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Aspects of Genetic Algorithm-Designed Fuzzy Logic Controllers

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UNITED STATES DEPARTMENT OF THE INTERIOR

BUREAU OF MINES

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UNIT OF MEASURE ABBREVIATIONS USED IN THIS REPORT

L	liter	(mL/s)/s	milliliter per second per second
M	molar	pct	percent
mL	milliliter	s	second
mL/s	milliliter per second		

ASPECTS OF GENETIC ALGORITHM-DESIGNED FUZZY LOGIC CONTROLLERS

By C. L. Karr,[1] J. W. Fleming,[2] and P. A. Vann[3]

ABSTRACT

U.S. Bureau of Mines (USBM) research has shown that fuzzy logic and genetic algorithms (GA's) have great potential for industrial process control. A technique has been developed by the USBM in which GA's are used to substantially reduce the time needed to design fuzzy logic controllers (FLC's). This technique shows promise as an efficient, robust approach to developing control systems. The research described in this report is twofold. First, the basic approach to developing an FLC using GA's is presented. The GA-designed FLC is developed for a specific physical system, a pH titration system. Second, empirical results are presented in which variations in the FLC implementation are compared. Specifically, the effects of altering five aspects of the pH FLC are considered: (1) membership function form, (2) number of fuzzy classes, (3) the center-of-area method, (4) implication operator, and (5) fuzzy rule form. Results indicate that (1) the technique in which GA's are used to design FLC's is effective, and (2) when this technique is used, variations in FLC implementation have little effect on FLC performance in a chosen control problem.

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INTRODUCTION

Rule-based systems have become increasingly popular as practical applications of artificial intelligence. These systems, commonly referred to as "expert systems," have performed as well as humans in several problem domains (*1*);[4] however, their lack of flexibility in representing the linguistic nature of human decision making (generally incorporated in "rules-of-thumb") has limited their performance in the area of process control. The flexibility inherent in human linguistic decision making can be incorporated into expert systems via fuzzy set theory (*2*). In fuzzy set theory, exact numbers can be represented with ***linguistic variables***, which are simply descriptive terms like "very high" and "not quite low." Linguistic variables are inviting because they allow a problem to be represented using the approach humans adopt, and they have been used in expert systems in the form of fuzzy logic controllers (FLC's) (*3-5*).

FLC's are rule-based systems that use fuzzy linguistic variables to model a human's rule-of-thumb approach to problem solving. These fuzzy expert systems include rules to direct the decision process, and membership functions to give some concrete meaning to the linguistic variables representing the pertinent subjective concepts. The rule set is gleaned from a human expert's experience and is composed of rules that include the type of subjective terms with which humans are generally comfortable. The membership functions are chosen by the FLC developer to represent the human expert's interpretation of the abstractions described by the linguistic variables. A change in the membership functions alters the performance of the controller because it is the membership functions that determine when a given rule is eligible to be put into effect. Thus, the performance of the FLC can be severely restricted by the choice of membership functions (given a set of rules). Viewed in another light, the performance of the FLC can be markedly improved by the choice of membership functions.

The first portion of this report deals with a technique for developing genetic algorithm- (GA-) designed FLC's (GA-FLC's). Researchers at the U.S. Bureau of Mines (USBM) have developed a powerful technique in which GA's are used to select high-performance membership functions for use in FLC's. GA's are search techniques that model the mechanics of natural genetics (*6*). They have received considerable attention in recent years as robust search algorithms—ones that are able to perform efficiently across a broad spectrum of search problems (*7*). Their strength lies in the ability to rapidly search poorly behaved spaces, requiring only objective function value information to guide them. Requiring nothing more than objective function information is an inviting characteristic since the majority of commonly used search techniques require derivative information, continuity of the search space, or complete knowledge of the objective function to guide their search. These restrictions can be quite inhibiting and sometimes insurmountable when viewed in the context of locating high-performance membership functions. Furthermore, since GA's are able to locate near-optimal solutions rapidly, they can be used to provide FLC's with adaptive capabilities in real time (*8-10*). These adaptive FLC's have received considerable attention from industry, and they show promise as efficient, robust control systems that could substantially improve the profitability of mineral processing systems.

The second portion of this report considers the consequences of altering the mechanics of GA-FLC's. To allow FLC's to reach their full potential in industrial settings, researchers need to utilize standard, efficient fuzzy logic techniques. Although the basic structure of an FLC is consistent across the board; i.e., a rule set is used in conjunction with a set of membership functions to arrive at a single concrete or "crisp" control decision, various researchers have employed dramatically different techniques when implementing an FLC in a particular problem. For instance, FLC's have been developed and successfully applied that use different membership function forms, including triangles (*11*), trapezoids (*12*), and sinusoidal functions (*13*). Moreover, researchers are beginning to consider new ways to execute many of the fuzzy mathematical computations that are performed by an FLC, such as the form of the implication operator used (*14*), the manner in which the rule form is referenced (*15*), and the way in which the one crisp action is selected (*16*). Each of these approaches to implementing an FLC has merit.

The purpose of the research described in this report was twofold. First, the USBM wanted to develop a powerful technique for combining GA's with FLC's. Although the technique was used to develop an FLC for a specific laboratory pH system, the implementation of this technique is presented in sufficient detail to serve as a general guideline for GA-FLC implementation. Second, the USBM wanted to investigate a number of different fuzzy techniques empirically. Specifically, the effects of altering five aspects of the pH FLC were considered: (1) membership function forms, (2) number of fuzzy classes, (3) the center-of-area method, (4) implication operators, and (5) fuzzy rule form. Thus, for the particular application to the control of a specific pH system, the most efficient FLC

[4] Italic numbers in parentheses refer to items in the list of references at the end of this report.

is developed. Aside from defining the best possible FLC for the pH system, this report provides an arena in which the performance characteristics of the fuzzy logic techniques can be observed and tested.

This research is aimed at improving the efficiency of mineral processing, in support of the USBM mission to help ensure that the United States has an adequate, dependable supply of minerals and materials.

GENETIC ALGORITHM-DESIGNED FUZZY LOGIC CONTROLLERS: BASIC APPROACH

USBM research has shown that fuzzy logic and GA's have great potential for industrial process control. This section presents the USBM-developed technique for producing a GA-FLC and includes discussions on the specifics of both designing an FLC and applying a GA. However, before the approach is presented, a particular laboratory pH titration system is introduced to serve as a vehicle for discussing GA-FLC's. This particular pH system is used in later sections as a forum for comparing the effectiveness of various FLC implementation characteristics.

PHYSICAL SYSTEM

A simple laboratory pH system is considered to present the USBM-developed technique for using GA's to design an FLC and for comparing various ways of implementing an FLC. A schematic of this pH system is shown in figure 1. The system consists of a beaker initially containing a given volume of a solution having a known pH. There are two valved input streams into the beaker. The valves on these two ***control input streams***, one a strong acid (0.1*M* HCl) and one a strong base (0.1*M* NaOH), can be adjusted to cause a change in the pH of the solution in the tank. The objective of the control problem is to neutralize the solution (i.e., drive the pH to 7) in the shortest time possible by adjusting the valves on the control input streams. Additionally, the valves on the control input

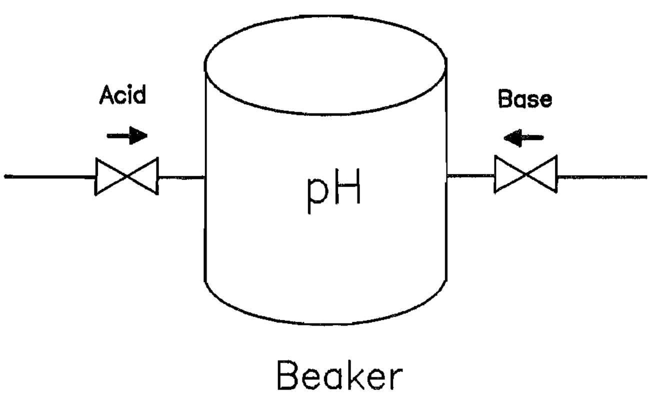

Figure 1.—Schematic of simple pH system.

streams are to be fully closed after the solution is neutralized. As a constraint on the control problem, the valves can only be adjusted a limited amount (0.5 (mL/s)/s, which is 20 pct of the maximum flow rate of 2.5 mL/s), to restrict pressure transients in the associated pumping systems.

The pH system was designed on a small scale so that experiments could be performed in a limited laboratory space. Titrations were performed in a 1-L beaker, and the solution was mixed with a magnetic stirring bar. Computer-driven peristaltic pumps were used for flow maintenance in each input stream. An industrial pH electrode and transmitter sent signals through an analog-to-digital board to a 386-based personal computer, which controlled the entire system.

A computer model of the physical system is required to develop an FLC using a GA. The computer model is used only by the GA; it is the physical pH system that is actually manipulated by the FLC. A model of the pH system is required because of the way in which a GA operates. GA's evaluate a number of possible solutions to the large-scale search problem in order to locate efficient membership functions. Some of the possible solutions the GA investigates are totally unacceptable: they represent preposterous control strategies. Therefore, the potential solutions are investigated on a computer simulation of the pH system.

Fortunately, the dynamics of the pH system are well understood and can be modeled for buffered reactions using conventional techniques (*17*). The result is the following cubic equation that must be solved for $[H_3O^+]$ ions, which directly yields the pH of the solution:

$$x^3 + Ax^2 + Bx + C = 0, \quad (1)$$

where
$x = [H_3O^+]$,

$A = k_a + [CH_3COONa] + [NaOH] - [HCl]$,

$B = k_a[NaOH] - k_a[HCl] - k_a[CH_3OOH] - k_w$,

$C = -k_a k_w$,

k_a = equilibrium constant for CH_3COOH, 1.8×10^{-5},

and k_w = equilibrium constant for H_2O, 1.0×10^{-14}.

Bracketed terms ([]) represent molar concentrations. In the pH system considered here, the development of a model of the physical system does not present an insurmountable obstacle. However, for many complex industrial systems, the development of an accurate computer model is an imposing task.

FUZZY LOGIC CONTROLLER DESIGN WITH GENETIC ALGORITHM

Controlling the laboratory pH system presented in the previous section is not difficult for a person with experience in performing titrations. However, for the purpose of demonstrating the effectiveness of the USBM-developed technique of combining GA's and FLC's, the simple pH system is sufficiently difficult to control, owing mainly to the nonlinear (logarithmic) nature of pH.

The use of GA's to select high-performance membership functions allows for the production of an efficient FLC that is able to establish and to maintain control of the laboratory system. The following discussion presents the details of FLC development. Some facets of the FLC described are central to the purpose of this report and will be expanded later. For instance, in this initial explanation, triangular membership functions are used to demonstrate the method of designing an FLC with GA's. Later, the performance of an FLC that employs triangular membership functions is compared with FLC's that use other membership function forms. Thus, the intent of the following discussion is to present a basic approach to FLC design using a GA.

Fuzzy Logic Controller Design

The first step in developing a pH FLC is to decide on the ***condition variables***. (These variables appear on the left side of the FLC rules, which are of the form: IF <condition> THEN <action>.) Certainly there are numerous condition variables that could be considered in the pH system (pH of solution in the tank, flow rates of the input streams, concentrations of input solutions, volume in the tank, and many others). However, it is important to limit the number of condition variables used to a small fundamental set because the size of the rule set increases multiplicatively with the number of condition variables. After a period of experimentation (an inevitable requirement for the development of a quality FLC), two condition variables were selected: the current value of pH (pH) in the beaker and the absolute value of the current time rate of change of the pH in the tank (ΔpH).

The second step is to determine the specific actions that can be taken on the system, i.e., the ***action variables***. In the pH system, the determination of the action variables is relatively straightforward. There are basically only two action variables that can be altered by the controller: the valve settings (and thus the flow rates) associated with the control input streams. Therefore, the two action variables were the flow rates for the strong acid (Q_{ACID}) and the strong base (Q_{BASE}), respectively, of the input streams. The selection of the action variables differs from the selection of the condition variables in that the number of action variables has no effect on the number of rules required.

The third step is to choose linguistic terms that represent each of the condition and action variables. Seven terms were used to describe pH, two terms were used to describe ΔpH, and five terms were used to describe both Q_{ACID} and Q_{BASE}. The specific linguistic terms used to describe the pertinent variables in the pH system follow:

pH: Very acidic (VA), acidic (A), mildly acidic (MA), neutral (N), mildly basic (MB), basic (B), and very basic (VB);

ΔpH: Small (S) and large (L);

Q_{ACID}: Zero (Z), very small (VS), small (S), medium (M), and large (L);

Q_{BASE}: Zero (Z), very small (VS), small (S), medium (M), and large (L).

All of these linguistic terms are subjective; i.e., the terms can mean different things to different people; but the developers (the authors) of the pH FLC have some conceptual meaning they associate with each of the terms.

The fourth step is to provide the selected linguistic terms with some concrete ("crisp") meaning. The linguistic terms are "defined" by membership functions. As with the requirement for selecting the necessary linguistic terms, there are no definite guidelines for constructing the membership functions; the terms are defined to represent the designers' general understanding of what the terms mean. The membership functions used in the author-developed FLC (AD-FLC) appear in figure 2. The membership functions chosen for the pH FLC at this point are triangular because that is in many respects the simplest membership function form that can be employed. The membership functions shown in figure 2 will soon be altered by a GA in an attempt to locate specific membership functions that provide near-optimal FLC performance. Alterations in these functions can dramatically change the performance characteristics of FLC's.

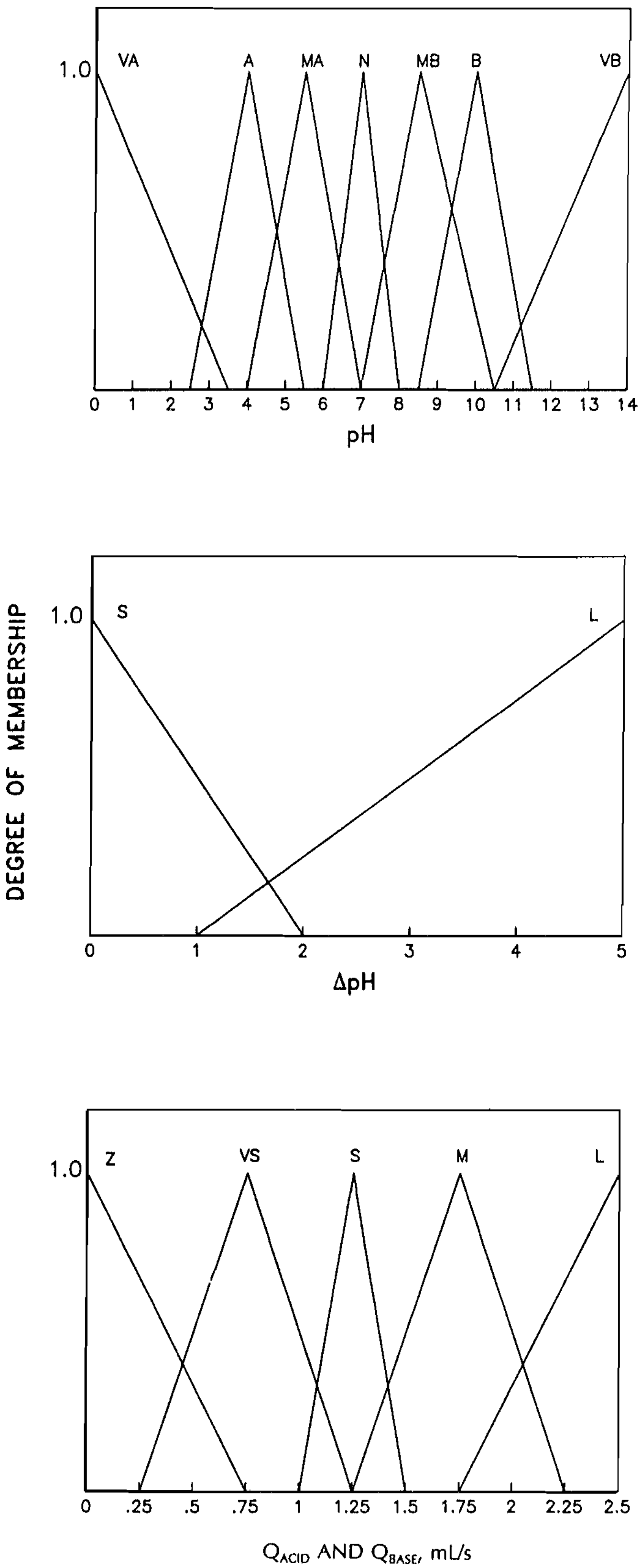

Figure 2.—Fuzzy membership functions for pH, ΔpH, and Q_{ACID} and Q_{BASE}.

The fifth step in the design of an FLC is the development of a rule set. The rule set in an FLC must include a rule for every possible combination of the controlled variables as they are described by the chosen linguistic terms. Thus, the pH FLC, as described to this point, will contain 2 × 7 = 14 rules to describe all of the possible conditions that could exist in the pH system as described by the linguistic terms represented by the membership functions shown in figure 2. The entire rule set for the pH FLC is shown in figure 3. For any combination of the condition variables, an appropriate choice of the action variables is prescribed. Owing to the nature of the linguistic terms, most of the actions needed for the 14 possible condition combinations are readily apparent. For instance, when pH is VA and ΔpH is S, then Q_{ACID} should be Z and Q_{BASE} should be L. However, there are some conditions for which the appropriate action is not readily apparent. In these instances, some experimentation is often needed.

Now that both the condition and action variables have been chosen and described with linguistic terms, and a rule set has been written that prescribes an appropriate action for every possible set of conditions, a single crisp value of the acid and base valve settings must be determined. This procedure for determining a single crisp value of the valve settings for the acid and base input streams is a concern because, unlike in traditional expert systems, more than one of the FLC's 14 rules can be applicable for a given state of the pH system. A common technique for accomplishing this task is the center-of-area (COA) method (sometimes called the centroid method). In the COA method, the action prescribed by each rule plays a part in the final crisp value of the valve settings. The contribution of each rule to the final value of Q_{ACID} and Q_{BASE} is proportional to the minimum confidence (the minimum value of the membership function values on the left side of

pH	ΔpH S	ΔpH L
VA	Z	Z
A	Z	Z
MA	Z	Z
N	Z	Z
MB	S	VS
B	M	S
VB	L	M

Q ACID

pH	ΔpH S	ΔpH L
VA	L	M
A	M	S
MA	S	VS
N	Z	Z
MB	Z	Z
B	Z	Z
VB	Z	Z

Q BASE

Figure 3.—Fourteen rules of pH FLC.

the rule) one has in that rule for the specific state of the physical system at the particular time.[5] This is equivalent to taking a weighted average of the prescribed actions. A more detailed explanation of the COA method is provided in a later section. For now, simply consider the COA method to be a means for determining a weighted average of all of the different prescribed actions that are applicable for a given state of the pH system.

One detail specific to the pH system should be considered here. There is a limit on the allowable change in the flow rates of the input streams; i.e., the flow rates cannot change by more than 0.5 (mL/s)/s. However, the membership functions used in the COA method (shown in figure 2) allow for values of Q_{ACID} and Q_{BASE} to range between 0.0 and 2.5 mL/s, irrespective of their current values. The constraint is imposed by making a slight adjustment to the value of the flow rates computed using the COA method. If the value computed using the COA method exceeds the maximum allowable change in flow rate, the flow rate is changed by the maximum allowable value of 0.5 mL/s (for either increases or decreases in flow rate). With the determination of a strategy for resolving "conflicts" in the actions prescribed by the individual rules, the FLC is complete.

Genetic Algorithm Application

The preceding section consisted of a general description of the elemental makeup of the pH FLC. This section discusses the use of a GA to select membership functions that provide the most efficient FLC. Certainly, there are numerous kinds or "flavors" of GA's; several genetic operators and variations of the basic scheme have been developed and implemented (*7*). Recent studies point to the effectiveness of a particular small-population GA, a micro GA (*8-10*). The following discussion concerning the use of a GA for selecting membership functions is kept intentionally generic with respect to the particular GA employed. This is due to the fact that virtually any GA will provide better FLC performance, although in some problem domains one particular GA scheme may outperform others. Once the details of the particular GA to be employed have been determined, there are basically two decisions to be made when utilizing a GA to select FLC membership functions: (1) how to code the possible choices of membership functions as finite bit strings, and (2) how to evaluate the performance of the FLC composed of the chosen membership functions.

Consider the selection of a coding scheme. To define an entire set of triangular membership functions (functions for pH, ΔpH, Q_{ACID}, and Q_{BASE}), several parameters must be selected. First, the distinction is made between the two types of triangles used (see figure 2). The right triangles (90° triangles) appearing on the left and right boundaries are termed "***extreme***" triangles, while the isosceles triangles appearing between the boundaries are termed "***interior***" triangles. To completely define an extreme triangle, only one point must be specified because the apex of the triangle is fixed at the associated extreme value of the condition or action variable (e.g., the maximum value of VA for the pH will always be at pH = 0). On the other hand, the complete definition of an interior triangle necessitates the specification of two points, given the constraint that the triangles must be isosceles; i.e., the apex is at the midpoint of the two points specified. Thus, for the complete definition of a set of triangular membership functions for the pH FLC as described above, 60 points had to be specified.

Certainly a 60-parameter search problem is challenging, to say the least. Fortunately, the search space associated with the selection of membership functions for the pH system can be pruned. The rule set presented above is symmetric because every condition wherein the pH is above the set point of 7 has an analogous condition wherein the pH is below the set point of 7. (This is true only when the concentrations of the acid and the base being used to titrate are the same.) Therefore, VA should be the mirror image of VB, A should be the mirror image of B, and so on for all of the membership functions. Also, the membership functions for Q_{ACID} and Q_{BASE} can be made identical because the concentrations of the input acid and input base are of the same strength. When these simplifications are made, the search space is reduced to 32 parameters. Thus, instead of finding 60 parameters, the GA is faced with the task of finding only the 32 points. Even though the original search space has been reduced by nearly a factor of two, a 32 parameter search problem is still of some consequence.

Now that the pertinent search parameters have been identified, a strategy for representing a set of these parameters as a finite bit string must be developed. One such strategy that is popular, flexible, and effective is ***concatenated, mapped, unsigned binary coding.*** In this coding scheme each individual parameter is discretized by mapping linearly from a minimum value (C_{min}) to a maximum value (C_{max}) using an m-bit, unsigned binary integer according to the equation:

$$C = C_{min} + \frac{b}{(2^m - 1)} \times (C_{max} - C_{min}), \qquad (2)$$

where C is the value of the parameter of interest, and b is the decimal value represented by an m-bit string ($m = 7$ in all of the examples presented). Representing more than one parameter (such as the 32 parameters necessary in the pH FLC) is accomplished simply by concatenating the individual 7-bit segments. Thus, in this example, a 224-bit string is necessary to represent an entire set of membership functions. This discretization of the problem produces a search space in which 2^{224} (= 2.696×10^{67}) possible solutions exist.

[5]Later in this report alternative methods for selecting the minimum value are investigated.

The second decision that must be made in a GA application is how the strings or the potential membership functions are to be evaluated. In the performance of the pH FLC, the controller must drive the pH in the beaker to a set point (pH = 7) in as short a time as possible, and keep it there. Also, membership functions must be selected that are capable of accomplishing this control objective from any of a number of initial conditions. Therefore, the actual objective function (sometimes called a *fitness function*) that the GA minimized is

$$f = \sum_{i=case1}^{i=case4} \sum_{j=0s}^{j=100s} (w_i \mid 7.0 - pH \mid$$

$$+ w_2(Q_{ACID} + Q_{BASE}), \qquad (3)$$

where w_1 and w_2 are weighting factors.

This membership function form takes into account the two most commonly considered aspects of a control system's ability to perform: (1) the amount by which the physical system overshoots the set point, and (2) the actual time necessary to attain and to maintain the set point. This fitness function penalizes a controller for both excessive overshoot and excessive time to attain and maintain the set point. In a later section, a specific fitness function value is considered relative to the overshoot and time to set point it entails.

The effect of using a micro GA to produce a pH FLC is summarized in figure 4. This figure compares the performance of an AD-FLC with the performance of a GA-FLC for the four initial condition cases used by the GA to evaluate the performance of its potential solutions. Clearly, the GA-FLC is more effective than the AD-FLC at driving the pH system to its set point.

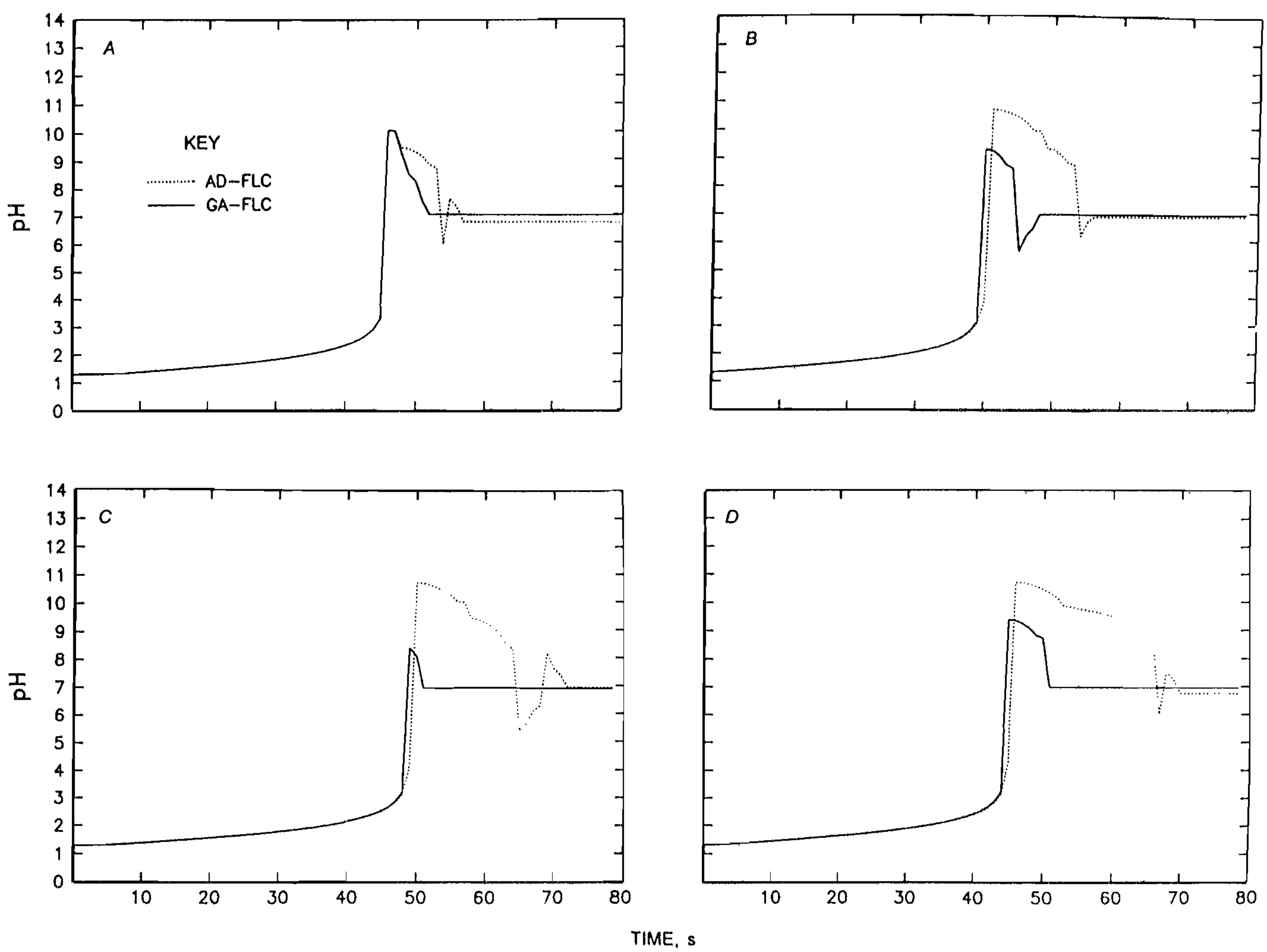

Figure 4.—Performance of AD-FLC versus GA-FLC for four initial conditions, at pH = 1.30. *A*, Q_{ACID} = 0.0, Q_{BASE} = 0.0; *B*, Q_{ACID} = 0.5, Q_{BASE} = 2.25; *C*, Q_{ACID} = 2.25, Q_{BASE} = 0.5; and *D*, Q_{ACID} = 2.25, Q_{BASE} = 2.25.

This section has provided an overview of one procedure for developing an FLC. Certainly, there are numerous alternatives to implementing an FLC. In the remainder of this report, some of these alternative techniques are investigated and compared for this one pH system. These results do not allow for a definitive conclusion as to which techniques are most effective for all control problems; such a conclusion must take into account the dynamics and characteristics of the system that is to be controlled. However, these results do provide key insights into the general performance of FLC's developed using the various techniques.

EMPIRICAL STUDIES

This section presents the results of empirical studies in which numerous variations in FLC implementation are compared. Specifically, the effects of altering five aspects of the pH GA-FLC developed in the previous section are considered: (1) membership function form, (2) number of fuzzy classes, (3) the COA method, (4) implication operator, and (5) fuzzy rule form. Results indicate that when a GA is used to design a particular FLC for the pH system, the specifics of the FLC implementation seem to make little difference in FLC performance.

In all of the results presented in this report, the controllers are evaluated, first and foremost, based on the fitness function described in this section. However, occasionally the authors make subjective judgments on the efficiency of the different FLC's presented. Moreover, each controller presented has been developed using a GA to select the optimum membership functions. This helps to ensure that each variation to a fuzzy logic technique has the same chance of producing the most efficient control system.

MEMBERSHIP FUNCTION FORMS: TRIANGLES, TRAPEZOIDS, AND SINUSOIDAL FUNCTIONS

There are a number of membership function forms that have been successfully used in the design of FLC's. The three most common membership function forms are by far (1) triangular, (2) trapezoidal, and (3) sinusoidal. An example of each of these membership function forms is shown in figure 5. This section compares the performance of GA-FLC's that utilize the three different membership function forms. All of the FLC's discussed in this section employed 14 rules of the form:

IF <pH and ΔpH> THEN <Q_{ACID} and Q_{BASE}>.

Furthermore, each of the FLC's used a GA with the fitness function form presented previously.

The following information is needed to fully describe each of the function forms. Both the triangular and trapezoidal membership function forms are conveniently described by declaring the location of points located on the different figures. Sinusoidal membership functions are more conveniently described by declaring the value of constants used in a functional relationship. As was stated in the previous section, extreme triangles require the definition of only a single point for complete description. Interior triangles, on the other hand, require the definition of two points. Trapezoidal membership functions require the definition of more points: extreme trapezoids need two points, while interior trapezoids need four points for complete definition. Sinusoidal membership functions, on the other hand, require the same number of parameters as the interior triangles. Sinusoidal membership functions are described by two constants: one that determines the center of the sine wave and one that determines the width of the sine wave. This information must be considered when using a GA to locate efficient membership function sets, because the number of parameters needed to completely define a set of membership functions specifies the size of the search space. The search space for the triangular and sinusoidal membership functions consists of 15 parameters (7 bits per parameter, forming 105-bit strings), while the search space for the trapezoidal membership functions consists of 32 parameters (forming 224-bit strings).

Figure 6 compares the performance of FLC's that utilize the three different membership function forms. The method of comparison used by the GA was simply the fitness function that considered the performance of the

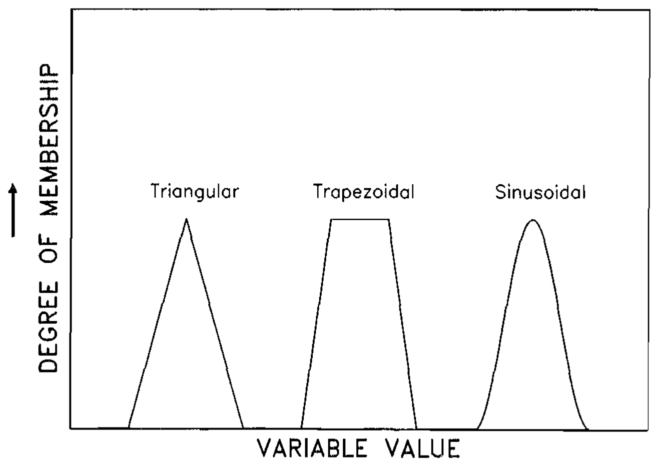

Figure 5.—Three most common membership function forms: triangular, trapezoidal, and sinusoidal.

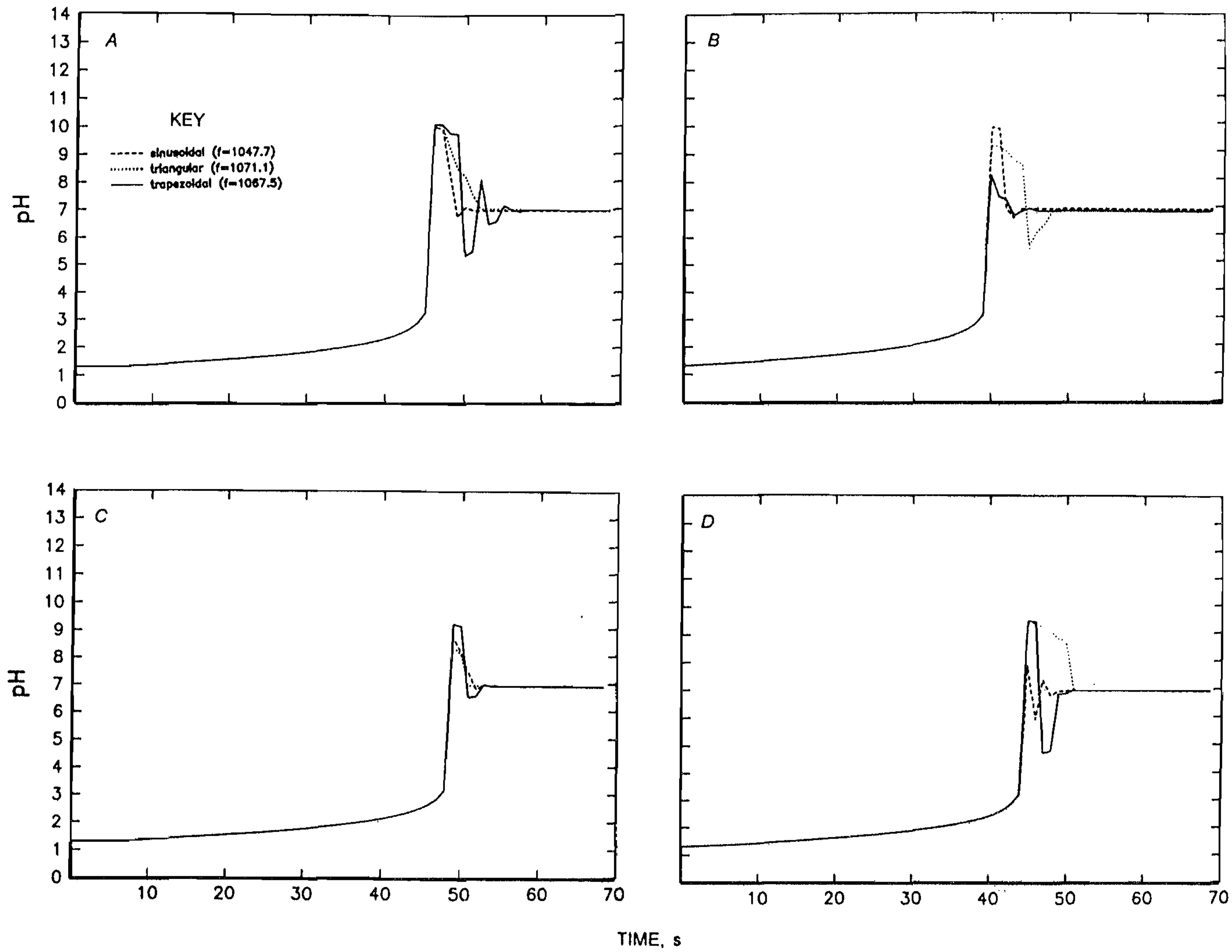

Figure 6.—Performance of GA-FLC's using three different membership function forms for initial conditions, at pH = 1.30. *A*, Q_{ACID} = 0.0, Q_{BASE} = 0.0; *B*, Q_{ACID} = 0.5, Q_{BASE} = 2.25; *C*, Q_{ACID} = 2.25, Q_{BASE} = 0.5; and *D*, Q_{ACID} = 2.25, Q_{BASE} = 2.25.

controller over four sets of initial conditions. Based on this criterion alone, the sinusoidal functions performed the best (fitness, f = 1,047.7). However, both the trapezoidal functions (f = 1,067.5) and the triangular functions (f = 1,071.1) produced FLC's that did adequate jobs of controlling the pH system. An important point concerning these fitness function values is that they are average values found over five independent GA runs. Average values are used because the solutions found by a GA are somewhat dependent on a "random seed" supplied by the user, which is simply a value used by pseudo-random number generators. However, an indication of the degree to which the GA depends on the random seed in a particular search environment is contained in the standard deviation (σ) of the five sample values. The standard deviations for the three different membership function forms are σ(f = 1,047.7) = 1.19, σ(f = 1,067.5) = 2.10, and σ(f = 1,071.1) = 0.88. Thus, all five of the independent GA runs are locating virtually the same set of membership functions for the triangles, the trapezoids, and the sinusoidal functions.

As stated earlier, the two most important aspects of the control system are the amount of overshoot and the time to set point. The amount of overshoot and the time to set point represented by each of the three fitness function values are summarized in table 1. Certainly, criteria other than the fitness function values could be used to judge the effectiveness of FLC's employing the different membership function forms. For instance, the controller using the sinusoidal membership functions often allowed for more overshoot than the controllers using either the triangles or the trapezoids. If the overshoot is more important than the time to set point, the fitness function can be changed. Based on figure 6, it can be concluded that all three membership function forms can be used to produce an efficient pH FLC, and, for the most part, there is little difference in the performance of the three controllers.

Table 1.—Fitness function values describing information concerning overshoot and time to set point

f value	Overshoot	Time to set point, s
	CASE 1	
1,047.7	2.99	51
1,067.5	3.21	63
1,071.1	3.08	52
	CASE 2	
1,047.7	2.96	44
1,067.5	2.66	49
1,071.1	1.31	48
	CASE 3	
1,047.7	1.87	53
1,067.5	4.14	52
1,071.1	1.43	51
	CASE 4	
1,047.7	0.92	49
1,067.5	2.98	60
1,071.1	2.43	51

EFFECT OF NUMBER OF FUZZY CLASSES ON CONTROLLER ACCURACY

This section focuses on alternative choices for the number of fuzzy classes selected for the condition and action variables. In all of the results presented, trapezoidal membership functions are used. It is quite possible that the selection of the number of fuzzy classes is dependent on the membership function form selected. However, as more and more alternatives for producing an FLC are considered, the number of possible combinations increases dramatically. Therefore, the performance of an FLC with an alternative choice for one facet of its design will be demonstrated for a situation in which the fuzzy characteristics that are not currently being considered are fixed. In other words, when the focus of a comparison is on the choice of the number of fuzzy classes, the membership function form will remain constant.

Five different combinations of choices for the number of fuzzy classes used to describe the variables included in the FLC are compared. These five combinations and their associated fitness values are summarized in table 2. Based strictly on the fitness values, case 3 is the top performer. However, all of the combinations produce FLC's that perform at an acceptable level. The performances of case 3 (f = 1,055.6) and case 4 (f = 1,096.8) are compared in figure 7. These are the most and least efficient of the five cases considered and provide the reader with an indication of the variation in performance represented by these different choices.

Intuitively, the reader may feel that the performance of an FLC should improve as the number of fuzzy classes increases. This intuition may be based on the idea that the more rules used, the finer the control. However, the results set forth in table 2 do not bear this out. Exactly why this is true is not readily apparent. At any rate, an FLC can contain too many fuzzy classes (because the large increase in computational time does not warrant the small increase in performance), and the appropriate choice can only be based on experience with both FLC development and experience with the problem environment (e.g., the pH system).

Table 2.—Five combinations of fuzzy classes for FLC variables

Case	pH	ΔpH	Q_{ACID}	Q_{BASE}	f value
1	5	2	4	4	1,082.2
2	7	2	5	5	1,067.5
3	7	3	6	6	1,055.6
4	9	2	6	6	1,096.8
5	9	3	7	7	1,091.9

THREE VARIATIONS IN CENTER-OF-AREA METHOD

As pointed out earlier, the COA method is a means for computing a weighted average. However, before variations of this approach are considered, the details of the COA method are presented by considering a specific example.

Consider the instance where pH = 3.0 and ΔpH = 1.5. The FLC must determine a single value of both Q_{ACID} and Q_{BASE} for these conditions. To do so, the membership function values (μ) associated with these conditions are computed. From figure 2, the value of $\mu_{pH}(VA) = 0.143$, $\mu_{pH}(A) = 0.33$, $\mu_{\Delta pH}(S) = 0.25$, $\mu_{\Delta pH}(L) = 0.125$, and all other membership function values are zero. These conditions, when compared with the rule set appearing in figure 3, indicate that there are four rules that are applicable, i.e., the rules for pH = VA, ΔpH = S; pH = VA, ΔpH = L; pH = A, ΔpH = S; and pH = A, ΔpH = L. Note that two of these rules call for Q_{BASE} to be M, while the other two prescribe S and L, respectively. The role of the COA is to combine the four actions prescribed by the four pertinent rules into a single value of Q_{BASE}. Each of the four rules also prescribes an appropriate action for Q_{ACID}, but for the purposes of explaining the details of the COA method, only the choice of flow rate for the base need be considered.

The COA method can be thought of as a graphical technique. The action prescribed by each of the four rules (S, M, and L) is plotted (according to its definition as described by its membership function seen in figure 2). However, the actions are not plotted with a height of 1.0 as they are defined. Instead, they are plotted with a height equal to the minimum value of μ on the condition side of the rule prescribing it, as shown in figure 8. The single

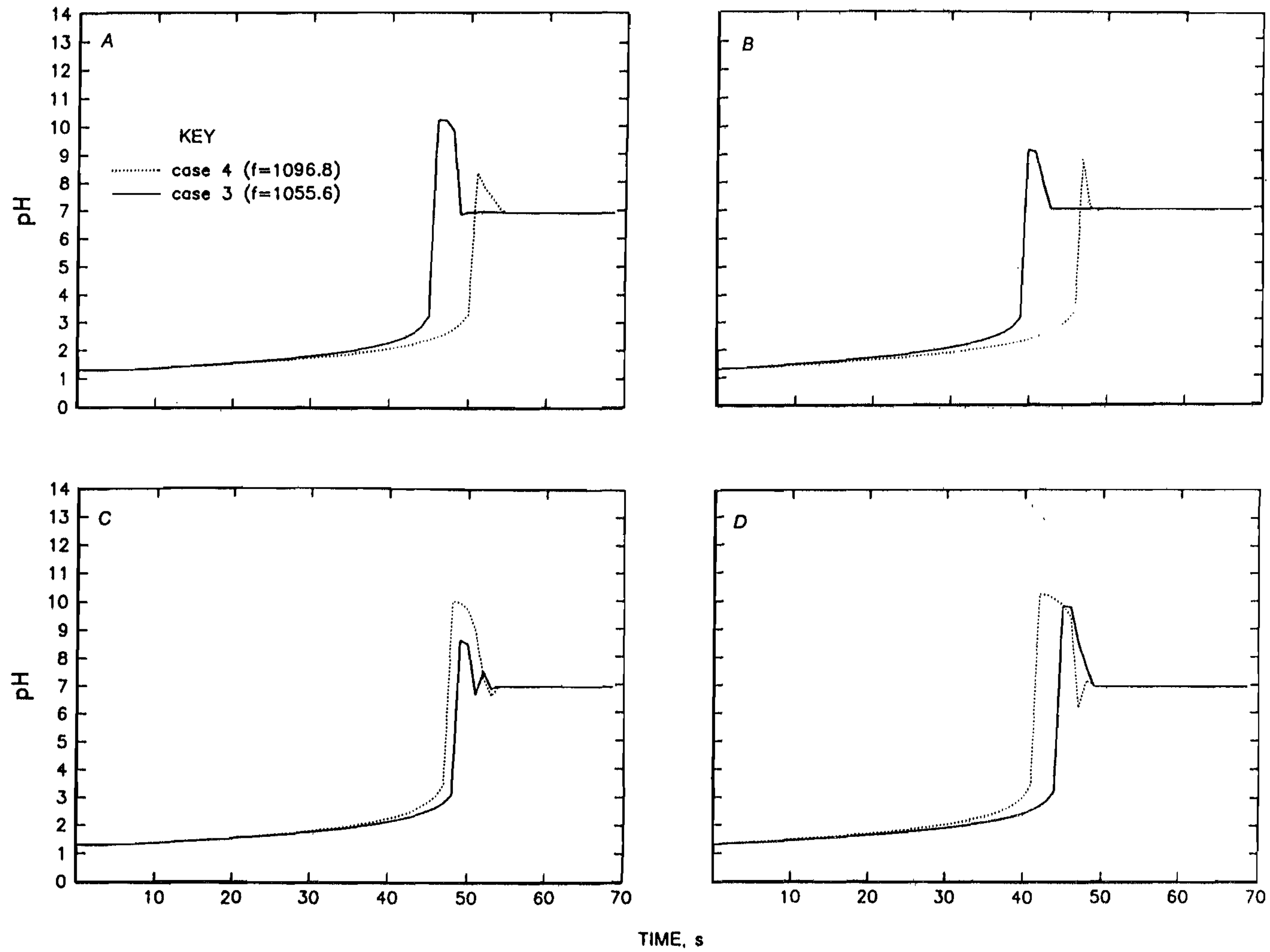

Figure 7.—Comparison of GA-FLC's that use varying numbers of linguistic variables for initial conditions, at pH = 1.30. *A*, Q_{ACID} = 0.0, Q_{BASE} = 0.0; *B*, Q_{ACID} = 0.5, Q_{BASE} = 2.25; *C*, Q_{ACID} = 2.25, Q_{BASE} = 0.5; and *D*, Q_{ACID} = 2.25, Q_{BASE} = 2.25.

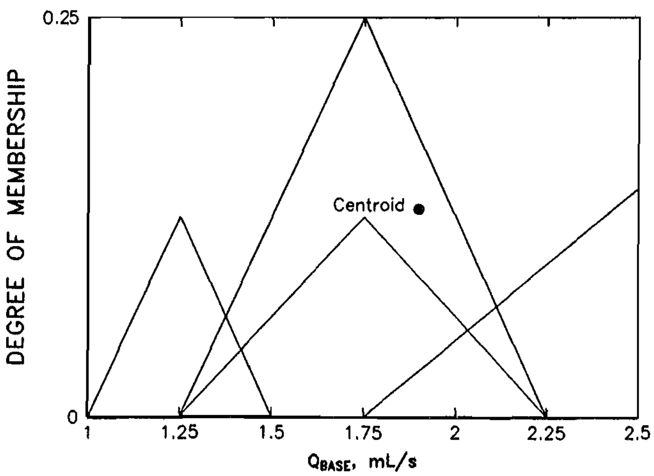

Figure 8.—COA method, used as graphical technique.

adjustment for Q_{BASE} is determined by finding the center of the area associated with the four membership functions that have been plotted.

There are a number of variations to the COA method, and all relate to the way in which the prescribed action membership functions are scaled. In this report, three such variations are considered as shown in figure 9. The first, termed "*traditional*," is as described above, where only the height of the membership function is scaled. The second, termed "*base_scale*," allows for the membership functions to be scaled in both height and width. The third, termed "*top_off*," allows for the top portion (proportional to μ) of the membership function to be removed. Each of these three alternatives has been applied to pH FLC's using triangular, trapezoidal, and sinusoidal membership function forms, producing nine different combinations.

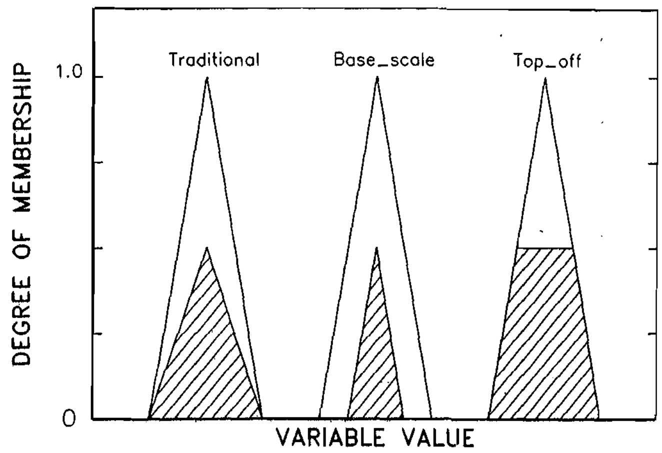

Figure 9.—Three variations in COA method, including *traditional, base_scale, top_off.*

The results of the nine FLC's are summarized in table 3. Based strictly on the fitness function values, the combination of sinusoidal membership functions and the *traditional* approach to the COA method produces the most efficient FLC (f = 1,047). However, as in previous sections, all of the nine FLC's were able to effectively control the pH system. Figure 10 compares the best and worst performers for the variations of the COA method.

Table 3.—The f values of nine FLC's constructed using different combinations of membership function forms and approaches to COA method

Membership function form	Traditional	Base_scale	Top_off
Triangular	f = 1,071	f = 1,078	f = 1,071
Trapezoidal	f = 1,097	f = 1,097	f = 1,095
Sinusoidal	f = 1,047	f = 1,050	f = 1,056

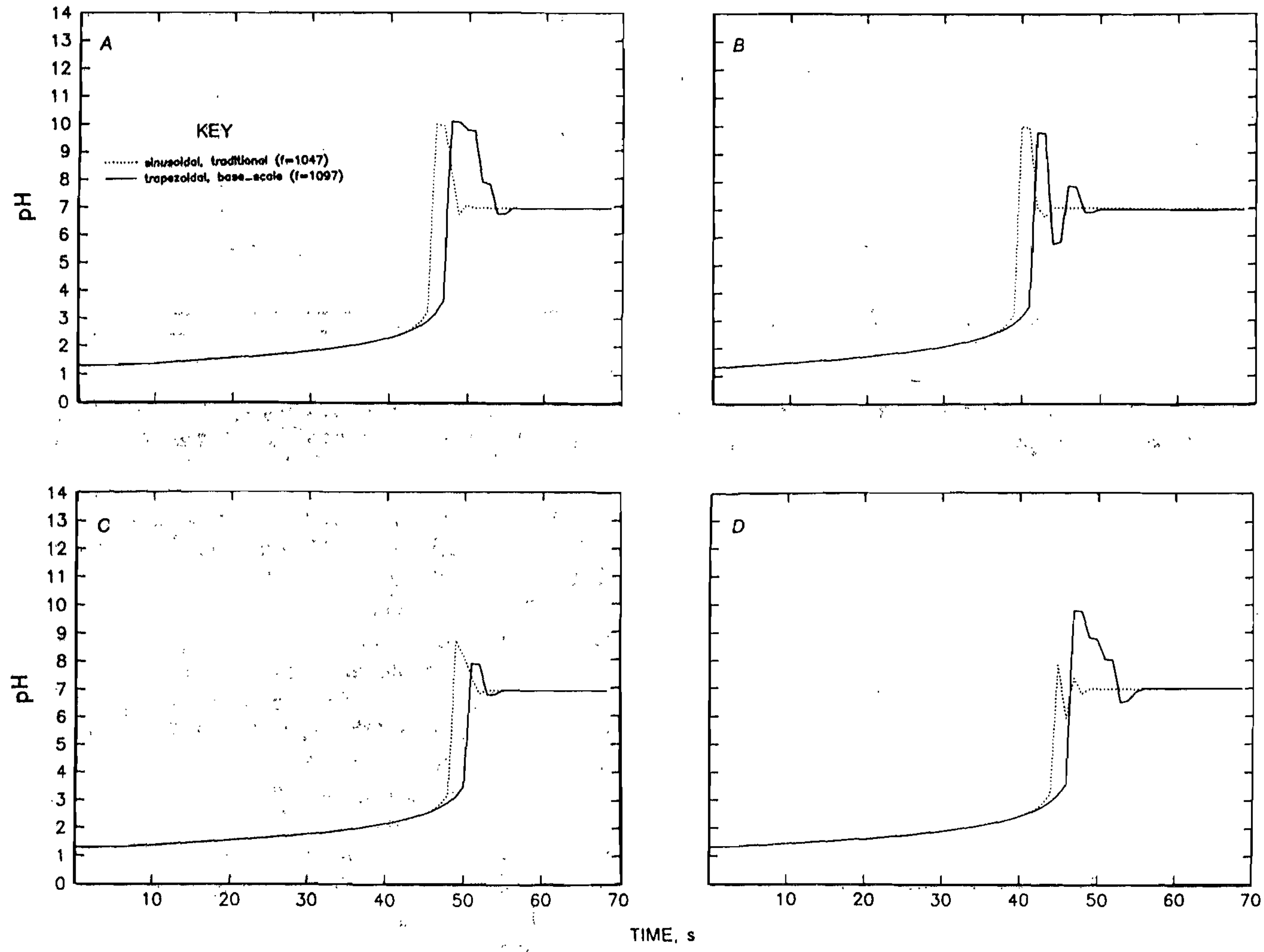

Figure 10.—Comparison of GA-FLC's that use variations of COA method for initial conditions, at pH = 1.30. *A*, Q_{ACID} = 0.0, Q_{BASE} = 0.0; *B*, Q_{ACID} = 0.5, Q_{BASE} = 2.25; *C*, Q_{ACID} = 2.25, Q_{BASE} = 0.5; and *D*, Q_{ACID} = 2.25, Q_{BASE} = 2.25.

IMPLICATION OPERATORS: "FUZZY AND" VERSUS "AND"

In the previous section, the COA method was presented as a graphical technique for determining a single crisp action to be taken on the pH system. The crisp value was computed using a *"fuzzy and"* implication operator, which means that the minimum value of μ associated with the condition side of a particular rule is used to scale the action prescribed by that rule. This is of course contrary to the *"and"* operator used in probability theory. In probability theory, the *"and"* operator indicates multiplication. In this section, the *"fuzzy and"* implication operator is compared with the *"and"* implication operator found in conventional set theory.

The distinction between these two implication operators is best demonstrated with the example presented in the previous section wherein pH = 3.0 and ΔpH = 1.5. For this particular system state, the associated membership functions are $\mu_{pH}(VA) = 0.143$, $\mu_{pH}(A) = 0.33$, $\mu_{\Delta pH}(S) = 0.25$, $\mu_{\Delta pH}(L) = 0.125$, and all other membership function values are zero. This means that the following four rules are applicable:

IF <pH is VA and ΔpH is S> THEN <Q_{ACID} is Z and Q_{BASE} is L>,

IF <pH is VA and ΔpH is L> THEN <Q_{ACID} is Z and Q_{BASE} is M>,

IF <pH is A and ΔpH is S> THEN <Q_{ACID} is Z and Q_{BASE} is M>,

and

IF <pH is A and ΔpH is L> THEN <Q_{ACID} is Z and Q_{BASE} is S>.

Figure 8 is a graphical representation of the COA method for this example using the *"fuzzy and"* (the *minimum* implication operator). The membership function L is plotted with a height equal to 0.143 (the minimum value of $\mu_{pH}(VA) = 0.143$ and $\mu_{\Delta pH}(S) = 0.25$). The membership function for the action portion of each of the other three rules is likewise plotted with a height equal to the minimum degree of membership associated with its particular condition portion. However, instead of using the minimum degree of membership associated with each rule to scale the prescribed action membership function, there is an acceptable alternative.

When the *"and"* operator in probability theory (the *"multiply"* implication operator) is used, a dramatically different result is obtained with the COA method. When the *multiply* implication operator is used, the membership function L is plotted with a height equal to 0.03575 (the value of $0.143 \times 0.25 = 0.03575$). Although the final results are different, both implication operators are perfectly valid.

The two implication operators described above have been used to produce FLC's for the pH system. The results of this comparative study are summarized in table 4. These results do not provide a clear-cut indication of which implication operator always produces the most efficient FLC. However, for the cases presented, the *minimum* operator performs better than or nearly as well as the *multiply* operator. So that readers can make their own subjective decision concerning the implication operators, the performance of the most and least efficient FLC's set forth in table 4 are plotted in figure 11 for the four initial condition cases considered in the GA's fitness function.

Table 4.—Selection of implication operators, which can alter performance of FLC

Membership function form	COA method	Implication operator	f value
Triangular	Top_off	Minimum	1,071
		Multiply	1,080
Trapezoidal	Traditional	Minimum	1,067
		Multiply	1,076
Sinusoidal	Base_scale	Minimum	1,050
		Multiply	1,049

RULE FORM VARIATIONS

The use of linguistic variables to describe the conditions in a problem environment dramatically eases the task of writing a rule set for controlling that problem environment. However, there are a number of instances in which the use of linguistic variables to describe appropriate actions would also ease the development of an effective rule set. There are some conditions for which the appropriate actions are not clear cut. For instance, in the pH system, what is the appropriate control action for the situation in which the pH is acidic (A) and the time rate of change of the pH is large (L)? Certainly this situation requires the addition of a base. However, making a determination as to the amount of base to add is not straightforward; should Q_{BASE} be made small (S), or should it be made very small (VS)? Before making a determination, one must make sure the nonlinearity of the pH scale is considered. Generally speaking, a human controller would like to make the flow rate of the base "somewhere in between" small and very small; i.e., a human controller would supply a "degree of fuzziness" to the action as well as to the condition.

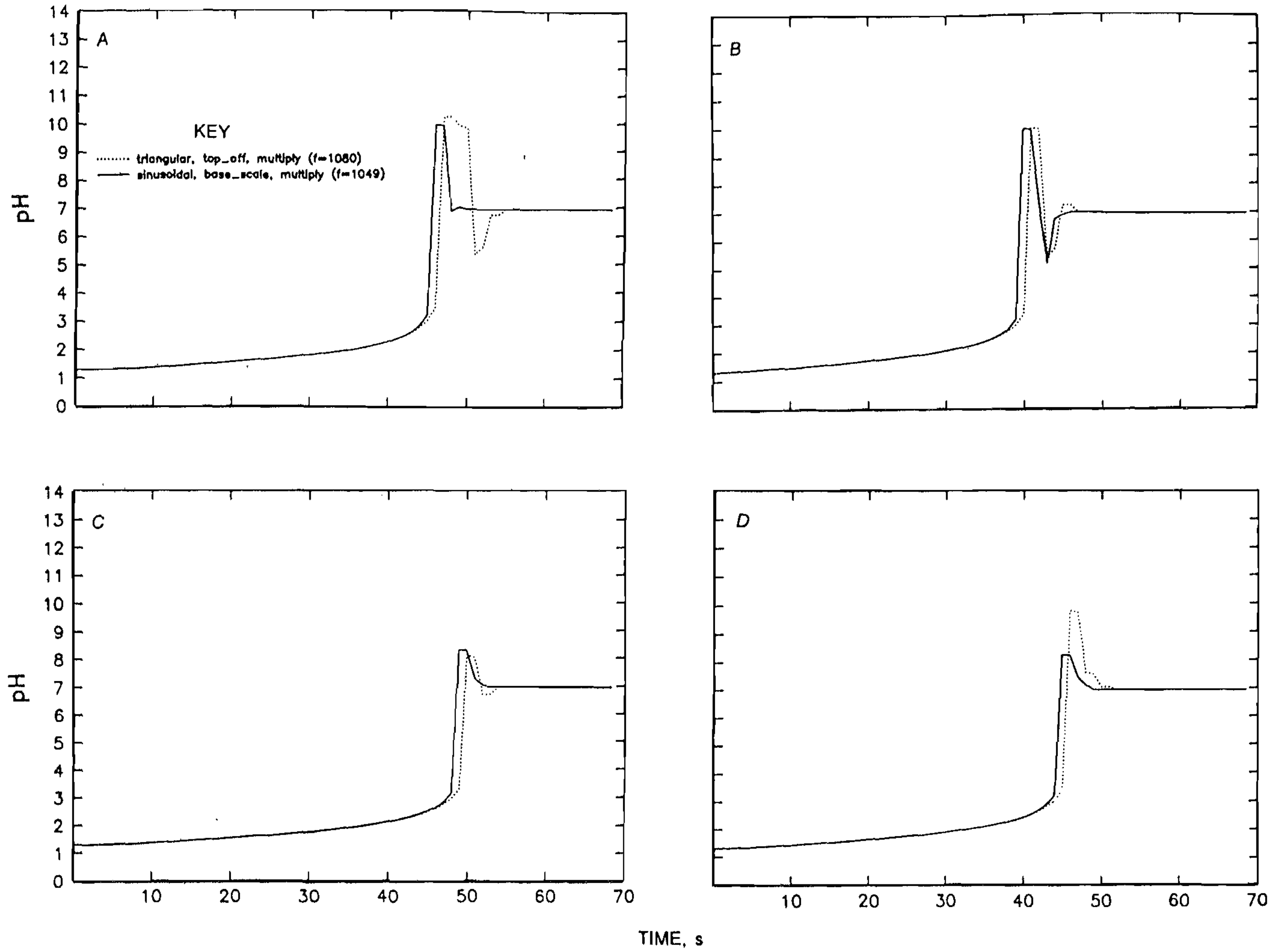

Figure 11.—Comparison of two GA-FLC's that use different membership function forms, COA methods, and implication operators for initial conditions, at pH = 1.30. ***A*****, Q_{ACID} = 0.0, Q_{BASE} = 0.0;** ***B*****, Q_{ACID} = 0.5, Q_{BASE} = 2.25;** ***C*****, Q_{ACID} = 2.25, Q_{BASE} = 0.5; and** ***D*****, Q_{ACID} = 2.25, Q_{BASE} = 2.25.**

An innovative approach to the design of FLC's developed by Cao and Kandel (*14-15*) allows for just such an inclusion of fuzzy terms, and it will be considered here as an alternative approach to FLC design. In the approach of Cao and Kandel, FLC rules take on the form:

IF <condition>

THEN <$action_1$ with a degree of certainty x_1,

$action_2$ with a degree of certainty x_2,

and

$action_n$ with a degree of certainty x_n>.

A specific rule for the pH system might then be

IF <pH is MA and ΔpH is L>

THEN <Q_{BASE} is Z with a degree of certainty of 0.60,

Q_{BASE} is VS with a degree of certainty of 0.27,

Q_{BASE} is S with a degree of certainty of 0.00,

Q_{BASE} is M with a degree of certainty of 0.00,

and

Q_{BASE} is L with a degree of certainty of 0.00>.

The use of this approach means the rule set depicted in figure 3 is transformed from a two-dimensional space to a three-dimensional space for both Q_{ACID} and Q_{BASE}. The skeleton of an appropriate rule set in which the above idea has been incorporated is shown in figure 12 (there are actually real-valued numbers between 0 and 1 in each box in the Q_{BASE} dimension, but they have not been included in the figure). There are still only 14 total rules, but now each rule has 5 prescribed actions, each to some degree.

The use of this approach to the rule development does not change the way in which single crisp action is selected; one can still employ the COA method. Now, however, each action membership function must be scaled both according to the degree of membership appearing on the condition side of the rule prescribing it (either with the *minimum* or the *multiply* implication operator) and with the degree of certainty appearing on the action portion of the rule. Although this may at first seem like an unnecessary complication, the "fuzzification" of the actions does not introduce any real difficulties, and like the other variations to the implementation of an FLC considered in the rest of this report, the use of this approach can and does alter the performance of the subsequent FLC.

This approach to FLC rule development has been incorporated into the pH FLC's considered to this point. In all cases considered, the inclusion of this approach has improved the performance of the FLC. Figure 13 shows

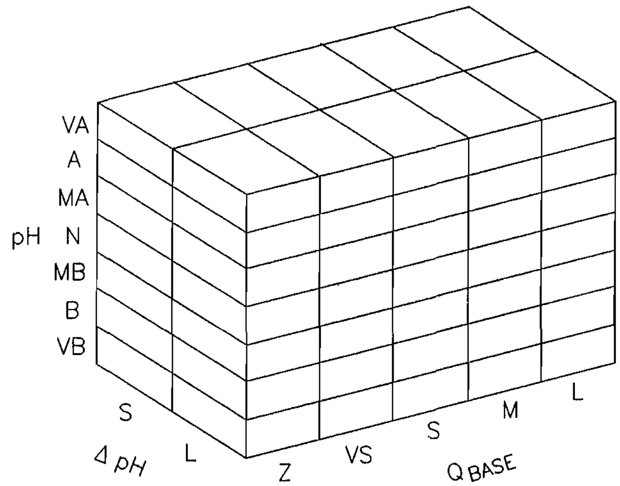

Figure 12.—Transformation to three-dimensional rule set caused by approach to rule development introduced by Cao and Kandel (*14-15*).

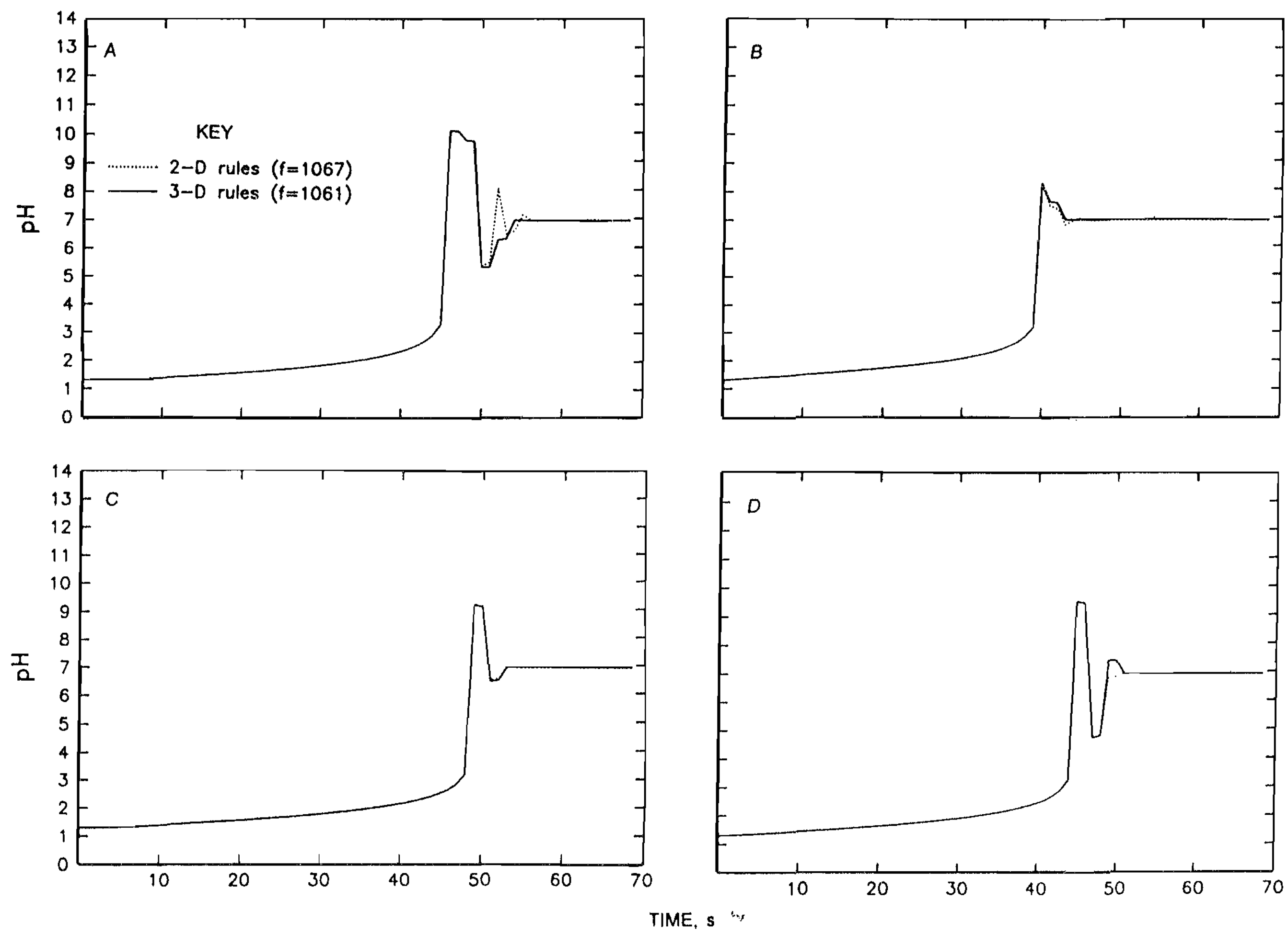

Figure 13.—Comparison of two GA-FLC's that use different rule forms for initial conditions, at pH = 1.30. *A*, Q_{ACID} = 0.0, Q_{BASE} = 0.0; *B*, Q_{ACID} = 0.5, Q_{BASE} = 2.25; *C*, Q_{ACID} = 2.25, Q_{BASE} = 0.5; and *D*, Q_{ACID} = 2.25, Q_{BASE} = 2.25.

a comparison of FLC's (trapezoidal membership functions, *minimum* implication operator, and *traditional* COA approach) that differ only in their rule form. The FLC that utilizes the approach introduced by Cao and Kandel (*14-15*) has a GA fitness function value of 1,061, while the controller that uses the more conventional rule forms has a GA fitness function value of 1,067. As in all of the other comparisons presented in this report, readers are invited to make their own subjective evaluation based on the results presented.

It is not surprising that the FLC's that employ Cao and Kandel's approach outperform the FLC's that employ the more conventional rule forms. In this new approach, the FLC developer has more flexibility in prescribing an appropriate action to be associated with a particular condition. However, when such an approach is employed, the development of the rule set becomes considerably more involved; like most things, it involves a tradeoff.

SUMMARY AND CONCLUSIONS

A technique for employing GA's to design FLC's has been described for controlling a particular laboratory pH system. The FLC designed using a GA consistently outperformed an FLC designed by the authors without the benefit of a GA.

Empirical results have been presented in which numerous variations in FLC implementation are compared. Specifically, the effects of altering five aspects of the pH FLC were considered: (1) membership function form, (2) number of fuzzy classes, (3) the COA method, (4) implication operator, and (5) fuzzy rule form. Results indicate that when a GA is used to select the fuzzy membership functions, variations in FLC implementation have little effect on FLC performance in a chosen control problem.

Despite the fact that the results of the empirical comparisons allow for the determination of the performance of an FLC only for the particular pH system considered, these results do provide insight on the general nature of the performance of FLC's using varying approaches. Thus, to maximize the effectiveness of a particular FLC, designers must still experiment with their particular problem environment. However, this report should provide some insight as to what to expect.

The popularity of FLC's and GA's has grown because of the tremendous ongoing push in the field of artificial intelligence. Both of these techniques have demonstrated abundant potential in the field of process control. The systems that are currently being developed that utilize the strengths of both of these techniques have the potential to outperform conventional process control systems. Thus, these techniques have the potential to dramatically improve the efficiency of the mineral processing industry.

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