--Continued

Record	Month	Time[1]	Degree of injury	Record	Month	Time[1]	Degree of injury
1982--Continued				1983--Continued			
434.............	10	22.33	6	477.............	4	8.42	3
435.............	10	25.39	3	478.............	4	10.46	3
436.............	11	2.05	6	479.............	4	27.35	3
437.............	11	4.35	2	480.............	4	29.46	6
438.............	11	9.42	6	481.............	5	2.58	6
439.............	11	15.08	3	482.............	5	11.43	6
440.............	11	15.10	6	483.............	5	11.46	6
441.............	11	18.92	6	484.............	5	12.50	3
442.............	11	23.79	3	485.............	5	20.38	3
443.............	11	28.18	6	486.............	5	24.97	6
444.............	11	29.54	6	487.............	5	26.39	3
445.............	11	29.83	6	488.............	6	7.58	3
446.............	11	30.54	6	489.............	6	9.79	3
447.............	11	30.75	6	490.............	6	10.38	3
448.............	12	3.39	3	491.............	6	16.75	3
449.............	12	6.85	5	492.............	6	16.88	6
450.............	12	7.42	6	493.............	6	27.38	3
451.............	12	7.42	3	494.............	7	2.10	3
452.............	12	10.38	3	495.............	7	11.17	3
453.............	12	10.94	5	496.............	7	11.59	3
454.............	12	11.04	4	497.............	7	21.21	3
455.............	12	13.42	3	498.............	7	28.68	3
456.............	12	15.29	6	499.............	7	29.76	3
457.............	12	17.43	3	500.............	8	1.55	6
458.............	12	17.68	5	501.............	8	2.84	3
459.............	12	19.42	6	502.............	8	4.54	3
460.............	12	20.42	3	503.............	8	9.67	3
1983				504.............	8	16.75	3
461.............	1	6.42	6	505.............	8	17.54	3
462.............	1	6.77	6	506.............	8	17.71	3
463.............	1	27.92	3	507.............	8	17.76	6
464.............	2	9.54	6	508.............	8	19.84	3
465.............	2	17.43	3	509.............	8	19.84	3
466.............	2	17.85	4	510.............	9	8.04	3
467.............	2	22.27	3	511.............	9	8.42	6
468.............	2	24.89	6	512.............	9	9.80	3
469.............	3	1.73	6	513.............	9	20.33	3
470.............	3	3.51	6	514.............	9	22.72	6
471.............	3	9.83	3	515.............	10	3.13	3
472.............	3	18.38	6	516.............	10	3.60	3
473.............	4	1.50	3	517.............	10	3.79	6
474.............	4	1.59	3	518.............	10	3.85	3
475.............	4	4.51	3	519.............	10	10.05	3
476.............	4	7.46	6	520.............	10	10.35	6

[1]Day and hundredth of a day.

TABLE A-1. - Listing of HSAC data--Continued

Record	Month	Time[1]	Degree of injury	Record	Month	Time[1]	Degree of injury
1983--Continued				1984--Continued			
521.............	10	11.72	3	556.............	2	1.01	3
522.............	10	16.05	6	557.............	2	8.34	3
523.............	10	17.76	3	558.............	2	8.40	3
524.............	10	18.75	3	559.............	2	9.33	3
525.............	10	19.13	3	560.............	2	9.77	3
526.............	10	26.89	3	561.............	2	14.38	3
527.............	10	29.31	6	562.............	2	20.80	3
528.............	11	9.22	3	563.............	2	28.54	3
529.............	11	15.42	6	564.............	3	13.59	3
530.............	11	21.58	6	565.............	3	14.08	3
531.............	12	4.33	0	566.............	3	16.38	3
532.............	12	5.42	3	567.............	3	16.92	3
533.............	12	8.88	3	568.............	3	19.13	3
534.............	12	13.47	6	569.............	3	21.71	3
535.............	12	13.51	3	570.............	3	26.97	3
536.............	12	13.55	6	571.............	3	28.43	2
537.............	12	13.72	3	572.............	3	30.14	3
538.............	12	19.63	3	573.............	3	30.83	3
539.............	12	20.38	6	574.............	4	3.55	3
540.............	12	20.76	3	575.............	4	3.77	3
541.............	12	21.34	0	576.............	4	16.38	3
542.............	12	21.46	3	577.............	4	16.76	3
543.............	12	22.54	3	578.............	4	27.68	6
544.............	12	27.51	3	579.............	5	8.50	3
545.............	12	29.35	3	580.............	5	15.05	6
1984				581.............	5	23.52	3
546.............	1	3.04	3	582.............	5	29.55	3
547.............	1	5.71	0	583.............	6	1.02	0
547.............	1	12.71	3	584.............	6	6.68	0
549.............	1	14.79	3	585.............	6	8.50	3
550.............	1	14.93	3	586.............	6	21.72	3
551.............	1	19.72	3	587.............	6	27.47	6
552.............	1	21.64	3	588.............	6	28.71	3
553.............	1	25.47	3	589.............	7	3.44	6
554.............	1	25.48	1	590.............	7	12.10	0
555.............	1	30.06	3	591.............	7	20.64	3

[1]Day and hundredth of a day.

APPENDIX B.--ELAPSED TIMES AND COMPUTER PROGRAM

The elapsed times between accidents are shown in table B-1. The statistical summary of the distribution is as follows:

Number	590
Mean	5.91
Median	3.65
Standard deviation	8.25
Standard error of the mean	0.34
Range:	
Maximum	61.00
Minimum	-1.80[1]
Interquartile range:	
Q_3	7.10
Q_1	1.20

The elapsed time was calculated using the procedure outlines in reference 4.

The following is a Fortran listing of the program used to calculate the elapsed days between accidents.

```
#FILE (CJK)ELAPSED/DAYS/LF ON SRC4

FILE  1(KIND=DISK, MAXRECSIZE=30, TITLE="LF75TO84/PLUCKED/SORTED",

     * FILETYPE=7,MYUSE=IN)

FILE  4 (KIND=DISK,MAXRECSIZE=14,TITLE="LF75TO84/ELAPSED/DAYS",

     *  FILETYPE=7,MYUSE=OUT)

FILE  8 (KIND=PRINTER,MAXRECSIZE=22, MYUSE=OUT)

FILE  5 (KIND=REMOTE,MAXRECSIZE=14,MYUSE=IO)

       DIMENSION X(700,62)

       OPEN(4)

         I=1

    5 CONTINUE

        READ(1,10,END=20)X(I,4),X(I,5)X(I,7)X(I,46)

    10  FORMAT(I2,1X,F5.2,1X,I2,1X,I1)

      I=I+1

       GO TO 5

    20 CONTINUE
```

[1]Incorrect data point included for completeness.

```
      NM=I-2
      DO 40 I=1,NM
      D=X(I,5)
      D2=X(I+1,5)
      M=X(I,4)
      M2=X(I+1,4)
      Y=X(I,7)
      Y2=X(I+1,7)
      IF(M.LE.2) GO TO 25
      W=INT((.4*M)+2.3)
      Z=Y
      GO TO 26
 25   W=0
      Z=Y-1
 26   D3=(365*Y)+(31*(M-1))+D+INT(Z/4)-W
      IF(M2.LE.2) GO TO 28
      W=INT((.4*M2)+2.3)
      Z=Y2
      GO TO 35
 28   W=0
      Z=Y2-1
 35   D4=(365*Y2)+(31*(M2-1))+D2+INT(Z/4)-W
      D5=D4-D3
      WRITE(4,30)D5
 30   FORMAT(F6.1)
      WRITE(5,30)D5
      WRITE(8,30)D5
 40   CONTINUE
      CLOSE (4,DISP=CRUNCH)
      END
```

TABLE B-1. - Chronological listing of elapsed times between accidents

Record	Elapsed time[1]	Record	Elapsed time[1]	Record	Elapsed time[1]	Record	Elapsed time[1]
1.........	3.9	49........	5.9	97........	4.3	145.......	3.5
2.........	17.6	50........	46.1	98........	3.6	146.......	24.3
3.........	5.2	51........	35.4	99........	8.0	147.......	.7
4.........	14.4	52........	32.1	100.......	1.3	148.......	6.1
5.........	8.0	53........	29.4	101.......	4.0	149.......	10.9
6.........	19.9	54........	46.0	102.......	.7	150.......	1.4
7.........	.7	55........	54.7	103.......	1.0	151.......	.2
8.........	6.2	56........	40.9	104.......	.3	152.......	8.3
9.........	.9	57........	58.0	105.......	12.0	153.......	1.0
10........	.3	58........	43.7	106.......	2.3	154.......	[2]-1.8
11........	6.9	59........	25.7	107.......	.8	155.......	8.0
12........	10.5	60........	7.8	108.......	3.5	156.......	1.1
13........	4.6	61........	6.2	109.......	7.6	157.......	3.0
14........	32.5	62........	12.7	110.......	7.8	158.......	13.3
15........	1.0	63........	57.4	111.......	3.0	159.......	6.6
16........	9.0	64........	37.2	112.......	3.0	160.......	1.0
17........	17.4	65........	3.5	113.......	.3	161.......	2.7
18........	.0	66........	45.5	114.......	6.2	162.......	4.1
19........	7.5	67........	.7	115.......	.0	163.......	7.7
20........	61.0	68........	5.8	116.......	4.2	164.......	13.0
21........	4.1	69........	5.0	117.......	4.0	165.......	2.6
22........	3.3	70........	3.0	118.......	.9	166.......	5.1
23........	.0	71........	4.1	119.......	12.9	167.......	13.8
24........	45.7	72........	1.2	120.......	4.1	168.......	.1
25........	6.2	73........	4.6	121.......	3.9	169.......	5.9
26........	14.9	74........	.4	122.......	1.1	170.......	.3
27........	10.8	75........	.1	123.......	1.4	171.......	1.7
28........	6.8	76........	5.6	124.......	2.6	172.......	2.1
29........	3.7	77........	1.9	125.......	.1	173.......	3.8
30........	4.0	78........	5.8	126.......	4.3	174.......	6.1
31........	14.0	79........	3.9	127.......	2.7	175.......	1.3
32........	13.4	80........	.5	128.......	.0	176.......	5.6
33........	10.4	81........	4.0	129.......	.9	177.......	2.9
34........	23.9	82........	2.0	130.......	.4	178.......	12.2
35........	4.3	83........	7.7	131.......	12.6	179.......	2.5
36........	14.8	84........	4.1	132.......	1.8	180.......	3.4
37........	10.6	85........	3.0	133.......	.1	181.......	.6
38........	1.4	86........	32.7	134.......	15.9	182.......	.6
39........	11.9	87........	.4	135.......	5.3	183.......	1.9
40........	.8	88........	1.1	136.......	8.5	184.......	7.0
41........	11.8	89........	.0	137.......	3.9	185.......	6.9
42........	9.1	90........	11.6	138.......	5.3	186.......	4.1
43........	5.4	91........	1.7	139.......	2.0	187.......	8.0
44........	17.1	92........	12.8	140.......	1.9	188.......	1.0
45........	18.6	93........	3.0	141.......	4.4	189.......	.8
46........	2.9	94........	12.9	142.......	2.2	190.......	.1
47........	6.5	95........	14.4	143.......	.8	191.......	.2
48........	.0	96........	3.7	144.......	8.5	192.......	.7

See footnotes at end of table.

TABLE B-1. - Chronological listing of elapsed times between accidents--Continued

Record	Elapsed time[1]	Record	Elapsed time[1]	Record	Elapsed time[1]	Record	Elapsed time[1]
193.......	5.2	243.......	3.2	293.......	1.5	343.......	9.3
194.......	7.1	244.......	2.4	394.......	1.4	344.......	1.2
195.......	4.9	245.......	.4	295.......	.6	345.......	5.2
196.......	.1	246.......	3.5	296.......	6.3	346.......	4.0
197.......	1.4	247.......	1.1	297.......	5.2	347.......	3.0
198.......	2.8	248.......	6.4	298.......	2.3	348.......	2.1
199.......	3.0	249.......	14.4	299.......	[2]-.8	349.......	9.5
200.......	7.4	250.......	5.8	300.......	6.1	350.......	11.2
201.......	1.3	251.......	.3	301.......	1.0	351.......	11.3
202.......	14.8	252.......	3.0	302.......	12.4	352.......	4.5
203.......	7.2	253.......	1.1	303.......	5.5	353.......	5.2
204.......	4.7	254.......	4.4	304.......	2.0	354.......	10.4
205.......	1.2	255.......	5.0	305.......	5.0	355.......	3.6
206.......	.2	256.......	3.0	306.......	11.1	356.......	.5
207.......	.0	257.......	7.6	307.......	11.0	357.......	12.4
208.......	5.4	258.......	4.8	308.......	6.8	358.......	5.0
209.......	.5	259.......	1.5	309.......	2.2	359.......	1.1
210.......	1.9	260.......	.9	310.......	7.0	360.......	6.1
211.......	6.7	261.......	6.2	311.......	6.2	361.......	9.0
212.......	13.0	262.......	8.6	312.......	.2	362.......	5.0
213.......	13.7	263.......	6.5	313.......	1.5	363.......	2.2
214.......	3.0	264.......	8.5	314.......	9.3	364.......	6.6
215.......	7.3	265.......	2.0	315.......	2.6	365.......	1.2
216.......	1.8	266.......	.0	316.......	8.0	366.......	4.3
217.......	5.7	267.......	1.1	317.......	5.3	367.......	1.9
218.......	4.8	268.......	5.8	318.......	.0	368.......	.0
219.......	.8	269.......	1.9	319.......	2.8	369.......	5.6
220.......	11.1	270.......	5.6	320.......	11.9	370.......	1.5
221.......	2.0	271.......	.8	321.......	.5	371.......	3.8
222.......	.4	272.......	5.7	322.......	7.4	372.......	1.8
223.......	3.6	273.......	.3	323.......	7.5	373.......	12.9
224.......	.3	274.......	1.1	324.......	4.6	374.......	1.1
225.......	6.5	275.......	1.5	325.......	.3	375.......	5.8
226.......	.2	276.......	6.4	326.......	1.0	376.......	3.0
227.......	.9	277.......	6.6	327.......	.0	377.......	22.6
228.......	31.4	278.......	.2	328.......	2.9	378.......	.1
229.......	10.7	279.......	.8	329.......	4.8	379.......	17.3
230.......	2.1	280.......	1.1	330.......	4.1	380.......	1.5
231.......	13.9	281.......	3.3	331.......	3.2	381.......	4.9
232.......	11.4	282.......	1.2	332.......	.7	382.......	2.1
233.......	1.7	283.......	.8	333.......	7.9	383.......	5.8
234.......	4.8	284.......	.3	334.......	5.1	384.......	6.0
235.......	2.6	285.......	5.4	335.......	4.8	385.......	9.6
236.......	1.5	286.......	1.1	336.......	53.6	386.......	10.4
237.......	3.3	287.......	.4	337.......	18.6	387.......	1.5
238.......	8.6	288.......	5.3	338.......	7.0	388.......	3.1
239.......	9.6	289.......	1.2	339.......	6.8	389.......	2.0
240.......	.0	290.......	5.3	340.......	.5	390.......	1.1
241.......	3.5	291.......	2.5	341.......	10.9	391.......	17.0
242.......	3.4	292.......	4.5	342.......	.4	392.......	3.2

TABLE B-1. - Chronological listing of elapsed times between accidents--Continued

Record	Elapsed time[1]	Record	Elapsed time[1]	Record	Elapsed time[1]	Record	Elapsed time[1]
393.......	0.3	442.......	4.4	491.......	0.1	541.......	0.1
394.......	.0	443.......	1.4	492.......	10.5	542.......	1.1
395.......	1.4	444.......	.3	493.......	4.7	543.......	5.0
396.......	2.2	445.......	.7	494.......	9.1	544.......	1.8
397.......	4.3	446.......	.2	495.......	.4	545.......	4.7
398.......	3.9	447.......	2.6	496.......	9.6	546.......	2.7
399.......	20.3	448.......	3.5	497.......	7.5	547.......	7.0
400.......	3.4	449.......	.6	498.......	1.1	548.......	2.1
401.......	9.8	450.......	.0	499.......	2.8	549.......	.1
402.......	3.4	451.......	3.0	500.......	1.3	550.......	4.8
403.......	8.2	452.......	.6	501.......	1.7	551.......	1.9
404.......	2.9	453.......	.1	502.......	5.1	552.......	3.8
405.......	5.8	454.......	2.4	503.......	7.1	553.......	.0
406.......	10.8	455.......	1.9	504.......	.8	554.......	4.6
407.......	.2	456.......	2.1	505.......	.2	555.......	1.9
408.......	1.1	457.......	.3	506.......	.1	556.......	7.3
409.......	2.1	457.......	1.7	507.......	2.1	557.......	.1
410.......	1.4	459.......	1.0	508.......	.0	558.......	.9
411.......	13.0	460.......	17.0	509.......	19.2	559.......	.4
412.......	4.0	461.......	.3	510.......	.4	560.......	4.6
413.......	.4	462.......	21.2	511.......	1.4	561.......	6.4
414.......	7.0	463.......	12.6	512.......	10.5	562.......	7.7
415.......	10.4	464.......	7.9	513.......	2.4	563.......	14.0
416.......	3.0	465.......	.4	514.......	10.4	564.......	.5
417.......	3.1	466.......	4.4	515.......	.5	565.......	2.3
418.......	4.9	467.......	2.6	516.......	.2	566.......	.5
419.......	3.1	468.......	4.8	517.......	.1	567.......	2.2
420.......	3.0	469.......	1.8	518.......	6.2	568.......	2.6
421.......	1.5	470.......	6.3	519.......	.3	569.......	5.3
422.......	6.0	471.......	8.6	520.......	1.4	570.......	1.5
423.......	2.1	472.......	14.1	521.......	4.3	571.......	1.7
424.......	1.7	473.......	.1	522.......	1.7	572.......	.7
425.......	7.1	474.......	2.9	523.......	1.0	573.......	3.7
426.......	9.2	475.......	2.9	524.......	.4	574.......	.2
427.......	1.4	476.......	1.0	525.......	7.8	575.......	12.6
428.......	6.6	477.......	2.0	526.......	2.4	576.......	.4
429.......	3.0	478.......	16.9	527.......	10.9	577.......	10.9
430.......	4.7	479.......	2.1	528.......	6.2	577.......	10.8
431.......	14.1	480.......	3.1	529.......	6.2	579.......	6.6
432.......	.3	481.......	8.9	530.......	12.8	580.......	8.5
433.......	8.8	482.......	.0	531.......	1.1	581.......	6.0
434.......	3.1	483.......	1.0	532.......	3.5	582.......	2.5
435.......	7.7	484.......	7.9	533.......	4.6	583.......	5.7
436.......	2.3	485.......	4.6	534.......	.0	584.......	1.8
437.......	5.1	486.......	1.4	535.......	.0	585.......	13.2
438.......	5.7	487.......	12.2	536.......	.2	586.......	5.8
439.......	.0	488.......	2.2	537.......	5.9	587.......	1.2
440.......	3.8	489.......	.6	538.......	.8	588.......	4.7
441.......	4.9	490.......	6.4	539.......	.4	589.......	8.7
				540.......	.6	590.......	8.5

[1]In days and tenths of days.
[2]Incorrect data point, included for completeness.

APPENDIX C.--INDEPENDENCE

Two tests were used to determine if the data (elapsed times) were random and therefore independent.

The first is a modified form of the chi-squared contingency test and was suggested by Tom McWilliams of Oregon State University. It tests whether successive occurrences are independent of each other. First, the data were ordered and divided into three groups of relatively equal size. Each point was assigned to one of the groups. Group 1 contains all elapsed times from 0 up to and including 1.9 days; group 2, from 2.0 up to and including 5.8 days; and group 3, from 5.9 up to and including 61.0 days. This gives 201 data points in group 1, 196 in group 2, and 193 in group 3 (table C-1).

TABLE C-1. - Numerically ordered listing of elapsed times between accidents

Elapsed time[1]	Number of observations	Elapsed time[1]	Number of observations	Elapsed time[1]	Number of observations
Group 1:		Group 2--Con.		Group 3--Con.	
-1.8[2]	1	3.1	5	6.3	2
-0.8[2]	1	3.2	3	6.4	4
0.0	20	3.3	3	6.5	3
0.1	15	3.4	4	6.6	5
0.2	12	3.5	7	6.7	1
0.3	15	3.6	3	6.8	3
0.4	15	3.7	3	6.9	2
0.5	8	3.8	4	7.0	5
0.6	7	3.9	5	7.1	3
0.7	8	4.0	6	7.2	1
0.8	10	4.1	7	7.3	2
0.9	6	4.2	1	7.4	2
1.1	11	4.3	6	7.5	3
1.2	16	4.4	4	7.6	2
1.3	7	4.5	2	7.7	4
1.4	4	4.6	7	7.8	3
1.5	12	4.7	5	7.9	3
1.6	10	4.8	7	8.0	5
1.7	8	4.9	4	8.2	1
1.8	6	5.0	6	8.3	1
1.9	9	5.1	4	8.5	5
Total...	201	5.2	5	8.6	3
Group 2:		5.3	6	8.7	1
2.0	7	5.4	3	8.8	1
2.1	10	5.5	1	8.9	1
2.2	6	5.6	4	9.0	2
2.3	4	5.7	4	9.1	2
2.4	4	5.8	8	9.2	1
2.5	3	Total...	196	9.3	2
2.6	7	Group 3:		9.5	1
2.7	3	5.9	3	9.6	3
2.8	3	6.0	3	9.8	1
2.9	6	6.1	4	10.4	5
3.0	16	6.2	9	10.5	3

See footnotes at end of table.

TABLE C-1. - Numerically ordered listing of elapsed times between accidents--Con.

Elapsed time[1]	Number of observations	Elapsed time[1]	Number of observations	Elapsed time[1]	Number of observations
Group 3--Con.		Group 3--Con.		Group 3--Con.	
10.6.......	1	13.4.......	1	23.9.......	1
10.7.......	1	13.7.......	1	24.3.......	1
10.8.......	3	13.8.......	1	25.7.......	1
10.9.......	4	13.9.......	1	29.4.......	1
11.0.......	1	14.0.......	2	31.4.......	1
11.1.......	2	14.1.......	2	32.1.......	1
11.2.......	1	14.4.......	3	32.5.......	1
11.3.......	1	14.8.......	2	32.7.......	1
11.4.......	1	14.9.......	1	35.4.......	1
11.6.......	1	15.9.......	1	37.2.......	1
11.8.......	1	16.9.......	1	40.9.......	1
11.9.......	2	17.0.......	2	43.7.......	1
12.0.......	1	17.1.......	1	45.5.......	1
12.2.......	2	17.3.......	1	45.7.......	1
12.4.......	2	17.4.......	1	46.0.......	1
12.6.......	3	17.6.......	1	46.1.......	1
12.7.......	1	18.6.......	2	53.6.......	1
12.8.......	2	19.2.......	1	54.7.......	1
12.9.......	3	19.9.......	1	57.4.......	1
13.0.......	3	20.3.......	1	58.0.......	1
13.2.......	1	21.2.......	1	61.0.......	1
13.3.......	1	22.6.......	1	Total....	193

[1]In days and tenths of days.
[2]Incorrect data point, included for completeness.

A transition matrix (table C-2) was made using the natural order of occurrence. That is, if in the chronological order of the elapsed times (appendix B), a point in group 1 is followed by a point in group 2, this was tallied for cell 2 in the matrix (first row, second column). For example, data point 1 is 3.9 (group 2) and data point 2 is 17.6 (group 3), so this was tallied to cell 3. A BASIC computer program used to make this tally is listed at the end of this appendix. These tallied values are then the observed values for the contingency table. Under the hypothesis of independence, each of the nine cells of table C-2 have equal probability of occurrence. The expected values are 590/9 = 65.5.

TABLE C-2. - Transition matrix--observed successive occurrence of accidents[1]

From group--	To group--		
	1	2	3
1..........................	69	67	67
2..........................	74	70	52
3..........................	58	58	76

[1]Expected number of occurrences for each cell is 65.5.

The contingency table produces a calculated chi-squared equal to 7.82. There are 8 degrees of freedom and if the level of significance, α, is chosen as 0.05, the chi-squared distribution table gives a value of 15.5073. Because 7.82 is not greater than 15.5073, the null hypothesis is accepted: that the magnitudes of elapsed times are equally likely and, therefore, that the accidents themselves are independent.

A second test for independence is the Runs test (6, p. 52). A run is a succession of identical symbols that are followed and preceded by different symbols or no symbols at all. For the data set of elapsed times, its median, 3.65, was subtracted from each value. Then the sign of the resulting value (+1 or -1) was recorded. The number of runs, as defined above, was 301, with 295 negative and 295 positive entries involved. Because of the large sample (590 data points), a normal curve approximation was used for the sampling distribution. The null hypothesis is that the order of pluses and minuses with respect to the median is random; that is, the elapsed times between accidents occurred in a random order, and therefore the accidents themselves are independent of one another. The alternative hypothesis is that the order of pluses and minuses with respect to the median is not random, and therefore the accidents are not independent.

The test statistic, Z, is equal to 0.4122. To reject the null hypothesis at a significance level of 0.05, Z would have to be less than or equal to -1.96 or greater than or equal to 1.96. Consequently, the Runs test also shows that the elapsed times are independent, and therefore, that the original accidents are independent. Thus, both the chi-squared and Runs test affirm the hypothesis that this data set satisfies the criterion of statistical independence.

The following is the program listing for computing the transition matrix:

```
  5 DIM E(589), D(200), F(190)

 20 OPEN "b:1f7584ed" FOR INPUT AS  1 LEN=40

 25 FOR N=0 TO 589

 30 INPUT#1,E(N)

 40 NEXT N

 60 FOR N=0 TO 588

 70 A=E(N)

 80 B=E(N+1)

 90 IF (A<=1.9) THEN A=1

100 IF (A>1.9) AND (A<=5.8) THEN A=2

110 IF (A>5.8) THEN A=3

120 IF (B<=1.9) THEN B=1

130 IF (B>1.9) AND (B<=5.8) THEN B=2

140 IF (B>5.8) THEN B=3

150 IF A=1 AND B=1 THEN C1 = C1+1

160 IF A=1 AND B=2 THEN C2 = C2+1

170 IF A=1 AND B=3 THEN C3 = C3+1
```

```
180 IF A=2 AND B=1 THEN C4 = C4+1
190 IF A=2 AND B=2 THEN C5 = C5+1
200 IF A=2 AND B=3 THEN C6 = C6+1
210 IF A=3 AND B=1 THEN C7 = C7+1
220 IF A=3 AND B=2 THEN C8 = C8+1
230 IF A=3 AND B=3 THEN C9 = C9+1
240 NEXT N
250 CLOSE 1
260 PRINT C1,C2,C3,C4,C5,C6,C7,C8,C9
270 END
```

APPENDIX D.--LEAST SQUARES FIT OF ELAPSED TIMES

The elapsed times were formed into the frequency distribution given in table D-1, with interval width equal to 1 and interval midpoints at 0.5, 1.5, etc. Two observations are below the first class.

From table D-1, the cumulative frequency distribution shown in table D-2 was formed.

From table D-2, the file of values listed in table D-3 was created for input into the least squares program for fitting a negative exponential cumulative distribution function.

TABLE D-1. - Frequency distribution of elapsed times

Middle of interval, days	Number of observations	Middle of interval, days	Number of observations
0.50	116	31.50	1
1.50	83	32.50	3
2.50	53	33.50	0
3.50	53	34.50	0
4.50	49	35.50	1
5.50	44	36.50	0
6.50	36	37.50	1
7.50	28	38.50	0
8.50	18	39.50	0
9.50	12	40.50	1
10.50	17	41.50	0
11.50	10	42.50	0
12.50	14	43.50	1
13.50	9	44.50	0
14.50	10	45.50	2
15.50	1	46.50	2
16.50	1	47.50	0
17.50	6	48.50	0
18.50	2	49.50	0
19.50	2	50.50	0
20.50	1	51.50	0
21.50	1	52.50	0
22.50	1	53.50	1
23.50	1	54.50	1
24.50	1	55.50	0
25.50	1	56.50	0
26.50	0	57.50	1
27.50	0	58.50	1
28.50	0	59.50	0
29.50	1	60.50	0
30.50	0	61.50	1

TABLE D-2. - Cumulative frequency of elapsed times

Middle of interval	Cumulative probability
1	0.34
3	.52
5	.67
7	.78
9	.83
11	.88
13	.92
15	.94
17	.95
19	.96
21	.96
21	.96
23	.96
25	.97
27	.97
29	.97
31	.97
33	.97
35	.98
37	.98
39	.98
41	.98
43	.98
45	.98
47	.99
49	.99
51	.99
53	.99
55	.99
57	.99
59	.99
61	.99

TABLE D-3. - Input values for least squares program

Record number	Cumulative probability
1	0.20
2	.34
3	.43
4	.52
5	.60
6	.67
7	.74
8	.78
9	.81
10	.83
11	.85
12	.88
13	.90
14	.92
15	.94
16	.94
17	.94
18	.95
19	.95
20	.96
21	.96
22	.96
23	.96
24	.96
25	.96
26	.97
30	.97
32	.97
33	.97
36	.98
38	.98
47	.99

The least squares program output follows (7):

```
CUM NEG. EXP FIT OF LF DATA 1975 TO 1984, ELAPSED TIMES, INPUT FILE NAME
IS HISTO/2/LF

VALUE OF DETERMINANT =    .4072401E+02
    6 ITERATIONS
NO/DATA PTS = 34                        WEIGHTED VARIANCE = 0.00
NO/IND VARIABLES = 1                    UNWTD SUM/SQ./DEV = 0.01
NO/PARAMETERS = 2 (1 HELD CONSTANT)    STD DEV/FIT = 0.02

CORRELATION COEFFICIENT = 0.66355399, INDEX/DETERMINATION = 0.99198650.

K GUESS OF FINAL VALUE S.D. OF K-TH PARAMETER EXACT LEAST SQ. EQ. [1Q.
K-TH PAR. OF K-TH PAR  PARAMETER   T-TEST     FITTED     INPUT
1   0.20000   0.18293    0.00285   64.24781    21.4367    21.4367
2   0.00000   0.00000    0.00000

MATRIX OF CORRELATIONS BETWEEN FREE PARAMETERS

1   1.000
```

	INDEPENDENT VARIABLE	DEPENDENT VARIABLE	CALCULATED FUNCTION
1	1.00	0.20	0.17
2	2.00	0.34	0.31
3	3.00	0.43	0.42
4	4.00	0.52	0.52
5	5.00	0.60	0.60
6	6.00	0.67	0.67
7	7.00	0.74	0.72
8	8.00	0.78	0.77
9	9.00	0.81	0.81

10	10.00	0.83	0.84
11	11.00	0.85	0.87
12	12.00	0.88	0.89
13	13.00	0.90	0.91
14	14.00	0.92	0.92
15	15.00	0.94	0.94
16	16.00	0.94	0.95
17	17.00	0.94	0.96
18	18.00	0.95	0.96
19	19.00	0.95	0.97
20	20.00	0.96	0.97
21	21.00	0.96	0.98
22	22.00	0.96	0.98
23	23.00	0.96	0.99
24	24.00	0.96	0.99
25	25.00	0.96	0.99
26	26.00	0.97	0.99
27	30.00	0.97	1.00
28	32.00	0.97	1.00
29	33.00	0.97	1.00
30	36.00	0.98	1.00
31	38.00	0.98	1.00
32	47.00	0.99	1.00
33	59.00	0.99	1.00
34	62.00	1.00	1.00

MEAN/IND.VAR = 20.2352941 VARIANCE = 229.2762923

MEAN/DEP.VAR = 0.8461765 VARIANCE = 0.0412001

Note that the fitted parameter's value for the cumulative negative exponential is 0.18293.

APPENDIX E.--GOODNESS-OF-FIT TESTS

The Kolmogornov-Smirnov goodness-of-fit tests (6, p. 47) was used to test the degree of agreement between the theoretical and the observed cumulative negative exponential frequency distributions.

Let D equal the maximum difference between the observed and the theoretical distributions. From the least squares fit (see appendix D), this value is 0.03, occurring at the first interval. At the 0.05 level of significance, the critical value for D is $1.36/(590)^{1/2} = 0.06$. Since 0.03 is less than 0.06, the hypothesis is accepted that the distribution is a negative exponential with parameter $P_1 = 0.18293$.

The second example shows that the theoretical Poisson model for the expected number of accidents per day, based upon $\lambda = 0.16681$ for the MLE method, agrees with the empirical data. The maximum difference between the cumulative model and the cumulative data occurs at zero accidents per day and is equal to 0.00682. Now, at the 0.05 level of significance, $D = 1.36/(584)^{1/2} = 0.056$, and 0.00682 is much less than 0.056. Thus, the hypothesis is accepted that the data does form a Poisson distrbution with $\lambda = 0.16681$.

APPENDIX F.--CONFIDENCE INTERVAL COMPUTATION

The following is a computation of the confidence interval (using the MLE of the Poisson) for the expected value of the first-order IA times between accidents (11):

$$\text{Now } p(\theta) = \theta e^{-\theta * t}, \qquad 0<t<\infty.$$

Let $\bar{x}$ equal the Poisson MLE, where $p(x) = e^{-\mu} \mu^x/x! = (\lambda t)^x e^{-(\lambda t)}/x!$ If t = 1 day or unit, then the equivalence, $\hat{\theta} = 1/\bar{x}$, results. Let n = the total number of occurrences over the entire time span. Then, the following defines the lower and upper confidence interval limits:

$$(\bar{\theta} = 2n\hat{\theta})/ \text{ (chi squared at } 1 - \alpha/2 \text{ with } (2n + 2) \text{ degrees of freedom)}$$

and $$(\underline{\theta} = 2n\hat{\theta} = 2n\theta)/ \text{ (chi squared of } \alpha/2 \text{ with } (2n) \text{ degrees of freedom).}$$

The large chi-squared values may be computed by the approximation of $0.5(z\beta + \sqrt{(2n - 1)^2}$ where $z\beta$ is the normal variate.

Applying the preceding formulas to the data gives the results tabulated in table F-1. The intervals are not symmetrical because the parent distribution is the chi-square distribution, which is asymmetrical.

TABLE F-1. - Confidence intervals for MLE estimation of expected time between accidents, days

Confidence, %	Lower limit	MLE	Upper limit
90..................	5.60	5.99	6.42
95..................	5.53	5.99	6.49

U.S. GOVERNMENT PRINTING OFFICE: 1987 - 606-017/60041

INT.-BU.OF MINES,PGH.,PA. 28476