RI 9411

REPORT OF INVESTIGATIONS/1992

BOMCRATR—A Curved Ray Tomographic Computer Program for Geophysical Applications

By D. R. Tweeton, M. J. Jackson, and K. S. Roessler

UNITED STATES DEPARTMENT OF THE INTERIOR

BUREAU OF MINES

Mission: As the Nation's principal conservation agency, the Department of the Interior has responsibility for most of our nationally-owned public lands and natural and cultural resources. This includes fostering wise use of our land and water resources, protecting our fish and wildlife, preserving the environmental and cultural values of our national parks and historical places, and providing for the enjoyment of life through outdoor recreation. The Department assesses our energy and mineral resources and works to assure that their development is in the best interests of all our people. The Department also promotes the goals of the Take Pride in America campaign by encouraging stewardship and citizen responsibility for the public lands and promoting citizen participation in their care. The Department also has a major responsibility for American Indian reservation communities and for people who live in Island Territories under U.S. Administration.

Report of Investigations 9411

BOMCRATR—A Curved Ray Tomographic Computer Program for Geophysical Applications

By D. R. Tweeton, M. J. Jackson, and K. S. Roessler

UNITED STATES DEPARTMENT OF THE INTERIOR
Manuel Lujan, Jr., Secretary

BUREAU OF MINES
T S Ary, Director

Library of Congress Cataloging in Publication Data:

Tweeton, Daryl R.
BOMCRATR : a curved ray tomographic computer program for geophysical applications / by D.R. Tweeton, M.J. Jackson, and K.S. Roessler.

p. cm. — (Report of investigations; 1990; 9411)

Includes bibliographical references (p. 34).

1. BOMCRATR. 2. Mining geology—Computer programs. 3. Geophysics—Computer programs. I. Jackson, M. J. (Michael James), 1954- . II. Roessler, K. S. (Kent S.) III. Title. IV. Series: Report of investigations (United States. Bureau of Mines); 1990; 9411.

TN23.U43 [TN260] 622 s—dc20 [622'.2'02855369] 92-7388 CIP

CONTENTS

ILLUSTRATIONS

CONTENTS—Continued

TABLES

UNIT OF MEASURE ABBREVIATIONS USED IN THIS REPORT

ft	foot	m/ms	meter per millisecond
ft/ms	foot per millisecond	(m/ms)/m	meter per millisecond per meter
kb	kilobyte	ms	millisecond
kHz	kilohertz	pct	percent
m	meter		

Disclaimer of Liability

BOMCRATR—A CURVED RAY TOMOGRAPHIC COMPUTER PROGRAM FOR GEOPHYSICAL APPLICATIONS

By D. R. Tweeton,[1] M. J. Jackson,[2] and K. S. Roessler[3]

ABSTRACT

The U.S. Bureau of Mines has developed tomographic computer programs for geophysical applications and is distributing them as part of a Bureau plan to facilitate the transfer of technology to potential users. These programs were developed for predicting and monitoring the flow pattern of leach solution during in situ mining. The Bureau has also applied them to assessing high-wall blast damage and examining the integrity of mine-related geological structures. This report describes the curved ray program BOMCRATR (Bureau of Mines curved ray tomographic reconstruction), tells how to run it, and gives examples with synthetic and field data.

BOMCRATR provides optional mathematical constraints to counteract the nonuniqueness of tomographic reconstructions with crosshole data. These constraints include limiting the maximum and minimum velocities, fixing the velocity at any point to any value, and establishing layers in which velocity does not vary with horizontal position. The user can test for nonuniqueness by varying the initial velocity pattern. The program can perform either curved or straight ray analysis. Ray paths are calculated analytically in triangular pixels, which is faster than incremental numerical approximations.

[1]Research physicist.
[2]Geophysicist.
[3]Mining engineer.
Twin Cities Research Center, U.S. Bureau of Mines, Minneapolis, MN.

INTRODUCTION

Tomography is a mathematical procedure for calculating the distribution of a parameter of interest within a region using measurements made at the edges of the region. The displayed calculated distribution of the parameter of interest is called a tomographic reconstruction or tomogram. For the seismic tomography described in this report, the travel times for seismic waves traveling from source locations in one borehole to receiver locations in another borehole are measured. These data are used in tomographic calculations to determine the distribution of seismic velocity between the boreholes.

Tomography can also be applied to calculate seismic attenuation (*1*),[4] though fewer studies have been performed with attenuation than with seismic velocity. Tomography has been applied extensively to electromagnetic systems, measuring velocity, attenuation, and phase (*2-4*). Several studies comparing electromagnetic and seismic tomographic results have been published (*5-6*). Tomography can, in principle, be applied to any measurement that is a line integral of some parameter between accessible boundaries.

The principles of geophysical tomography and tests of applications are described in many publications, such as the pioneering research by Dines and Lytle (*7*), a presentation of the mathematical aspects of tomography by Natterer (*8*), a description of the limitations of using ray theory for seismic ray tracing by Chapman (*9*), and many articles giving results of recent tomographic research in the Expanded Abstracts of the Society of Exploration Geophysicists (SEG) 1991 meeting (*10*).

BUREAU INTEREST IN TOMOGRAPHY

U.S. Bureau of Mines tomographic research includes applications to in situ mining, blast assessment, rock-quality evaluation, and assessment of coal pillars. The research is a part of the Bureau's effort to enhance mining productivity and worker safety. For in situ mining applications, the goal was to develop improved methods for predicting the flow pattern of leach solution before injection and for locating the solution after injection. Seismic tomography seemed promising because of its ability to provide a two-dimensional picture of the distribution of seismic velocity between boreholes. Fractured regions where leach solution is likely to flow produce low seismic velocities. Where leach solution is injected above the water table, the increase in saturation increases the compressional wave velocity. Field tests were successful at locating fractured zones and water simulating leach solution injected above the water table (*11*). For blast assessment, tomographic analysis has shown the change in fracture pattern from preblast to postblast (*12*). The region with the greatest decrease in seismic velocity was in a position to experience the greatest blast damage. For evaluating the rock quality of dimension stone, a low seismic velocity can indicate a badly fractured region that would render the rock unsuitable for this purpose. Ground-penetrating radar tomography located relatively low-velocity material in a coal pillar. A clay vein coincided with the image of the low-velocity material in the tomogram (*2*).

NEED FOR CURVED RAY TOMOGRAPHY

Seismic ray paths may bend if the velocity varies with location. Curved ray tomography is required if the bending cannot be ignored without a significant effect on the tomogram. There may be noticeable bending of ray paths without significantly affecting the calculated velocities. The amount of bending depends on the magnitude of the velocity gradient and on the angle between the gradient and the ray path. A velocity gradient parallel to the path does not bend it.

The degree of velocity contrast that requires curved ray tomography depends on several factors, including the answers to the following questions:

1. Do anomalies need to be located as accurately as possible, or is it adequate to know whether they exist within the tomographic plane?
2. How accurate are the data? A small effect that is observable with perfectly accurate synthetic data may not be significant with typical field data.
3. How far apart are the boreholes? For a given velocity gradient, there will be less bending between closely spaced boreholes.
4. What is the usual angular range between the gradient and the ray paths? The ray bending is a maximum when the gradient is perpendicular to the ray paths. Thus, a vertical gradient would cause greater bending than a horizontal gradient for crosshole rays in a vertical tomographic plane.

Based on the results of Bureau calculations, straight ray analysis is adequate when the greatest velocity is no greater than 120 pct of the lowest velocity and is not adequate when the greatest velocity most rays encounter is double the lowest velocity. For intermediate ranges, straight ray tomography may or may not be adequate, depending on the answers to the four questions listed above.

NEED FOR BOMCRATR

Bureau tomographic research has used straight ray tracing with the program BOMTOM (Bureau of Mines

[4]Italic numbers in parentheses refer to items in the list of references preceding the appendix at the end of this report.

tomography) (*13*). However, the velocity contrast at some sites was large enough to make curved ray tracing desirable. Commercial curved ray programs did not provide all the desired mathematical constraints to counteract the problem of nonuniqueness and did not provide a source code to allow the Bureau to add them.

Accordingly, the Bureau created a tomographic program by combining algorithms from several sources and refining the combination. The curved ray tracing algorithm is from a program used by Weber (*14*) for the Gaussian beam method of computing synthetic seismograms, though the Bureau program does not use the Gaussian beam method. The algorithm uses triangular pixels with a constant gradient within each pixel, which allows the ray paths to be calculated analytically as arcs of circles or straight lines, depending on the gradient. Müeller (*15*) had previously pointed out the efficiency of using such triangles to calculate the paths. Bregman (*16*) also applied triangular pixels to tomography, based on earlier work in ray tracing by Chapman (*9*). Chris Calnan of the University of Utah wrote an algorithm for associating source takeoff angles and particular receivers. A different algorithm written by Bureau researchers was substituted later for use with more pixels and greater velocity contrasts. The simultaneous iterative reconstruction technique (SIRT) algorithm for applying iterative corrections was taken from the straight ray program BOMTOM (*13*). The BOMTOM SIRT algorithm was based on research by Dines and Lytle (*7*). The optional constraints were also taken from BOMTOM. Some new options based on reviewers' comments and Bureau research were added. These options are described in the section titled "Nonuniqueness of Tomographic Reconstructions."

These tomography programs are available through a Bureau plan to facilitate transfer of technology to industry. FORTRAN source and executable codes are available.[5]

LIMITATIONS OF TOMOGRAPHY AND METHODS TO COUNTERACT THEM

Problems in tomographic calculations can reduce the reliability of reconstructions. Besides the obvious problem of inaccurate data, other detrimental factors persist, even with accurate data. Several of these problems are described below. Where BOMCRATR provides features to help counteract the problems, the features are described.

UNREALISTIC RAY PATHS

One problem with the curved ray tracing method used by BOMCRATR is that shadow zones may appear, in which a receiver is not reached by any ray path. One situation that may create a shadow zone is a horizontal line of high velocity. Rays above the line tend to curve upward and rays below it tend to curve downward, as shown in figure 1. Thus, receivers near the high-velocity line may not be reached by any of the mathematical ray paths, even though experimental travel times were measured for them. This problem is fundamental to this type of ray tracing. Alternatives that do not create such shadow zones are being investigated.

Another problem is that a ray path may be bent so that it travels repeatedly over the same pixels and never encounters a side of the grid. The program may continue tracing the same ray indefinitely. This problem occurred several times with earlier versions of BOMCRATR while analyzing field data having strong contrasts in velocity. The program now checks whether a ray has turned around and is going backward. If it is, BOMCRATR stops calculations with it and goes on to the next ray.

NONUNIQUENESS OF TOMOGRAPHIC RECONSTRUCTIONS

As used here, a unique tomographic reconstruction is a calculated set of seismic velocities for the pixel grid that minimizes the errors between calculated and measured travel times better than any other set of velocities. The error minimization can be any of several types. Two examples are the minimization of the sum of the squares of the differences between the measured and calculated travel times and the minimization of the sum of the absolute values of those differences.

Crosshole data alone cannot provide a unique reconstruction, no matter how accurate or extensive the data.

[5]A copy of the programs can be obtained upon request from the Twin Cities Research Center, U.S. Bureau of Mines, 5629 Minnehaha Avenue, South, Minneapolis, MN 55417. Please reference this publication.

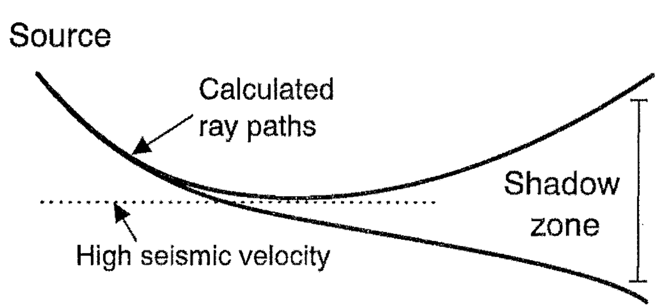

Figure 1.—Ray path shadow zone.

This problem of nonuniqueness was discussed by Ivansson (*17*), who pointed out that if a vertical strip of contrasting velocity passes through the tomographic plane, its location and thickness cannot be determined. Examples were given by Tweeton (*13*) in which the resulting tomogram depended on the velocity pattern used to start the iterative solution.

Uniqueness can be tested in several ways. A visualization method to help predict whether a reconstruction based on a certain set of data and constraints will be unique is to ask whether more than one distribution of velocities would give the same travel times. If the answer is yes, then the reconstruction will not be unique. For example, if there are only crosshole data with no constraints, then two vertical strips of contrasting velocity could be added so that the effects of the strips would cancel and the travel times would be the same with and without the contrasting strips. This method can predict nonuniqueness, but cannot guarantee uniqueness.

Stroud and Dennen (*18*) described a matrix method for determining if a unique direct solution, that is, one obtainable by direct matrix inversion, exists. They used the row-echelon reduction method to determine how many of the equations representing rays with measured travel times are independent. If the number of independent equations is less than the number of pixels or points for which the velocity is to be determined, then the reconstruction will not be unique.

BOMCRATR allows the user to test uniqueness by varying the initial velocity pattern used to start the iterative solution procedure. A unique reconstruction is independent of the initial velocity pattern.

In practice, reconstructions with geophysical data will seldom be truly unique. The important question is how much can the reconstruction vary without significantly affecting the errors between calculated and measured travel times. Varying the initial guess for the velocity distribution can help to answer that question, even when the reconstruction is not unique in the strict mathematical sense.

BOMCRATR provides optional mathematical constraints to help counteract the nonuniqueness of reconstructions that occurs with crosshole data. These constraints are similar to those in BOMTOM and include the following options:

1. Minimum and maximum calculated velocities. The constraint of a realistic maximum velocity based on measurements in intact rock can be very beneficial in counteracting nonuniqueness. It is more difficult to justify a minimum because rock can be fractured to an unknown degree. These limits are set with VELMIN and VELMAX in the input file.

2. Known, fixed velocities in boreholes, such as from sonic logs. The velocity is fixed in NFXLFT and NFXRHT columns of nodes at the left and right sides, respectively, of the grid.

3. Fixing the velocity to the initial value at any point or combination of points. Each point has an associated logical variable LGLVAR that can be set to TRUE to allow the velocity to vary or to FALSE to fix the velocity to the initial value. The velocity at any point can be set to any value because the initial velocity can be input separately for every point.

4. Horizontal layers near the top and/or bottom of the investigated area in which seismic velocity does not vary with horizontal position. These constraints can be realistic if the region of interest extends to layers where the lateral variations can be ignored. After each iteration, the velocities in each specified row are set to the average for that row, so that the row is laterally invariant. NLATOP and NLABOT designate the number of laterally invariant rows near the grid top and bottom, respectively.

5. Smoothing, in which the calculated velocity at a node is influenced by the velocities at neighboring nodes. It replaces the velocity at a node with a weighted average of the velocities in that node and neighboring nodes. The weighting is 20 times the velocity in that node, 4 times the velocity in each of the adjoining 4 nodes, and 1 time the velocity in each of the 4 diagonal corner neighbors. It is normalized by dividing by 40. Bureau researchers have found that this option usually has less effect with curved rays than with straight rays. Curved ray tomography provides some inherent smoothing as the ray path changes between iterations, which reduces the effect of added smoothing.

In addition to those constraints on velocity, the ray path spatial coordinates are constrained to lie within the grid region. A ray path stops when it hits a boundary. If curved ray tomography is unconstrained in both velocity and space, the program may produce good fits using unrealistically long paths with unrealistically high velocities.

TWO-DIMENSIONAL TOMOGRAPHIC PLANE

A more fundamental problem, one that BOMCRATR does not correct, is that the mathematical ray paths are two dimensional. Curved ray tomography allows the two-dimensional bending of rays in the tomographic plane, but in this formulation, does not consider the three-dimensional bending out of the plane. Thus, curved ray tomography can describe a more realistic path if the greatest velocity gradient is in the tomographic plane, which will tend to be true for horizontal layering in a vertical tomographic plane. However, if the layering is steeply dipping and the strike is not perpendicular to the tomographic plane, then there can be significant bending that will not be described by curved rays in the tomographic plane (*19*). The three-dimensional character of the true ray paths and the two-dimensional character of the calculated curved ray tomography paths should be kept in mind when interpreting tomographic analyses.

OVERVIEW OF TOMOGRAPHIC PROGRAM SET—PREBCONV, PREBMCRA, BOMCRATR, AND TOMOPLOT

The curved ray tomographic calculations require two sequential programs. Figure 2 shows the relationship of the preprocessor PREBMCRA and the main tomographic program BOMCRATR, and the input and output files. A program PREBCONV is also supplied for converting data from the format used by the straight ray program BOMTOM to the format used by BOMCRATR. Output from BOMCRATR can be contoured with the program TOMOPLOT. These programs can be run on a 386 microcomputer with 640-kb randon access memory (RAM) when using typical data sets.

Revisions to PREBMCRA, PREBCONV, BOMCRATR, and TOMOPLOT made after the publication of this report will be explained on the distribution diskettes in files READPCRA.DOC, READPCON.DOC, READBOMC.DOC, and READTOMO.DOC, respectively. Any READ*.DOC files on the distribution diskettes should be read before running the programs.

As shown in figure 2, the user prepares an input file for PREBMCRA. Running PREBMCRA or PREBCONV creates an input file for BOMCRATR. Having two separate programs saves computer time because the grid calculations do not need to be repeated for each run. The velocity is specified at node points that are the corners of the triangular pixels. The velocity is assumed to vary linearly within each pixel. The velocity is continuous over the grid, but the gradient of velocity can change at the sides of pixels.

BOMCRATR reads the file produced by PREBMCRA or PREBCONV and performs the tomographic calculations. An initial guess of the velocity distribution is made first. Then the solution is obtained iteratively by applying velocity corrections determined from the differences between calculated and measured travel times. The assumption of linear velocity variation allows the ray path in each pixel to be calculated analytically, which provides faster calculations with less numerical error than stepwise numerical approximations to the path. BOMCRATR has a precalculation menu to help examine and edit parameters controlling the program. It also has postcalculation aids to help examine the results, including a tomogram output using the shadings of extended American Standard Code for Information Interchange (ASCII) characters to display the distribution of the velocity.

BOMCRATR produces several output files. The summary output file *.SUM can serve as an input file for the display program TOMOPLOT. TOMOPLOT provides a contoured display of the results. It allows users to select values for the contour levels and colors. It goes beyond the usual contouring program in assisting the user to analyze and display the difference between two tomograms. It displays the distribution of the differences to help the user decide what is a significant change in velocity. Instructions for running TOMOPLOT are given in the READTOMO.DOC file on the TOMOPLOT program diskette. More details of PREBCONV, PREBMCRA, and BOMCRATR are provided in the following sections.

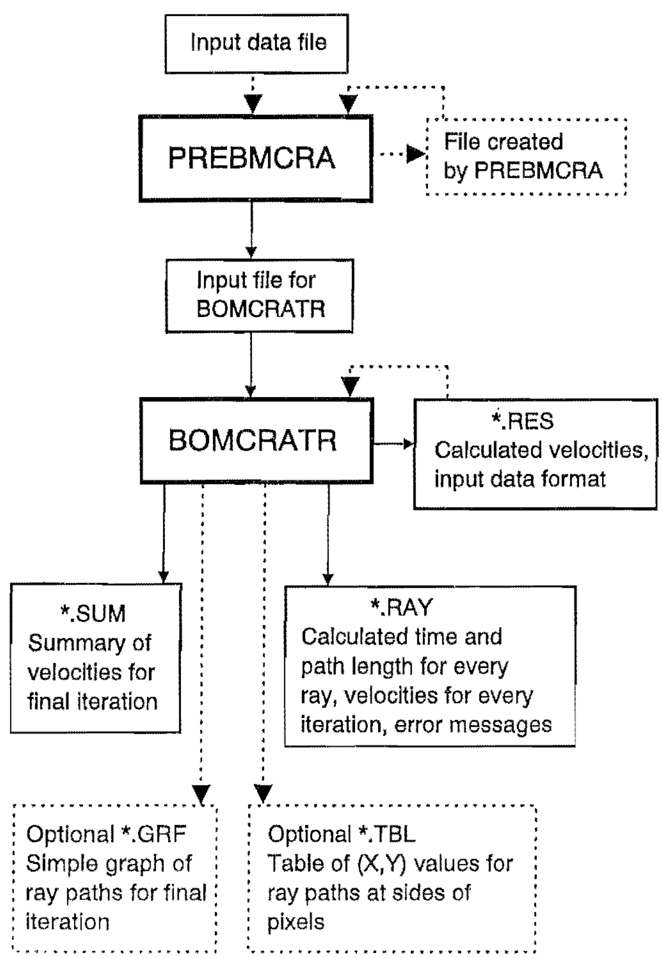

Figure 2.—Tomographic program set. (Dotted lines indicate optional output or user action.)

BOMTOM TO BOMCRATR CONVERSION PROGRAM PREBCONV

PREBCONV reads the same input file format as used by the straight ray program BOMTOM and converts it into the proper format for BOMCRATR. BOMCRATR is less flexible than BOMTOM, so certain restrictions apply. PREBCONV ignores the y coordinates, so all y coordinates should be 0.0 in BOMTOM. In BOMCRATR, all receivers must be along boundaries of the pixel grid. In BOMCRATR, sources must be inside a pixel, so sources should not be exactly on a boundary of the pixel grid, and care must be taken so that sources inside the grid are not inadvertently on a diagonal side of a triangular pixel. The source positions or the boundaries of the pixel grid can be moved slightly if necessary to satisfy that requirement. Deviated boreholes in BOMTOM are represented by the source and receiver locations within a rectangular pixel grid, but in BOMCRATR, a deviated borehole containing receivers must be represented by slanting a boundary of the grid. Thus, a deviated receiver borehole must be deviated manually in the BOMCRATR input file after running PREBCONV. As will be explained later, BOMCRATR uses the boundary of the grid as the receiver locations to allow more efficient interpolation to match ray takeoff angle and arrival at a particular receiver. After starting the program by entering "PREBCONV" and entering the input and output file names when prompted, selecting "1. RUN PROGRAM" from the menu results in an input file for BOMCRATR.

PREPROCESSOR PREBMCRA

PREBMCRA prepares the pixel grid input file for BOMCRATR. The grid is composed of triangular pixels, with each pixel and each corner numbered, as shown in figure 3. Each corner forms a node in the grid at which the velocity is specified. The direction of the diagonals in the triangular pixels alternates to avoid systematic effects.

PREPARATION OF INPUT FILE FOR PREBMCRA

The input file for PREBMCRA can be prepared in two ways: (1) with any word processor that can prepare an ASCII file and (2) interactively with PREBMCRA. A list and description of the input parameters will be given first. Then an example will be given for creating a data file interactively. Users unfamiliar with the programs may want to read the list of parameters to help understand them, but then create the file by running PREBMCRA and using the interactive help the program can provide for generating an input file.

Input Parameter List for Preparing File Using Word Processor

The input format used for PREBMCRA is described in the comment section of the program. The comment section is listed below, beginning at the row of asterisks. It can be used as a guide in preparing a data file with any word processor that can generate ASCII files.

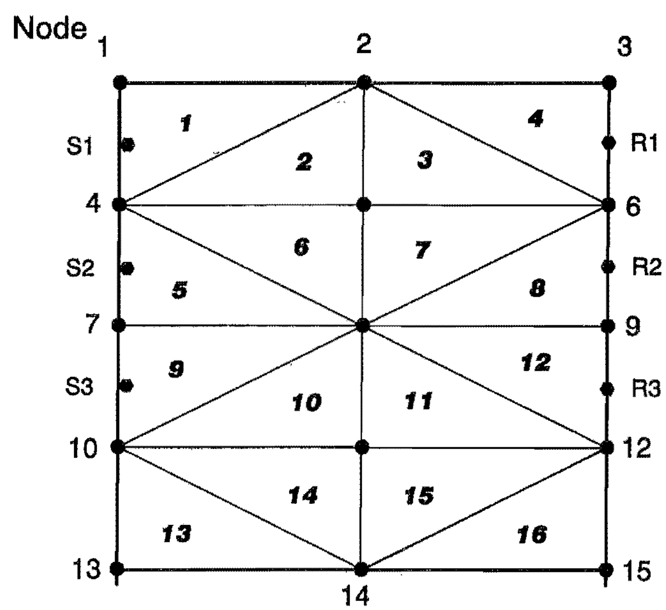

Figure 3.—Tomographic node grid. (Nodes are circular dots, with node number nearby. Pixels are numbered in italics. Source and receiver locations are shown as S and R, respectively.)

****
Units for length, travel times, and velocities must be self-consistent.
Usual units are length in meters, time in milliseconds, velocity in m/ms.
For labeling in the program, sources are assumed to be on the left and
receivers on the right side.
Pixel 1 is at top left. Pixel 2 is next pixel in top row.

Data are read in list-directed format. Input numbers must be separated
by a space or comma. They do not need to be in particular columns.
****
HEADER

One line of comments for the top of output file.
****
ITRMAX MSMTH MSTRT

ITRMAX = maximum number of iterations allowed.
Set MSMTH=1 for smoothing after each iteration. Weighting is 20, 4, 1
for pixel, its nearest neighbors, and corner neighbors, respectively.
Set MSTRT=1 to force all rays to be straight.
****
IPDAT IPITR IPSEG IRAYGRF IRAYTBL

IPDAT=1 causes printing of positions and travel times in *.RAY.
IPITR=1 causes printing of ray information in *.RAY for all iter-
ations instead of just the final one. It can make a very large file.
IPSEG=1 causes printing of segment lengths for each path and pixel in
*.RAY, making a very large output file useful mostly for debugging.
IRAYGRF=1 makes output file *.GRF of graphical display of ray paths.
IRAYTBL=1 makes output file *.TBL of x,z at pixel sides along ray paths.
Any value other than 1 suppresses the printing or display.
****
XTOPL ZTOPL XBOTL ZBOTL

XTOPL and ZTOPL are the x and z coordinates of the top left node.
XBOTL and ZBOTL are the x and z coordinates of the bottom left node.
The coordinate z increases with depth.
They should be set to minimize the intersection of nodes by rays, by
approximately centering source and receiver positions vertically in
pixels. If pixel 1 is too far above or to the left of the paths,
then indices might become larger than the dimension. If pixel 1 is
below or to the right of a ray path, then that ray cannot be traced.
The borders are assumed to be straight lines for this calculation. The
node positions on the borders can be adjusted to account for curved
boreholes.
****
XTOPR ZTOPR XBOTR ZBOTR

XTOPR and ZTOPR are the x and z coordinates of the top right node.
XBOTR and ZBOTR are the x and z coordinates of the bottom right node.
****
NNODH NNODV NLATOP NLABOT NFXLFT NFXRHT

NNODH=number of columns of nodes (counting horizontally).
NNODV=number of rows of nodes (counting vertically).

If desired pixel height is EPIXV, set NNODV=1+(ZBOTR-ZTOPL)/EPIXV.
NLATOP and NLABOT are the number of laterally invariant rows at top and bottom of the grid. The velocity in each of those rows is set equal to the average for that row after each iteration.
NFXLFT and NFXRHT are the number of columns at left and right sides of the grid in which the velocities are fixed at the initial values.
****
IDSOR(ISOR) XSOR(ISOR) ZSOR(ISOR)
One line for each ISOR = 1 to NSOR; NSOR is the number of sources.
End with -99 -99. -99.
IDSOR(ISOR) is the identification number of the source position ISOR. They do not need to be sequential.
XSOR and ZSOR are x and z coordinates of source position ISOR. The coordinate z increases with depth.
Sources should not be exactly on a pixel side.
****
IDREC(IREC) XREC(IREC) ZREC(IREC)
One line for each IREC = 1 to NREC; NREC is the number of receivers.
End with -99 -99. -99.
IDREC(IREC) is the identification number of the receiver position IREC.
XREC and ZREC are x and z coordinates of receiver position IREC.
Receivers must be on a grid border, but not in a corner.
****
ADJTTM

ADJTTM is added to all experimental travel times.
****
IDSORT(IEXPTM) IDRECT(IEXPTM) EXPTT(IEXPTM)
One line for each IEXPTM = 1 to NEXPTM; NEXPTM = num of travel times.
End with -99 -99 -99.
IDSORT and IDRECT are the identification numbers of the source and receiver positions for the travel time EXPTM(IEXPTM).
****
SMFPLL VELMIN VELMAX INVOPT

SMFPLL is the lower limit for the sum of fractions of path lengths in a pixel (segment length in pixel/length of path from source to rec). If the sum is less than or equal SMFPLL, then the velocity for the pixel is displayed as ###, and is not calculated.

VELMIN and VELMAX are low and high limits for the calculated velocities. If a velocity is out of that range, it is set = the limit. If no limit is desired, set the limit to -99.

INVOPT specifies the option for inputting initial guesses of velocities.

If INVOPT = 1, input is uniform and the format is:
VEL(1).
BOMCRATR sets initial velocity = VEL(1) for every node.

If INVOPT = 2, input is row by row and the format is:
IROWV VEL(IROWV). (One line for each row, NNODV lines.)
IROWV is the row number. Rows can be input in any order.

BOMCRATR sets initial velocities=VEL(IROWV) for all nodes in that row. This option facilitates varying the pattern of initial guesses to test the uniqueness of a solution.

If INVOPT = 3, input is node by node and the format is:
IROWV (VEL(I),I=1,NNODH) (One line for each row, NNODV lines.)
IROWV is the row number.
VEL(I) is the initial velocity for row IROWV, column I.
This option is used when resuming a run using a data file as input.
******END OF DESCRIPTION OF DATA READ FROM INPUT FILE******

Output into BOMCRATR.IN or a different selected file consists of all the input, plus the following.
*****
I XNOD(I) ZNOD(I) (One line for each node.)

I is the node number, I = 1 to NNODE. The maximum allowed I = MAXNOD.
XNOD(I),ZNOD(I) are x,z coordinates of node I.

End of this set with -99 -99. -99.
****

XNOD(ICOL), every second position, at top of grid
ZNOD(INODV) LGLVAR(ICOL + (INODV-1)*NNODH), ICOL=1,number of node col.
One line for each row of velocity node points.
XNOD(ICOL), every second position, at bottom of grid

This matrix of logical variables is for conveniently specifying whether the velocity should be fixed to the initial value (F) or allowed to vary (T). XNOD and ZNOD are displayed around the matrix to facilitate selecting the velocity points to fix, and are read into an array that is temporarily not in use, not read as new data.
Fixing velocities with NFXLFT or NFXRHT overrides T in this matrix.
****

J K(J,1) K(J,2) K(J,3) LJ(J,1) LJ(J,2) LJ(J,3)
(One line for each triangular pixel.)
J is the triangular pixel number, starting with J=1, monotonically increasing, maximum J = MAXTRI (see parameter definition).

K(J,1)
K(J,2) are nodepoints (I) of this triangle, in mathematically
k(J,3) positive direction, i.e., counterclockwise

LJ(J,1) LJ are indices of neighboring triangles of triangle J
LJ = -32111,-32222,-32333,-32444 indicate left,
LJ(J,2) bottom, right, top borders of grid, respectively.
LJ given in counterclockwise direction. LJ(J,1) is the
LJ(J,3) index of the neighboring triangle across the triangle
side with the nodes K(J,1) and K(J,2), etc.

End of this set is -99 -99 -99 -99 -99 -99 -99
****

Example of Preparing File With Interactive Help From PREBMCRA

To begin interactive generation of an input file with PREBMCRA, request an input file that does not exist and then answer affirmatively when asked if the file should be created. For most data sets, following the on-screen instructions and referring to the list of input parameters given above will be adequate. If a problem occurs and the program aborts with only part of the data file prepared or with a data file prepared incorrectly, the faulty or incomplete data file should be deleted to avoid reading it inadvertently at some later time.

To help the user deal with more unusual data sets, an illustrative example will be given incorporating several complicating factors. These factors include generation of synthetic data, a combination of crosshole and reverse vertical seismic profiling (RVSP) source-receiver configurations, and extremely crooked boreholes.

Extremely crooked boreholes complicate the use of BOMCRATR more than BOMTOM because BOMCRATR calculates times for ray paths to the edge of the grid, not to the receivers themselves. Therefore, receivers must be along a grid border, and a crooked receiver borehole requires a crooked grid border. For most situations, this can be done easily by making the grid border follow the borehole. The departure of the receivers from the borehole on short, straight line segments between node points will be negligible for typical borehole deviations. The illustrative example includes a case where it is not negligible because of a sharp bend.

BOMCRATR calculates ray path times to the grid borders instead of to receivers to improve computational efficiency and reliability when using many receiver locations. There are two methods of determining the takeoff angle at the source position that makes a ray arrive at a particular receiver. The first method is to consider one receiver location at a time, bracket it with rays, and find the takeoff angle that sends a ray to it. This method allows the receiver to be anywhere, and is efficient when there are few receivers. But it may not find the shortest path if it merely finds the takeoff angles that give rays above and below the receiver, and then finds one takeoff angle that sends the ray to the receiver. Many rays should be traced to be sure that a shorter time path does not exist. This problem is more severe with larger contrasts in velocity because the greater bending allows more paths from a source to a particular receiver. Thus, considering one receiver location at a time is best suited for few receivers and smaller velocity contrasts.

The second method, the one used by BOMCRATR, is to cover an entire grid boundary with rays and to determine which rays go between which receivers. This method becomes more advantageous as the number of receiver locations on the grid boundary increases. It does require that the receiver locations be on the edge of the grid to allow interpolation between locations, however. Receivers can be on any of the four edges of the grid, or any combination of edges.

Suppose that the user has the following configuration of source and receiver locations, as shown in figure 4: Coordinates are designated by (x,z) pairs, with (0,0) arbitrarily at the top of the left borehole. For the crosshole data, assume that the user has sources S1 (1,10), S2 (2,20), S3 (3,30), and receivers R1 (40,11), R2 (40,20), R3 (41,30), all in meters. Thus, the left borehole is deviated but straight, the right borehole is crooked, and not all source-receiver pairs are evenly spaced. Assume that the user also has two RVSP receivers on the surface, with the surface sloping so that the receivers are at (10,0.5) and (30,3).

The first step is to choose a pixel grid. A rule of thumb used by Bureau researchers to avoid excessive underdetermination of the reconstruction is not to have many more horizontal nodes than the number of source or receiver locations. For this example, use three nodes horizontally.

Choose the vertical height of the pixels equal to the vertical spacing between source or receiver locations, so that there is one source-receiver pair in each row of pixels. Thus, use a pixel height of 10 m, as shown in figure 5. The ray tracing is most reliable if ray paths do not make grazing angles with pixel boundaries, so approximately center the source-receiver locations vertically in the pixel rows. Therefore, the nodes near the source-receiver locations in the boreholes should be at z = 5, 15, 25, and

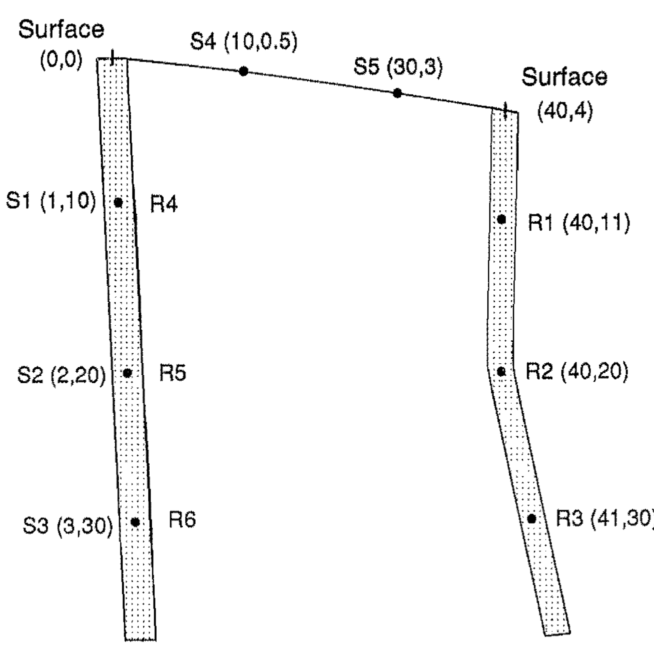

Figure 4.—Synthetic data source and receiver locations. (Locations are shown as (x,z) coordinates.)

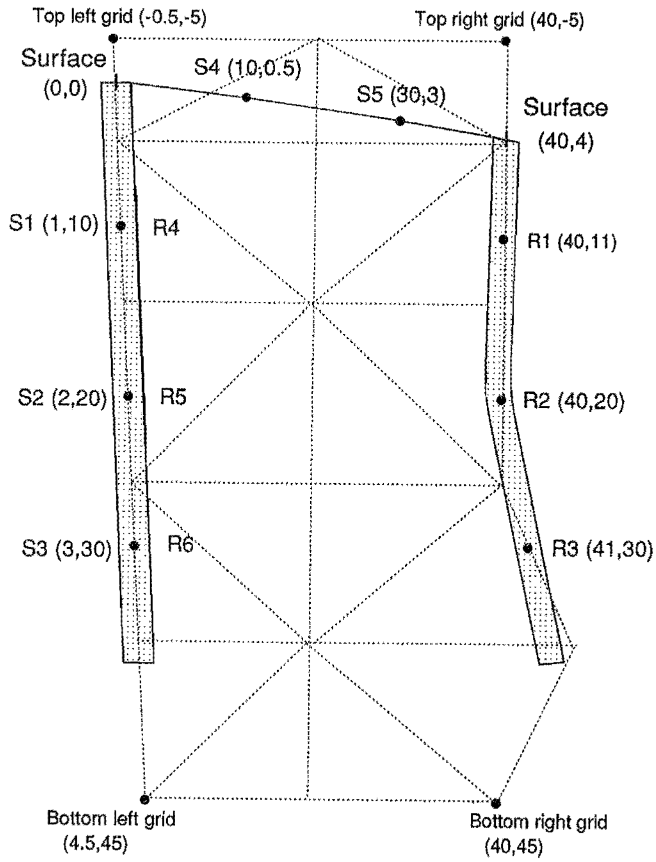

Figure 5.—Synthetic data node grid. (Locations are shown as (x,z) coordinates.)

35 m. The source borehole is slanted but straight, so it is simplest to keep all node points on the same line. Thus, nodes near sources will be at (0.5,5), (1.5,15), (2.5,25), and (3.5,35). BOMCRATR places receivers on the sides of pixels, so the simplest way to obtain the receiver positions is to have nodes in the receiver borehole at (40,5), (40,15), (40,25), and (42,35). The last point is chosen so that a line between node points passes through the receiver position at (41,30).

To include the surface receivers, the grid needs to be extended at least to the surface. The user could extend the top row just to the surface and make it narrow and changing in thickness, but it is easier to extend it a full uniform row to above the surface. The physical receivers will be treated as mathematical sources, so they can be anywhere within pixels. To keep all rows the same height, extend the grid upward and put nodes at (-0.5,-5) and (40, -5).

An additional row should be added to the bottom because rays may bend enough to need another row below the bottom source or bottom receiver. Therefore, extend the grid linearly in the source borehole and add a node at (4.5,45). In the receiver borehole, it is simplest to add a node at (40,45). Only the corner nodes are specified directly in the input file to PREBMCRA, so this choice of bottom nodes minimizes the number of node positions that must be changed manually in the output file from PREBMCRA.

Thus, the top left of the grid is at (-0.5,-5), the bottom left at (4.5,45), the top right at (40,-5), and the bottom right at (40,45). As stated above, there are three nodes horizontally. There are six vertically, to give a node every 10 m from -5 to 45 m, as shown in figure 5. The user does not know before the tomographic calculation if the bottom row will contain any ray paths.

A step-by-step example of the user input will be given. The user knows source and receiver locations, as described above. In addition, the user has either measured travel times between most of the source-receiver pairs and wants to image the velocity, or wants to calculate synthetic travel times between the source-receiver pairs for a specified velocity distribution. Both cases will be described. Synthetic data will be generated first, then those synthetic travel times will be fit by BOMCRATR to create a tomogram. The agreement between the tomogram and the original velocity model will test the uniqueness of the solution.

To generate the input file for PREBMCRA interactively, start the program by entering "PREBMCRA." When the program asks for the input file, enter any legal name of a file (up to eight characters for the file name plus up to three characters for the extension) that does not exist. PREBMCRA will then ask if the file should be created. Answer "Y," and an interactive session to provide the input file will begin. The example will show the message from PREBMCRA indented two spaces, followed by an explanation.

  Use PREBMCRA.IN for input file? (Y/N)= = >

This name is the default name for the input file. There is a sample PREBMCRA.IN and a duplicate of it named "SYNTHDAT.PRE" on the program diskette, which contains synthetic travel times that will be generated in this example. To create a new file, enter "N" or "n."

  Name of file to use? = = >

Enter any legal name, like "SYNTHGEN.PRE". You can also specify a drive and path, using up to 23 total characters.

  File SYNTHGEN.PRE not found. Create file? (Y/N)= = >

Enter "Y" or "y" to begin interactive creation of data file.

Enter one line of description.

Enter up to 78 characters describing the run. For this example, enter "Synthetic crosshole and RVSP data. Units m and ms. Crooked boreholes."

ITRMAX, MSMTH, MSTRT will be 1 0 0

The number of iterations is set to one here, but can be changed later. Only one iteration is needed to generate synthetic data. Five to ten iterations are typical for calculating tomograms from travel times. The default MSMTH will not smooth the velocity among adjoining pixels. The default MSTRT does not force all rays to be straight. Reasonable default values are generated by PREBMCRA to reduce the amount of manual input. They can be changed later, interactively, while starting PREBMCRA or BOMCRATR.

IPDAT,IPITR,IPSEG,IRAYGRF,IRAYTBL will be 1 0 0 1 0

These are default values of output control parameters defined previously in the listing of the program comments. These values mean that initial data will be printed, information on individual rays will be printed only for the final iteration, path segment lengths will not be printed, a simple character-based graph of ray paths will be printed, and the table of ray positions at pixel sides will not be printed.

Enter top left X,Z; bottom left X,Z grid coordinates.

The user should enter "-0.5,-5.0,4.5,45.0." The example source and receiver locations are shown in figures 4 and 5. With sources at z = 10, 20, and 30, the nodes should be at 5, 15, 25, and 35. Add one more row of nodes at the top at z = -5, and one more row at the bottom at z = 45.

Enter top right X,Z; bottom right X,Z grid coordinates.

The user should enter "40.0,-5.0,40.0,45.0."

Enter number of nodes in the horizontal, vertical directions.

The user should enter "3,6."

There are three borehole source locations and three borehole receiver locations, so three nodes horizontally is about right. With straight rays, four nodes would be adequate vertically. However, with curved rays, and not knowing how much the rays will curve, add one extra node at the top and at the bottom to give six nodes vertically.

NLATOP,NLABOT,NFXLFT,NFXRHT will be 0 0 0 0

None of these constraints, described in the program comments, will be imposed.

Enter ident num, X,Z coordinates of source position (IDSOR,XSOR,ZSOR).

Enter -99,-99.,-99. to end source input.

The sources must not be on a side of a pixel because BOMCRATR needs to identify the pixel containing the source. If a source is on a side of a pixel, it is not inside any pixel, and the test by BOMCRATR to identify the source pixel may fail. The sources can be shifted 0.01 m to the right to keep them off the pixel sides. Enter "1,1.01,10.0" for the first source. The identification numbers can be in any order. The next entry is "2,2.01,20.0," and "3,3.01,30.0" is the third. The RVSP receivers are treated as mathematical sources, so enter "4,10.0,0.5" and "5,30.0,3.0" to include them. Then enter "-99,-99.,-99." to end the source input.

Enter ident num, X,Z coordinates of receiver position (IDREC,XREC,ZREC).

Enter -99,-99.,-99. to end receiver input.

Enter "1,40.0,11.0" for the first receiver. The next entry is "2,40.0,20.0," and "3,41.0,30.0" is the third. To perform RVSP with the mathematical sources near the surface, allow the locations in the left borehole to be both sources and receivers. Enter "4,1.0,10.0," then "5,2.0,20.0," and finally "6,3.0,30.0" to place receivers in the left borehole. Then enter "-99,-99.,-99." to end the receiver input.

Time correction to be added to each travel time (ADJTTM) = 0.0

You can add a correction to all travel times when the interactive menu is displayed after starting PREBMCRA. The interactive file generation sets it to 0 by default.

Enter source ident num, rec ident num, measured time (IDSORT,IDRECT,EXPTM).

Enter -99,-99,-99. to end time input.

Enter -1,-1,timall to set time = timall for all source-rec pairs separated by more than 0.01 of the average grid width.

The user should enter "-1,-1,8.0."

If the travel times have been measured experimentally, they can be given. For this sample, synthetic data will be generated, so enter "-1,-1,8.0" to set the time = 8.0 for all pairs that are separated by more than 0.01 of the average width of the pixel grid. This separation requirement prevents arithmetic problems with sources and receivers that are at or very close to the same location. BOMCRATR will use only the pairs for which a time is given, so if synthetic times are to be calculated, then some value of time must be given for each pair. If measured times were available, then the entry for a travel time of 15.1 ms from the source, having identification number 1 to receiver, having identification number 1, would be 1,1,15.1, and so on. The source-receiver pairs do not need to be listed in any particular order. If measured times are given, then end the time data input by entering "-99,-99,-99." If -1,-1,timall is given, the input ends automatically.

SMFPLL will be 0.005000.

This value is reasonable for SMFPLL. Any value greater than the accuracy of floating point numbers could be used.

No low or high limits for calculated velocity (VELMIN,VELMAX).

The corresponding default values are -99., so the lower and upper limits of the calculated velocity are not constrained. Velocity constraints can be imposed later.

Enter guess for uniform initial velocity (VEL).

For this sample, enter "5.0." When first generating this file, PREBMCRA uses INVOPT = 1, and so uses the same velocity for all nodes. The velocity distribution can be changed later.

Use BOMCRATR.IN for output file? (Y/N)= = >

There is a sample file named "BOMCRATR.IN" and a duplicate of it named "SYNTHDAT.BOM" on the program diskette, which contains synthetic travel times generated in this example. To make a new file, enter "N" or "n."

Name of file to use? = = >

The user should enter "SYNTHGEN.BOM." PREBMCRA then displays the following menu:

```
CHANGE INPUT? RUN PROGRAM OR ENTER NUMBER OF ANY OPTION TO CHANGE.
NUM OPTION
 1  RUN PROGRAM
 2  DESCRIPTION
Synthetic crosshole and RVSP data. Units m and ms. Crooked boreholes.
 3  ITRMAX (ITERATION LIMIT) = 1
 4  MSMTH = 0, NO SMOOTHING
 5  IPDAT,IPITR,IPSEG,IRAYGRF,IRAYTBL (1 TO PRINT: DATA;RAYS ALL ITER;
       SEG LENGTH;RAY GRAPHS,RAY X,Z TABLES) = 1 0 0 1 0
 6  GRID TOP LEFT X,Z =  -0.500  -5.000; BOT LEFT X,Z =  4.500  45.000
 7  GRID TOP RIGHT X,Z =  40.000 -5.000; BOT RHT X,Z =  40.000 45.000
 8  NUMBER OF NODES HORIZONTALLY, VERTICALLY =   3      6
       PIXEL WIDTH, HEIGHT AT TOP LEFT =  20.2500     10.0000
 9  NLATOP,NLABOT (NUM OF LATERALLY INVARIANT ROWS AT TOP,BOTTOM)=0 0
10  NFXLFT,NFXRHT (NUM OF COL WITH FIXED VEL AT LEFT,RIGHT SIDES)=0 0
11  LGLVAR (FIX OR VARY VELOCITY AT A NODE).
12  ADJTMM (TIME ADDED TO ALL TRAVEL TIMES) = 0.0000
13  SMFPLL (LOWER LIMIT OF PATH FRACTION FOR DISPLAY) = 0.0050
14  VELMIN,VELMAX (LIMITS OF CALC VELOCITY,UNLESS = -99.)=-99.0-99.0
15  INVOPT (INPUT OF INITIAL VEL: 1,UNIFORM; 2,ROW; 3 NODE) 1
16  SAVE THESE PARAMETERS IN AN INPUT FILE FOR PREBMCRA.
17  CHANGE THE OUTPUT (BOMCRATR INPUT) FILE NAME FROM SYNTHGEN.BOM
18  QUIT WITHOUT RUNNING PROGRAM.
       ENTER OPTION NUMBER, 1 - 18.
```

Most of these parameters were explained in the program comments.

The velocity should be changed from being a uniform 5.0 to having a gradient in the z direction to give curved rays for this example. Enter "15" to change INVOPT. To provide an example with curved rays, take the velocity as V = 3.0 + z/10 m/ms. Then the velocities for the six rows of nodes will be 2.5, 3.5, 4.5, 5.5, 6.5, and 7.5 m/ms. These values will give a noticeable curve to the rays.

CURRENT INVOPT IS 1

ENTER NEW INVOPT (1 UNIFORM; 2 ROW BY ROW; OR 3 NODE BY NODE)

The user should enter "2."

ENTER NOD ROW NUMBER. NEG ENTRY ENDS THIS INPUT.

The user should enter "1."

CURRENT INITIAL GUESS IS 5.000

ENTER NEW INITIAL GUESS

The user should enter "2.5."

Continue entering with row 2 = 3.5, row 3 = 4.5, row 4 = 5.5, row 5 = 6.5, and row 6 = 7.5. End the input by entering -1 for a row number. The menu will then reappear with option 15 INVOPT = 2.

Choosing option 1 runs PREBMCRA and creates an input file for BOMCRATR. The screen first displays "Program is entering subroutine NEIBOR." and near the end of the calculations displays "Program has left subroutine NEIBOR."

Options 6 and 7 ask for the four corners of the grid separately because the grid does not have to be rectangular. Recall that borehole deviation can be taken into account in this way.

Option 11 changes the logical variable LGLVAR(INODE) to designate whether the velocity at node INODE is fixed or is allowed to vary in BOMCRATR. If LGLVAR(INODE) is TRUE, the velocity at INODE is allowed to vary. The default is TRUE, but the user can change any to FALSE. The user can then input known velocities for some locations and keep those velocities fixed. Velocities might be known from borehole sonic logs. If that option is selected, the program will ask which node velocity to fix or vary and whether to fix or vary it.

The array LGLVAR is displayed as a matrix of T's and F's in the output file (the input file for BOMCRATR) with node coordinates above and below and to the side of it. This array is convenient for visualizing which nodes are fixed and which are varied, and to change LGLVAR in selected areas with a word processor. Option 10 directly above the LGLVAR option also fixes the velocity in selected nodes, but in NFXLFT and NFXRHT columns at the left and right sides of the grid to enable convenient fixing of velocities in the boreholes if they are known from sonic logs. NFXLFT and NFXRHT override a T in the LGLVAR array.

Option 16 saves an input file for PREBMCRA with the new options and gives a choice of writing over the current input file or writing a new one.

EXAMPLE OF INPUT FILE FOR PREBMCRA

This file is the synthetic data file SYNTHGEN.PRE for PREBMCRA created interactively by PREBMCRA using the SYNTHGEN example. Authors' comments and explanations are enclosed in square brackets ([ ]).

Synthetic crosshole and RVSP data. Units m and ms. Crooked boreholes.

1	0	0			
1	0	0	1	0	
-0.5000	-5.0000	4.5000	45.0000	[Grid	
40.0000	-5.0000	40.0000	45.0000	corners]	
3	6	0	0	0	0
1	1.0100	10.0000			
2	2.0100	20.0000		[Sources]	
3	3.0100	30.0000			
4	10.0000	0.5000			
5	30.0000	3.0000			
-99	-99.0000	-99.0000			
1	40.0000	11.0000			
2	40.0000	20.0000		[Receivers]	
3	41.0000	30.0000			

4	1.0000	10.0000		
5	2.0000	20.0000		
6	3.0000	30.0000		
-99	-99.0000	-99.0000		
0.00000				
1	1	8.00000		
1	2	8.00000		[All times are set to 8.0 to serve as place-holders, so BOMCRATR will use these source-receiver pairs in synthetic time calculations.]
1	3	8.00000		
1	5	8.00000		
1	6	8.00000		
2	1	8.00000		
2	2	8.00000		
2	3	8.00000		
2	4	8.00000		
2	6	8.00000		
3	1	8.00000		
3	2	8.00000		
3	3	8.00000		
3	4	8.00000		
3	5	8.00000		
4	1	8.00000		
4	2	8.00000		
4	3	8.00000		
4	4	8.00000		
4	5	8.00000		
4	6	8.00000		
5	1	8.00000		
5	2	8.00000		
5	3	8.00000		
5	4	8.00000		
5	5	8.00000		
5	6	8.00000		
-99	-99	-99.00000		
0.00500	-99.0000	-99.0000	2	
1	2.5000			
2	3.5000			[Rows and velocities]
3	4.5000			
4	5.5000			
5	6.5000			
6	7.5000			

TOMOGRAPHIC PROGRAM BOMCRATR

BOMCRATR calculates the distribution of velocities that gives the best fit to the measured travel times.

EXAMPLE OF INPUT FILE FOR BOMCRATR (OUTPUT FROM PREBMCRA)

The modified output file of PREBMCRA, SYNTHGEN.BOM, that is the input file for BOMCRATR, is listed below. The indicated row was changed to move a node at (40.0,35.0) to (42.0,35.0) to place the receiver above it at (41.0,35.0). This change can be done with any word processor that can read and write ASCII files. It is important to remember to change the file manually because the program traces rays to the edge of the grid, not to the positions of the receivers. It assumes that the receivers are on the border.

One other change was necessary. Travel times for source-receiver pairs that were on the same border were removed. When generating the data file interactively with the -1,-1,timall user input, PREBMCRA removes all source-receiver pairs where the source and receiver locations are closer than 0.01 of the average grid width. It does not remove source-receiver pairs that are separated but on the same border. Those pairs must be removed from the BOMCRATR input file by the user. This manual removal of source-receiver pairs is necessary only when synthetic RVSP data are generated with the timall input parameter.

Synthetic crosshole and RVSP data. Units m and ms. Crooked boreholes.

1	0	0			
1	0	0	1	0	
-0.5000	-5.0000	4.5000	45.0000		
40.0000	-5.0000	40.0000	45.0000		
3	6	0	0	0	0
1	1.0100	10.0000			
2	2.0100	20.0000			
3	3.0100	30.0000			
4	10.0000	0.5000			
5	30.0000	3.0000			
-99	-99.0000	-99.0000			
1	40.0000	11.0000			
2	40.0000	20.0000			
3	41.0000	30.0000			
4	1.0000	10.0000			
5	2.0000	20.0000			
6	3.0000	30.0000			
-99	-99.0000	-99.0000			
0.00000					
1	1	8.00000			
1	2	8.00000			
1	3	8.00000			
2	1	8.00000	[Source-receiver pairs 1-5, 1-6,		
2	2	8.00000	2-4, 2-6, 3-4, 3-5 were removed		
2	3	8.00000	manually. Pairs 1-4, 2-5, 3-6		
3	1	8.00000	were removed automatically		
3	2	8.00000	because they were close		
3	3	8.00000	together.]		
4	1	8.00000			
4	2	8.00000			
4	3	8.00000			
4	4	8.00000			
4	5	8.00000			
4	6	8.00000			
5	1	8.00000			
5	2	8.00000			
5	3	8.00000			
5	4	8.00000			
5	5	8.00000			
5	6	8.00000			
-99	-99	-99.00000			
0.00500	-99.0000	-99.0000	2		
1	2.5000				
2	3.5000		[Node rows, velocities]		
3	4.5000				
4	5.5000				

```
        5     6.5000
        6     7.5000
        1    -0.5000    -5.0000
        2    19.7500    -5.0000
        3    40.0000    -5.0000          [Node coordinates]
        4     0.5000     5.0000
        5    20.2500     5.0000
        6    40.0000     5.0000
        7     1.5000    15.0000
        8    20.7500    15.0000
        9    40.0000    15.0000
       10     2.5000    25.0000
       11    21.2500    25.0000
       12    40.0000    25.0000
       13     3.5000    35.0000
       14    21.7500    35.0000
       15    42.0000    35.0000          [This was 15  40.0000   35.0000]
       16     4.5000    45.0000
       17    22.2500    45.0000
       18    40.0000    45.0000
      -99   -99.0000   -99.0000
    -0.50      40.00
-5.00  T  T  T   [T to vary velocity,
 5.00  T  T  T   F to fix velocity
15.00  T  T  T   at these nodes.]
25.00  T  T  T        [The lines below designate pixel numbers,
35.00  T  T  T        corner numbers, and neighboring pixel
45.00  T  T  T        numbers.  -32111, -32222, -32333, and -32444
     4.50      40.00  designate left, bottom, right, top borders.]
        1          1          4          2     -32111          2     -32444
        2          2          4          5          1          6          3
        3          2          5          6          2          7          4
        4          2          6          3          3     -32333     -32444
        5          4          7          8     -32111          9          6
        6          4          8          5          5          7          2
        7          5          8          6          6          8          3
        8          6          8          9          7         12     -32333
        9          7         10          8     -32111         10          5
       10          8         10         11          9         14         11
       11          8         11         12         10         15         12
       12          8         12          9         11     -32333          8
       13         10         13         14     -32111         17         14
       14         10         14         11         13         15         10
       15         11         14         12         14         16         11
       16         12         14         15         15         20     -32333
       17         13         16         14     -32111         18         13
       18         14         16         17         17     -32222         19
       19         14         17         18         18     -32222         20
       20         14         18         15         19     -32333         16
      -99        -99        -99        -99        -99        -99        -99
```

STEPS IN CALCULATIONS

Figure 6 is a flowchart showing the main sections of BOMCRATR. The steps to calculate the velocity are as follows:

1. Read the input.
   a. Control parameters such as the number of iterations.
   b. Identification numbers and locations of sources and receivers.
   c. Travel times between source-receiver pairs, specified by the identification numbers.
   d. Initial guess for velocities.
   e. Pixel grid specifications, locations of nodes that form the triangular pixels, and which node velocities should be varied. The left, bottom, right, and top borders of the grid are specified by -32111, -32222, -32333, and -32444, respectively, to allow I*2 (2-byte) integers to be used in computations. Using I*2 integers reduces memory requirements and allows more data points to be read.

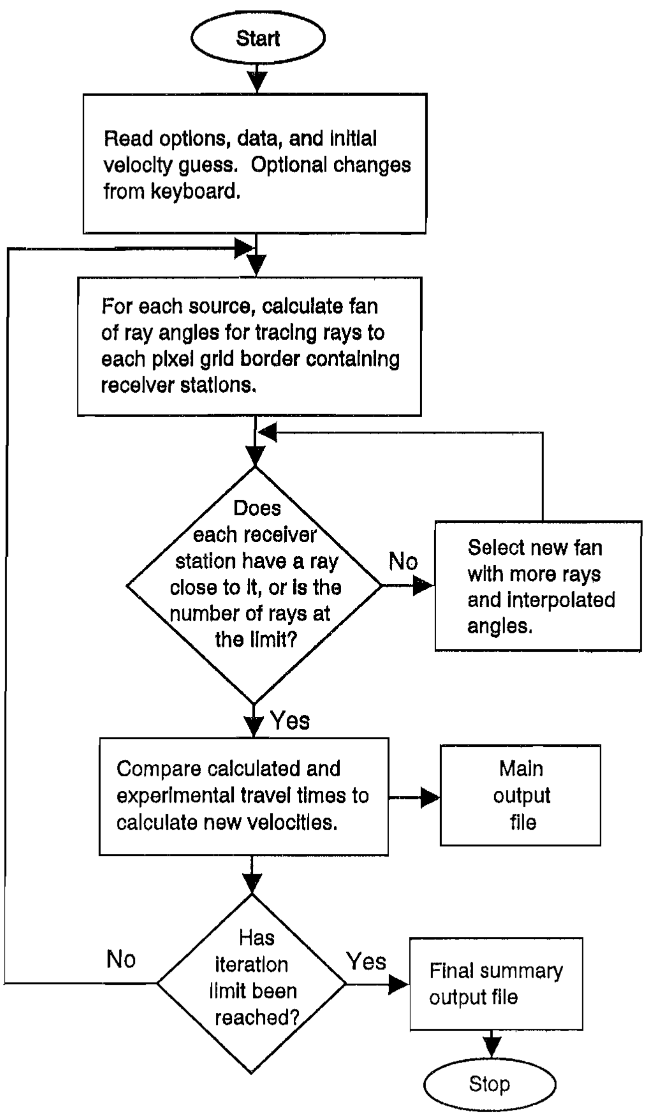

Figure 6.—BOMCRATR flowchart.

2. Allow interactive modification of input parameters. The menu that starts BOMCRATR is similar to the PREBMCRA menu, but it does not allow changing the node grid because PREBMCRA must be run to calculate the positions of the nodes and the configuration of the triangular pixels. If the pixel grid is to be changed, rerun PREBMCRA.

3. Determine in which pixel the first source is located. If the source is on or very close to the side of a pixel, the pixel cannot be determined. The error message shown below will appear, and the program will stop. The source identification number is input by the user as part of the data (see program comments).

> ERROR AT BOMCRATR $2400. SOURCE IDENT 1 NOT INSIDE OF A PIXEL. This problem is usually caused by the source being on a side of a pixel. Move source or nodes slightly to make the source inside a pixel.

4. With straight rays, skip to step 6. With curved rays, calculate upper and lower limits for the fan of initial angles for the ray paths. The integer flag COVFLG indicates the progress of establishing a ray path takeoff angle for a ray that connects a source location with a particular receiver position. COVFLG is set to 0 at the start of each iteration. Starting with the first source and the first grid side for which there are travel times to receivers, BOMCRATR calculates the limits to the fan of angles for the ray tracing. The fan limits are determined by calculating the straight rays to the first and last receiver along that grid side and by increasing the fan size by 15° at each side because rays may bend. If both the first and last receivers are covered by the fan of rays, the program proceeds to the next step. If they are not, the program expands the fan at the appropriate side several times. COVFLG is set to 1 after the fan limits are selected. In some cases, no calculated rays can reach certain receiver locations. That result is a deficiency of this ray tracing approach in general, and Bureau researchers are pursuing an alternative approach.

5. Conduct ray tracing over the fan limits at the first number of angles, set to (30 plus the number of receivers on that grid side) times five. If all receivers within the fan range have at least one ray on each side, the program goes to the next step. If some receivers do not have at least one ray on each side, then the number of angles is increased and the program repeats the calculation. If the limit for the number of angles, MAXRAY, is reached, the

program goes to the next step. COVFLG is set to 2 before going to the next step.

6. Conduct final ray tracing from source to receiver positions. With straight rays, the takeoff angle that should make the ray go to a particular receiver is calculated directly from geometry. With curved rays, it is interpolated from the rays above and below the receiver, or from the closest rays. If the ray passes within XTOL or ZTOL (depending on the border) of the receiver, the corresponding time is stored as a possible travel time. With curved rays, several rays from various takeoff angles may pass close to a given receiver. The travel time for a particular receiver is the shortest time for rays passing within XTOL or ZTOL of the receiver.

If the first interpolated ray is not sufficiently close to the receiver, then the program interpolates between the two rays that form the closest pair out of the original rays and the interpolated ray. The program tries that procedure up to five times and then goes to the next step. The time is used if the closest ray is within the distance limit or the time between the two closest rays is within the time limit. Those calculations are necessary because the ray position at the receiver boundary can be a strongly fluctuating nonlinear function of the takeoff angle. The most computer time is consumed in ray tracing to find the ray takeoff angles corresponding to particular receivers.

As either straight or curved rays move from pixel to pixel, the next pixel number is determined by calculating which side of the current pixel the ray intercepts, and then using the list containing the neighboring pixel number for that side. If the ray intercepts a node, the program may not be able to calculate a side, and the tracing may fail for that ray. If that happens, one of the following error messages will appear, along with information about the ray and position:

AT RAYTRC $380, PROGRAM COULD NOT DETERMINE RAY EXIT POINT ON PIXEL SIDE. Continuous recurrence at one pixel may imply input problem, such as a source on a pixel side or arrays passed incorrectly.

AT RAYTRC $460, COULD NOT DETERMINE SIDE OF PIXEL FOR STRAIGHT RAY EXIT. Continuous recurrence at one pixel may imply input problem, such as a source on a pixel side or a source-rec pair on same grid border.

AT RAYTRC $615, COULD NOT DETERMINE SIDE OF PIXEL FOR CURVED RAY EXIT. Continuous recurrence at one pixel may imply input problem, such as a source on a pixel side or a source-rec pair on same grid border.

It is not serious if these messages appear several times. It only means that certain rays have intercepted a node and have not reached a grid side. Rays with slightly larger or smaller takeoff angles will not intercept that node and so will reach a grid side. However, if any of these messages appear repeatedly for the same pixel and the program cannot make progress, the indicated problem may have occurred and the data should be checked. If the number of successful source-receiver rays is less than expected because of node interceptions, the number may increase if the grid or sources are moved slightly.

7. Calculate correction factors for node velocities. A velocity correction must first be calculated for each pixel on the ray path, and then a correction must be calculated for the nodes. The pixel correction is distributed equally among all three of the nodes forming the triangle. The corrections are weighted according to the fraction of the ray path that passed through the pixel, not according to the length of the path segment in the pixel. If the weighting was based directly on the length of the segment in the pixel, then diagonal ray paths with greater total lengths would have a greater effect on the weighting than horizontal paths. The correction factors are accumulated for each node.

8. Repeat the process for each grid side for rays from that source.

9. Repeat the process for each source.

10. After all grid sides for all sources have been considered, normalize the accumulated correction factors for each node and apply a correction factor to each velocity.

11. Start a new iteration with the new velocities. The total process is repeated until the iteration limit is reached.

12. Generate velocity output in tabular form and in a simple graphical form using extended ASCII characters for shading.

EXAMPLE OF OUTPUT FILES FROM BOMCRATR FROM GENERATING SYNTHETIC TRAVEL TIMES

The output files listed below were generated by BOMCRATR using the synthetic data input file SYNTHGEN.BOM described previously. BOMCRATR can be run with either curved or straight rays, depending on MSTRT. The results listed below were with curved rays. The results that readers obtain may look slightly different if the format is changed in later versions of the program. The numbers may vary slightly with the precision of the computer.

The file listed below is the main output file SYNTHGEN.RAY for one iteration. Synthetic calculated travel times are in the second column to the right in rows, such as

SOR ID,REC ID,PATH LEN,CAL TIME,EXP-CAL TIME= 1 1 40.36 9.40 -1.40,

where the synthetic travel time is 9.40. They can be cut from the output with any word processor that allows columns to be cut, or they can be copied manually.

```
BOMCRATR MAIN OUTPUT FILE SYNTHGEN.RAY
INPUT FILE WAS SYNTHGEN.BOM
Synthetic crosshole and RVSP data. Units m and ms. Crooked boreholes.
ITERATION LIMIT =    1
MSMTH = 0     MSTRT = 0
IPDAT,IPITR,IPSEG,IRAYGRF,IRAYTBL =    1   0   0   1   0
XTOPL,ZTOPL,XBOTL,ZBOTL   =   -0.5000  -5.0000   4.5000  45.0000
XTOPR,ZTOPR,XBOTR,ZBOTR   =   40.0000  -5.0000  40.0000  45.0000
NNODH,NNODV,NLATOP,NLABOT,NFXLFT,NFXRHT =  3  6  0  0  0  0
IDSOR(ISOR),XSOR(ISOR),ZSOR(ISOR)   =      1   1.0100  10.0000
IDSOR(ISOR),XSOR(ISOR),ZSOR(ISOR)   =      2   2.0100  20.0000
IDSOR(ISOR),XSOR(ISOR),ZSOR(ISOR)   =      3   3.0100  30.0000
IDSOR(ISOR),XSOR(ISOR),ZSOR(ISOR)   =      4  10.0000   0.5000
IDSOR(ISOR),XSOR(ISOR),ZSOR(ISOR)   =      5  30.0000   3.0000
                                         -99 -99.0000 -99.0000
IDREC(IREC),XREC(IREC),ZREC(IREC)   =      1  40.0000  11.0000
IDREC(IREC),XREC(IREC),ZREC(IREC)   =      2  40.0000  20.0000
IDREC(IREC),XREC(IREC),ZREC(IREC)   =      3  41.0000  30.0000
IDREC(IREC),XREC(IREC),ZREC(IREC)   =      4   1.0000  10.0000
IDREC(IREC),XREC(IREC),ZREC(IREC)   =      5   2.0000  20.0000
IDREC(IREC),XREC(IREC),ZREC(IREC)   =      6   3.0000  30.0000
                                         -99 -99.0000 -99.0000
ADJTTM =    0.00000
IDSORT, IDRECT, EXPTM(IEXPTM)  =    1     1   8.0000
IDSORT, IDRECT, EXPTM(IEXPTM)  =    1     2   8.0000  [These times
IDSORT, IDRECT, EXPTM(IEXPTM)  =    1     3   8.0000  are not
IDSORT, IDRECT, EXPTM(IEXPTM)  =    2     1   8.0000  meaningful
IDSORT, IDRECT, EXPTM(IEXPTM)  =    2     2   8.0000  when
IDSORT, IDRECT, EXPTM(IEXPTM)  =    2     3   8.0000  generating
IDSORT, IDRECT, EXPTM(IEXPTM)  =    3     1   8.0000  synthetic
IDSORT, IDRECT, EXPTM(IEXPTM)  =    3     2   8.0000  data, and
IDSORT, IDRECT, EXPTM(IEXPTM)  =    3     3   8.0000  could be
IDSORT, IDRECT, EXPTM(IEXPTM)  =    4     1   8.0000  anything.]
IDSORT, IDRECT, EXPTM(IEXPTM)  =    4     2   8.0000
IDSORT, IDRECT, EXPTM(IEXPTM)  =    4     3   8.0000
IDSORT, IDRECT, EXPTM(IEXPTM)  =    4     4   8.0000
IDSORT, IDRECT, EXPTM(IEXPTM)  =    4     5   8.0000
IDSORT, IDRECT, EXPTM(IEXPTM)  =    4     6   8.0000
IDSORT, IDRECT, EXPTM(IEXPTM)  =    5     1   8.0000
IDSORT, IDRECT, EXPTM(IEXPTM)  =    5     2   8.0000
IDSORT, IDRECT, EXPTM(IEXPTM)  =    5     3   8.0000
IDSORT, IDRECT, EXPTM(IEXPTM)  =    5     4   8.0000
IDSORT, IDRECT, EXPTM(IEXPTM)  =    5     5   8.0000
IDSORT, IDRECT, EXPTM(IEXPTM)  =    5     6   8.0000
                                  -99   -99 -99.0000
SMFPLL,VELMIN,VELMAX,INVOPT  =   0.00500 -99.0000 -99.0000  2
ROW, INITIAL VELOCITY FOR THAT ROW =   1   2.500
ROW, INITIAL VELOCITY FOR THAT ROW =   2   3.500
ROW, INITIAL VELOCITY FOR THAT ROW =   3   4.500
ROW, INITIAL VELOCITY FOR THAT ROW =   4   5.500
ROW, INITIAL VELOCITY FOR THAT ROW =   5   6.500
ROW, INITIAL VELOCITY FOR THAT ROW =   6   7.500
```

VELOCITIES VARIED (T) AND FIXED (F). NFXLFT OR NFXRHT
OVERRIDE T.

```
      -0.50 40.00
 -5.00  T  T  T
  5.00  T  T  T
 15.00  T  T  T
 25.00  T  T  T
 35.00  T  T  T
 45.00  T  T  T
       4.50 40.00
```

```
ITER,SOURCE ID,PIXEL,VEL AT SOURCE=  1   1   5  4.000
NO TRAVEL TIMES FROM SOURCE ID  1 TO LEFT SIDE [Synthetic travel times
NO TRAVEL TIMES FROM SOURCE ID  1 TO BOTTOM are in |this| col.]
SOR ID,REC ID,PATH LEN,CAL TIME,EXP-CAL TIME=1 1  40.343  9.293 -1.293
SOR ID,REC ID,PATH LEN,CAL TIME,EXP-CAL TIME=1 2  41.395  8.722 -0.722
SOR ID,REC ID,PATH LEN,CAL TIME,EXP-CAL TIME=1 3  45.809  8.837 -0.837
NO TRAVEL TIMES FROM SOURCE ID    1 TO TOP

ITER,SOURCE ID,PIXEL,VEL AT SOURCE=  1   2   9  5.000
NO TRAVEL TIMES FROM SOURCE ID    2 TO LEFT SIDE
NO TRAVEL TIMES FROM SOURCE ID    2 TO BOTTOM
SOR ID,REC ID,PATH LEN,CAL TIME,EXP-CAL TIME=2 1  40.078  8.376 -0.376
SOR ID,REC ID,PATH LEN,CAL TIME,EXP-CAL TIME=2 2  38.838  7.426  0.574
SOR ID,REC ID,PATH LEN,CAL TIME,EXP-CAL TIME=2 3  41.041  7.193  0.807
NO TRAVEL TIMES FROM SOURCE ID    2 TO TOP

ITER,SOURCE ID,PIXEL,VEL AT SOURCE=  1   3  13  6.000
NO TRAVEL TIMES FROM SOURCE ID    3 TO LEFT SIDE
NO TRAVEL TIMES FROM SOURCE ID    3 TO BOTTOM
SOR ID,REC ID,PATH LEN,CAL TIME,EXP-CAL TIME=3 1  42.451  8.156 -0.156
SOR ID,REC ID,PATH LEN,CAL TIME,EXP-CAL TIME=3 2  38.998  6.861  1.139
SOR ID,REC ID,PATH LEN,CAL TIME,EXP-CAL TIME=3 3  38.592  6.230  1.770
NO TRAVEL TIMES FROM SOURCE ID    3 TO TOP

ITER,SOURCE ID,PIXEL,VEL AT SOURCE=  1   4   2  3.050
SOR ID,REC ID,PATH LEN,CAL TIME,EXP-CAL TIME=4 4  13.121  3.725  4.275
SOR ID,REC ID,PATH LEN,CAL TIME,EXP-CAL TIME=4 5  21.111  5.334  2.666
SOR ID,REC ID,PATH LEN,CAL TIME,EXP-CAL TIME=4 6  30.349  6.947  1.053
NO TRAVEL TIMES FROM SOURCE ID    4 TO BOTTOM
SOR ID,REC ID,PATH LEN,CAL TIME,EXP-CAL TIME=4 1  32.636  8.710 -0.710
SOR ID,REC ID,PATH LEN,CAL TIME,EXP-CAL TIME=4 2  36.551  8.869 -0.869
SOR ID,REC ID,PATH LEN,CAL TIME,EXP-CAL TIME=4 3  43.579  9.627 -1.627
NO TRAVEL TIMES FROM SOURCE ID    4 TO TOP

ITER,SOURCE ID,PIXEL,VEL AT SOURCE=  1   5   3  3.300
SOR ID,REC ID,PATH LEN,CAL TIME,EXP-CAL TIME=5 4  30.556  7.996  0.004
SOR ID,REC ID,PATH LEN,CAL TIME,EXP-CAL TIME=5 5  33.342  7.860  0.140
SOR ID,REC ID,PATH LEN,CAL TIME,EXP-CAL TIME=5 6  38.696  8.338 -0.338
NO TRAVEL TIMES FROM SOURCE ID    5 TO BOTTOM
SOR ID,REC ID,PATH LEN,CAL TIME,EXP-CAL TIME=5 1  12.845  3.464  4.536
SOR ID,REC ID,PATH LEN,CAL TIME,EXP-CAL TIME=5 2  19.770  4.809  3.191
SOR ID,REC ID,PATH LEN,CAL TIME,EXP-CAL TIME=5 3  29.219  6.440  1.560
NO TRAVEL TIMES FROM SOURCE ID    5 TO TOP
```

```
NODE VELOCITIES AFTER ITERATION     1
  2.50  1.88  2.50
  2.78  2.68  2.65      [These are not meaningful when
  3.74  3.75  3.50       generating synthetic data.]
  5.08  5.42  5.08
  5.55  5.80  5.71
  7.50  7.50  7.50
AVERAGE NODE VELOCITY =    4.124
NUMBER OF TRAVEL TIMES =    21  SUCCESSFUL SOURCE-REC RAYS =
  21
 AVE DELT =   0.7042  AVE ABS(DELT) =   1.3639 RMS DELT =   1.8541
```

[AVE DELT is the average experimental - calculated travel times, using signs; it should be near zero for real data with enough iterations.]

```
// SUM OF PATH FRACTIONS FOR NODES.
  0.00  1.99  0.00
  5.14  5.52  4.88      [The total sum of path fractions for nodes is
  4.05 14.81  4.21       3 times the number of source to receiver
  5.44  5.05  5.44      paths because each pixel has 3 nodes.]
  1.04  4.20  1.13
  0.00  0.00  0.00

// SUM OF PATH FRACTIONS FOR PIXELS
  0.00  1.33  0.67  0.00
  2.21  1.60  1.92  2.29   [The total sum of path fractions for pixels
  1.84  1.51  1.51  1.91    equals the number of source to receiver
  1.04  1.04  0.99  1.13    paths.]
  0.00  0.00  0.00  0.00
```

The output also includes a graph of the node velocities represented by shadings of extended ASCII characters (not shown here).

The output listed below is part of the file SYNTHGEN.GRF that gives a simple graph of ray paths using text characters. This text output provides a way of quickly observing how much ray paths bend without slowing the calculations very much. The graphs show pixels that the ray path has passed through as "*," and other pixels "·." Only rows that have at least one pixel with a path through it are shown. Notice that the display is of pixels, not of nodes. For example, the three horizontal nodes give four triangular pixels per row.

```
// Synthetic crosshole and RVSP data. Units m and ms. Crooked boreholes.
 RAY PATHS IN PIXELS AT ITERATION    1

 SOR ID,REC ID,PATH LEN,CAL TIME,EXP-CAL= 1  1  40.34   9.29  -1.29
****

 SOR ID,REC ID,PATH LEN,CAL TIME,EXP-CAL= 1  2  41.40   8.72  -0.72
*···
****

 SOR ID,REC ID,PATH LEN,CAL TIME,EXP-CAL= 1  3  45.81   8.84  -0.84
*···
***·
··**
```

BOMCRATR produces several other output files. The output file SYNTHGEN.SUM is a summary of the main output file for the final iteration. It can be used as the input file for the contouring program TOMOPLOT. The //'s in that file indicate lines that are to be skipped when read by TOMOPLOT. The output file SYNTHGEN.TBL provides x,y coordinates of the intersections of rays with pixel sides that can be used as input to a graphing program. The remaining output file, SYNTHGEN.RES, gives the results of the calculations of the velocities in the same format as an input file for BOMCRATR so a run can be resumed easily without rerunning the earlier iterations. This ability is important for curved ray programs because they take so long to run.

COMPARISONS OF TOMOGRAPHIC RECONSTRUCTIONS USING SYNTHETIC DATA

An important question in tomography is how much results are improved by using the more complicated constrained curved ray analysis instead of unconstrained or straight ray analyses. One comparison that is possible with either field or synthetic data is the examination of the differences between the travel times read as data and the travel times calculated by the program as it tries to fit the data. These time differences are designated by "EXP-CAL TIME" in the *.RAY output file from BOMCRATR, representing experimental minus calculated travel times. Large time errors could imply assumptions in the program that are inconsistent with the data.

The output from BOMCRATR lists other related errors. AVE DELT is the average value of EXP-CAL TIME, not taking absolute values. It should be close to 0 if the program ran correctly with enough iterations. AVE ABS(DELT) is the average of the absolute values of EXP-CAL TIME and shows how well the program could fit the measured travel times. AVE ABS(DELT) is designated by average time error in the figures that follow. RMS DELT in the program output is the root-mean-square (RMS) time error.

Another approach to comparing methods is to generate synthetic travel times using a model velocity distribution, to fit those travel times with various tomographic methods, and to observe the agreement between the tomographic velocity reconstruction and the velocity model. The corresponding error, model minus calculated velocity, shows how well constraints succeed in selecting the correct velocity distribution from the infinite number (for crosshole data) that would fit the data equally well. Large velocity errors can occur even with perfect data and perfect fits to the data. For this approach to provide a meaningful test, the initial guess for the velocity distribution must be different from the velocity model. If the initial guess is similar to the model, the reconstruction will be close to the model, even without constraints.

To apply this approach, the synthetic calculated travel times in SYNTHGEN.RAY were used to create an input file for PREBMCRA by substituting the calculated times for the 8.00's. PREBMCRA was run to generate an input file for BOMCRATR. Reconstructions obtained with curved versus straight rays and constrained versus unconstrained calculations were then compared with the corresponding model velocity distribution. The model velocity was $v = 3.0 + z/10$ m/ms. The initial guess was a uniform 4.0 m/ms for all calculations with synthetic data. The iteration limit was 40 to ensure that poor fits were not caused by an insufficient number of iterations. In all the calculations with synthetic data, all 21 source-receiver combinations were successfully connected by ray paths. Shadow zones did not occur with the uniform gradient of the velocity model.

The time errors were examined to measure how much they increased because of using straight rays when curved rays were appropriate. The velocity errors were examined to measure the effectiveness of constraints in reducing such errors. The errors were presented in terms of the average absolute value of the differences, designated as average time error and average velocity error in the figures that follow. This form was chosen rather than the RMS error because the latter can be strongly affected by a few large errors.

BOMCRATR WITH CURVED RAYS

With $v = 3.0 + z/10$, the model velocity for BOMCRATR was as shown in the left section of figure 7. A bottom row with all entries = 7.50 had no paths and is not shown. The center section of figure 7 shows the velocities calculated by BOMCRATR, using curved rays and no constraints. Here, ### designates nodes lacking ray paths, where the velocity could not be calculated. The nodes having no ray paths can be identified from the SUM OF PATH FRACTIONS FOR NODES output (not shown here). BOMCRATR was then run with two constraints. The top two rows were laterally invariant, and the maximum velocity was constrained to 6.5 m/ms. The results were as shown in the right section of figure 7.

Velocity model	Unconstrained	Constrained
2.50 2.50 2.50	### 3.17 ###	### 2.86 ###
3.50 3.50 3.50	3.71 3.56 3.75	3.50 3.50 3.50
4.50 4.50 4.50	4.50 4.41 4.44	4.51 4.38 4.58
5.50 5.50 5.50	5.52 5.59 5.55	5.55 5.63 5.50
6.50 6.50 6.50	6.66 6.35 6.61	6.50 6.36 6.50
	Average vel = 4.91 m/ms	Average vel = 4.84 m/ms
	Average time error = 0.023 ms	Average time error = 0.032 ms

Figure 7.—Velocities calculated by BOMCRATR with curved rays and synthetic data. (### indicates no ray paths for corresponding node.)

BOMTOM WITH STRAIGHT RAYS

Because BOMTOM specifies velocity at the centers of pixels instead of at the corners, the velocity model for BOMTOM was shifted by half a row, and one less row was required. Thus, the model velocity distribution for BOMTOM was as shown in the left section in figure 8. When the data were analyzed with the straight ray program BOMTOM and no constraints, the results were as shown in the center section of figure 8. BOMTOM was then run with the constraints corresponding to those used with BOMCRATR. Only one top row was made laterally invariant because there was one less row with BOMTOM. The maximum calculated velocity was 6.0 because BOMTOM specifies the velocity in constant velocity pixels instead of at the corners. The results are shown in the right section of figure 8.

BOMCRATR WITH STRAIGHT RAYS

When BOMCRATR and BOMTOM results were compared, two factors were different. Besides the straight rays, BOMTOM uses pixels having a constant velocity. To separate the effects of the straight rays from the effects of the constant-velocity pixels, the data were reanalyzed with BOMCRATR modified to use only straight rays. Then the difference between the curved ray and straight ray BOMCRATR results was caused by using straight rays, and the difference between the straight ray BOMCRATR and the BOMTOM results was caused by the constant velocity pixels.

The velocity model was the same as for the curved ray BOMCRATR calculation, as shown in the left section of figure 9. The results with the straight ray BOMCRATR and no constraints are in figure 9 in the center section. BOMCRATR was then run with straight rays and the two constraints used with curved rays. As before, the top two rows were laterally invariant, and the maximum velocity was constrained to 6.5 m/ms. The results are shown in the right section of figure 9.

SUMMARY OF COMPARISONS WITH SYNTHETIC VELOCITY MODEL

To summarize the comparisons, the model minus calculated velocities are shown in figure 10 for all the reconstructions with synthetic data and the corresponding errors are listed in table 1. For BOMCRATR, only the four rows with velocities in all three columns are shown in figure 10.

Table 1.—Summary of errors with synthetic data

Program	Constrained	Maximum vel error, m/ms	Average vel error, m/ms	Average time error, ms
BOMCRATR curved ray . . .	N	0.25	0.10	0.023
	Y	.14	.04	.032
BOMTOM straight ray	N	.48	.18	.038
	Y	.27	.07	.052
BOMCRATR straight ray . . .	N	.39	.18	.094
	Y	.40	.15	.072

N No.
Y Yes.

Velocity model	Unconstrained	Constrained
3.00 3.00 3.00 4.00 4.00 4.00 5.00 5.00 5.00 6.00 6.00 6.00	3.07 3.09 2.77 4.08 4.20 4.09 4.93 5.33 4.95 5.64 6.48 5.84 Average vel = 4.54 m/ms Average time error = 0.038 ms	3.01 3.01 3.01 4.11 4.24 4.05 4.94 5.27 5.03 6.00 6.00 6.00 Average vel = 4.56 m/ms Average time error = 0.052 ms

Figure 8.—Velocities calculated by BOMTOM with straight rays and synthetic data.

Velocity model	Unconstrained	Constrained
2.50 2.50 2.50 3.50 3.50 3.50 4.50 4.50 4.50 5.50 5.50 5.50 6.50 6.50 6.50	### 2.79 ### 3.59 3.42 3.70 4.59 4.52 4.47 5.61 5.88 5.74 6.11 6.34 6.12 Average vel = 4.84 m/ms Average time error = 0.094 ms	### 2.72 ### 3.54 3.54 3.54 4.62 4.53 4.63 5.56 5.88 5.64 6.10 6.39 6.17 Average vel = 4.84 m/ms Average time error = 0.072 ms

Figure 9.—Velocities calculated by BOMCRATR with straight rays and synthetic data. (### indicates no ray paths for corresponding node.)

	Model minus calculated velocities, m/ms	
Program	Unconstrained	Constrained
BOMCRATR curved ray	-0.21 -0.06 -0.25 0.00 0.09 0.06 -0.02 -0.09 -0.05 -0.16 0.15 -0.11	0.00 0.00 0.00 0.01 -0.12 0.08 0.05 0.13 0.00 0.00 -0.14 0.00
BOMTOM straight ray	-0.07 -0.09 0.13 -0.08 -0.20 -0.09 -0.07 -0.33 0.05 -0.36 -0.48 0.16	0.01 0.01 0.01 0.11 0.24 0.05 0.06 0.27 0.03 0.00 0.00 0.00
BOMCRATR straight ray	-0.09 0.08 -0.20 -0.09 -0.02 0.03 -0.11 -0.38 -0.24 0.39 0.16 0.38	0.04 0.04 0.04 0.12 0.03 0.14 0.06 0.38 0.14 0.40 -0.11 -0.33

Figure 10.—Model minus calculated velocity errors with synthetic data.

These comparisons showed that the curved ray results were significantly closer to the model velocity distribution, and that for curved rays, the application of constraints improved the agreement with the velocity model. The best agreement was given by the constrained curved ray fit. The constrained straight ray BOMTOM calculation gave a slightly smaller average velocity error but a slightly larger maximum error than the unconstrained curved ray calculation. For these numerical examples, the average velocity error was affected more by using straight rays than by using constant velocity pixels.

The comparisons also indicated that while the curved ray analysis matched the model velocity significantly better, both had the same general pattern of increasing velocity with depth. The straight ray results could be adequate for some applications, even though the velocity gradient was rather large.

COMPARISONS OF TOMOGRAPHIC RECONSTRUCTIONS USING FIELD DATA

Comparisons were also made between curved and straight ray calculations using field data. All calculations were performed with five iterations and an initial velocity guess of 14 ft/ms.

FIELD DATA COLLECTION

Seismic crosshole field data were collected at the Colorado School of Mines Experimental Mine (Edgar Mine) near Idaho Springs, CO, in a joint project (*20*) with the Bureau's Minneapolis, MN, and Denver, CO, Research Centers. The purpose of the experiment was to examine the procedures required for solution control in a stope leaching operation. Research focused on locating and characterizing joints and other discontinuities that may carry fluid through the rock. The predominant structural features that, in an actual stope leaching operation, would contribute to solution loss in the rock mass were joints and foliations. The predominant rock types were Precambrian metamorphosed sedimentary rocks, metamorphosed igneous rocks, and igneous rocks.

One method evaluated for geophysical imaging of the discontinuities was seismic crosshole tomography. The data shown here are from boreholes 2 and 3, with a borehole separation of 13.38 ft. The vertical distance between source locations and also between receiver locations was 1.0 ft. The piezoelectric seismic source operated at 20 kHz, with the frequency of the received signals centered at 3 kHz. The accuracy of the travel times was estimated to be 0.01 ms.

The input file for PREBMCRA, FIELDDAT.PRE on the program diskette, was as follows:

```
Field crosshole data. Units ft and ms. Straight vertical boreholes.
        5         0          0
        1         0          0          1          1
  -0.0001    8.5000    -0.0001    21.5000
  13.3800    8.5000    13.3800    21.5000
       11        14          0          0          0          0
       10    0.0000    10.0000
       11    0.0000    11.0000    [Sources. Note that the left edge
       12    0.0000    12.0000    of the grid was set to -0.0001 to
       13    0.0000    13.0000    avoid having the sources on a pixel
       14    0.0000    14.0000    side. For convenience, the source
       15    0.0000    15.0000    identification numbers were set
       16    0.0000    16.0000    equal to the depth.]
       17    0.0000    17.0000
       18    0.0000    18.0000
       19    0.0000    19.0000
       20    0.0000    20.0000
      -99  -99.0000   -99.0000
       10   13.3800    10.0000
       11   13.3800    11.0000    [Receivers are on the right edge of the
       12   13.3800    12.0000    grid. The receiver identification
       13   13.3800    13.0000    numbers were set equal to the depth.]
       14   13.3800    14.0000
       15   13.3800    15.0000
```

16	13.3800	16.0000
17	13.3800	17.0000
18	13.3800	18.0000
19	13.3800	19.0000
20	13.3800	20.0000
-99	-99.0000	-99.0000
0.00000		

[These measured travel times are listed in two sets of columns to save space.]

10	11	0.75700			
10	12	0.81650			
10	13	0.89100			
10	14	1.06000			
10	15	1.16100			
10	16	1.24700			
10	17	1.39000			
10	18	1.62100			
10	19	1.37300			
10	20	1.21000			
11	10	0.76000	16	10	0.86150
11	11	0.76150	16	11	0.83200
11	12	0.86100	16	12	0.90700
11	13	0.86100	16	13	1.04100
11	14	1.08850	16	14	0.81700
11	15	1.26900	16	15	1.02900
11	16	1.40350	16	16	0.99800
11	17	1.48000	16	17	1.14000
11	18	1.27100	16	18	0.99600
11	19	1.32050	16	19	0.93250
11	20	1.35200	16	20	0.95900
12	10	0.77850	17	10	0.88900
12	11	0.76700	17	11	0.86400
12	12	0.84150	17	12	0.93350
12	13	1.05600	17	13	0.91400
12	14	1.06100	17	14	1.12400
12	15	1.29200	17	15	1.21400
12	16	1.28800	17	16	1.11400
12	17	1.47200	17	17	1.08800
12	18	1.45700	17	18	0.91650
12	19	1.39100	17	19	1.05200
12	20	1.38200	17	20	0.91100
13	10	0.79400	18	10	0.92850
13	11	0.77300	18	11	0.90000
13	12	0.77300	18	12	0.96700
13	13	0.84200	18	13	1.08800
13	14	1.11900	18	14	1.22850
13	15	0.95700	18	15	1.40050
13	16	1.15300	18	16	1.27500
13	17	1.28400	18	17	1.34700
13	18	1.03600	18	18	1.01250
13	19	1.02800	18	19	1.11200
13	20	1.04900	18	20	1.12150
14	10	0.81100	19	10	0.93300
14	11	0.78500	19	11	1.08100
14	12	0.85000	19	12	1.01600
14	13	0.85300	19	13	0.99400

```
14    14    1.13500        19    14    1.30300
14    15    1.20700        19    15    0.86200
14    16    1.13300        19    16    0.86900
14    17    1.37500        19    17    0.87700
14    18    1.29200        19    18    0.76900
14    19    1.01500        19    19    0.86000
14    20    1.02800        19    20    0.85900
15    10    0.82900        20    10    0.98300
15    11    0.81000        20    11    1.08800
15    12    0.88700        20    12    1.05600
15    13    1.01150        20    13    1.05400
15    14    1.14550        20    14    0.94300
15    15    0.97500        20    15    0.95200
15    16    1.07900        20    16    0.97300
15    17    1.17000        20    17    0.94600
15    18    0.94400        20    18    0.87200
15    19    0.94800        20    19    0.86900
15    20    1.01400        20    20    0.85800
                           -99   -99   -99.00000
0.00100   6.0000   20.0000      1
14.0000
```

The output from PREBMCRA (not shown here) was then used as the input to BOMCRATR. The travel times were also entered into an input file for BOMTOM.

EFFECT OF CONSTRAINING MAXIMUM VELOCITY

To help obtain a reliable reconstruction, a realistic velocity constraint was imposed. Sonic borehole logs[6] for shear waves showed a maximum velocity of 11.6 ft/ms. Taking the ratio of shear wave velocity to compressional wave velocity as no less than 0.58 gave an upper limit of 20.0 ft/ms. The upper velocity limit was also determined by measuring the velocity in an intact core, cut to form a sphere. The velocity was measured in many directions through the sphere. The highest velocity found was 20.0 ft/ms. This constraint was applied to the maximum velocity. The minimum velocity was constrained to 6.0 ft/ms, but this value did not affect the tomogram because the lowest calculated velocity was greater than that. The lower velocity constraint was retained for consistency in the displays.

BOMCRATR With Curved Rays and Constrained Velocity

The first and last parts of the output file *.RAY from BOMCRATR with the curved ray option and the velocity constraint are shown on the following page, with the calculated velocities placed in figure 11. The top and bottom node rows were removed because they had no ray paths. The contoured velocity tomogram is shown in figure 12.

[6]The authors thank James Snodgrass, geophysicist, Denver Research Center, for supplying the shear wave sonic borehole logs.

BOMCRATR With Curved Rays and No Constraints

The analysis was then run with no constraints. The calculated velocities near the upper right of the pixel grid were unrealistically high, but the results are listed in figure 13 and displayed in the tomogram in figure 14 to illustrate the need for constraints.

The maximum calculated velocity was 29.80 ft/ms, which is unrealistically high for this rock. The average time error of 0.098 ms was not lower than the time error obtained with a velocity limit. Therefore, constraining the velocity gave more realistic results without decreasing the goodness of fit.

EFFECT OF STRAIGHT INSTEAD OF CURVED RAYS

BOMTOM With Straight Rays and Constrained Velocity

For comparison, the data were analyzed with BOMTOM. Recall that BOMTOM specifies the velocity at the centers of pixels instead of at the corners, so the general tomogram image obtained with BOMTOM should be compared with that from BOMCRATR, without expecting a pixel-by-pixel agreement. The results with BOMTOM are listed in figure 15 and shown as a contoured tomogram in figure 16.

Comparing these results with those from BOMCRATR shows the same general pattern, but the straight ray graph shows greater contrasts in the velocity. The lowest calculated node velocity was 8.41 ft/ms, whereas for the other two programs it was 8.96 ft/ms.

```
BOMCRATR MAIN OUTPUT FILE FIELDDAT .RAY
INPUT FILE WAS FIELDDAT.BOM
Field crosshole data. Units ft and ms. Straight vertical boreholes.
ITERATION LIMIT =    5
MSMTH = 0     MSTRT = 0
IPDAT,IPITR,IPSEG,IRAYGRF,IRAYTBL =    1   0   0   1   1
XTOPL,ZTOPL,XBOTL,ZBOTL =    -0.0001    8.5000  -0.0001   21.5000
XTOPR,ZTOPR,XBOTR,ZBOTR =    13.3800    8.5000   13.3800   21.5000
NNODH,NNODV,NLATOP,NLABOT,NFXLFT,NFXRHT = 11   14   0   0   0   0
IDSOR(ISOR),XSOR(ISOR),ZSOR(ISOR) =        10    0.0000   10.0000

[Most of the file has been deleted to save space.]

ITER,SOURCE ID,PIXEL,VEL AT SOURCE=  5   20  221  18.611
NO TRAVEL TIMES FROM SOURCE ID   20 TO LEFT SIDE
NO TRAVEL TIMES FROM SOURCE ID   20 TO BOTTOM
SOR ID,REC ID,PATH LEN,CAL TIME,EXP-CAL TIME=20 10 17.115 1.085 -0.102
SOR ID,REC ID,PATH LEN,CAL TIME,EXP-CAL TIME=20 11 16.698 1.075  0.013
SOR ID,REC ID,PATH LEN,CAL TIME,EXP-CAL TIME=20 12 16.773 1.084 -0.028
SOR ID,REC ID,PATH LEN,CAL TIME,EXP-CAL TIME=20 13 16.815 1.120 -0.066
SOR ID,REC ID,PATH LEN,CAL TIME,EXP-CAL TIME=20 14 16.354 1.187 -0.244
SOR ID,REC ID,PATH LEN,CAL TIME,EXP-CAL TIME=20 15 15.054 1.222 -0.270
SOR ID,REC ID,PATH LEN,CAL TIME,EXP-CAL TIME=20 16 15.037 1.236 -0.263
IDREC   17 DOES NOT HAVE RAYS ON BOTH SIDES.[Note that these are
IDREC   18 DOES NOT HAVE RAYS ON BOTH SIDES. source-receiver pairs for
IDREC   19 DOES NOT HAVE RAYS ON BOTH SIDES. which a corresponding ray
IDREC   20 DOES NOT HAVE RAYS ON BOTH SIDES. path was not found.]
NO TRAVEL TIMES FROM SOURCE ID   20 TO TOP
```

BOMCRATR With Straight Rays and Constrained Velocity

As with synthetic data, BOMCRATR was run with straight rays. The results are shown in figure 17 and in a tomogram in figure 18.

SUMMARY OF COMPARISONS USING FIELD DATA

Table 2 summarizes the errors with field data. It shows that the straight ray programs fit the data as well as the curved ray programs, giving a slightly lower average absolute value of the measured minus calculated travel times. The unconstrained curved ray fit found the fewest source-receiver ray paths, 101 of 120 travel times. This loss of 19 usable travel times represents measurements that did not influence the calculated velocities, with a possible bias in the results.

All four tomograms had a generally similar pattern, with a strong high-velocity zone in the upper right, a weaker high-velocity zone in the lower left, and a large low-velocity zone near the right center. There were differences, however. The unconstrained curved calculation produced unrealistically high velocities. The BOMTOM straight ray fit with constant velocity pixels produced small zones of contrasting velocity that did not seem realistic. The BOMCRATR constrained curved and straight ray analyses were similar, though the straight ray fit produced somewhat larger zones of high velocity.

Table 2.—Summary of errors with field data

Program	Constrained	Successful source-to-receiver paths[1]	Average time error, ms
BOMCRATR curved ray . .	Y	109	0.098
	N	101	.098
BOMTOM straight ray	Y	120	.074
BOMCRATR straight ray . .	Y	120	.085

N No.
Y Yes.
[1]There is a possibility of 120 paths.

In general, the differences between the straight and curved ray fits were less significant with these field data than with the synthetic data, though the velocity contrasts were similar.

10.00	11.82	13.41	14.70	15.83	15.63	20.00	20.00	20.00	20.00	20.00
10.71	10.83	12.01	12.99	14.75	16.52	19.35	20.00	20.00	20.00	2.00
10.09	10.74	11.56	12.73	14.58	15.92	17.28	18.17	19.11	20.00	20.00
11.03	11.81	13.19	13.32	13.99	15.02	15.59	14.66	13.33	14.59	16.33
13.65	13.24	12.56	12.45	13.90	14.61	13.10	11.33	11.11	10.49	11.39
13.35	13.59	13.11	13.95	14.64	13.67	11.67	11.07	10.01	9.98	10.32
16.18	16.31	15.03	14.61	13.44	12.57	11.73	10.80	10.00	10.20	10.21
15.67	15.78	14.68	14.01	10.55	10.68	10.76	9.91	9.69	9.54	8.96
12.46	14.13	14.53	14.10	12.94	12.00	11.62	11.46	10.74	9.69	9.57
11.85	13.40	15.07	13.39	14.25	14.27	14.29	13.58	13.11	12.45	12.83
19.47	18.24	15.57	14.70	14.99	15.18	15.05	15.27	14.45	13.97	13.72
20.00	20.00	20.00	16.85	16.84	15.08	14.63	14.83	14.81	14.35	14.07

Average node vel = 14.09 ft/ms
Number of travel times = 120   Successful source-rec rays = 109
[Note that not all travel times were successfully matched with
rays between the source and receiver.]
Average time error = 0.098 ms

Figure 11.—Velocities calculated by BOMCRATR with curved rays, field data, and maximum velocity constrained to 20 ft/ms.

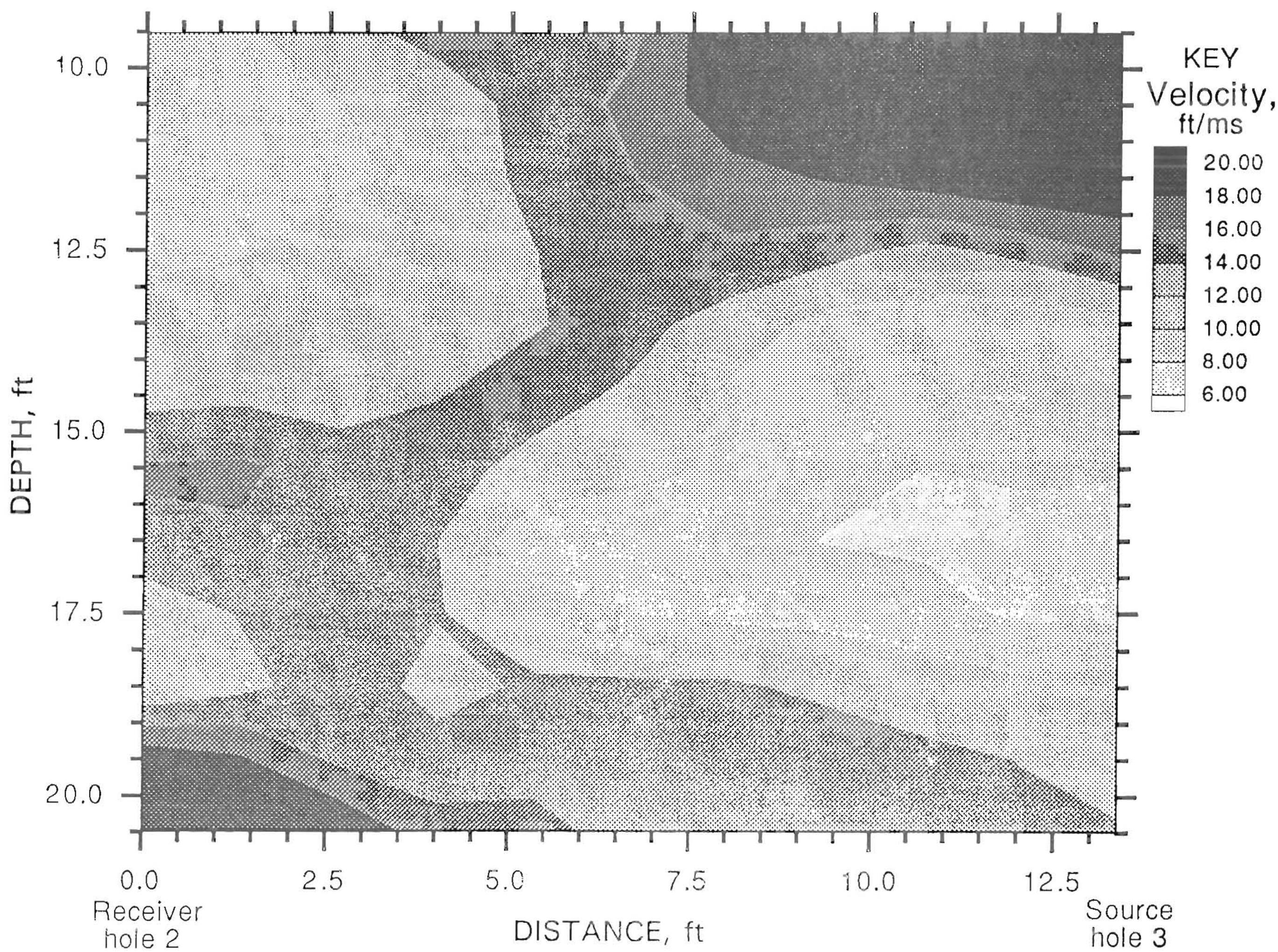

Figure 12.—Velocity tomogram from BOMCRATR with curved rays, field data, and maximum velocity constrained to 20 ft/ms.

9.70	11.33	12.90	13.86	15.01	14.64	19.74	24.44	27.25	29.27	29.80
10.43	10.58	11.92	12.88	14.40	15.92	18.81	20.98	25.22	26.17	29.18
9.90	10.61	11.42	13.03	14.74	16.04	17.16	17.58	18.41	19.97	21.34
10.73	11.53	12.83	13.01	13.72	14.80	15.47	14.44	13.20	14.36	16.09
13.32	12.91	12.44	12.32	13.42	14.01	12.84	11.48	11.07	10.41	11.34
13.39	13.34	12.58	13.68	14.26	13.11	11.31	10.78	9.94	10.02	10.29
16.08	16.22	14.54	13.99	12.97	11.63	10.90	10.43	9.84	10.17	10.23
15.53	15.61	14.51	13.11	9.24	10.54	10.76	10.02	10.46	9.63	8.92
12.20	13.95	14.14	13.65	12.51	11.46	11.68	11.84	10.89	9.76	9.61
11.68	12.72	13.70	13.69	14.27	14.19	14.12	13.55	12.97	12.46	12.74
18.44	17.10	15.54	15.08	15.52	15.74	14.96	15.09	14.26	13.89	13.38
22.17	23.03	22.95	17.24	16.84	15.05	13.16	14.30	13.55	13.50	13.17

Average node vel = 14.32 ft/ms
Number of travel times = 120 Successful source-rec rays = 101
Average time error = 0.098 ms

Figure 13.—Velocities calculated by BOMCRATR with curved rays, field data, and no constraints.

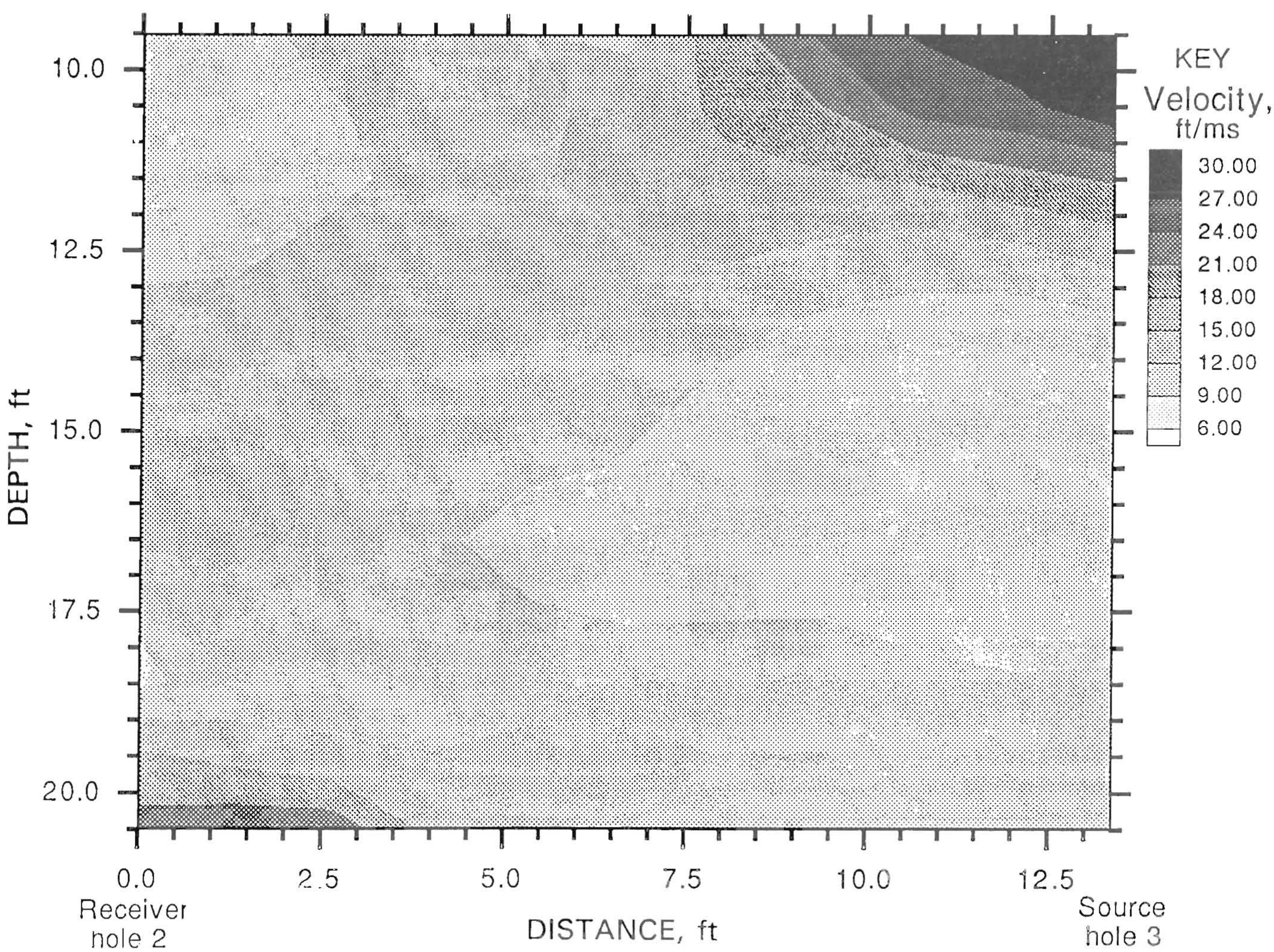

Figure 14.—Velocity tomogram from BOMCRATR with curved rays, field data, and no constraints.

10.57	12.02	16.53	18.68	18.68	20.00	20.00	20.00	20.00	20.00	20.00
10.82	10.98	12.21	15.52	17.83	20.00	20.00	20.00	20.00	20.00	20.00
9.55	10.30	11.07	11.72	14.80	17.99	20.00	20.00	20.00	20.00	20.00
18.23	12.96	11.91	12.22	11.71	14.29	16.74	18.22	16.65	17.29	17.22
12.53	13.49	13.87	11.92	12.64	11.68	12.19	12.02	12.56	11.92	11.28
15.62	15.71	13.04	14.41	13.50	13.28	11.63	10.77	10.46	10.66	11.28
17.49	17.55	18.03	15.63	14.04	11.41	10.39	10.13	9.45	10.78	10.99
14.69	14.92	13.32	13.09	13.03	14.09	13.21	11.47	10.63	8.75	8.41
10.16	10.46	13.01	15.36	14.63	14.85	16.27	15.41	14.99	13.22	11.87
20.00	20.00	20.00	16.38	15.77	14.43	13.44	13.70	13.47	13.13	12.52
20.00	18.41	17.36	16.64	16.64	16.18	15.75	15.75	12.26	13.75	13.33

Average pixel vel = 14.74 m/ms
Number of travel times = 120 Successful source-rec rays = 120
Average time error = 0.074 ms

Figure 15.—Velocities calculated by BOMTOM with straight rays, field data, and maximum velocity constrained to 20 ft/ms.

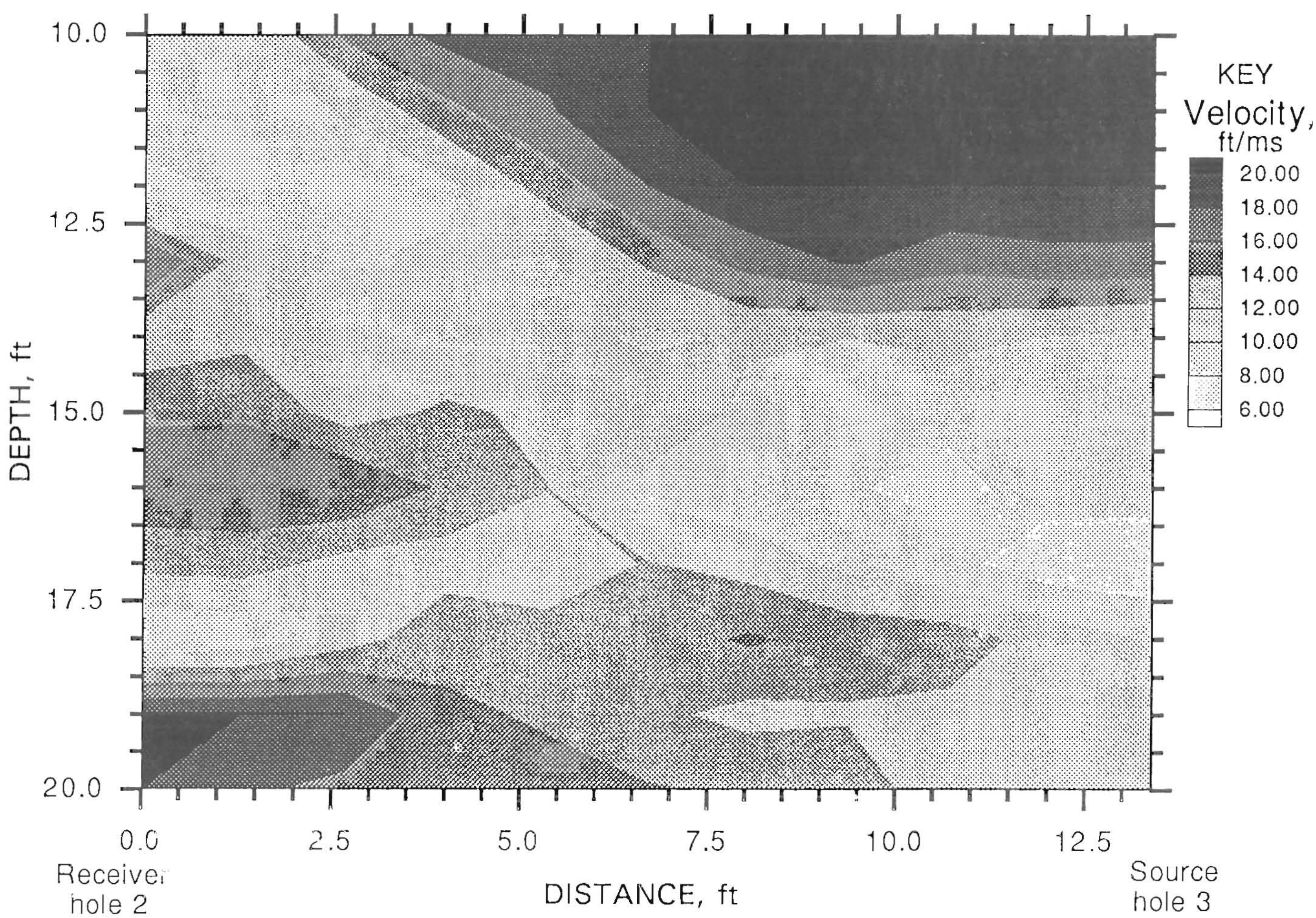

Figure 16.—Velocity tomogram from BOMTOM with straight rays, field data, and maximum velocity constrained to 20 ft/ms.

10.88	15.85	16.88	20.00	20.00	14.00	20.00	20.00	20.00	20.00	20.00
10.62	11.23	13.85	16.07	20.00	20.00	20.00	20.00	20.00	20.00	20.00
10.05	10.94	11.33	13.34	15.50	20.00	20.00	20.00	20.00	20.00	20.00
10.80	11.55	10.81	11.71	12.54	15.45	18.99	20.00	20.00	19.60	19.61
14.29	14.56	12.60	12.26	12.06	12.37	13.55	14.28	14.21	14.15	13.61
13.52	14.08	13.20	13.04	12.37	12.35	11.89	11.35	11.27	11.05	10.81
17.02	17.01	15.62	16.44	13.82	12.25	10.84	9.63	9.95	10.76	10.97
16.61	16.28	15.32	14.33	12.76	12.37	11.54	10.47	9.65	9.53	9.00
11.87	11.37	12.75	12.93	13.58	15.36	14.57	13.41	12.20	10.86	9.79
14.62	14.42	17.05	15.64	15.59	14.78	14.80	15.01	14.56	13.28	12.83
20.00	20.00	20.00	17.21	16.15	14.71	13.48	13.74	13.26	13.32	13.14
19.61	18.72	16.34	16.38	16.08	16.60	16.33	14.85	14.61	13.45	13.53

Average node vel = 14.85 ft/ms
Number of travel times = 120 Successful source-rec rays = 120
Average time error = 0.085 ms

Figure 17.—Velocities calculated by BOMCRATR with straight rays, field data, and maximum velocity constrained to 20 ft/ms.

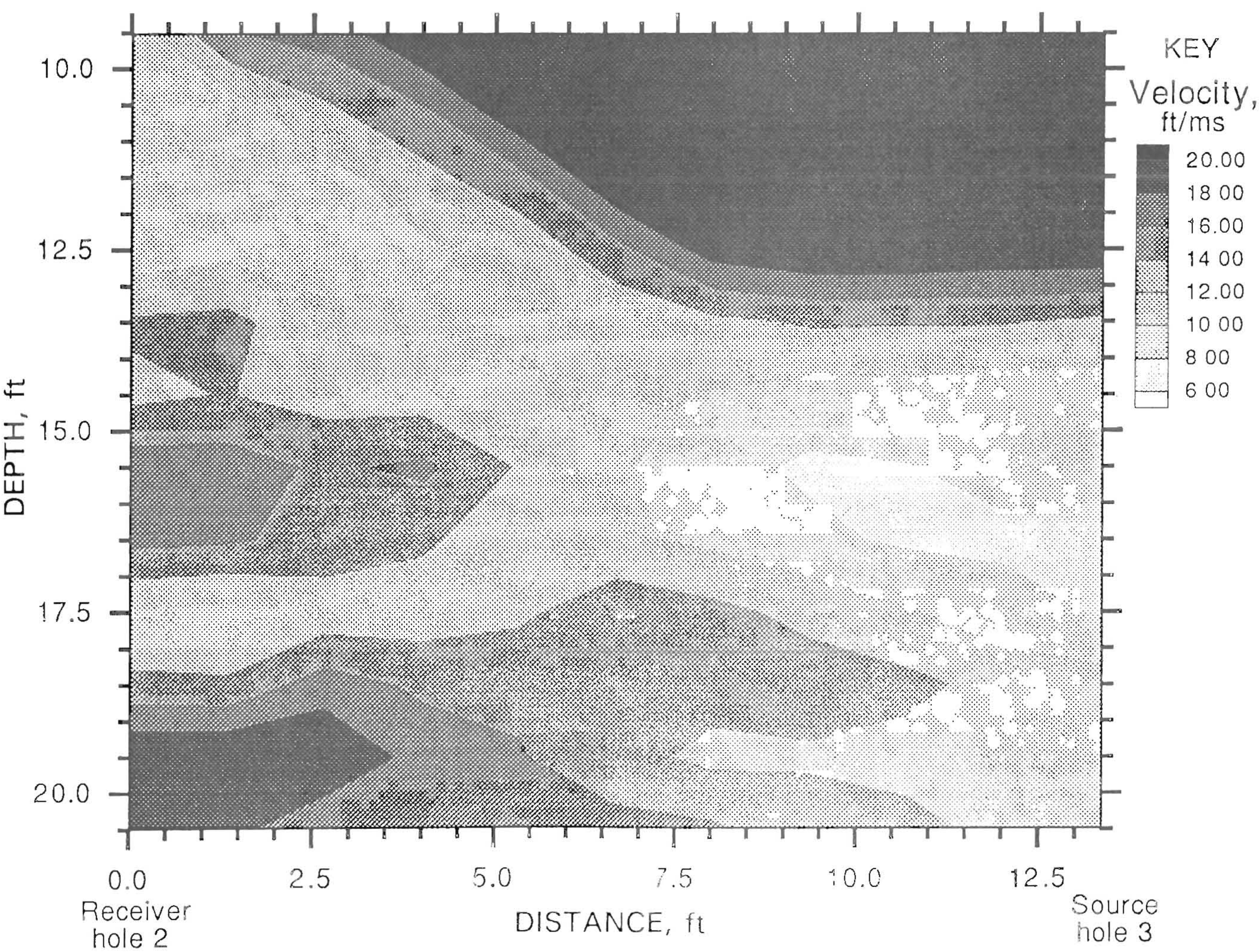

Figure 18.—Velocity tomogram from BOMCRATR with straight rays, field data, and maximum velocity constrained to 20 ft/ms.

CONCLUSIONS

The curved ray tomographic program BOMCRATR traces rays analytically in triangular pixels with the velocity specified at the corners, by assuming that the velocity varies linearly with position in each pixel. It uses the ray shooting method to establish the ray path from a source to a particular receiver. It then uses SIRT to determine the two-dimensional distribution of velocities that provides the best fit to the measured travel times.

BOMCRATR offers the following features:

1. Constraints to help counteract the nonuniqueness of tomographic reconstructions of the velocity pattern when using only crosshole data.
2. The ability to change the pattern of initial velocity guesses to help test the uniqueness of reconstructions.
3. The ability to perform either curved or straight ray analysis, which is useful for determining when the slower, curved ray analysis is necessary.
4. Interactive setting of control parameters, such as the number of iterations and constraints.
5. Ability to skew the grid to deal efficiently with slanted or crooked boreholes.

The use of curved rays adds complications to tomographic analysis. An example of a problem unique to curved ray analysis with the shooting method is the shadow zone near a line of high velocity, where receivers are not reached by any of the mathematical ray paths.

Comparisons with synthetic and field data were made using these approaches:

1. Curved ray analysis using BOMCRATR with and without constraints.
2. Straight ray analysis with constant-velocity rectangular pixels, using the previously published program BOMTOM.
3. Straight ray analysis using BOMCRATR.

The synthetic velocity model varied from 2.5 to 6.5 m/ms over the active path region, sufficient to make the rays curve significantly. The curved ray fit was noticeably better than the two straight ray fits. The two straight ray fits were similar to each other. Though the straight ray fits had a larger average velocity error than the curved ray fits, the general pattern of layering was correct and would be adequate for many applications.

The combination of constraining the velocity and making laterally invariant rows near the top of the grid significantly improved the fit when using curved rays. It halved the average velocity error without significantly increasing the average time error.

With synthetic data, there were significant differences between the curved ray and the straight ray results. The differences were much less significant with field data, even though the velocity contrasts were similar.

With field data, all the methods showed a large high-velocity zone in the upper right and a smaller high-velocity zone in the lower left. The average time errors were slightly smaller with the straight ray fits, possibly because some travel times were not being successfully used in the curved ray analysis. Only 101 of 120 source-receiver pairs were successfully connected by a ray path with an unconstrained curved ray fit, and 109 were connected by a constrained curved ray fit. Constraining the velocity upper limit to 20 ft/ms made the results more realistic without harming the fit to the data. The curved and straight ray fits were similar enough that, for most applications, either would be adequate.

BOMTOM produced more contrast in velocities between neighboring pixels than BOMCRATR, and the tomogram did not seem quite as realistic. The left high-velocity zone extended higher and there was a small low-velocity zone in with the high-velocity zone that did not occur with BOMCRATR.

These tomography programs are available through a Bureau plan to facilitate transfer of technology to industry. FORTRAN source and executable codes are available free of charge from the authors on IBM-compatible PC diskettes.

REFERENCES

1. Wong, J., N. D. Bregman, G. West, and P. A. Hurley. Cross-Hole Seismic Scanning and Tomography. Geophys.: The Leading Edge of Explor., v. 6, Jan. 1987, pp. 36-41.

2. Dennen, R. S., and W. P. Stroud. Radar Hazard Detection in a Coal Structure. Min. Eng. (Littleton, CO), v. 43, No. 4, 1991, pp. 413-418.

3. Stolarczk, L. G. Definition Imaging of an Orebody With the Radio Imaging Method (RIM). Paper in Proceedings of the Tenth WVU International Mining Electrotechnology Conference (Morgantown, WV, July 24-27, 1990). WV Univ., Morgantown, WV, 1990, pp. 140-147.

4. Shope, S. M. Electromagnetic Coal Seam Tomography. Ph.D. Thesis, PA State Univ., University Park, PA, 1987, 293 pp.

5. Martel, S. J., and J. E. Peterson, Jr. Interdisciplinary Characterization of Fracture Systems at the US/BK Site, Grimsel Laboratory, Switzerland. Int. J. Rock Mech. and Min. Sci. & Geomech. Abst., v. 28, No. 4, 1991, pp. 295-323.

6. Elbring, G. J., S. M. Shope, J. R. Wayland, D. O. Lee, S. Ballard, and K. L. Zonge. Comparison of Seismic and Electromagnetic Inversion at the Idaho Springs Tunnel Detection Test Facility, Idaho Springs, Colorado. Paper in 1990 Technical Program, Volume 1. Expanded

Abstracts With Biographics (60th Annu. Int. Mcct., San Francisco, CA, Sept. 23-27, 1990). Soc. Explor. Geophys., 1990, pp. 435-438.

7. Dines, K. A., and R. J. Lytle. Computerized Geophysical Tomography. Proc. IEEE, v. 67, No. 7, 1979, pp. 1065-1073.

8. Natterer, F. The Mathematics of Computerized Tomography. Wiley, 1986, 222 pp.

9. Chapman, C. H. Ray Theory and Its Extensions: WKBJ and Maslov Seismograms. J. Geophys., v. 58, Nos. 1-3, 1985, pp. 27-43.

10. Society of Exploration Geophysicists. Proceedings of 1991 Technical Program. Expanded Abstracts With Biographies (61st Annu. Int. Meet., Houston, TX, Nov. 10-14, 1991). 1991, 1707 pp.

11. Tweeton, D. R., C. L. Cumerlato, J. C. Hanson, and H. L. Kuhlman. Field Tests of Seismic Tomography for Fracture and Fluid Detection. Paper in Proceedings of the Tenth WVU International Mining Electrotechnology Conference (Morgantown, WV, July 24-27, 1990). WV Univ., Morgantown, WV, 1990, pp. 134-139.

12. Cumerlato, C. L., V. J. Stachura, and D. R. Tweeton. Application of Refraction Tomography To Map the Extent of Blast-Induced Fracturing. Paper in Key Questions in Rock Mechanics: Proceedings of 29th U.S. Symposium (Minneapolis, MN, June 3-15, 1988). Balkema, 1988, pp. 691-698.

13. Tweeton, D. R. A Tomographic Computer Program With Constraints To Improve Reconstructions for Monitoring In Situ Mining Leachate. BuMines RI 9159, 1988, 70 pp.

14. Weber, M. Computation of Body-Wave Seismograms in Absorbing 2-D Media Using the Gaussian Beam Method: Comparison With Exact Methods. Geophys. J., v. 92, No. 1, 1988, pp. 9-24.

15. Müeller, G. Efficient Calculation of Gaussian-Beam Seismograms for Two-Dimensional Inhomogeneous Media. Geophys. J. R. Astron. Soc., v. 79, No. 1, 1984, pp. 153-166.

16. Bregman, N. D., R. C. Bailey, and C. H. Chapman. Crosshole Seismic Tomography. Geophysics., v. 54, No. 2, 1989, pp. 200-215.

17. Ivansson, S. Seismic Borehole Tomography--Theory and Computational Methods. Proc. IEEE, v. 74, No. 2, 1986, pp. 328-338.

18. Stroud, W. P., and R. S. Dennen. Direct Tomographic Reconstruction and Applications to Mining. BuMines RI 9361, 1991, 17 pp.

19. Gibson, R. L., Jr., A. G. Sena, and M. N. Toksöz. Paraxial Ray Tracing in 3D Inhomogeneous, Anisotropic Media. Geophys. Prospect., v. 39, 1991, pp. 473-504.

20. Boreck, D., F. Djahanguiri, N. Miller, J. Snodgrass, and R. Speirer. Rock Mass Characterization for Designing Underground Leaching Stopes (Pres. at 1991 SME-AIME Annu. Meet., Denver, CO, Feb. 25-28, 1991). SME-AIME preprint 91-191, 1991, 11 pp.

APPENDIX.—DERIVATION OF FORMULA FOR SHAPE AND TRAVEL TIME OF RAY PATH SEGMENT IN ONE PIXEL

The following derivation shows the shape of a ray segment in one pixel and the corresponding travel time. The assumption that the velocity gradient is constant within each pixel allows the path to be calculated analytically. If the velocity gradient is small or the ray is nearly parallel to the gradient, then the ray path is nearly straight. Otherwise, the ray path is the arc of a circle. The radius is determined by the magnitude of the gradient and by the angle between the gradient and the initial direction of the segment in the pixel. The center of the circle is chosen so that the arc is tangent to the ray path upon entering the pixel.

To simplify the derivation, first assume that the gradient dv/dz is parallel to the z axis. If the gradient has components in both the x and z directions, the coordinate system can be rotated to make the gradient parallel to a new z' axis.

Consider a ray path segment with initial point P. This initial point can be either a source within the pixel or the entrance point of the ray entering the side of the triangular pixel. To be consistent with the notation used in BOMCRATR, denote the coordinates of P as (XP,ZP) and let the exit point be Q, with coordinates (XQ,ZQ). Specify the direction of the ray path segment by θ. Let the reference line for angles be the positive z axis, pointing downward. Then, a ray traveling downward has the path angle $\theta = 0°$, and a ray traveling horizontally to the right, in the positive x direction, has $\theta = 90°$. Denote the velocity at P as VP.

With our assumptions that the velocity gradient is constant within the pixel and the gradient is just dv/dz, the velocity v at z can be expressed as $v(z) = VP + (dv/dz)(z - ZP)$.

To find the shape of the ray path, use

$$XQ = XP + \int_{ZP}^{ZQ} \frac{dx}{dz} dz.$$

The reference line for angles is the z axis, so $dx/dz = \tan\theta = \sin\theta/\cos\theta$. Designate $\sin\theta$ at P as SINP, a constant. Designate $\cos\theta$ at P as COSP, a constant. Snell's law states that $(1/v)\sin\theta = (1/VP)\,SINP$, so $\sin\theta = v\,(SINP/VP)$. Therefore, the equation above yields

$$XQ = XP + \int_{ZP}^{ZQ} \frac{v\,(SINP/VP)\,dz}{\sqrt{1 - v^2 (SINP/VP)^2}}.$$

Substituting $v = VP + (dv/dz)(z - ZP)$ into the integral and integrating gives

$$XQ = XP - \frac{VP}{(dv/dz)SINP} \sqrt{1 - [VP + (dv/dz)(z - ZP)]^2 (SINP/VP)^2}\ \Big|_{ZP}^{ZQ}.$$

Evaluating the expression and rearranging terms gives

$$XQ - \left[XP + \frac{VP\,COSP}{(dv/dz)SINP}\right] = - \sqrt{\left[\frac{VP}{(dv/dz)\,SINP}\right]^2 - \left[ZQ - \left(ZP - \frac{VP}{dv/dz}\right)\right]^2}.$$

This form shows that the ray path is the arc of a circle with radius R,

$$R = \frac{VP}{(dv/dz)\,SINP},$$

and center (XM,ZM) with

$$XM = XP + \frac{VP\,COSP}{(dv/dz)\,SINP}$$

and

$$ZM = ZP - \frac{VP}{(dv/dz)}.$$

XQ can then be calculated as the intersection of that arc and a straight line forming a side of the pixel.

In BOMCRATR, the radius of the circle RADVEC is designated by

$$RADVEC = VP/ALIGN.$$

ALIGN indicates how close the ray path and the velocity gradient are to parallel. If they are parallel, then ALIGN is 0, the radius is infinite, and the ray path is straight.

$$ALIGN = \frac{dv}{dz}TPX - \frac{dv}{dx}TPZ,$$

where TPX and TPZ are the unit vector x,z components of the ray path direction at the entry point of the pixel. Thus, ALIGN is greatest in magnitude when the ray path and velocity gradient are perpendicular. As an example, when dv/dz = 0.1 (m/ms)/m and v = 4.0 m/ms in the x direction,

$$ALIGN = (0.1\frac{m/ms}{m})\,(1.0) - (0.0\frac{m/ms}{m})\,(0.0).$$

Then,

$$RADVEC = (4.0\,m/ms)/(0.1\frac{m/ms}{m}) = 40\,m.$$

With the center designated by M,

$$XM = XP + (RADVEC)\,(TPZ)$$

and

$$ZM = ZP - (RADVEC)\,(TPX)$$

to make the arc tangent to the ray path at the entry point.

The length L of the path along the arc of a circle with center at (XM,ZM), the start at (XP,ZP), the end at (XQ,ZQ), and radius R is given by

$$L = 2R\,ARCSIN\frac{H}{2R},$$

where

$$H = \sqrt{(XQ - XP)^2 + (ZQ - ZP)^2}\,.$$

H is the length of the chord of the circle from (XP,ZP) to (ZP,ZQ). Half the chord and the radius form a right triangle, with the angle at the center of the circle equal arcsin(H/2R). Thus, the angle between the radius to (XP,ZP) and the radius to (XQ,ZQ) is 2 arcsin(H/2R). Therefore, the length of the corresponding arc is 2R arcsin(H/2R).

The program calculates the ray exit directions TQX and TQZ as the unit vectors for the perpendicular to the radius that goes to (TQX,TQZ). Since the path segment is the arc of a circle and the center (XM,ZM) is known, the perpendicular to the radius can be calculated. If the (X,Z) components of the radius are EX and EZ, then TQX = EZ and TQZ = -EX. If the velocity gradient is small or nearly parallel to the ray, then TQX = TPX and TQZ = TPZ.

To calculate the travel time T, use dt = dl/v, where l is the path segment length and v is the velocity. Then,

$$T = \int_{ZP}^{ZQ} (1/v)\,(dl/dz)\,dz.$$

Because the reference line for the angle is the z axis, dz/dl = cos Θ. Therefore,

$$\frac{dl}{dz} = \frac{1}{\cos\Theta} = \frac{1}{\sqrt{1 - \sin^2\Theta}} = \frac{1}{\sqrt{1 - v^2(SINP/VP)^2}}.$$

Again, substituting v = VP + (dv/dz)(z - ZP) into the integral and integrating gives

$$T = \frac{1}{(dv/dz)} \ln \frac{1 + \sqrt{1 - [VP + (dv/dz)(z - ZP)]^2 (SINP/VP)^2}}{[VP + (dv/dz)(z - ZP)]\,(SINP/VP)} \Big|_{ZP}^{ZQ}.$$

ZQ is known, and therefore, VQ = VP + (dv/dz)(ZQ - ZP) can be calculated. Substituting VQ allows the evaluated expression to be simplified to

$$T = -\frac{1}{(dv/dz)} \ln \left[1 + \frac{\sqrt{1 - (VQ)^2 (SINP/VP)^2}}{VQ\ SINP/VP} \frac{SINP}{1 + \sqrt{1 - SINP^2}} \right].$$

Snell's law states that SINP/VP = SINQ/VQ, which allows the expression to be simplified to

$$T = -\frac{1}{(dv/dz)} \ln \left[\frac{1 + COSQ}{SINQ} \frac{SINP}{1 + COSP} \right].$$

BOMCRATR uses the form

$$T = \frac{0.5}{(dv/dz)} \ln \left[\frac{1 - COSQ}{1 + COSQ} \frac{1 + COSP}{1 - COSP} \right],$$

which is more efficient computationally. It is equivalent because the sine terms can be written as

$$\sin\Theta = \sqrt{(1 + \cos\Theta)\,(1 - \cos\Theta)}.$$

If the initial direction of the ray path is parallel to the gradient, the ray path will continue in a straight line. Then, when the time is calculated from

$$T = \int_{ZP}^{ZQ} (1/v)\,(dl/dz)\,dz,$$

the dl/dz term is a constant (1/COSP). COSP is 1 when the ray travels in the same direction as the gradient and -1 when the ray travels in the opposite direction. When v = VP + (dv/dz)(z - ZP) is substituted into the integral and the results are expressed in terms of VP and VQ, the are result is

$$T = \frac{(1/COSP)}{(dv/dz)} \ln \frac{VQ}{VP}.$$

If the gradient is 0, then the ray path is a straight line and the velocity is a constant VP, so that the time is just the distance from P to Q divided by VP.

To generalize to gradients that are not parallel to the z axis, the derivation is similar, with the coordinate system rotated to a set of coordinates (x',z') with z' parallel to the gradient. Then, the reference line for angles is the direction of the gradient, so that COSP is the angle between the ray at P and the gradient direction. The magnitude of the gradient dv/dz' is given by

$$dv/dz' = \sqrt{(dv/dz)^2 + (dv/dx)^2}.$$

Then, the formula for T has the same form as given above, with dv/dz' substituted for dv/dz, and COSP and COSQ referring to the pixel entry and exit angles between the ray and the direction of the generalized gradient.

To show the general expression for the velocity gradient as calculated by BOMCRATR, denote the velocity at corner 1 of a pixel as V1, and the x and z coordinates as X1 and Z1. The velocity gradients in the pixel are

$$\frac{dv}{dx} = \frac{(V2 - V1)(Z3 - Z1) - (V3 - V1)(Z2 - Z1)}{(X2 - X1)(Z3 - Z1) - (X3 - X1)(Z2 - Z1)}$$

and

$$\frac{dv}{dz} = \frac{(V3 - V1)(X2 - X1) - (V2 - V1)(X3 - X1)}{(Z3 - Z1)(X2 - X1) - (Z2 - Z1)(X3 - X1)}.$$

The generalized expression used by BOMCRATR is

$$T = \frac{0.5}{A} LN \frac{(1 - COSQ)(1 + COSP)}{(1 + COSQ)(1 - COSP)},$$

where A is the total velocity gradient,

$$A = \sqrt{B^2 + C^2},$$

B is the velocity gradient in the x direction, and C is the velocity gradient in the z direction.

$$COSQ = (B/A)(TQX) + (C/A)(TQZ).$$

$$COSP = (B/A)(TPX) + (C/A)(TPZ).$$

Additional explanations concerning the calculation methods used by BOMCRATR are in the program comments.

INT.BU.OF MINES,PGH.,PA 29536