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Rotational Failure Mechanisms Using Multiple Friction Circles for Stability Analysis

By Douglas A. Crum

UNITED STATES DEPARTMENT OF THE INTERIOR



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UNIT OF MEASURE ABBREVIATIONS USED IN THIS REPORT			
deg	degree	MHz	megahertz
kN/m ³	kilonewton per cubic meter	min	minute
m	meter	pct	percent

ROTATIONAL FAILURE MECHANISMS USING MULTIPLE FRICTION CIRCLES FOR STABILITY ANALYSIS

By Douglas A. Crum¹

ABSTRACT

The Taylor's friction circle method for slope stability analysis can be extended to incorporate multiple circles defining the slip surface in a failure mechanism so that it is a workable method for bearing capacity calculations of foundations and rotational failures of retaining walls and similar structures. The U.S. Bureau of Mines has further extended the multiple-circle method to include friction in the material strength and to utilize computational schemes such that this method can be programmed with optimization capabilities rather than being used as a graphical or semigraphical method. Some examples are given that demonstrate ultimate bearing capacity calculations of embedded foundations with eccentric loads and nearby slopes.

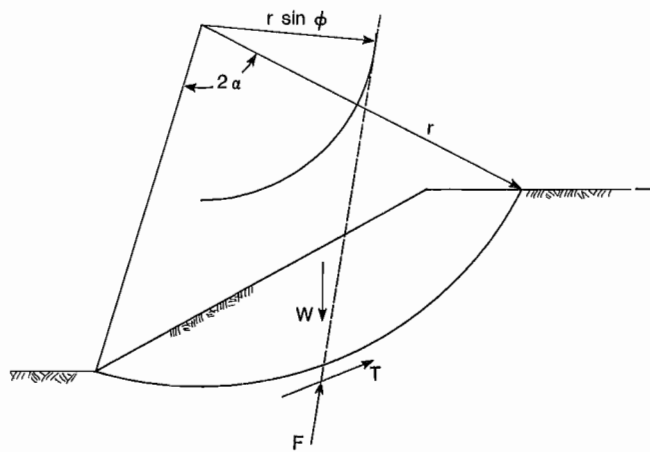
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INTRODUCTION

As part of its program to enhance occupational health and safety in open pit mines, the U.S. Bureau of Mines conducted this research as part of its investigation of dump-point accidents. Through full-scale model tests and analysis by conventional methods, the dump-point problem (of a haul truck near the crest of an embankment) was found to be influenced by three-dimensional effects (1)² and eccentric loads (2). The friction circle method addresses the latter. This research was pursued in an attempt to establish an easy method to determine the effect of eccentric loads (or a load coupled with a moment) on bearing capacities of structures near slopes.

Taylor's friction circle method derives its name from D. W. Taylor who first used the method to produce slope stability charts (3), and from the circles that are associated with the graphical method. The friction circle method has been used primarily to determine the stability of slopes. The method considers a circular failure surface in the two-dimensional plane (fig. 1), which can be viewed as a cross section of an infinitely long cylinder in three dimensions.

²Italic numbers in parentheses refer to items in the list of references preceding the appendixes at the end of this report.



KEY

- α Angle of half the arc defined by slip circle
- φ Internal angle of friction of the soil
- r Radius of slip circle
- F Resultant of normal force and frictional resistance
- T Tangential force due to cohesion
- W Weight of soil wedge

Figure 1.—Circular failure surface in friction circle method.

The advantage of the friction circle method is that it is sensitive to the location of forces. This makes it ideal for investigation of rotational failure mechanisms influenced by eccentrically loaded footings or retaining walls.

The friction circle method has limited applications since it assumes a homogeneous slope such that soil strength parameters are constant along the slip surface. This method is not very adaptable to effective stress analysis or slopes with a phreatic surface; although this method has been demonstrated for completely submerged slopes and slopes experiencing an instantaneous complete drawdown. In this report, the discussion is limited to total stress analysis.

Collapse mechanisms composed of a single circular failure surface (fig. 1) are quite limited in their ability to follow rupture surfaces generated in the collapse of structural components. Gudehus (4) showed that composite failure mechanisms can be constructed of more than one friction circle if the centers of each three intersecting circles lie on a line. This construction of composite failure mechanisms greatly enhances the power of the friction circle method, allowing more complex structures to be analyzed. As two examples, consider the collapse mechanism of a foundation near a retaining wall (fig. 2) or the rotational collapse of the same retaining wall where the soil ruptures and slides along five circular arcs (fig. 3). For these collapse scenarios, a single circle would yield a poor approximation of an expected rupture surface in the field.

Provided that the *hodograph*³ can be constructed, analysis of composite failure mechanisms is relatively easy. However, this is only valid for purely cohesive ($\phi = 0$) soils.⁴ Although Gudehus has demonstrated that the system of forces on the composite failure mechanisms can be resolved by statics, his method is not straightforward. Gudehus introduced simplifications as necessary, such as assuming locations for points of application of forces, neglecting forces, and expressing separate cohesive and frictional forces as a single resultant to reduce the number of unknowns.

In this study, a more general approach is presented that is an expansion of Gudehus' method. Since evaluation of the equations is very time consuming, the method has been incorporated in a computer code OPTIC that will optimize the configuration of the circles and determine the value of cohesion required in a soil for a given limit load. Demonstration of the program is given for eccentrically loaded footings, embedded footings, and bearing capacity calculations.

³Italicized terms are defined in appendix A.

⁴Purely cohesive soils refer to soils with an internal angle of friction equal to zero ($\phi = 0$).

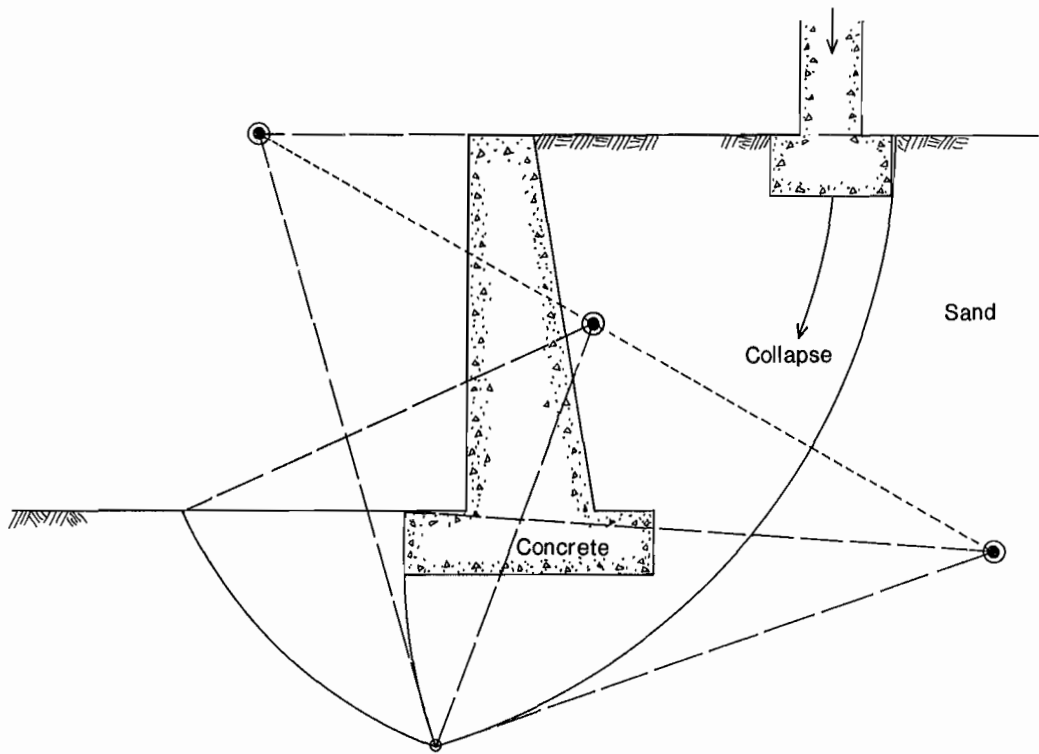


Figure 2.—Three-circle mechanism for foundation collapse.

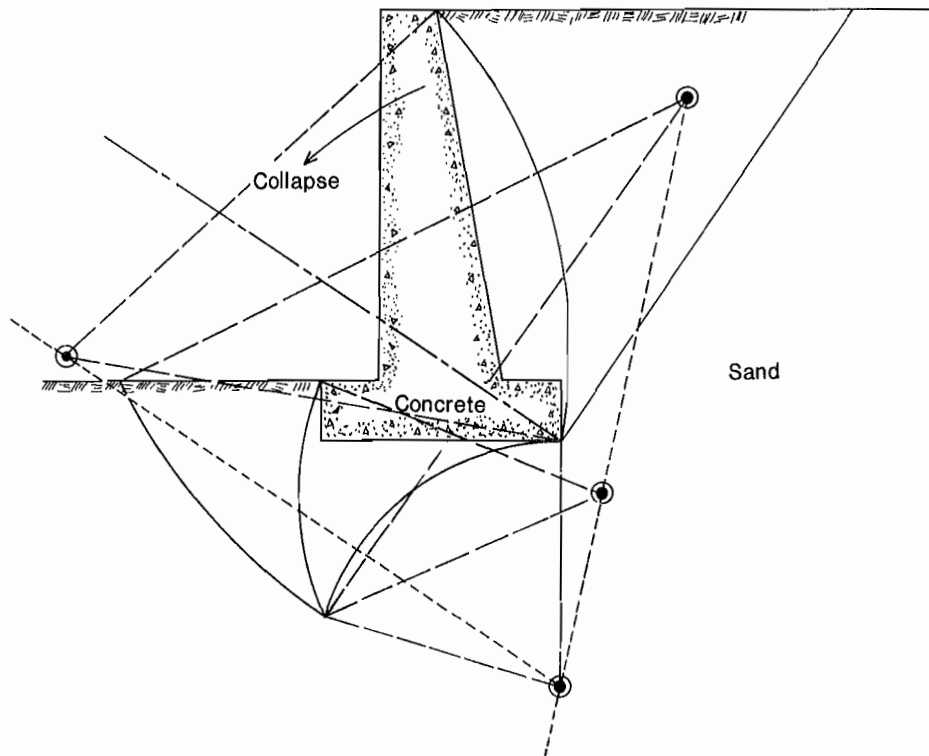


Figure 3.—Five-circle mechanism for active retaining wall.

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GENERAL METHOD AND DERIVATION OF EQUATIONS⁵

The friction circle method relies on the ability to express the stresses in the soil mass as two resultant forces (a cohesive and a frictional force) resisting motion for each circular arc. If these forces can be calculated by static equilibrium, then the stability can be assessed for any soil strength parameters, c and ϕ , where c is the cohesion in the soil. However, if the soil can be approximated as purely cohesive, then assessment of stability can be made by consideration of *kinematics* and resolution of forces is not necessary.

RESULTANT COHESIVE FORCE

Assume that the cohesive shear stress can be described as a function of position along the slip circle by $t(\theta)$, where θ is an angle measured counterclockwise from the center of the slip circle. The orientation and location of the resultant cohesive force can be determined by integrating $t(\theta)$ along the circular arc. But if $t(\theta)$ is not constant, the integral will have to be performed for every trial failure surface. Thus, the friction circle method assumes a homogeneous soil such that the cohesion is constant along the entire length of the slip surface; i.e., $t(\theta) = t$. This creates a constant shearing stress tangent to the slip surface.

An expression for the orientation and location of the resultant cohesive force T , where T is equivalent to the resisting $t(\theta)$ along the circular arc, BC (see figure 4), is obtained by considering equilibrium of forces and moments. Consider a local coordinate system (x', y') with its origin at the circle's center (fig. 4). The resultant cohesive force is assumed to act along the x' axis such that equilibrium of vertical forces is satisfied,

$$\sum F_y = \int_{-\alpha}^{\alpha} t \sin \theta \, d\theta = 0. \quad (1)$$

Since vertical equilibrium is unconditionally satisfied, consideration of only horizontal forces,

$$\sum F_x = T - \int_{-\alpha}^{\alpha} t \cos \theta \, d\theta = 0, \quad (2)$$

and equilibrium of moments (M) about some arbitrary point, A ,

$$\sum M_A = T(R + V) - \int_{-\alpha}^{\alpha} t(r + v \cos \theta - u \sin \theta) \, d\theta = 0, \quad (3)$$

is necessary; where point A is offset from the origin an arbitrary distance (u, v) , R is the radial distance to the cohesion resultant (fig. 4) and angle $\alpha < 90^\circ$. Combining equations 2 and 3 leads to

$$(R + V) \int_{-\alpha}^{\alpha} \cos \theta \, d\theta = \int_{-\alpha}^{\alpha} (r + v \cos \theta - u \sin \theta) \, d\theta. \quad (4)$$

Integration and cancellation of like terms leads to an expression that shows the resultant cohesive force must be moved out from the arc,

$$\boxed{R/r = \alpha/\sin \alpha}, \quad (5)$$

valid for $-90^\circ < \alpha < 90^\circ$. The result simplifies the resistance of cohesion to a force that can be used in the static resolution of the forces and is valid for equilibrium about any point in the plane.

The total effect of the cohesive resistance is related to the magnitude of the resultant, T , by the same ratio:

$$T\alpha/\sin \alpha = \int_{-\alpha}^{\alpha} t(\theta) \, d\theta. \quad (6)$$

⁵Boxed equations indicate equations directly used in program OPTIC. Unboxed equations are included for derivations.

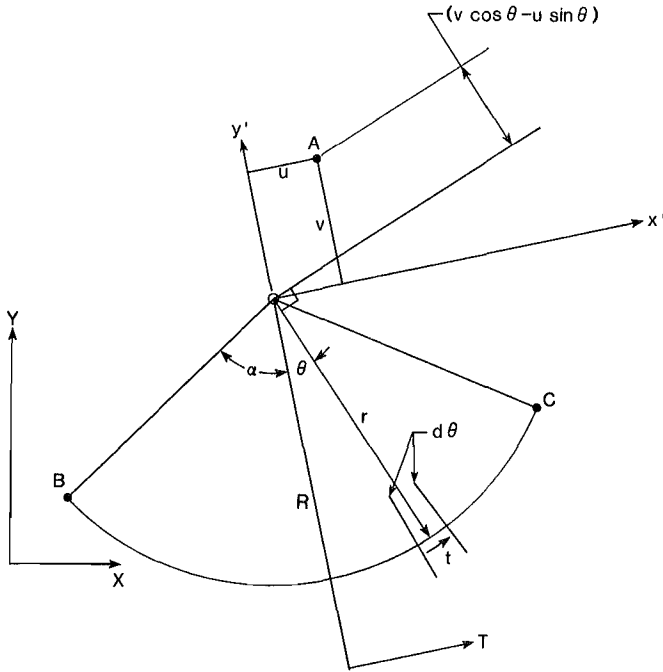


Figure 4.—Resultant cohesive force along circular arc.

RESULTANT FRICTION FORCE

Similar to the cohesive force, the friction force must be expressed as a resultant. The friction force is taken to be the resultant of the normal force and the resistance due to friction. For cohesive materials, the friction force degenerates to a normal to the sliding surface. Assuming a mobilized friction angle along a moving rupture surface, the friction force is inclined to the base at an angle ϕ so that the line of action is everywhere tangent to a smaller circle with radius of $r_\phi = r \sin \phi$. This smaller circle is known as the friction circle.

As with the cohesive force, the friction force will be some distribution $[f(\theta)]$ along the arc. Assuming a local coordinate system oriented with the arc, as shown in figure 5, the horizontal and vertical components of the friction force are

$$\begin{aligned} f_{x'}(\theta) &= f(\theta) \sin(\phi - \theta), \\ f_{y'}(\theta) &= f(\theta) \cos(\phi - \theta). \end{aligned} \quad (7)$$

The horizontal and vertical components of the resultant cohesive force are found by integrating equation 7:

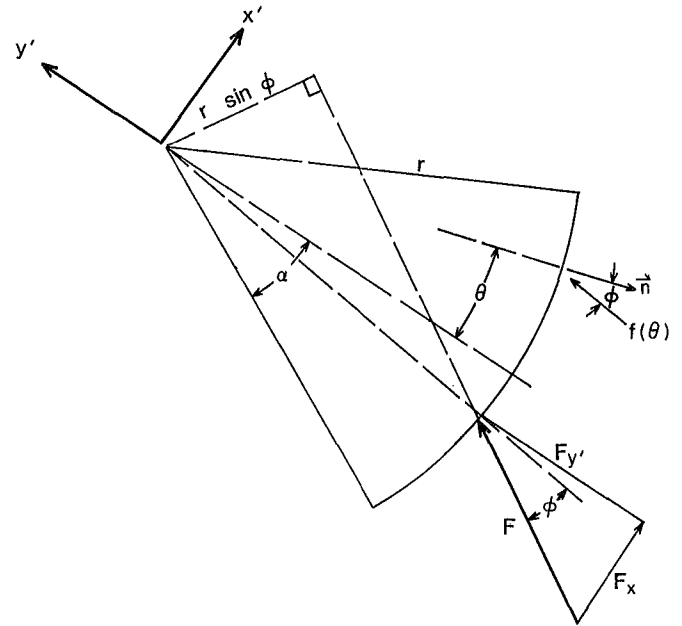


Figure 5.—Resultant friction force on circular arc.

$$F_{x'} = \int_{-\alpha}^{\alpha} f(\theta) \sin(\phi - \theta) r d\theta,$$

$$F_{y'} = \int_{-\alpha}^{\alpha} f(\theta) \cos(\phi - \theta) r d\theta. \quad (8)$$

The moment generated by the force distribution is

$$M = \int_0^{2\alpha} (r \sin \phi) f(\theta) r d\theta. \quad (9)$$

There are four possible methods to solve equations 8 and 9. The first method is to introduce an unknown for the location of the point of action of the friction force, F , on the circular arc. This is undesirable since it would be tedious, messy, and would probably limit the ability to create composite failure mechanisms where the friction forces are oriented in a logical manner. The second method is the special case where the arc approaches a straight line ($\alpha \rightarrow 0$). This is not especially useful since it simply produces rigid-block mechanisms. The third method is the special case of zero friction ($\phi = 0$). This is not useful either since the kinematic approach for $\phi = 0$ can be simply used. The last alternative is to assume $f(\theta)$ can be approximated by the resultant friction force (a single force

F located at θ^* , where $-\alpha \leq \theta^* \leq \alpha$). This can be written mathematically via the Dirac delta function $[\delta(x)]$:

$$F = f(\theta)r\delta(\theta^*). \quad (10)$$

By substitution of equation 10 into equations 8 and 9, the resultant friction force is defined in the local coordinate system,

$$\begin{aligned} F_x &= F \sin(\phi - \theta^*), \\ F_y &= F \cos(\phi - \theta^*), \\ M &= Fr \sin\phi. \end{aligned} \quad (11)$$

Vardoulakis (5) solved equation 9 by making the assumption of a purely frictional soil ($c = 0$), such that equilibrium requires $F = F_y$, where there are no horizontal actuating forces. He then introduced a modifier (k) to the radius of the friction circle such that the moment $Fkr\sin\phi$ is equivalent to equation 9.

In practice, the radius of the slip circle, r , and the location of the friction force, θ^* , will be known so that the only unknown in equation 11 is the value of the resultant, F . Furthermore, since the angle $(\phi - \theta^*)$ is the orientation of the resultant, F , in the local coordinate system, a new angle (η) can be defined that is the orientation of the resultant, F , in the global coordinate system (x,y), which is done in the following section.

RESOLUTION OF FORCES

In the preceding two sections, it was shown how the stress distribution in the soil mass may be approximated by two resultant forces (F and T) on each circular arc. Creating a composite failure mechanism of three circular arcs (fig. 6) produces a failure mass of two soil wedges. Satisfying equilibrium of forces and moments for each soil

wedge requires six equilibrium equations for six unknowns ($F_1, T_1, F_2, T_2, F_3, T_3$). In figure 6, F_1 and T_1 are equilibrium forces; Q, W_L , and W_R are actuating forces; A_3, B_1, D_2, M, K , and Q_1 are strategic points where the subscripts refer to the circular arc they lie on; $\eta_i, \theta_i, \alpha_i$, and $\Delta\alpha_i$ are angles, and r_1 and r_1^ϕ are radii of the slip circle and friction circle, respectively.

In the case of a single friction circle (fig. 1), the system of forces consists of only three: friction force, F , cohesive force, T ; and resultant actuating forces, $Q + W$, where Q is a load applied on the footing. In the case of only three force vectors, it is required that they must pass through a point to satisfy moments. In this case, the problem is simplified since the point of application of the friction force is fixed. In a two-wedge mechanism, there are a minimum of four forces acting on a wedge (fig. 7). In the case where the unit weight of the material is not zero, there will be five or more forces acting on each wedge. The significance of this is that the points of application of the friction forces are not specified. Thus, points Q_i , which define the locations of the friction forces, are said to be located off the center of each arc by an angle $\Delta\alpha_i$.

This introduces an additional three unknowns. The values of $\Delta\alpha_i$ will affect the orientations of the forces so that the force polygon (fig. 7) is not unique for a given configuration of the circle centers. Thus, it is necessary to optimize the angles, $\Delta\alpha_i$, to determine the optimum orientations of the forces.

The matrix of equation 12 was formulated to solve the six remaining unknowns for the case of only vertically applied loads and body forces. Notation is shown in figure 6, where B_i is the solution vector, a_{ij} are the matrix coefficients, e is the load eccentricity, and η_i and θ_i are the orientations of the friction forces, F_i , and the cohesive forces, T_i , respectively. By using equilibrium of forces of the global system and right wedge, and taking moments about point Q_3 and the second circle center,

$$\begin{bmatrix} \sin\eta_3 & \sin\theta_3 & \sin\eta_2 & \sin\theta_2 & 0 & 0 \\ \cos\eta_3 & \cos\theta_3 & \cos\eta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & \sin\eta_2 & \sin\theta_2 & \sin\eta_1 & \sin\theta_1 \\ 0 & 0 & \cos\eta_2 & \cos\theta_2 & \cos\eta_1 & \cos\theta_1 \\ 0 & 0 & 0 & 0 & a_{55} & a_{65} \\ a_{16} & a_{26} & a_{36} & a_{46} & 0 & 0 \end{bmatrix} \begin{Bmatrix} F_3 \\ T_3 \\ F_2 \\ T_2 \\ F_1 \\ T_1 \end{Bmatrix} = \begin{Bmatrix} Q + W_R + W_L \\ 0 \\ Q + W_R \\ 0 \\ B_5 \\ B_6 \end{Bmatrix}, \quad (12)$$

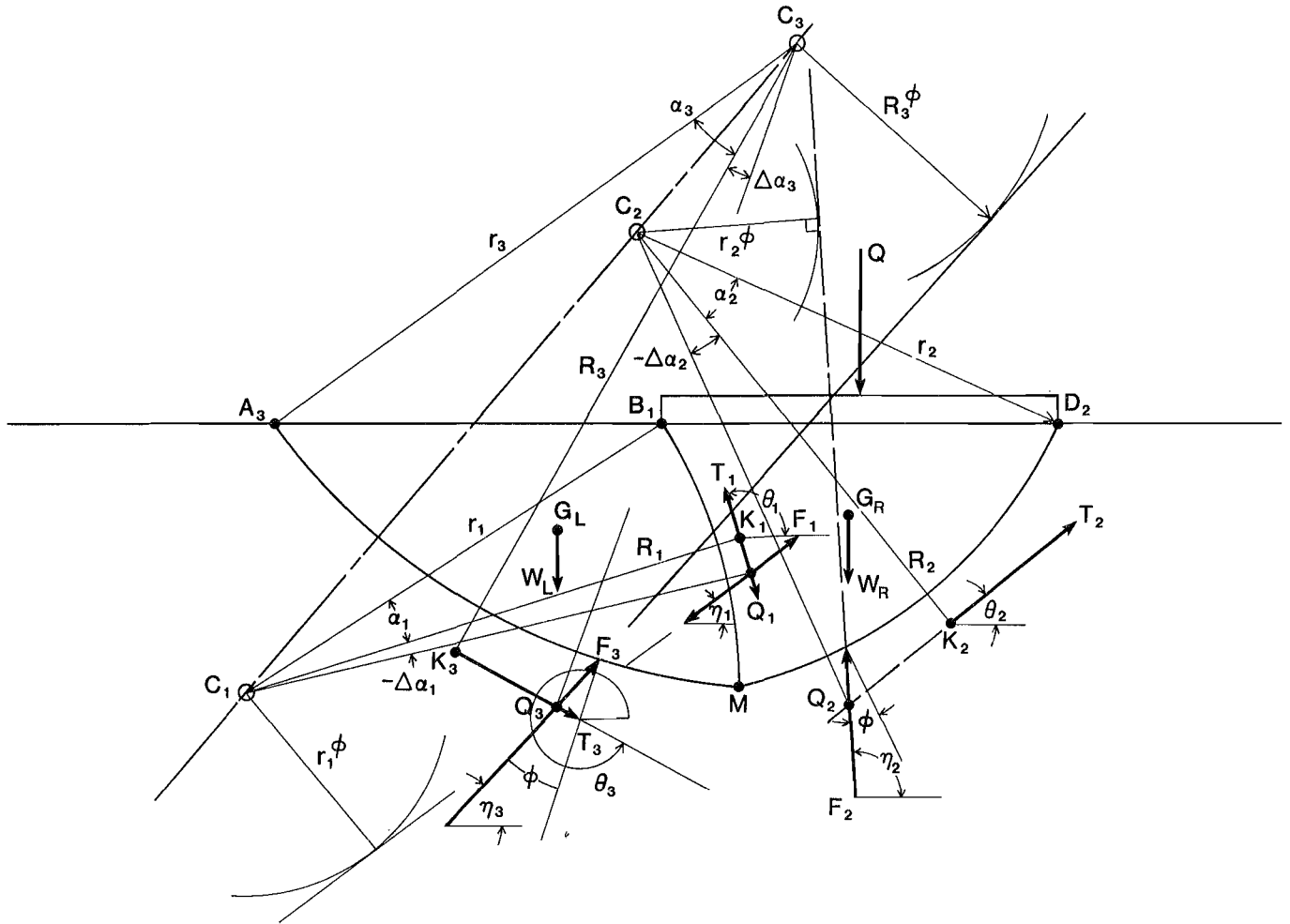


Figure 6.—General system of forces on two-wedge mechanism.

where

$$B_5 = W_L(Q_{3x} - G_{Lx}),$$

$$B_6 = W_R(C_{2x} - G_{Rx}) + Q(C_{2x} - b/2 - e),$$

$$a_{36} = -R_2 \sin \phi,$$

$$a_{46} = -R_2 \alpha_2 / \sin \alpha_2,$$

and the moment arms a_{55} , a_{65} , a_{16} , a_{26} are the orthogonal distances from point C_2 and Q_3 to the lines of action of the forces. The moment arms are positive for forces acting clockwise about the points and negative for forces acting counterclockwise. If the second circular arc is reversed, so it is convex instead of concave, then the parameters a_{36} and a_{46} are positive for forces acting counterclockwise and negative for forces acting clockwise. Points G_L and G_R are the centers of gravity of the left and right soil wedges.

KINEMATICS

For an admissible collapse mechanism, it must be possible to describe the motion that takes place upon collapse. This can be presented in a hodograph.

The assumption of circular sliding surfaces made in the friction circle mechanism is only valid for no friction. Rigid wedges with friction along curved surfaces must have a surface with the shape of a logarithmic spiral to be admissible kinematically. Thus, the hodograph of the friction circle method is rigorously correct for purely cohesive materials, but not for frictional materials ($\phi > 0$). In the case of material with friction, the assumption must be made that the mechanism will slide along circular surfaces in order to apply the friction circle method; so kinematic admissibility is checked via the hodograph regardless of the value of friction.

Construction of the hodograph is based on drawing the velocities so that they are tangent to the sliding surfaces. For the friction circle, this can be accomplished most easily by drawing the velocities perpendicular to the circle radii. When the hodograph is completed, each solid wedge of soil is represented by an identically shaped wedge of arbitrary size rotated 90°.

The rotation of the wedge onto its perspective orientation in the hodograph can be clockwise or counterclockwise and can be determined by observing the relative

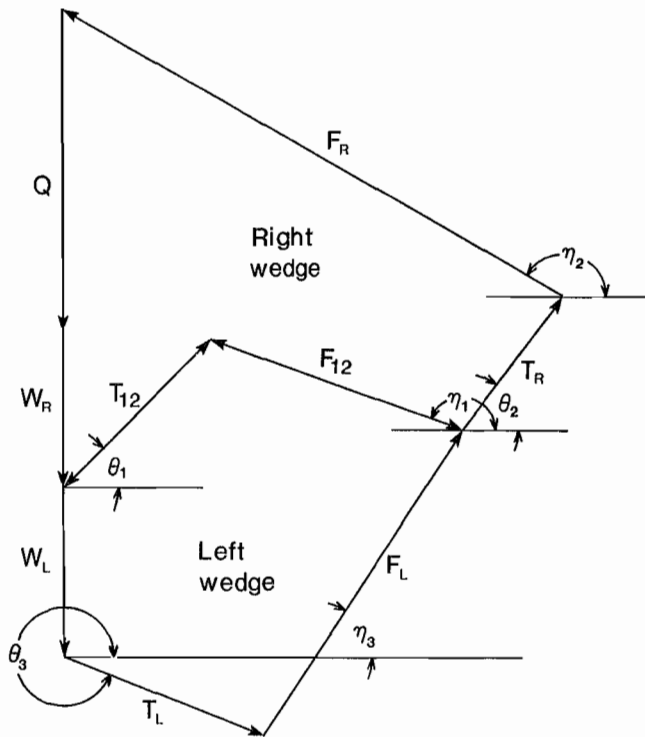


Figure 7.—Force polygon for two-wedge mechanism.

motion of the wedge as it slides. For example, in figure 8A, the general direction of the wedge is shown by the vector V_P , where V is the velocity at the point P . As the block slides along the arc it will rotate clockwise. Thus, the wedge is rotated 90° clockwise on the hodograph (fig. 9A). Figures 8B and 9B show a counterclockwise rotation. The hodograph does not define the actual magnitude of velocities, but only ratios of velocities: (V_P/V_Q) , (V_A/V_Q) , etc.

The construction of the hodograph for a mechanism composed of multiple circles is more complex. Figure 10 shows a hodograph for the Prandtl mechanism. A step-by-step procedure is outlined for the construction of the hodograph for a three-circle mechanism:

1. A point is picked to represent zero velocity of fixed earth (point $C_2^h = C_3^h$ in figure 10). When the hodograph is completed, a vector drawn from the stationary point to some other point, P , will represent the magnitude and orientation of the velocity vector of point P with respect to a stationary point.

2. A line perpendicular to the circle centerline is drawn, passing through the stationary point. Also, rays are drawn for the velocities of each point on the exterior surface (points A_3 , M , and D_2). This is done in figure 10 where rays r_1^\perp , r_2^\perp , r_3^\perp , and r_4^\perp on the hodograph correspond to rays r_1 , r_2 , r_3 , and r_4 on the mechanism. It is assumed the centers of circles 2 and 3 are represented on the hodograph at the stationary point. This is because circles 2 and 3 define exterior surfaces where there is a moving wedge sliding against fixed material.

3. Centers of circles that define interface surfaces between two moving blocks do not have centers at the stationary points. An arbitrary point, C_1^h , on the perpendicular to the circle centerline is selected to represent point C_1 . The location of this point determines the size of the

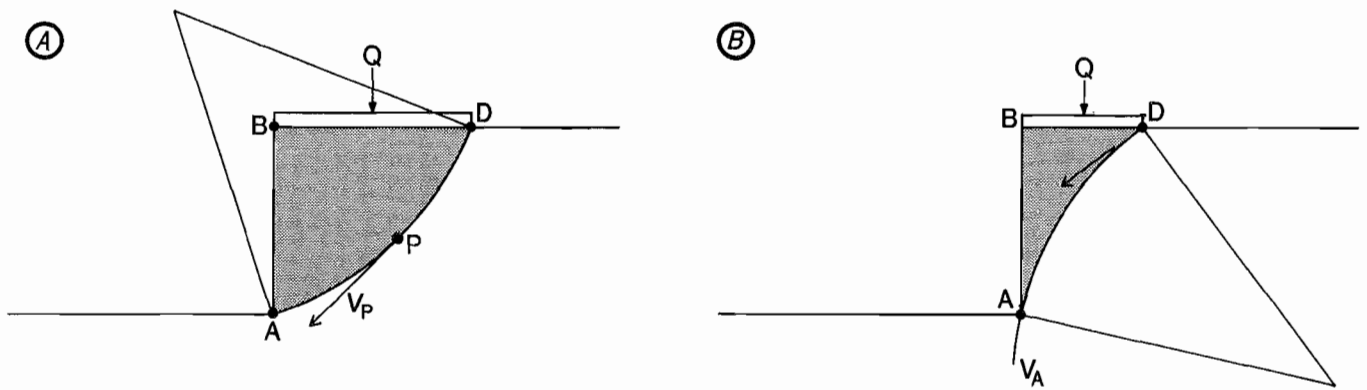


Figure 8.—Rotating wedges. A, Clockwise rotation; B, counterclockwise rotation.

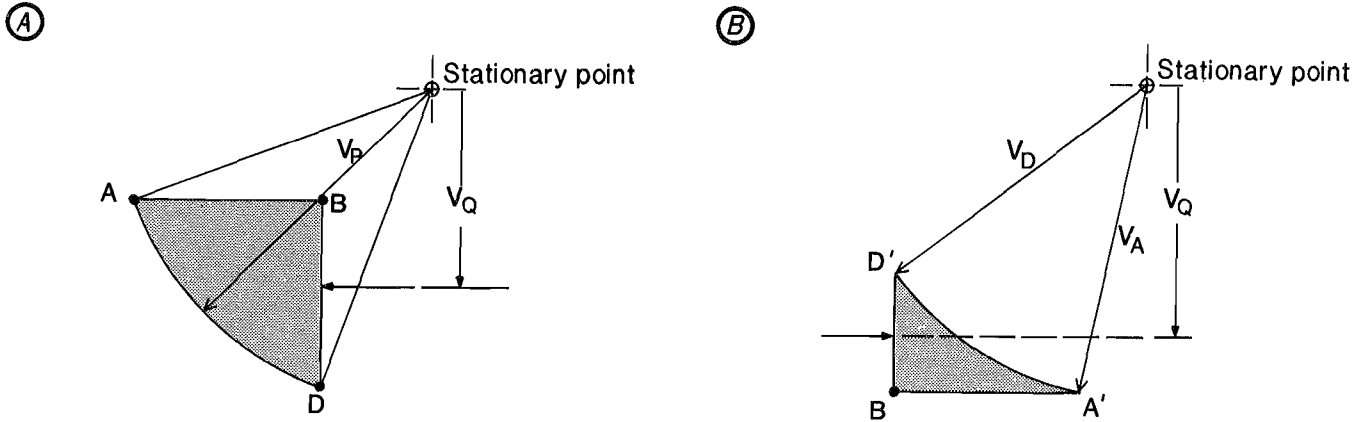


Figure 9.—Hodographs of rotating wedges. A, Clockwise rotation; B, counterclockwise rotation.

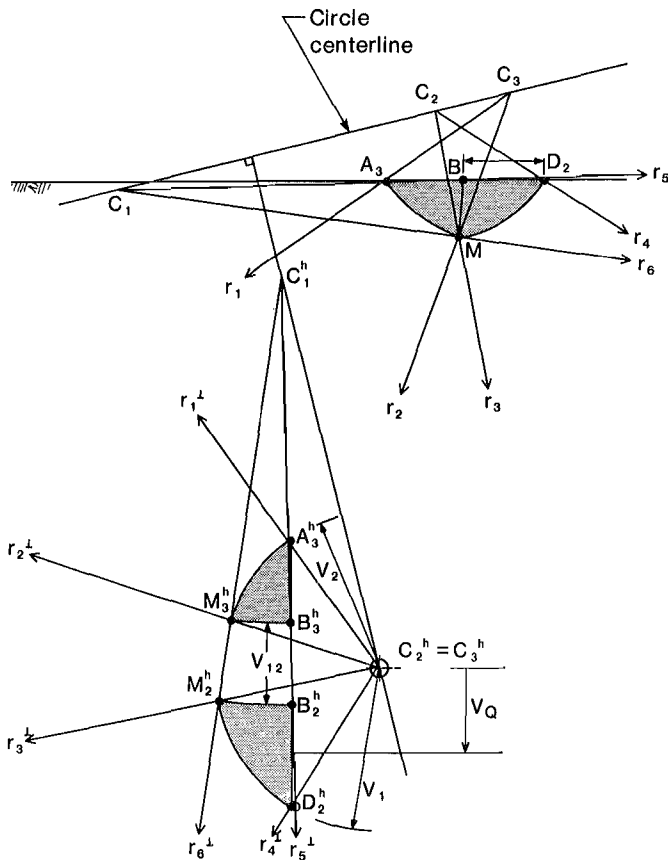


Figure 10.—Construction of hodograph for mechanism composed of three friction circles.

hodograph. Some other starting point could have been picked, but this one is convenient.

4. Rays r_5^\perp and r_6^\perp are drawn (perpendicular to rays r_5 and r_6 , respectively, passing through point C_1^h) to represent relative velocities between the two moving blocks along the interface.

5. Strategic points can now be located on the hodograph. Point M, which must be located first, is at the intersection of rays $r_2 \cap r_6$ on the left wedge and rays $r_3 \cap r_6$ on the right wedge. Points M_2^h and M_3^h on the hodograph are at the corresponding perpendicular rays. Points A_3^h , B_3^h , D_2^h , and B_2^h can be located by rotating about the circle centers from point M^h to the appropriate ray the point lies on. (For example, point D_2 lies on ray r_4 and is connected to point M by an arc rotated about point C_2 . Thus, point D_2^h is located by rotating about point C_2^h from point M_2^h . The direction of rotation should be obvious if the perpendicular rays are drawn correctly.) The rotations define four arcs on the hodograph that represent the three circular surfaces.

6. The remaining segments (A_3B and D_2B) can be connected, because the wedges in the hodograph should have the same shape as the wedges in the mechanism. The hodograph construction may be completed by scaling off relative distances, but in the case of figure 10, the segments are simply straight lines.

Once the hodograph is completed, the relative velocities of the forces can be measured by determining the point of application of the force on the body in the hodograph.

The direction and relative magnitude of the velocity of any point in the moving body can be measured by drawing a vector from the stationary point to the point of interest.

The velocities of all the friction forces are orthogonal to the force vectors for the case of $\phi = 0$, such that no work is done by them on collapse. The velocity vectors of the cohesive forces are the velocities along the arcs. The relative velocity between the two moving wedges, everywhere tangent to the interior surface, and its relative magnitude (V_{12}) is the distance between the two wedges. The case where $V_{12} = 0$ is evident by a single wedge on the hodograph ($M_2 = M_3$, and $B_2 = B_3$) and circle centers of the mechanism C_2 and C_3 , which correspond.

FACTORS OF SAFETY

Since methods for calculating the forces and velocities of the friction circle mechanisms have been developed, expressions for the relative stability of the mechanisms can now be developed. The friction circle method has adopted the factor of safety (FS) based on energies

$$FS = \delta D / \delta A, \quad (13)$$

where δD is the dissipated energy of resisting forces and δA is the actuating energy of existing loads. By using different calculations for the energies (δD and δA), it is possible to formulate several expressions for the FS that are useful.

Friction

The friction circle method assumes the friction force is oriented at the angle of friction to the surface normal (see figures 1 and 5). This angle of friction that is used to represent a collapse mechanism is the *critical* angle of friction at failure (ϕ^*). Since ϕ^* differs from the measured angle of friction (ϕ) and ϕ^* is required to orient the forces, it is necessary to separate FS's for friction and cohesion.

To obtain an FS consistent with equation 13, consider figure 11. The only actuating force is Q . The friction and cohesion resultants (F and T) can be determined, and the available shearing resistance to limit motion (T_L) is found by taking moments about the circle center,

$$T_L r = Fr \sin \phi + TR. \quad (14)$$

Similarly, the equilibrium shearing resistance (T_{eq}) is

$$T_{eq} r = QL. \quad (15)$$

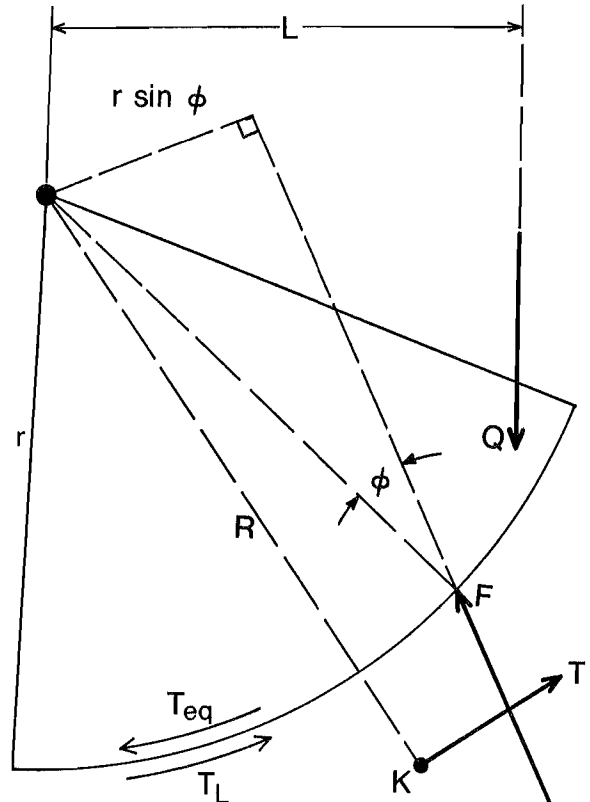


Figure 11.—Shearing resistances.

Since the shearing resistances (T_L and T_{eq}) are defined along the same surface, their displacements and velocities are equal and the FS (equation 13) reduces to a ratio of moments:

$$FS = T_L / T_{eq}. \quad (16)$$

Substituting equations 5, 14, and 15 into 16 leads to

$$FS = \frac{F \sin \phi + T \alpha / \sin \alpha}{Q(L/r)}. \quad (17)$$

Equation 17 is not particularly useful since it cannot be solved explicitly. Since T is a function of F , F is a function of ϕ , and ϕ is a function of FS, equation 17 is implicit. However, the expression must hold as cohesion becomes small ($T \rightarrow 0$). Then, by equilibrium of forces, F must equal Q , be vertical, and be applied to a point on the slip circle directly under Q . In this case, $(L/r) = \sin \phi^*$ and equation 17 reduces to

$$FS_\phi = \sin \phi / \sin \phi^*. \quad (18)$$

Thus, ϕ^* to be used in the analysis can be calculated from ϕ and a desired FS.

Cohesion

Since the factor of safety for friction (FS_ϕ) must be incorporated into the geometry of the problem via equation 11, it is only necessary to solve for the factor of safety for cohesion (FS_c). The shearing resistances for $\phi = 0$ can be written

$$\begin{aligned} T_L &= (2\alpha r)c, \\ T_{eq} r &= TR. \end{aligned} \quad (19)$$

By using the location of the cohesive force resultant (equation 5) and the above expressions for the shearing resistances (equation 19), the FS (equation 16) can be expressed in the following usable form:

$$FS_c = 2cr \sin\alpha / T. \quad (20)$$

This expression is valid for a single circular arc or can be taken as a summation of the numerator and denominator for an average FS for the mechanism.

An alternative to equation 20 is to express δA in terms of the actuating force, Q , instead of the shearing resistance (T_{eq}). Since the velocities of the forces are not equal, they are retained in the expressions for the energies

$$\begin{aligned} \delta D &= (2\alpha rc) \cdot V_c, \\ \delta A &= Q \cdot V_Q. \end{aligned} \quad (21)$$

Substituting equation 21 into the FS (equation 13) leads to

$$FS_c = [2\alpha rc \cdot V_c] / [Q \cdot V_Q]. \quad (22)$$

The advantages of equation 22 are that the system of forces does not need to be solved and that it yields an *upper-bound* kinematic solution as postulated by Drucker (6). For composite failure mechanisms, the numerator is summed over the circular arcs and the denominator summed over each actuating force, which may also include the weights of the soil wedges in addition to the load, Q . The disadvantage of equation 22 is that it is not affected by friction and thus always gives a solution for purely cohesive material.

Multisurface Mechanisms

During optimization of a mechanism, the configuration is adjusted such that the FS is minimized. Selection of a particular expression for the FS as an optimization parameter can have a drastic influence on the optimization path. The FS_c of equation 20 can be written for any individual surface generated by the i 'th circle as

$$FS_c^i = 2cr_i \sin\alpha_i / T_i, \quad (23)$$

or as a summation for the mechanisms as a whole:

$$FS_c^{av} = \frac{2c \sum (r_i \sin\alpha_i)}{\sum T_i}. \quad (24)$$

For frictional materials ($\phi > 0$), the following assumptions were postulated:

Assumption 1: ϕ and c are fully mobilized on all surfaces where movement occurs.

Assumption 2: The FS is taken as a summation over surfaces where movement occurs.

Although the assumptions are straightforward, they have some implications that are not immediately obvious and are necessary to determine treatment of the interior circular arc.

Velocities on exterior surfaces (arcs generated by circles 2 and 3 on figure 6) are never zero except for the trivial case of no failure. Assuming limit load calculation, the FS on exterior surfaces is 1 ($FS_\phi = FS_c = 1$).

Velocity on the interior surface (the arc generated by circle 1 in figure 6) may be zero if the centers of the second and third circles correspond. In this case, the subscript i takes on values 1,2 in equation 24. Since there is no movement, $FS_c^1 \geq 1$ (via equation 20) and $FS_\phi^1 \geq 1$ (via equation 18). A value of $FS_\phi > 1$ means that the radius of the friction circle may be less than critical; i.e., $0 < r_\phi^1 < r^1 \sin\phi^*$. If the velocity between the two soil wedges along the interior surface is not zero, the subscript i takes values 1,2,3.

OPTIMIZATION

Optimization of the average factor of safety for cohesion (FS_c^{av}) resulted in erroneous results characterized by local FS's for individual arcs that varied widely and sometimes

a wide variation in velocities of forces. Optimization of $FS_{c,av}$ involves two somewhat unrelated summations of minimizing the circle surfaces and maximizing the total cohesive forces necessary for equilibrium. Apparently, the maximum value for cohesive forces (ΣT_i) is that where two forces approach zero, forcing the third cohesive force to be large.

Optimization of FS_c via kinetics (equation 22) did not experience such problems. The mechanism consistently converged to an optimum configuration. The stability of the optimization with equation 22 as an optimization parameter is related to the influence of the hodograph.

Because use of equation 24 as an optimization parameter was not sufficient to ensure a stable convergence, a third assumption was made:

Assumption 3: The minimum FS is found where the local FS's are also minimum and equal on each surface where movement occurs.

This assumption guarantees that the maximum amount of shearing resistance is fully utilized on each surface.

Assumption 3 was administered by periodically minimizing the largest local FS (FS_i) and then converging the FS's such that they were equal. Convergence was done by choosing the maximum of $|FS_1 - FS_2|$, $|FS_1 - FS_3|$, and $|FS_2 - FS_3|$ as the optimization parameter to be minimized. Thus, the optimization procedure adjusts the mechanism with respect to two separate parameters.

In some cases, optimization with respect to two separate parameters leads to convergence toward two distinct configurations. This results in an oscillation between two convergence points and results in a solution that is not optimum and may be inadmissible if some local FS is < 1 . Generally, the optimization was found to converge toward one distinct configuration. However, convergence was found to be slower and in some cases not possible as the friction angle was increased. Also, a large load, Q , with respect to soil weight was usually more likely to result in all positive forces (F_i and T_i) for a trial mechanism, so that convergence was faster and more likely to be completed.

The optimization scheme that was utilized in code OPTIC uses a modified term based on the direct search method proposed by Hooke and Jeeves (7). A flowchart of the direct search method is given in figure 12 for optimization of an arbitrary function. The base point and head point refer to the set of variables that define the value of the function for the initial trial mechanism and the latest optimization. Otherwise, the flowcharts are somewhat generalized and self-explanatory.

The direct search method was selected since it does not require derivatives and it moves one parameter at a time

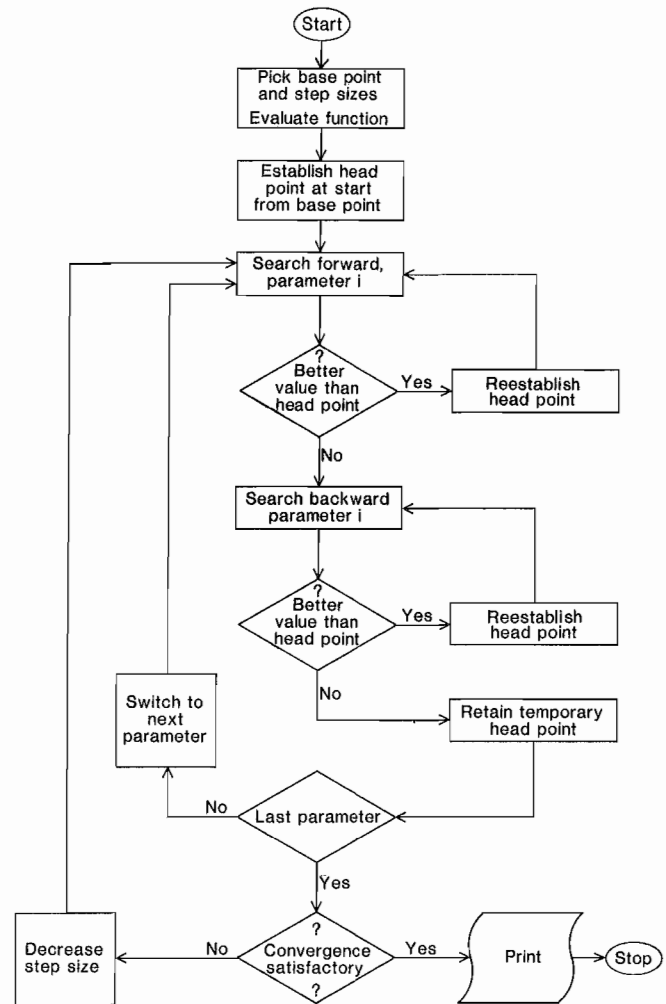


Figure 12.—Flowchart for Hooke and Jeeves method.

in search for an optimum configuration. Although other optimization schemes were not attempted, it is reasoned that a simplistic approach would be least affected by inadmissible configurations. Due to the vast amount of constraints in the friction circle method, the amount of admissible configurations is limited, making it difficult to optimize.

The FS for the friction circle may be expressed as a function:

$$FS = F\{\Delta\alpha_1, \Delta\alpha_2, \Delta\alpha_3, A_{3x}, C_{1x}, C_{1y}, C_{2x}, C_{2y}\}, \quad (25)$$

where the variables $\Delta\alpha_i$ refer to modification factors to the locations of points Q_i and the remaining variables are coordinates of the points, as shown in figure 6. The

optimization may be visualized as finding the minimum value on a surface in nine-dimensional space. This surface for the friction circle method is found to be defined over a finite extent and usually contains numerous local minima. Furthermore, it appears to contain jagged edges and holes where configurations are not admissible.

A flowchart of the modified scheme utilized in code OPTIC (see appendix B) is given in figure 13. In the modified scheme, the locations of the forces, by adjusting the value of $\Delta\alpha_i$, are moved first so that the optimum force polygon is considered for each configuration of circles. Then point A_{3x} is optimized for each movement of points C_1 and C_2 . This is done because only point A_3 is

needed to control the shape of the left wedge once the shape of the right wedge has been determined by points C_1 and C_2 . Thus, in effect, the optimum configuration of the left wedge is considered for each trial configuration of the right wedge.

The method is demonstrated in a computer program OPTIC, which is written in compiled BASIC and is relatively user friendly (see appendix B). Run time of the program varies widely, dependent on the input parameters. However, most solutions are obtained in 2 to 3 min, running on an IBM Corp. compatible personal computer equipped with an Intel 80286 chip and 80287 math coprocessor, both having a clock speed of 16 MHz.

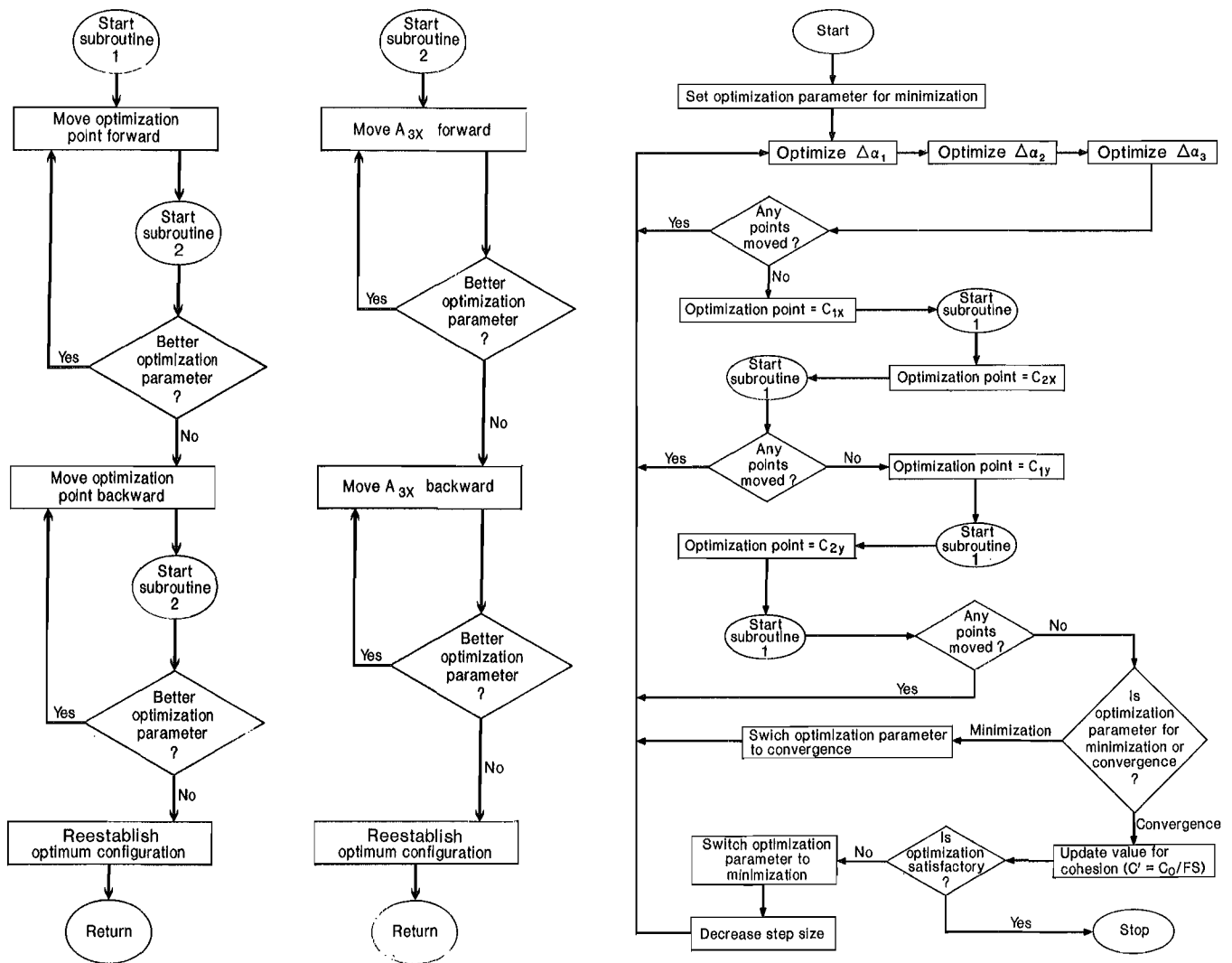


Figure 13.—Flowchart for optimization procedure in code OPTIC.

ECCENTRIC LOADS

Because eccentric loads will tend to cause rotational failure mechanisms, the friction circle method is a useful method for investigating the effect of eccentricities.

The commonly adopted method of treating eccentricities in foundation design is to assume the footing to be a width, $b_o - 2e$, where b_o is the actual width of the footing. As shown in figure 14, the assumed footing remains centrally loaded since a width of $2e$ is assumed not to exist. If $(q/c)^*$ is the nondimensional load capacity of the eccentrically loaded footing over a width, b_o (fig. 14) and $(q/c)_o$ is the nondimensional load capacity of a centrally loaded footing over a width, b_o (not shown), then the method of equivalent footings leads to the expression

$$\boxed{(q/c)^* = (q/c)_o [1 - 2e/b_o]} \quad (26)$$

Similar expressions can be derived from the friction circle method.

SINGLE-CIRCLE MECHANISM WITHOUT UPLIFT

The effect of eccentric loads is considered here for a strip footing that is not allowed to tilt up. The roller (shown in figure 15A) allows the left edge of the footing to translate horizontally, but not vertically. This constraint considered in the hodograph (fig. 15B) requires that the center of the circle must be above the edge of the footing.

Consider a purely cohesive soil so that the friction force, F , passes through the circle center. To satisfy moments, the friction force must also pass through the intersection of the force vectors, Q and T . Then, from the geometry of the mechanism (fig. 15A),

$$\tan \theta = R/(b/2 + e), \quad (27)$$

and from the hodograph (fig. 15B),

$$\sin \alpha = 2 \frac{V_Q}{V_c} \frac{b/2}{b/2 + e}, \quad (28)$$

and from the force polygon (fig. 15C),

$$\tan \theta = Q/T. \quad (29)$$

Expressing the load as a resultant of the distributed load, $Q = qb$, the FS can be obtained by substituting equation 28 in the kinematic expression (equation 22) or

by substituting equations 27 and 29 into the *force equilibrium* expression (equation 20). Either method reduces to

$$\frac{q}{c} = \frac{4\alpha r}{(b + 2e)\sin \alpha}. \quad (30)$$

Making use of the relation $\sin \alpha = b/r$, equation 30 can be expressed in a form analogous to the expression derived by consideration of equivalent footings:

$$\boxed{(q/c)^* = (q/c)_o [1 + 2e/b_o]^{-1}}, \quad (31)$$

where

$$(q/c)_o = 4\alpha/\sin^2 \alpha. \quad (32)$$

Equation 31 is of a form similar to the equivalent footings expression (equation 26). (Incidentally, an optimum value of $(q/c)_o \approx 5.52$ is obtained from a value of $\alpha \approx 1.1656$ radians.)

SINGLE-CIRCLE MECHANISM WITH UPLIFT

If the roller support is removed from the scenario in figure 15A so that footing uplift is allowed, the circle center is not necessarily above the edge of the actual footing. Here, the assumption is made that the circle center lies a distance $2e$ to the right of the footing edge, as shown in figure 16A. The hodograph is shown in figure 16B.

The friction force again passes through the circle center and the intersection of force vectors, Q and T . From the geometry of the mechanism,

$$\tan \theta = R/(b/2 - e). \quad (33)$$

The force polygon (fig. 16C) is the same as the previous case without uplift.

The ultimate load is obtained by substituting equations 29 and 33 into the force equilibrium expression for the FS (equation 20),

$$(q/c) = 4\alpha r/bs \sin \alpha. \quad (34)$$

Substituting $\sin \alpha = (b - 2e)/r$ into equation 34 and rearranging leads to the very same expression derived considering equivalent footings (equation 26). This is to be expected since the size of the friction circle (fig. 16A) is proportional to the footing width by the same ratio as the equivalent footing (refer to discussion concerning equation 26).

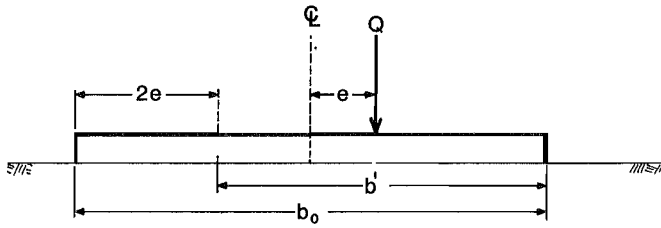


Figure 14.—Equivalent footings.

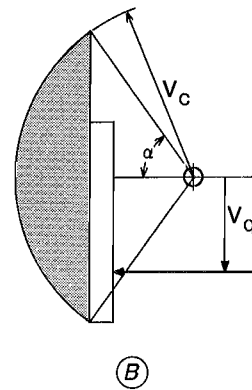
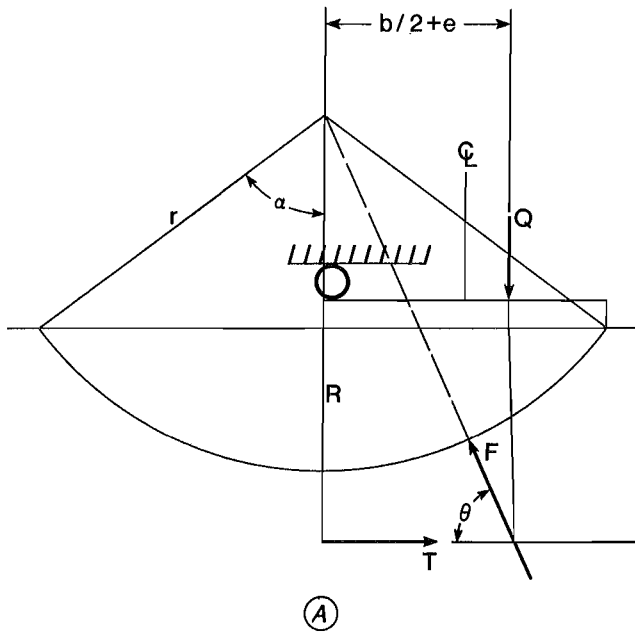
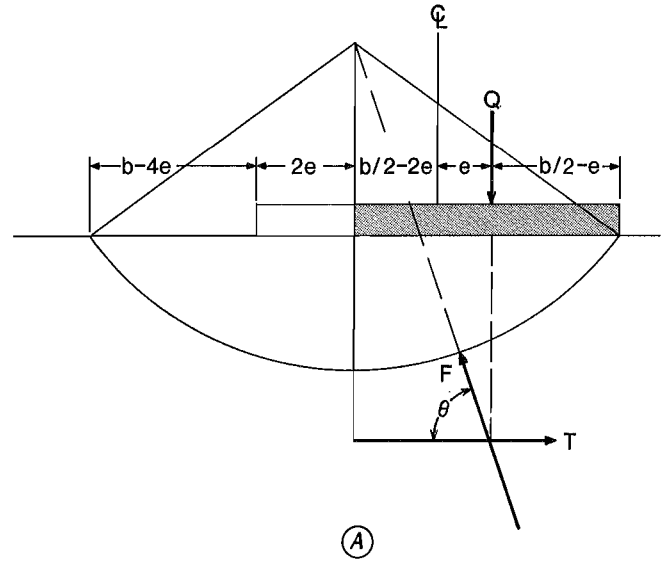


Figure 16.—Single-circle mechanism with uplift. A, Force diagram; B, hodograph; C, force polygon.

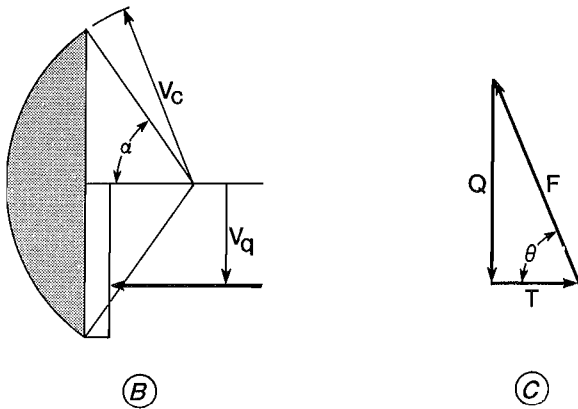


Figure 15.—Single-circle mechanism without uplift. A, Force diagram; B, hodograph; C, force polygon.

THREE-CIRCLE MECHANISM

Code OPTIC was used to compute results for the eccentrically loaded footing by the kinematic method. The three-circle mechanism was allowed to move freely, unlike the single-circle examples that were computed analytically, where a mechanism is partially assumed by specifying the horizontal coordinate of the circle center. The interesting result of the three-circle mechanism is that, over a moderate range of eccentricities, it converges to the single-circle mechanism with uplift (as shown in figure 16A).

The equations derived from the equivalent footing assumption or single circle with uplift (equation 26) and

single circle without uplift (equation 31) are plotted in figure 17. Comparison shows that when the footing is restrained such that it does not rotate upward, a significant increase in strength occurs.

The three-circle mechanism solution to Prandtl's problem is $(q/c) \approx 5.30$, which is closer to the closed form solution of $(q/c) = 2 + \pi$ than the single-circle solution $(q/c) \approx 5.52$. However, for eccentric loads, the three-circle mechanism converges to the equivalent footings expression (equation 26). The transition point where the single circle and composite mechanism diverge was found to be $(e/b) \approx 0.09$. The mechanisms obtained from the convergence points of code OPTIC for eccentricities of $e = 0$ and $(e/b) \approx 0.09$ are shown in figures 18A and 18B, respectively. The improvement of the solution by the three-circle mechanism over equation 26, for small eccentricities of $(e/b) < 0.9$, is shown in the upper left portion of figure 17.

The three-circle mechanism also diverges from equation 26 for eccentricities $(e/b) > 0.25$. This anomaly is produced by a constraint in the OPTIC code. The failure surface termination point A_3 (see figure 6) is not allowed under the footing, so that the failure surface runs a minimum length of the footing. However, this anomaly lies in the range of eccentricities that is typically not allowable in practice.

The conclusion is that a factor for bearing-capacity reduction, based on the three-circle mechanism, due to eccentric loads would result in deviations from the present method that would be negligible. Also, the present method of equivalent footings is on the conservative side.

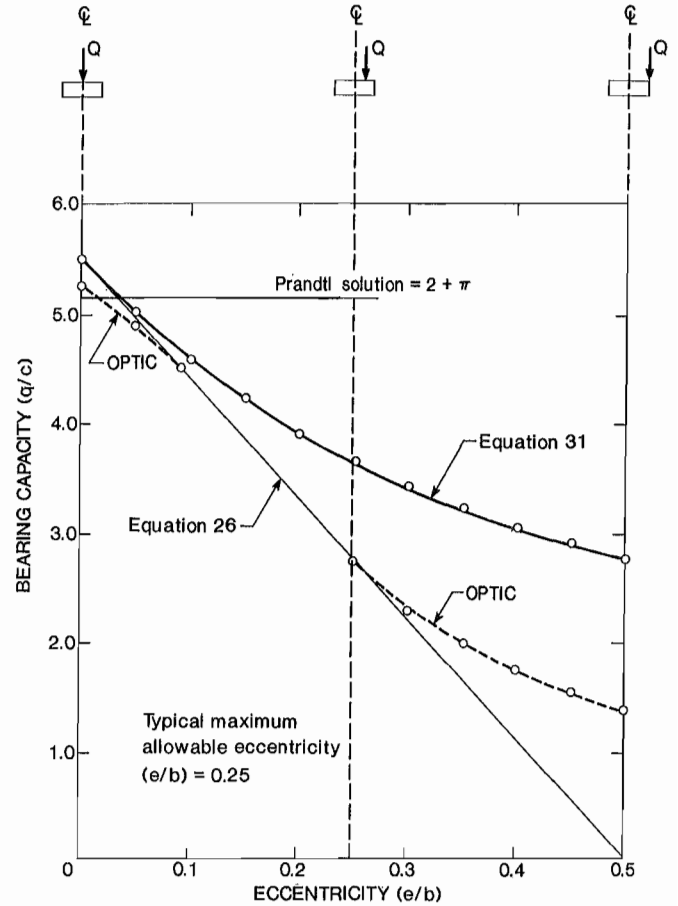


Figure 17.—Effect of eccentricity on bearing capacity.

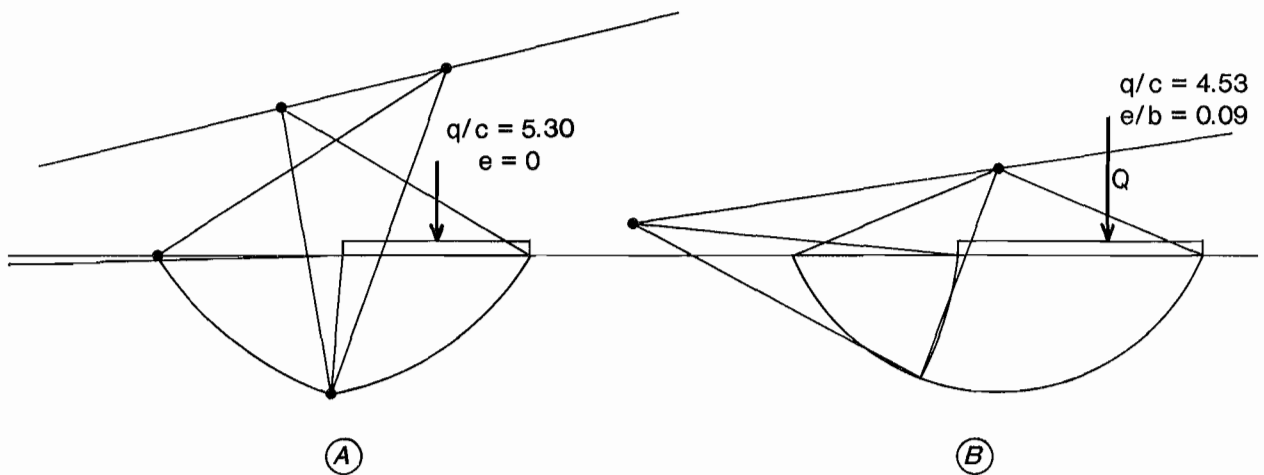


Figure 18.—Failure mechanisms of strip footings. A, Centrally loaded; B, eccentrically loaded.

EFFECT OF EMBEDMENT

Calculation of ultimate load of foundations may be given by the Buisman-Terzaghi equation (8):

$$q = cN_c + q_oN_q + \frac{1}{2}\gamma bN_\gamma \tag{35}$$

where γ is the unit weight and N is the bearing capacity factor. Values of N_c , N_q , and N_γ , as given in table 1 are calculated by

$$\begin{aligned} N_q &= e^{\pi \tan \phi} \tan^2(\pi/4 + \phi/2), \\ N_c &= (N_q - 1) \cot \phi, \\ N_\gamma &= 2(N_q + 1) \tan \phi, \end{aligned} \tag{36}$$

and are said to lead to errors not exceeding 17 to 20 pct for frictional materials ($30^\circ < \phi < 40^\circ$), while the error is zero for cohesive materials. Equation 36 represents curve fits to data that are obtained by the method of characteristics. The analyses by method of characteristics assume a mechanism, as shown in figure 19, where the footing is even with the ground and there is a distributed surcharge load, q_o .

In reality, the foundation is rarely even with the ground surface as posed in figure 19. For embedded footings, as shown in figure 20, the soil weight over the depth of embedment (δ) is taken as the surcharge load, $q_o = \gamma \delta$. With this simplification, the shearing resistance between the foundation and soil (segment AD in figure 20) and between the overburden and supported soil (segment BC in figure 20) is neglected so that the bearing capacity factor (N_c) becomes conservative as δ is increased. The composite friction circle method does take account of this additional shearing resistance, which is a particular advantage of the friction circle method.

An attempt was made to establish a relationship between ultimate load and δ . However, this is complicated by two fundamental problems. The first problem is that, while the bearing capacity factors are approximated as a function of the friction angle only, the effect of embedment (influenced by the shearing resistance along the neglected segments AD and BC of figure 20) is dependent on both friction and cohesion. The second problem is the inherent complication associated with the friction circle method as the cohesion becomes small, which places a limitation on the range of values for soil strength where the effect of embedment could be investigated. Thus, establishing a relationship between ultimate load and embedment over a broad range of c and ϕ could take a considerable amount of time and effort.

Although the composite friction circle method becomes conservative as the cohesion approaches zero, this does not reduce the applicability of the method for higher values of cohesion. The calculation of bearing capacity for a particular problem is a much easier task than trying to fit a curve to the nonlinear variation of bearing capacity as a function of embedment.

Table 1.—Partial listing of bearing capacity factors

Friction angle (ϕ), deg	N_c	N_q	N_γ
0	5.14	1.00	0.00
5	6.49	1.57	.45
10	8.35	2.47	1.22
20	14.83	6.40	5.39
30	30.14	18.40	22.40
40	75.31	64.20	109.41

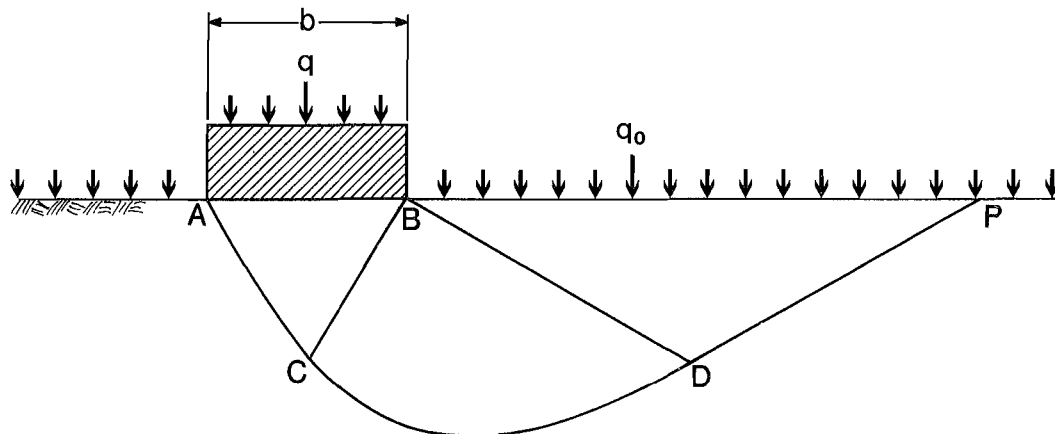


Figure 19.—Assumed mechanism for bearing capacity factors.

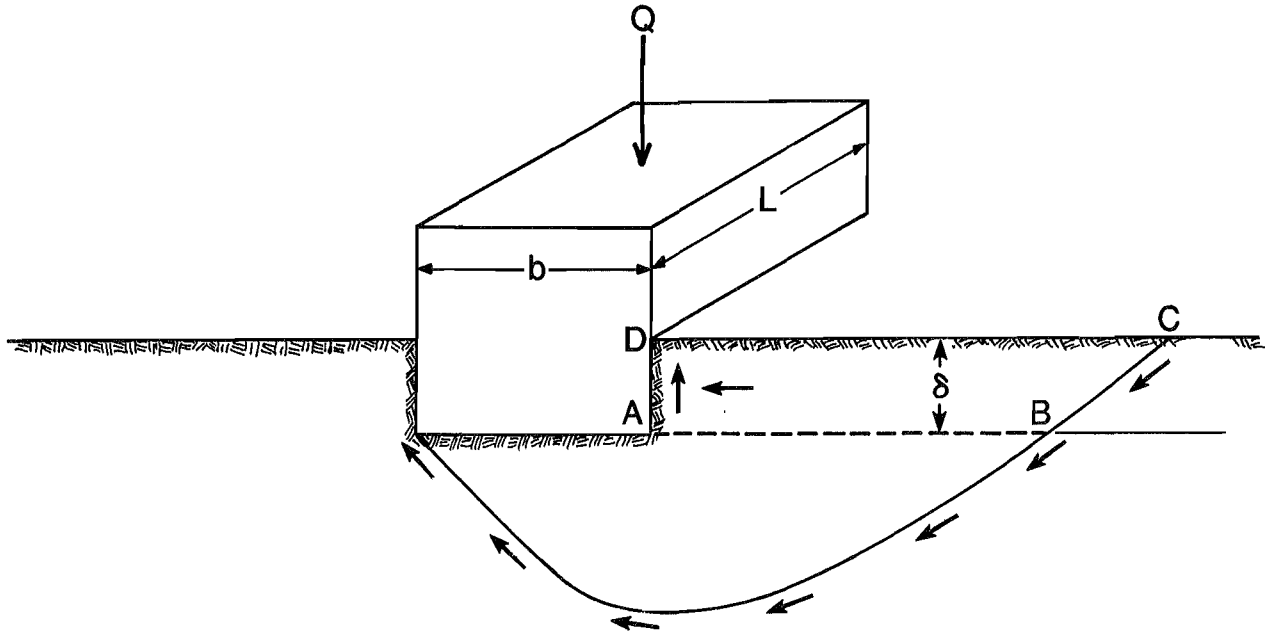


Figure 20.—Analysis of embedded footing with bearing capacity factors.

BEARING CAPACITY CALCULATIONS NEAR SLOPES

The friction circle method can be used for bearing capacity calculations of footings that are embedded, have eccentric and/or inclined loads, and are near slopes. Some demonstrations of the OPTIC code are given here.

Comparison is made with the kinematic rigid-block method. Papanastiou (9) used a mechanism composed of two rigid blocks (shown in figure 21) to produce a dimensionless *nomograph* for a centrally loaded surface footing near a slope. The limit load, Q_u , is then a function of the soil properties, γ , c , and ϕ ; the footing width, b ; and the slope angle (β). For constant values of β and ϕ , the dimensionless term $Q/\gamma b^2$ varies almost linearly with the dimensionless term $c/\gamma b$. Typical values of $\gamma = 15 \text{ kN/m}^3$, $\phi = 20^\circ$, and $b = 5 \text{ m}$ were chosen, and β was varied from 0° to 50° in increments of 10° . The nomograph given by Papanastiou was reproduced and results from the composite friction circle method were superimposed on it in figure 22.

The major disadvantage of Taylor's friction circle method (composed of a single circle for slope stability without any surcharge loads) is that when ϕ exceeds β , the equilibrium cohesion necessary to maintain stability is negative. This violates the kinematics of the problem since a negative cohesion force does not oppose motion; i.e., it cannot be developed for the assumed collapse mechanism. This basic problem is also inherent to the composite

friction circle method with three circles. For small values of cohesion it becomes increasingly difficult to find a mechanism where all three equilibrium cohesion forces are positive. Thus, the result is that the rigid-block mechanism is typically *lower bound* to the composite friction circle method for values of $(c/\gamma b) < 0.2$ and $\phi = 20^\circ$.

In the upper right portion of the nomograph, where β 's are small and equilibrium cohesive forces are large, the friction circle showed a significant improvement over the rigid two-block mechanism. However, this is primarily due to the limiting ability of only two blocks to conform to a smooth failure surface for small β 's. A rigid-block mechanism composed of four or more blocks showed a significant improvement over the two-block mechanism for β 's $< 20^\circ$ and was slightly lower bound to the composite friction circle method.

The example of the collapse mechanism in the nomograph does not take advantage of several features of the composite friction circle method. A slightly more complex problem of an embedded foundation with an eccentric load is analyzed in figure 23. This is an example where many methods, such as the method of slices, would be limited in their ability to assess the influence of the embedment and eccentric loads. With only slight modification, the OPTIC code could also consider inclined eccentric loads.

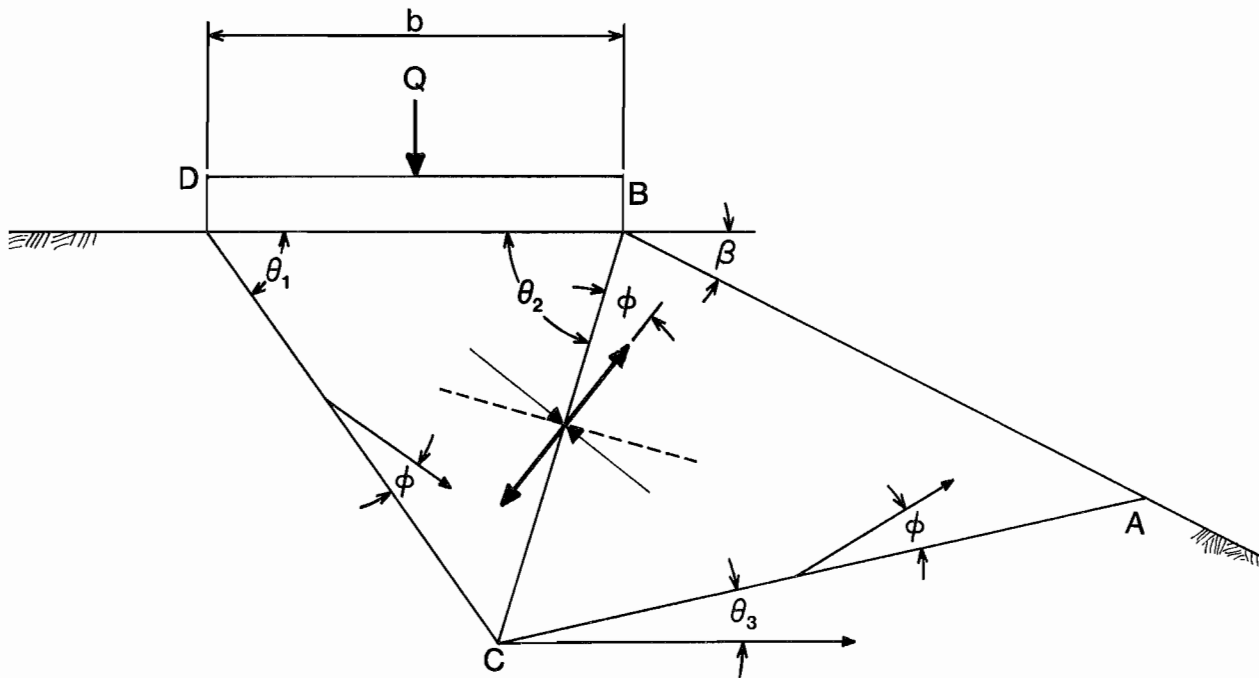


Figure 21.—Kinematic rigid-block mechanism.

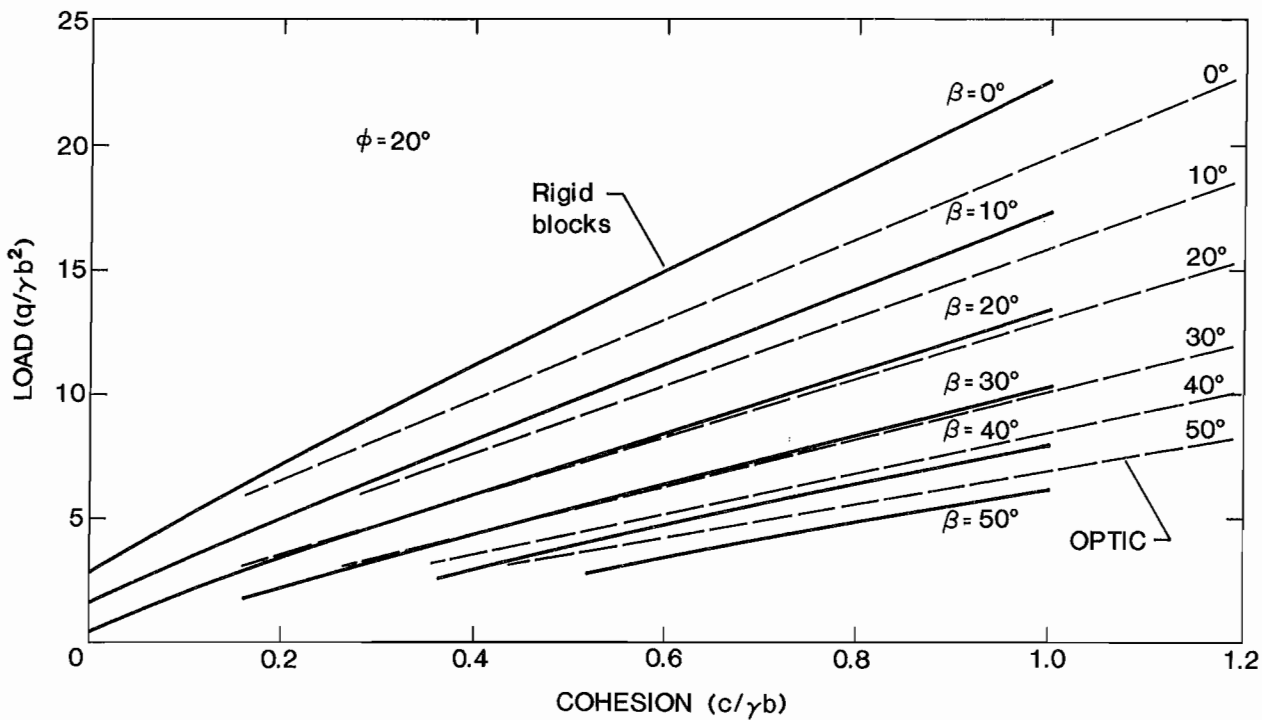


Figure 22.—Comparison of bearing capacity factors by two-wedge rigid-block method and two-wedge friction circle method.

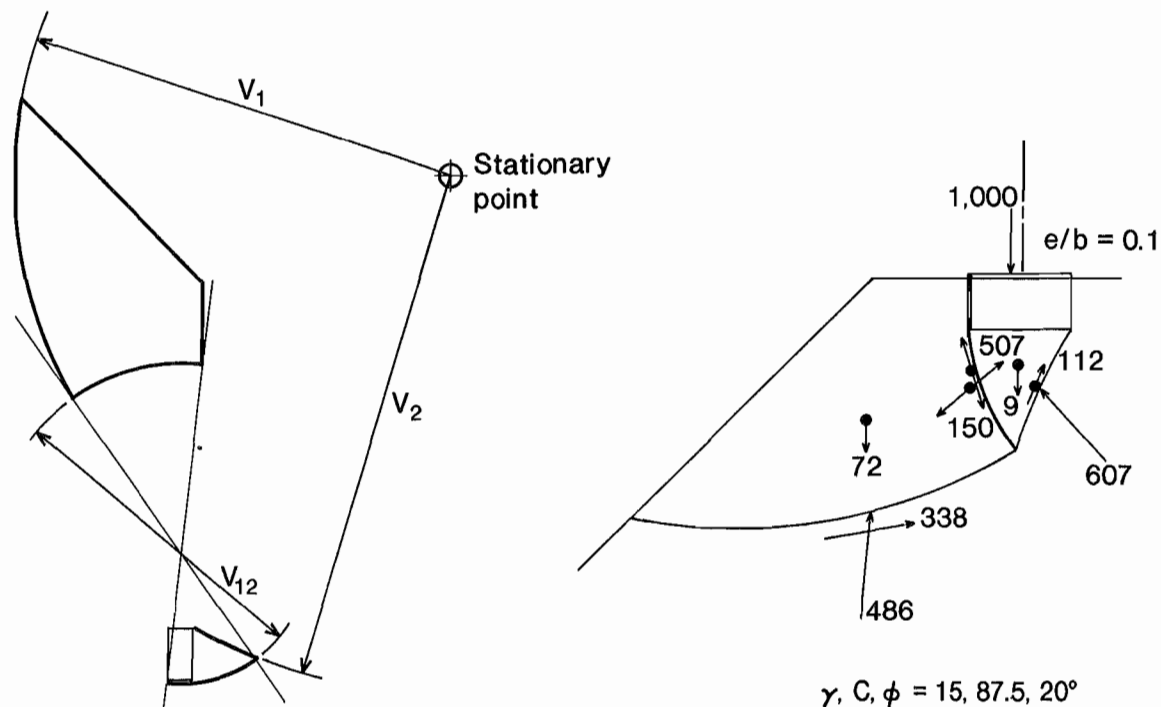


Figure 23.—Collapse mechanism for embedded foundation with eccentric load near slope.

The weight of the foundation has been neglected in the OPTIC code. It is assumed that the weight of the foundation is included in the load, Q . Since the foundation is assumed to act as an integral portion of the right soil

wedge, a unit weight of concrete could be incorporated into the code. Although, for large loads, this would probably have a negligible influence.

CONCLUSIONS

A method for utilizing composite failure mechanisms constructed of three friction circles is presented. This method can be used for determining stability of some simple structures on homogeneous soils. The method is an extension of the well-known Taylor friction circle method and follows the work of Gudehus (4). The method has the following characteristics:

1. The FS is assumed to be equal to 1 for *limit analysis*. This effectively makes the use of the kinematic FS based on energy equilibrium, used in derivation of the friction circle method, irrelevant, so that the more commonly adopted FS,

$$FS = c/c^* = \tan\phi/\tan\phi^*, \quad (37)$$

may be utilized.

2. Due to constraints of the method, it is not possible to optimize the mechanism and converge toward a minimum FS. Thus, the value of cohesion at limiting equilibrium is determined as an alternative. If the cohesion is known a priori, as is usually the case, a limit load must be determined by trial and error.

3. The value of cohesion in a soil mass cannot be negative. Since the method calculates the value of cohesion at limiting equilibrium, it can only be used when a value of cohesion >0 is required for stability.

4. Although the method generally utilizes equilibrium of forces to calculate the FS, an FS based on kinematics can be utilized for $\phi = 0$. In this case, the method yields an upper-bound kinematic solution as postulated by Drucker (6). This FS based on kinetics is always less than or equal to the FS based on equilibrium of forces.

Comparison of bearing capacities with other methods shows the method yields results that are somewhat conservative for small values of cohesion and converge closer to closed-form solutions for larger values of cohesion with respect to soil weight and friction angle. The major hindrances to the method are local minima obtained in optimization, requirement of a homogeneous soil mass, and the difficulty of obtaining an initial mechanism as a starting point prior to optimization. Presently, one must obtain input parameters by using a compass and straight edge and drawing an admissible mechanism on paper.

There are several areas for further research and improvements in the OPTIC code. The weaknesses of the code include the optimization procedure and searching routines for an admissible mechanism, which are areas for possible refinement. Possible additions to the code include horizontal loads on the footing. An option of horizontal gravitational loads on the footing would enable the code to be used for analyses of retaining walls. Inclusion of the weight of concrete in the foundation would allow analysis of gravity walls and dams.

REFERENCES

1. Degroot, R. L. Dump Point Stability Analysis. M.S. Thesis, Univ. MN, 1986, 66 pp.
2. Crum, D. A. Application of a Full Scale Dump Point Stability Experiment to Engineering Analysis. M.Ge.E. Thesis, Univ. MN, 1988, 77 pp.
3. Taylor, D. W. Fundamentals of Soil Mechanics. Wiley, 1948, 700 pp.
4. Gudehus, G. Engineering Approximations for Some Stability Problems in Geomechanics. Paper in Advances in Analysis of Geotechnical Instabilities. Univ. Waterloo Press, 1978, pp. 1-23.
5. Vardoulakis, I. Lecture Foundation Engineering. Univ. MN, Minneapolis, MN, 1985-90, 100 pp.; available from I. Vardoulakis, Univ. MN.
6. Drucker, D. C., H. J. Greenberg, and W. Prager. Extended Limit Design Theorems for Continuous Media. Q. Appl. Math., v. 9, No. 4, 1952, pp. 381-389.
7. Kuester, J. L., and J. H. Mize. Optimization Techniques With FORTRAN. McGraw-Hill, 1973, 256 pp.
8. Winterkorn, H. F., and H. Y. Fang. Foundation Engineering Handbook. Van Nostrand Reinhold, 1975, 75 pp.
9. Papanastiou, P. C. Passive Earth Pressure and Bearing Capacity Computations With Microcomputers. M.S. Thesis, Univ. MN, 1986, 76 pp.

BIBLIOGRAPHY

- Huang, Y. H., and C. M. Avery. Stability of Slopes by Logarithmic Spiral Method. J. Geotech. Eng., ASCE GT1, v. 102, Jan. 1976, pp. 41-49.
- Huntington, W. C. Earth Pressures and Retaining Walls. Wiley, 1957, 534 pp.

APPENDIX A.—NOTATION¹

ENGLISH	(x,y) (x',y')	coordinates in global system coordinates in local system
A,B,C,D,M,P	points	
a_{ij}	matrix coefficients	
b	width of footing [DISTANCE]	
c	cohesion in soil [FORCE divided by AREA]	
e	eccentricity	
F_1	resultant of normal force and frictional resistance [FORCE]	
FS	factor of safety	
$f(\theta)$	frictional resistance and normal stress as function of position on slip circle [FORCE divided by AREA]	
G	center of gravity	
K_i	intersection points of resultant cohesion force T and central bisector of circular arc	
k	a constant	
L	length [DISTANCE]	
M	moment [FORCE multiplied by DISTANCE]	
N	bearing capacity factor	
\bar{n}	outward normal	
Q	load [FORCE]	
Q_i	intersection points of force vectors F_1 and T_1	
q	distributed load [FORCE divided by AREA]	
R	radial distance to cohesion resultant, T_1 [DISTANCE]	
r_i	radius of slip circle or ray related to hodograph [DISTANCE]	
T_1	tangential force due to cohesion [FORCE]	
t	constant value of cohesive shear stress [FORCE divided by AREA]	
$t(\theta)$	cohesive shear stress as function of position on slip circle [FORCE divided by AREA]	
(u,v)	coordinates in local system	
V	velocity [DISTANCE divided by TIME]	
W	weight of soil wedge [FORCE]	

GREEK	
α	angle of half the arc defined by slip circle
$\Delta\alpha_i$	angle locating point Q_i
β	slope angle
δ	depth of embedment [DISTANCE]
$\delta(x)$	Dirac delta function
γ	unit weight [FORCE divided by VOLUME]
η_i	orientation of friction resultant, F_1
ϕ	internal angle of friction of the soil
θ	angle measured counterclockwise from center of slip circle
θ_i	orientation of cohesion resultant, T_1

SUBSCRIPTS	
A,P	points
c	cohesion
eq	equilibrium, from actuating forces
i,j	unspecified circle or matrix coefficient index
L	limiting, from dissipating forces
L,R	left and right soil wedge
o	initial state
Q	load
q	surcharge load
x	coordinate along x axis, horizontal
y	coordinate along y axis, vertical
1,2,3	one of three circles
1-6	matrix or solution vector coefficients
ϕ	friction

SUPERSCRIPTS	
av	average
h	hodograph
*	critical, at point of failure or having singularity point
'	local
\perp	perpendicular

¹Terms in brackets define dimensions of selected variables.

APPENDIX B.—GLOSSARY

Critical.—The point at which the stability of the structure changes from a stable, safe state of being to a collapse, where the factor of safety is 1.

Force equilibrium.—Refers to calculations based on admissible and limiting states of stress in the structure.

Hodograph.—A velocity diagram where the relative velocities of moving objects are mapped.

Kinematic (moving).—Refers to calculations based on velocities.

Limit analysis.—Analysis of a structure at its limiting or critical state of impending failure.

Lower bound.—A value that is on the "safe" side. A method that yields an ultimate load less than some base value, where the base value is taken to be the "true value" if not otherwise specified.

Nomograph.—A graph containing multiple curves, constructed for use in determining values of some unknown based on two other known values.

Upper bound.—A value that is on the "unsafe" side. A method that yields an ultimate load higher than some base value, where the base value is taken to be the "true value" if not otherwise specified.

APPENDIX C.—OPTIC CODE LISTING

The OPTIC code was written using Turbo Basic¹ software by Borland International, Inc. (version 1.1). The code is designed to be compiled and executed on IBM compatible microcomputers. Since there are no graphics in the code, it should run on almost any system.

The operation of the code is relatively self-explanatory, provided that the user is familiar with the friction circle method as described in this report. Use is made of the cursor location keys for moving around in the menus as well as typing the first letter of a command line. The user must use the DOS screen dump option ([CNTR]-[PRTSCN]) to obtain hard copies of output. Also, the F1 function key will terminate optimization and return the user to the main menu. However, the F1 key should be used with caution since the code may jump out of the optimization loop with a nonadmissible mechanism.

Some helpful hints when running the program include using a saved data file with similar geometries as the problem the user is trying to solve and converging it through a series of small changes and optimizations. Creating new geometries is usually most efficient if the following procedure is utilized: (1) Use a compass and paper to draw a "reasonable mechanism," (2) input and optimize the mechanism with the kinematic option regardless of the angle of friction, (3) switch to force equilibrium if the angle of friction is >0 and optimize, (4) if the constraint is violated because of negative forces, run the "admissible search" option in the main menu, (5) check results by other methods or by moving some input parameters back and forth so that the convergence is started from some different configurations. The admissible search option should also be used with caution since it sometimes tends to move the mechanism to a configuration that hinders subsequent optimization.

¹Reference to specific products does not imply endorsement by the U.S. Bureau of Mines.

Figure C-1 shows the values of the input parameter MECH that determines the concave or convex orientation of each of the circles.

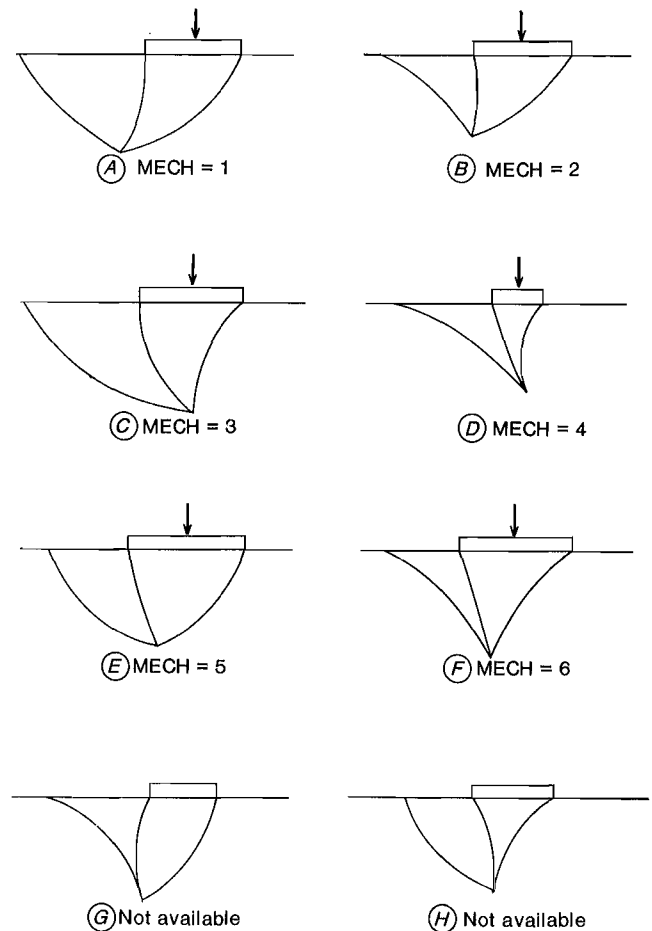


Figure C-1.—Values for Input parameter to choose mechanism in code OPTIC.

'Foundation-Slope Stability Program

'spring, 1989

CLS : PRINT Program "OPTIC", a "FOUNDATION - SLOPE STABILITY PROGRAM"

PRINT "This program uses a Friction Circle method consisting of 3 circles."

PRINT "It calculates the optimum limit load by adjusting the value of"

PRINT "cohesion for a given factor of safety. The program assumes a footing"

PRINT "near an infinite slope."

PRINT "-----"

1 V\$ = INKEY\$: IF V\$="" THEN 1

OPTION BASE 1 : KEY OFF

DIM LX(19), LY(19), VAR(19), MAT(7,6), B(6), X(6), CH\$(13)

DIM PX(20), PY(20), A(8), GX(8), GY(8), DELTA(4), FS(4), Q(4)

PI=4*ATN(1) : RNNUM=0 : FSCOH=1 : KINMAT=0 : TRAP\$="OFF"

ON ERROR GOTO 9900

KEY(1) ON : ON KEY(1) GOSUB Kbreak

GOTO 150

'-----

Kbreak:

RETURN 150

Okey:

IF (ROW=8) OR (ROW=9) THEN

IF TRAP\$="ON" THEN

COLOR 10,0 : LOCATE LY(8),LX(8) : PRINT CH\$(8)

COLOR 12,0 : LOCATE LY(9),LX(9) : PRINT CH\$(9)

ELSE

COLOR 10,0 : LOCATE LY(9),LX(9) : PRINT CH\$(9)

COLOR 12,0 : LOCATE LY(8),LX(8) : PRINT CH\$(8)

END IF

ELSEIF (ROW=10) OR (ROW=11) THEN

IF KINMAT=1 THEN

COLOR 10,0 : LOCATE LY(10),LX(10) : PRINT CH\$(10)

COLOR 12,0 : LOCATE LY(11),LX(11) : PRINT CH\$(11)

ELSE

COLOR 10,0 : LOCATE LY(11),LX(11) : PRINT CH\$(11)

COLOR 12,0 : LOCATE LY(10),LX(10) : PRINT CH\$(10)

END IF

END IF

RETURN

Upkey:

KEY(11) OFF : GOSUB Okey : LROW = ROW : ROW = ROW-1

IF (ROW=8) OR (ROW=10) THEN ROW = ROW-1

IF (ROW=9) AND (TRAP\$="ON") THEN ROW = 8

IF (ROW=11) AND (KINMAT=1) THEN ROW=10

IF ROW = 0 THEN ROW = 1

COLOR 0,7 : LOCATE LY(ROW),LX(ROW) : PRINT CH\$(ROW)

IF (ROW>9) OR (ROW<7) THEN

COLOR 12,0 : LOCATE LY(LROW),LX(LROW) : PRINT CH\$(LROW)

END IF

COLOR 7,0 : KEY(11) ON

RETURN 400

Downkey:

```
KEY(14) OFF : GOSUB Okey : LROW = ROW : ROW = ROW+1
IF (ROW<9) THEN COLOR 12,0 : LOCATE LY(LROW),LX(LROW) : PRINT CH$(LROW)
IF (ROW=9) OR (ROW=11) THEN ROW = ROW+1
IF (ROW=8) AND (TRAP$="OFF") THEN ROW= 9
IF (ROW=10) AND (KINMAT=0) THEN ROW=11
IF ROW = 13 THEN ROW = 12
COLOR 0,7 : LOCATE LY(ROW),LX(ROW) : PRINT CH$(ROW)
COLOR 7,0 : KEY(14) ON
RETURN 400
```

```
400 IF (ROW=7) OR (ROW=12) THEN LROW=ROW
IF (ROW=12) OR (ROW<8) THEN 160
KEY(12) ON : KEY(13) ON
```

Trap:

GOTO Trap:

Sideway:

```
LROW = ROW : ROW = ROW + 1
IF (ROW=10) OR (ROW=12) THEN ROW = ROW - 2
RETURN 160
```

```
150 CLS : LROW = 1
GOSUB 8000 : ROW = 1 '[Main menu screen setup]
KEY(11) ON : ON KEY(11) GOSUB Upkey
KEY(12) ON : ON KEY(12) GOSUB Sideway
KEY(13) ON : ON KEY(13) GOSUB Sideway
KEY(14) ON : ON KEY(14) GOSUB Downkey
```

```
160 COLOR 12,0 : LOCATE LY(LROW),LX(LROW) : PRINT CH$(LROW)
COLOR 0,7 : LOCATE LY(ROW),LX(ROW) : PRINT CH$(ROW)
COLOR 7,0
```

```
162 V$ = INKEY$ : IF V$ = "" THEN 162
IF LEN(V$) = 2 THEN V$ = RIGHT$(V$,1)
IF (V$="I") OR (V$="i") THEN
LROW = ROW : ROW = 1 : GOTO 160
ELSEIF (V$="D") OR (V$="d") THEN
LROW = ROW : ROW = 2 : GOTO 160
ELSEIF (V$="F") OR (V$="f") THEN
LROW = ROW : ROW = 3 : GOTO 160
ELSEIF (V$="R") OR (V$="r") THEN
LROW = ROW : ROW = 4 : GOTO 160
ELSEIF (V$="O") OR (V$="o") THEN
LROW = ROW : ROW = 5 : GOTO 160
ELSEIF (V$="A") OR (V$="a") THEN
LROW = ROW : ROW = 6 : GOTO 160
ELSEIF (V$="P") OR (V$="p") THEN
LROW = ROW : ROW = 7 : GOTO 160
ELSEIF (V$="2") THEN
```



```

    LROW = ROW : ROW = 8 : GOTO 160
ELSEIF (V$="3") THEN
    LROW = ROW : ROW = 9 : GOTO 160
ELSEIF (V$="K") OR (V$="k") THEN
    LROW = ROW : ROW = 10 : GOTO 160
ELSEIF (V$="L") OR (V$="l") THEN
    LROW = ROW : ROW = 11 : GOTO 160
ELSEIF (V$="E") OR (V$="e") THEN
    LROW = ROW : ROW = 12 : GOTO 160
ELSEIF ASC(V$) = 13 THEN
    ON ROW GOTO 201,202,203,204,205,206,207,208,208,210,210,212
201  GOSUB 4000 : GOTO 150      'Interactive Input
202  GOSUB 4100 : GOTO 150      'Data file input
203  GOSUB 4200 : GOTO 150      'File save
204  CLS : RN$ = "RUN" : GOSUB 6300 '[INITIALIZE]
    GOSUB 1000 : GOSUB 1000
    IF LIM=0 THEN PRINT"*** Constraint #";cnum;" Violated *****"
    PRINT"Kine. = ";FSKIN
    PRINT"Limi. = ";FSCL1,FSCL2,FSCL3
Trap3:
    IF INKEY$="" THEN Trap3:
    GOTO 150
205  RN$ = "OPTI"      : GOSUB 2000 : GOTO 150
206  RN$ = "SEARCH"   : GOSUB 2200 : GOTO 150
207  KEY(11) OFF : KEY(12) OFF : KEY(13) OFF : KEY(14) OFF
    GOSUB 5000 : GOTO 150      'Print results
208  IF ROW = 8 THEN
    TRAP$ = "ON"
    COLOR 10,0 : LOCATE LY(8),LX(8) : PRINT CH$(8)
    COLOR 12,0 : LOCATE LY(9),LX(9) : PRINT CH$(9)
    ELSE 'row=9
    TRAP$ = "OFF"
    COLOR 12,0 : LOCATE LY(8),LX(8) : PRINT CH$(8)
    COLOR 10,0 : LOCATE LY(9),LX(9) : PRINT CH$(9)
    END IF
    GOTO 162
210  IF ROW = 10 THEN
    KINMAT = 1
    COLOR 10,0 : LOCATE LY(10),LX(10) : PRINT CH$(10)
    COLOR 12,0 : LOCATE LY(11),LX(11) : PRINT CH$(11)
    COLOR 7,0
    ELSE 'row=11
    KINMAT = 0
    COLOR 12,0 : LOCATE LY(10),LX(10) : PRINT CH$(10)
    COLOR 10,0 : LOCATE LY(11),LX(11) : PRINT CH$(11)
    COLOR 7,0
    END IF
    GOTO 162
212  STOP : END
    ELSE
    LOCATE 23,20 : PRINT "Huh ?" : GOTO 160
    END IF

```

1000 'Subroutine FACTOR

KEY(1) ON : LIM=1 : RNNUM=RNNUM+1
 IF A3X => BF THEN LIM=0 : CNUM=6 : GOTO 1987 'Constraint #6
 IF C1Y=C2Y THEN LIM=0 : CNUM=0 : GOTO 1987

'----- Compute Strategic Points & Radii -----'

B1X = BF : B1Y = 0
 B2X = B1X + BW : B2Y=-EMBED
 SELECT CASE MECH

CASE 1
 IF C1X>B1X OR C2X>B2X THEN LIM=0 : CNUM=8 : GOTO 1987
 CASE 2
 IF C1X>B1X OR C2X>B2X THEN LIM=0 : CNUM=8 : GOTO 1987
 CASE 3
 IF C1X<B1X OR C2X<B2X THEN LIM=0 : CNUM=8 : GOTO 1987
 CASE 4
 IF C1X<B1X OR C2X<B2X THEN LIM=0 : CNUM=8 : GOTO 1987
 CASE 5
 IF C1X<B1X OR C2X>B2X THEN LIM=0 : CNUM=8 : GOTO 1987
 CASE 6
 IF C1X>B1X OR C2X<B2X THEN LIM=0 : CNUM=8 : GOTO 1987
 END SELECT

A3Y = A3X*TAN(BETA)
 R1=SQR((C1X-B1X)^2 + (C1Y-B1Y)^2)
 R2=SQR((C2X-B2X)^2 + (C2Y-B2Y)^2)

V1 = (C2X-C1X)/(C2Y-C1Y) : V3 = V1^2 + 1
 V2 = (C1X^2 - R1^2 - C2X^2 + R2^2 + (C2Y-C1Y)^2)/(2*(C2Y-C1Y))
 V4 = 2*V1*V2 - 2*C2X : V5 = V2^2 - R2^2 + C2X^2
 IF (V4^2 - 4*V3*V5)<0 THEN LIM=0 : CNUM=1 : GOTO 1987 'Constraint #1
 IF EMBED>0 THEN
 IF MECH=3 OR MECH=4 THEN
 V1 = SQR((C1X-B1X)^2 + (C1Y+EMBED)^2)
 IF V1>R1 THEN LIM=0 : CNUM=0 : GOTO 1987 'thru foundation
 END IF
 END IF

SELECT CASE MECH
 CASE 1
 MX = (-V4 + SQR(V4^2 - 4*V3*V5))/(2*V3)
 CASE 2
 MX = (-V4 + SQR(V4^2 - 4*V3*V5))/(2*V3)
 CASE 3
 MX = (-V4 - SQR(V4^2 - 4*V3*V5))/(2*V3)
 CASE 4
 MX = (-V4 - SQR(V4^2 - 4*V3*V5))/(2*V3)
 CASE 5
 MX = (-V4 - SQR(V4^2 - 4*V3*V5))/(2*V3)

CASE 6

MX = $(-V4 + \text{SQR}(V4^2 - 4*V3*V5))/(2*V3)$
 END SELECT

TEMP = $R1^2 - (MX-C1X)^2$: IF TEMP < 0 THEN LIM=0 : GOTO 1987

My = $C1y + \text{SQR}(TEMP)$

1010 V1 = $\text{SQR}((MX-C2x)^2 + (My-C2y)^2) - R2$: IF INT(ABS(V1*100))=0 THEN 1011

My = $C1y - \text{SQR}(TEMP)$: GOTO 1010

1011 'endif

IF MY > -EMBED THEN LIM=0 : CNUM=2 : GOTO 1987 'Constraint #2

IF MY > MX*TAN(BETA) THEN LIM=0 : CNUM=3 : GOTO 1987 'Constraint #3

2 IF (TRAP\$="ON") AND (BETA=0) THEN

C3X = C2X : C3Y = C2Y

A3X = $C2X - \text{SQR}(R2^2 - C2Y^2)$: A3Y = 0

ELSEIF TRAP\$="ON" THEN

C3X = C2X : C3Y = C2Y

V1 = $1 + 1/\text{TAN}(BETA)^2$: V2 = $-2*(C2Y + C2X/\text{TAN}(BETA))$

V3 = $C2X^2 + C2Y^2 - R2^2$

A3Y = $(-V2 - \text{SQR}(V2^2 - 4*V1*V3))/(2*V1)$: A3X = $A3Y/\text{TAN}(BETA)$

ELSEIF C1X=C2X THEN

X = $(MX + A3X)/2$: Y = $(MY + A3Y)/2$

V1 = $(MY-A3Y)/(MX-A3X)$: V1 = $-1/V1$: V2 = $Y - V1*X$

C3X = C1X : C3Y = $V1*C3X + V2$

ELSEIF MX=A3X THEN

M = $(C2Y-C1Y)/(C2X-C1X)$: B = $C1Y - M*C1X$

C3Y = $(MY + A3Y)/2$: C3X = $(C3Y - B)/M$

ELSE

M = $(C2Y-C1Y)/(C2X-C1X)$: B = $C1Y - M*C1X$ '1st Line

X = $(MX + A3X)/2$: Y = $(MY + A3Y)/2$

V1 = $(MY-A3Y)/(MX-A3X)$: V1 = $-1/V1$: V2 = $Y - V1*X$ '2nd Line

C3X = $(B-V2)/(V1-M)$: C3Y = $M*C3X + B$ 'Intersection

END IF

R3 = $\text{SQR}((C3Y-MY)^2 + (C3X-MX)^2)$

IF MECH=3 AND C3Y < MY THEN LIM=0 : CNUM=0 : GOTO 1987

IF MECH=4 AND C3Y > MY THEN LIM=0 : CNUM=0 : GOTO 1987

' Constraint #10----->

IF MECH=2 OR MECH=4 OR MECH=6 THEN

IF BETA = 0 THEN

IF (C3X > A3X) THEN LIM=0 : CNUM=10 : GOTO 1987

ELSE

M = $-1/\text{TAN}(BETA)$: B = $A3Y - M*A3X$

IF C3X > (C3Y-B)/M THEN LIM=0 : CNUM=10 : GOTO 1987

END IF

END IF

' Constraint #4----->

PT2X=C3X : PT2Y=C3Y : PT1X=A3X : PT1Y=A3Y : GOSUB 3000 : OMEGA3=PSI

PT1X=MX : PT1Y=MY : GOSUB 3000 : OMEGM=PSI

PT1X=(MX+A3X)/2 : PT1Y=(MY+A3Y)/2 : GOSUB 3000 : OMEGBI=PSI

```

IF MECH=2 OR MECH=4 OR MECH=6 THEN
  IF OMEGM > PI THEN OMEGM = OMEGM - 2*PI
END IF
IF INT(OMEGBI*1000) < > INT((OMEGA3+OMEGM)*500) THEN LIM=0 : CNUM=4 : GOTO 1987
,----- compute centroids & areas of sliding wedges -----
P1X=B1X : P1Y=B1Y : P2X=MX : P2Y=MY : P3X=C1X : P3Y=C1Y : R=R1
GOSUB 3300          '[ARC]
A(1)=A : GX(1)=GX : GY(1)=GY : ALPH1=ALPHA : THET1=THETA : P1THET=PSI
,
P1X=B2X : P1Y=B2Y : P2X=MX : P2Y=MY : P3X=C2X : P3Y=C2Y : R=R2
GOSUB 3300          '[ARC]
A(2)=A : GX(2)=GX : GY(2)=GY : ALPH2=ALPHA : THET2=THETA : P2THET=PSI
,
P1X=MX : P1Y=MY : P2X=A3X : P2Y=A3Y : P3X=C3X : P3Y=C3Y : R=R3
GOSUB 3300          '[ARC]
A(3)=A : GX(3)=GX : GY(3)=GY : ALPH3=ALPHA : THET3=THETA : P3THET=PSI

IF ABS(DALPH1)>ALPH1 OR ABS(DALPH3)>ALPH3 THEN LIM=0 : CNUM=0 : GOTO 1987
IF ABS(DALPH2)>ALPH2 THEN LIM=0 : CNUM=0 : GOTO 1987

X1=B1X : Y1=B1Y : X2=B2X : Y2=B2Y : X3=MX : Y3=MY
GOSUB 3200 : A(4) = AREA : GX(4) = GX : GY(4) = GY
,
X2=0 : Y2=0 : GOSUB 3200          '[TRIANGLE]
A(5) = AREA : GX(5) = GX : GY(5) = GY
X2=A3X : Y2=A3Y : GOSUB 3200          '[TRIANGLE]
GX(5) = (GX(5)*A(5) + GX*AREA)/(A(5) + AREA)
GY(5) = (GY(5)*A(5) + GY*AREA)/(A(5) + AREA)
A(5) = A(5) + AREA

SELECT CASE MECH
CASE 2
  A(3)=-A(3)
CASE 3
  A(1)=-A(1) : A(2)=-A(2)
CASE 4
  A(1)=-A(1) : A(2)=-A(2) : A(3)=-A(3)
CASE 5
  A(1)=-A(1)
CASE 6
  A(2)=-A(2) : A(3)=-A(3)
END SELECT

AR=A(2)+A(4)-A(1)-BW*EMBED/2 : WR=GAMA*AR
AL=A(1)+A(3)+A(5) : WL=GAMA*AL : IF AL=0 THEN LIM=0 : GOTO 1987
GRX=(A(2)*GX(2) + A(4)*GX(4) - A(1)*GX(1) - BW*EMBED*(B1X+B2X)/6)/AR
GRY=(A(2)*GY(2) + A(4)*GY(4) - A(1)*GY(1) + BW*EMBED*EMBED/3)/AR
GLX=(A(1)*GX(1) + A(3)*GX(3) + A(5)*GX(5))/AL
GLY=(A(1)*GY(1) + A(3)*GY(3) + A(5)*GY(5))/AL
A(1)=ABS(A(1)) : A(2)=ABS(A(2)) : A(3)=ABS(A(3))
,-----

```

```
PT1X=GRX : PT1Y=GRY : PT2X=C2X : PT2Y=C2Y
GOSUB 3000 : ANGWR = PSI '[ANGLE 360]
RWR = SQR(((GRX-C2X)^2 + (GRY-C2Y)^2)
```

```
PT1X=GLX : PT1Y=GLY : PT2X=C3X : PT2Y=C3Y
GOSUB 3000 : ANGWL = PSI '[ANGLE 360]
RWL = SQR(((GLX-C3X)^2 + (GLY-C3Y)^2)
```

```
GOSUB 3600          '[HODOGRAPH]
IF LIM=0 THEN CNUM=11 : GOTO 1987
IF (VEL12>.0001) AND (M3YHD<M2YHD) THEN LIM=0 : CNUM=12 : GOTO 1987
IF (VEL12>.0001) AND (B3YHD<B2YHD) THEN LIM=0 : CNUM=12 : GOTO 1987
```

```
'Kinematic solution
```

```
IF KINMAT=1 THEN
  DELTAD = 2*(R1*ALPH1*VEL12 + R2*ALPH2*VEL1 + R3*ALPH3*VEL2)
  DELTAA = ABS(Q*VELQ + WR*VELWR + WL*VELWL)
  IF OP$ = "COHESION" THEN
    FSKIN = DELTAD*COH/DELTAA : COH = COH/FSKIN
    FSKIN = DELTAD*COH/DELTAA : OPTPAR=FSKIN : OPT1=OPTPAR
    FSCOH=0 : FSCL1=0 : FSCL2=0 : FSCL3=0
  ELSE
    IF LIM=1 THEN FSKIN=DELTAD*COH/DELTAA ELSE FSKIN=0
    OPTPAR = FSKIN
  END IF
  GOTO 1987
END IF ' - Continue for limit equilibrium solution
```

```
'----- Compute Locations & Orientations of Forces -----
```

```
OPTN=1 : R=R1 : CX=C1X : CY=C1Y          '1st Circle
ALPHA=ALPH1 : DALPHA=DALPH1 : THETA=THET1 : PTHETA=P1THET
GOSUB 3400 '[ETA]
ETA1=(ETA-PI) : K1X=KX : K1Y=KY : Q1X=QX : Q1Y=QY : QP1X=QPX : QP1Y=QPY
```

```
OPTN=2 : R=R2 : CX=C2X : CY=C2Y          '2nd Circle
ALPHA=ALPH2 : DALPHA=DALPH2 : THETA=THET2 : PTHETA=P2THET
GOSUB 3400 '[ETA]
ETA2=ETA : K2X=KX : K2Y=KY : Q2X=QX : Q2Y=QY : QP2X=QPX : QP2Y=QPY
```

```
OPTN=3 : R=R3 : CX=C3X : CY=C3Y : GX=GLX  '3rd Circle
ALPHA=ALPH3 : DALPHA=DALPH3 : THETA=THET3 : PTHETA=P3THET
GOSUB 3400 '[ETA]
ETA3=ETA : K3X=KX : K3Y=KY : Q3X=QX : Q3Y=QY : QP3X=QPX : QP3Y=QPY
```

```
SELECT CASE MECH
```

```
CASE 2
```

```
  ETA3 = ETA3-PI
```

```
CASE 3
```

```
  ETA1 = ETA1-PI : ETA2 = ETA2-PI
```

```
CASE 4
```

```
  ETA1 = ETA1-PI : ETA2 = ETA2-PI : ETA3 = ETA3-PI
```

```
CASE 5
```

```

ETA1 = ETA1-PI
CASE 6
  ETA2 = ETA2-PI : ETA3 = ETA3-PI
END SELECT
IF ETA1 < -PI THEN ETA1 = ETA1 + 2*PI

'      Constraints #7 & 9 ----->
CPTX=MX : CPTY=MY : VRX=Q3X : VRY=Q3Y : VFX=QP3X : VFY=QP3Y
GOSUB 3500 : CLCK7$ = CLCK$
CPTX=MX : CPTY=MY : VRX=Q2X : VRY=Q2Y : VFX=QP2X : VFY=QP2Y
GOSUB 3500 : CLCK9$ = CLCK$
SELECT CASE MECH
CASE 1
  IF CLCK7$="CNTR" OR CLCK9$="CLCK" THEN LIM=0 : CNUM=79 : GOTO 1987
CASE 2
  IF CLCK7$="CLCK" OR CLCK9$="CLCK" THEN LIM=0 : CNUM=79 : GOTO 1987
CASE 3
  IF CLCK7$="CNTR" OR CLCK9$="CNTR" THEN LIM=0 : CNUM=79 : GOTO 1987
CASE 4
  IF CLCK7$="CLCK" OR CLCK9$="CNTR" THEN LIM=0 : CNUM=79 : GOTO 1987
CASE 5
  IF CLCK7$="CNTR" OR CLCK9$="CLCK" THEN LIM=0 : CNUM=79 : GOTO 1987
CASE 6
  IF CLCK7$="CLCK" OR CLCK9$="CNTR" THEN LIM=0 : CNUM=79 : GOTO 1987
END SELECT

'----- Compute Moment Arms -----
ORTHO=0 : X0=Q3X : Y0=Q3Y : X1=Q1X : Y1=Q1Y : X2=QP1X : Y2=QP1Y
GOSUB 3100 : ARMLF=DIST      '[Subroutine DISTANCE]
ORTHO=1 : X0=Q3X : Y0=Q3Y : X1=Q1X : Y1=Q1Y : X2=C1X : Y2=C1Y
GOSUB 3100 : ARMLT=DIST      '[Subroutine DISTANCE]
M=TAN(ETA3) : ARMRF = ABS(M*(C2X-Q3X) + Q3Y - C2Y)/SQR(1 + M^2)
M=TAN(THET3) : ARMRT = ABS(M*(C2X-Q3X) + Q3Y - C2Y)/SQR(1 + M^2)

CPTx=Q3x : CPTy=Q3y : VRx=Q1x : VRY=Q1y : OMEGA = (ETA1+PI)
GOSUB 3501 : IF CLCK$="CNTR" THEN ARMLF = -ARMLF
OMEGA = (THET1+PI)
GOSUB 3501 : IF CLCK$="CNTR" THEN ARMLT = -ARMLT
CPTx=C2x : CPTy=C2y : VRX=Q3X : VRY=Q3Y : OMEGA = ETA3
GOSUB 3501 : IF CLCK$="CNTR" THEN ARMRF = -ARMRF
OMEGA = THET3
GOSUB 3501 : IF CLCK$="CNTR" THEN ARMRT = -ARMRT

'----- Compute Forces -----
GOSUB 6200 '[MATRIX-IN]
GOSUB 3700 '[GAUSS]
F1 = MAT(7,1) : F2 = MAT(7,3) : F12 = MAT(7,5)
T1 = MAT(7,2) : T2 = MAT(7,4) : T12 = MAT(7,6)

IF T1<0 AND T2<0 AND T12<0 THEN
  IF F1>0 AND F2>0 AND F12>0 THEN
    CLS : LOCATE 12,10 : COLOR 28,0

```

```

      PRINT"Mechanism is stable with FS>1 for zero cohesion!!"
Trap19:
      IF INKEY$ = "" THEN Trap19:
      RETURN 150
      END IF
      END IF

      IF T1<0 OR T2<0 OR T12<0 THEN
      IF RN$ <> "SEARCH" THEN LIM=0 : CNUM=5 : GOTO 1987 'Constraints #5
      ELSEIF F1<0 OR F2<0 OR F12<0 THEN
      IF RN$ <> "SEARCH" THEN LIM=0 : CNUM=5 : GOTO 1987 'Constraints #5
      END IF
1901 GOTO 1960

      '----- Compute Factors of Safety -----
1950 'MINI-SUB
      FSKIN = VEL12*T12*ALPH1/SIN(ALPH1) + VEL1*T2*ALPH2/SIN(ALPH2)
      FSKIN = (FSKIN + VEL2*T1*ALPH3/SIN(ALPH3))/(-VELO*Q)
      FSCL1 = (2*R1*SIN(ALPH1)*COH)/T12
      FSCL2 = (2*R2*SIN(ALPH2)*COH)/T2
      FSCL3 = (2*R3*SIN(ALPH3)*COH)/T1
      IF TRAP$ = "ON" THEN
      FSCOH = (FSCL2*T2 + FSCL3*T1)/(T2 + T1)
      ELSE
      FSCOH = (FSCL1*T12 + FSCL2*T2 + FSCL3*T1)/(T12 + T2 + T1)
      END IF

      IF FSCL3 > FSCL2 THEN V1 = FSCL3 ELSE V1 = FSCL2
      IF (TRAP$ = "OFF") AND (FSCL1 > V1) THEN V1 = FSCL1
      FSLIMM = V1 'minimization parameter

      V1 = ABS(FSCL3 - 1)
      IF ABS(FSCL2-1) > V1 THEN V1 = ABS(FSCL2-1)
      IF ABS(FSCL1-1) > V1 AND TRAP$="OFF" THEN V1 = ABS(FSCL1-1)
      FSLIMC = V1 'convergence parameter

      RETURN
      '-----
1960 IF RN$ = "SEARCH" THEN
      IF F1 < 0 THEN NFOR = F1 ELSE NFOR = 0
      IF F2 < 0 THEN NFOR = NFOR + F2
      IF F12 < 0 THEN NFOR = NFOR + F12
      IF T1 < 0 THEN NFOR = NFOR + T1
      IF T2 < 0 THEN NFOR = NFOR + T2
      IF T12 < 0 THEN NFOR = NFOR + T12
      OPTPAR = -NFOR : GOTO 1987
      ELSEIF RN$ = "RUN" THEN
      GOSUB 1950
      ELSEIF RN$ = "OPTI" THEN
      IF OP$ = "MINIMIZE" THEN
      GOSUB 1950 : OPTPAR = FSLIMM
      ELSEIF OP$ = "CONVERGE" THEN

```

```

      GOSUB 1950 : OPTPAR = FSLIMC
    ELSEIF OP$ = "CONV1" THEN
      GOSUB 1950 : OPTPAR = FSLIMC : OPT1 = OPTPAR
    ELSEIF OP$ = "COHESION" THEN
      GOSUB 1950 : COH = COH/FSCOH
      GOSUB 1950 : OPTPAR = FSLIMM : OPT1 = OPTPAR
    ELSE
      PRINT "OP$ VARIABLE = ?" : STOP
    END IF
  END IF
1987 KEY(1) OFF : IF RN$="OPTI" THEN LOCATE 12,25 : PRINT RNNUM
RETURN

*****

2000 'Subroutine OPTIMIZE
  DELTA = 1 : OP$ = "MINIMIZE"
  RNNUM=0 : UPDATE = 0 : NUMPR=0 : GOSUB 1000
  GOSUB 1000
  IF LIM=0 THEN PRINT"NON-ADMISSIBLE STARTING POINT" : RETURN
  OPT1 = OPTPAR : OPT2 = OPTPAR : GOSUB 8200 : GOSUB 5100
  ,
  OP$ = "COHESION" : GOSUB 1000 : OP$ = "MINIMIZE"           '[OPTI-COH]
2030 UPDATE = 0 : IF KINMAT=1 THEN 2034
2031 GOSUB 2470                                               '[OPTI-Q1]
  GOSUB 2480                                                 '[OPTI-Q2]
  GOSUB 2490 : IF UPDATE=1 THEN UPDATE=0 : GOTO 2031        '[OPTI-Q3]
2034 UPDATE = 0 : GOSUB 2500                                  '[OPTI-C1x]
  GOSUB 2600 : IF UPDATE=1 THEN UPDATE=0 : GOTO 2030        '[OPTI-C2x]
  UPDATE = 0 : GOSUB 2700                                     '[OPTI-C1y]
  GOSUB 2800 : IF UPDATE=1 THEN UPDATE=0 : GOTO 2030        '[OPTI-C2y]

  IF OP$ = "MINIMIZE" THEN
    OP$ = "CONV1" : GOSUB 1000 : OP$ = "CONVERGE"
  ELSEIF OP$ = "CONVERGE" THEN
    OP$ = "COHESION" : GOSUB 1000 : OP$ = "MINIMIZE"
  ELSE
    PRINT "OP$ variable = ?" : STOP
  END IF

  GOSUB 5100 'PARTIAL PRT]
  IF OP$ = "MINIMIZE" THEN DELTA = DELTA/2
  IF DELTA > (.0001/2) THEN 2030
  ,
  BEEP : BEEP
  LOCATE 5,60 : PRINT" Done "
  LOCATE 23,10 : PRINT"Hit any Key to Return"
2170 IF INKEY$ = "" THEN 2170
RETURN [return]

*****

```


2200 'Subroutine SEARCH

```

RNNUM=0 : UPDATE = 0 : DELTA = 1 : NUMPR=0 : GOSUB 1000
IF LIM=0 THEN PRINT"NON-ADMISSIBLE STARTING POINT"
IF LIM=0 THEN PRINT"Constraint # =";CNUM : RETURN
OPT1 = OPTPAR : OPT2 = OPTPAR

```

2230 IF NFOR = 0 THEN PRINT"ADMISSIBLE POINT FOUND" : RETURN

```

GOSUB 2470 '[OPTI-Q1]
IF UPDATE=1 THEN UPDATE=0 : GOTO 2230
GOSUB 2480 '[OPTI-Q2]
IF UPDATE=1 THEN UPDATE=0 : GOTO 2230
GOSUB 2490 '[OPTI-Q3]
IF UPDATE=1 THEN UPDATE=0 : GOTO 2230
GOSUB 2500 '[OPTI-C1x]
IF UPDATE=1 THEN UPDATE=0 : GOTO 2230
GOSUB 2600 '[OPTI-C2x]
IF UPDATE=1 THEN UPDATE=0 : GOTO 2230
GOSUB 2700 '[OPTI-C1y]
IF UPDATE=1 THEN UPDATE=0 : GOTO 2230
GOSUB 2800 '[OPTI-C2y]
IF UPDATE=1 THEN UPDATE=0 : GOTO 2230
PRINT"NFOR=";NFOR;" RNNUM=";RNNUM;" DELTA=";DELTA
DELTA = DELTA/2 : IF DELTA > (.001/2) THEN 2230

```

```

BEEP : BEEP : PRINT"NOT FOUND" : RETURN '[return]

```

```

*****

```

2400 'Subroutine OPTI-A3

```

UPDATE=0

```

2410 A3X = A3X + DELTA : GOSUB 1000 'Forward Try

```

IF (LIM=1) AND (OPTPAR<OPT2) THEN
OPT2=OPTPAR : UPDATE=1 : GOTO 2410
END IF

```

```

A3X = A3X - DELTA : IF UPDATE=1 THEN RETURN

```

2460 A3X = A3X - DELTA : GOSUB 1000 'Backward Try

```

IF (LIM=1) AND (OPTPAR<OPT2) THEN
OPT2=OPTPAR : UPDATE=1 : GOTO 2460
END IF

```

```

A3X = A3X + DELTA : RETURN

```

```

*****

```

2470 'Subroutine OPTI-Q1

```

OPT2=OPT1

```

```

DALPH1 = DALPH1 + 5*DELTA*(PI/180) : GOSUB 2400 'Forward Try

```

```

IF (OPT2<OPT1) THEN OPT1=OPT2 : UPDATE=1 : RETURN

```

```

DALPH1 = DALPH1 -10*DELTA*(PI/180) : GOSUB 2400 'Backward Try

```

```

IF (OPT2<OPT1) THEN OPT1=OPT2 : UPDATE=1 : RETURN

```

```

,
5*DELTA*(PI/180) : RETURN 'Set back

```

```

*****

```

```

2480 'Subroutine OPTI-Q2

```

```

OPT2=OPT1
DALPH2 = DALPH2 + 5*DELTA*(PI/180) : GOSUB 2400 'Forward Try
IF (OPT2<OPT1) THEN OPT1=OPT2 : UPDATE=1 : RETURN
,
DALPH2 = DALPH2 -10*DELTA*(PI/180) : GOSUB 2400 'Backward Try
IF (OPT2<OPT1) THEN OPT1=OPT2 : UPDATE=1 : RETURN
,
DALPH2 = DALPH2 + 5*DELTA*(PI/180) : RETURN 'Set back

```

```

*****

```

```

2490 'Subroutine OPTI-Q3

```

```

OPT2=OPT1
DALPH3 = DALPH3 + 5*DELTA*(PI/180) : GOSUB 2400 'Forward Try
IF (OPT2<OPT1) THEN OPT1=OPT2 : UPDATE=1 : RETURN
,
DALPH3 = DALPH3 -10*DELTA*(PI/180) : GOSUB 2400 'Backward Try
IF (OPT2<OPT1) THEN OPT1=OPT2 : UPDATE=1 : RETURN
,
DALPH3 = DALPH3 + 5*DELTA*(PI/180) : RETURN 'Set back

```

```

*****

```

```

2500 'Subroutine OPTI-C1x

```

```

OPT2=OPT1
C1X = C1X+DELTA : GOSUB 2400 'Forward Try
IF (OPT2<OPT1) THEN OPT1=OPT2 : UPDATE=1 : RETURN
,
C1X = C1X - 2*DELTA : GOSUB 2400 'Backward Try
IF (OPT2<OPT1) THEN OPT1=OPT2 : UPDATE=1 : RETURN
,
C1X = C1X + DELTA : RETURN 'Set back to initial

```

```

*****

```

```

2600 'Subroutine OPTI-C2x

```

```

OPT2=OPT1
C2X = C2X+DELTA : GOSUB 2400 'Forward Try
IF (OPT2<OPT1) THEN OPT1=OPT2 : UPDATE=1 : RETURN
,
C2X = C2X - 2*DELTA : GOSUB 2400 'Backward Try
IF (OPT2<OPT1) THEN OPT1=OPT2 : UPDATE=1 : RETURN
,
C2X = C2X + DELTA : UPDATE=0 : RETURN 'Set back to initial

```

```

*****

```

```

2700 'Subroutine OPTI-C1y
  OPT2=OPT1
  C1Y = C1Y+DELTA      : GOSUB 2400          'Forward Try
  IF (OPT2<OPT1) THEN OPT1=OPT2 : UPDATE=1 : RETURN
  ,
  C1Y = C1Y - 2*DELTA  : GOSUB 2400          'Backward Try
  IF (OPT2<OPT1) THEN OPT1=OPT2 : UPDATE=1 : RETURN
  ,
  C1Y = C1Y + DELTA    : RETURN              'Set back to initial

*****

2800 'Subroutine OPTI-C2y
  OPT2=OPT1
  C2Y = C2Y + DELTA    : GOSUB 2400          'Forward Try
  IF (OPT2<OPT1) THEN OPT1=OPT2 : UPDATE=1 : RETURN
  ,
  C2Y = C2Y - 2*DELTA  : GOSUB 2400          'Backward Try
  IF (OPT2<OPT1) THEN OPT1=OPT2 : UPDATE=1 : RETURN
  ,
  C2Y = C2Y + DELTA    : UPDATE=0 : RETURN    'Set back to initial

*****

3000 'Subroutine ANGLE360 - calculates angle w.r.t. 360 degrees
  ' Input - [pt1,pt2]      Output - [psi]
  RISE=PT1Y-PT2Y : HOR=PT1X-PT2X
  IF HOR = 0 AND RISE > 0 THEN PSI = PI/2 : RETURN
  IF HOR = 0 AND RISE < 0 THEN PSI = 3*PI/2 : RETURN
  PSI=ATN(ABS(RISE/HOR))
  IF RISE>0 THEN
    IF HOR>0 THEN PSI=PSI ELSE PSI=PI-PSI
  ELSE
    IF HOR>0 THEN PSI=2*PI-PSI ELSE PSI=PI+PSI
  END IF
  RETURN

*****

3100 'Subroutine DISTANCE
  'Input - ORTHO - 0 if dist to line; 1 if dist to othogonal line
  '      (X0,Y0)-the point
  '      (X1,Y1)-pt. that line passes through
  '      (X2,Y2)-2nd point on line or orthogonal line
  'Output - DIST, (X,Y)
  IF ORTHO=0 THEN
    M=(Y2-Y1)/(X2-X1) : B=Y1-M*X1
    X=((B-Y0)*(Y1-Y2) + X0*(X2-X1))/(M*(Y2-Y1) + X2 - X1)
    Y=M*X + B : DIST=SQR((X-X0)^2 + (Y-Y0)^2)
  ELSE
    M=(X1-X2)/(Y2-Y1) : B=Y1-M*X1
    X=((B-Y0)*(Y1-B) - X0*X1)/(M*(B-Y1) - X1)

```

```

    Y=M*X + B : DIST=SQR((X-X0)^2 + (Y-Y0)^2)
END IF
RETURN

```

```

*****

```

3200 'Subroutine TRIANGLE

```

'Input - POINTS (X1,Y1) (X2,Y2) (X3,Y3)
'Output - area, center or gravity
DX(1) = ABS(X2-X1) : DX(2) = ABS(X3-X2) : DX(3) = ABS(X1-X3)
DY(1) = ABS(Y2-Y1) : DY(2) = ABS(Y3-Y2) : DY(3) = ABS(Y1-Y3)

```

```

DXMAX = 0 : DYMAX = 0 : TEMP = 0
FOR I = 1 TO 3
  IF DX(I) > DXMAX THEN DXMAX = DX(I)
  IF DY(I) > DYMAX THEN DYMAX = DY(I)
  TEMP = TEMP + DX(I)*DY(I)
NEXT I

```

```

AREA = DXMAX*DYMAX - TEMP/2
GX = (X1 + X2 + X3)/3 : GY = (Y1 + Y2 + Y3)/3
RETURN

```

```

*****

```

3300 'subroutine ARC - calculates area & centroid of circular arcs

```

'input: [P1,P2,P3,R]      output: [A,GX,GY,THETA,ALPHA]
V1=SQR((P1X-P2X)^2 + (P1Y-P2Y)^2)
IF (V1/2)/R => 1 THEN LIM=0 : GOTO 1987      'precision constraint
ALPHA=(V1/2)/R : ALPHA=ATN(ALPHA/SQR(1-ALPHA^2)) ' [arcsin]
PT1X=P1X : PT1Y=P1Y : PT2X=P2X : PT2Y=P2Y
GOSUB 3000 : THETA=PSI                       '[ANGLE360]
V3=(2/3)*R*SIN(ALPHA)^3/(ALPHA-SIN(ALPHA)*COS(ALPHA))
A=R^2*(ALPHA-SIN(2*ALPHA)/2)
PT1X=(P1X+P2X)/2 : PT1Y=(P1Y+P2Y)/2 : PT2X=P3X : PT2Y=P3Y
GOSUB 3000                                    '[angle360]
GX = P3X + ABS(V3*SIN(THETA))*SGN(COS(PSI))
GY = P3Y + ABS(V3*COS(THETA))*SGN(SIN(PSI))
RETURN

```

```

*****

```

3400 ' subroutine ETA - calculate angle eta

```

' Input: [theta,R,(Cx,Cy),alpha,Gx]      Output: [eta,K,Q,QP]
RT = R*ALPHA/SIN(ALPHA)
RPHI = R*SIN(PHI) : IF PHI=0 THEN RPHI=1E-09
KX = CX + ABS(RT*SIN(THETA))*SGN(COS(PTHETA))
KY = CY + ABS(RT*COS(THETA))*SGN(SIN(PTHETA))

TEMP = R*(ALPHA/SIN(ALPHA))*TAN(DALPHA)
QX = KX + TEMP*COS(THETA) : QY = KY + TEMP*SIN(THETA)
B = SQRT((CX-QX)^2 + (CY-QY)^2)

```

```

OMEGA = ATN(SQR(1-(RPHI/B)^2)/(RPHI/B))           'arccos(R/B)
IF QX=CX THEN OMEGB=PI/2 : GOTO 3410
OMEGB = ATN((CY-QY)/(QX-CX)) : IF QX<CX THEN OMEGB = PI+OMEGB
3410  SELECT CASE OPTN
      CASE 1
        QPX=CX-RPHI*COS(PI-OMEGB-OMEGA) : QPY = CY-RPHI*SIN(PI-OMEGB-OMEGA)
      CASE 2
        QPX = CX+RPHI*COS(OMEGB-OMEGA) : QPY = CY-RPHI*SIN(OMEGB-OMEGA)
      CASE 3
        QPX = CX+RPHI*COS(OMEGB-OMEGA) : QPY = CY-RPHI*SIN(OMEGB-OMEGA)
      END SELECT

```

```

PT1X=QPX : PT1Y=QPY : PT2X=QX : PT2Y=QY
GOSUB 3000 : ETA=PSI           '[ANGLE360]
RETURN
,

```

```

*****

```

```

3500 'Subroutine CLOCKWISE

```

```

'Input : (CPTx,CPTy) - center pt.
'        (VFX,VFY)  - vector front
'        (VRx,VRy)  - vector rear
'output : CLCK$ = "CLCK wise" or "CNTR clockwise"

```

```

IF VFY=VRY THEN
  IF (VFY-CPTY) > (VRY-CPTY) THEN CLCK$="CNTR" ELSE CLCK$="CLCK"
  RETURN
END IF
IF VFX=VRX THEN
  B = VFX
ELSE
  M = (VFY-VRY)/(VFX-VRX) : B = VFX - (VFY-CPTY)/M
END IF
IF (B-CPTX) > 0 AND SGN(VFY-CPTY) <> SGN(VRY-CPTY) THEN
  IF (VFY-CPTY) > (VRY-CPTY) THEN CLCK$="CNTR" ELSE CLCK$="CLCK"
  RETURN
ELSE
  PT2X=CPTX : PT2Y=CPTY : PT1X=VFX : PT1Y=VFY : GOSUB 3000 : OMEGF=PSI
  PT1X=VRX : PT1Y=VRY : GOSUB 3000 : OMEGR=PSI
  IF (OMEGR - OMEGF) > 0 THEN CLCK$="CLCK" ELSE CLCK$="CNTR"
  RETURN
END IF
,-----

```

```

3501 'Subroutine CLOCKWISE - 2nd Option

```

```

'Input : (CPTx,CPTy) - center pt.
'        (VRx,VRy) ; OMEGA - vector point & orientation
'output : CLCK$ = "CLCK wise" or "CNTR clockwise"

```

```

IF OMEGA = 0 THEN
  IF (CPTy > VRy) THEN CLCK$="CNTR" ELSE CLCK$="CLCK"
  RETURN
ELSEIF OMEGA = PI THEN

```

```

    IF (CPTy < VRy) THEN CLCK$="CNTR" ELSE CLCK$="CLKK"
    RETURN
END IF
IF OMEGA = > PI THEN OMEGA = OMEGA - 2*PI
IF OMEGA = PI/2 THEN
    IF (CPTx < VRx) THEN CLCK$="CNTR" ELSE CLCK$="CLKK"
    RETURN
ELSEIF OMEGA = -PI/2 THEN
    IF (CPTx > VRx) THEN CLCK$="CNTR" ELSE CLCK$="CLKK"
    RETURN
END IF

```

```

M = TAN(OMEGA) : B = VRy - M*VRx
IF ABS(OMEGA) < PI/2 THEN
    IF (CPTy > CPTX*M + B) THEN CLCK$="CNTR" ELSE CLCK$="CLKK"
ELSE
    IF (CPTy < CPTX*M + B) THEN CLCK$="CNTR" ELSE CLCK$="CLKK"
END IF
RETURN

```

3600 'Subroutine HODOGRAPH

```

'INPUT: MECH, PI, THET1-2-3, ALPH1-2-3, C1 & C3
'LOCAL: ANGLES: LIN-M1-M2-M3-B1-D2-A3
'      POINTS: C1xHD, M2xHD, M3xHD, B2xHD, B3xHD,D2xHD,A3xHD
'OUTPUT: VEL-0-1-2-12
'-----

```

```

'Calculate Angles:
IF C1X=C3X THEN
    ANGLIN = PI
ELSE
    ANGLIN = ATN((C1Y-C3Y)/(C1X-C3X)) + PI/2
END IF

```

SELECT CASE MECH

CASE 1

```

    ANGM1 = THET1 - ALPH1 + PI : ANGB1 = THET1 + ALPH1 + PI
    ANGM2 = THET2 - ALPH2 + PI ; ANGD2 = THET2 + ALPH2 + PI
    ANGM3 = THET3 + ALPH3 - PI : ANGA3 = THET3 - ALPH3 - PI
    ANGWR = ANGWR - PI/2      : ANGWL = ANGWL - PI/2

```

CASE 2

```

    ANGM1 = THET1 - ALPH1 + PI : ANGB1 = THET1 + ALPH1 + PI
    ANGM2 = THET2 - ALPH2 + PI : ANGD2 = THET2 + ALPH2 + PI
    ANGM3 = THET3 - ALPH3 - PI : ANGA3 = THET3 + ALPH3 - PI
    ANGWR = ANGWR - PI/2      : ANGWL = ANGWL + PI/2

```

CASE 3

```

    ANGM1 = THET1 + ALPH1 + PI : ANGB1 = THET1 - ALPH1 + PI
    ANGM2 = THET2 + ALPH2 + PI : ANGD2 = THET2 - ALPH2 + PI
    ANGM3 = THET3 + ALPH3 - PI : ANGA3 = THET3 - ALPH3 - PI
    ANGWR = ANGWR + PI/2      : ANGWL = ANGWL - PI/2

```

CASE 4

ANGM1 = THET1 + ALPH1 + PI : ANGB1 = THET1 - ALPH1 + PI
 ANGM2 = THET2 + ALPH2 + PI : ANGD2 = THET2 - ALPH2 + PI
 ANGM3 = THET3 - ALPH3 - PI : ANGA3 = THET3 + ALPH3 - PI
 ANGWR = ANGWR + PI/2 : ANGWL = ANGWL + PI/2

CASE 5

ANGM1 = THET1 + ALPH1 : ANGB1 = THET1 - ALPH1
 ANGM2 = THET2 - ALPH2 + PI : ANGD2 = THET2 + ALPH2 + PI
 ANGM3 = THET3 + ALPH3 - PI : ANGA3 = THET3 - ALPH3 - PI
 ANGWR = ANGWR - PI/2 : ANGWL = ANGWL - PI/2

CASE 6

ANGM1 = THET1 - ALPH1 + PI : ANGB1 = THET1 + ALPH1 + PI
 ANGM2 = THET2 + ALPH2 + PI : ANGD2 = THET2 - ALPH2 + PI
 ANGM3 = THET3 - ALPH3 - PI : ANGA3 = THET3 + ALPH3 - PI
 ANGWR = ANGWR + PI/2 : ANGWL = ANGWL + PI/2

END SELECT

'Assume origin of hodograph @ (0,0) & Hr2 = 10

'Find points on first wedge (M2, B2, D2):

HR2 = 10

D2XHD = HR2*COS(ANGD2) : D2YHD = HR2*SIN(ANGD2)

M2XHD = HR2*COS(ANGM2) : M2YHD = HR2*SIN(ANGM2)

M = TAN(ANGLIN) : B = 0

V1 = TAN(ANGM1) : TEMP1 = M2YHD - V1*M2XHD

C1XHD = TEMP1/(M-V1) : C1YHD = M*C1XHD + B

HR1 = SQR((C1XHD-M2XHD)^2 + (C1YHD-M2YHD)^2)

B2XHD = C1XHD + HR1*COS(ANGB1) : B2YHD = C1YHD + HR1*SIN(ANGB1)

'Find points on 2nd wedge (M3, A3, B3):

M = TAN(ANGM3) : B = 0

V1 = TAN(ANGM1) : TEMP1 = M2YHD - V1*M2XHD

M3XHD = TEMP1/(M-V1) : M3YHD = M*M3XHD + B

HR3 = SQR(M3XHD^2 + M3YHD^2)

A3XHD = HR3*COS(ANGA3) : A3YHD = HR3*SIN(ANGA3)

HR1 = SQR((C1XHD-M3XHD)^2 + (C1YHD-M3YHD)^2)

IF MECH=2 OR MECH=3 THEN ANGB1 = ANGB1-PI

B3XHD = C1XHD + HR1*COS(ANGB1) : B3YHD = C1YHD + HR1*SIN(ANGB1)

'Check Constraints - enclosure around ea. wedge

IF BETA = 0 THEN

TEMP = B3XHD

ELSE

M = 1/TAN(BETA+PI/2) : B = B3XHD - M*(B3YHD + BF*HR3/R3)

TEMP = M*A3YHD + B

END IF

IF ABS(A3XHD-TEMP) > .01 THEN LIM=0 : RETURN 'left wedge segment

IF MECH=1 OR MECH=2 OR MECH=5 THEN

TEMP = D2XHD + EMBED*HR2/R2

ELSE

TEMP = D2XHD - EMBED*HR2/R2

END IF

IF ABS(B2XHD-TEMP) > .01 THEN LIM=0 : RETURN 'right wedge segment

VELQ = (B2YHD + D2YHD)/2 + E*(D2YHD - B2YHD)/(B2X - B1X)

VEL1 = SQR(M2XHD^2 + M2YHD^2) : VEL2 = SQR(M3XHD^2 + M3YHD^2)

VEL12 = SQR((M3XHD-M2XHD)^2 + (M3YHD-M2YHD)^2)

VELWR = RWR*(HR2/R2)*SIN(ANGWR) : VELWL = RWL*(HR3/R3)*SIN(ANGWL)

RETURN

,

3700 'Subroutine MATRIX - gauss elimination by pivot points

'MAT(i,j) - square matrix of coefficients

'B(i) - vector of constants on right side of equations

'X(i) - solution vector ([MAT]{X} = {B})

,

N=6

FOR I=1 TO N : MAT(7,I)=B(I) : NEXT I

FOR PVR = 1 TO N

PPT = MAT(PVR,PVR)

FOR I = 1 TO (N+1) : MAT(I,PVR) = MAT(I,PVR)/PPT : NEXT I

PPT = MAT(PVR,PVR)

FOR J = 1 TO N

IF J = PVR THEN 3710

ELIM = MAT(PVR,J)

FOR I = 1 TO (N+1)

MAT(I,J) = PPT*MAT(I,J) - ELIM*MAT(I,PVR)

NEXT I

3710 'ENDIF

NEXT J

NEXT PVR

RETURN

4000 'Subroutine INTERACTIVE INPUT

KEY OFF : CLS : GOSUB 6000 : ROW=1 : I=0 : DOT=0

GOSUB 8100 '[input screen setup]

KEY(11) ON : ON KEY(11) GOSUB 4020 'cursor up

KEY(14) ON : ON KEY(14) GOSUB 4030 'cursor down

COLOR 14,0 : FOR ROW = 2 TO 18

LOCATE LY(ROW),LX(ROW) : PRINT USING"####.#### ";VAR(ROW)

NEXT ROW : ROW = 1 : COLOR 0,7

LOCATE LY(ROW),LX(ROW) : PRINT USING"####.#### ";VAR(ROW)

COLOR 7,0

4010 V\$ = INKEY\$: IF V\$ = "" THEN 4010

RST\$ = "Y" : GOTO 4060

,

```

'mini-Subroutine KEYOPTION
4020 LOCATE LY(ROW),LX(ROW)                'cursor up
      IF ROW = 19 THEN PRINT"RETURN"
      IF ROW < 19 THEN PRINT USING"####.####" ;VAR(ROW)
      IF ROW <> 1 THEN ROW = ROW - 1
      RST$="Y" : RETURN 4040
,
4030 IF ROW=19 THEN RETURN 4040           'cursor down
      LOCATE LY(ROW),LX(ROW) : PRINT USING"####.####" ;VAR(ROW)
      ROW = ROW + 1 : RST$="Y" : RETURN 4040
,-----
4040 COLOR 0,7 : LOCATE LY(ROW),LX(ROW)
      IF ROW < 19 THEN PRINT USING"####.####" ;VAR(ROW)
      IF ROW = 19 THEN PRINT" RETURN ";
      COLOR 7,0
,
4050 V$=INKEY$ : IF V$="" THEN 4050
4060 IF LEN(V$)=2 THEN V$ = RIGHT$(V$,1)
      IF ASC(V$) <> 13 THEN
          IF RST$="Y" THEN VAR(ROW)=1E-30 : DOT=0 : RST$="N" : NEGA=1
          CHCK$ = "NUM"
          IF ASC(V$) < 45 THEN CHCK$ = "LET" : BEEP
          IF ASC(V$) > 57 THEN CHCK$ = "LET" : BEEP
          IF ASC(V$) = 47 THEN CHCK$ = "LET" : BEEP
          IF ASC(V$) = 46 THEN CHCK$ = "DOT" : DOT = DOT + 1
          IF ASC(V$) = 45 THEN CHCK$ = "MIN" : NEGA = -NEGA
,
          IF DOT=0 AND CHCK$="NUM" THEN VAR(ROW) = VAR(ROW)*10 + NEGA*VAL(V$)
          IF DOT=1 AND CHCK$="DOT" THEN I = 0
          IF DOT>0 AND CHCK$="NUM" THEN I=I-1 : VAR(ROW)=VAR(ROW)+NEGA*VAL(V$)*10^I
          IF DOT>1 AND CHCK$="DOT" THEN BEEP
          IF CHCK$ = "MIN" THEN VAR(ROW) = -VAR(ROW)
          GOTO 4040
      END IF
,
      IF ROW=19 THEN
          KEY(11) OFF : KEY(14) OFF
      FOR I = 1 TO 15
          IF ABS(VAR(I)) < 1E-20 THEN VAR(I) = FIX(VAR(I))
      NEXT I
          GOSUB 6100 : RETURN
      END IF
      LOCATE LY(ROW),LX(ROW) : PRINT USING"####.####" ;VAR(ROW)
      RST$="Y" : ROW = ROW + 1
      GOTO 4040

*****

4100 'Subroutine FILE INPUT
      COLOR 0,7,1 : LOCATE 23,10 : INPUT " d:fname.ext = "; F$
      COLOR 7,0 : LOCATE 23,10 : PRINT "
      OPEN F$ FOR INPUT AS #1

```

```

INPUT #1, GAMA, COH, PHI, FSPHI
INPUT #1, BETA, BF, BW
INPUT #1, Q, E, EMBED
INPUT #1, C1X, C1Y, C2X, C2Y
INPUT #1, A3X,MECH
INPUT #1, DALPH1,DALPH2,DALPH3
CLOSE #1
  PHI=PHI*(PI/180)   :   BETA = BETA*(PI/180)
  IF GAMA=0 THEN GAMA = 1E-30
RETURN

```

4200 'Subroutine file OUTput

```

  PHI=PHI*(180/PI)   :   BETA = BETA*(180/PI)
OPEN "OPTIC.LOG" FOR OUTPUT AS #2
WRITE #2, GAMA, COH, PHI, FSPHI
WRITE #2, BETA, BF, BW
WRITE #2, Q, E, EMBED
WRITE #2, C1X, C1Y, C2X, C2Y
WRITE #2, A3X,MECH
WRITE #2, DALPH1,DALPH2,DALPH3
CLOSE #2
  PHI=PHI*(PI/180)   :   BETA = BETA*(PI/180)
RETURN

```

5000 'Subroutine PRINT

```

CLS
LOCATE 3,20 : PRINT"NUMBER OF ITERATIONS = ";RNNUM
LOCATE 4,10 : PRINT"Mechanism #";MECH
LOCATE 4,40 : PRINT DATE$,TIME$
LOCATE 5,5 : PRINT

```

```

" _____
|
| "
|   FOR I=6 TO 15
|     LOCATE I,5 : PRINT" | " : LOCATE I,75 : PRINT" | "
|   NEXT I
|   LOCATE 15,5 : PRINT
" _____
|
| "

```

```

LOCATE 17,1 : PRINT "Geometry:"
PRINT"Circle # ----- (CX,CY) ----- R ---- ALPHA --- THETA -- ETA--"
PRINT" 1      (;"
PRINT USING"###.###,";C1X;      :   PRINT USING"###.###) ";C1Y;
PRINT USING"###.### ";R1;ALPH1*(180/PI);THET1*(180/PI);ETA1*(180/PI)
PRINT" 2      (;"
PRINT USING"###.###,";C2X;      :   PRINT USING"###.###) ";C2Y;
PRINT USING"###.### ";R2;ALPH2*(180/PI);THET2*(180/PI);ETA2*(180/PI)
PRINT" 3      (;"
PRINT USING"###.###,";C3X;      :   PRINT USING"###.###) ";C3Y;

```

```
PRINT USING"###.###";R3;ALPH3*(180/PI);THET3*(180/PI);ETA3*(180/PI)
```

```
LOCATE 6,10 : PRINT "Factors of Safety: "
LOCATE 7,16 : PRINT USING"Limiting Cohesion = ###.###";FSCL1
LOCATE 7,46 : PRINT USING"###.###";FSCL2
LOCATE 7,58 : PRINT USING"###.###";FSCL3
LOCATE 8,16 : PRINT USING"Kinetic Cohesion = ###.###";FSKIN
LOCATE 10,8 : PRINT "Footing Load,Eccentricity & Embedment = "
LOCATE 10,45 : PRINT USING"#####.###";Q;E;EMBED
LOCATE 11,8 : PRINT "Slope Angle = "
LOCATE 11,35 : PRINT USING"###.###, ";BETA*(180/PI)
LOCATE 12,8 : PRINT"Width, & Distance from Crest = "
LOCATE 12,40 : PRINT USING"#####.###";BW,BF
LOCATE 14,10 : PRINT "Soil Weight, cohesion, & Friction Angle = "
LOCATE 14,49 : PRINT USING"#####.###";GAMA;COH;PHI*(180/PI)
```

```
COLOR 29 : LOCATE 24,50 : PRINT"Press any key to continue" : COLOR 2,0
```

```
TrapA:
```

```
IF INKEY$="" THEN TrapA:
```

```
CLS
```

```
PRINT "Summary of Areas & Weights:"
```

```
PRINT "Region # --- Area ---- (Center of Gravity) ----- weight ----"
```

```
H$(1)="ARC 1 " : H$(2)="ARC 2 " : H$(3)="ARC 3 "
```

```
H$(4)="Rt Triag" : H$(5)="Lt Poly "
```

```
H$(6)="Rt Wedge" : H$(7)="Lt Wedge"
```

```
A(6)=AR : GX(6)=GRX : GY(6)=GRY : A(7)=AL : GX(7)=GLX : GY(7)=GLY
```

```
FOR I=1 TO 7
```

```
PRINT H$(I);
```

```
PRINT USING"###.###" (;A(I);
```

```
PRINT USING"###.###";GX(I); : PRINT USING"###.###";GY(I);
```

```
PRINT USING"###.###";A(I)*GAMA
```

```
NEXT I
```

```
PRINT "summary of forces and force geometry:"
```

```
PRINT"Circle # -- force # -- Friction Force --- Cohesion Force -----"
```

```
PRINT " 1 12 ";
```

```
PRINT USING"###.###"; F12;T12
```

```
PRINT " 2 2 ";
```

```
PRINT USING"###.###"; F2;T2
```

```
PRINT " 3 1 ";
```

```
PRINT USING"###.###"; F1;T1
```

```
PRINT "Circle # ----- pt K ----- pt Q ----- pt QP ----"
```

```
PRINT" 1 ";
```

```
PRINT USING"(###.###"; K1X; : PRINT USING"###.###" "; K1Y;
```

```
PRINT USING"(###.###"; Q1X; : PRINT USING"###.###" "; Q1Y;
```

```
PRINT USING"(###.###"; QP1X; : PRINT USING"###.###"; QP1Y
```

```
PRINT" 2 ";
```

```
PRINT USING"(###.###"; K2X; : PRINT USING"###.###" "; K2Y;
```

```
PRINT USING"(###.###"; Q2X; : PRINT USING"###.###" "; Q2Y;
```

```
PRINT USING"(###.###"; QP2X; : PRINT USING"###.###"; QP2Y
```

```
PRINT" 3 ";
```

```

PRINT USING"(###.###"; K3X; : PRINT USING"###.###) "; K3Y;
PRINT USING"(###.###"; Q3X; : PRINT USING"###.###) "; Q3Y;
PRINT USING"(###.###"; QP3X; : PRINT USING"###.###); QP3Y

```

```

PRINT USING"point A3 = (###.###"; A3X; : PRINT USING",###.###); A3Y
PRINT USING"point M = (###.###"; MX; : PRINT USING",###.###); MY
PRINT ARMRF,ARMRT,ARMLF,ARMLT

```

TrapB:

```
IF INKEY$="" THEN TrapB:
```

```
CLS : LOCATE 1,15 : PRINT"----- HODOGRAPH DATA: -----"
```

```
PRINT"Circle #      1          2          3"
```

```
PRINT"Center Pt." : PRINT"Point M"
```

```
PRINT"Point B" : PRINT"Point D" : PRINT"Point A"
```

```
PRINT"Angle M" : PRINT"Angle B" : PRINT"Angle D"
```

```
PRINT"Angle A"
```

```
LOCATE 3,12 : PRINT USING"###.###";C1XHD; : PRINT USING"###.###";C1YHD
```

```
LOCATE 3,36 : PRINT "(0,0)          (0,0)"
```

```
LOCATE 4,32 : PRINT USING"###.###";M2XHD; : PRINT USING"###.###";M2YHD
```

```
LOCATE 5,32 : PRINT USING"###.###";B2XHD; : PRINT USING"###.###";B2YHD
```

```
LOCATE 6,32 : PRINT USING"###.###";D2XHD; : PRINT USING"###.###";D2YHD
```

```
LOCATE 4,52 : PRINT USING"###.###";M3XHD; : PRINT USING"###.###";M3YHD
```

```
LOCATE 5,52 : PRINT USING"###.###";B3XHD; : PRINT USING"###.###";B3YHD
```

```
LOCATE 7,52 : PRINT USING"###.###";A3XHD; : PRINT USING"###.###";A3YHD
```

```
LOCATE 8,15 : PRINT USING"###.###";ANGM1*(180/PI);
```

```
LOCATE 8,34 : PRINT USING"###.###";ANGM2*(180/PI);
```

```
LOCATE 8,54 : PRINT USING"###.###";ANGM3*(180/PI);
```

```
LOCATE 9,15 : PRINT USING"###.###";ANGB1*(180/PI);
```

```
LOCATE 10,34 : PRINT USING"###.###";ANGD2*(180/PI);
```

```
LOCATE 11,54 : PRINT USING"###.###";ANGA3*(180/PI)
```

```
PRINT USING" Angle of circle centers line = ###.###";ANGLIN*(180/PI)
```

```
PRINT"-----"
```

```
PRINT USING"VELQ = ###.###";VELQ
```

```
PRINT USING"VEL1 = ###.###";VEL1
```

```
PRINT USING"VEL2 = ###.###";VEL2
```

```
PRINT USING"VEL12= ###.###";VEL12
```

```
PRINT USING"VELWR = ###.###";VELWR
```

```
PRINT USING"VELWL = ###.###";VELWL
```

```
PRINT : PRINT"τ = ";GAMA
```

```
COLOR 7,0
```

TrapC:

```
IF INKEY$="" THEN TrapC:
```

```
IF T1<0 THEN 5010
```

```
IF T2<0 THEN 5010
```

```
IF T12<0 THEN 5010
```

```
RETURN
```

5010 PRINT "****Equilibrium Cohesive Force is Negative....."

```
PRINT "Slope is Stable, but Factor of Safety is not Correct! ****"
```

TrapD:

```
IF INKEY$="" THEN TrapD:
```

```
RETURN
```

5100 'Subroutine PARTIAL-PRINT

```

NUMPR=NUMPR+1 : IF NUMPR>4 THEN NUMPR=4
FOR I=3 TO 1 STEP (-1)
  FSC(I+1)=FSC(I) : COH(I+1)=COH(I) : DELTA(I+1) = DELTA(I)
  FS1(I+1)=FS1(I) : FS2(I+1)=FS2(I) : FS3(I+1)=FS3(I) : FSK(I+1)=FSK(I)
NEXT I
FSC(1)=FSCOH : COH(1)=COH : DELTA(1) = DELTA
FS1(1)=FSCL1 : FS2(1)=FSCL2 : FS3(1)=FSCL3 : FSK(1)=FSKIN
,
FOR I=1 TO NUMPR
  LOCATE (I+15),8 : PRINT USING"###.###"; DELTA(I)
  LOCATE (I+15),18 : PRINT USING"###.###"; FS1(I)
  LOCATE (I+15),25 : PRINT USING"###.###"; FS2(I)
  LOCATE (I+15),31 : PRINT USING"###.###"; FS3(I)
  LOCATE (I+15),40 : PRINT USING"###.###";FSC(I)
  LOCATE (I+15),52 : PRINT USING"###.###";FSK(I)
  LOCATE (I+15),65 : PRINT USING"###.###"; COH(I)
NEXT I
RETURN

```

6000 'Subroutine ARRAY-IN

```

PHI = PHI * (180/PI) : BETA = BETA * (180/PI)
DALPH1=DALPH1*(180/PI) : DALPH2=DALPH2*(180/PI) : DALPH3=DALPH3*(180/PI)
VAR(1) = GAMA : VAR(2) = COH : VAR(3) = PHI
VAR(4) = BETA : VAR(5) = BF : VAR(6) = BW
VAR(7) = Q : VAR(8) = E : VAR(9) = EMBED
VAR(10) = C1x : VAR(11) = C1y : VAR(12) = C2x
VAR(13) = C2y : VAR(14) = A3x : VAR(15) = MECH
VAR(16) = DALPH1 : VAR(17) = DALPH2 : VAR(18) = DALPH3
RETURN

```

6100 'Subroutine ARRAY-OUT

```

GAMA = VAR(1) : COH = VAR(2) : PHI = VAR(3)
BETA = VAR(4) : BF = VAR(5) : BW = VAR(6)
Q = VAR(7) : E = VAR(8) : EMBED = ABS(VAR(9))
C1x = VAR(10) : C1y = VAR(11) : C2x = VAR(12)
C2y = VAR(13) : A3x = VAR(14) : MECH = VAR(15)
DALPH1 = VAR(16) : DALPH2 = VAR(17) : DALPH3 = VAR(18)
DALPH1=DALPH1*(PI/180) : DALPH2=DALPH2*(PI/180) : DALPH3=DALPH3*(PI/180)
PHI = PHI*(PI/180) : BETA = BETA*(PI/180)
RETURN

```

6200 'Subroutine [MATRIX-IN]

```

FOR I = 1 TO 6
  FOR J=1 TO 7 : MAT(J,I)=0 : NEXT J
  B(I) = 0
NEXT I
MAT(1,1)=SIN(ETA3) : MAT(2,1)=SIN(THET3) : MAT(3,1)=SIN(ETA2)
MAT(1,2)=COS(ETA3) : MAT(2,2)=COS(THET3) : MAT(3,2)=COS(ETA2)
MAT(4,1)=SIN(THET2) : MAT(4,2)=COS(THET2)
MAT(3,3)=SIN(ETA2) : MAT(4,3)=SIN(THET2) : MAT(5,3)=SIN(ETA1)
MAT(3,4)=COS(ETA2) : MAT(4,4)=COS(THET2) : MAT(5,4)=COS(ETA1)
MAT(6,3)=SIN(THET1) : MAT(6,4)=COS(THET1)
MAT(5,5)=ARMLF : MAT(6,5)=ARMLT
MAT(1,6)=ARMRF : MAT(2,6)=ARMRT
  IF MECH=1 OR MECH=2 OR MECH=5 THEN
    MAT(3,6)=-R2*SIN(PHI) : MAT(4,6)=-R2*(ALPH2/SIN(ALPH2))
  ELSE
    MAT(3,6)= R2*SIN(PHI) : MAT(4,6)= R2*(ALPH2/SIN(ALPH2))
  END IF
B(1) = Q + WR + WL : B(3) = Q + WR
B(6) = -(Q*((B1X+B2X)/2+E-C2X) + WR*(GRX-C2X) + WL*(GLX-C2X))
RETURN

```

6300 'Subroutine INITIALIZE

```

FOR i=1 TO 6 : A(i)=0 : Gx(i)=0 : Gy(i)=0 : NEXT i
R1=0 : R2=0 : R3=0 : C3X=0 : C3Y=0 : ALPH1=0 : ALPH2=0 : ALPH3=0
THET1=0 : THET2=0 : THET3=0 : ETA1=0 : ETA2=0 : ETA3=0
F1=0 : F2=0 : F12=0 : T1=0 : T2=0 : T12=0
K1X=0 : K1Y=0 : K2X=0 : K2Y=0 : K3X=0 : K3Y=0
Q1X=0 : Q1Y=0 : Q2X=0 : Q2Y=0 : Q3X=0 : Q3Y=0
QP1X=0 : QP1Y=0 : QP2X=0 : QP2Y=0 : QP3X=0 : QP3Y=0
MX=0 : MY=0 : A3Y=0 : FSCL1=0 : FSCL2=0 : FSCL3=0
RNNUM=0
RETURN

```

8000 'Subroutine MAIN MENU SCREEN SETUP

'configure cursor locations

```

LX(1)=30 : LY(1)= 4 : LX(5)=30 : LY(5) =10 : LX(9) = 50 : LY(9) =17
LX(2)=30 : LY(2)= 5 : LX(6)=30 : LY(6) =11 : LX(10)= 20 : LY(10)=18
LX(3)=30 : LY(3)= 6 : LX(7)=30 : LY(7) =14 : LX(11)= 50 : LY(11)=18
LX(4)=30 : LY(4)= 9 : LX(8)=20 : LY(8) =17 : LX(12)= 30 : LY(12)=20

```

COLOR 13,0 : LOCATE 2,10 : PRINT"----- MAIN MENU -----"

COLOR 14,0 'Headings

```

LOCATE 3,10 : PRINT"1) Data Manipulation..."
LOCATE 8,10 : PRINT"2) Processing....."
LOCATE 13,10 : PRINT"3) Output....."
LOCATE 16,10 : PRINT"4) Options....."

```

'Choices ----->

```

CH$(1)= " < Interactive input > " : CH$(2)= " < Data file input > "
CH$(3)= " < File save > " : CH$(4)= " < Run present config. > "
CH$(5)= " < Optimize config. > " : CH$(6)= " < Admissible search > "
CH$(7)= " < Print results > " : CH$(8)= " < 2 Surface solution > "
CH$(9)= " < 3 Surface solution > " : CH$(10)= " < Kinematic solution > "
CH$(11)= " < Limit analysis sol. > " : CH$(12)= " < EXIT PROGRAM > "

```

```

COLOR 12,0 : FOR ROW = 1 TO 12
  LOCATE LY(ROW),LX(ROW) : PRINT CH$(ROW) : NEXT ROW

```

```

COLOR 10,0
IF TRAP$="ON" THEN
  LOCATE LY(8),LX(8) : PRINT CH$(8)
ELSE
  LOCATE LY(9),LX(9) : PRINT CH$(9)
END IF
IF KINMAT=1 THEN
  LOCATE LY(10),LX(10) : PRINT CH$(10)
ELSE
  LOCATE LY(11),LX(11) : PRINT CH$(11)
END IF
COLOR 0,7
RETURN

```

8100 'subroutine INPUT SCREEN SETUP

'Configure cursor locations

```

LX(1)=49 : LY(1)=3 : LX(6)=54 : LY(6)=10 : LX(11)=50 : LY(11)=17
LX(2)=49 : LY(2)=4 : LX(7)=54 : LY(7)=11 : LX(12)=50 : LY(12)=18
LX(3)=49 : LY(3)=5 : LX(8)=54 : LY(8)=12 : LX(13)=50 : LY(13)=19
LX(4)=54 : LY(4)=8 : LX(9)=54 : LY(9)=13 : LX(14)=50 : LY(14)=20
LX(5)=54 : LY(5)=9 : LX(10)=50 : LY(10)=16 : LX(15)=50 : LY(15)=21
LX(16)=30 : LY(16)=22 : LX(17)=45 : LY(17)=22
LX(18)=60 : LY(18)=22 : LX(19)=30 : LY(19)=24

```

CLS : COLOR 2,0

```

  LOCATE 2,20 : PRINT" Soil Parameters:  "
  LOCATE 7,20 : PRINT" Slope Geometry:  "
  LOCATE 15,20 : PRINT" Failure Mechanism: "
  LOCATE 24,30 : PRINT"RETURN";

```

COLOR 12,0

```

LOCATE 3,10 : PRINT"Unit Weight of Soil -          τ ="
LOCATE 4,10 : PRINT"Cohesion -                      c ="
LOCATE 5,10 : PRINT"Friction Angle -                 φ ="
LOCATE 8,10 : PRINT"Slope Angle -
LOCATE 9,10 : PRINT"Distance of Footing from Slope Crest - β ="
LOCATE 10,10 : PRINT"Width of Footing -                b ="
LOCATE 11,10 : PRINT"Load on Footing (force/unit length) - w ="
LOCATE 12,10 : PRINT"Load Eccentricity -              Q ="
LOCATE 13,10 : PRINT"Footing Embedment depth -        e ="
LOCATE 16,10 : PRINT"Center of First Circle -          δ ="

```

C1x ="

```

LOCATE 17,10 : PRINT"
LOCATE 18,10 : PRINT"Center of Second Circle -
LOCATE 19,10 : PRINT"
LOCATE 20,10 : PRINT"Toe of Failure Mechanism -
LOCATE 21,10 : PRINT"Mechanism configuration -
LOCATE 22,10 : PRINT"dα = "
COLOR 7,0
RETURN

```

```

C1y = "
C2x = "
C2y = "
A3x = "
MECH = "

```

8200 'Subroutine PARTIAL RESULTS SETUP

```

CLS : COLOR 31 : LOCATE 5,60 : PRINT "Optimizing..." : COLOR 7
LOCATE 10,20 : PRINT "Partial Results"
LOCATE 13,6 :

```

```

PRINT"
LOCATE 14,6 : PRINT" || Opt. Step | FS1 FS2 FS3 | Limiting FSc | Kinetic
FSc | Cohesion ||"
LOCATE 15,6 :
PRINT"
LOCATE 16,6 : PRINT" || | | | |
LOCATE 17,6 : PRINT" || | | | |
LOCATE 18,6 : PRINT" || | | | |
LOCATE 19,6 : PRINT" || | | | |
LOCATE 20,6 :
PRINT"
RETURN

```

9900 'Subroutine ERROR

```

CLS : PRINT
PRINT"IBM BASIC 3.0 - Error Number = ";ERR
IF (ERR=5) OR (ERR=6) OR (ERR=11) THEN
PRINT"Incomplete input data has been specified or program is"
PRINT"not capable of specified geometry & mechanism"
END IF
Trap4:
IF INKEY$="" THEN Trap4
9999 RESUME 150

```
