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UNITED STATES DEPARTMENT OF THE INTERIOR Donald Paul Hodel, Secretary

## BUREAU OF MINES

Robert C. Horton, Director

## Library of Congress Cataloging in Publication Data:

Conical bit rotation as a function of selected cutting parameters.
(Report of investigations / United States Department of the Interior, Bureau of Mines ; 8983)

Bibliography: p. 13.
Supt. of Docs. no.: I 28.23: 8983.

1. Bits (Drilling and boring)-Testing. 2. Coal mining machineryTesting. I. Wingquist, Carl F. II. Series: Report of investigations (United States. Bureau of Mines) ; 8983.

TN23،U43 [TN279] 622s [622'.23] 85-600 120
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#### Abstract

The Bureau of Mines is engaged in research to evaluate the effects of cutter bit wear at coal mine faces. This paper addresses one element of conical bit wear, bit rotation. A discussion on mounting configuration and bit forces and their effects on rotation is presented. The effects on rotation and bit forces of bit attack angle, cutting depth, and skew are determined for two types of conical bits during linear cutting of sandstone-inclusive rock. The results indicate that a $10^{\circ}$ negative skew angle and $35^{\circ}$ attack angle produce the maximum rotation of $17.5^{\circ}$ for each foot of cutting. Data on bit rotation and bit forces are presented in an appendix.


[^0]
## INTRODUCTION

This laboratory study of conical bit rotation is part of an extensive ongoing Bureau research program on coal cutting technology in which the coal cutting system is being fully evaluated to determine those factors that affect the health and safety of the miners. The primary considerations are respirable dust generated by cutting and methane ignition due to frictional heating at the tool-mineral interface. It has been shown ( $1-4)^{5}$ that bit wear increases the like $\overline{1} \mathrm{i}$ hood of methane ignition in gassy areas. It is also known that worn bits require a greater normal force (5) and thus tend to cut shallower; for a given machine thrust, than new bits. Since it has been demonstrated (6) that deep cutting lowers specific dust, it follows that bit wear leads to higher dust production. Although conical tools are intended to rotate freely so they will wear symmetrically, they commonly do not rotate effectively.

The primary purpose of this study was to investigate the relationship between conical bit rotation and bit attack angle, skew angle, bit type, cutting depth, and intercut spacing. Effective tool use
requires that rotation of the bit occurs, particularly when cutting hard inclusive material such as sandstone and shale. Nonrotation results in rapid asymmetric wear of the tool tip and consequently, high normal force, shallow cutting, high specific dust, and premature tool failure. The information generated by this study should be of particular interest to tool and machine designers and mine operators using continuous mining machines since only by choosing tool mounting configurations and operating practices that enhance rotation can the full potential of the tool and machine be realized.

The initial step in this study was to identify those parameters felt to be relevant to bit rotation and then create a factorial experimental design or test plan based on those parameters. Cutting tests were then performed in which measurements of bit rotation and bit force were taken for each test condition. The resulting data were analyzed to determine the degree to which each parameter influenced rotation and bit forces. Additional tests, based on a central composite design, were carried out to provide a basis for predicting rotation.

## ACKNOWLEDGEMENT

The authors wish to acknowledge Kennametal, Inc., of Latrobe, PA, for assistance in planning the testing and for
providing the cutting tools used in the study.

## bit force and bit angle nomenclature and definitions

During cutting tests, bit forces were measured in three mutually perpendicular directions defined in reference to a rectangular ( $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ ) coordinate system oriented so that the XY plane is parallel to the sample cutting face with the $Y$ axis parallel to the direction of cutting. These forces, which are named lateral force ( $\mathrm{FX}_{\mathrm{X}}$, cutting force ( $\mathrm{F}_{\mathrm{y}}$ ), and

[^1]normal force ( $\mathrm{F}_{\mathrm{Z}}$ ), are shown in figure 1 and are reported in pound-force (1bf) in this paper.

The attack and skew angles used to describe the mounting configuration of the bit are shown in figure 1 and defined in reference to the rectangular ( $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ ) coordinate system. The attack angle ( $\theta$ ) ranges from $30^{\circ}$ to $50^{\circ}$ and is defined as the angle between the longitudinal axis of the bit and the projection of the longitudinal axis into the XZ plane. The skew angle ( $\phi$ ) ranges from $-10^{\circ}$ to $+10^{\circ}$


FIGURE 1. - Bit forces and attack and skew angles shown in reference to coordinate system.
and is defined as the angle between the projection of the longitudinal axis of the bit onto the XZ plane and the Z axis. Stated another way, if the longitudinal axis of the bit passes through the origin and point $\mathrm{P}(\mathrm{X}, \mathrm{Y}, \mathrm{Z})$, then the attack angle is given by

$$
\theta=\arcsin \frac{Y}{\left(X^{2}+Y^{2}+Z^{2}\right)^{1 / 2}}
$$

The skew angle is given by

$$
\phi=\arctan \frac{X}{Z} .
$$

Skew is regarded as negative or positive depending on whether the bit is inclined away from or toward the uncut material respectively, as shown in figure 2 .

CUTTING--INDUCED BIT ROTATION

Although an analysis of the mechanics of bit rotation is beyond the scope of this paper, it is obvious that if a bit is to rotate, a moment, or torque, about the longitudinal or rotational axis of the bit must be applied. Referring to figure 3, it can be seen that such a moment results when an unbalanced or net lateral force is present. To simplify
illustration of the concept, the lateral force, $\mathrm{F}_{\mathrm{X}}$, is assumed to be applied at a single point on the tip of the bit. An unbalanced lateral force is generally a result of "asymmetrical" employment of the cutting tool such as nonzero bit skew or interactive cutting (in which the breakout zones of neighboring cuts overlap). The frictional force resulting


FIGURE 2. - Top view of bit showing ( $A$ ) negative skew and ( $B$ ) positive skew.
from the bit sliding against the rock is given by $\mu \mathrm{FX}$, where $\mu$ is the coefficient of friction between rock and steel. This force is directed opposite to the direction of cut. The component of this force acting at right angles to the longitudinal axis of the bit is given by $\mu \mathrm{F}_{\mathrm{X}}$ cos $\theta$, when $\theta$ is the angle of attack. The torque ( $T$ ) or moment about the axis of rotation is thus given by $T=R \mu F_{X} \cos \theta$.

The quantity R is the moment arm or the perpendicular distance in inches from the axis of rotation to the line of action of the force $\mu \mathrm{F}_{\mathrm{X}} \cos \theta$.

It follows from the above analysis that the direction of rotation is dependent on the direction of the net lateral force. Thus, as shown in figure $3 A$, a positive lateral force (directed to the left) produces a negative (counterclockwise) rotation.

The above equation suggests that cutting with high lateral force and low angles of attack will insure rotation; however, the frictional binding that occurs due to high moments at the rock-bit and bit-holder interfaces resists rotation. For rotation to occur, it is necessary for the moment ( $T$ ) to overcome frictional binding, and it was the primary objective of this study to identify which modes of


FIGURE 3. - Generation of rotational moment. $A$, Top view of bit showing application of lateral force; $B$, side view of bitshowing frictional force and its component perpendicular to axis of rotation; $C$, end view of bit showing moment about rotational axis.
cutting favor the generation of a rotational moment while maintaining reasonable cutting and normal forces and minimal binding.

It is anticipated that future work will include an analysis of the mechanics of bit rotation to establish the relationships between rotational moment, binding friction, and bit forces for various bit mounting configurations. Fast Fourier transforms of the bit force data will be studied to determine if any correlation exists between the frequency and amplitude of bit force fluctuation and rotation.

## EXPERIMENTAL DESIGN

The experimental test plan was divided into two phases. First, a full factorial experiment was run using bit type, attack angle, skew angle, cut depth, and spacing as the independent variables. Each of the five variables was run at two levels (table 1). The two bits used are shown in figure 4. For each test, the amount of rotation per foot of cut and the lateral, cutting, and normal forces were measured. Standard analysis of variance (ANOVA) techniques were used to determine


FIGURE 4. - Bit types tested. Both bits are approximately 5 in long.
the significance of the five independent variables. The second phase of the test plan was to develop a central composite design, dropping any variables not showing any significant effects. The central composite design allows quantitative analysis of the relationships between the variables. A central composite design is composed of a factorial portion and an axial portion. Bit type did not affect any of the four dependent variables and was not included in the central composite design. The arrangement of the remaining four independent variables is shown in table 2. A more detafled explanation of both the factorial and the central composite experiments is found in appendix A.

TABLE 1. - Values of independent variables for factorial experiment

| Bit <br> type | At tack <br> angle, <br> deg | Skew <br> angle, <br> deg | Cut <br> depth, <br> in | Spacing, <br> in |
| :--- | :---: | :---: | :---: | :---: |
| A | 35 | -10 | 0.25 | 0.50 |
| B | 45 | +10 | .50 | 1.00 |

TABLE 2. - Values of independent variables for central composite experiment

| Attack <br> angle, <br> deg | Skew <br> angle, deg | Cut <br> depth, in | Spacing, <br> in |
| :---: | :---: | :---: | :---: |
| FACTORIAL PORTION ${ }^{1}$ |  |  |  |
| 35 | -5 | 0.25 | 0.50 |
| 45 | +5 | .50 | 1.00 |
| AXIAL PORTION |  |  |  |
| 40 | 0 | 0.375 | 0.75 |
| 30 | 0 | .375 | .75 |
| 50 | 0 | .375 | .75 |
| 40 | -10 | .375 | .75 |
| 40 | +10 | .375 | .75 |
| 40 | 0 | .125 | .75 |
| 40 | 0 | .625 | .75 |
| 40 | 0 | .375 | .25 |
| 40 | 0 | .375 | 1.25 |

${ }^{\prime}$ All 16 possible combinations were run.

The modified Rockford model SA vertical slotter that was used to perfcrm the cutting tests is shown in figure 5, Cutting speed is variable between 5 and 20 in/s with a maximum force capability of 11.000 lbf. The lateral motion of the worktable is synchronized with the vertical movement of the cutter ram so that crossfeed of a preset distance occurs between cuts. The crossfeed increment (dis.tance between cuts) can be adjusted from $1 / 8$ in to 2 in. The sample holder: into which the three-axis force dynamometer is
incorporated, was designed specifically to hold the $1-f t$ cube rock samples used for these tests. The worktable is infed toward the cutter to obtain the desired depth of cut. The cutter ram, after engagement, cycles up and down vertically, producing a linear cut in the sample with each downstroke, with the bit retracting on the upstroke. A complete set of tool hclders was designed and fabricated in. house to provide the several attack and skew angles required for the tests

## iviensurerient sistem

A schematic representation of the measurement system is shown in figure 6.

Bit rotation was measured with a multiturn, low-torque rotary potentiometer, mechanically connected by a flexible coupling, to a $1 / 4 \cdots$ in shaft extending from the rear of the bit as shown in figure 7. Potentiometer excitation was supplied by a dual-voltage ( $\pm 18 \mathrm{~V}$ dc) power
supply to provide zero output when the porentiometer was at the center of its mechanical range of rotation. Signal polarity then indicated whether net rotation was clockwise or counterclockwise. Because of the relatively high impedance of the rotary potentiometer (10,000 ohms), a noninverting voltage follower operational-amplifier circuit was used as


FIGURE 5. - Vertical slotter and sample holder,


FIGURE 6. - Schematic diagram of measurement system.


FIGURE 7. - Bit, holder, and rotary potentiometer.
a buffer between the potentiometer and the multichannel magnetic tape recorder used to record rotation and bit force signals. A digital voltmeter was used to monitor the rotation signal while the test was in progress.

Bit forces were measured in three mutually perpendicular directions. The sample holder was coupled to the worktable of the cutter machine through four threeaxis piezoelectric load cells. The four outputs for each axis were connected in
parallel to the input of a charge amplifier. A multichanne1 FM magnetic tape recorder was used to record the analog signals from the rotation and force measurement systems. Periodically, as the testing progressed, the tape was played into a four-channel strip chart recorder to produce a hard copy of the data for visual inspection and analysis. Additional processing and analysis were accomplished by the cutting laboratory computer system.

## SAMPLE MATERIAL

The rock samples used for these cutting tests were cut from the Berea Sandstone Formation near Cleveland, OH. This rock is abrasive ( 78 pct quartz) with a compressive strength of $4,200 \mathrm{psi}$ and is

## EXPERIMENTAL PROCEDURE

A tool holder and new bit are installed using the attack angle, skew angle, and bit type required for the particular test. The rock sample is placed in the sample holder and oriented so that the bedding planes of the rock are parallel to the direction of cutting but perpendicular to the $Z$ axis (normal force direction). After the sample is securely clamped in place, a series of shallow interactive conditioning cuts are made on the smooth saw-cut surface. This trues up the cutting surface of the sample so it is paralle1 with the direction of crossfeed and also provides a rougher (more natural) test surface. The table is infed to produce the desired cut depth and moved laterally so that the first cut will occur at the extreme edge of the sample. The cross-feed increment control is then adjusted for the desired spacing between cuts, and the bit is rotated in its holder until a zero output is obtained from the multiturn rotary potentiometer. A voice announcement of the test
typical of inclusive rock encountered in coal cutting; such rock is the primary agent of bit wear rather than the coal, which causes minimal wear.
number and test conditions is placed on one channel of the tape recorder at playback speed (3-3/4 in/s). After the recorder is brought up to data recording speed ( $15 \mathrm{in} / \mathrm{s}$ ), the cutter ram is engaged and a series of equally spaced vertical cuts are made as the sample is automatically cross-fed in step with the cutter ram. After each test is completed, samples of the data on the tape are viewed on a storage oscilloscope to verify that no malfunctions occurred. A test is defined as the series of cuts resulting from one pass across the face of the sample. Although the wear resulting from one test is slight, a new bit is used for each test. The number of cuts in the test depends on the spacing required between cuts. The cutting speed on all tests was limited to $12 \mathrm{in} / \mathrm{s}$ since faster speeds would not allow sufficient time for sample crossfeed. Figure 8 shows a test sample following a test. The test results are given in tables $A-1$ and A-2 (appendix A).

## DISCUSSION OF RESULTS

## FACTORIAL EXPERIMENT

Bit type was the only independent variable that did not show any significant effect on any of the dependent variables.

The other four affected at least two of four dependent variables measured. Tables 3-6 show the ANOVA results for rotation, cutting force, normal force, and lateral force. The two factors affecting


FIGURE 8. - Test sample after test.
rotation were attack angle and skew angle. As can be seen in table 7, using a $35^{\circ}$ attack angle and a $-10^{\circ}$ skew angle produced an average rotation of $8.4^{\circ}$, compared with an average of $2.1^{\circ}$ for a $45^{\circ}$ attack angle and a $+10^{\circ}$ skew angle. Examination of the data in tables $\mathrm{A}-1$ and $A-2$ confirms that the direction of rotation is determined by the direction of the lateral force. Both cutting force and normal force were affected by attack angle, skew angle, cut depth, and spacing. As expected, both forces increased
with increasing cut depth and increasing spacing. Table 7 shows how these forces were affected by attack angle and skew angle. The force values at the $35^{\circ}$ attack angle and $-10^{\circ}$ skew angle combina-tion (maximum rotation) are slightly lower than those at the $45^{\circ}$ attack angle and $+10^{\circ}$ skew angle combination (minimum rotation). Lateral force was affected by attack angle, skew angle, cut depth, and spacing. Lateral force increased with increasing cut depth and decreased with increasing spacing.

TABLE 3. - ANOVA results for rotation

|  | Sum of <br> squares | Degrees of <br> freedom | Mean <br> square | F-value |
| :--- | ---: | :---: | ---: | ---: |
| Bit type...... | 18.907 | 1 | 18.907 | 0.02 |
| Attack angle... | 144.554 | 1 | 144.554 | 18.14 |
| Skew angle..... | 352.669 | 1 | 352.669 | 119.85 |
| Cut depth...... | .637 | 1 | .637 | .04 |
| Spacing........ | 1.817 | 1 | 1.817 | .10 |
| Error......... | $1,225.722$ | 69 | 17.764 | NAp |

${ }^{{ }^{N} \text { SAp Not applicable. }}$.

TABLE 4. - ANOVA results for cutting force

|  | Sum of squares | Degrees of freedom | Mean square | F-value |
| :---: | :---: | :---: | :---: | :---: |
| Bit type................... | 52.0 | 1 | 52.0 | 0.005 |
| Attack angle............... | 1,185,036.0 | 1 | 1,185,036.0 | ${ }^{1} 107.6$ |
| Skew angle................ | 882,428.0 | 1 | 882,428.0 | 180.1 |
| Cut depth................... | 2,166,604.0 | 1 | 2,166,604.0 | ${ }^{1} 196.6$ |
| Spacing.................... | 173,400.0 | 1 | 173,400.0 | ${ }^{1} 15.7$ |
| Error...................... | 760,228.0 | 69 | 11,017.8 | NAp |

NAp Not applicable.
'Significant at the $99-$ pet level of confidence.
TABLE 5. - ANOVA results for normal force

|  | Sum of <br> squares | Degrees of <br> freedom | Mean <br> square | F-value |
| :--- | ---: | :---: | :---: | ---: |
| Bit type.......... | $1,952.0$ | 1 | $1,952.0$ | 0.1 |
| Attack angle..... | $2,072,992.0$ | 1 | $2,072,992.0$ | 155.8 |
| Skew angle...... | $1,395,632.0$ | 1 | $1,395,632.0$ | 104.9 |
| Cut depth........ | $2,247,568.0$ | 1 | $2,247,568.0$ | 168.9 |
| Spacing.......... | $134,024.0$ | 1 | $134,024.0$ | 10.1 |
| Error........... | $918,120.0$ | 69 | $13,306.1$ | NAp |

NAp Not applicable.
${ }^{1}$ Significant at the $95-$ pct level of confidence.
TABLE 6. - ANOVA results for lateral force

|  | Sum of squares | Degrees of freedom | Mean square | F-value |
| :---: | :---: | :---: | :---: | :---: |
| Bit type.. | 14,876.0 | 1 | 14,876.0 | 2.0 |
| Attack angle...... | 28,394.0 | 1 | 28,394.0 | ${ }^{1} 3.8$ |
| Skew angle........ | 829,746.0 | 1 | 829,746.0 | ${ }^{2} 112.1$ |
| Cut depth......... | 1,406,261.0 | 1 | 1,406,261.0 | ${ }^{2} 190.1$ |
| Spacing........... | 206,370.0 | 1 | 206,370.0 | 227.9 |
| Error...... | 510,439.0 | 69 | 7,397.7 | NAp |

NAp Not applicable.
${ }^{\prime}$ Significant at the 95 -pct level of confidence.
${ }^{2}$ Significant at the 99-pct level of confidence.
TABLE 7. - Factorial experiment results

| Skew angle................... | Attack angle |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $35^{\circ}$ |  | $45^{\circ}$ |  |
|  | $-10^{\circ}$ | $+10^{\circ}$ | $-10^{\circ}$ | $+10^{\circ}$ |
| Rotation'.............deg.. | 8.4 | 5.9 | 7.2 | 2.1 |
| Force, lbf: |  |  |  |  |
| Cutting.................... | 588 | 368 | 782 | 618 |
| Normal...................... | 672 | 392 | 927 | 725 |
| Lateral... | 363 | 152 | 372 | 211 |

${ }^{1}$ Per $12-\mathrm{in}$ cut (absolute value).
NOTE.--The values presented are averaged over all levels of spacing and cut depth.

## CENTRAL COMPOSITE EXPERIMENT

The central composite design allows a determination of the functional relationship between the variables of interest. The form of that relationship is

$$
\begin{aligned}
Y=A_{o} & +\sum_{i=1}^{4}\left(A_{i} X_{i}+A_{i j} X_{i}{ }^{2}\right) \\
& +\sum_{i=1}^{3} \sum_{j=i+1}^{4} A_{i j} X_{i} X_{j}
\end{aligned}
$$

where $\quad Y=$ dependent variable (i.e., rotation),
$X_{i}=$ independent variables (i.e., attack angle),
and $\quad A_{i}, A_{i}$, and $A_{i j}=$ coefficients.
Computational procedures required the coding of the independent variables. The $X_{1}$ 's in the above equation are the coded values. Table 8 gives the relationship betwen the $X_{i}$ 's and the four independent variables. A sample calculation using the model is given in appendix $B$.

Results for the central composite design are given in tables 9-12. With the exception of lateral force, the model presented above gave a reasonable fit to

TABLE 8. - Coding values for $X_{i}$ 's

| X $_{\text {I }}$ | Attack <br> ang1e, <br> deg | Skew <br> ang1e, <br> deg | Cut <br> depth, <br> in | Spacing, <br> in |
| :---: | :---: | :---: | :---: | :---: |
| -2 | 30 | -10 | 0.125 | 0.25 |
| -1 | 35 | -5 | .25 | .50 |
| 0 | 40 | 0 | .375 | .75 |
| 1 | 45 | 5 | .50 | 1.00 |
| 2 | 50 | 10 | .625 | 1.25 |

the data. The coefficients for rotation, cutting force, and normal force are given in table 13.

As previously stated, the central com.posite model can be used to analyze the relationships between the independent variables. Table 14 shows an example of this type of analysis. In the example, attack angle and cut depth are held constant at $35^{\circ}$ and 0.5 in, respectively. As shown in the table, with the skew angle at $-10^{\circ}$, increasing the spacing causes an increase in rotation. However, with the skew angle at $+10^{\circ}$ increasing spacing results in a decrease in rotation. This brief analysis illustrates the complex nature of the interactions between the variables.

It should be noted that the model is not valid outside the limits of the factorial portion of the experiment (table $2)$.

TABLE 9. - ANOVA for rotation - central composite design

|  | Sum of squares | $\begin{aligned} & \text { Degrees } \\ & \text { of } \\ & \text { freedom } \\ & \hline \end{aligned}$ | Mean square | F-value |
| :---: | :---: | :---: | :---: | :---: |
| lst-order term | 125.30 | 4 | 31.33 | ${ }^{1} 20.06$ |
| 2d-order terms | 129.20 | 10 | 12.92 | 28.27 |
| Lack of fit | 76.21 | 10 | 7.62 | 34.88 |
| Blocks | 50.71 | 2 | 25.36 | ${ }^{1} 16.24$ |
| Error................... | 4.68 | 3 | 1.56 | NAp |

NAp Not applicable.
${ }^{1}$ Significant at the $95-$ pct level of confidence.
${ }^{2}$ Significant at the $90-$ pct level of confidence.
${ }^{3}$ Not significant at the $90-$ pct level of confidence.

TABLE 10. - ANOVA for cutting force - central composite design

|  | Sum of squares | $\begin{aligned} & \text { Degrees of } \\ & \text { freedom } \end{aligned}$ | Mean square | F -value |
| :---: | :---: | :---: | :---: | :---: |
| 1st-order terms..................... | 591,857.0 | 4 | 147,964.3 | 147.14 |
| 2d-order terms........................ | 161,835.2 | 10 | 16,183.5 | 25.16 |
| Lack of fit.. | 67,432.8 | 10 | 6,743.3 | ${ }^{3} 2.15$ |
| Blocks.................................. | 545.0 | 2 | 272.5 | 3.09 |
| Error................................. | 9,417.0 | 3 | 3,139.0 | NAp |

NAp Not applicable.
${ }^{1}$ Significant at the $95-$ pct level of confidence.
${ }^{2}$ Significant at the 90 pct level of confidence.
${ }^{3}$ Not significant at the 75 -pct level of confidence.
TABLE 11. - ANOVA for normal force - central composite design

|  | Sum of <br> squares | Degrees <br> of <br> freedom | Mean <br> square | F-value |
| :--- | ---: | :---: | :---: | ---: |
| lst-order terms.... | $611,186.4$ | 4 | $152,796.6$ | 140.76 |
| 2d-order terms..... | $247,837.6$ | 10 | $24,783.8$ | 26.61 |
| Lack of fit........ | $114,929.8$ | 10 | $11,493.0$ | 33.07 |
| Blocks............ | 216.8 | 2 | 108.4 | 4.03 |
| Error............. | $11,245.5$ | 3 | $3,748.5$ | NAp |

NAp Not applicable.
${ }^{1}$ Significant at the $95-$ pct level of confidence.
${ }^{2}$ Significant at the $90 \sim$ pct level of confidence.
${ }^{3}$ Not significant at the $90-\mathrm{pct}$ level of confidence.
${ }^{4}$ Not significant at the 75 -pct level of confidence.
TABLE 12. - ANOVA for lateral force - central composite design

|  | Sum of <br> squares | Degrees <br> of <br> freedom | Mean <br> square | F-value |
| :--- | ---: | :---: | :---: | :---: |
| lst-order terms..... | $695,153.4$ | 4 | $173,788.4$ | 12.04 |
| 2 d-order terms..... | $369,304.9$ | 10 | $36,930.5$ | 1.43 |
| Lack of fit........ | $130,523.7$ | 10 | $13,052.4$ | 1.15 |
| Blocks........... | $14,625.8$ | 2 | $7,312.9$ | 1.9 |
| Error............. | $255,565.5$ | 3 | $85,188.5$ | NAp |

NAp Not applicable.
${ }^{1}$ Not significant at the $75-\mathrm{pct}$ level of confidence.

TABLE 13. - Coefficients from central composite design analysis

| Coeff | Rotation | Cutting <br> force | Normal <br> force | Coeff | Rotation | Cutting <br> force | Normal <br> force |
| :--- | ---: | ---: | ---: | :---: | :---: | ---: | ---: |
| $\mathrm{A}_{0}$ | 1.9167 | 429.3333 | 397.1667 | $\mathrm{~A}_{44}$ | 0.2229 | -6.2292 | -10.3438 |
| $\mathrm{~A}_{1}$ | -1.3833 | 90.4167 | 118.1250 | $\mathrm{~A}_{12}$ | 1.3750 | 36.6250 | 39.1875 |
| $\mathrm{~A}_{2}$ | -1.7250 | -28.4167 | -37.8750 | $\mathrm{~A}_{13}$ | -.0625 | -38.5000 | -39.6875 |
| $\mathrm{~A}_{3}$ | .0333 | 124.5000 | 100.3750 | $\mathrm{~A}_{14}$ | -.6500 | 7.0000 | 2.9375 |
| $\mathrm{~A}_{4}$ | .5750 | 13.3333 | -1.7083 | $\mathrm{~A}_{23}$ | .9625 | -13.7500 | -24.3125 |
| $\mathrm{~A}_{11}$ | .2604 | 40.0208 | 54.2813 | $\mathrm{~A}_{24}$ | -1.7500 | 55.0000 | 64.0625 |
| $\mathrm{~A}_{22}$ | .6229 | 1.5208 | 6.4063 | $\mathrm{~A}_{34}$ | -1.0125 | -29.8750 | -41.8125 |
| $\mathrm{~A}_{33}$ | .1729 | -4.4792 | -7.3438 |  |  |  |  |

${ }^{1}$ In the subscripts to the coefficients, $1=$ attack angle, $2=$ skew angle, 3
$=$ depth, and 4 = spacing.
TABLE 14. - Predicted values for rotation

| Skew angle...... | Rotation, deg/ft of cut |  |
| :--- | :---: | :---: |
|  | $-10^{\circ}$ | $+10^{\circ}$ |
| 1.0 in spacing.. | 8.8 | 4.1 |

## SUMMARY

The results of this work show that rotation of conical bits is chiefly affected by the attack angle and the skew angle of the bit. A $35^{\circ}$ attack angle produced more rotation than a $45^{\circ}$ attack angle in both the factorial and the central composite experiments. Similarly, a negative skew angle produced more rotation than a positive skew angle. The central composite experiment also indicated that maximum rotation is obtained
with a 0.25 -in cut depth and 1.00 in spacing when the attack angle and skew angle are set at their most efficient levels. The direction of rotation depends on the sign or direction of the net lateral force. The cutting and normal forces for the maximum rotation conditions are 301 and 294 lb , respectively. Maximum rotation is obtained at reasonable levels of cutting and normal force.

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## APPENDIX A

For the factorial segment of the experiment, a full $2^{5}$ factorial design with block confounding was used. Block confounding was necessary because all 32 combinations could not be run on a single sandstone block. The factorial was divided into four blocks using the bit $\times$ attack angle $x$ skew angle, the bit $x$ spacing $x$ depth, and the attack angle $\times$ skew angle $\times$ spacing $\times$ depth interactions as confounded effects. Both the order of the eight tests on each block and the order of each block were randomized.

The central composite design also had to be run in different blocks. The factorial portion was run on two blocks using the attack angle $\times$ skew angle $\times$ spacing $x$ depth interaction as the confounded effect. The axial portion was run on a third block. Two center points were run on each of the three blocks.

The data for both the full factorial and the central composite designs are shown in tables $\mathrm{A}-1$ and $\mathrm{A}-2$.

TABLE A-1. - Test results for factorial experiment

| ATTACK | SKEW | CUT | SPACIMG, | BLOLK | RGTATION, | AVERGIE FORCE, LE |  |  | RMS FORCE, LE |  |  | FEAT FGPEE LE |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ANGLE. DEG | AMGLE. DEG | $\begin{gathered} \text { DEFTH. } \\ \text { IN } \\ \hline \end{gathered}$ | IN | No, | DEG | Lateral | CUTTIHG | NORMAL | LATERAL | CUTT IHG | MORMAL | LATERAL | CUTTINE | HGRTMAL |
| - BIT TYPE A |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 35 | $-10$ | 0.25 | 0.50 | 1 | 13.9 | -349 | 574 | 678 | 399 | 643 | 754 | $-913$ | 1420 | 1646 |
|  |  |  |  |  | 2.3 | -172 | 336 | 375 | 228 | 402 | 476 | -710 | 10.42 | 12 Cl |
|  |  |  |  |  | 4.0 | -284 | 487 | 604 | 336 | 551 | 660 | -811 | 1224 | 1378 |
|  |  |  | 1.00 | 3 | 17,4 | -195 | 492 | 586 | 266 | 611 | 725 | -820 | 1740 | 1859 |
|  |  |  |  |  | 22.6 | -160 | 390 | 403 | 229 | 478 | 508 | -573 | 1294 | 1411 |
|  |  |  |  |  | 12.5 | -190 | 375 | 453 | 286 | 463 | 555 | -351 | 1263 | 1476 |
|  |  | 0.50 | 0.50 | 3 | 3.5 | -688 | 900 | 1069 | 732 | 959 | 1116 | $-1405$ | 15411 | 1924 |
|  |  |  |  |  | 2.5 | -528 | 811 | 748 | 586 | 894 | 813 | - 125 | 2046 | 1685 |
|  |  |  |  |  | 4.3 | -446 | 612 | 769 | 489 | 693 | 829 | $-1050$ | 1573 | 1632 |
|  |  |  | 1.00 | 1 | 1.2 | -658 | 984 | 1151 | 712 | 1084 | 1225 | $-1406$ | 2ers | 237 |
|  |  |  |  |  | 3.0 | -368 | 601 | 548 | 377 | 715 | 641 | -929 | 1875 | 1525 |
|  |  |  |  |  | 4.5 | -441 | 685 | 348 | 504 | 783 | 906 | $-1237$ | 1992 | 1586 |
|  | $+10$ | 0.25 | 0.50 | 2 | $-5.5$ | 96 | 162 | 252 | 151 | 243 | 316 | 483 | 804 | 98\% |
|  |  |  |  |  | $-4.8$ | 102 | 233 | 236 | 172 | 303 | 299 | 596 | 598 | 873 |
|  |  |  |  |  | -4.1 | 96 | 248 | 280 | 180 | 323 | 353 | 393 | 1091 | 1115 |
|  |  |  | 1.00 | 4 | 0.6 | $-53$ | 380 | 444 | 145 | 535 | 574 | -318 | 1702 | 1675 |
|  |  |  |  |  | 2.8 | -7 | 274 | 247 | 125 | 374 | 352 | -31 | 1291 | 1205 |
|  |  |  |  |  | 3.2 | -46 | 209 | 242 | 151 | 311 | 340 | -159 | 1255 | 1208 |
|  |  | 0.50 | 0.50 | 4 | $-19.5$ | 305 | 440 | 537 | 391 | 540 | 597 | 932 | 144 | 1257 |
|  |  |  |  |  | $-6.3$ | 279 | 419 | 338 | 363 | 521 | 415 | 1020 | 155 | 1144 |
|  |  |  |  |  | $-10.5$ | 235 | 332 | 414 | 335 | 452 | 546 | 1044 | 146 | 1329 |
|  |  |  | 1.00 | 2 | -8.3 | 163 | 394 | 543 | 268 | 583 | 635 | 895 | 1985 | 1752 |
|  |  |  |  |  | -4.0 | 251 | 592 | 498 | 355 | 795 | 630 | 1032 | 2473 | 2051 |
|  |  |  |  |  | $-5.7$ | 190 | 912 | 808 | 499 | 1123 | 949 | 671 | 3001 | 2484 |
| 45 | $-10$ | 0.25 | 0.50 | 2 | 0.0 | -164 | 497 | 700 | 206 | 567 | 787 | $-59 ?$ | 1396 | 1796 |
|  |  |  |  |  | 4.1 | $-150$ | 447 | 531 | 206 | 535 | 636 | -6.73 | 1426 | 1651 |
|  |  |  |  |  | 16.5 | -374 | 825 | 940 | 409 | 889 | 998 | -901 | 1804 | 1967 |
|  |  |  | 1.00 | 4 | 7.8 | $-183$ | 762 | 908 | 290 | 875 | 1009 | -859 | 1977 | 2542 |
|  |  |  |  |  | 6.3 | $-168$ | 442 | 477 | 224 | 517 | 552 | $-633$ | 1459 | 1253 |
|  |  |  |  |  | 0.0 | -161 | 416 | 535 | 220 | 495 | 626 | -70e | 1451 | 16.71 |
|  |  | 0.50 | 0.50 | 4 | 12.0 | -601 | 1019 | 1243 | 639 | 1085 | 1297 | -1322 | 2057 | 2133 |
|  |  |  |  |  | 13.4 | -189 | 301 | 321 | 215 | 331 | 351 | - 518 | 498 | 7 F 5 |
|  |  |  |  |  | 9.0 | -449 | 679 | 823 | 451 | 768 | 684 | -1058 | 1767 | 1770 |
|  |  |  | 1.00 | 2 | 8.6 | -428 | 872 | 1188 | 468 | 1000 | 1278 | -1052 | 2570 | 2657 |
|  |  |  |  |  | 14.8 | $-402$ | 1119 | 1183 | 502 | 1271 | 1298 | -1407 | 3101 | 2748 |
|  |  |  |  |  | 15.0 | $-784$ | 1350 | 1410 | 844 | 1478 | 1521 | -1659 | 3206 | 2971 |
|  | +10 | 0.25 | 0.50 | 1 | $-1.1$ | 157 | 383 | 498 | 254 | 486 | 614 | 515 | 1402 | 1722 |
|  |  |  |  |  | $-1.2$ | 34 | 229 | 247 | 176 | 306 | 345 | 755 | 1117 | 1267 |
|  |  |  |  |  | 0.0 | 151 | 317 | 362 | 235 | 402 | 454 | 724 | 1334 | 1372 |
|  |  |  | 1.00 | 3 | 2.4 | -1 | 869 | 1075 | 240 | 1012 | 1204 | -462 | 2047 | 2045 |
|  |  |  |  |  | 1.0 | 19 | 507 | 581 | 159 | 504 | 685 | 81 | 1723 | 1909 |
|  |  |  |  |  | -2.1 | -34 | 415 | 534 | 214 | 508 | 621 | $-376$ | 1452 | 1554 |
|  |  | 0.50 | 0.50 | 3 | $-4.9$ | 556 | 936 | 1114 | 652 | 1040 | 1216 | 1516 | 2259 | 2423 |
|  |  |  |  |  | $-3.2$ | 420 | 723 | 735 | 570 | 841 | 899 | 1503 | 2021 | 2361 |
|  |  |  |  |  | $-3.9$ | 305 | 592 | 685 | 424 | 723 | 791 | 1123 | 1931 | 1965 |
|  |  |  | 1.00 | 1 | $-4.3$ | 253 | 1232 | 1531 | 505 | 1428 | 1688 | 1032 | 34185 | 3458 |
|  |  |  |  |  | $-1.6$ | 184 | 726 | 765 | 290 | 842 | 882 | 8アE | 2225 | 217 |
|  |  |  |  |  | 0.0 | 118 | 725 | 924 | 336 | 905 | 1109 | 776 | 2654 | 2325 |

TABLE A－1．－Test results for factorial experiment－－Continued

| ATTACK | SKEld | CUT | SPACING， | BLOCK | ROTATION， | AYERAGE FORCE，LE |  |  | RUS FORCE，LE |  |  | FEAK FORCE，LP |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| GNGLE， DEG | ANGLE， DEG | $\begin{gathered} \text { UEPTH. } \\ \text { IHj } \end{gathered}$ | IN | No． | DEG | LATERAL | CUTTING | NOPMAL | LATEFAL | CUTTINİ | NÖRMAL | LATEFAL | EUTTING | NOFFそく |
| EIT TYFE B |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 35 | $-10$ | 0.25 | 0.50 | 4 | 9.6 | －330 | 514 | 633 | 363 | 568 | 687 | －880 | 1236 | 1478 |
|  |  |  |  |  | 16．3 | －227 | 428 | 442 | 283 | 483 | 516 | －792 | 1109 | 1215 |
|  |  |  |  |  | 17.3 | －227 | 355 | 447 | 269 | 414 | 516 | －711 | 1056 | 1238 |
|  |  |  | 1．00 | 2. | 13.8 | －191 | 322 | 452 | 236 | 398 | 532 | －625 | 1084 | 135.3 |
|  |  |  |  |  | 10.8 | －125 | 375 | 377 | 200 | 449 | 449 | －7こ1 | 1233 | 11.54 |
|  |  |  |  |  | 19.2 | －201 | 496 | 568 | 292 | 603 | 682 | －917 | 1662 | 1815 |
|  |  | 0.50 | 0.50 | 2 | 1.3 | －437 | 459 | 732 | 473 | 534 | 781 | －9．94 | 1305 | 1479 |
|  |  |  |  |  | 5，0 | －490 | 730 | 754 | 557 | 827 | 842 | －1277 | 1875 | 1793 |
|  |  |  |  |  | 0.0 | －822 | 1105 | 1292 | 860 | 1165 | 1343 | －15．54 | 2123 | 2275 |
|  |  |  | 1.00 | 4 | 2.2 | －568． | 890 | 1025 | 650 | 1027 | 1118 | $-1333$ | 2375 | こここも |
|  |  |  |  |  | 8.3 | －288 | 552 | 514 | 357 | 662 | 613 | $-1043$ | 1829 | 1513 |
|  |  |  |  |  | 5.5 | －381 | 629 | 563 | 461 | 775 | 751 | $-1154$ | 2175 | 1730 |
|  | $+10$ | 0.25 | 0.50 | 3 | －4．7 | 165 | 310 | 360 | 240 | 392 | 437 | 732 | 1156 | 112 y |
|  |  |  |  |  | －3．9 | 78 | 207 | 212 | 165 | 288 | 305 | 45.5 | 954 | 1098 |
|  |  |  |  |  | －2．3 | 73 | 179 | 221 | 151 | 252 | 282 | 427 | 885 | 854 |
|  |  |  | 1．00 | 1 | 7．0 | －1：7 | 257 | 361 | 181 | 385 | 494 | －588 | 1317 | 15シ6． |
|  |  |  |  |  | －2．2 | 42 | 218 | 199 | 127 | 290 | 273 | 454 | 98.5 | 951 |
|  |  |  |  |  | 2， 5 | －17 | 219 | 258 | 153 | 315 | 329 | －269 | 1206 | 1007 |
|  |  | 0.50 | 0.50 | 1 | $-5.6$ | 469 | 581 | 659 | 565 | 579 | 728 | 1360 | 1514 | 1623 |
|  |  |  |  |  | －7．6 | 284 | 412 | 351 | 366 | 524 | 443 | 1015 | 1581 | 1215 |
|  |  |  |  |  | －6， 5 | 330 | 444 | 519 | 425 | 554 | 596 | 1145 | 1611 | 1415 |
|  |  |  | $\bigcirc .00$ | 3 | －4．0 | 54 | 558 | 634 | 352 | 726 | 734 | 358 | 2122 | 1907 |
|  |  |  |  |  | －4，7 | 143 | 432 | 345 | 251 | 615 | 475 | 775 | 2423 | 1511 |
|  |  |  |  |  | －6．4 | 48 | 40.3 | 45.3 | 252 | 504 | 555 | 250 | 2086 | $17 \% 4$ |
|  | $-10$ | 0.25 | 0,50 | 3 | 3.3 | －407 | 883 | 1115 | 456 | 940 | 1175 | －991 | 1782 | 2008 |
| 45 |  |  |  |  | 7.3 | －276 | 590 | 656 | 324 | 657 | 729 | －744 | 1351 | 1523 |
|  |  |  |  |  | 2.9 | －216 | 427 | 579 | 267 | 500 | 691 | －65E | 1259 |  |
|  |  |  | 1.00 | 1 | 1.2 | －184 | 747 | 964 | 272 | 838 | 1051 | －802 | 1345 | 2045 |
|  |  |  |  |  | 1.9 | －229 | 590 | 659 | 289 | 559 | 730 | －777 | 1563 | 15.3 .3 |
|  |  |  |  |  | 2.3 | －156 | 676 | 879 | 263 | 762 | 955 | －854 | 1915 | 1961 |
|  |  | 0.50 | 0.50 | 1 | 6．0 | －697 | 1123 | 1456 | 736 | 1183 | 1511 | －1475 | こ128 | 2331 |
|  |  |  |  |  | 3.1 | －409 | 775 | 834 | 4.59 | 851 | 907 | －994 | 1775 | 1732 |
|  |  |  |  |  | 8.5 | －585 | 96.3 | 1182 | 634 | 1032 | 1238 | $-1200$ | 2113 | 2130 |
|  |  |  | 1．00 | 3 | 18．1 | －718 | 1337 | 1545 | 772 | 1437 | 1545 | －1501 | 2840 | 2856 |
|  |  |  |  |  | 7.6 | －493 | 1026 | 1029 | 568 | 1157 | 1127 | －1304 | 2979 | 2302 |
|  |  |  |  |  | 8.2 | －508 | 894 | 1093 | 565 | 993 | 1156 | $-114.3$ | 2518 | 2145 |
|  | $+10$ | 0.25 | 0.50 | 4 | 0.0 | 174 | 575 | 776 | 254 | 652 | 850 | 832 | 1519 | 1810 |
|  |  |  |  |  | 0.0 | 115 | 353 | 413 | 195 | 420 | 512 | 75. | 1201 | 1508 |
|  |  |  |  |  | －1．5 | 150 | 316 | 425 | 247 | 408 | 520 | 797 | 1139 | 1422 |
|  |  |  | 1.00 | 2 | 0.0 | 53 | 484 | 634 | 163 | 581 | 729 | 395 | 1591 | 1806 |
|  |  |  |  |  | 1.0 | －25 | 524 | $54 \%$ | 215 | 645 | 664 | －576． | 1373 | 1713 |
|  |  |  |  |  | 3.7 | －191 | 725 | 887 | 324 | 829 | 982 | －981 | 1933 | 1399 |
|  |  | 0.50 | 0.50 | 2 | －2．4 | 203 | 403 | 694 | 30.5 | 499 | 765 | 1023 | 1384 | 1777 |
|  |  |  |  |  | $-7.6$ | 385 | 545 | 572 | 486 | 781 | 677 | 1195 | 2211 | 1675 |
|  |  |  |  |  | －3．1 | 671 | 941 | 983 | 737 | 1031 | 1056 | 1349 | 2406 | ご192 |
|  |  |  | 1.00 | 4 | －2．2 | 425 | 1098 | 1210 | 550 | 1242 | 1308 | 1519 | 30017 | 2732 |
|  |  |  |  |  | －1．4 | 215 | 573 | 514 | 328 | 727 | 621 | 1068 | 24139 | 1667 |
|  |  |  |  |  | －2．1 | 180 | 542 | 694 | 338 | 728 | 832 | 855 | 2563 | 2282 |

TABLE A-2. - Test results for central composite design

| ATTACK ANGLE, DEG | SKEld f ANGLE, DEG | $\begin{aligned} & \text { CUT } \\ & \text { DEFTH. } \\ & \text { IN. } \end{aligned}$ | $\begin{aligned} & \text { SPACING, } \\ & \text { IN } \end{aligned}$ | $\begin{aligned} & \text { BLÕCK } \\ & \text { HO. } \end{aligned}$ | $\begin{gathered} \text { ROTATION, } \\ \text { DEG } \end{gathered}$ | AVERAIE FORCE, LB |  |  | RMS FORCE, LB |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | LATERAL | CUTTING | NÜRMAL | LATERAL | EUTTING | NüRMAL | LATEFAL | CUTTINA | HOETHEL |
| 35 | -5 | 0. 25 | 0.50 | 1 | 6.0 | -140 | 262 | 268 | 189 | 315 | 333 | -574 | 8013 | 533 |
|  |  |  | 1. 00 | 2 | 16.6 | -127 | 301 | 294 | 208 | 371 | 417 | -733 | 1071 | 1391 |
|  |  | 0.50 | 0.50 |  | 6.2 | -373 | 548 | 598 | 418 | 719 | 667 | -94i3 | 1564 | 1379 |
|  |  |  | 1. 00 | 1 | 11.6 | -269 | 596 | 550 | 336 | 678 | 630 | -9.34 | 1526 | 1422 |
|  | +5 | 0.25 | 0.50 | 2 | -. 5 | 56 | 139 | 121 | 105 | 185 | 169 | 354 | 754 | 587 |
|  |  |  | 1. ${ }^{\text {un }}$ | 1 | 2.1 | -5 | 221 | 191 | 102 | 291 | 257 | -259 |  | Syi |
|  |  | 0.50 | 0.50 |  | -7.7 | 336 | 503 | 446 | 389 | 576 | 508 | 593 | 137 | 1117 |
|  |  |  | 1. 00 | 2 | -. 8 | 221 | 453 | 367 | 290 | 569 | 457 | 840 | $1: 559$ | 1352 |
| 45 | -5 | 0.25 | 0.50 |  | 2.5 | -189 | 480 | 514 | 245 | 549 | 626 | -727 | 1352 | 1544 |
|  |  |  | 1.00 | 1 | 4.6 | -152 | 351 | 351 | 188 | 400 | 415 | -489 | 1090 | 1029 |
|  |  | 0.50 | 0.50 |  | 1.7 | -303 | 726 | 789 | 340 | 791 | 840 | -74こ | 1725 | 163E |
|  |  |  | 1.00 | 2 | 3.1 | -261 | 503 | 454 | 308 | 588 | 522 | -7.26 | $15 \cdot 1$ | $11 \overline{8}$ |
|  | +5 | 0.25 | 0.50 | 1 | -. 9 | 115 | 312 | 322 | 174 | 364 | 396 | 559 | 984 | 1123 |
|  |  |  | 1.00 | 2 | , 2 | 91 | 634 | 716 | 161 | 725 | 803 | 608 | 1213 | 1555 |
|  |  | 0.50 | 0.50 |  | -3.0 | 323 | 524 | 496 | 367 | 576 | 559 | 772 | 1.34 .3 | 1251 |
|  |  |  | 1.00 | 1 | -. 5 | 278 | 685 | 616 | 357 | 805 | 712 | 914 | 2901 | 1551 |
| 30 | 0 | 0.38 | 0.75 | 3 | . 4 | 155 | 323 | 255 | 220 | 417 | 335 | 218 | 1310 | 1 ¢12 |
| 50 |  |  |  |  | 1.3 | 214 | 862 | 961 | 336 | 960 | 1077 | 977 | 2252 | 2311 |
| 40 | -10 |  |  |  | 3.5 | -240 | 510 | 508 | 292 | 572 | 564 | -705 | $1 \geq 26$ | 1204 |
|  | $+10$ |  |  |  | 1.1 | 168 | 367 | 325 | 258 | 479 | 452 | 570 | 1523 | 1474 |
|  | 0 | 0.13 |  |  | $-.6$ | -23 | 152 | 144 | 68 | 193 | 189 | -1013 | 509 | 624 |
|  |  | 0.63 |  |  | . 4 | -373 | 677 | 579 | 427 | 761 | 629 | -962 | 1945 | 129 ? |
|  |  | 0.38 | 0.25 |  | 0.0 | 241 | 365 | 356 | 279 | 406 | 398 | 553 | 275 | 554 |
|  |  |  | $1.25$ |  | -1.4 | 142 | 450 | 343 | 239 | 585 | 462 | 798 | 1973 | 1457 |
|  |  |  | 0.75 | 1 | 1.2 | -224 | 477 | 442 | 271 | 542 | 496 | -540 | 1375 | 1115 |
|  |  |  |  |  | -3.8 | 215 | 413 | 377 | 271 | 487 | 453 | 771 | 13.3 | 1148 |
|  |  |  |  | 2 | -2.8 | 170 | 365 | 338 | 226 | 425 | 415 | 584 | 1253 | 1155 |
|  |  |  |  |  | 1.3 | -227 | 438 | 409 | 276 | 504 | 465 | -701 | 1405 | 1189 |
|  |  |  |  | 3 | -. 9 | -236 | 490 | 466 | 273 | 545 | 510 | -607 | 1462 | 1158 |
|  |  |  |  |  | -1.5 | 165 | 393 | 351 | 222 | 470 | 428 | 515 | 1325 | 1223 |

## APPENDIX B

The procedure to follow in using the empirical model on page 11 to predict rotation, normal force, or cutting force for a given attack angle, skew angle, cut depth, and spacing is illustrated by the
following example: Given attack angle $=30^{\circ}$, skew angle $=-5^{\circ}$, depth $=0.5 \mathrm{in}$, and spacing $=0.75 \mathrm{in}$, find rotation. First the condensed form of the equation

$$
Y=A_{0}+\sum_{i=1}^{4}\left(A_{i} X_{i}+A_{i} X_{1}{ }^{2}\right)+\sum_{i=1}^{3} \sum_{j=i+1}^{4} A_{i j} X_{i} X_{j}
$$

should be expanded to the form:

$$
\begin{aligned}
Y= & A_{0}+A_{1} X_{1}+A_{11} X_{1}{ }^{2}+A_{2} X_{2}+A_{22} X_{2}{ }^{2}+A_{3} X_{3}+A_{33} X_{3}{ }^{2}+A_{4} X_{4}+A_{44} X_{4}{ }^{2} \\
& +A_{12} X_{1} X_{2}+A_{13} X_{1} X_{3}+A_{14} X_{1} X_{4}+A_{23} X_{2} X_{3}+A_{24} X_{2} X_{4}+A_{34} X_{3} X_{4}
\end{aligned}
$$

where the subscripted A's are the coefficients from table 13 and $X_{1}, X_{2}, X_{3}$, and $X_{4}$ are the independent variables attack angle, skew ang1e, depth, and spacing respectively.

Next, using table 8, determine the coded values corresponding to each independent variable $X_{1}$. Thus for an attack
angle of $30^{\circ}$, a skew angle of $-5^{\circ}$, a cutting depth of 0.5 in , and a spacing of 0.75 in, the values of $X_{1}, X_{2}, X_{3}$, and $X_{4}$ are $-2,-1,1$, and 0 respectively. Substitution of the coded values for the independent variables and the coefficients from the rotation column of table 13 into the expanded form of the equation yields

$$
\begin{aligned}
\mathrm{Y}= & 1.92+(-1.38)(-2)+(0.26)(.2)^{2}+(-1.72)(-1)+(0.62)(-1)^{2}+(0.03)(1) \\
& +(0.17)(1)^{2}+(0.58)(0)+(0.22)(0)^{2}+(1.38)(-2)(-1)+(-0.06)(-2)(1) \\
& +(-0.65)(-2)(0)+(0.96)(-1)(1)+(-1.75)(-1)(0)+(-1.01)(1)(0) \\
& \text { or } Y=10.18 \text { deg/ft. }
\end{aligned}
$$


[^0]:    ${ }^{1}$ Physicist.
    ${ }^{2}$ Physical scientist.
    ${ }^{3}$ Supervisory physical scientist.
    $4_{\text {Mining }}$ engineering technician.
    Twin Cities Research Center, Bureau of Mines, Minneapolis, mN.

[^1]:    ${ }^{5}$ Underlined numbers in parentheses refer to items in the list of references preceding the appendixes.

