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## **Determining Horizontal Displacement and Strains Due to Subsidence**

**By Sathit Tandanand and Larry R. Powell**

UNITED STATES DEPARTMENT OF THE INTERIOR



BUREAU OF MINES

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## CONTENTS

	<i>Page</i>
Abstract .....	1
Introduction .....	2
Characterization of subsidence trough .....	2
Slope .....	2
Curvature .....	3
Profile curve .....	5
Horizontal displacement and ground strains .....	6
Horizontal displacement .....	6
Ground strains .....	7
Analysis of subsidence profile .....	7
Discussion .....	9
Conclusions .....	9
References .....	9

## ILLUSTRATIONS

1. Characterization of extraction areas .....	3
2. Horizontal and vertical sections of curved surface .....	4
3. Mohr's circle for curvature .....	5
4. One-half profile curve, horizontal displacement, and slope .....	6
5. Values of $c/H$ with respect to limit angle at various ratios of $u_o/S_m$ .....	8

## TABLE

1. Subsidence parameters of various coalfields in Europe .....	8
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## UNIT OF MEASURE ABBREVIATIONS USED IN THIS REPORT

deg	degree	$m^{-1}$	per meter
m	meter		

## SYMBOLS USED IN THIS REPORT

$x, y, z$	Rectangular coordinates	$S_m$	Maximum possible or full subsidence, m
a-n, a-t	Arbitrary directions in the x-y plane	$S'$	Slope of profile curve
c	Tilt number, m	$S''$	Curvature of profile curve $m^{-1}$
f, F	Function	$S''_{max}$	Maximum curvature $m^{-1}$
s	Subsidence, m	$\alpha$	Angle, deg
u, v, w	Components of displacements	$\gamma$	Limit angle, deg
$u_o$	Maximum horizontal displacement	$\gamma_{xy}$	Shearing strain component in the x-y plane
B	Critical radius, m	$\epsilon_x, \epsilon_y$	Strains or unit elongations in the x and y directions
H	Extraction depth, m	$\epsilon_{max}$	Maximum strain
$K_x, K_y, K_{xy}$	Curvature of the surface with respect to the x and y axes	$\phi_o$	Tilt or maximum slope, deg
S	Maximum subsidence		

# DETERMINING HORIZONTAL DISPLACEMENT AND STRAINS DUE TO SUBSIDENCE

By Sathit Tandanand<sup>1</sup> and Larry R. Powell<sup>2</sup>

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## ABSTRACT

Horizontal displacements and ground strains induced by mine subsidence are significant information needed for calculating damage and developing precautions against subsidence effects on surface structures. To devise a simple method for determining the surface horizontal displacements and strains simultaneously with the subsidence prediction, the U.S. Bureau of Mines examined the significance of the tilt number, which is the proportionality constant in the relationship between the horizontal displacement and the slope of the subsidence profile. The ratio of the tilt number to the critical radius of the subsidence trough is identical to the ratio of the maximum possible horizontal displacement to the full subsidence, which is found to be constant in most European coalfields. If this ratio is known for a particular minesite in the United States, then horizontal displacement and ground strains can be readily obtained from the primary subsidence data.

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## INTRODUCTION

Protection of surface structures from severe damage due to subsidence relies on careful mine planning, knowledge of ground movements, and subsidence prediction capability. Usually, when a subsidence trough is formed, surface points move downward and inward toward the center of the trough. Thus, a horizontal displacement always accompanies subsidence. Slope, curvature, horizontal displacement, and ground strains are needed to design precautions against subsidence effects on surface structures. Measuring horizontal displacements is more time consuming and tedious than measuring vertical displacements.

Because subsidence is a time-dependent event, the surface conditions and surveying monuments are subjected to environmental changes. Variations of monument intervals, topography, and calculating procedures tend to affect the calculated results. Although the data reduction methods for the subsidence parameters have been established with the subsidence prediction methodology (1-5), imperfect

modeling and uncertainty of field measurements seem inevitable.

In subsidence prediction, the horizontal displacement cannot be pre-calculated from the primary data without using a site constant. Currently, this constant is not available for the coal regions in the United States. To establish a simple method for calculating the horizontal displacement and ground strains, a proportionality constant, known as tilt number ( $I$ ), was suggested in the relationship between the horizontal displacement and the slope of the subsidence trough. The ratio of the tilt number to the critical radius was found to be identical to the ratio of the maximum horizontal displacement to the full subsidence. The latter has been found to be a site constant that varies between 0.3 to 0.44 in most European coal fields (2). The calculations of the derived subsidence parameters would be simplified if this ratio is known and the tilt number at the minesite is determined from the primary subsidence data.

## CHARACTERIZATION OF SUBSIDENCE TROUGH

The slope of a typical subsidence trough is governed by the size and shape of the extraction area. Basically, for a rectangular extraction, the maximum possible or full subsidence is controlled by the critical area of extraction. If full subsidence occurs at more than one point in the trough, then the extraction area is regarded as supercritical. Otherwise, the extraction area is subcritical, if the magnitude of the maximum subsidence is less than the full subsidence (fig. 1). In a vertical section across the panel, the limit angle helps to define the subsidence limit graphically. Likewise, the angle of break locates the projection of the maximum tensile strain onto the subsidence profile. In analysis, the surface trough is described by the general equation  $F(x,y,z)$  from which  $f(x,z)$  and  $f(y,z)$  represent the continuous profiles in the  $x$  and  $y$  directions. Hence, the slope and curvature around a point in the trough are bound vectors defined by the first and second derivatives of the profile equations. The concepts, geometrical properties, and relationships of the slope, curvature, and relevant parameters are discussed as follows.

### SLOPE

The slope of a curve is the rate of change in the elevation between adjacent points. The slope of a surface is a vector that changes its magnitude with the direction in the tangent plane to the surface. In the  $x$  direction (fig. 2), the slope is  $\partial w/\partial x$ , and in the  $y$  direction,  $\partial w/\partial y$ . The difference in the elevations of adjacent points is expressed by

$$dw = \frac{\partial w}{\partial x} dx + \frac{\partial w}{\partial y} dy. \quad (1)$$

If  $\alpha$  is the angle between the  $a-n$  direction and the  $x$  axis, then the corresponding slope is

$$\frac{dw}{dn} = \frac{\partial w}{\partial x} \cos \alpha + \frac{\partial w}{\partial y} \sin \alpha, \quad (2)$$

where

$$\cos \alpha = \frac{dx}{dn} \text{ and } \sin \alpha = \frac{dy}{dn}. \quad (3)$$

The slope of the surface at a given point, therefore, depends on the directional derivatives. The maximum slope is obtained from  $d(dw/dn)/d\alpha = 0$ , which gives the direction  $\alpha_1$ , defined by

$$\tan \alpha_1 = \frac{\partial w}{\partial y} / \frac{\partial w}{\partial x}, \quad (4)$$

where

$$\sin \alpha_1 = \frac{\partial w}{\partial y} / \left[ \left( \frac{\partial w}{\partial x} \right)^2 + \left( \frac{\partial w}{\partial y} \right)^2 \right]^{1/2} \quad (5)$$

and

$$\cos \alpha_1 = \frac{\partial w}{\partial x} / \left[ \left( \frac{\partial w}{\partial x} \right)^2 + \left( \frac{\partial w}{\partial y} \right)^2 \right]^{1/2}$$



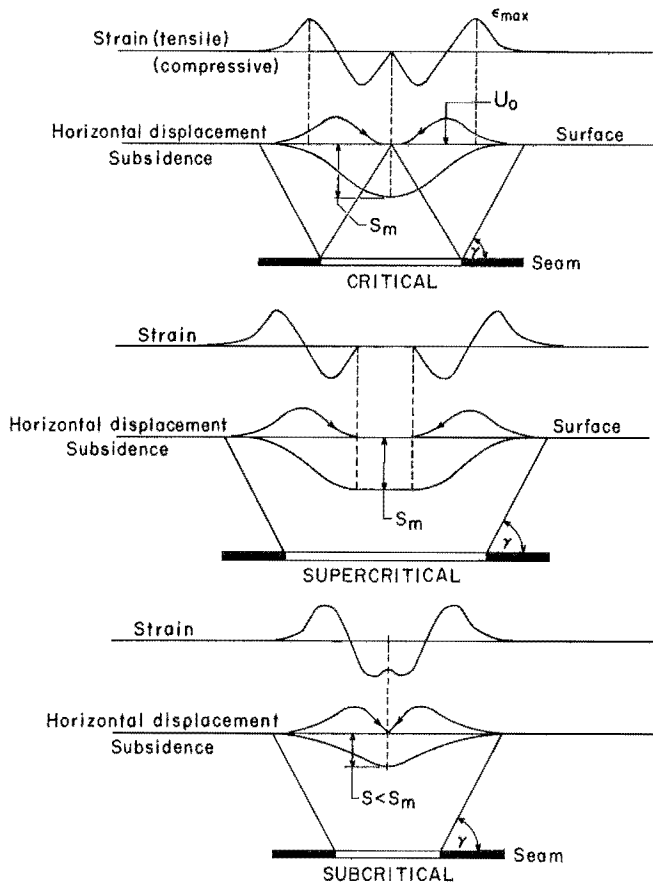


Figure 1.—Characterization of extraction areas.

It follows that the maximum slope is

$$\left(\frac{dw}{dn}\right)_{\max} = \left[ \left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 \right]^{1/2}. \quad (6)$$

The direction  $\alpha_2$  of zero slope is also obtained from equation 1, and setting  $dw/dn = 0$ ,

$$\tan \alpha_2 = - \frac{dw}{dx} / \frac{dw}{dy}. \quad (7)$$

Since  $\tan \alpha_1 \tan \alpha_2 = -1$ , it is implied that the direction of the zero slope and the maximum slope are perpendicular. A locus of regular points of equal elevation is a contour line on the surface. The line normal to the contour line is the principal direction of tilt, and the maximum slope is sometimes called the principal tilt. Geologically, the maximum slope is called the dip; the direction of zero slope is called the direction of strike.

## CURVATURE

The curvature of a trough is a vector. In any direction of the surface, the curvature is the rate of change of the slopes at two adjacent points with respect to the arc length. In the  $x$  direction, the expression for curvature is

$$K_x = \frac{d}{da} (\tan \alpha) = \frac{d}{da} \left( \frac{dw}{dx} \right), \quad (8)$$

and for the arc length

$$da = \left[ (dx)^2 + (dw)^2 \right]^{1/2}, \quad (9)$$

which gives

$$\frac{da}{dx} = \left[ 1 + \left( \frac{dw}{dx} \right)^2 \right]^{1/2}. \quad (10)$$

For a small slope,  $(dw/dx)^2$  is negligible compared to unity; thus, the curvature  $K_x$  is reduced to

$$K_x = \frac{d}{da} \left( \frac{dw}{dx} \right) \frac{da}{dx} = \frac{d^2w}{dx^2}. \quad (11)$$

Likewise, in the  $y$  direction

$$K_y = \frac{d^2w}{dy^2}, \quad (12)$$

and the twist of the surface with respect to the  $x$  and  $y$  axes

$$K_{xy} = \frac{d^2w}{dxdy}. \quad (13)$$

The reciprocal of curvature is called the radius of curvature.

At a point on the surface, the curvature is dependent on its position and direction. In considering the curvatures in detail, the reader is referred to the literature (6). In brief, at point  $a$ , in the direction  $a-n$  (fig. 2), the curvature is expressed as

$$K_n = \frac{\partial}{\partial n} \left( \frac{\partial w}{\partial n} \right). \quad (14)$$

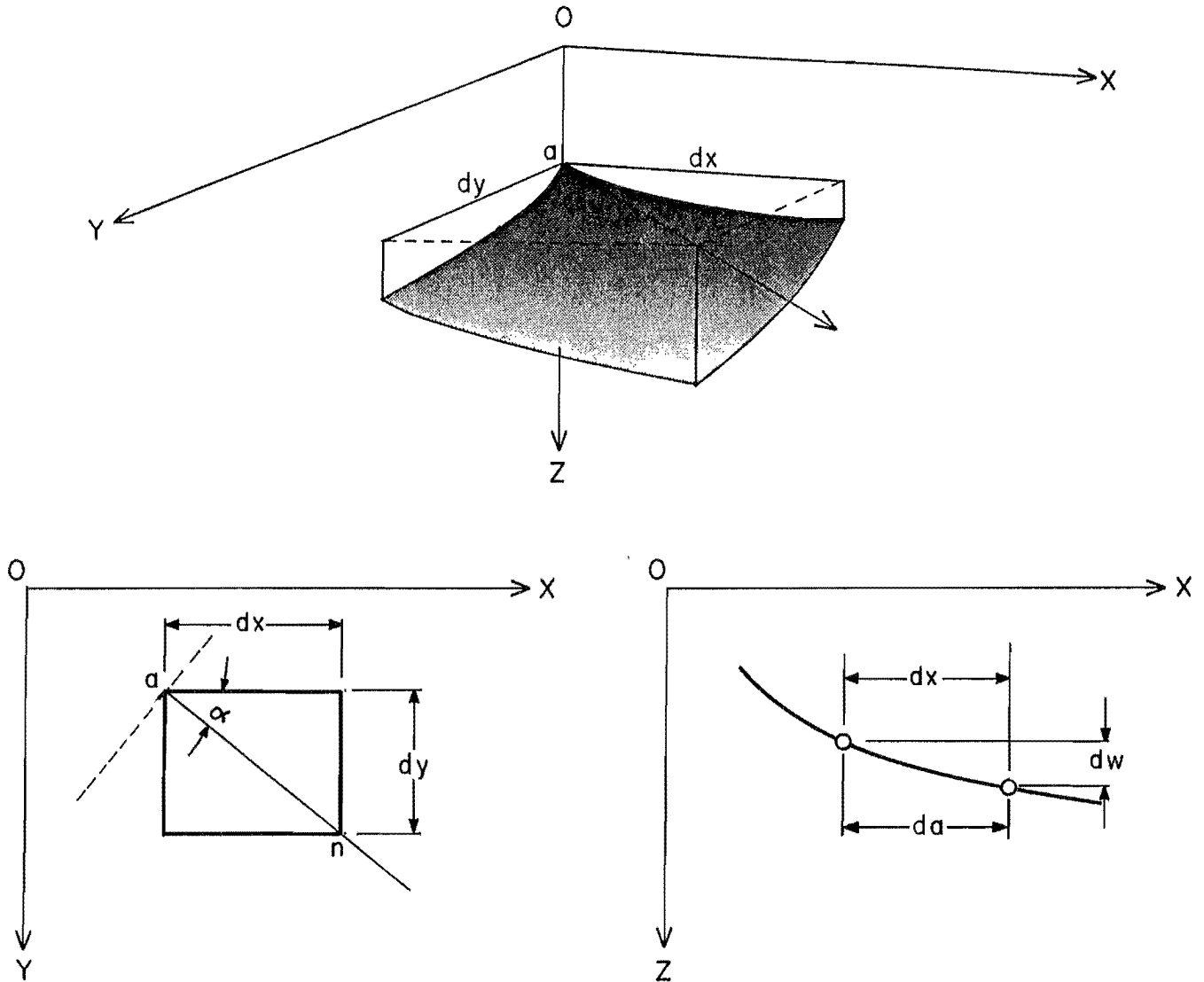


Figure 2.—Horizontal and vertical sections of curved surface.

With  $\partial w/\partial n$  in equation 2 and  $\partial/\partial n = \partial/\partial x \cos \alpha + \partial/\partial y \sin \alpha$ ,  $K_n$  expressed in terms of  $K_x$ ,  $K_y$ ,  $K_{xy}$  is

$$K_n = K_x \cos^2 \alpha + K_{xy} \sin 2\alpha + K_y \sin^2 \alpha. \quad (15)$$

The twist of the surface at a with respect to direction a-t perpendicular to a-n is

$$K_{nt} = -\frac{\partial}{\partial t} \left[ \frac{\partial w}{\partial n} \right]. \quad (16)$$

In the same manner,  $K_{nt}$  can be obtained by substituting  $(\pi/2 + \alpha)$  for  $\alpha$  in equation 1 and with the operator  $\partial/\partial n$ ,

$$K_{nt} = 1/2 (K_x - K_y) \sin 2\alpha - K_{xy} \cos 2\alpha. \quad (17)$$

The maximum and minimum curvatures of the surface at a and their directions can be determined. By taking the derivative of equation 15 with respect to  $\alpha$ , equating to zero, and solving for  $\alpha$ ,

$$\tan 2\alpha = \frac{2K_{xy}}{K_x - K_y}. \quad (18)$$

The two values of  $\alpha$  represent the directions of the maximum and minimum curvatures of  $K_n$  at point a.

The principal curvatures can be obtained when  $K_{xy} = 0$ . The curvatures of a trough can be graphically represented using the Mohr circle similar to the graphical representation of the state of stress at a point. Assuming that two vertical planes x-z and y-z are parallel to the principal

planes of curvature in which  $K_{xy} = 0$  at the point A, then the principal curvature  $K_n$  for any angle  $\alpha$  is

$$K_n = K_x \cos^2 \alpha + K_y \sin^2 \alpha, \quad (19)$$

and the twist of the surface is

$$K_{nt} = 1/2 (K_x - K_y) \sin 2\alpha. \quad (20)$$

Taking the curvatures as abscissas and the twist as ordinates, and using  $(K_x - K_y)/2$  as the radius, the Mohr circle for curvatures can be drawn as shown in figure 3.

### PROFILE CURVE

In subsidence calculations, the profile curve is expressed in rectangular coordinates. One-half of the curve is normally used because the entire curve, for regular extractions, is symmetrical. The horizontal axis represents the intact surface and, for convenience, the vertical displacement is considered positive. The vertical axis coincides

with the inflection point, thus dividing the curve so that the subsidence limit and the point of full subsidence appear in two opposite quadrants (fig. 4). The slope of the curve increases from zero on each end point and reaches its maximum at the inflection point. The distance from this point to the subsidence limit is the critical radius.

The curvature of the profile is convex on the right segment, but concave on the left. The characteristic of the curve implies the sense of deformation according to the geometry of line segment and the coordinates. Graphically, concavity and convexity can be distinguished by the relative position of tangent with respect to the chord of the segment, or by the rotation of the tangent vector. In the analysis of a deformable system, the sign convention simplifies the operation. In general, the selection of signs is a matter of convenience (7). Because displacement, slope, and curvature are vectors, the sign convention can be selected according to the vector concepts and that of the coordinate system. However, consistency is necessary to maintain the geometrical meaning of each entity unless the change is explicitly stated.

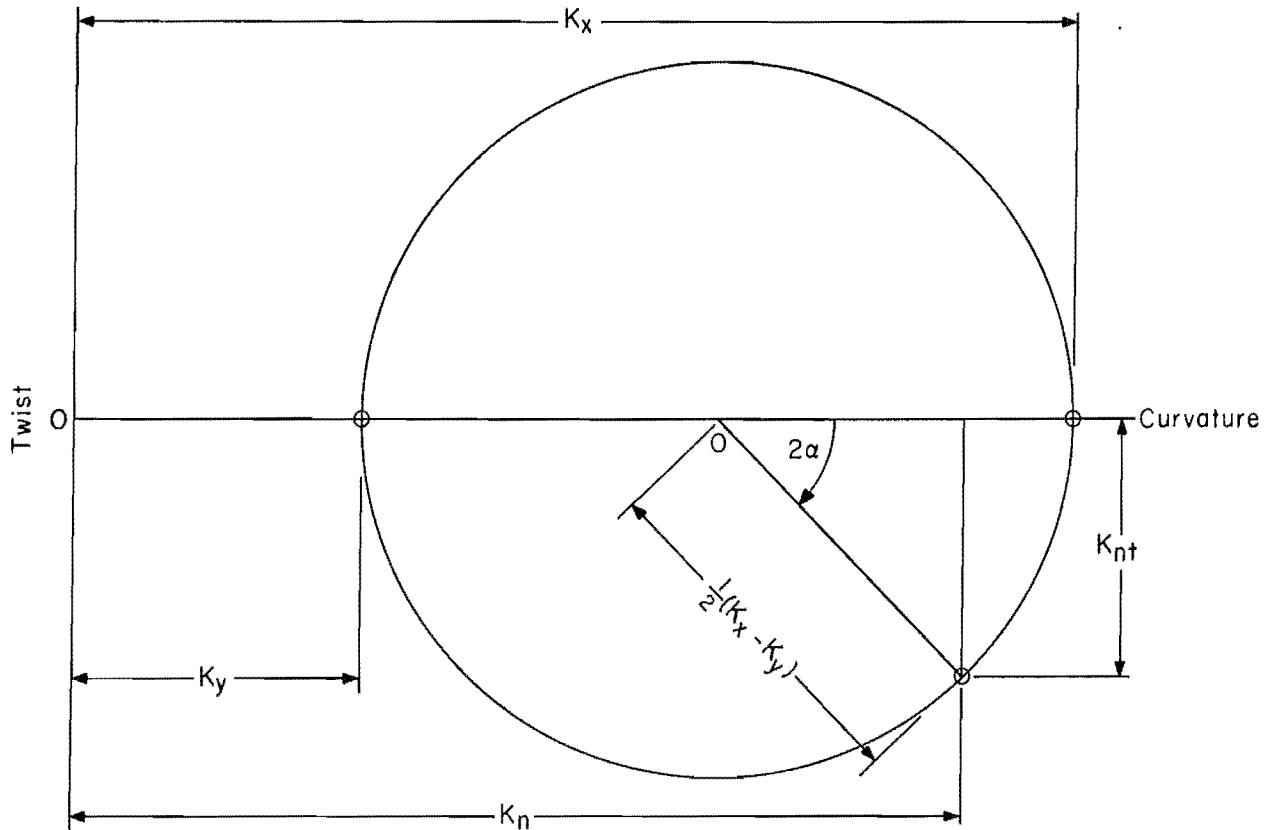


Figure 3.—Mohr's circle for curvature.

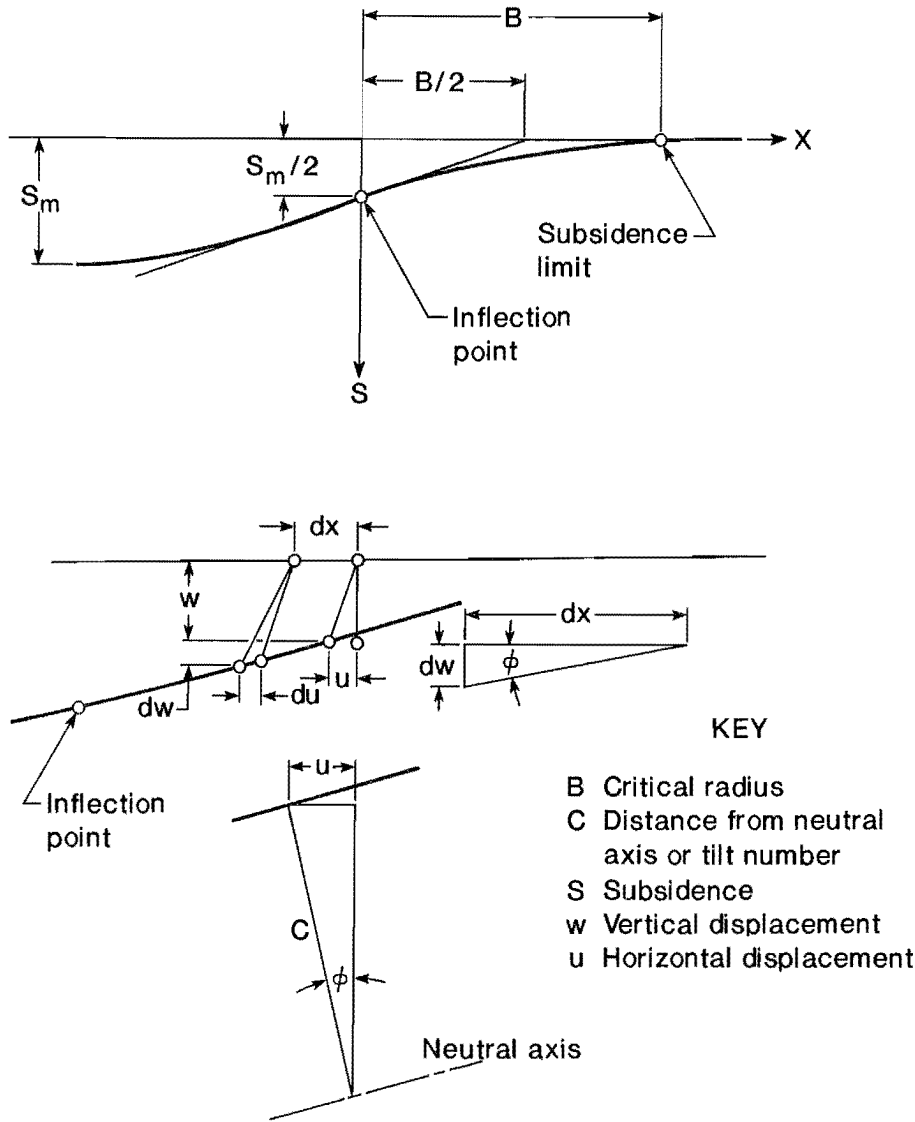


Figure 4.—One-half profile curve, horizontal displacement, and slope.

## HORIZONTAL DISPLACEMENT AND GROUND STRAINS

The overburden above an extracted seam can be divided into three zones according to the nature of the strata movements. Strata immediately above the coal seam collapse and form rubble in the caved zone. This eventually gives support to the overlying intermediate strata, which experience bending, fracturing, and bed separations. Above the intermediate zone, surficial strata of incompetent weathered rocks, unconsolidated sediments, and soils will deform with the underlying strata. Surface points in the subsidence trough tend to be displaced toward the lowest point in the trough. The displacement vector of the surface point can be resolved into vertical and horizontal components.

### HORIZONTAL DISPLACEMENT

Now, consider the upper strata as a beam, subjected to end constraint beyond the edges of the extraction. This beam deflects due to the movements of the underlying strata. The relationship between the horizontal displacement and the slope of the profile curve can be shown graphically (fig. 4). For a small angle between the vertical axis and the normal of the neutral axis,  $\sin \phi = \tan \phi$ ; thus,

$$u = c \sin \phi = c \frac{dw}{dx}. \tag{21}$$

Here, the horizontal displacement is linearly proportional to the slope. The proportionality constant,  $c$ , is the depth of the neutral axis below the deformed surface and concisely termed "tilt number" (5). If the ground is isotropic, then the displacement in the  $y$  direction is

$$v = c \frac{\partial w}{\partial y}. \quad (22)$$

When  $\partial w/\partial x$  and  $\partial w/\partial y$  are maximum,  $u$  and  $v$  are maximum accordingly.

### GROUND STRAINS

Ground strains are related to the unit deformation and distortion at a point on the ground during the subsidence event. To describe the unit deformation, considered a linear element in the  $x$  direction,  $dx$ , which is displaced simultaneously by  $u$  in the horizontal and  $w$  in the vertical direction. The elongation (or contraction) of  $dx$  due to the horizontal elongation,  $du$ , is  $(\partial u/\partial x) dx$ , and due to the vertical elongation,  $dw$ , is  $(\partial w/\partial x)dx$ . Therefore, total unit elongation that makes up the strain in the  $x$  direction is

$$\epsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2. \quad (23)$$

Likewise, the strain in the  $y$  direction is

$$\epsilon_y = \frac{\partial v}{\partial y} + \frac{1}{2} \left( \frac{\partial w}{\partial y} \right)^2. \quad (24)$$

The shearing strain that characterizes the distortion of the surface element due to the displacement  $u$  and  $w$  is

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial y} \frac{\partial w}{\partial x}. \quad (25)$$

However, the foregoing unit elongations can be simplified because  $\partial u/\partial x$  and  $\partial v/\partial y$  correspond to the tangent of small tilt angles. Thus, the square term and the product of two small slopes of the surface are very small. The calculation of strains can be simplified by neglecting the square terms of slope so that the components of ground strain can be determined directly from the curvatures of the subsidence trough:

$$\epsilon_x = \frac{\partial u}{\partial x} = c \frac{\partial^2 w}{\partial x^2}, \quad (26)$$

$$\epsilon_y = \frac{\partial v}{\partial y} = c \frac{\partial^2 w}{\partial y^2}, \quad (27)$$

$$\text{and } \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 2c \frac{\partial^2 w}{\partial x \partial y}. \quad (28)$$

### ANALYSIS OF SUBSIDENCE PROFILE

A trigonometric profile function for critical extraction, which is used as the profile function for the Donets coalfield in Europe (1, 5), is selected:

$$s = \frac{S_m}{2} \left[ 1 - \frac{x}{B} - \frac{1}{\pi} \sin \frac{\pi x}{B} \right], \quad (29)$$

where  $s$  = subsidence,

$S_m$  = maximum possible subsidence,

and  $B$  = critical radius.

Note that  $s$  is the same as the vertical displacement,  $w$ , in the preceding equations. The origin of the coordinates for equation 29 is directly above the inflection point where  $x = 0$ , and  $S_0 = S_m/2$ . The critical radius,  $B$ , is obtained graphically or by

$$B = H \cot \gamma \quad (30)$$

where  $H$  is extraction depth, and  $\gamma$  is the limit angle. It follows that the slope of the profile curve is

$$S' = -\frac{S_m}{2B} \left[ 1 + \cos \frac{\pi x}{B} \right], \quad (31)$$

which is negative for  $-B \leq x \leq B$ , according to the positive assignment for  $s$ .

By multiplying the slope with the tilt number, the horizontal displacement is obtained:

$$u = -\frac{cS_m}{2B} \left[ 1 + \cos \frac{\pi x}{B} \right]. \quad (32)$$

Here, the negative sign indicates that the displacement vector is opposite to the  $x$  direction, i.e. toward the center of the trough. At  $x = 0$ , the absolute value of the maximum displacement  $u_0$  is  $u_0 = cS_m/B$ , which gives

$$\frac{c}{B} = \frac{u_0}{S_m}. \quad (33)$$

The ratio  $u_0/S_m$  for various coalfields in Europe appears to vary slightly as shown in table 1. Thus, there is an implication that for a particular depth of extraction,

$u_0/S_m$  is very likely to depend on lithology, which may control the subsidence limit. The plots of  $c/H$  as a function of the limit angle for various values of  $u_0/S_m$  is shown in figure 5. For a given depth, the tilt number decreases as the limit angle increases.

**Table 1.—Subsidence parameters of various coalfields in Europe (7)**

Location	Susidence factor (a)	Limit angle ( $\gamma$ )	$u_0/S_m^1$	$\cot\gamma/a$
France: Northern . . . . .	0.88	55°	0.4	0.80
Germany, Federal Republic of: Ruhr . . . . .	.9	53°	.4	.84
Netherlands: Limberg (8) . . . . .	.9	50°	.44	.93
U.S.S.R.:				
Chelyabinsk . . . . .	.9	60°	.3	.64
Donetz . . . . .	.6	60°	.3	.96
Karaganda . . . . .	.7	60°	.3	.83
Kizelov . . . . .	.7	60°	.3	.83
Kutznetsk . . . . .	.7	60°	.3	.83

<sup>1</sup>Maximum horizontal displacement ( $u_0$ ) divided by full subsidence ( $S_m$ ).

To verify that the square term of the maximum slope is very small, the following field data for the Illinois coal basin are used:

The average seam thickness is 2.38 m (9), the limit angle is 60°, and the subsidence factor is 0.6.

For now, 0.38 is used as the ratio of  $u_0/S_m$ , taken from the average value of those in table 1. Hence, one can find:

The maximum subsidence is 1.43 m, the critical radius is 105.6 m, and the tilt number is 40.1 m.

From these calculated data, the following subsidence parameters are obtained:

The maximum slope of 0.0135, and the maximum horizontal displacement of 0.541 m.

Hence, the square of the maximum slope is 0.0002, and the second term in equation 23 is 0.0001.

The curvature of the subsidence profile is

$$S'' = \frac{\pi S_m}{2B^2} \sin \frac{\pi}{B} x. \tag{34}$$

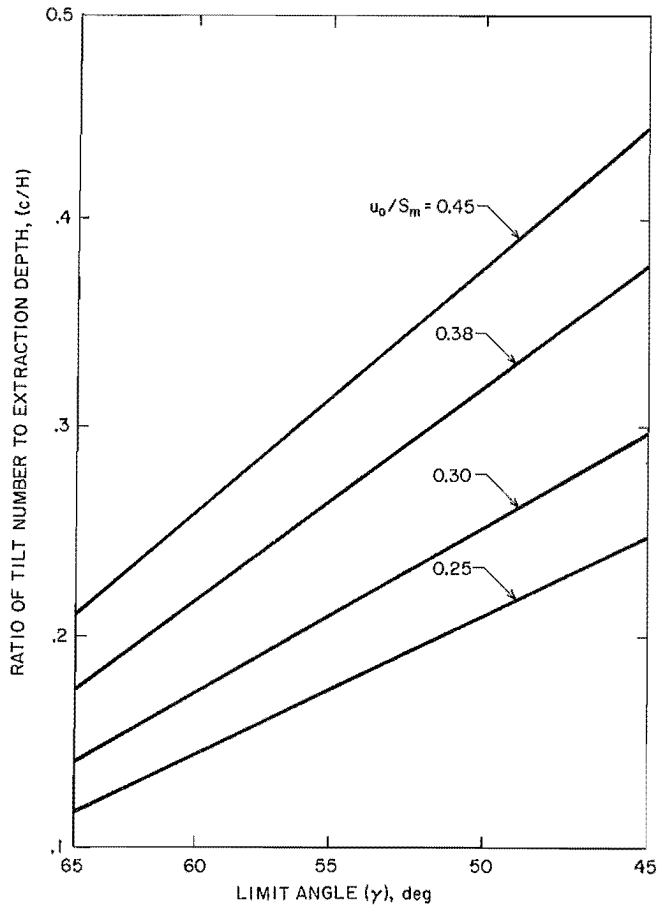
Hence, the strain is

$$\epsilon_x = \frac{c\pi S_m}{2B^2} \sin \frac{\pi}{B} x. \tag{35}$$

Since the maximum curvature is at  $x = B/2$ ,  $S_{max} = \pi/2 S_m/B^2$ , and the maximum strain is

$$\epsilon_{max} = 1.571 c \frac{S_m}{B^2}. \tag{36}$$

Based on the foregoing data, the maximum tensile strain is 0.0081. If the second term in equation 23 is included (i.e. 0.0001), it is insignificant when compared with the magnitude of the maximum calculated strain,  $\epsilon_{max}$ .



**Figure 5.—Values of  $c/H$  with respect to limit angle at various ratios of  $u_0/S_m$ .**

## DISCUSSION

The calculations for the horizontal displacement and the strain of a subsidence trough can be simplified if the tilt number for the site is known. As shown (figure 4 and equation 21), this number is obtained graphically from an idealized profile curve. Because of the linearity of equation 21, the maximum horizontal displacement,  $u_o$ , is also proportional to the maximum slope,  $\tan \phi_o$ , provided that  $\phi_o$  is a small angle. Usually, most profile functions predict the vertical displacement at the inflection point to be  $S_m/2$ . If a tangent to the profile curve is drawn to the inflection point to intersect the abscissa at an angle, say  $\alpha$ , then the intersection point on the coordinate axis would measure  $B/2$ , and  $\tan \phi_o = S_m/B$ . Hence, the critical radius and the tilt number are

$$B = S_m / \tan \phi_o, \quad c = u_o / \tan \phi_o, \quad (37)$$

and 
$$\frac{c}{B} = \frac{u_o}{S_m}, \quad (38)$$

which can also be obtained from equation 29.

A question is raised here if the same result would be obtained from any profile functions  $s = f(x)$ , which are exponential and asymptotic in  $x$ . Apparently, Knothe (10) derived the profile function from the Gaussian distribution function. With the same consideration that the horizontal displacement is proportional to the slope of the profile curve, the ratio  $U_o/S_m$  was found to be 0.4. This implies that the constant in the relationship is independent on the mathematical form whether it is a trigonometric or exponential function. The constant,  $u_o/S_m$ , is derived and valid for flat-seam extractions of the critical area. Special attention must be given to steep seams and terrains where natural uphill and downhill slopes will affect the field results (11-12).

## CONCLUSIONS

Slope and curvature can be obtained readily from the profile function used in subsidence prediction if the function is mathematically continuous. In practice, the values of horizontal displacement and ground strains can be easily obtained if the constant  $u_o/S_m$  and the tilt

number for a particular coal mining region are known. These parameters are meaningful whenever the maximum horizontal displacement and critical ground strains are used for designing subsidence control measures and precautions against damage to surface structures.

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